Dedication

To my parents , Family , Friends and colleagues.

Acknowledgments

At first I would like to thank Allah who gives me the ability to complete this work.

I would like to express my deep thanks to my supervisor

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Abstract

We show the sampling theorems for multivariate , nonuniform average sampling , reconstruction in multiply generated , Riesz bases in Hilbert space related to sampling , dual frames in Hilbert space connected with generalized sampling , multivariate generalized sampling with approximation properties in shift-invariant spaces .We consider the average sampling in spline subspaces and in shift-invariant subspaces with symmetric averaging functions .We investigate the channeled sampling in translation and in shift-invariant spaces .We study approximating , by using very coarsely quantized data and sampling with recovery of multidimensional bandlimited functions . We discuss the sampling expansion and asymmetric multi-channel sampling in shift- invariant spaces.

الخلاصة

أوضحنا مبرهنات المعاينة للتغاير المتعدد و المعاينة المتوسطة غير المنتظمة واعادة التشييد في مولد الضرب واساس ريس في فضاء هلبرت ذو العلاقة الي المعاينة و الاطارات الثنائية وفي فضاء هلبرت المتصل مع المعاينة المعممة و المعاينة المعممة للتغاير المتعدد مع خواص التقريب في ازاحة – الفضاءات غير المتغيرة . تم اعتبار المعاينة المتوسطة في الفضاءات الجزئية الاسبلاين وفي ازاحة الفضاءات الجزئية غير المتغيرة مع الدوال المتوسطة المتماثلة . ناقشنا معاينة القناة في الانسحاب وفي ازاحة – الفضاءات غير المتغيرة .درسنا التقريب بواسطة استخدام بيانات التكميم الخشن جداً و المعاينة مع التحسن لدوال الشريط المحدود متعدد البعد . تم دراسة مفكوك المعاينة وتماثل تعدد معاينة القناة في ازاحة – الفضاءات غير المتغيرة .

Introduction

A sampling theorem for regular sampling in shift invariant subspaces is established. The sufficient-necessary condition for which it holds is found. Then, the theorem is modified to the shift sampling in shift - invariant subspaces by using the Zak transform . We show that every function in a spline subspace is uniquely determined and can be reconstructed by its local averages near certain points.

We show single and two-channel sampling formula in the translation invariant subspaces in the multi resolution analysis of wavelet theory. Regular and irregular sampling theorems for multivariate shift invariant subspaces are studied. We give a characterization of regular points and an irregular sampling theorem, which covers many known results . We give a sufficient condition for a function to belong to some sampling space. We prove that a sampling space must be contained in another one which has a Riesz basis consisting of integer translations of a base function. Examples are given.

Fast approximation-projection iterative reconstruction algorithms are developed. The performance of the algorithms are analyzed when the data is corrupted by noise. Estimates are derived for the convergence rates of the algorithms in terms of the sampling density, the generators of the shift-invariant space, and the sampling functionals (ψ_{x_j}). We study the reconstruction of functions in shift invariant subspaces from local averages with symmetric averaging functions. We present an average sampling theorem for shift invariant subspaces and give quantitative results on the aliasing error and the truncation error. The Fourier duality is an elegant technique to obtain sampling formulas in Paley–Wiener spaces. It is show that there exists an analogue of the Fourier duality technique in the setting of shift-invariant spaces.

The aim is to derive stable generalized sampling in a shift-invariant space by using some special dual frames in $L^2(0,1)$. These sampling formulas involve samples of filtered versions of the functions in the shift-invariant space . Nowadays the topic of sampling in a shift-invariant space is having a significant impact: it avoids most of the problems associated with classical Shannon's theory. Under appropriate hypotheses, any multivariate function in a shift-invariant space can be recovered from its samples at $\mathbb{Z}^d$. However, in many common situations the available data are samples of some convolution operators acting on the function itself: this leads to the problem of multivariate generalized sampling in shift-invariant spaces. This extra information on the functions in the shift-invariant space will allow to sample in an appropriate sublattice of $\mathbb{Z}^d$

We investigate First order $\Sigma\Delta$ and Higher order $\Sigma\Delta$ -quantization circumvents by providing a simple iterative algorithm in which the q_n^λ are constructed by taking into account not only $f\left(\frac{n}{\lambda}\right)$ but also past $f\left(\frac{n}{\lambda}\right)$, we restrict our attention to a particular class of A/D conversion schemes adapted to audio signals. We investigate frames for $L_2\,[-\pi,\pi]^d$ consisting of exponential functions in connection to oversampling and nonuniform sampling of bandlimited functions. We derive a multidimensional nonuniform over sampling formula for bandlimited functions with a fairly general frequency domain. The stability of said formula under various perturbations in the sampled data is investigated, and a computationally manageable simplification of the main oversampling theorem is given.

We show sampling expansion formulas on the shift invariant closed subspace $V(\phi)$ of $L^2(\mathbb{R})$ generated by a frame or a Riesz generator $\phi(t)$. For any $\phi(t)$ in $L^2(\mathbb{R})$, let $V\,(\phi)$ be the closed shift invariant subspace of $L^2(\mathbb{R})$ spanned by integer translates $\{\phi(t-n):n\in\mathbb{Z}\}$ of $\phi(t)$. Assuming that $\phi(t)$ is a frame or a Riesz generator of $V\,(\phi)$, we first find conditions under which $V\,(\phi)$ becomes a reproducing kernel Hilbert space. We show an asymmetric multi-channel sampling on a shift invariant space $V(\phi)$ with a Riesz generator $\phi(t)$ in $L^2(\mathbb{R})$, where each channeled signal is assigned a uniform but distinct sampling rate.

The Contents