Dedication

This thesis is dedicated to:

My mother ... for her ultimate carefulness of every single detailin my life.

My father... for his great support may Allah rest his soul in peace.

My wife...

My daughter...

My brothers and sisters.

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Abstract

An extension of a bound on functions in Sobolev spaces with Scattered zeroes and applications to radial basis function surface fitting,spline interpolation,smoothing and polynomial are shown.The characterizations of interpolation of Bosevspaces and on domains in Euclidean spaces are discussed, sampling inequalities to Sobolev semi-norms,spaces of fractional order and derivatives data are determined.We use the method of Carlesonmeasures and operator theoretic differences between Hardy and spaces of Dirichlettype.The duality of an inequality, embedding derivatives with related inequalities one given.

الخلاصة

أوضحنا تمديد حدية الدوال في فضاءات سوبوليف مع الأصفار المبعثرة والتطبيقات الى ملائمة سطح دالة الأساس القطرية وإستكمال الشريحة والملسان وكثيرة الحدود. ثُرست تشخيصات الإستكمال لفضاءات بيسوف والمجالات في الفضاء الإقليدي ، وحُددت متباينات الاستكمال لشبه نظائم سوبوليف وفضاءات الرتبة الكسرية وبيانات المشتقة ، استخدمنا طريقة قياسات كارلسون وفروقات نظرية المؤثر بين فضاءات هاردي وفضاءات نوع درشلت.أُعطيت ازدواجية المتتالية واشتقاقات الطمر مع المتباينات ذات العلاقة.

Introduction

We discuss Sobolev bounds on functions that vanish at scattered points in a bounded, Lipschitz domain that satisfies a uniform interior cone condition. The Sobolev spaces involved may have fractional as well as integer order.Given a function on a bounded open subset of the Euclidean space with a Lipschitz-continuous boundary, we obtain a Sobolev bound involving the values off at finitely many points of the closure of the bounded open subset .This result improves previous ones due to Narcowichet al and Wendland and Rieger.

Defining the function kernel as μ-measure of a hyperbolic ball of fixed radius centered at a point ,we may describe the characterization here briefly, if opaquely ,in terms of member ship of the kernel ; in a 'weighted tent space' or an L^{∞}-analogue of one (depending on the size of q) .In the course of the proof there is developed a theory of 'Tent spaces' with respect to arbitrary measures on R_{+}^{n+1} A consequence of the theory is an interpolation theorem for the values of derivatives of H^{p}-functions at the points of an 'η -lattice'. Constructive proofs and several generalizations of approximation results of J. H. Bramble and S. R. Hubert are presented. Using an averaged Taylor series, we represent a function as a polynomial plus a remainder. The remainder can be manipulated in many ways to give different types of bounds .We investigate Besov spaces and their connection with dyadic spline approximation in $L_{p}(\Omega)$, $0 < p \leq \infty$.

We show that there is an extension operator ξ which is a bounded mapping from $B_{q}^{\alpha}(L_{p}(\Omega))$ onto $B_{q}^{\alpha}(L_{p}(R^{d}))$. We devoted to sampling inequalities, provides some extensions of previous results by Arcangéli et al. Given a function in a suitable Sobolev space defined on a domain in Euclidean space , sampling inequalities typically yield bounds of integer order Sobolev semi-norms of the function in terms of a higher order Sobolev semi-norm of the function, the fill distanced between the domain and a discrete set of the closure of the domain, and the values of the function on the subspace.

The structure of the sampling inequalities in spaces $W^{r,p}(\Omega)$ is analysed, their role in the study of the interpolation error by spline functions is recalled, and the analogy between these inequalities and those relative to intermediate semi-norms in spaces$W^{r,p}(\Omega)$ .If $0 < p < \infty$and $\alpha > -1$, the space $\mathcal{D}_{\alpha}^{p}$ consists of those functions f which are analytic in the unit disc $\mathbb{D}$ and have the property that f' belongs to the weighted Bergman space A_{α}^{p} . In 1999, Z. Wu obtained a characterization of the Carleson measures for the spaces $\mathcal{D}_{\alpha}^{p}$ for certain values of $p$and α. In particular, he proved that, for $0 < p \leq 2$, the Carleson measures for the space $\mathcal{D}_{p-1}^{p}$ are precisely the classical Carleson measures. Wu also conjectured that this result remains true for $2 < p < \infty$.

For $0 < p < \infty$,the Dirichlet-type space $\mathcal{D}_{p-1}^{p}$ consists of the analytic functions f in the unit disc $\mathbb{D}$ such that $\int_{\mathbb{D}} |f'(z)|^p (1-|z|)^{p-1}\, dA(z) < \infty$. Motivated by operator theoretic differences between the Hardy space H^p and

$\mathcal{D}_{p-1}^{p}$, the integral operator

$$T_g(f)(z) = \int_0^z f(\zeta) g'(\zeta) d\zeta \quad , z \in \mathbb{D} \ .$$

acting from one of these spaces to another is studied . We show a fractional version of Poincaré inequalities in the context of R^n endowed with a fairly general measure. Namely we show a control of anL^2 norm by a non-local quantity, which plays the role of the gradient in the standard Poincaré inequality. The assumption on the measure is the fact that it satisfies the classical Poincaré inequality, so that our result is an improvement of the latter inequality. We present various results on the equivalence and mapping properties under affine trans-formations of fractional-order Sobolev norms and semi-norms of orders between zero and one.

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