

List of Symbol

Symbol		Page
L^p	Lebesgue Space	1
\oplus	Orthogonal sum	1
Var	Total Variation	2
BV	Bounded Variation	2
Min	mimimum	4
Sup	Supremum	6
arg	argumenl	9
L^1	Lebesgue on the real line	17
L^2	Hilbert Space	24
Sgn	Sgnature	28
max	maximum	32
Re	real	33
BIP	bounded imaginary power	33
Loc	local	48
a.e	almost every where	49
$W^{1,p}$	Sobolev Space	53
l^2	Hilbert Space	61
Ker	kernel	81
SVEP	single-valued extension property	85
ind	index	86
rng	range	99
tr	trace	104
VI	variation inequality	124
GVI	General variation inequality	151
Im	imaginary	163
det	determinal	174
dim	dimension	178
col	closure	178
mes	measure	179

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