Chapter One

(1.1) Introduction:

The basic unit of matter is atom, the three particles of the atom are: protons, neutrons, and electrons. The central portion of the atom is nucleus which it consist of protons and neutrons. Electrons orbit the nucleus. If the nucleus consists of excess mass or excess energy, it gets rid of them by emitting α or γ radiation.

Alpha (α) radiation is the Helium nucleus, while gamma (γ) radiation is a stream of photons. β rays which are electrons or anti electrons can be emitted. They are emitted if the number of neutrons is not equal to the number of protons, the nucleus will not be stable. The unstable atom will try to become stable by converting the excess neutrons or protons to protons or neutrons respectively. The materials that emit radiation are called radioactive materials [1, 2].

Radio activity is widely used in medicine in curing some diseases like cancer, beside applications in diagnosis. It is also used in non-distractive testing and mineral exploration [3, 4].

(1.2) The Problem of the Thesis:

The radioactive decay law is not related to the laws of quantum mechanics. The nucleus which is responsible for decay consists of sub atomic particles which are described by quantum laws. This is obvious as far as the nucleuseswhich are responsible for nuclear radiation, is described by the laws of quantum mechanics, and have nothing to do with classical laws [5, 6, 7].

(1.3) Literature Review:

Different attempts were made to construct new quantum laws [7, 8] and statistical laws to describe physical phenomena [9, 10]. But most of them are directed to describe scattering processes or atomic spectra. No well-established work is done for nuclear reaction.

(1.4) The Aim of the Thesis:

The aim of this work is to derive the radioactive decay laws from the laws of quantum machines and statistical laws. It also needs relating the half-life to the relaxation time.

(1.5) Thesis out Line:

The thesis consists of four chapters .Chapter one is the introduction, and chapter two is concerned with Radioactive decay Law .Chapter three is the literature review, while chapter four is devoted for contribution.

Chapter Two

Nuclear Reaction

(2.1) Introduction

Early Greek philosophers speculated that the earth was made up of different combinations of basic substances, or elements. They considered these basic elements to be earth, air, water, and fire. Modern science shows that the early Greeks held the correct concept that matter consists of a combination of basic elements, but they incorrectly identified the elements [11].

In 1661 the English chemist Robert Boyle published the modern criterion for an element. He defined an element to be a basic substance that cannot be broken down into any simpler substance after it is isolated from a compound, but can be combined with other elements to form compounds. To date, 105 different elements have been confirmed to exist, and researchers claim to have discovered three additional elements. Of the 105 confirmed elements, 90 exist in nature and 15 are man-made [12,13,14].

Another basic concept of matter that the Greeks debated was whether matter was continuous or discrete. That is, whether matter could be continuously divided and subdivided into ever smaller particles or whether eventually an indivisible particle would be encountered. Democritus in about 450 B.C. argued that substances were ultimately composed of small, indivisible particles that he labeled atoms. He further suggested that different substances were composed of different atoms or combinations of atoms, and that one substance could be converted into another by rearranging the atoms. It was impossible to conclusively prove or disprove this proposal for more than 2000 years. The modern proof for the atomic nature of matter was first proposed by the English chemist John Dalton in 1803. Dalton stated that each chemical element possesses a particular kind of atom, and any quantity of the element is made up of identical atoms of this kind [15,16]. What distinguishes one element from another element is the kind of atom of which it consists, and the basic physical difference between kinds of atoms is their weight.

(2.2)Composition of Atomic Nucleus - Neutrons and Protons:

Nuclides are nucleus of different isotopes, which is represented by the symbol $\frac{4}{2}X$ - X being symbol of the element [17]. A is the mass number, which is the sum of number of protons & neutrons; Z is the atomic number, the number of protons, which also represents the nuclear charge. Number of neutrons = (A – Z). For example, the symbol of carbon-14 nuclide is $\frac{14}{6}C$, which means a carbon-14 nuclide has 6 protons and 8 neutrons. The nuclide symbol of lead-206 is $\frac{206}{82}$ Pb.

(2.3) Radioactive Decay:

Nuclei with unsuitable compositions are unstable and they undergo spontaneous disintegration or nuclear decay. Nuclides that undergo spontaneous decay are said to be radioactive. Nuclear decay changes a radioactive nuclide into a stable one, which also change the identity of the nuclide. For example, when a carbon-14 nuclide decays by emitting a beta particle, it becomes nitrogen-14 (which is stable) [18,19,20].

 $^{14}_{6}C \rightarrow ^{14}_{7}N + ^{0}_{-1}e(2.3.1)$

Therefore, unlike a chemical reaction, nuclear reactions alter the identity of the atom.

There are three Types of Radioactive Decay Alpha Decay, Beta Decay and Gamma Decay.

(2.3.1) Alpha Decays:

There are several different types of radioactive decay [21]. One frequently observed decay process involves production of an alpha (α) particle, which is a helium nucleus and assigned the $\frac{4}{2}\alpha$. This is the common mode of decay for heavy radioactive nuclides (those with atomic number, Z > 83)[22,23]. When a nucleus loses an alpha particle its mass number (A) decreases by 4 units and the atomic number (Z) decreases by 2 units. The product of an alpha decay is a nuclide with two less protons and neutrons, respectively, than the original nuclide.

$$A_Z^A X \to Y_{Z-2}^{A-4} + \alpha(2.3.2)$$
$$\alpha = {}_2^4 He$$

For example, uranium-238 ($^{238}_{92}$ U) and radium-226 ($^{226}_{88}$ Ra) decay by alpha particle emission, which change them to thorium-234 and radon-222, respectively.

 ${}^{238}_{92}\text{U} \rightarrow {}^{234}_{90}\text{Th} + {}^{4}_{2}\alpha; \qquad {}^{226}_{88}\text{Ra} \rightarrow {}^{222}_{86}\text{Rn} + {}^{4}_{2}\alpha(2.3.3)$

(2.3.2) Beta Decays:

Another common nuclear decay involves the production of a beta (β) particle, which is a nuclear particle identical to an electron in mass and charge, and is assigned the symbol ${}^{0}_{-1}\beta$ [24,25]

$${}^{A}_{Z}X \to Y^{A}_{Z+1} + \beta^{-} + v^{-}(2.3.4)$$

For example, radioactive nuclides such as cobalt-60 ($^{60}_{27}$ Co) and carbon-14 ($^{14}_{6}$ C), decay by emitting β particle.

$${}^{60}_{27}\text{Co} \rightarrow {}^{60}_{28}\text{Ni} + {}^{0}_{-1}\beta; \qquad {}^{14}_{6}\text{C} \rightarrow {}^{14}_{7}\text{N} + {}^{0}_{-1}\beta; (2.3.5)$$

The lost of a beta particle convert a neutron to a proton: ${}_{0}^{1}n \rightarrow {}_{1}^{1}H + {}_{-1}^{0}\beta$. Thus, the product of beta decay is a nuclide with the same mass number (A), but with an atomic number one unit higher than the original nuclide [26, 27]. Both decay processes (as well as other types of radioactive decay) are accompanied by the production of gamma (γ) rays, which are high-energy photons of electromagnetic radiation that has a very short wavelength ($\lambda \sim 10^{-13}$ m) and a very high frequency ($\nu \sim 10^{21}$ s⁻¹). The new nuclide formed in the decay process goes to an excited nuclear energy state, which then relaxes to the ground state and releases the excess nuclear energy as γ radiation. For example, when uranium-238 decays, two types of γ rays of different energy are produced in addition to alpha particle ($\frac{4}{2}\alpha$):

 $^{238}_{92}$ U $\rightarrow ^{234}_{90}$ Th + $^{4}_{2}\alpha$ + 2 $^{0}_{0}\gamma$;(2.3.6)

 γ Rays are photons, which have zero charge and mass. The production of γ ray does not change the mass number (A) or the atomic number (Z).

(2.3.3) Positron Emission:

A positron is identical to an electron in mass, but it is positively charged. The symbol used for positron is ${}_{1}^{0}\beta$ [28.29]

 $^A_Z X \rightarrow Y^A_{Z+1} + \beta^+ + \nu(2.3.7)$

For example, carbon-11 decays by a positron emission:

 ${}^{11}_{6}C \rightarrow {}^{11}_{5}B + {}^{0}_{1}\beta + v(2.3.8)$

When a nuclide emits a positron, a proton is converted to neutron:

$$^{1}_{1}p \rightarrow ^{1}_{0}n + ^{0}_{1}\beta(2.3.9)$$

Thus, the product of positron emission is a nuclide with the same mass number (A), but with one unit lower in atomic number relative to the original nuclide.

Positron is not easily detected because as soon as it is released it will encounter an electron outside the nucleus and they annihilate each other, producing characteristic γ rays that go in opposite directions:

 ${}^{0}_{1}\beta + {}^{0}_{-1}\beta \rightarrow 2\gamma. (2.3.10)$

This annihilation process is an example whereby matter is converted to energy, and positron is often referred to as antimatter.

(2.3.4) Electron Capture:

Some radioactive nuclides decay by the electron capture (EC) process; that is, an electron from the innermost shell (n = 1) is absorbed by the nucleus, where it combines with a proton to form a neutron:

$$^{1}_{1}p + ^{0}_{-1}\beta \rightarrow ^{1}_{0}n$$
 (2.3.11)

As in the positron emission, an electron capture produces a nuclide with the same mass number (A), but with one unit less in atomic number (Z) [30]. For example:

 $^{195}_{79}$ Au $+ ^{0}_{-1}e \rightarrow ^{195}_{78}$ Pt(2.3.12)

(2.3.5)Gamma Decay:

Gamma rays are a type of electromagnetic radiation that results from a redistribution of electric charge within a nucleus. A γ ray is a high energy photon. The only thing which distinguishes a γ ray from the visible photons emitted by a light bulb is its wavelength; the γ ray's wavelength is much shorter. For complex nuclei there are many different possible ways in which the neutrons and protons can be arranged within the nucleus. Gamma rays can be emitted when a nucleus undergoes a transition from one such configuration to another. For example, this can occur when the shape of the nucleus undergoes a change. Neither the mass number nor the atomic number is changed when a nucleus emits a γ ray in the reaction [31].

(2.3.6) Radioactive Decay Series:

Heavy radioactive nuclides, such as radium-226 and uranium-238, cannot gain stability by emission of a single radioactive particle. They undergo a decay series, producing either an alpha or a beta particle and gamma radiation during each step, until a stable nuclide is formed [32]. For example, uranium-238 undergoes a decay series to form a stable lead-206.

$$\overset{238}{_{92}}U \rightarrow \overset{234}{_{90}}Th \rightarrow \overset{234}{_{91}}Pa \rightarrow \overset{234}{_{92}}U \rightarrow \overset{230}{_{90}}Th \rightarrow \overset{226}{_{88}}Ra \rightarrow \overset{222}{_{86}}Rn \rightarrow \overset{218}{_{84}}Po \rightarrow \overset{214}{_{82}}Pb \rightarrow \overset{214}{_{83}}Bi \rightarrow \overset{214}{_{84}}Po \rightarrow \overset{210}{_{82}}Pb \rightarrow \overset{210}{_{83}}Bi \rightarrow \overset{210}{_{84}}Po \rightarrow \overset{206}{_{82}}Pb(2.3.13)$$

In nuclear reactions, equations are balanced by matching the atomic number (that is, the charges) and the mass number on both side of the equation. Once the atomic number of the product nuclide is determined, it must be assigned proper symbol. For example, polonium-212 decays by an alpha emission. Identify the new nuclide formed.

$^{212}_{84}$ Po $\rightarrow ^{4}_{2}\alpha + ?$

To identify the second product, its mass number and atomic number must be determined. That is, the new nuclide has an atomic number = 84 - 2 = 82 (which is lead) and a mass number = 212 - 4 = 208. The nuclide symbol of the new nuclide is S (208, 82)Pb. The nuclear equation is written as:

 $^{212}_{84}$ Po $\rightarrow \alpha$ + $^{208}_{82}$ Pb(2.3.14)

(2.3.7) Nuclear Transformation or Transmutation:

Transmutation is an induced nuclear reaction that results when a target nucleus is bombarded with a fast moving particle or nuclear particle called projectile [33]. The first transmutation process was observed by Rutherford in 1919, who noticed that when nitrogen-14 was bombarded with an alpha particle an oxygen-17 is formed:

 $^{14}_{7}N + ^{4}_{2}\alpha \rightarrow ^{17}_{8}O + ^{1}_{1}p(2.3.15)$

About 14 years later, Irene Curie and her husband, Frederick Joliot, observed a similar transformation of aluminum to phosphorus when bombarded by alpha particles.

$$^{27}_{13}\text{Al} + ^{4}_{2} \alpha \rightarrow ^{30}_{15}\text{P} + ^{1}_{0}\text{n}(2.3.16)$$

Transmutation can also be carried out by bombarding the target with other positive ions. Since the bombarding particles (the projectile) are positively charged and strongly repelled by the target nuclei, they must be accelerated to a very high speed in order to acquire sufficient kinetic energy that would overcome the repulsion force. This is accomplished in various types of particle accelerators, such as the Stanford Linear Accelerator, the cyclotron at UC Berkeley Lawrence Livermore Lab, and other particle accelerators.

All transuranium elements (those with Z > 93) are produced by transmutation processes. For example, element-106 (Seaborgium) was formed by bombarding californium-249 with oxygen-18 nuclei, the

projectiles are accelerated in a linear accelerator, called Superhilac.

$$^{249}_{98}$$
Cf + $^{18}_{8}$ O $\rightarrow ^{263}_{106}$ Sg + 4 $^{1}_{0}$ n (2.3.17)

Element 110, which is not yet named, was produced by bombarding lead-208 with nickel-64 ions:

$$^{208}_{82}$$
Pb + $^{64}_{28}$ Ni⁺ \rightarrow $^{271}_{110}$ E + $^{1}_{0}$ n(2.3.18)

(*E is not actual symbol)

Neutrons are also used as projectiles in many transmutation processes. Since neutrons are neutral particles, they are not strongly repelled by target nuclei and do not to be accelerated as fast as positively charged particles. Instead, neutrons are accelerated by heat, thus called thermal neutrons, which gives them enough energy to enter target nuclei. The reaction is called neutron capture:

$${}^{59}_{27}\text{Co} + {}^{1}_{0}n \rightarrow {}^{60}_{27}\text{Co} + {}^{0}_{0}\gamma (2.3.19)$$

 152 Dy* ----> 152 Dy + γ (2.3.20)

(2.4) Radioactive Decay law

(2.4.1) Mathematical Consideration of Radioactive Decay:

Consider the number of nuclei (dN) decaying in a short time dt [34].The decaying rate is given by : $(dN/dt) = -\lambda N(2.4.1)$

Where: N is the number of radioactive nuclei present at that moment dtis the time over which the measurement is made the element, represented by a constant (λ) called the disintegration (or

decay)constant.

So:

 $dN/dt = -\lambda N$

the minus sign is there because the number of radioactive nuclei decreases as time increases. The quantity dN/dt is the rate of decay of the source or the activity of the source and is the number of disintegrations per second. This is measured in units called Becquerels (Bq) where 1 Bq = 1 disintegration per second.

A larger and more traditional unit is the Curie (Ci) 1 Ci = 3.7×10^{10} Bq. The disintegration constant or decay constant (λ) can be defined as the probability of a nucleon decaying in the next second, and we can use it to find out the mass of a given source if we know its activity.

Returning to the formula

 $dN/dt = -\lambda N(2.4.2)$

Rearranging gives:

 $dN/N = -\lambda dt(2.4.3)$

Which when integrated between the limits $N = N_o$ and N = N for the number of nuclei at time 0 and t gives:

$$\int_{N_0}^{N} \frac{dN}{N} = \lambda \int_{0}^{t} dt$$
$$ln \frac{N}{N_0} = -\lambda t$$

 $N = N_0 e^{-\lambda t} \ (2.4.4)$

If we plot ln(N) against t we have a straight line graph with gradient $(-\lambda)$ and an intercept on the ln(N) axis of $ln(N_0)$. Fig (2.1)It is this sort of graph that would be most helpful in finding the half-life (T) by

measuring the gradient and then using the relation between the halflife and the disintegration constant.



Fig: (2.1)

Returning to the equation and taking anti logs of both sides of equations

gives:

 $N = N_o e^{-\lambda t} (2.4.5)$

This is known as radioactive decay law .

The activity A is defined as a decay rate per unit time, thus it is given by:

 $A = -dN/dt = \lambda N_0 e^{-\lambda t} = \lambda N(2.4.6)$

(2.4.2)Half-life and the Radioactive Decay Constant:

We can now get a much better idea of the meaning of not only the half-life (T) but also of the decay constant (λ) [34]. When

 $N = N_0/2(2.4.7)$

The number of radioactive nuclei will have halved and so one halflives will have passed. Therefore when

t = T

$$N = N_o/2 = N_o e^{-\lambda T} (2.4.8)$$

And so:

$$1/2 = e^{-\lambda T}(2.4.9)$$

Taking the inverse gives:

$$2 = e^{\lambda T}$$

And so:

$$\ln(2) = 0.693 = \lambda T$$

 $\lambda = 0.693/T (2.4.10)$

Where: λ is the decay constant

The following table (2.1) gives some values of half-lives and decay constants. Notice that short half-lives go with large decay constants - a radioactive material with a short half-life will obviously lose its radioactivity rapidly.

Isotope	Half life	Decay
		constant
		(s ⁻¹)
Uranium	4.5x10 ⁹ years	5.0x10 ⁻¹⁸
238		
Plutonium	$2.4 \text{x} 10^4 \text{ years}$	9.2x10 ⁻¹³
239		
Carbon	5570 years	3.9x10 ⁻¹²
14		
Radium	1622 years	1.35x10 ¹¹
226		
Free	15 minutes	1.1x10 ⁻³
neutron		
239		
Radon	52 seconds	1.33x10 ⁻²
220		
Lithium 8	0.84 seconds	0.825
Bismuth	1.6×10^4 seconds	4.33x10 ³
214		
Lithium 8	6x10 ⁻²⁰ seconds	$1.2 x 10^{19}$

To Proof that:

 $A = A_o/2^n$

Start with the standard radioactive decay law and take logs to the base:

$$A = A_o e^{-\lambda t}$$

 $\ln A = \ln_{o} -\lambda t = \ln A_{o} - \ln 2 \ (t/T)(2.4.11)$

Where: T is the half-life.

Therefore:

 $\ln A = \ln(A_0/2^n) (2.4.12)$

Where:

n = t/T(2.4.13)

And so:

 $A = A_0/2^n (2.4.14)$

(2.5) Radioactivity Dating

(2.5.1) The Detection of Radioactivity:

Radioactivity or the rate of nuclear decay is measured in units called becquerel (Bq), where 1 Bq = 1 event (disintegration) per second. Another commonly used unit of radioactivity is the Curie (Ci), where $1 \text{ Ci} = 3.70 \times 10^{10} \text{Bq}$.

Radioactivity may be measured using a Geiger counter or scintillation detector. A Geiger counter counts activities by measuring the pulses of electric current produced by ionized gas particles in the probe. The probe in a Geiger counter is filled with argon gas. When the high-energy radioactive particles, such alpha or beta particles, enter the probe, they collide with argon atoms and ionize the latter. The argon ions create an electrical potential and produce electrical pulses which are detected and counted by the detector [35].

In the scintillation detector, the probe uses a substance such as sodium iodide that produces flashes of light when struck by radioactive particles. These flashes produce photoelectric current, which are detected and measured by the detector. Unlike a Geiger counter which only measures the rate of nuclear decay, a scintillation detector also measures radiation energy [36].

The biological units for radiation dose are roentgen (R), rad (D), gray (Gy), and rem (roentgen equivalent man). One roentgen is the quantity of radiation that will produce 2.1×10^9 ion pairs in 1 cm³ of dry air at

normal temperature and pressure, or generates about 1.8×10^{12} ion pairs per gram of tissue.

The rad and gray are biological units to describe the effects of radiation dose in terms of the amount of radiation energy absorbed by the affected tissue. 1 rad = 1.0×10^{-2} J (or 2.4 x 10^{-3} cal) of radiation energy per kilogram of tissue, 1 gray = 1 J per kg tissue. The rem is a biological radiation unit that takes into account the different effects of various types of radiation. A 1-rad dose of gamma radiation does not produce the same health effects as a 1-rad of alpha radiation. Since alpha particles are more massive, at the same energy, alpha radiation would cause a greater damage to tissue than either beta or gamma radiation. Therefore, the unit rem is created as a standard comparative unit, such that one rem of any type of radiation has the same health effect as 1 roentgen of gamma rays or X-rays [36].

Archeologists and geologists use radioactive substances to determine the ages and accurate dates of artifacts and rocks. Artifacts that were made from woods or plant materials can be dated using carbon-14 dating. Rocks can be dated using uranium or other radioactive substances present in them.

Since the rate of radioactive decay is constant and the half-life of a given radioactive substance is fixed, it can serve as a dating clock. For example, carbon-14 is continuously being formed in the upper atmosphere by the bombardment of nitrogen-14 with neutrons from cosmic rays.

 $^{14}_{7}N + ^{1}_{0}n \rightarrow ^{14}_{6}C + ^{1}_{1}p(2.5.1)$

Carbon-14 then undergoes beta-decay to nitrogen-14:

 $^{14}_{6}C \rightarrow ^{14}_{7}N + ^{0}_{-1}\beta$

After millions of years, the rate of formation and decay of carbon-14 reach a steady state level, and the natural abundance of carbon-14 becomes constant. The steady state level for carbon-14 in a fresh plant material is about 15.0 dis. per min. per gram of total carbon (dpm/g C). This level continues to decrease and after several hundred to a few thousand years, only a fraction of the activity remains. From the ratio of the residual activity to the steady state activity of carbon-14, and the half-life of carbon-14, the age of the artifact can be estimated. The following equation is used to calculate the age of artifacts or rocks:

$$ln(N_t/N_0) = \frac{-0.693t}{t_{1/2}}(2.5.2)$$

Where: N_t = radioactivity at the present time or at the time the artifact was discovered

 N_0 = the initial or steady state activity;

 $t_{1/2}$ = half-life of the radioisotope;

t = the age of the artifact. The Radioactive Decay Law

Exponential decay law

(2.5.2)Half-Lives of Radioactivity:

The activity of a radioactive substance is normally given in the form of its half-live, which the time is taken for 50% of the original amount of the radioactive substance to decay. For example, the half-life of carbon-14 is 5730 years, which means that, if a sample originally contains 100 million carbon-14 nuclides, the first 50 millionwill decay in 5730 years, the next 25 million nuclei decay in another 5730 years, the next 12.5 million nuclides decay in another 5730 years, and so on.

The half-life of iodine-131, which is used in radiotherapy for thyroid cancer, is 8.0 days. If a sample contains 100 millions I-131 nuclides, it will take only 8.0 days for the first 50 million nuclides to decay, and the next 25 million will take the same number of days to decay, and so on. On the other hand, a sample containing the same number of uranium-238 nuclides, which has a half-life 4.5 x 10^9 years, will take 4.5 x 10^9 years for the first 50% of the sample to decay [37].

5730 y 5730 y 100 millions ${}^{14}_{6}C \rightarrow 50$ millions ${}^{14}_{6}C \rightarrow 25$ millions ${}^{14}_{6}C$

8.0 d 100 millions ${}^{131}_{53}I \rightarrow 50$ millions ${}^{131}_{53}I \rightarrow 25$ millions ${}^{131}_{53}I$

 $4.5 \times 10^9 \text{ y}$ $4.5 \times 10^9 \text{ y}$

100 millions ${}^{238}_{92}$ U \rightarrow 50 millions ${}^{238}_{92}$ U \rightarrow 25 millions ${}^{238}_{92}$ U

Thus radioactive elements with short half-lives have higher activity (decay at faster rates) than those with longer half-lives. It also means that, radioactive materials with longer half-lives remain active for longer periods than those with short half-lives. Nuclides with very long half-lives also means that they are relatively more stable and nuclides with half-lives longer than 10^{11} years are considered as stable.

(2.6) Nuclear Binding Energy:

The total mass of a stable nucleus is always less than the sum of the masses of its separate protons and neutrons.

It has become energy, such as radiation or kinetic energy, released during the formation of the nucleus.

This difference between the total mass of the constituents and the mass of the nucleus is called the total binding energy of the nucleus.

To compare how tightly bound different nuclei are, we divide the binding energy by Ato get the binding energy per nucleon [38].See Fig (2.2)



Fig: (2.2)

The higher the binding energy per nucleon, the more stable the nucleus. More massive nuclei require extra neutrons to overcome the

Coulomb repulsion of the protons in order to be stable.



Fig: (2.3)

The force that binds the nucleons together is called the strong nuclear force. It is a very strong, but short-range, force. It is essentially zero if the nucleons are more than about 10-15m apart. The Coulomb force is long-range; this is why extra neutrons are needed for stability in high-*Z*nuclei, see fig (2.3).

Nuclei that are unstable decay; many such decays are governed by another force called the weak nuclear force.

(2.7) Nuclear Fission Process:

The most important nuclear fission is the fission of uranium-235 when bombarded with thermal neutrons:

 ${}^{235}_{92}\text{U} + {}^{1}_{0}n \rightarrow {}^{236}_{92}\text{U}^{*} \rightarrow {}^{141}_{56}\text{Ba} + {}^{92}_{36}\text{Kr} + {}^{3}_{0}n (2.7.1)$

Bombarding uranium-235 nuclides with a slow (thermal) neutron produces an excited uranium-236, called a compound nucleus. The unstable uranium-236 then undergoes fission to form two "daughter nuclei" and more neutrons. The fission is a random process that is all types of daughter nuclei will be formed. The neutrons produced in one fission reaction can be used to cause further fission on other nuclei, which causes the number of neutrons and fission reactions to increase very rapidly, producing branching chain reactions [38].

The extent of chain reactions depends on the size of the target sample. A critical sample size is needed to maintain the chain reactions in a controllable fashion. If the size is too large (supercritical size) the number of neutrons produced increases very rapidly and it will cause a chain reaction in an uncontrolled manner. An enormous amount of energy is released in a very short time and a tremendous explosion may occur.

Since slow or thermal neutrons are needed to bring about nuclear fission on uranium-235, a moderator, such as liquid sodium, is used to slow down neutrons produced by the fission process. These are then fed back into the reactor core to bring about more fission. The core in a nuclear reactor contains the fuel and the control rods. The control rods usually contain cadmium or boron carbide, absorbs slow neutrons effectively. If the number of neutrons produced from fission increases, the control rods are lowered in between the fuel rods to absorb excess neutrons, and this will slow down the fission reactions. When the number of neutrons becomes too low, the control rods are raised and the fission reactions increase.

(2.8)Nuclear Fusion Process:

Fusion is the combination of nuclides of low mass numbers. A simple example is the fusion of hydrogen-2 to form hydrogen-3 or helium-3:

$${}^{2}_{1}H + {}^{2}_{1}H \rightarrow {}^{3}_{1}H + {}^{1}_{1}p(2.8.1)$$
$${}^{2}_{1}H + {}^{2}_{1}H \rightarrow {}^{3}_{2}He + {}^{1}_{0}n(2.8.2)$$

For fusion to occur, the nuclei must be accelerated to a very high speed so that they have enough kinetic energy to overcome the nuclear repulsion and to bind the nuclei together. The process requires a temperature in the order of $10^6 - 10^7$ K.

The following fusion reactions are believed to occur in the interior of the sun, which produces in the order of 10^{19} kJ/day of solar energy [38].

$$2 {}_{1}^{1}p + 2 {}_{1}^{1}p \rightarrow 2 {}_{1}^{2}H + 2 {}_{1}^{0}\beta (2.8.3)$$

$$2 {}_{1}^{1}p + 2 {}_{1}^{2}H \rightarrow 2 {}_{2}^{3}He + 2 \gamma (2.8.4)$$

$${}_{2}^{3}He + {}_{2}^{3}He \rightarrow {}_{2}^{4}He + 2 {}_{1}^{1}p (2.8.5)$$

Net:

 $4_{1}^{1}p \rightarrow {}_{2}^{4}\text{He} + 2_{1}^{0}\beta + 2\gamma(2.8.6)$

(2.9) Problem Associated with Nuclear Power:

The supply of uranium-235 is very limited. The relative natural abundance of uranium-235 is only about 0.7%; naturally occurring uranium is mostly uranium-238, which does not undergo fission reaction. Therefore, a uranium sample has to go through an extensive enrichment process, to increase the concentration of uranium-235 to about 3%, before it can be used as fuel in a nuclear reactor. The spent fuel, which is still highly radioactive, must be properly stored [38].

(2.10) Application of Nuclear Physics:

Technological advances spurred by the demands of nuclear research have led directly to the creation of research and analytical tools in fields ranging from medicine and environmental science to art and archaeology. These new technologies have also found practical applications ranging from integrated circuit production to weapons verification. Emerging applications of nuclear technology show great potential for addressing the future needs of the nation [39].

(2.10.1)Medical Diagnosis and Treatment:

Nuclear diagnostic techniques have revolutionized medicine by providing ways to see inside the body without surgery. Today, 3500 hospital-based nuclear medicine departments in the U.S. perform 10 million nuclear medicine procedures each year, generating about \$1 billion in business and saving countless lives. In many cases, the practitioners are nuclear physicists who cooperate with physicians to develop and apply the techniques [42].

(2.10.2) Radioactive Isotopes:

Many medical procedures require radioactive isotopes. A radiopharmaceutical is a drug containing a radioactive isotope, an unstable nucleus. The isotope concentrates in the relevant part of the body and emits small amounts of radiation, which is sensed by a detector known as a "gamma camera."

Radioimmunoassay is an in vitro procedure which combines radio chemicals and antibodies to detect trace quantities of hormones, vitamins, or drugs in a patient's blood. Physicians rely on radioimmunoassay to monitor the concentration of digitalis, a medication that slows the heart rate, in the blood of coronary patients [41].

(2.10.3) Positron Emission Tomography:

Positron emission tomography (PET) is a medical imaging technique which reveals dynamic effects such as blood flow. The patient ingests a radiopharmaceutical which emits a form of anti-matter, a positively charged electron called a positron.

When the positron meets a normal electron, the two annihilate each other, emitting a pair of gamma rays in opposite directions. A circle of detectors pinpoints the location of each annihilation event. In the image below left, a PET scan of a brain, the bright red and yellow colors indicate the presence of malignant head and neck tumors. [39].

(2.10.4)Cancer Treatment:

Radiation with X-rays is a conventional treatment for cancer. The Xrays are generated by microwave linear accelerators, which are the descendants of nuclear physics research tools. Physicists cooperated with radiologists to optimize versions of these systems for medical treatment.

X-rays deposit most of their energy where they enter the body, then successively less until they leave the body. As a result, normal tissues near the X-ray source receive higher doses of radiation than the tumor itself. The sensitivity of normal tissues to radiation limits the dose which radiologists can safely deliver, especially when a tumor lies near vital organs.

To solve this problem, nuclear physicists and radiologists have developed a new treatment known as proton therapy. Protons penetrate only to a controllable depth, and they deposit most of their energy at the end of their range. They enable radiologists to increase the dose to the tumor while reducing the dose to normal tissues [42].

(2.10.5) Environmental Science:

Accelerator mass spectrometry (AMS), a new technique which can find any nucleus in concentrations below 1 part per trillion, is making important contributions to environmental science. AMS has revolutionized carbon-14 dating, which can determine the age of organic material up to 50,000 years old. Traditional techniques measure the decay rate of radioactive carbon-14. AMS is more sensitive because it counts individual carbon-14 nuclei. As a result, AMS can analyze samples a thousand times smaller [39].

(2.10.6)Ocean Circulation Studies and Global Warming:

Radioactive dating of the oceans by AMS is helping researchers understand ocean circulation patterns. Carbon-14 atoms, produced in the upper atmosphere when cosmic rays strike nitrogen nuclei, join with oxygen atoms to form carbon dioxide (CO_2). The atmosphere exchanges CO_2 with the ocean, which tends to inhale CO_2 near the poles and to exhale it near the equator. As seawater ages, the carbon14 content of its CO_2 decreases, Researchers are creating a 3dimensional map of the age of the oceans based on AMS studies of seawater samples taken at various depths, latitudes, and longitudes. These studies are helping researchers to understand the oceans' largescale circulation patterns and the earth's weather patterns.

Many people are concerned that man-made CO_2 contributes to global warming. Since the atmosphere exchanges CO_2 with the ocean, the 3-dimensional map of oceanic carbon-14 is also helping researchers learn about the natural fluctuations of the earth's CO_2 cycle -- an essential step toward understanding the significance of man-made CO_2 in the atmosphere [41].

(2.10.7)Water Resources:

The National Park Service asked hydrologists to evaluate water supply alternatives in the Wawona area of Yosemite National Park. In cooperation with physicists and nuclear geochemists, the hydrologists found that the ground water in Wawona's fractured granitic rocks is vertically segregated.

AMS measurements of carbon-14 showed that rainfall recently recharged the shallow ground water. But the deeper zone of the aquifer contains a mixture of water from a deep saline source and water from ancient rainfall; it was last recharged about 6,000 years ago. The deep and shallow zones are not hydraulically connected. Since the deep zone recharges slowly, the scientists recommended high altitude springs as a more reliable source of water for Wawonathan deep wells [40].

(2.10.8) Air Quality:

Since wood contains carbon-14 and fossil fuels do not, AMS studies of particulates in smog can identify the relative contributions of wood burning and fossil fuel burning. These studies have shown that wood burning is the major source of air pollution during winters in Albuquerque and Las Vegas.

Nuclear physicists are studying air pollution in the National Parks by proton-induced X-ray emission (PIXE). Since PIXE can detect constituents of the haze in concentrations below 1 part per trillion, the physicists can often identify the source of the pollution. They identified the Navajo Generating Station, a coal-fired power plant, as the main source of air pollution in the Grand Canyon. Their data convinced plant operators to install scrubbing equipment to reduce emissions by 90% [41].

(2.10.9) Stratospheric Ozone Depletion:

Man-made chlorofluorocarbons in the atmosphere have depleted the ozone layer over Antarctica. In the spring, half the ozone over the South Pole disappears, including nearly 100% of the ozone at altitudes between 25 and 45 kilometers. Since ozone screens the sun's ultraviolet rays, its depletion over populated areas could increase cancer rates.

AMS studies of radioisotopes such as beryllium-7 and beryllium-10 are contributing to an understanding of ozone depletion. These

beryllium isotopes are created in the stratosphere when cosmic rays strike nitrogen atoms. AMS researchers are studying the concentration of these isotopes in falling snow and in air samples collected by highaltitude aircraft. Since beryllium isotopes attach readily to aerosols, they are helping scientists to understand aerosol movement in the upper atmosphere. Aerosol particles serve as host sites for chemical reactions which create the forms of chlorine that destroy ozone [41].

(2.10.10) ENERGY- Nuclear Power:

Nuclear power plants generate 22% of the electricity in the U.S. To enhance the safety of these plants, nuclear physicists are

in reactor pressure vessels by a technique known as neutron diffraction. Their results are helping plant manufacturers to refine the analytical models that predict plant performance.

Other nuclear physicists are developing a process for treating radioactive waste from power plants. By bombarding waste with neutrons, they hope to transmute certain radioactive nuclei into either stable nuclei or nuclei with a shorter half-life that would require relatively brief storage Subcritical power plants, designed for maximum safety and now under development, may someday be essential to replace our dwindling oil reserves.

On a much smaller scale, nuclear "batteries" have served as the power sources of heart pacemakers. Similar nuclear sources are found in other common devices such as home smoke detectors [5].

(2.10.11) On-Line Analysis of Coal:

The coal and electric utility industries have installed 600 on-line analyzers which determine the chemical composition of coal by nuclear techniques. On-line analyzers monitor the quality of coal at the mine, sort and blend coal, and streamline the operation of power plants. They are helping the coal and utility industries to reduce air pollution [40].

(2.10.12)Nuclear Fusion:

The U.S. and several other countries have established long-range plans to generate electricity with commercial nuclear fusion reactors. These reactors fuse hydrogen nuclei to create helium, thereby liberating energy in a process similar to nuclear reactions in the sun [38].

(2.10.13)MATERIALS - Ion Implantation:

Chip manufacturers create integrated circuits by doping silicon wafers with boron or phosphorous ions. Ion implantation systems load several wafers onto a wheel and rotate the wheel in front of an ion beam see [40]. They accelerate the dopant ions to high energies and shoot them into the wafers. The ion accelerators in these systems are descendants of nuclear physics research tools.

Nitrogen ions implanted into surgical alloys help prevents repeated surgery to replace hip prostheses by reducing wear and corrosion from normal body fluids [40].

(2.10.14) RBS and Channeling:

Rutherford back scattering (RBS) and channeling are quality assurance techniques in the semiconductor industry. Both techniques accelerate alpha particles (helium nuclei) toward a chip. RBS experiments study the reflected alpha particles to measure levels of impurities. Channeling experiments check the effectiveness of ion implantation. Implanted boron and phosphorous ions serve their intended purpose as electron donors or receptors only if they sit on a silicon site in the crystal lattice, not if they occupy random interstitial sites. Since interstitial ions block the transmission of alpha particles through channels in the lattice, channeling experiments can detect them [40].

(2.11) The Ordinary Schrödinger Equation:

(2.11.1) The Time-Dependent Schrödinger Equation:

The Schrödinger equation is the key equation of quantum mechanics [43]. This second order, partial differential equation determines the spatial shape and temporal evolvement of a wave function in a given potential and for given boundary conditions. The one-dimensional Schrödinger equation is used when the particle of interest is confined to one spatial dimension .To derive the one-dimensional Schrödinger equation, we start with the total energy equation, i.e. the sum of kinetic and potential energy.

$$E = \frac{P^2}{2m} + V(2.11.1)$$

Multiplying both sides by Ψ

$$E\Psi = \frac{P^2}{2m}\Psi + V\Psi (2.11.2)$$

The wave function of free particle is given by

$$\Psi = e^{i(kx - wt)} \left(2.11.3\right)$$

But according to Planck theory

$$E = \hbar w$$

$$P = \hbar k$$

$$\Psi = Ae^{\frac{i}{\hbar}(Px - Et)}(2.11.4)$$
Therefore
$$\frac{\partial \Psi}{\partial t} = -\frac{i}{\hbar}E\Psi(2.11.5)$$

$$\frac{i}{\hbar}\frac{\partial \Psi}{\partial t} = E\Psi$$

$$\frac{\partial \Psi}{\partial x} = \frac{i}{\hbar}P\Psi$$

$$\frac{i}{\hbar}\frac{\partial \Psi}{\partial x} = P\Psi$$

$$\frac{\partial^2 \Psi}{\partial x^2} = \left(\frac{i}{\hbar}P\right)^2 \Psi = -\frac{1}{\hbar^2}P^2\Psi$$

$$-\hbar^2\frac{\partial^2 \Psi}{\partial x^2} = P^2\Psi$$

In three dimension

$$\frac{\hbar}{i}\nabla\Psi = P\Psi$$
$$-\hbar^2\nabla^2\Psi = P^2\Psi(2.11.6)$$

Sub equation (2.11.5) and (2.11.6) in equation (2.11.2) yields

$$\frac{i}{\hbar}\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\Psi + V\Psi(2.11.7)$$

This is the ordinary Schrödinger equation.

(2.11.2) The Time-Independent Schrödinger Equation:

For static potential which is independent of time, one can separate the wavefunction into time and time independent part:

 $\Psi(x,t) = \Psi(x)f(t)(2.11.8)$

Where $\Psi(x)$ depend only on x and f(t) depend only on t. Insertion of Eq. (2.11.8) into the Schrödinger equation yields

$$\frac{1}{\Psi(x)}H\Psi(x) = \frac{i\hbar}{f(t)}\frac{d}{dt}f(t)(2.11.9)$$

The left side of this equation depends on x only, while the right side depends on t, because x and t are completely independent variables, the equation can be true only if both sides are constants

$$\frac{i\hbar}{f(t)}\frac{d}{dt}f(t) = \text{const}$$
(2.11.10)

Tentatively this constant is designated as constant E where the meaning of E become evident below, integration of Equation (2.11.10) yields

$$f(x) = e^{-iEt/\hbar}(2.11.11)$$

Insertion of this result in to Equation (2.11.8) yields the timedependent wave function

$$\Psi(x,t) = \Psi(x)e^{-iEt/\hbar}(2.11.12)$$

If *E* is real, then the wave function has amplitude $\Psi(x)$ and a phase exp $(-iEt/\hbar)[43]$. The amplitude and a phase representation are convenient for many applications. To find the physical meaning of the real quantity *E*,we calculate expectation value of the total energy using the wave function obtained from the product method [43,44]

$$E_{total} = e^{iEt/\hbar} e^{-iEt/\hbar} E \int_{-\infty}^{\infty} \Psi^*(x) \Psi(x) dx = E(2.11.13)$$

Because the wave function is normalized that is

 $\langle \Psi(x), \Psi(x) \rangle = 1$ (2.11.14)

The designated as *E* the expectation value of the total energy.

The time –independent Schrödinger equation is obtained by inserting the wave function obtained from the product method. Equation (2.11.12) into the time- independent Schrödinger equation one obtains

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi(x) + U(x)\Psi(x) = E\Psi(x)(2.11.15)$$

This is the time-independent Schrödinger equation. Using the Hamiltonian operator, one obtains

$H\Psi(x) = E\Psi(x)(2.11.16)$

Since H is an operator and E is real number, the Schrödinger equation has the form of an eigenvalue equation. The Eigen function $\Psi_n(x)$ and the eigenvalue E_n are found by solving the Schrödinger equation.

The Eigenvalues of the Schrödinger equation E_n are discrete, that is only certain energy values are allowed, all other energies are disallowed or forbidden. The energy Eigen values or Eigen energies, the lowest Eigen state energy is ground state energy. All higher energies are called of excited state energies.

(2.11.3)Probability Current Density:

The Probability Current Density is given by:

$$\frac{\partial}{\partial t} \int p d^3 r = \int \frac{\partial |\Psi|^2}{\partial t} d^3 r = \int \frac{\partial}{\partial t} \Psi^* \Psi d^3 r$$
$$= \int \left[\Psi^* \frac{\partial \Psi}{\partial t} + \frac{\partial \Psi^*}{\partial t} \Psi \right] d^3 r (2.11.17)$$

But from Schrödinger equation:

$$\frac{i\partial\Psi}{\partial t} = \frac{-\hbar^2}{2m} \nabla^2 \Psi + U\Psi \quad , \quad \frac{-i\hbar\partial\Psi^*}{\partial t} = \frac{-\hbar^2}{2m} \nabla^2 \Psi^* + U\Psi^*$$
$$\frac{\partial\Psi}{\partial t} = \frac{-i\hbar^2}{2m} \nabla^2 \Psi - iU\Psi \quad , \quad \frac{\hbar\partial\Psi^*}{\partial t} = \frac{-i\hbar^2}{2m} \nabla^2 \Psi^* + iU\Psi^* (2.11.18)$$

Thus:

$$\frac{\partial}{\partial t} \int p d^3 r = \frac{i\hbar}{2m} \int \Psi^* \nabla^2 \Psi - \Psi \nabla^2 \Psi^* d^3 r$$
$$= \frac{i\hbar}{2m} \int \nabla \cdot [\Psi^* \nabla \Psi - (\nabla \Psi^*) \Psi] d^3 r$$
$$= \frac{i\hbar}{2m} \int \nabla \cdot S d^3 r = \frac{i\hbar}{2m} \int \nabla \cdot [\Psi^* \nabla \Psi - (\nabla \Psi^*) \Psi] dA(2.11.19)$$

But the continuity equation reads:

$$\frac{\partial \rho}{\partial t} + \nabla J = 0 \tag{2.11.20}$$

In view of (2.11.19):

$$\frac{\partial \rho}{\partial t} + \nabla . S = 0 \tag{2.11.21}$$

Thus S represents the Intensity of particles, i .e the flux of particles crossing unit area per unit time, where:

$$S = \frac{i\hbar}{2m} \left[\Psi^* \nabla \Psi - \Psi (\nabla \Psi^*) \right] (2.11.22)$$

It is well known that the momentum operator is hermition .

Hence:

$$\int \hat{P}\overline{\Psi} \Psi d^{3}r = \int \overline{\Psi} \hat{P} \Psi d^{3}r$$
$$-\frac{\hbar}{i} \int \nabla \Psi^{*} \Psi d^{3}r = -\frac{\hbar}{i} \int \Psi^{*} \nabla \Psi d^{3}r$$

Hence:

$$\Psi \overline{\nabla \Psi^*} = -\overline{\Psi^*} \nabla \Psi$$

Therefore:

$$S = \frac{i\hbar}{2m} \left[\overline{2\Psi^*} \nabla \Psi \right]$$

 $S = \frac{i\hbar}{m} \overline{\Psi^*} \nabla \Psi(2.11.23)$ Generally =Real[$\frac{i\hbar}{m} \overline{\Psi^*} \nabla \Psi$]

(2.12)Quantum Tunneling of Radiation:

Although the walls of the potential well of finite height, they were assumed to be infinitely thick. As a result the particle was trapped forever even though it could penetrate the walls. We next look at the situation of a particle that strikes a potential barrier of height U, again with E < U, but here the barrier has a finite width what we will find is that the particle has a certain probability not necessarily great, but not zero either, of passing through the barrier and emerging on the other side. The particle lacks the energy to go over the top of the barrier, but it can nevertheless tunnel through it, so to speak. Not surprisingly, the higher the barrier and the wider it is, the less the chance that the particle can get through [43]. The tunnel effect actually occurs, notably in the case of the alpha particles emitted by certain radioactive nuclei. an alpha particle whose kinetic energy is only a few MeV is able to escape from a nucleus whose potential wall is perhaps 25 MeV high. The probability of escape is so small that the alpha particle might have to strike the wall 1038 or more times before it emerges, but sooner or later it does get out. Tunneling also occurs in the operation of certain semiconductor diodes in which electrons pass through potential barriers even though their kinetic energies are smaller than the barrier heights. Let us consider a beam of identical particles all of which have the kinetic energy E. The beam is incident from the left on a potential barrier of height U and width L, as in on both sides of the barrier U = 0, which means that no forces act on the particles there. The wave function region I represents the incoming particles moving to the right and region–Irepresents the reflected particles moving to the left,regionIII represents the transmitted particles moving to the right. The wave function region II represents the particles inside the barrier, some of which end up in region III while the others return to region I. The transmission probability T for a particle to pass through the barrier is equal to the fraction of the incident beam that gets through the barrier see fig (2.12.1).



Fig (2.12.1)

The approximate value of the probability is calculated by [43]:

$$T = e^{-2k_2L}$$

Where:

$$k_2 = \frac{\sqrt{2m(U-L)}}{\hbar}$$

(2.12.1) Nuclear Quantum Tunnelling:

The radioactive decay of nuclear particles can be considered as resulting from tunneling of them through the finite potential barrier see fig (2.12.2).


Fig (2.12.2)

Schrödinger equation takes the form

$$\frac{\partial^2 \Psi}{\partial^2 x} + \frac{2m}{\hbar^2} (E - V) \Psi = 0$$
$$\frac{\partial^2 \Psi}{\partial^2 x} + k^2 \Psi = 0 (2.12.1)$$

Where:

$$k^{2} = \frac{2m}{\hbar^{2}} (E - V)(2.12.2)$$

Consider the barrier as shown in fig (2.12.3). In the first and third region

V = 0



Fig (2.12.3)

And Schrödinger equation becomes:

$$\frac{\partial^2 \Psi_1}{\partial^2 x} + k_0 \Psi_1 = 0(2.12.3)$$
$$k_0^2 = \frac{2m}{\hbar^2} E_0(2.12.4)$$

$$\frac{\partial^2 \Psi_3}{\partial^2 x} + \mathbf{k_0} \Psi_3 = 0$$
(2.12.5)
$$E_0 = T_0 = Kineticenergy$$
(2.12.6)

Where Ψ_1 represents incident and reflected waves.

Hence:

$$\Psi_1 = Ae^{ik_0x} + Be^{-ik_0x}(2.12.7)$$

Where Ψ_3 represents transmitted wave, thus:

$$\Psi_3 = Ge^{ik_0x}(2.12.8)$$

For particles inside the barrier:

$$k_{2=}^2 \frac{2m}{\hbar^2} (T-V)$$

 $k_b = -ik_2$

The wave function becomes:

$$\Psi_1 = Ce^{-k_b x} + De^{k_b x} (2.12.9)$$

The boundary conditions at:

$$x = 0$$
 and $x = L$

Requires:

$$x = 0$$

$$\Psi_{1} = \Psi_{2}$$

$$\Psi'_{1} = \Psi'_{2}$$

At

$$x = L$$

$$\Psi_{2} = \Psi_{3}$$

$$\Psi'_{2} = \Psi'_{3}$$

This finally gives transmission probability P given by:

$$P = \frac{|E|^2}{|A|^2} = e^{-2k_b L}$$

The decay constant λ is found to be equal:

$$\lambda = fP = \frac{v}{2R}P$$
$$ln\lambda = \ln\left(\frac{v}{2R_0}\right) + 2.97Z^{1/2}R_0^{1/2} - 3.95T^{-1/2}(2.12.10)$$

Where: R_0 , Z and T is the nuclear radius, atomic number and kinetic energy of alpha particle respectively and *L* is the width of the barrier.

Chapter three

Literature Review

(3.1) Introduction:

Different attempts were used to construct new quantum laws accounting for the effect of friction on particles moving in a medium. These laws succeeded in explaining some physical phenomena which cannot be explained easily by ordinary quantum equations.

This chapter exhibits some of these attempts. It also exhibits some new statistical mechanics laws which solve some long standing problems.

(3.2) Modification of Schrödinger Equation in a Media:

A collision phenomenon is one of the oldest quantum mechanical problems. It includes scattering process in which a particle or a beam of particles is scattered by a medium. The scattering quantum theory is very complex [44, 45, 46, 47]. Therefore it is very difficult to solve scattering quantum equations without doing certain approximations, or doing special treatments. For example the inelastic scattering process is explained by the so called optical potential in which an imaginary potential is inserted by hand in the energy expression [48,49, 50, 51] These problems motivateK.Haroun and M.Dirar to propose a new

quantum mechanical equation for scattering process [52].

In this model new quantum Schrodinger Equations derived by using the expression of the electric decaying wave in a conducting medium [53,54]. This expression is based on Maxwell's equations. In this model the wave function for particles in a media of conductivity σ , refractive index n_1 a magnetic permeability μ and electic permittivity ε is given by:

$$\Psi = Ae^{-\alpha x}e^{-\beta x}e^{\frac{i}{\hbar}(Px-Et)}(3.2.1)$$

Where :

$$\begin{cases} \alpha = \frac{2\pi\mu c\sigma}{n_1} \\ \beta = \frac{\sigma}{\varepsilon} \end{cases} (3.2.2)$$

With C standing for speed of light using this wave function the new Schrödingerequation is given by:

$$i\hbar\frac{\partial\Psi}{\partial t} + i\hbar\beta\Psi = -\frac{\hbar^2 c_1}{2mn_1^2}\nabla^2\Psi - \frac{\hbar^2\alpha c_1}{2mn_1^2}\nabla\Psi - \frac{\hbar^2\alpha^2}{2mn_1^2}c_1\Psi$$
(3.2.3)

Where:

$$c_1 = \left(1 - \frac{x}{2\varepsilon}\right)$$

This equation succeeds in explaining inelastic scattering process easily compared to ordinary Schrödinger equation.

(3.3) Quantization of Friction for Non Isolated Systems:

A useful expression for frictional quantum system was proposed by SwasanElhouri, M.Dirar and others [55].

In this model the plasma equation for a fluid having a pressure P is given by:

$$mn[\dot{v} + v.\nabla v] = +F - \nabla P - F_r(3.3.1)$$

With F, F_r Standing for field and frictional forces respectively, where $F = -n\nabla V(3.3.2)$

n here is the particle number density and V is the potential per unit particle.

According to this equation the particle energy in the presence of pressure and friction is given by:

$$E = \frac{P^2}{2m} + V + kT - \frac{i\hbar^3}{2\tau m^2 c^2} \Psi(3.3.3)$$

 τ here is the relaxation time and T is the absolute temperature.

According to this energy equation the new Schrödinger equation is given by:

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\Psi + kT\Psi - \frac{i\hbar^3}{2\tau m^2 c^2}\Psi(3.3.4)$$

For particle in a box, the friction energy E_2 is quantized, where:

$$E_2 = \frac{n^2 h^2}{8L^2 \tau m^2 c^2}$$

(3.4) Derivation of Klein-Gordon Equation from

Maxwell's Electric Equation:

K. H. Elegilnai and others derived Klein-Gordon equation by using Maxwell's equation [56].

Maxwell's equations for free particle are given by:

$$-\nabla^2 E + \mu \varepsilon \frac{\partial^2 E}{\partial t^2} = -\mu \frac{\partial^2 P}{\partial t^2} (3.4.1)$$

The polarization term can be reduced to be:

$$-\mu \frac{\partial^2 P}{\partial t^2} = -\mu \varepsilon \omega^2 E = -k_m^2 E(3.4.2)$$

 k_m is the medium wave number.

From (3.4.1) and (3.4.2):

$$-\nabla^2 E + \mu \varepsilon \frac{\partial^2 E}{\partial t^2} = -k_m^2 E(3.4.3)$$

Multiplying both sides by \hbar^2 yields:

$$-c^{2}\hbar^{2}\nabla^{2}E + \hbar^{2}\frac{\partial^{2}E}{\partial t^{2}} = -c^{2}\hbar^{2}k_{m}^{2}E = -c^{2}m_{0}^{2}c^{2}E(3.4.4)$$

Where the medium polarization takes the form:

$$P_m^2 = \hbar^2 k_m^2 = m_0^2 c^2 (3.4.5)$$

Thus:

$$-\hbar^2 \frac{\partial^2 E}{\partial t^2} = -c^2 \hbar^2 \nabla^2 E + m_0^2 c^4 E(3.4.6)$$

The ordinary Klein-Gordon equation can be found by replacing E by Ψ , where $|E|^2$, $|\Psi|^2$ are both proportional to the photon intensity, thus:

$$-\hbar^2 \frac{\partial^2 \Psi}{\partial t^2} = -c^2 \hbar^2 \nabla^2 \Psi + m_0^2 c^4 \Psi (3.4.7)$$

(3.5)Interpretation of the change of Intensity and Spectral line width for Bhutan, Neon, Fluorine, and Chlorine by Using Complex Statistical Distribution and Quantum harmonic Oscillator Model:

The wavelength of the emitted photons changes due to some interactions .For example the thermal motion of atoms can produce Doppler shift due to the effect of velocity [57]. Collision of atoms due to thermal vibration can also change the wavelength due to Compton scattering [58]. These thermal effects on the spectrum wavelength for some gases is investigated and studied in this work [59].

(3.5.1) Materials and Methods:

The following Apparatus and Instruments and Gases are used in the experiment:

1- Apparatus and Instruments:

USB2000 Fiber Optic Spectrometer (Ocean Optics), Heater, Thermometer, Laptop, Source of Helium Neon laser -133, Power supply, Connect cable. Test tubes (Borosilicate Glass).

2- Gases:

Bhutan (C4₄H₁₀), Neon (Ne), Fluorine (F₂) and Chlorine (CL₂).

3- Experiment set up:

Glass tube is filled by gases, each gas should be heated in steps about one or two degrees and the spectrum are recorded at each degree by using thermometer. Helium-133 beams is directed to incident on the glass tube, the spectrum of each gas including the intensity and band width is recorded for each temperature by using USB2000 Fiber Optic Spectrometer. The relations are drawn between intensity and line width of the transmitted radiation from each gas and temperature.

4- **Results**

Table (3.5.1): Spectrum of Bhutan (C_4H_{10}) at Different

Temperatures

T(K)	λ (nm)	A(m ²)	W (nm)	I(a.u)
300	630.73	4390.43	6.92	126.8
301	630.78	2536.69	6.96	125.89
303	630.83	2282.57	7.07	125.74
305	630.79	1645.95	7.03	125.79
307	630.78	1645.9	7.02	125.79
309	630.84	1855.9	7.16	125.13
311	630.78	1888.66	7.1	124.64
313	630	1698.66	7.03	124.79
315	630.83	1482.01	7.003	124.53
317	630.86	1560.78	7.03	123.96
318	630.84	1717.5	7.04	123.5
319	630.89	2037.01	7.08	123.46
320	630.95	1310.88	6.95	117.62
323	631.01	1265.28	7.02	116.95
324	631.01	1276.02	7.05	116.65

T = temperature, λ = wavelength, I = Intensity, A = area W=width



Fig (3.5.1.1) Relation between Intensity and Temperature



Fig (3.5.1.2) Relation between Width and Temperature

T(K)	λ (nm)	A(m ²)	W(nm)	I(a.u)
307	630.66	6071.9	6.41	102.94
312	630.7	2287.59	6.48	103.25
313	630.76	1941.9	6.51	103.04
315	630.74	2111.14	6.46	103.45
317	630.73	3099.17	6.51	103.21
319	630.77	2276.22	6.53	103.26
320	630.78	3762.85	6.46	103.34
321	630.8	3515.7	6.51	103.39
322	630.77	2276.22	6.53	103.25
323	630.77	3179.53	6.48	103.4

Table (3.5.2): Spectrum of Neon (Ne) atDifferent Temperatures



Fig (3.5.2.1) Relation between Intensity and Temperature



Fig (3.5.2.2) Relation between Width and Temperature

T(K)	λ (nm)	A(m ²)	W(nm)	I(a.u)
304	631.02	6337.97	7.04	100.25
305	630.81	5964.04	6.44	98.63
307	630.9	5565.97	7.11	102.13
309	630.01	4442.99	7.16	102
311	631.01	5302.56	7.23	101.88
312	630.98	5332.11	7.25	101.52
314	631.06	4669.27	7.23	101.95
316	631.05	5193.09	7.16	101.82
317	631.08	5193.92	7.19	101.37
319	631.04	4549.7	7.2	101.66
321	631.11	3541.63	7.21	102.04
323	631.07	3351.55	7.13	102.3
325	631.08	4702.13	7.18	102.5

Table (3.5.3): Spectrum of Fluorine (F_2) atDifferent



Temperatures





Fig (3.5.3.2) Relation between Width and Temperature

T(K)	λ (nm)	A(m ²)	W(nm)	I(a.u)
301	630.86	15934.5	6.92	122.23
303	630.83	7706.45	7.03	130.73
306	630.96	7478.8	7	124.24
308	630.95	8890.52	7.01	124.92
311	630.88	5738.32	7.08	126.31
313	630.94	5137.66	7.11	127.67
315	630.87	4995.6	7.01	127.81
317	630.88	4764.68	7.11	128.59
319	630.85	6501.06	7.006	129.27
321	630.93	4897.88	7.09	130.43
323	630.72	7777.74	6.88	131.22
325	630.77	6840.38	7.012	132.51
327	630.75	4907.18	6.92	134.15
329	630.75	5803.33	7.001	135.56
330	630.77	2866.74	7.05	137.006
331	630.83	2045.54	7.09	137.77

Table (3.5.4): Spectrum of Chlorine (CL₂) at Different Temperatures



Fig (3.5.4.1) Relation between Intensity and Temperature



Fig (3.5.4.2) Relation between Width and Temperature

(3.5.2) Theoretical Interpretation

Three scenarios are proposed to explain the temperature effect on the atomic spectra

(3.5.2.1)Complex Energy Statistical Distribution:

Heat energy results from collision of particles which lead to loss of energy that appears as heat thermal energy. In some theories, like optical theorem the energy is written as a summation of real and imaginary part. The real part stands for the particle energy, while the imaginary part represents the energy lost by the particle due to its interaction with the surrounding medium.

$$E = E_1 + iE_2(3.5.1)$$

If the uniform back ground is the attractive gas ions potential. Then $\overline{E} = -V_m(3.5.2)$

According to Shair .S model for non-equilibrium is given by system, the distribution:

$$n=n_0e^{-\frac{H}{H}}$$

Where *E* represents the non-uniform particle energy, where as \overline{E} stands for uniform particle energy thus substituting this in the equation:

$$n = n_0 e^{\frac{E_1 + iE_2}{V_m}}$$

$$n = n_0 e^{\frac{E_1}{V_m} e^{\frac{iE_2}{V_m}}}$$

$$n = n_0 e^{\left(\frac{E_1}{V_m}\right)} \left[\cos\frac{E_2}{V_m} + i\sin\frac{E_2}{V_m}\right] (3.5.3)$$

Considering the complex term standing for thermal energy:

$$n = n_0 e^{\left(\frac{E_1}{V_m}\right)} \left[sin \frac{E_2}{V_m} \right] (3.5.4)$$

In view of equations (3.5.4):

$$I = I_0 e^{\left(\frac{E_1}{V_m}\right)} \left[\sin \frac{E_2}{V_m} \right] (3.5.5)$$

Assuming E_1 to be kinetic thermal energy of the particle and E_2 be the lost thermal energy such that:

$$E_2 = C_0 E_1 = C_0 kT(3.5.6)$$

 $V_m = k(3.5.7)$
It follows that:
 $I = I_0 e^{T} \sin C_0 T(3.5.8)$

 $C_0 = \pi$



Fig (3.5.2)

However if the uniform background is the negative repulsive electron gas .In this case:

to:

$$\overline{E} = V_m(3.5.9)$$
Thus equation (3.5.7) and (3.5.8) reduces
$$n = n_0 e^{\left(\frac{-E_1}{V_m}\right)} \left[\cos \frac{E_2}{V_m} - i \sin \frac{E_2}{V_m} \right] (3.5.10)$$

$$I = I_0 e^{\left(\frac{-E_1}{V_m}\right)} \left[\sin \frac{E_2}{V_m} \right] (3.5.11)$$

In view of equations (3.5.8) and (3.5.7) $I = I_0 e^{-T} \sin C_0 T$ (3.5.12)

 $C_0 = \pi$



Fig (3.5.3)

Using Maxwell Boltzmann distribution for electron:

ne =ge^{- α}e^{- β (E₁+iE₂)(3.5.13)}

The imaginary part stands for absorption from medium or emission to the medium:

$$E_e = E_1 + iE_2(3.5.14)$$

For photon the number of photons is given by:

$$n_{\rm p} = e^{-\beta(E_3 + iE_4)}(3.5.15)$$

 $E_{p} = E_{3} + i E_{4}$

The for composite system medium which consists of electrons emitting photons, the energy is given by:

$$E = E_e + E_p(3.5.16)$$

Where E is the energy of the electron in excited state E_e is the electron energy in the ground state, while E_p is the photon energy thus the number of photon which is equal to the number of excited electrons is given by:

 $n = e^{-\beta(E)} = e^{-\beta(Ee+Ep)}(3.5.17)$

For electron and photons colliding with particles medium, the energy lost appears as an imaginary part.

Thus:

n=
$$e^{-\beta(E_1+E_3)}e^{-j\beta(E_2)}e^{-j(\beta E_4)}$$

For $E_1 \rightarrow 0$ $E_3 \rightarrow 0$
n= $e^{-j\beta(E_2)}e^{-j(\beta E_4)}(3.5.18)$
n = $(\cos\beta E_2 - i \sin\beta E_2)(\cos\beta E_4 + i \sin\beta E_4)$ (3.5.19)
Taking real part yields:
n= $\cos\beta E_2 \cos\beta E_4(3.5.20)$
n = $\cos\theta_2 \cos\theta_4(3.5.21)$
I= $\cos\theta_2 \cos\theta_4$
If one assumes energy lost by the electron is 10 times that lost by

If one assumes energy lost by the electron is 10 times that lost by the photon, it follows that

$$I = \cos 10\theta_4 \cos \theta_4 \theta_2 = 10\theta_4$$





Fig (3.5.4)

(3.5.2.2) Quantum Mechanical Approach:

According to the laws of quantum mechanics, the density and number of particles n are related to the wave function Ψ according to the relation:

 $n = |\Psi|^2 (3.5.22)$

But the intensities of radiation I is related to the total number of excited electrons per unit volume n

$$I = Cn_p = C C_0 n(3.5.23)$$

But the number of electrons n is given by:

n= number of atoms excited per second X number of electrons excited per second:

$$n = \frac{dn_a}{dt} \times \frac{dn_e}{dt} = \left(\frac{d|\Psi_a|^2}{dt}\right) \left(\frac{d|\Psi_e|^2}{dt}\right) (3.5.24)$$

Now consider simple case of free particles, In this case such equation. Reads:

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} = E\Psi(3.5.25)$$

The solution will be:

$$\Psi = Asin\alpha x, \Psi = -\alpha^2 \Psi(3.5.26)$$
$$\frac{\hbar^2}{2m} \alpha^2 \Psi = E\Psi, \alpha^2 = \frac{2mE}{\hbar^2} (3.5.27)$$

The probability current density is given by equation to be:

$$S = \frac{\hbar^2}{m} [\Psi^* \nabla \Psi] = A^2 \frac{\hbar^2}{m} [\propto sinaxcosax]$$
$$\frac{dm}{dt} = s = \frac{A^2 \propto \hbar^2}{2m} sin2 \propto x = c_0 sin2 \propto x (3.5.28)$$

But since:

v = 0

$$E = \frac{mv^2}{2} = \frac{p^2}{2m}$$

$$2mE = p^2 = \hbar^2 k^2 (3.5.29)$$
From (3.5.27):

$$\alpha^2 = k^2$$

$$\propto = k = \frac{2\pi}{\lambda} = \frac{2\pi f}{\lambda f} = \frac{\omega}{v}$$

$$\propto = \frac{\hbar\omega}{\hbar v} (3.5.30)$$

If one heat particles are harmonic oscillator, thus according to equation (3.5.29) the kinetic thermal energy becomes:

$$KT = E = \hbar\omega(3.5.31)$$
$$\propto = \frac{KT}{\hbar\nu}(3.5.32)$$

Thus for atoms and electrons see equation (3.5.27):

$$\frac{dn_a}{dt} = c_a sin2 \frac{KT}{\hbar v_a} x(3.5.33)$$
$$\frac{dn_e}{dt} = c_e sin2 \frac{KT}{\hbar v_e} x(3.5.34)$$

In view of equation (3.5.23):

$$I = CC_0 n = CC_0 C_a C_e \sin 2\frac{\kappa T x}{\hbar v_a} \sin 2\frac{\kappa T x}{\hbar v_e} (3.5.35)$$

t a certain position x_0 :

$$I = C_a C_e sin2 \frac{KTx_0}{\hbar v_a} sin2 \frac{KTx_0}{\hbar v_e} (3.5.36)$$
$$I = C_T sin2 \frac{KTx_0}{\hbar v_a} sin2 \frac{KTx_0}{\hbar v_e} (3.5.37)$$

For Simi plicate let:

$$20Kx_0 = \hbar v_0(3.5.38)$$
$$\frac{2Kx_0}{\hbar} = \frac{v_0}{10}(3.5.39)$$

Thus:

$$I = C_T \sin \frac{Tv_0}{10v_a} \sin \frac{Tv_0}{10v_e} (3.5.40)$$

Let also $:v_a = \frac{1}{3}v_e = \frac{1}{3}v_0$
 $I = C_T \sin 3T \sin T = C_T \sin T \sin 3T (3.5.41)$
 $C_T = 10$





(3.5.2.3)Semi Classical Harmonic Oscillator Mod:

Consider on electric field that causes oscillation of atoms and electrons to emit radiation. The equation of motion of the oscillating particle is given by:

ma= -F

The force acting on the electron or atom is the electric field. Thus:

F = Ee(3.5.43)

The displacement is given by:

$$x = x_0 e^{-i\omega_0 t}$$

Hence, the speed and acceleration are given by:

$$v = \dot{x} = -i\omega_0 x$$
, $a = \ddot{x} = -\omega_0^2 x$ (3.5.44)
Inserting (3.5.44) and (3.5.43) in (3.5.42):

 $-m\omega_0^2 x = -eE(3.5.45)$

If resistive force for a medium of particles density n is the form:

$$F_r = \frac{-nmv}{\tau} = \frac{nim}{\tau} \,\omega x(3.5.46)$$

The frequency change from ω_0 to ω , thus the equation of motion becomes:

$$ma = -eE - F_r(3.5.47)$$

With:

$$x = x_0 e^{-i\omega t}$$

 $v = \dot{x} = -i\omega x$, $a = \ddot{x} = -\omega^2 x(3.5.48)$
Therefore, inserting (3-3-4),(3-3-5) and(3-3-7) in(3-3-6)yields:

$$-m\omega^{2}x = -m\omega_{0}^{2}x + \frac{inm\omega x}{\tau}$$

$$\omega_{0}^{2} - \omega^{2} = \frac{in\omega}{\tau}$$

$$(\omega + \omega_{0})(\omega_{0} - \omega) = \frac{in\omega}{\tau}(3.5.49)$$
If:

$$\omega \approx \omega_{0} , \omega + \omega_{0} \approx 2\omega, \omega_{0} - \omega = \Delta\omega(3.5.50)$$

Thus:

$$(2\omega)(\Delta\omega) = \frac{in\omega}{\tau}$$
$$(\Delta\omega) = \frac{in}{2\tau}(3.5.51)$$

According to quantum harmonic oscillator model, if one treat the electrons and atoms as harmonic oscillators, their energy is given by:

$$E_0 = \hbar \omega_0$$
, $E = \hbar \omega (3.5.52)$

.

The energy difference due to friction is thus given by:

$$\Delta E = E_0 - E = \hbar (\omega_0 - \omega) = \hbar \Delta \omega = \frac{i\hbar n}{2\tau} (3.5.53)$$

The imaginary term is not surprising as far as the inelastic scattering is described by imaginary potential. This is known as optical theorem, in which inelastic scattering, where particles loose energy by collision, is described by a complex potential.

In atomic spectra thus thermal energy leads to lost or gain of energy by collision leading numerically to the change of frequency in the from:

$$\Delta f = \frac{\Delta \omega}{2\pi} = \frac{n}{4\pi\tau} (3.5.54)$$

The cores ponding change of length takes the form:

$$\Delta f = f_0 - f = \frac{c}{\lambda_0} - \frac{c}{\lambda} = \frac{c(\lambda - \lambda_0)}{\lambda \lambda_0}$$
$$= \frac{c\Delta\lambda}{\lambda^2} = \frac{c^2}{c\lambda^2} \Delta\lambda = \frac{f^2}{c} \Delta\lambda(3.5.55)$$

Where:

$$\lambda \approx \lambda_0 \Delta \lambda = \lambda - \lambda_0 (3.5.56)$$

Thus, in view of (3.5.56), equation (3.5.45) gives:

$$\Delta \lambda = \frac{nc}{4\pi f^2 \tau} = \frac{\pi nc}{\omega^2 \tau} (3.5.57)$$

But since the number density is related to the wave function according to the relation:

$$n=|\Psi|^2$$

Thus:

$$\Delta \lambda = \frac{\pi c |\Psi|^2}{\omega^2 \tau} (3.5.58)$$

Using the complex energy statistical distribution in equation:

$$n = n_0 e^{E_1/V_m} \sin \frac{E_2}{V_m} (3.5.59)$$

Following the same procedures in equation from (3.5.6) to (3.5.12) one gets the line width in the form:

w~ $\Delta\lambda$ ~ $e^{-T}sin\pi t(3.5.60)$



Fig (3.5.6)

If one using the expression for the total number of electrons resulting from multiplying the number of electrons in each atom by the number of atoms, one gets:

```
{\rm w}{\sim}\Delta\lambda{\sim}cos\beta E_2cos\beta E_4
```

 $\sim cos10TcosT(3.5.61)$



Fig (3.5.7)

Where one follows the same procedures used for the intensity expression derived for complex energy by using Maxwell's distribution (see equation (3.5.20))

The use of quantum expression in equations (3.5.45) together with equation (3.5.58) leads to

 $W \sim \Delta \lambda \sim sinTsin3T(3.5.62)$



Fig (3.5.8)

(3.5.3)Discussion

The relation between Intensity and temperature for Neon (Ne) see fig. (3.5.21) resembles that obtained theoretically in equations (3.5.5) and (3.5.8), for the case when the temperature Is non-uniform[see fig. (3.5.2)]. Thus:

$$E_1, E_2 \sim kT$$

And the electric static potential is assumed to be uniform. This agrees with the fact that the gas is heated at the bottom, where it is very hot, while its temperature at the top is less. The same empirical relation for Ne can be explained by using quantum mechanics model in Fig (3.5.6).

The model based on semi classical harmonic oscillator and quantum mechanics explains the effect of temperatures on the line width of the spectrum for the gases Bhutan (C_4H_{10}), Neon (Ne), Fluorine (F₂) and chlorine (CL₂).

The comparison of Figs (3.5.1.2) with Fig (3.5.7), (3.5.2.2) with Fig (3.5.8), (3.5.4.2) with Fig (3.5.7) and (3.5.3.2) with Fig (3.5.3) shows that the theoretical relations of line width with temperatures resembles the corresponding empirical relation

(3.6)Explanation of Intensity Spectral change of Bhutan, Carbon dioxide , Carbon Monoxide, Oxygen, Nitrogen Gases on the basis of Non Equilibrium Statistical Distribution:

The experimental work shows parabolic increase of wave length and decrease of peak separation with temperatures. These changes were explained on the relation between change of refractive index and wavelength change, beside cavity length [60]. In another work Bragg grating central wave length changes with temperatures is also investigated [61.62]. The experimental work shows increase of Bragg wave length with temperature, this change was explained on the base is of the relation of wave length change with refractive index and thermal expansion to temperature, fortunately no explanation was done by using quantum or statistical laws which are suitable for the micro world [63].

(3.6.1)StatisticalDistribution Law of Non- Equilibrium Statistical System:

The plasma equation of motion of particles in the presence of a field potential per particle V and a pressures force P beside a resistive force F_r is given by:

 $nm\frac{dv}{dt} = -\nabla p - Fr - \nabla nV(3.6.1)$

Where n, m stands for particle number density and particle mass respectively considering the motion to be in one dimension along the *x*-axis the equation of motion becomes:

$$nm\frac{dv}{dx}\frac{dx}{dt} = -\frac{dp}{dx} - \frac{d(nV)}{dx} - F_r(3.6.2)$$

$$nmv\frac{dv}{dx} = -\frac{dp}{dx} - \frac{dV}{dx} - F_r$$
$$n\frac{d(\frac{1}{2}mv^2)}{dx} = -\frac{dp}{dx} - \frac{d(nV)}{dx} - F_r(3.6.3)$$

The term T stands for the kinetic energy of a single particle and can be written as:

$$T = \frac{1}{2}mv^2 = E_0(3.6.4)$$

The pressure *P* can also split in to thermal P_t and non thermal P_0 to be in the form:

$$P = P_t + P_0 = nkT + nP_p(3.6.5)$$

Where P_p in the non-thermal pressure for one particle

(3.6.2) Plasma Statistical Equation in the Presence of Potential Field Only:

When the potential is only present beside the thermal pressure term the equation of motion (3.6.3) read:

$$n\frac{dE_0}{dx} = -\frac{d(nV)}{dx} - \frac{d(nkT)}{dx}(3.6.6)$$

If one assumes to change with (X) due to the change of (n) only then equation (3.6.2) to:

$$n\frac{dE_0}{dx} = -kT\frac{dn}{dx} - \frac{d(nV)}{dx}(3.6.7)$$

The temperature here is assumed to be uniform; here one has two cases either $V_T = nV$ changes with respect to X due to the change of Vonly. In this case equation (3.6.7) reads:

$$n\frac{dE_0}{dx} = -kT\frac{dn}{dx} - n\frac{dV}{dx}(3.6.8)$$
$$n\frac{d(E_0+V)}{dx} = -kT\frac{dn}{dx}(3.6.9)$$

The total energy is given by:

 $E = T + V = E_0 + V(3.6.10)$

Therefore (3.6.9) becomes:

$$n\frac{dE}{dx} = -kT\frac{dn}{dx}$$
$$ndE = -kTdn$$

Integration both sides yields:

$$-\int \frac{dE}{dx} = \int \frac{dn}{n}$$
$$lnn = -\frac{E}{kT} + C_0$$
$$n = Ce^{\frac{-E}{kT}}$$
(3.6.11)

This is the ordinary Maxwell-Boltzmann distribution. But if V_T changes due to change of (n) only, then equation (3.6.7) reads:

$$n\frac{dE_0}{dx} = -kT\frac{dn}{dx} - V\frac{dn}{dx}$$
$$n\frac{dE_0}{dx} = -(kT + V)\frac{dn}{dx}$$
$$-\frac{dE_0}{(KT + V)} = \frac{dn}{n}$$

Integration both sides yields:

$$\int \frac{dn}{n} = -\int \frac{dE_0}{(kT+V)}$$

$$lnn = -\frac{E_0}{kT+V} + C_0$$

$$n = Ce^{\frac{-E_0}{(KT+V)}}$$
(3.6.12)

The energy E_0 here stands for the kinetic energy only as shown by equation (3.6.4).

2-2 Plasma Statistical Equation When Thermal Pressure Changes Due to the Temperature Change

When the thermal pressure change due to the temperature change:

$$\frac{dP_t}{dx} = n \frac{d(kT)}{dx} (3.6.13)$$

In this case the plasma equation (3.6.3) in the absence of a resistive force is given by:

$$n\frac{dE_0}{dx} = -\frac{dp_t}{dx} - \frac{d(nV)}{dx}$$
$$n\frac{dE_0}{dx} = -n\frac{d(kT)}{dx} - \frac{d(nV)}{dx}(3.6.14)$$

Where the pressure here is assumed to be due to the thermal pressure only, if the total potential V_T is assumed to be related to the rate of change at V only, I.e.

$$\frac{dV_T}{dx} = \frac{d(nV)}{dx} = n\frac{dV}{dx}(3.6.15)$$

In this case equation (3.6.14) reads:

$$n\frac{dE_0}{dx} = -n\frac{d(kT)}{dx} - n\frac{dV}{dx}$$
$$E_0 = -KT - V + C_0$$

Thus:

 $C_0 = E_0 + KT + V$

One can easily deduce that C_0 is equal to the total energy *E*,I.e $E = E_0 + V + kT(3.6.16)$

I.e. the total energy is equal to kinetic energy E_0 beside potential energy V and thermal energy kT, But if V_T change due to the rate of change of n only, I.e.

$$\frac{dV_T}{dx} = V \frac{dn}{dx} (3.6.17)$$

Equation (3.6.14) thus reads:

$$n\frac{dE_0}{dx} = -n\frac{d(kT)}{dx} - V\frac{dn}{dx}$$
$$n(dE_0 + dkT) = -Vdn$$
$$\int \frac{dn}{n} = -\int \frac{(dE_0 + dkT)}{V}$$
$$lnn = -\frac{E}{kT} + C_0$$
$$n = Ce^{\frac{-(E_0 + kT)}{V}} (3.6.18)$$

I f the change of V_T with respect to (x) is due to the change of both (n) and (V) with respect to (x): $\frac{dV_T}{dx} = \frac{d(nV)}{dx} = n \frac{dV}{dx} + V \frac{dn}{dx}(3.6.19)$ Inserting (3.6.19) in (3.6.14) yields: $n \frac{dE_0}{dx} = -n \frac{d(kT)}{dx} - n \frac{dV}{dx} - V \frac{dn}{dx}$ $nd(E_0 + kT + V) = -Vdn$ $\int \frac{dn}{n} = -\int \frac{d(E_0 + kT + V)}{V}$ $lnn = -\frac{(E_0 + kT + V)}{V} + C_0$ $n = Ce^{\frac{-(E_0 + kT + V)}{V}}(3.6.20)$

Thus for non-uniform temperature systems, and non-uniform potential energy per particle, the statistical distribution low is described by (3.6.20). This relation is different from that obtained in (3.6.11), where the temperature is assumed to be uniform.

2-3 Plasma Statistical Equation When Thermal Pressure Change Due to the Change of Both (T) and (n) When the thermal pressure changes due to the change of both (n) and (T), in this case the plasma equation (3.6.14) is given by:

$$n\frac{dE_0}{dx} = -n\frac{d(kT)}{dx} - kT\frac{dn}{dx} - \frac{d(nV)}{dx}(3.6.21)$$

If the total potential V_T is assumed to be related to the rate of change of (V) only ,I.e

$$\frac{dV_T}{dx} = \frac{d(nV)}{dx} = n\frac{dV}{dx}(3.6.22)$$

In this case equation (3.6.21) reads:

$$n\frac{dE_0}{dx} = -n\frac{d(kT)}{dx} - kT\frac{dn}{dx} - n\frac{dV}{dx}$$
$$n(dE_0 + d(kT) + dV) = -kTdn$$
$$\int \frac{dn}{n} = -\int \frac{(dE_0 + d(kT) + dV)}{kT}$$
$$lnn = -\frac{(E_0 + kT + V)}{kT} + C_0$$
$$n = Ce^{\frac{-(E_0 + kT + V)}{kT}} (3.6.23)$$

But if change due to the rate of change on n only, I.e.

$$\frac{dV_T}{dx} = \frac{d(nV)}{dx} = V \frac{dn}{dx} (3.6.24)$$

In this case equation (3.6.21) reads:

$$n\frac{dE_0}{dx} = -n\frac{d(kT)}{dx} - kT\frac{dn}{dx} - V\frac{dn}{dx}$$
$$n(dE_0 + d(kT)) = -(kT + V)dn$$
$$\frac{dn}{n} = -\frac{(dE_0 + d(kT))}{(kT + V)}$$
$$\int \frac{dn}{n} = -\int \frac{(dE_0 + d(kT))}{(kT + V)}$$
$$lnn = -\frac{(E_0 + kT)}{(kT + V)} + C_0$$

 $n = Ce^{\frac{-(E_0 + kT)}{(kT + V)}}$ (3.6.25)

If the change of V_T with respect to x is due to the change of both n and V with respect to, then:

$$\frac{dV_T}{dx} = \frac{d(nV)}{dx} = n\frac{dV}{dx} + V\frac{dn}{dx}(3.6.26)$$

Inserting (3.6.26) in (3.6.21) yields:

$$n\frac{dE_{0}}{dx} = -n\frac{d(kT)}{dx} - kT\frac{dn}{dx} - n\frac{dV}{dx} - V\frac{dn}{dx}$$
$$n(dE_{0} + dV + d(kT)) = -(kT + V)dn$$
$$\int \frac{dn}{n} = -\int \frac{(dE_{0} + dV + d(kT))}{(kT + V)}$$
$$lnn = -\frac{(E_{0} + kT + V)}{(kT + V)} + C_{0}$$
$$n = Ce^{\frac{-(E_{0} + kT + V)}{(kT + V)}} (3.6.27)$$

(3.6.3) Variation of Spectral Intensity of Some Gases with

Temperature:

1- Materials and Methods:

The following Apparatus and Instruments are used in the experiment

2- Apparatus and Instruments:

USB2000 Fiber Optic Spectrometer (Ocean Optics), Thermometer, Heater, Laptop, Source of Helium Neon laser -133, Power supply, Connect cable, Origin program which specialist draw, analyze and address the different data and Test tubes (Borosilicate Glass).

3-Gases:

Bhutan $(C4_4H_{10})$, carbon dioxide (CO_2) ,Carbon Monoxide (CO), nitrogen (N_2) ,oxygen (O_2)

4-Experiment Set Up

Glass tube is filled by gases, each gas should be heated in steps about one or two degrees and the spectrum are recorded at each degree by using thermometer. Helium-133 beams is directed to incident on the glass tube, the spectrum of each gas including the intensity and band width is recorded for each temperature by using USB2000 Fiber Optic Spectrometer. The relations are drawn between intensity and line width of the transmitted radiation from each gas and temperature.

5- Results

Table (3.6.1): Spectrum of Bhutan (C_4H_{10}) at different temperatures

T = temperature, λ = wavelength, I = Intensity, A = area

W=width

T(K)	λ (nm)	A(m ²)	W (nm)	I(a.u)
300	630.73	4390.43	6.92	126.8
301	630.78	2536.69	6.96	125.89
303	630.83	2282.57	7.07	125.74
305	630.79	1645.95	7.03	125.79
307	630.78	1645.9	7.02	125.79
309	630.84	1855.9	7.16	125.13
311	630.78	1888.66	7.1	124.64
313	630	1698.66	7.03	124.79
315	630.83	1482.01	7.003	124.53
317	630.86	1560.78	7.03	123.96
318	630.84	1717.5	7.04	123.5
319	630.89	2037.01	7.08	123.46
320	630.95	1310.88	6.95	117.62
323	631.01	1265.28	7.02	116.95
324	631.01	1276.02	7.05	116.65



Fig (3.6.1) Relation between Intensity and temperature

Table (3.6.2):	spectrum	of	Carbon	dioxide	(CO_2)	at	different
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T(K)	λ(nm)	A(m ²)	W(nm)	I(a.u)
308	631.09	6721.97	7.09	104.78
310	630.69	3971.93	6.56	104.82
312	630.71	2990.83	6.61	104.98
314	630.68	2313.09	6.67	105.16
318	630.67	2672	6.59	105.105
319	630.73	2562.34	6.62	105.16
321	630.77	2788.17	6.63	104.95
323	630.75	2604.82	6.71	105.38
325	630.81	2746.3	6.76	105.25
327	630.75	2630.97	6.68	105.37
329	630.77	2896.99	6.58	105.56
331	630.66	3235.75	6.72	107.02
333	630.7	2852.53	6.67	109.16
336	630.64	2867.93	6.73	112.15

temperatures



Fig(3.6.2) relationship between intensity and temperature

 Table (3.6.3): spectrum of Carbon Monoxide (CO) at different

 temperatures

T(K)	λ (nm)	A(m ²)	W(nm)	I(a.u)
305	630.81	7727.23	7.19	127.88
308	630.83	17902.95	7.52	128.35
310	631.15	23879.64	7.95	127.71
314	631.24	28752	8.02	127.72
316	630.95	35431.03	7.82	128.63
317	630.87	33455.02	7.63	129.07
318	630.97	35343.99	7.67	128.96
319	630.75	36682	7.59	128.89
320	630.5	31212.8	6.31	128.81
321	630.54	30925.51	6.29	129.12
323	630.53	35011.42	6.37	129.37
324	630.62	29236.35	6.28	132.05
325	630.62	30482.95	6.83	132.47
326	630.61	32561.45	6.4	134.94
328	630.6	28986.9	6.43	136.16



Fig(3.6.3) relationship between intensity and temperature

Tabla ((361)	• snoetrum	of Oxygon	(O.) at	different tem	noraturas
Table ((3.0.4)): spectrum	of Oxygen	(\mathbf{O}_2) at	unierent tem	peratures

T(K)	λ (nm)	A(m ²)	W (nm)	I(a.u)
307	630.38	6.33	8357.87	121.82
309	630.39	6.45	8237.15	124.16
310	630.36	6.29	10776.64	105.39
312	630.39	6.34	9984.4	107.28
313	630.33	6.26	10333.1	108.35
317	630.36	6.36	10055.14	111.84
319	630.25	6.29	8871.58	114.79
320	630.1	6.37	10285.94	126.56
322	630.17	6.31	6953.2	126.2
323	630.14	6.37	7661.5	126.12



Fig(3.6.4) relationship between intensity and temperature

Table (3.6.5)	: spectrum	of Nitrogen	(N ₂) at	different	temperatures
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T(K)	λ (nm)	A(m2)	W(nm)	I(a.u)
307	630.57	13210.75	6.46	106.37
308	631.05	22845	7	106.55
309	631.17	19689.5	7.03	105.14
313	631.14	26622.9	7.06	102.97
318	631.03	37660.7	7.02	101.47
323	631.04	22161.25	7.04	100.71
325	631.02	17831.8	6.99	100.73
326	631.15	20802.4	6.99	100.46
327	631.07	20115.56	7.05	100.21
328	631.05	23555.87	7.12	99.97
329	631.07	13968.94	6.99	101.27
331	631.1	13849.26	7.07	101.94
333	631.13	13601.3	7.07	102
334	631.15	14948.05	7.06	102.37
337	631.01	13328.25	7.02	103.29



Fig (3.6.5) relationship between intensity and temperature
(3.6.4) Theoretical Interpretation:

The behavior of gases can be explained by using statistical physics .This explanation is related to the fact. That gas consists of a large number of atoms and molecules. The electrons and atoms of gases can also be explained by using the laws of quantum mechanics. This is not surprising, since atomic and sub atomic microscopic particles are explained by using the laws of quantum mechanics.

(3.6.4.1) Thermal Equilibrium Statistical Interpretation:

According to Maxwell distribution the density of particles is given by: $n=n_0e^{-\frac{E}{KT}}(3.6.28)$

It is quite natural to assume that the density of photons emitted n_p is proportional to the exited atoms or electrons density I. e

$$n_p = C_0 n = C_0 n_0 e^{-\frac{E}{KT}} (3.6.29)$$

Where C_0 is a constant

Assume that the spectrum is formed due to the emission of free electrons surrounding the positive ion of the gas. In this case the potential is negative and attractive. By neglecting kinetic term, when the potential is very high in this case

$$E = -V_0(3.6.30)$$

Therefore equation becomes (3.6.29):

$$n_p = C_0 n_0 e^{\frac{V_0}{KT}} (3.6.31)$$

$$V_0 \sim \frac{9 \times 10^9 \times (1.6)^2 \times 10^{-38}}{r_0} \sim \frac{10^{-28}}{r_0}$$

$$\frac{V_0}{k} \sim \frac{10^{-5}}{r_0}$$

$$Forr_{0} \sim 10^{-5} m \frac{V_0}{k} \sim 1$$
 (3.6.32)

The light intensity of the emitted photons is given by:

$$I = Cn_p = CC_0 n_0 e^{\frac{V_0}{KT}} = I_0 e^{\frac{V_0}{KT}} (3.6.33)$$

By a suitable choice of (3.6.32) and using (3.6.33) parameters one can choose:





(3.6.4.2)Non- Equilibrium Statistical System:

The work done by SuhairSalihMakawySuliman [64] shows that for non-Equilibrium systems where the temperature, potential are no longer constant, but worry with position, the number density is given by:

$$n=n_0e^{-\frac{E}{\overline{E}}}(3.6.35)$$

Where E stands for non-uniform varying energy, while \overline{E} represents the uniform energy .If one assumes that the electrons are affected by the bulk potential of positive ions, which is attractive, then: $\overline{E} = -V_0(3.536)$ E = kT(3.5.37)

Sub situating (43.5.29) and (3.5.37) in (3.5.35) yields:

 $n = n_0 e^{\frac{KT}{V_0}}(3.6.38)$

In view of equations (3.6.29), (3.6.33) and (3.6.34) .The light intensity is given by:

$$I = I_0 e^{\frac{KT}{V_0}} (3.6.39)$$

Using (3.6.32):

$$\frac{V_0}{k} \sim 1$$

When:

 $V_0 \sim k \sim 1 \times 10^{-22} \tag{3.6.40}$

In this case equation (3.6.39) becomes :

 $I = I_0 e^T I_0 = 10(3.6.41)$



Fig (3.6.7)

When the electron energy is equal to thermal energy, beside energy V_r resulting from repulsive force.

In this case:

 $E = KT + V_r$ (3.6.42)

If the back ground is the attractive uniform potential as in equation

(3.6.29) then:

 $n=n_0e^{\frac{V_r+KT}{V_0}}(3.6.43)$

Assuming that:

 $V_{r \sim} 100V_0$ $V_{0 \sim} 100K \sim 10^{-21}(3.6.44)$

Using relations (3.6.29), (3.6.33), (3.5.43) and (3.544) yields:



Fig (3.6.8)

If the energy resulting from the repulsive electron gas is assumed to be uniform, then:

 $\overline{E} = V_r(3.6.46)$

When the electron energy is the thermal, beside the energy resulting

from attractive ions V_0 then:

 $E = KT - V_0(3.6.47)$

In this case equation (3.5.35) reads:

$$n = n_0 e^{\frac{V_0 - KT}{V_r}} = n_0 e^{\frac{V_0}{V_r} - \frac{kT}{V_r}} (3.6.48)$$

In view of equations (3.5.29), (3.5.33) and (3.6.48), one gets:

$$I = I_0 e^{\frac{V_0}{V_r} - \frac{kT}{V_r}} (3.6.49)$$

Assuming:

 $V_0 \sim 5V_r$ $V_r \sim k(3.6.50)$

One gets the light intensity in the form:

 $I = I_0 e^{5-T} I_0 = 2 \ (3.6.51)$





And assuming electron repulsive back ground

 $E = V_r$

Thus equation (3.5.35) reads:

$$n = n_0 e^{\frac{-V_r}{KT}} (3.5.52)$$

For:

 $V_r \sim k(3.6.53)$

Using also equations (3.5.29) and (3.5.33) one gets:

$$I = I_0 e^{-\frac{1}{T}} (3.5.54)$$

Set:
$$I_0 = 10e$$
$$I = 10e e^{-\frac{1}{T}} = 10e^{1-\frac{1}{T}} (3.6.55)$$



Fig (3.6.10)

(3.6.5)Dissection:

Fig (3.6.1) shows the Relation between Intensity and temperature for Bhutan (4_4H_{10}) , the curve of this relation resembles that of fig(3.6.9). This means the Bhutan gas the homogenous repulsive electron gas potential is almost constant compared to attractive ion potential and temperature.

Fig (3.6.2) shows the Relation between Intensity and temperature for Carbon dioxide (CO₂) which has a curve that resembles the curve in figs (3.6.7) and (3.6.8). This indicates that the attractive positive ions looks homogeneous and stable compared to temperature. This is not surprising since the gas was heated at the bottom by a heater. This means that the bottom is hotter than the top part of the gas. Thus the temperature is not uniformities

In Fig (3.6.3) the Relation between Intensity and temperature for Carbon Monoxide (CO) which is displayed in this fig, resembles the curve of figs (3.6.7) and (3.6.8). This is not surprising, since for both figures the statistical distribution is based on the homogeneity of the ionic field and non-homogeneity of temperature. The nonhomogeneity of temperature results again from the fact that the bottom of the gas exposed to a heater is very hot compared to the top of the gas.

However Fig (3.6.4) shows that the Relation between Intensity and temperature for Oxygen (O_2) can be easily explained by fig (3.6.10) where the repulsive homogeneous electron field dominates, compared the attractive ionic field. Fig (3.6.5) shows for Nitrogen (N2) it spectrum is displayed by the Relation between Intensity and temperature. The curve of this relation resembles fig (3.6.6) which shows homogeneity of temperature compared to less homogenous attractive crystal field.

(3.6.6)Conclusion:

The new statistical non-equilibrium model based on plasma equation that relates to the intensity of atomic spectra to the gas temperature can successfully describe the dependence of atomic spectra on temperature. The promotion of these models in the future may successfully be capable of determining the atmospheric temperatures of deferent layers.

(3.7) Summary and Critique:

In these attempts new Schrödinger equation was derived to account for friction. But most of them are complex and does not base on harmonic oscillator model which is a corner stone of conventional ordinary Schrödinger equation.

It is important to note that none of them derives radioactive decay law on the basis of quantum mechanical laws.

Chapter 4

Schrödinger Quantum Frictional Equation for Nucleus

(4.1)Introduction:

When any particle enters a certain medium it suffers from friction which decreases energy and momentum. Such particle cannot be described by using ordinary Schrödinger equation. Thus one needs new quantum equation that accounts for the effect of friction. This chapter is concerned with this new equation. This equation is used to derive radioactive decay law. A new statistical physical law for particle in a field is also derived and used for describing nuclear behavior.

(4.2) Relaxation time and Friction:

For any particle having mass m and velocity v the force *F* exerted on it can be described by the equation:

$$m\frac{dv}{dt} = F(4.2.1)$$

Considering the particle as harmonic oscillator the velocity v is given by :

 $v = v_0 e^{iw_0 t} (4.2.2)$

Where

 w_0 is the angular frequency v_0 is the maximum velocity Substituting equation (4.2.2) in (4.2.1) yields $imw_0v_0 = F$ (4.2.3) If the particle moves in a resistive medium of coefficient γ the equation of motion becomes

$$m\frac{dv}{dt} = F - \gamma mv(4.2.4)$$

According to equation (4.2.3) the force affect the velocity amplitude and frequency. The force also changes the energy of the system.

Assuming that the frictional force affects the frequency only, one can assume

$$v = v_0 e^{iwt} (4.2.5)$$

If one treat the particle as a harmonic oscillator, the energies for w_0 , and w are given by:

$$E_0 = \hbar w_0, E = \hbar w(4.2.6)$$

This means that E is affected by the frequency only. This conforms to our assumption that F affect the frequency only as far as F affect E. Substituting equation (4.2.3) in equation (4.2.4) yields:

$$imwv_0 = imw_0v_0 - \gamma mv_0$$

Cancelling similar terms and multiplying both sides by *i* yields

$$w - w_0 = i\gamma = \frac{i}{\tau}(4.2.7)$$

Thus the energy loss is given by

$$\Delta E = \hbar w_0 - \hbar w = -i\gamma\hbar = \frac{-i\hbar}{\tau}(4.2.8)$$

Thus the new energy of the system affected by friction is given by:

$$E = E_0 + \Delta E = E_0 - i\gamma\hbar(4.2.9)$$

Thus the wave equation can be written

$$\Psi = A e^{\frac{i}{\hbar} [Px - (E_0 - i\hbar\gamma)t]}$$
$$\Psi = A e^{\frac{i}{\hbar} (Px - E_0 t)} e^{-\frac{\gamma\hbar t}{\hbar}} (4.2.10)$$
$$\Psi = A e^{-\gamma t} e^{\frac{i}{\hbar} (Px - E_0 t)} (4.2.11)$$

It is very striking to observe that the imaginary friction term in equations (4.2.7) and (4.2.8) appears in equation (4.2.11) to make the amplitude of Ψ decay with time.

Therefore the average energy \overline{E} which is equal total classical value, i.e $\overline{E} = \int \overline{\Psi} E \Psi dt = E e^{-2\gamma t} (4.2.12)$

Indicates that the energy decays with time .This agrees with the fact that friction causes particle energy to decrease

The relation time from uncertainty principle is given by

 $\Delta E \Delta t = \hbar (4.2.13)$

With

$$\Delta E = \frac{\hbar}{\Delta t} = \frac{\hbar}{\tau} (4.2.14)$$

Where $\Delta t = \tau$

It's very interesting to note that equations (4.2.8) and (4.2.14) give the same numerical values. But the expression (4..2.8) is more convenient, as far as it is insertion in the wave function predicts time decaying energy .This means that treating particles as harmonic oscillators gives quantum results similar to the classical one.

(4.3)Schrödinger Equation for Frictional Media:

The ordinary Schrödinger equation does not account for the effect of frictional force. For example consider two particles; both of them are affected by a field of potential V. Let the first one moves in free space, while the other move in a medium like water. Schrödinger equation for the two particles gives the same wave function, with the same average wave function, with the average momentum and energy. This situation is in direct conflict with experiments and common sense. This is due to the fact that the second particles have less momentum and energy because it loses them due to friction effect of the medium [50].

This means that one needs new Schrödinger equation accounting for the effect of friction. This can be done with the aid of equation (4.2.9), where

$$E = E_0 - i\gamma\hbar(4.3.1)$$

 E_0 is the energy when no friction exists. For classical system this energy is given by:

$$E_0 = \frac{P^2}{2m} + V \ (4.3.2)$$

Hence the complete expression of energy in the presence of friction is given by:

$$E = \frac{P^2}{2m} + V - i\gamma\hbar(4.3.3)$$

To derive the new Schrödinger equation multiplying both sides of equations (4.3.3) by Ψ to get:

$$E\Psi = \frac{P^2}{2m}\Psi + V\Psi - i\gamma\hbar(4.3.4)$$

For free particle the wave function is given by

$$\Psi = Ae^{i(kx-wt)} = Ae^{\frac{i}{\hbar}(Px-Et)}$$
(4.3.5)

Differentiating both sides with respect to time and coordinate gives:

$$\frac{\partial \Psi}{\partial t} = -\frac{i}{\hbar} Et$$
$$i\hbar \frac{\partial \Psi}{\partial t} = E\Psi(4.3.6)$$
$$\frac{\partial \Psi}{\partial x} = \frac{i}{\hbar} P\Psi$$
$$\frac{\partial^2 \Psi}{\partial x^2} = -\frac{P^2}{\hbar^2} \Psi$$

$$-\hbar^2 \frac{\partial^2 \Psi}{\partial x^2} = -\hbar^2 \nabla^2 \Psi = P^2 \Psi (4.3.7)$$

Inserting equations (4.3.6) and (4.3.5) in (4.3.4) yields:

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\Psi + V\Psi - i\gamma\hbar\Psi(4.3.8)$$

This equation is the Schrödinger equation for frictional medium.

(4.4) The Wave Function of Free Particle in Frictional

Medium:

For free particle the potential vanishes, i.e

$$V=0$$
 (4.4.1)

For ordinary Schrödinger equation the equation reduced to:

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\Psi(4.4.2)$$

The solution of this equation is given:

$$\Psi = Ae^{\frac{\iota}{\hbar}(Px-Et)}(4.4.3)$$

.

However for the new Schrödinger equation the equation reduced to:

$$-\frac{\hbar^2}{2m}\nabla^2\Psi - i\gamma\hbar\Psi = i\hbar\frac{\partial\Psi}{\partial t}(4.4.4)$$

This can be solved by assuming:

$$\Psi = Ae^{-i\beta t + ikx}(4.4.5)$$

Thus

$$\begin{cases} \nabla^2 \Psi = k^2 \Psi \\ \frac{\partial \Psi}{\partial t} = -i\beta \Psi \end{cases} (4.4.6)$$

Substituting (4.4.5) and (4.4.6) in (4.4.4) yields

$$\left[\frac{\hbar^2 k^2}{2m} - i\gamma\hbar\right]\Psi = \hbar\beta\Psi(4.4.7)$$

Since from free particle:

$$P = \hbar k$$

$$E = \frac{P^2}{2m}(4.4.8)$$

Therefore:

$$\hbar\beta = \frac{P^2}{2m} - i\gamma\hbar = E - i\gamma\hbar$$
$$\beta = \frac{E}{\hbar} - i\gamma(4.4.9)$$

Substituting (4.4.9) in (4.4.5) yields:

$$\Psi = Ae^{-i\gamma}e^{\frac{i}{\hbar}(Px-Et)}(4.4.10)$$

The average energy for this particle given to be:

$$\langle E \rangle = \int \overline{\Psi} \widehat{H} \Psi dr = \int \overline{\Psi} E \Psi dr = E \int \overline{\Psi} \Psi dr = E e^{-2\gamma t} \qquad (4.4.11)$$

(4.5)Quantum Radioactive Decay Law:

A simple expression for radioactive decay law can be obtained by using equation (4.4.10) .The number of particles is given according to equation (4.4.10):

$$N = \Psi \overline{\Psi} = A e^{-2\gamma t} (4.5.1)$$

Since at

$$t=0$$
 , $N=N_0$

Thus equation (4.5.1) gives:

 $N_0 = A$

Therefore equation (4.5.1) becomes:

$$N = N_0 e^{-2\gamma t} (4.5.2)$$

By setting:

$$2\gamma = \lambda$$

$$N = N_0 e^{-\lambda t} (4.5.3)$$

This is the ordinary radioactive decay law

The radiation emitted by unstable nuclei is due to the fact that these nuclei are in an excited state .This can be shown with the aid of equations (4.2.8), (4.2.14) and (4.5.2), where:

$$\Delta E = \frac{\hbar}{\tau} = \gamma \hbar (4.5.4)$$

Thus:

$$N = N_0 e^{-2\frac{\Delta E}{\hbar}t} (4.5.5)$$

This shown that nuclear decay is due to nuclear excitation .i.e. the existence of the nucleus in an excited state.

According to the classical harmonic oscillator model, equation (4.2.8) shows that the energy loss due to frictional force is shown to be related to the reciprocal of the relaxation time. The relaxation time here measures the delay in particle motion .It is very striking to find that typical expression for energy loss by excited particle is obtained by using quantum uncertainty principle according to equation (4.2.14) .Here again τ represents time taken by a particle in an excited state . If a photon is absorbed by a particle it become excited for τ seconds , then it return back to the ground state after re-emitting a photon with time delay τ seconds.

However the imaginary term in the classical expression (4.2.8) make it give a direct physical meaning of the role of friction in causing energy losses according to equation (4.2.12). It is also very interesting to note that the wave function resulting from the energy expression for resistive media in equation (4.2.11) can be utilized to derive a simple expression for the radioactive decay law.

Fortunately this mew expression shows that decay of particles results from nuclear excitation .This is since the original energy does not appear, while excitation energy appears in decay expression as shown by equation (4.5.5).

The harmonic oscillator model which is related to the string theory appears to be successful in describing the interaction of particles with bulk matter. The classical and quantum expression for the energy lost due to this interaction are numerically the same. This model succeeded in deriving very simple direct radioactive decay law.

(4.6)Nuclear Reaction on the Basis of Statistical distribution Based on Nuclear Potential:

. The nucleus is held by the forces which protect them from the enormous repulsion forces of the positive protons. It is a force with short range and not similar to the electromagnetic force. It is well known that the nucleus is consistingof protons and neutrons. These are formed from quarks which are held together with strong force. This strong force is residual color force. The basic exchange particle is called gluon which works as mediator forces between quarks. Both the particles; gluons and quarks are present in protons and neutrons. The range of force between particles is not determined by the mass of particles. Thus, the force which balanced the repulsion force between the positively charged particles protons is a nuclear attraction which overcomes the electric repulsion force.

Nuclear Force is defined as the force exerted between numbers of nucleons. This force is attractive in nature and binds protons and neutrons in the nucleus together. Since the protons are of same positive charge they exert a repulsive force among them.

Because of this attractive Nuclear Force, the total mass of the nucleus is less than the summation of masses of nucleons that is protons and neutrons. This force is highly attractive between nucleons at a distance of 10^{-15} m or 1 femtometer (fm) approximately from their centers. There are two types of nuclear forces, strong and weak nuclear force. Nuclear forces are independent of the charge of protons and neutrons. This property of nuclear force is called charge independence. It depends on the spins of the nucleons that is whether they are parallel

or no and also on the non-central or tensor component of nucleons.

The short range nuclear force field does not exist outside the nucleus. However the gravity and electric beside magnetic fields can distribute themselves around the nucleus affecting the surrounding electrons. In general the effect of gravity on electrons can be neglected compared to the electrostatic effects. But the gravitational field becomes important for some astronomical objects like neutrons stars. The gravity and electromagnetic fields manifests themselves as macroscopic potential, while nuclear short range field is a microscopic field.

In the statistical physics the role of macroscopic fields and their generation are not widely studied. Some attempts were made to accounts for the effect of potential on statistical distribution. But no detailed studies were made to use statistical laws to explain generation of macroscopic fields by the nucleus and neutron stars.

(4.6.1)Newtonian Statistical Distribution Laws for Particles in a Field:

According to Newton laws, the total energy E is given by:

$$E = \int E_n dP dV$$

For one particle the total energy takes the form:

$$E = \frac{P^2}{2m} + V(4.6.1)$$

Where: P is the momentum and Vis the potential ,thus for n particles the total energy is given by:

$$\mathbf{E} = \int \left(\frac{\mathbf{P}^2}{2\mathbf{m}} + \mathbf{V}\right) e^{-\beta \left[\left(\frac{P^2}{2m}\right) + V\right]} dP dV (4.6.2)$$

Thus the average energy is given by:

$$\langle E \rangle = average energy = \frac{\iint_{0}^{\infty} \left(\frac{P^{2}}{2m} + V\right) e^{-\beta \left(\frac{P^{2}}{2m} + V\right)} dP dV}{\iint_{0}^{\infty} e^{-\beta \left(\frac{P^{2}}{2m} + V\right)} dP dV} (4.6.3)$$

$$\langle E \rangle = \frac{\int_{0}^{\infty} V e^{-\beta V} dV}{\int_{0}^{\infty} e^{-\beta V} dV} + \frac{\int_{0}^{\infty} \frac{P^{2}}{2m} e^{-\beta \frac{P^{2}}{2m}} dP}{\int_{0}^{\infty} e^{-\beta \frac{P^{2}}{2m}} dP} = \frac{I_{1}}{I_{2}} + \frac{I_{3}}{I_{4}}$$

Taking the integral:

$$\int_{0}^{\infty} V e^{-\beta v} dV(4.6.4)$$

By integrating by parts:
$$u_{1} = V \rightarrow du_{1} = dV$$
$$dV_{1} = e^{-\beta v} dV \rightarrow V_{1} = -\frac{1}{\beta} e^{-\beta V}$$
$$\int_{0}^{\infty} u_{1} dV = u_{1}V_{1-} \int_{0}^{\infty} v_{1} du_{1}$$
Then:
$$I_{1} = \int_{0}^{\infty} V e^{-\beta V} dV = -\left[\frac{V}{\beta} e^{-\beta V}\right]_{0}^{\infty} - \int_{0}^{\infty} -\frac{1}{\beta} e^{-\beta V} dV$$
$$I_{1} = 0 + \frac{1}{\beta} \int e^{-\beta V} dV = \frac{1}{\beta} \left[-\frac{1}{\beta} e^{-\beta V}\right]_{0}^{\infty}$$

$$I_1 = \frac{-1}{\beta^2} [0 - 1] = \frac{1}{\beta^2} (4.6.5)$$

$$I_1 = \frac{1}{\beta^2} (4.6.6)$$

The second integral is given by:

$$I_{2} = \int_{0}^{\infty} e^{-\beta V} dV = -\left[\frac{1}{\beta}e^{-\beta V}\right]_{0}^{\infty}$$
$$I_{2} = \frac{-1}{\beta} [0 - 1] = \frac{1}{\beta} (4.6.7)$$

Thus from (4.6.7) and (4.6.6) one gets:

$$\frac{I_1}{I_2} = \frac{\int_0^\infty V e^{-\beta V} dV}{\int_0^\infty e^{-\beta V} dV} = \frac{\frac{1}{\beta^2}}{\frac{1}{\beta}} = \frac{\beta}{\beta^2} = \frac{1}{\beta}$$
$$\frac{I_1}{I_2} = \frac{\int_0^\infty v e^{-\beta v} dv}{\int_0^\infty e^{-\beta v} dv} = \frac{1}{\beta} (4.6.8)$$

The third integral is given by:

$$I_{3} = \int_{0}^{\infty} \frac{P^{2}}{2m} e^{-\beta \frac{P^{2}}{2m}} dP(4.6.9)$$

Let $x = \beta \frac{p^{2}}{2m} \rightarrow (\frac{2m}{\beta} x)^{-1/2} = P(4.6.10)$
 $\rightarrow dP = \frac{1}{2} (\frac{2m}{\beta} x)^{-1/2} (\frac{2m}{\beta}) dx = \frac{m}{\beta} (2m/P)^{-1/2} x^{-1/2} dx$
At:

$$P = 0 \to x = 0, P = \infty \to x = \infty$$
$$I_3 = \int_0^\infty \frac{x}{\beta} e^{-x} \frac{m}{\beta} (\frac{2m}{\beta})^{-1/2} x^{-1/2} dx$$
$$I_3 = \frac{m}{\beta^2} (\frac{2m}{\beta})^{-1/2} \int_0^\infty x^{-1/2} e^{-x} dx (4.6.11)$$

By using Gamma Function integrations:

$$[(n) = \int_0^\infty x^{n-1} e^{-x} dx (4.6.12)$$

$$n - 1 = 1/2 \to n = 3/2$$

$$\int_0^\infty x^{-1/2} e^{-x} dx = \left[\left(\frac{3}{2}\right) = \frac{1}{2} \left[\left(\frac{1}{2}\right) = \sqrt{\pi}/2 \right]$$
 (4.6.13)

Then

$$I_3 = \frac{m}{\beta^2} \left(\frac{2m}{\beta}\right)^{-1/2} \sqrt{\pi/2} (4.6.14)$$

The forth integral is also given by:

$$I_{4} = \int_{0}^{\infty} e^{-\beta \frac{P^{2}}{2m} dP = \frac{m}{\beta} \left(\frac{2m}{\beta}\right)^{-1/2}} \left[\left(\frac{1}{2}\right) I_{4} = \frac{m}{\beta} \left(\frac{2m}{\beta}\right)^{-1/2} \sqrt{\pi} (4.6.15) \right]$$

Then from (4.2.1.14) and (4.2.1.15):

$$\frac{I_3}{I_4} = \frac{\frac{m}{\beta^2} \left(\frac{2m}{\beta}\right)^{-1/2} \sqrt{\pi}}{\frac{m}{\beta} \left(\frac{2m}{\beta}\right)^{-1/2} \sqrt{\pi}} = \frac{1}{2\beta} (4.6.16)$$

Thus inserting equations (4.6.8) and (4. 6.6) in equation (4.6.3) yields: $\langle E \rangle = \frac{1}{\beta} + \frac{1}{2\beta} = \frac{2+1}{2\beta} = \frac{3}{2\beta} (4.6.17)$

According to liquid drop model, the nuclear can treated as consisting of a large number of small tiny particles like massive photons. Thus the use of statisticalphysics in describing its behavior is justifiable.

Thus if these particles that consciences the nucleus are re-distributed to be at infinity, then no field is observed. But when a work done to assemble and collect these tiny particles by bringing them from infinity the nucleus produces macroscopic gravity field, beside electrostatic field, with field strengths equal to E_g and E_e respectively. Thus the total macroscopic energy produced by the nucleus is:

$$E = \left(\varepsilon E_e^2 + \frac{1}{4\pi G} E_g^2\right) \left(\frac{4\pi}{3} R_0^3\right) (4.6.18)$$

Where E_g and E_e are the gravity and electric field strengths just outside the nucleus, But according to equation (4.6.17) to be:

$$E = \frac{3N}{2\beta} (4.6.19)$$

Where N are the number of particles forming the nucleus

Comparing equations (4.6.19) and (4.6.18):

$$\frac{3N}{2\beta} = \left(\varepsilon E_e^2 + \frac{1}{4\pi G} E_g^2\right) \left(\frac{4\pi}{3} R_0^3\right)$$
$$\beta = \frac{3}{2} \frac{N}{\left(\varepsilon E_e^2 + \frac{1}{4\pi G} E_g^2\right) \left(\frac{4\pi}{3} R_0^3\right)} (4.6.20)$$

This parameter is related to the macroscopic energy produced by nucleus. While the potential appearing in equations (4.6.1) and (4.6.3) is the microscopic potential which may have functional form and thus nature different from the macroscopic nucleus.

(4.6.2) Massive and Super Massive Astronomical Objects:

Consider a massive astronomical body like planets or stars or super massive neutron stars. For such objects the macroscopic field produced is the gravitational field. When any object is formed by assembling far tiny particles located at infinity to form this astronomical objects the work done to move them from infinity requires giving them a kinetic energy. A work is also done against the field existed. Thus the total energy given by equation (4.6.17) is transformed to gravitational energy given by equation (4.6.18), thus:

$$E = \frac{3N}{2\beta} = \frac{E_g^2 R^3}{3G} (4.6.21)$$

Where R is the radius of the star

(4.6.3) Statistical Laws Based on Generalized Special Relativity:

According to generalized special relativity:

$$E = mc^2(4.6.22)$$

The average energy is given by:

$$\langle E \rangle = \frac{\int_0^\infty E e^{-\beta E} dE}{\int_0^\infty e^{-\beta E} dE} (4.6.23)$$

$$\langle E \rangle = \frac{\int_0^\infty mc^2 e^{-\beta mc^2} dmc^2}{\int_0^\infty e^{-\beta mc^2} dmc^2} (4.6.24)$$

$$I_1 = \int mc^2 e^{-\beta mc^2} dmc^2 (4.6.25)$$

Let:

$$x = \beta m c^2 \rightarrow \frac{x}{\beta c^2} = m \rightarrow dm = \frac{dx}{\beta c^2}$$

Then:

$$\int \frac{x}{\beta} e^{-x} \frac{dx}{\beta} = \int \frac{x}{\beta^2} e^{-x} dx = \frac{1}{\beta^2} \int x e^{-x} dx$$

Use integration by parts let:

$$u = x \to du = dx, dv = e^{-x} dx \to v = -e^{-x}$$
$$\int xe^{-x} dx = -xe^{-x} + \int e^{-x} dx = -xe^{-x} - e^{-x}$$
$$\therefore I_1 = \frac{1}{\beta^2 c^2} [-xe^{-x} - e^{-x}] = \frac{1}{\beta^2 c^2} (4.6.26)$$

Let:

$$I_2 = \int e^{-\beta c^2 m} dm c^2$$

Let:

$$\begin{aligned} x &= \beta c^2 m \to m = \frac{x}{\beta c^2} \to dm = \frac{dx}{\beta c} \\ I_2 &= \int e^{-x} \frac{dx}{\beta c^2} = \frac{1}{\beta c^2} \int e^{-x} dx = \left[-\frac{e^{-x}}{\beta c^2} \right]_0^\infty = \frac{1}{\beta c^2} (4.6.27) \\ \langle E \rangle &= \frac{I_1}{I_2} = \frac{1}{\beta} (4.6.28) \\ \langle E \rangle &= \frac{\frac{1}{\beta^2} [-x e^{-x} - e^{-x}]_0^\infty}{\frac{1}{\beta} [e^{-x}]_0^\infty} = \frac{1}{\beta} \frac{[-(0+0) + (0+1)]}{1} \\ \langle E \rangle &= \frac{1}{\beta} (4.6.29) \end{aligned}$$

Where $\frac{1}{\beta}$ the energy per particle is constant

$$m_0 c^2 + kT + \frac{\varepsilon E^2}{n} (4.6.30)$$

For relativistic particles producing nucleus

$$E = N\langle E \rangle = \frac{N}{\beta} = N \left(m_0 c^2 + kT + \varepsilon E^2_e + \frac{E_g}{4\pi\theta} \right) \left(\frac{4\pi}{3} R_0^2 \right) (4.6.31)$$

(4.7) Discussion:

According to the classical harmonic oscillator model, equation (4.2.8) shows that the energy loss due to frictional force is shown to be related to the reciprocal of the relaxation time. The relaxation time here measures the delay in particle motion .It is very striking to find that typical expression for energy loss by excited particle is obtained by using quantum uncertainty principle according to equation (4.2.14). Here again τ represents time taken by a particle in an excited state. If a photon is absorbed by a particle it become excited for τ seconds , then it return back to the ground state after re-emitting a photon with time delay τ seconds.

However the imaginary term in the classical expression (4.2.8) make it give a direct physical meaning of the role of friction in causing energy losses according to equation (4.2.12). It is also very interesting to note that the wave function resulting from the energy expression for resistive media in equation (4.2.11) can be utilized to derive a simple expression for the radioactive decay law.

Fortunately this mew expression shows that decay of particles results from nuclear excitation .This is since the original energy does not appear, while excitation energy appears in decay expression as shown by equation (4.5.5). The ordinary Schrödinger equation cannot accounts for affected of frictional for particles moving in a resistive medium.

This is since the average energy, which is related to the classical energy, is given by:

$$\langle E \rangle = \int \overline{\Psi} i\hbar \frac{\partial \Psi}{\partial t} d\underline{r} = \int \overline{\Psi} E \Psi dt = E \int \overline{\Psi} \Psi d\underline{r} = E (4.7.1)$$

Is constant, this is in direct conflict with experiments which shows decrease of energy and momentum with time.

However the situation is different for the new Schrödinger equation (4.3.8) which has an imaginary term consisting of friction coefficient γ . This equation when solved for particle in a resistive medium shows time decaying energy as shown by equation (4.4.11). This result confirms with experiments which shows energy decrease with time.

The total energy and average energy of statistical systems consisting of particles having both kinetic as well as potential energy was derived as shown by equation (4.6.17) and (4.6.19).

The parameter β is related to the macroscopic energy as in the conventional statistical laws. In the case of nucleus the macroscopic energy includes gravity and electric fields produced by the nucleus as shown by equation (4.6.18). Thus β is given by (4.6.20). For Astronomical objects the β is related to the gravity field as shown by equation (2.6.21).when relativistic effects are taken in to account β includes also rest mass energy.

The new statistical law which incorporates potential energy beside kinetic one can be used to find new statistical laws that can describe the generation of macroscopic fields.

(4.8) Conclusion:

The new quantum and statistical models are promising, since they can easily describe radioactive decay phenomena. It also enables statistical laws to describe macroscopic electric and gravity field beside rest mass energy.

(4.9) Recommendation:

- 1. The new quantum model needs to be applied for neutron stars and black holes.
- 2. The scattering process this new quantum model should be also established.
- 3. The new statistical model can be used for stars evolution.

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