Dedication

To my Family , Friends and colleagues.

Acknowledgments

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Abstract

We deal with pseudodifferential operators with smooth symbols and Weierstras's theorem in weighted Sobolev spaces. We describe the zeros, critical points, zero location and nth root asymptotics of Sobolev orthogonal polynomials, we also show the convergence in the mean and necessary conditions for weighted mean convergence of Fourier series in orthogonal polynomials.We consider Sobolev embeddings , concentration-compactness, alternative, Gagliardo-Nirenberg, composition, products, Bourgain-Brezis-Mironescu theorem concerning limiting embeddings and Hitchhiker's guide of fractional Sobolev spaces, we also determine the best constants for Sobolev inequalities for higher order fractional derivatives and how to recognize constant functions connections with Sobolev spaces. The structures of the relative asymptotics, asymptotic properties and Fourier series of orthogonal polynomials with a discrete and non-discrete Gegenbauer-Sobolev inner products are investigated,we also show the $W^{1,p}$-convergence of Fourier–Sobolev expansions.