

## REFERENCES

- [1] S. C. Brenner and L. R. Scott, The mathematical theory of finite element methods, Springer, New York, 1994. MR 95f:65001
- [2] R. Brownlee and W. Light, Approximation orders for interpolation by surface splines to rough functions, IMA J.Numer. Anal., 24(2004), 179–192.
- [3] C. de Boor, R. A. DeVore, and A. Ron, Approximation from shift-invariant subspaces of  $L^2(\mathbb{R}^d)$ , Trans. Amer. Math. Soc., 341(1994), 787–806. MR 94d:41028
- [4] R. A. DeVore and R. C. Sharpley, Besov spaces on domains in  $\mathbb{R}^d$ , Trans. Amer. Math. Soc., 335(1993), 843–864. MR 93d:46051
- [5] J. Duchon, Sur l'erreur d'interpolation des fonctions de plusieurs variables par les Dmsplines, Rev. Fran,caise Automat. Informat.Rech.Oper.Anal. Numer., 12(1978), 325–334. MR 80j:41052
- [6] W.R.Madych and S.A.Nelson, Multivariate interpolation and conditionally positive definite functions, Approximation Theory Appl., 4(1988), 77–89. MR 90e:41006
- [7] , Multivariate interpolation and conditionally positive definite functions II, Math.Comp., 54(1990), 211–230. MR 90e:41007
- [8] W. R. Madych and E. H. Potter, An estimate for multivariate interpolation, J. Approx.Theory, 43(1985), 132–139. MR 86g:65022
- [9] C. A. Micchelli, Interpolation of scattered data: Distance matrices and conditionally positive definite functions, Constr. Approx., 2(1986), 11–22. MR 88d:65016
- [10] F. J. Narcowich and J. D. Ward, Scattered-data interpolation on  $\mathbb{R}^n$ : Error estimates for radial basis and band-limited functions, SIAM J. Math. Anal., to appear. License or copyright restrictions may apply to redistribution; see <http://www.ams.org/journal-terms-of-use> SOBOLEV BOUNDS ON FUNCTIONS WITH SCATTERED ZEROS AND RBFS 763
- [11] F. J. Narcowich, J. D. Ward, and H. Wendland, Refined error estimates for radial basis function interpolation, Constr. Approx., 19(2003), 541–564.
- [12] A. Ron, The  $L^2$ -approximation orders of principal shift-invariant spaces generated by a radial basis function, Numerical Methods in Approximation Theory. Vol. 9: Proceedings of the conference held in Oberwolfach, Germany, November 24–30, 1991 (Basel) (D. Braess et al., eds.), Int. Ser. Numer.Math., vol. 105, Birkh" auser, 1992, pp. 245–268. MR 95d:41035
- [13] R. Schaback, Approximation by radial basis functions with finitely many centers, Constr. Approx., 12(1996), 331–340. MR 97d:41013

- [14] E. M. Stein, *Singular Integrals and Differentiability Properties of Functions*, Princeton University Press, Princeton, New Jersey, 1971. MR 44:7280
- [15] H. Wendland, Piecewise polynomial, positive definite and compactly supported radial functions of minimal degree, *Adv. Comp. Math.*, 4(1995), 389–396. MR 96h:41025
- [16] , Meshless Galerkin methods using radial basis functions, *Math.Comp.*, 68(1999), 1521–1531. MR 99m:65221
- [17] Local polynomial reproduction and moving least squares approximation, *IMAJ. Numer. Anal.*, 21(2001), 285–300. MR 2002a:65025 .
- [18] F.J. Narcowich, J.D. Ward, H. Wendland, Sobolev bounds on functions with scattered zeros, with applications to radial basis function surface fitting, *Math. Comp.* 74 (2005) 743–763.
- [19] R. Arcangéli, M.C. L’opez de Silanes, J.J. Torrens, An extension of a bound for functions in Sobolev spaces, with applications to  $(m, s)$ -spline interpolation and smoothing, *Numer. Math.* 107 (2) (2007) 181–211.
- [20] Adams, R.A.: *Sobolev Spaces*. Academic, New York (1975)
- [21] Adams, R.A., Fournier, J.J.F.: *Sobolev Spaces*, 2nd edn. Academic, New York (2003)
- [22] Arcangéli, R., López de Silanes, M.C., Torrens, J.J.: *Multidimensional Minimizing Splines. Theory and Applications*. Grenoble Science. Kluwer Academic Publishers, Boston (2004)
- [23] Arcangéli, R., Ycart, B.: Almost sure convergence of smoothing  $D^m$ -splines for noisy data. *Numer. Math.* 66, 281–294 (1993)
- [24] Bezhaev, A.Y., Vasilenko, V.A.: *Variational Theory of Splines*. Kluwer Academic/Plenum Publishers, New York (2001)
- [25] Bouleau, N.: *Probabilités de l’Ingénieur*. Hermann, Paris (1986)
- [26] Ciarlet, P.G.: *The Finite Element Method for Elliptic Problems*, *Classics in Applied Mathematics*, vol. 40. SIAM, Philadelphia (2002). Firstly published by North-Holland, Amsterdam (1978)
- [27] Cox, D.: Multivariate smoothing spline functions. *SIAM J. Numer. Anal.* 21(4), 789–813 (1984)
- [28] Craven, P., Wahba, G.: Smoothing noisy data with spline functions. *Numer. Math.* 31, 377–403 (1979)

- [29] Duchon, J.: Splines minimizing rotation-invariant semi-norms in Sobolev spaces. Lecture Notes Math.571, 85–100 (1977)
- [30] Duchon, J.: Sur l’erreur d’interpolation des fonctions de plusieurs variables par les  $D^m$ -splines. RAIRO Anal. Numer.12(4), 325–334 (1978)
- [31] Grisvard, P.: Elliptic Problems in Nonsmooth Domains. Pitman, Boston (1985)
- [32] Johnson, M.J.: An error analysis for radial basis function interpolation. Numer. Math.98, 675– 694 (2004)
- [33] Johnson, M.J.: The  $L_p$  –approximation order of surface spline interpolation for  $1 \leq p \leq 2$ . Constr. Approx.20, 303–324 (2004)
- [34] Le Gia, Q.T., Narcowich, F.J., Ward, J.D., Wendland, H.: Continuous and discrete least-squares approximation by radial basis functions on spheres. J. Approx. Theory143, 124–133 (2006)
- [35] Light, W., Wayne, H.: On power functions and error estimates for radial basis function interpolation. J. Approx. Theory92, 245–266 (1998)
- [36] López de Silanes, M.C., Arcangéli, R.: Estimations de l’erreur d’approximation par splines d’interpolation et d’ajustement d’ordre(m,s). Numer.Math. 56, 449–467 (1989)
- [37] López de Silanes, M.C., Arcangéli, R.: Sur la convergence des  $D^m$ -splines d’ajustement pour des données exactes ou bruitées. Rev. Mat. Univ. Complut.Madrid4(2–3), 279–294 (1991)
- [38] Madych, W.R.: An estimate for multivariate interpolation II. J. Approx. Theory142, 116–128 (2006)
- [39] Madych, W.R., Potter, E.H.: An estimate for multivariate interpolation. J. Approx. Theory43, 132–139 (1985)
- [40] Narcowich, F.J., Ward, J.D., Wendland, H.: Sobolev bounds on functions with scattered zeros, with applications to radial basis function surface fitting. Math. Comp.74, 743–763 (2005)
- [41] Narcowich, F.J., Ward, J.D., Wendland, H.: Sobolev error estimates and a Bernstein inequality for scattered data interpolation via radial basis functions. Constr. Approx.24, 175–186 (2006)
- [42] Nečas, J.: Les Méthodes Directes en Théorie des Équations Elliptiques. Masson, Paris (1967)
- [43] Ragozin, D.: Error bounds for derivative estimates based on spline smoothing of exact or noisy data. J. Approx. Theory37, 335–355 (1983)

- [44] Sanchez, A.M.: Sur l'estimation des erreurs d'approximation et md'interpolation polynomiales dans lesespaces de Sobolev d'ordre non entier. Thèse de 3e cycle, Université de Pau et des Pays de l'Adour(1984)
- [45] Sanchez, A.M., Arcangéli, R.: Estimations des erreurs de meilleure approximation polynomiale et d'interpolation de Lagrange dans les espaces de Sobolev d'ordre non entier. Numer. Math.45, 301–321 (1984)
- [46] Stein, E.: Singular Integrals and Differentiability Properties of Functions. Princenton University Press, Princenton, New Jersey (1970)
- [47] Strang, G.: Approximation in the Finite Element Method. Numer. Math.19, 81–98 (1972)
- [48] Utreras, F.: Convergence rates for multivariate smoothing spline functions. J. Approx. Theory52, 1–27 (1988)
- [49] Wahba, G.: Smoothing noisy data by spline functions. Numer. Math.24, 383–393 (1975)
- [50] Wendland, H., Rieger, C.: Approximate interpolation with applications to selecting smoothing parameters. Numer. Math.101, 643–662 (2005)
- [51] Wu, Z.M., Schaback, R.: Local error estimates for radial basis function interpolation to scattered data. IMA J. Numer. Anal.13, 13–27 (1993) 123
- [52] T.M.Flett, The dual of an inequality of Hardy and Littlewood and some related inequalities, J. Math. Anal. Appl. 38(1972) 756–765.
- [53] T. hl. FLETT, A note on some inequalities, Proc. Glasgow Math. Assoc. 4 (1958), 7-15.
- [54] T. M. FLETT, Mean values of power series, Pucijic J. Math.25 (1968), 463-494.
- [55] T. M. FLETT, Inequalities for the  $p$ -th mean values of harmonic and subharmonic functions with  $p \leq 1$ , Proc. London Math. Sot. (3) 20 (1970), 249-275.
- [56] T. M. FLETT, "A theorem concerning the real part of a power series," Mathematical essays dedicated to A. J. MacIntyre (Ed. H. Shankar), Ohio, 1970, 135-143.
- [57] T. M. FLETT, On the rate of growth of mean values of holomorphic and harmonic functions, PYOC. London Math. Sot. (3) 20 (1970), 749-768.

- [58] T. M. FLETT, Temperatures, Bessel potentials, and Lipschitz spaces, Proc. London Math. Soc. (3) 22 (1971), 385-451.
- [59] A. E. GWILLIAM, On Lipschitz conditions, Proc. London Math. Soc. (2) 40 (1936), 353-364.
- [60] G. H. HARDY AND J. E. LITTLEWOOD, Some new properties of Fourier constants, Math. Ann. 97 (1926), 159-209.
- [61] G. H. HARDY AND J. E. LITTLEWOOD, Elementary theorems concerning power series with positive coefficients and moment constants of positive functions, J. Math. 157 (1927), 141-158.
- [62] G. H. HARDY AND J. E. LITTLEWOOD, A convergence criterion for Fourier series, Math. Z. 28 (1928), 612-634.
- [63] G. H. HARDY AND J. E. LITTLEWOOD, Some properties of conjugate functions, J. Math. 167 (1931), 405-423.
- [64] G. H. HARDY AND J. E. LITTLEWOOD, Some properties of fractional integrals. II, Math. Z. 34 (1932), 403-439.
- [65] G. H. HARDY AND J. E. LITTLEWOOD, Theorems concerning mean values of analytic or harmonic functions, Quart. J. Math. 12 (1941), 221-256.
- [66] J. E. LITTLEWOOD, "Lectures on the theory of functions," Oxford, 1944.
- [67] H. P. MULHOLLAND, Some theorems on Dirichlet series with positive coefficients and related integrals, Proc. London Math. Soc. (2) 29 (1929), 281-292.
- [68] M. H. ROSENBLUM, On the theory of Lipschitz spaces of distributions on Euclidean  $n$ -space. I. Principal properties; II. Translation invariant operators, duality and interpolation; III. Smoothness and integrability of Fourier transforms, smoothness of convolution kernels, J. Math. Mech. 13 (1964), 407-479; 14 (1965), 821-839; 15 (1966), 973-981.
- [69] D. H. LUECKING, Embedding derivatives of Hardy spaces into Lebesgue spaces, Proc. Lond. Math. Soc. 63 (1991) 565-619.
- [70] A. BENEDEK and R. PANZONE, 'The spaces  $L^p$  with mixed norm', Duke Math. J. 28 (1961) 301-324.
- [71] L. CARLESON, 'Interpolation by bounded analytic functions and the corona problem', Ann. of Math. 76 (1962) 547-559.
- [72] R. R. COIFMAN, Y. MEYER, and E. M. STEIN, 'Some new function spaces and their applications to harmonic analysis', J. Funct. Anal. 62 (1985) 304-335.

- [73] R.R.COIFMAN and G.WEISS, 'Extension of Hardy spaces and their use in analysis', *Bull. Amer. Math. Soc.* 83 (1977) 569-645.
- [74] P.DUREN, 'Extension of a theorem of Carleson', *Bull. Amer. Math. Soc.* 75 (1969) 143-146.
- [75] C.FEFFERMAN and E.M.STEIN, 'Some maximal inequalities', *Amer. J. Math.* 93 (1971) 107-115.
- [76] C.FEFFERMAN and E.M.STEIN, ' $H^p$  spaces of several variables', *Acta Math.* 129 (1972) 137-193.
- [77] E.HARBOURE, J.TORREA, and B.VIVIANI, 'A vector-valued approach to tent spaces', *Anal. Math.*, to appear.
- [78] J.-P.KAHANE, 'Some random series of functions' (D.C.Heath, Lexington, Mass., 1968).
- [79] D.LUECKING, 'Forward and reverse Carleson inequalities for functions in the Bergman spaces and their derivatives', *Amer. J. Math.* 107 (1985) 85-111.
- [80] D.LUECKING, 'Representation and duality in weighted spaces of analytic functions', *Indiana Univ. Math. J.* 34 (1985) 319-336. Erratum, 35 (1986) 927-928.
- [81] D.LUECKING, 'Dominating measures for spaces of analytic functions', *Illinois J. Math.* 32 (1988) 23-39.
- [82] R.ROCHBERG and S.SEMMES, 'A decomposition theorem for BMO and applications', *J. Funct. Anal.* 67 (1986) 228-263.
- [83] N.A.SHIROKOV, 'Some generalizations of the Littlewood-Paley Theorem', *Zap. Nauch. Sem. LOMI* 39 (1974) 162-175; *Soviet Math.* 8 (1977) 119-129.
- [84] N.A.SHIROKOV, 'Some embedding theorems for spaces of harmonic functions', *Zap. Nauch. Sem. LOMI* 56 (1976) 191-194; *J. Soviet Math.* 14 (1980) 1173-1176.
- [85] E.M.STEIN, 'Singular integrals and differentiability properties of functions' (Princeton University Press, 1970).
- [86] I.V.VIDENSKII, 'On an analogue of Carleson measures', *Dokl. Akad. Nauk SSSR* 298 (1988) 1042-1047; *Soviet Math. Dokl.* 37 (1988) 186-190.
- [87] T. Dupont and R. Scott, 'Polynomial approximation of functions in Sobolev spaces', *Math. Comp.*, 34 (1980), pp. 441-463.

- [88] S. AGMON, Lectures on Elliptic Boundary Value Problems, Van Nostrand, Princeton, N. J., 1965.
- [89] R. ARCANGELI & J. L. GOUT, "Sur l'évaluation de l'erreur d'interpolation de Lagrange dans un ouvert de  $R^n$ ," RAIRO Analyse Numérique, v. 10, 1976, pp. 5-27.
- [90] A. BERGER, R. SCOTT & G. STRANG, "Approximate boundary conditions in the finite element method," Symposia Mathematica X, Academic Press, New York, 1972, pp. 295-313.
- [91] J. H. BRAMBLE & S. R. HILBERT, "Estimation of linear functionals on Sobolev spaces with applications to Fourier transforms and spline interpolation," SIAM J. Numer. Anal., v. 7, 1970, pp. 112-124.
- [92] J. H. BRAMBLE & S. R. HILBERT, "Bounds for a class of linear functionals with applications to Hermite interpolation," Numer. Math., v. 16, 1971, pp. 362-369.
- [93] V. I. BURENKOV, "Sobolev's integral representation and Taylor's formula," Trudy Mat. Inst. Steklov., v. 131, 1974, pp. 33-38.
- [94] P. G. CIARLET & C. WAGSCHAL, "Multipoint Taylor formulas and applications to the finite element method," Numer. Math., v. 17, 1971, pp. 84-100.
- [95] J. DOUGLAS, JR., T. DUPONT, P. PERCELL & R. SCOTT, "A family of  $C_1$  finite elements with optimal approximation properties for various Galerkin methods for 2nd and 4th order problems," RAIRO Analyse Numérique, v. 13, 1979, pp. 227-255.
- [96] T. DUPONT & R. SCOTT, "Constructive polynomial approximation in Sobolev spaces," Recent Advances in Numerical Analysis (C. de Boor and G. Golub, Eds.), Academic Press, New York, 1978.
- [97] R. E. EWING, "Alternating direction Galerkin methods for some third and fourth order equations." (To appear.)
- [98] A. FRIEDMAN, Partial Differential Equations, Holt, Rinehart, and Winston, New York, 1969.
- [99] P. GRISVARD, "Behavior of solutions of an elliptic boundary value problem in poly-gonal or polyhedral domains," Numerical Solution of Partial Differential Equations-III (Synspade 1975) (B. Hubbard, Ed.), Academic Press, New York, 1976, pp. 207-274.
- [100] L. HÖRMANDER, Linear Partial Differential Operators, Springer-Verlag, Berlin, 1963.

- [101] P. JAMET, "Estimation de l'erreur d'interpolation dans un domaine variable et application aux éléments finis quadrilatéraux dégénérés," *Méthodes Numérique en Mathématiques Appliquées*, Presses de l'Université de Montréal, 1977, pp. 55 — 100.
- [102] J. MEINGUET, "Structure et estimations de coefficients d'erreurs," *RAIRO Analyse Numérique*, v. 11, 1977, pp. 355-368.
- [103] C. B. MORREY, *Multiple Integrals in the Calculus of Variations*, Springer-Verlag, Berlin, 1966.
- [104] R. SCOTT, "Interpolated boundary conditions in the finite element method," *SIAM J. Numer. Anal.*, v. 12, 1975, pp. 404-427.
- [105] S. L. SOBOLEV, *Applications of Functional Analysis in Mathematical Physics*, Transl. Math. Monographs, Vol. 7, Amer. Math. Soc, Providence, R. I., 1963.
- [106] E. M. STEIN, *Singular Integrals and Differentiability Properties of Functions*, Princeton Univ. Press, Princeton, N. J., 1970.
- [107] B. L. VAN DER WAERDEN, *Modern Algebra H*, 2nd ed., Ungar, New York, 1950.
- [108] A. ZYGMUND, *Trigonometric Series (2 volumes)*, 2nd ed., Cambridge Univ. Press, London, 1959.
- [109] R.A.Devore, V.A.Popov, *Transaction American Math* vol.305,no.1,1988
- [110] C. de Boor, The quasi-interpolant as a tool in elementary polynomial spline theory, *Approximation Theory (G. G. Lorentz, ed.)*, Academic Press, New York, 1973, pp. 269-276.
- [111] C. de Boor and G. F. Fix, Spline approximation by quasi-interpolants, *J. Approx. Theory* 8(1973), 19-45.
- [112] Yu. Brudnyi, Approximation of functions of  $n$ -variables by quasi-polynomials, *Math. USSR Izv.* 4 (1970), 568-586.
- [113] P.L.Butzer, H.Berens, *Sem-groups of operators and approximation*, Springer-Verlag, New York, 1
- [114] Z. Ciesielski, Constructive function theory and spline systems, *Studia Math.* 52(1973), 277-302.
- [115] R. DeVore, Degree of approximation, *Approximation Theory, II (G. G. Lorentz, C. K. Chui, L. L. Schumaker, eds.)*, Academic Press, New York, 1976, pp. 117-162.



- [116] R. DeVore and V. Popov, Interpolation and non-linear approximation, Proc. Conf. on interpolation and Allied Topics in Analysis, Lund, 1986 (to appear).
- [117] R. DeVore and V. A. Popov, Free multivariate splines, Constructive Approximation 3(1987), 239-248.
- [118] R. DeVore and K. Scherer, A constructive theory for approximation by splines with an arbitrary sequence of knot sets, Approximation Theory, Lecture Notes in Math., vol. 556, Springer-Verlag, New York, 1976.
- [119] R. DeVore and R. Sharpley, Maximal functions measuring smoothness, Mem. Amer. Math. Soc., vol. 47, no. 293, 1984.
- [120] M. Frazier and B. Jawerth, Decomposition of Besov spaces, preprint.
- [121] J. Peetre, New thoughts on Besov spaces, Duke Univ. Math. Ser. I, Durham, N.C., 1976.
- [122] P. Petrushev, Direct and converse theorems for spline and rational approximation and Besov spaces, Proc. Conf. on Interpolation and Applied Topics in Analysis, Lund, 1986 (to appear).
- [123] E. A. Storozhenko and P. Oswald, Jackson's theorem in the spaces  $L_p(\mathbb{R}^k)$ ,  $0 < p < 1$ , Siberian Math. J. 19 (1978), 630-639.
- [124] V. A. Popov and P. Petrushev, Rational approximation of real valued functions, Encyclopedia of Math. and Applications, vol. 28, Cambridge Univ. Press, Cambridge, 1987.
- [125] R. A. DeVore and R. C. Sharpley, Besov spaces on domains in  $\mathbb{R}^d$ , Trans. Amer. Math. Soc., 335 (1993), pp. 843-864.
- [126] M. Christ, The extension problem for certain function spaces involving fractional orders of differentiability, Ark. Mat. 22 (1984), 63-81.
- [127] R. DeVore and V. Popov, Interpolation of Besov spaces, Trans. Amer. Math. Soc. 305 (1988), 397-414.
- [128] Free multivariate splines, Constr. Approx. 3 (1987), 239-248.
- [129] R. DeVore and R. Sharpley, Maximal functions measuring smoothness, Mem. Amer. Math. Soc., No. 293, 1983.
- [130] H. John and K. Scherer, On the equivalence of the K-functional and moduli of continuity and some applications, Lecture Notes in Math., vol. 571, Springer-Verlag, Berlin, 1976, 119-140.
- [131] P. W. Jones, Quasiconformal mappings and extendability of functions in Sobolev spaces, Acta Math. 147 (1981), 71-88.

- [132] E. M. Stein, Singular integrals and differentiability properties of functions, Princeton Univ.Press, Princeton, N.J., 1970.
- [133] E. A. Storozhenko and P. Oswald, Jackson's theorem in the spaces  $L_p(\mathbb{R}^k)$ ,  $0 < p < 1$ , Siberian Math. J. 19 (1978), 630-639.
- [134] R. Arcangé li, M.C. L ´ opez de Silanes, J.J. Torrens, Extension of sampling inequalities to Sobolev semi-norms of
- [135] Adams, R.A., Fournier, J.J.F.: Sobolev Spaces. 2nd edn. Academic Press, New York (2003)
- [136] Arcangéli, R., López de Silanes, M.C., Torrens, J.J.: An extension of a bound for functions in Sobolevspaces, with applications to(m,s)-spline interpolation and smoothing. Numer.Math.107(2), 181–211 (2007)
- [137] Arcangéli, R., López de Silanes, M.C., Torrens, J.J.: Estimates for functions in Sobolev spaces definedon unbounded domains. J. Approx. Theory161, 198–212 (2009)
- [138]Arcangéli, R., Torrens, J.J.: Limiting behaviour of intrinsic semi-norms in fractional order Sobolevspaces. arXiv:1111.7301v1 [math.FA] (2011)
- [139] Bergh, J., Löfström, J.: Interpolation Spaces. An Introduction.Grundlehren der Mathematischen Wis-senschaften, No. 223. Springer-Verlag, Berlin (1976)
- [140] Bourgain, J., Brézis, H., Mironescu, P.: Another look at Sobolev spaces. In: Menaldi, J.L., Rofman, E.,Sulem, A. (eds.) Optimal Control and Partial Differential Equations. Volume in honour of ProfessorAlain Bensoussan's 60th Birthday, pp. 439–455. IOS Press, Amsterdam (2001)
- [141] Brenner, S.C., Scott, L.R.: The Mathematical Theory of Finite Element Methods. Springer-Verlag,New York (2002)
- [142] Corrigan, A., Wallin, J., Wanner, T.: A sampling inequality for fractional order Sobolev semi-norms using arbitrary order data. arXiv:0801.4097v3 [math.NA] (2009)
- [143] Duchon, J.: Sur l'erreur d'interpolation des fonctions de plusieurs variables par les $D^m$ -splines. RAIROAnal. Numer.12(4), 325–334 (1978)
- [144] Krebs, J., Louis, A.K., Wendland, H.: Sobolev error estimates and a priori parameter selection forsemi-discrete Tikhonov regularization. J. Inv. Ill-Posed Probl.17(9), 845–869 (2009)
- [145] Madych, W.R.: An estimate for multivariate interpolation II. J. Approx. Theory142, 116–128 (2006)
- [146]Madych, W.R., Potter, E.H.: An estimate for multivariate interpolation. J. Approx. Theory43,132–139 (1985)

- [147] Maz'ya, V., Shaposhnikova, T.: On the Bourgain, Brezis, and Mironescu theorem concerning limiting embeddings of fractional Sobolev spaces. *J. Funct. Anal.*195(2), 230–238 (2002)
- [148] Milman, M.: Notes on limits of Sobolev spaces and the continuity of interpolation scales. *Trans. Am. Math. Soc.*357(9), 3425–3442 (electronic) (2005)
- [149] Narcowich, F.J., Ward, J.D., Wendland, H.: Sobolev bounds on functions with scattered zeros, with applications to radial basis function surface fitting. *Math. Comput.*74, 743–763 (2005)
- [150] Nečas, J.: *Les Méthodes Directes en Théorie des Équations Elliptiques*. Masson, Paris (1967)
- [151] Rieger, C., Schaback, R., Zwicknagl, B.: Sampling and stability. In: *Mathematical Methods for Curves and Surfaces*. *Lect. Notes Comput. Sci.*, vol. 5862, pp. 347–369. Springer, Berlin/Heidelberg (2010)
- [152] Schaback, R.: Convergence of unsymmetric kernel-based meshless collocation methods. *SIAM J. Numer. Anal.*45(1), 333–351 (electronic) (2007)
- [153] Stein, E.M.: *Singular integrals and differentiability properties of functions*, Princeton Mathematical Series, No. 30. Princeton University Press, Princeton (1970)
- [154] Tartar, L.: *An introduction to Sobolev spaces and interpolation spaces*. *Lecture Notes of the Unione Matematica Italiana*, vol. 3. Springer, Berlin (2007)
- [155] Wendland, H.: *Scattered data approximation*. *Cambridge Monographs on Applied and Computational Mathematics*, vol. 17. Cambridge University Press, Cambridge (2005)
- [156] Wendland, H.: Divergence-free kernel methods for approximating the Stokes problem. *SIAM J. Numer. Anal.*47(4), 3158–3179 (2009)
- [157] Wendland, H., Rieger, C.: Approximate interpolation with applications to selecting smoothing parameters. *Numer. Math.*101, 643–662 (2005)
- [158] R. Arcangeli, J.J. Torrens, Sampling inequalities in Sobolev spaces, *Journal of Approximation Theory* 182 (2014) 18–28.
- [159] R.A. Adams, J.J.F. Fournier, *Sobolev Spaces*, second ed., Academic Press, New York, 2003.
- [160] R. Arcangeli, M.C. López de Silanes, J.J. Torrens, *Multidimensional Minimizing Splines. Theory and Applications*, in: *Grenoble Sciences*, Kluwer Academic Publishers, Boston, 2004.

- [161] R. Arcangeli, M.C. López de Silanes, J.J. Torrens, Estimates for functions in Sobolev spaces defined on unbounded domains, *J. Approx. Theory* 161 (2009) 198–212.
- [162] J. Bergh, J. Löfström, *Interpolation Spaces, An Introduction*, Springer-Verlag, Berlin, 1976.
- [163] S.C. Brenner, L.R. Scott, *The Mathematical Theory of Finite Element Methods*, Springer-Verlag, New York, 2002.
- [164] J. Duchon, Sur l'erreur d'interpolation des fonctions de plusieurs variables par les  $D^m$ -splines, *RAIRO Anal.Numer.* 12 (4) (1978) 325–334.
- [165] E. Fuselier, G.B. Wright, Scattered data interpolation on embedded submanifolds with restricted positive definite kernels: Sobolev error estimates, *SIAM J. Numer. Anal.* 50 (3) (2012) 1753–1776.
- [166] T. Hangelbroek, F.J. Narcowich, J.D. Ward, Polyharmonic and related kernels on manifolds: interpolation and approximation, *Found. Comput.Math.* 12 (2012) 625–670.
- [167] M.B. Lee, Y.J. Lee, J. Yoon, Sobolev-type  $L_p$ -approximation orders of radial basis function interpolation to functions in fractional Sobolev spaces, *IMA J. Numer. Anal.* 32 (2012) 279–293.
- [168] W.R. Madych, An estimate for multivariate interpolation II, *J. Approx. Theory* 142 (2006) 116–228.
- [169] C. Rieger, *Sampling inequalities and applications*, Thesis, Göttingen, 2008.
- [170] C. Rieger, R. Schaback, B. Zwicknagl, Sampling and stability, in: *Mathematical Methods for Curves and Surfaces*, in: *Lecture Notes in Computer Science*, vol. 5862, 2010, pp. 347–369.
- [171] C. Rieger, B. Zwicknagl, Sampling inequalities for infinitely smooth functions, with applications to interpolation and machine learning, *Adv. Comput. Math.* 32 (2010) 103–129.
- [172] C. Rieger, B. Zwicknagl, Improved exponential convergence rates by oversampling near the boundary, *Constr.Approx.* (2013) 1–19.
- [173] D. Schröder, H. Wendland, An extended error analysis for a meshfree discretization method of Darcy's problem, *SIAM J. Numer. Anal.* 50 (2012) 838–857.
- [174] E.M. Stein, *Singular Integrals and Differentiability Properties of Functions*, in: *Princeton Mathematical Series*, vol.30, Princeton University Press, Princeton, New Jersey, 1970.
- [175] H. Wendland, C. Rieger, Approximate interpolation with applications to selecting smoothing parameters, *Numer.Math.* 101 (2005) 729–748.

- [176] D.Girela, J.A. Peláez, Carleson measures for spaces of Dirichlet type, *Integral Equations Operator Theory* 55 (3) (2006) 415–427.
- [177] N. Arcozzi, R. Rochberg and E. Sawyer, Carleson measures for analytic Besov spaces, *Rev. Mat. Iberoamericana* 18(2002), no. 2, 443–510.
- [178] R. Aulaskari and P. Lappan, Criteria for an analytic function to be Bloch and aharmonic or meromorphic function to be normal, In: *Complex Analysis and its Applications* (Hong Kong, 1993), pp. 136-146, Pitman Res. Notes Math. Ser., 305, Longman Scientific and Technical, Harlow 1994.
- [179] R. Aulaskari, D. A. Stegenga, and J. Xiao, Some subclasses of BMOA and their characterizations in terms of Carleson measures, *Rocky Mountain J. Math.* 26(1996), no. 2, 485-506.
- [180] S.M. Buckley, P. Koskela and D. Vukotić, Fractional integration, differentiation, and weighted Bergman spaces, *Math. Proc. Cambridge Philos. Soc.* 126(1999), 369-385.
- [181] L. Carleson, Interpolation by bounded analytic functions and the corona problem, *Ann. of Math.* 76(1962), 547-559.
- [182] P. L. Duren, *Theory of  $H^p$  Spaces* (Academic Press, New York-London 1970. Reprint: Dover, Mineola, New York 2000).
- [183] P.L. Duren and A.P. Schuster, *Bergman Spaces, Mathematical Surveys and Monographs*, 100, American Mathematical Society, Providence, (2004).
- [184] M. Essen and J. Xiao, Some results on  $Q_p$  spaces,  $0 < p < 1$ , *J. Reine Angew. Math.* 485(1997), 173-195.
- [185] T.M. Flett, The dual of an inequality of Hardy and Littlewood and some related inequalities, *J. Math. Anal. Appl.* 38(1972), 746–765.
- [186] J. B. Garnett, *Bounded Analytic Functions* (Academic Press, New York, etc. 1981).
- [187] D. Girela, Mean growth of the derivative of certain classes of analytic functions, *Math. Proc. Camb. Phil. Soc.* 112(1992), 335-342.
- [188] D. Girela and J. A. Peláez, Growth properties and sequences of zeros of analytic functions in spaces of Dirichlet type, to appear in *J. Austral. Math. Soc.*
- [189] D. Girela and J. A. Peláez, Integral means of analytic functions, *Ann. Acad. Sci. Fenn. Math.* 29(2004), 459-469.
- [190] W. W. Hastings, A Carleson measure theorem for Bergman spaces, *Proc. Amer. Math. Soc.* 52(1975), 237-241.

[191] H. Hedenmalm, B. Korenblum and K. Zhu, Theory of Bergman Spaces, Graduate Texts in Mathematics 199, Springer, New York, Berlin, etc. (2000).

Vol. 55 (2006) Carleson Measures for Spaces of Dirichlet Type 427

[192] D. H. Luecking, Inequalities on Bergman spaces, Illinois J. Math. 25(1981), 1-11.

[193] D. H. Luecking, A technique for characterizing Carleson measures on Bergman spaces, Proc. Amer. Math. Soc., 87(1983), 656-660.

[194] M. Mateljevic and M. Pavlovic,  $L_p$ -behaviour of power series with positive coefficients and Hardy spaces, Proc. Amer. Math. Soc., 87(1983), 309-316.

[195] V. L. Oleinik, Embedding theorems for weighted classes of harmonic and analytic functions, (in russian), Zap. Nauch. Sem. LOMI Steklov 47(1974), 120-137; English translation in J. Soviet Math. 9(1978), 228-243.

[196] V. L. Oleinik and B. S. Pavlov, Embedding theorems for weighted classes of harmonic and analytic functions, (in russian), Zap. Nauch. Sem. LOMI Steklov 22(1971), 94-102; english translation in J. Soviet Math. 2(1974), 135-142.

[197] D. Stegenga, Multipliers of the Dirichlet spaces, Illinois J. Math. 24(1980), 113-139.

[198] S. A. Vinogradov, Multiplication and division in the space of analytic functions with area integrable derivative, and in some related spaces, (in russian), Zap. Nauch. Sem. S.-Peterburg. Otdel. Mat. Inst. Steklov. (POMI) 222(1995), Issled. po Linein. Oper. i Teor. Funktsii 23, 45-77, 308; english translation in J. Math. Sci. (New York) 87, no. 5 (1997), 3806-3827.

[199] Z. Wu, Carleson measures and multipliers for Dirichlet spaces, J. Funct. Anal. 169(1999), 148-163.

[200] J. Xiao, Carleson measure, atomic decomposition and free interpolation from Bloch space, Ann. Acad. Sci. Fenn. Ser. A I Math. 19(1994), no. 1, 35-46.

[201] J. Xiao, Holomorphic  $Q$ -classes, Lecture Notes in Mathematics 1767, Springer-Verlag, Berlin (2001).

[202] A. Zygmund, Trigonometric Series, Vol. I and II, second edition, Cambridge Univ. Press, Cambridge (1959).

[203] José Ángel Peláez, Fernando Pérez-González, Jouni Rättyä, Operator theoretic differences between Hardy and Dirichlet-type spaces  $\star$ , J. Math. Anal. Appl. 418 (2014) 387-402

- [204] A. Aleman, J.A. Cima, An integral operator on  $H^p$  and Hardy's inequality, *J. Anal.Math.* 85 (2001) 157–176.
- [205] A.Aleman, A. Siskakis, An integral operator on  $H^p$ , *Complex Var. Theory Appl.* 28 (1995) 149–158.402 J.Á. Peláez et al. / *J. Math.Anal. Appl.* 418 (2014) 387–402
- [206] A. Aleman, A. Siskakis, Integration operators on Bergman spaces, *Indiana Univ. Math. J.* 46 (1997) 337–356.
- [207] N.Arcozzi, R. Rochberg, E. Sawyer, Carleson measures for analytic Besov spaces, *Rev. Mat.Iberoam.* 18 (2002) 443–510.
- [208] A.Calderón, Commutators of singular integral operators, *Proc. Natl. Acad.Sci.* 53 (1965) 1092–1099.
- [209] L.Carleson, An interpolation problem for bounded analytic functions, *Amer. J. Math.* 80 (1958) 921–930.
- [210] L.Carleson, Interpolations by bounded functions and the corona problem, *Ann. of Math.* 76 (1962) 547–559.
- [211] B.R.Choe, H. Koo, W. Smith, Composition operators acting on Sobolev holomorphic spaces, *Trans. Amer. Math. Soc.*(2003) 2829–2855.
- [212] P. Duren, *Theory of  $H^p$  Spaces*, Academic Press, New York–London, 1970.
- [213] P. Galanopoulos, D. Girela, J.A. Peláez, A. Siskakis, Generalized Hilbert operators on classical spaces of analytic functions, *Ann.Acad. Sci. Fenn. Math.* 39 (1) (2014) 231–258, <http://dx.doi.org/10.5186/aasfm.2014.3912>.
- [214] J.Garnett, *Bounded Analytic Functions*, Academic Press, New York, 1981.
- [215] D.Girela, M. Pavlović, J.A. Peláez, Spaces of analytic functions of Hardy–Bloch type, *J. Anal.Math.* 100 (2006) 53–81.
- [216] D.Girela, J.A. Peláez, Growth properties and sequences of zeros of analytic functions in spaces of Dirichlet type, *J. Aust.Math. Soc.* 80 (2006) 397–418.
- [217] D.Girela, J.A. Peláez, Carleson measures for spaces of Dirichlet type, *Integral Equations Operator Theory* 55 (3) (2006) 415–427.
- [218] D.Girela, J.A. Peláez, Carleson measures, multipliers and integration operators for spaces of Dirichlet type, *J. Funct.Anal.* 241 (1) (2006) 334–358.
- [219] J.E.Littlewood, R.E.A.C. Paley, Theorems on Fourier series and power series. II, *Proc. Lond. Math. Soc.* 42 (1936) 52–89.
- [220] D.H.Luecking, Embedding derivatives of Hardy spaces into Lebesgue spaces, *Proc. Lond.Math. Soc.* 63 (1991) 565–619.

- [221] J.A.Peláez, J. Rättyä, Weighted Bergman spaces induced by rapidly increasing weights, *Mem. Amer. Math. Soc.* 227 (1066)(2014).
- [222] C.Pommerenke, Schlichte funktionen und analytische funktionen von beschränkter mittlerer oszillation, *Comment.Math.Helv.* 52 (1977) 591–602.
- [223] Z.Wu, Carleson measures and multipliers for Dirichlet spaces, *J. Funct.Anal.* 169 (1999) 148–163.
- [224] C. Mouhot, E. Russ, and Y. Sire, Fractional Poincaré inequalities for general measures, *J. Math. Pures Appl.*, 95 (2011), pp. 72–84.
- [225] R. Adams, J. Fournier, *Sobolev Spaces*, second edition, *Pure Appl. Math.(Amst.)*, vol. 140, Elsevier/Academic Press, Amsterdam, 2003.
- [226] D. del Castillo-Negrete, B. Carreras, V. Lynch, Nondiffusive transport in plasma turbulence: a fractional diffusion approach, *Phys. Rev. Lett.* 94(2005) 065003.
- [227] L. Caffarelli, A. Vasseur, Drift diffusion equations with fractional diffusion and the quasi-geostrophic equation, *Ann. of Math.* 171 (3) (2010)1903–1930.
- [228] C. Villani, On a new class of weak solutions to the spatially homogeneous Boltzmann and Landau equations, *Arch. Ration. Mech. Anal.* 143(1998) 273–307.
- [229] C. Mouhot, Explicit coercivity estimates for the linearized Boltzmann and Landau operators, *Comm. Partial Differential Equations* 31 (2006)1321–1348.
- [230] C. Mouhot, R.M. Strain, Spectral gap and coercivity estimates for linearized Boltzmann collision operators without angular cutoff, *J. Math.Pures Appl.* (9) 87 (2007) 515–535.
- [231] P. Gressman, R. Strain, Global classical solutions of the Boltzmann equation with long-range interactions, *Proc. Natl. Acad. Sci. USA* 107(2010) 5744–5749.
- [232] G. Di Nunno, B. Øksendal, F. Proske, *Malliavin Calculus for Lévy Processes with Applications to Finance*, Universitext, Springer-Verlag, Berlin, 2009.
- [233] W. Feller, *An Introduction to Probability Theory and Its Applications. Vol. II*, second edition, John Wiley & Sons Inc., New York, 1971.
- [234] A. Signorini, Questioni di elasticità non linearizzata e semilinearizzata, *Rend. Mat. Appl.* (5) 18 (1959) 95–139.
- [235] L. Silvestre, Regularity of the obstacle problem for a fractional power of the Laplace operator, *Comm. Pure Appl. Math.* 60 (2007) 67–112.
- [236] L. Caffarelli, A. Figalli, Regularity of solutions to the parabolic fractional obstacle problem, preprint, 2010.



- [237] D. Bakry, F. Barthe, P. Cattiaux, A. Guillin, A simple proof of the Poincaré inequality for a large class of probability measures including the log-concave case, *Electron. Commun. Probab.* 13 (2008) 60–66.
- [238] C. Villani, Hypocoercivity, *Mem. Amer. Math. Soc.* 202 (2009) 1–141.
- [239] J.-D. Deuschel, D.W. Stroock, Hypercontractivity and spectral gap of symmetric diffusions with applications to the stochastic Ising models, *J. Funct. Anal.* 92 (1990) 30–48.
- [240] M. Ledoux, *The Concentration of Measure Phenomenon*, Amer. Math. Soc., 2001.
- [241] L. Wu, A new modified logarithmic Sobolev inequality for Poisson point processes and several applications, *Probab. Theory Related Fields* 118(2000) 427–438.
- [242] D. Chafaï, Entropies, convexity, and functional inequalities: on  $\Phi$ -entropies and  $\Phi$ -Sobolev inequalities, *J. Math. Kyoto Univ.* 44 (2004) 325–363.
- [243] I. Gentil, C. Imbert, The Lévy–Fokker–Planck equation:  $\Phi$ -entropies and convergence to equilibrium, *Asymptot. Anal.* 59 (2008) 125–138.
- [244] N.S. Landkof, *Foundations of Modern Potential Theory*, Grundlehren Math. Wiss., Band 180, Springer-Verlag, New York, 1972, translated from the Russian by A.P. Doohovskoy.
- [245] D. Bakry, M. Émery, Propaganda for  $\Gamma_2$ , in: *From Local Times to Global Geometry, Control and Physics*, Coventry, 1984/85, in: Pitman Res. Notes Math. Ser., vol. 150, Longman Sci. Tech., Harlow, 1986, pp. 39–46.
- [246] M. Gaffney, The conservation property of the heat equation on Riemannian manifolds, *Comm. Pure Appl. Math.* 12 (1959) 1–11.
- [247] P. Auscher, On Necessary and Sufficient Conditions for  $L_p$  Estimates of Riesz Transforms Associated to Elliptic Operators on  $R^n$  and Related Estimates, *Mem. Amer. Math. Soc.*, vol. 186, Amer. Math. Soc., 2007.
- [248] P. Auscher, S. Hofmann, M. Lacey, A. McIntosh, P. Tchamitchian, The solution of the Kato square root problem for second order elliptic operators on  $R^n$ , *Ann. of Math.* (2) 156 (2002) 633–654.
- [249] P. Auscher, A. McIntosh, E. Russ, Hardy spaces of differential forms on Riemannian manifolds, *J. Geom. Anal.* 18 (2008) 192–248.
- [250] D. Henry, *Geometric Theory of Semilinear Parabolic Equations*, second edition, *Lecture Notes in Math.*, vol. 840, Springer-Verlag, Berlin/New York, 1981.
- [251] R. Strichartz, Multipliers on fractional Sobolev spaces, *J. Math. Mech.* 16 (1967) 1031–1060.

- [252] E. Stein, The characterization of functions arising as potentials I, *Bull. Amer. Math. Soc.* 67 (1961) 102–104.
- [253] T. Coulhon, E. Russ, V. Tardivel-Nachef, Sobolev algebras on Lie groups and Riemannian manifolds, *Amer. J. Math.* 123 (2001) 283–342.
- [254] E. Davies, *One-Parameter Semigroups*, second edition, London Math. Soc. Monogr. Ser., vol. 15, Academic Press Inc., London/New York, 1980.
- [255] M. Kanai, Rough isometries and combinatorial approximations of geometries of noncompact Riemannian manifolds, *J. Math. Soc. Japan* 37(1985) 391–413
- [256] Norbert Heuer, On the equivalence of fractional-order Sobolev semi-norms arXiv:1211.0340v2 [math.FA] 21 Mar 2014
- [257] R. Adams, *Sobolev Spaces*, Academic press, New York, San Francisco, London, 1979.
- [258] J. Bergh and J. Löfström, *Interpolation Spaces*, no. 223 in *Grundlehren der mathematischen Wissenschaften*, Springer-Verlag, Berlin, 1976.
- [259] D. Braess, *Finite Elements: Theory, Fast Solvers, and Applications in Solid Mechanics*, Cambridge University Press, Cambridge, 1997.
- [260] S. C. Brenner and L. R. Scott, *The Mathematical Theory of Finite Element Methods*, no. 15 in *Texts in Applied Mathematics*, Springer-Verlag, New York, 1994.
- [261] I. Drelichman and R. G. Durán, Improved Poincaré inequalities with weights, *J. Math. Anal. Appl.*, 347 (2008), pp. 286–293.
- [262] V. J. Ervin and N. Heuer, An adaptive boundary element method for the exterior Stokes problem in three dimensions, *IMA J. Numer. Anal.*, 26 (2006), pp. 297–325.
- [263] B. Faermann, Localization of the Aronszajn-Slobodeckij norm and application to adaptive boundary element methods, Part II. The three-dimensional case, *Numer. Math.*, 92 (2002), pp. 467–499.
- [264] P. Grisvard, *Elliptic Problems in Nonsmooth Domains*, Pitman Publishing Inc., Boston, 1985.
- [265] N. Heuer, Additive Schwarz method for the p-version of the boundary element method for the single layer potential operator on a plane screen, *Numer. Math.*, 88 (2001), pp. 485–511.
- [266] G. C. Hsiao and W. L. Wendland, *Boundary Integral Equations*, Springer, 2008.
- [267] J.-L. Lions and E. Magenes, *Non-Homogeneous Boundary Value Problems and Applications I*, Springer-Verlag, New York, 1972.

[268] W. McLean, Strongly Elliptic Systems and Boundary Integral Equations, Cambridge University Press, 2000.

[269] J. Nečkas, Les Méthodes Directes en Théorie des Equations Elliptiques, Academia, Prague, 1967.15

[270] L. E. Payne and H. F. Weinberger, An optimal Poincaré-inequality for convex domains, Arch. Rational Mech. Anal., 5 (1960), pp. 286–292.

[271] R. Verfürth, A note on polynomial approximation in Sobolev spaces, M2AN Math. Model. Numer. Anal., 33 (1999), pp. 715–719.

[272] Shawgy Hussein Abd Alla and Abdelilah Kamal Hassan Sedeeg, Sampling Inequalities and Fractional-Order Sobolev Semi-norms with Operator Theoretical Differences P.hd. Thesis, Sudan University of Science and Technology, 2015.