Dedication

To soul of my father, my beloved mother, brothers, sisters , wife and daughters .

Acknowledgements

At first I would to thank Allah who gives me the ability to complete this work.

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Abstract

We show the equivalence relations, distances between Hilbert frames, ellipsoidal tight frames completions with prescribed norms and projection decompositions of operators. We characterize the generalization of Gram–Schmidt orthogonalization generating all Parseval frames and verify the Schur-Horn theorem for operators and frames. We study the spectra of contractions belonging to spectral classes and the hyperinvariant subspace problem for asymptotically nonvanishing contractions, with invariant subspaces for power-bounded operator of class C_1. We discuss the equal-norm Parseval frames and constructing finite frames of a given spectrum and set of lengths. We show the shift-type invariant subspaces of contractions quasianalytic contractions, function algebras and the compression of quasianalytic spectral sets of cyclic contractions.

الخلاصـــــــــة

تم ايضاح العلاقات المكافئه واطارات هلبرت بين المسافات وتمامات الاطارات المحكمة الناقصية مع النظائم الموصوفة وتفكيكات الاسقاط للمؤثرات. شخصنا تعميم ناظمية تعامدية جرام – شميدت المولدة لكل اطارات بارسيفال وتحقق مبرهنة سشر – هورن للمؤثرات والاطارات. درسنا طيف الانكماشات المنتمية الي العائلات الطيفية ومسالة الفضاء الجزئ الفوقى اللامتغيرلاجل الانكماشات غير المتلاشية التقاربية مع الفضاءات الجزئية اللامتغيرة لاجل مؤثر القوى – المحدود للعائلة C_1. تمت دراسة تساوى –نظيم اطارات بارسيفال والاطارات المنتهية البناء لطيف معطى وفئة الاطوال. اوضحنا الفضاءات الجزئية اللامتغيرة نوع – الازاحة للانكماشات والانكماشات شبه التحليلة وجبريات الدالة وانضغاط للفئات الطيفية شبه التحليلية للانكماشات الدوارة .

Introduction

We study some equivalency relations between Hilbert frames and closed subspaces of L^2 (1). We define also a distance between frames and we establish the geometric meaning of this metric. We show the existence of tight frames whose elements lie on an arbitrary ellipsoidal surface within a real or complex separable Hilbert space $\mathcal{H}$, and we analyze the set of attainable frame bounds. In the case where $\mathcal{H}$ is real and has finite dimension, we give an algorithmic proof.

Given an arbitrary finite sequence of vectors in a finite–dimensional Hilbert space, we describe an algorithm, which computes a Parseval frame for the subspace generated by the input vectors while preserving redundancy exactly.
We compute the minimum $r \in N \cup \{\infty\}$ with this property. Using recent results on the Schur-Horn theorem, we also obtain a not so optimal but algorithmic computable (in a finite numbers of steps) tight completion sequence $\mathcal{G}$.

Sz.-Nagy and Foia? classified the contractions according to the asymptotic behaviour of their iterates. We obtain new information on the structure of contractions of calss $C_{1,}$ and to develop new ways for obtaining hyperinvariant subspaces for these operators.

Connections with the questions of convergence of T^n to 0 in the strong operator topology and of cyclicity of power-bounded operators of class C_1 are discussed**.** We introduce a new equivalence relation, ampliation quasisimilarity, onLðHÞ; more general than quasisimilarity, that preserves the existence of nontrivial hyperinvariant subspaces.

We relate the existence of frames with the Schur-Horn theorem of majorization, and give a reformulation of the extended version of Schur-Horn theorem, due to A. Neumann. We use this to get necessary conditions (and to generalize known sufficient conditions) for a pair (S, c) to be frame admissible. The construction of equal-norm Parseval frames is fundamental for many applications of frame theory.We present a construction method based on a system of ordinary differential equations, which generates a flow on the set of Parseval frames that converges to equal-norm Parseval frames.We developed this method to address a question posed by Vern Paulsen: How close is a nearly equal-norm, nearly Parseval frame to an equal-norm Parseval frame? The distance estimate derived here can be used to substantiate numerically found, approximate constructions of equal-norm Parseval frames. When constructing finite frames for a given application, the most important consideration is the spectrum of the frame operator.

Using the Sz.-Nagy–Foias functional model it was shown that under certain conditions on a contraction T the natural embedding of a Hardy space of vector-valued functions into the corresponding$\mathcal{L}_2$ space can be factored into the product of two transformations, intertwining T with a unilateral shift and with an absolutely continuous unitary operator, respectively. Completing former results the effect of the

Sz.-Nagy–Foias functional calculus on the unitary asymptote of a contraction is described. The hyperinvariant subspace problem for a class of cyclic, quasianalytic $C10$-contractions is reduced to the particular case, when the quasianalytic spectral set coincides with the unit circle T.The class $\mathcal{L}_0(\mathcal{H})$ of cyclic quasianalytic contractions was studied in Kérchy. The subclass$\mathcal{L}_1(\mathcal{H})$ consists of those operators T in $\mathcal{L}_0(\mathcal{H})$ whose quasianalytic spectral set $\pi(T)$ covers the unit circle $\mathbb{T}$. The contractions in $\mathcal{L}_1(\mathcal{H})$ have rich invariant subspace lattices.

The Contents