ABSTRACT

The main purpose of this work is to study the applications of Lie algebras up to Conservation Laws of Physics, Here we use the idea of symmetry Lie algebras to exploit a new method for solving partial differential equations, by calculating symmetry of PDE's system in terms of its Lie's algorithm.

In fact when the equation admits solvable Lie algebras, the ideals of the algebra can be used to perform the reduction.

الخلاصة

الهدف الأساسي من هذه الرسالة هو دراسة تطبيقات جبر لي في القوانين المحافظة للفيزياء ، حيث إستخدمنا فكرة التماثل لزمر لي لإستنباط طريقة حديثة لحل المعادلات التفاضلية الجزئية التي تتعلق بقوانين الحفظ. تمت حسابات التماثل لهذه المعادلات بدلالة خوارزميات لي .

في الواقع عندما تسمح المعادلات بجبر لي القابلة للحل فإن المثاليات في هذا الجبر يمكن إستخدامها لتنفيذ الإختزال .

TO …….

My Father's Spirit

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My Brother's who helped and supported me with a lot of patience

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Contents

Introduction

1. Chapter One : Lie groups

1.1. Topological Manifolds …… 3

1.2. Differnetial Manifold structure …… 6

1.3. Sub manifold ……... 8

1.4. Lie groups ……..... 10

1.5. Some Differential geometry ……..... 17

1.6. Action of groups …….. 28

1.7. Left invariant vector field ……... 30

1.8. Lie group homomorphism ……..… 35

1.9. The Lie algebras of a Lie group ……..… 36

1.10. Exponential Map ……40

1.11. Representation of Lie group …….. 43

1.12. Operation on representations ……. 47

2. Chapter Two: Lie algebras

2.1. Introduction …….. 52

2.2. one parameter and Local one-parameter groups action on Manifold ….. 55

2.3. The Lie algebras of vector fields on a Manifold ……. 62

2.4. Lie derivative …… 66

2.5. Matrix groups …….. 72

2.6. Representations of Lie algebrs …… 77

2.7. Nilpotent Lie algebras ……. 80

2.8. Solvable Lie algebras …….. 82

2.9. Semisimple Lie algebras ……..86

2.10. Root decomposition and root systems ……92

2.11. Automorphisms and Weyl group ……..101

2.12. Weyl chambers …….. 111

2.13. Simple reflections ……. 112

2.14. Dynkin diagrams and classification of root systems ……114

3. Chapter Three : Conservation Laws of Physics

3.1. Introduction ……. 118

3.2. Concept of a Conservation Law …….. 118

3.3. Hamilton's principles and the Euler-Lagrange equations …… 120

3.4. The action principle and derivation of the Euler-Lagrange equation ….. 121

3.5. Hamiltonian's equation ……..... 122

3.6. Symmetry and Conservation Laws ……..……124

3.7. Alternative Derivation ……. 128

3.8. Displacement and Energy Conservation …… 129

3.9. Continuous Symmetry Implies Conserved Charges ……... 130

3.10. Conservation of Linear angular momentum ……. 133

3.11. The Hamiltonian …… 135

3.12. Charged particle in a Magnetic field ……. 140

3.13. fast perturbations : rapidly Oscillating fields ……... 142

3.14. Field Theory ……. 146

3.15. Gross- Pitaevskii Model ……150

3.16. Momentum and aAngular Momentum ……..152

3.17. Translation Invariance in space ……153

3.18. Rotational Invariance ……154

3.19. Center of Mass Theorem ……155

3.20. Conservation Laws resulting from Lorentz Invariance ……..157

3.21. Generating the Symmetries …… 160

3.22. Canonical Energy Momentum Tensor ……162

3.23. Electromagnetism ……...163

3.24. Dirac field ……..164

3.25. Angular Momentum ……..166

3.26. Electromagnetic field ……170

3.27. Dirac file ……173

3.28. Internal symmetries ……...175

3.29. Generating Symmetry Transformations on Quantum fields ……..…177

4. Chapter Four: Symmetries and Lie algebras

4.1. Introduction …… ……180

4.2. Symmetric group ……183

4.3. Symmetry Sets ……184

4.4. Homomorphisms of product sets ……185

4.5. Reflection sets and Symmetry sets ……..185

4.6. Classical Lie algebras ……191

4.7. Root systems ……193

4.8. Classifications of Lie root system of rank 2 ……197

4.9. Base and closure ……201

4.10. Rigidity and Collapse under Core ……...205

4.11. Lie root system Excluding T_2 ……206

4.12. Lie algebras ……..211

4.13. Reflective and Toral Lie algebras ……215

4.14. The nilpotent roots of L ……221

4.15. Exclusion of sub types of R and L ……...228

5. Chapter Five : Applications of Lie algebras

5.1. Symmetry of differential equation ……230

5.2. Some applications …… 242

Conclusion ……..…… 255

Bibliography ……..…… 256