

# *Dedication*

*To my parents,*

*Wife,*

*Children,*

*And family.*

## *Acknowledgments*

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## Abstract

We study the Lyapunov inequalities for the time scales, for discrete linear Hamiltonian systems on the time scales. We show the strong instability and stability standing waves for nonlinear Klein – Gordon equations and Klein – Gordon- Zakharov system. We also show the stability and instability of standing waves for one- dimensional nonlinear Schrodinger equation with multiple – power nonlinearity and stability of bound states of Hamiltonian partial differential equations in the degenerated cases. The moment identities for Skorohod integrals, random Hermite polynomials with Girsanov identities on the Wiener space are characterized. The transfer principle method from Wiener to Poisson space and measure invariance on the Lie- Wiener path space are discussed. We show the Laplace transform identities and describe the measure – preserving transformations on the Lie- Wiener – Poisson spaces.

## الخلاصة

تمت دراسة متباينات ليايينوف لتدرجات الزمن لأنظمة هملتونيان الخطية المتقطعة على تدرجات الزمن. أوضحنا عدم الاستقرار القوي، والموجات القائمة المستقرة لمعادلات كلين - جوردون غير الخطية ونظام كلين - جوردون - زاكاروف. أيضاً أوضحنا الاستقرار وعدم الاستقرار للموجات القائمة لمعادلة شرودينجر غير الخطية ذات الرتبة الأولى مع مضاعفة - عدم خطية القوة واستقرارية الحالات الحدية للمعادلات التفاضلية الجزئية لهملتونيان في حالات تفكيك التوليد. تم تشخيص متطابقات العزم لتكاملات سكوروهود وكثيرات حدود هيرمايت العشوائية مع متطابقات جيرسونوف على فضاء واينر. تمت دراسة طريقة مبدأ التحويل من فضاء واينر الى فضاء بويسون وعدم تباين القياس على فضاء مسار لي - واينر. أوضحنا متطابقات تحويل لابلاس ووصفنا قياس التحويلات الحافظة على فضاءات لي - واينر - بويسون.

## Introduction

The theory of time scales has been introduced and characterized in order to unify discrete and continuous analysis. We present a Lyapunov inequality for Sturm-Liouville dynamic equations of second order on such time scales. We also extend the presented material to the case of a general linear Hamiltonian dynamic system on time scales. Applying these inequalities, we obtain some disconjugacy and stability criteria for discrete Hamiltonian systems.

We establish a sharp instability theorem for the bound states of lowest energy of the nonlinear Klein-Gordon equation,  $u_{tt} - \Delta u + f(u) = 0$ , and the nonlinear Schrödinger equation,  $-iu_t - \Delta u + f(u) = 0$ . We show a moment identity on the Wiener space that extends the Skorohod isometry to arbitrary powers of the Skorohod integral on the Wiener space. As simple consequences of this identity we obtain sufficient conditions for the Gaussianity of the law of the Skorohod integral and a recurrence relation for the moments of second order Wiener integrals.

We derive a formula for the expectation of random Hermite polynomials in Skorohod integrals, extending classical results in the adapted case. We establish several new Lyapunov type inequalities for linear Hamiltonian systems on an arbitrary time scale when the end-points are not necessarily usual zeroes, but rather, generalized zeroes, which generalize and improve all related existing ones including the continuous, when the time scale is a real set, and discrete cases, when the time scale is a set a finless.

The orbital instability of ground state standing waves  $e^{i\omega t} \phi_\omega(x)$  for the nonlinear Klein-Gordon equation has been known in

the domain of all frequencies  $\omega$  for the supercritical case and for frequencies strictly less than a critical sequence of frequency  $\omega_c$  in the subcritical case. We show the strong instability of ground state standing waves for the entire domain above. We apply our theorem to the single power nonlinear Klein-Gordon equation and the double power nonlinear Schrödinger equation.

We construct the stochastic calculus of variations on Poisson space and some of its applications via the stochastic analysis on Wiener space. We define a new gradient operator on Wiener space, whose adjoint extends the Poisson stochastic integral. We extend some recent results on moment identities, Hermite polynomials, and measure invariance properties on the Wiener space, to the setting of path spaces over Lie groups.

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