Dedication

To my parents,

husband,

children,

and family.

Acknowledgment

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Abstract

We show the operators with singular continuous spectrum : ranke one operators and perturbations, Hausdorff dimensions and localization. We study the rank one perturbations of self-adjoint, diagonal and normal operators with generalization of projection constants. Minimal volume shadows and projections of cubes and totally unimodular matrices are considered. We discuss the characteristic functions for infinite sequences of non-commuting operators and joint invariant subspaces. We show the sufficient enlargements of minimal volume for projections and infinite dimensional normed linear spaces.

الخلاصة

أوضحنا المؤثرات مع الطيف المستمر الشاذ: مؤثرات الرتبة الواحدة والإرتجاجات وأبعاد هاوسدورف والموضعية. تم دراسة إرتجاجات الرتبة الواحدة لمؤثرات المرافق-الذاتي والقطري والناظم مع تعميم ثوابت الاسقاط. إعتبرنا ظلال الحجم الأصغري والإسقاطات للمكعبات والمصفوفات آحادية القيمة الكلية . تم دراسة الدوال المميزة للمتتاليات اللانهائية للمؤثرات غير التبديلية والفضاءات الجزئية اللامتغيرة معاً. تم توضيح التوسيعات الكافية للحجم الأصغري للأسقاطات والفضاءات الخطية المنتظمة ذات البعد اللانهائي

Introduction

For an operator, A, with cyclic vector φ, we study $A + \lambda P$, where P is the rank one projection onto multiples of φ. Let A be a selfadjoint operator in a Hilbert space $\boldsymbol{H}$. Its rank one perturbations $A + \tau(\cdot, \omega)\omega$, $\tau \in \boldsymbol{R}$, are studied when ω belongs to the scale space $\boldsymbol{H}_{-2}$ associated with $\boldsymbol{H}_{+2} = \mathrm{dom}\, A$ and $(\cdot,\cdot)$ is the corresponding duality. If A is nonnegative and ω belongs to the scale space $\boldsymbol{H}_{-1}$, Gesztesy and Simon prove that the spectral measures of $A(\tau), \tau \in \boldsymbol{R}$, converge weakly to the spectral measure of the limiting perturbation $A(\infty)$. In fact $A(\infty)$ can be identified as a Friedrichs extension. Further results for nonnegative operators A were obtained by Kiselev and Simon by allowing $\omega \in \boldsymbol{H}_{-2}$.

Let A be a selfadjoint operator in a Hilbert space $\boldsymbol{H}$ with inner product $[\cdot,\cdot]$. The rank one perturbations of A have the form $A + \tau[\cdot, \omega]\omega$, $\tau \in \boldsymbol{R}$, for some elements $\omega \in \boldsymbol{H}$. The main goal is to look at the singular spectrum produced by rank one perturbations from this point of view.

It is very natural to consider the following question: How small can be made the set $P\big(B(X)\big)$ under a proper choice of P? We study the shape of minimal-volume shadows of a cube in a given subspace. First we show an essentially known result that for every subspace L the set of minimal-volume shadows in L contains a parallelepiped.

We study rank-one perturbations of diagonal Hilbert space operators mainly from the standpoint of invariant subspace problem. In addition to showing some general properties of these operators, we identify the normal operators and contractions in this class. We concerned with operators on Hilbert space of the form $T = D + u \otimes v$ where D is a diagonalizable normal operator and $u \otimes v$ is a rank-one operator.

We deal with the "characteristic function" of an infinite sequence $\boldsymbol{T} = \{T_n\}_{n=1}^{\infty}$ of noncommuting operators on a Hilbert space H, when the matrix $[T_1, T_2, \dots]$ is a contraction. In connexion with this, we extend to the setting the results for two operators and many of the results for one operator. Let $T \coloneqq [T_1, \dots, T_n]$ be an n-tuple of operators on a Hilbert space such that T is a completely non-coisometric row contraction. We establish the existence of a “one-to-one” correspondence between the joint invariant subspaces under $T_1, \dots, T_n$, and the regular factorizations of the characteristic function Θ_T associated with T. In particular, we show that there is a non-trivial joint invariant subspace under the operators $T_1, \dots, T_n$, if and only if there is a non-trivial regular factorization of Θ_T. We also provide a functional model for the joint invariant subspaces in terms of the regular factorizations of the characteristic function, and show the existence of joint invariant subspaces for certain classes of n-tuples of operators.

The main purpose is to continue investigation of sufficient enlargements started in the before cited. Among all linear projections onto a given linear subspace L in $\boldsymbol{R}^n$ we select those that minimize the volume of the image of the cube $\{x: |x_i| \leq 1\}$. Let B_Y denote the unit ball of a normed linear space Y. A symmetric, bounded, closed, convex set A in a finite-dimensional normed linear space X is called a sufficient enlargement for X if, for an arbitrary isometric embedding of X into a Banach space Y , there exists a linear projection $P: Y \to X$ such that $P(B_Y) \subset A$.

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