Sudan University of Sciences and Technology

College of Graduate Studies

Expression of Energy and Lasing in Generalized Special Relativity

تعبير الطاقة والفعل الليزري في النسبية الخاصة المعممة

A Thesis submitted to the College of Graduate Studies as A fulfilment of the requirement for the degree of Doctor of Philosophy in theoretical physics

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Dedicated

To my Family

To my Mother, to memory of my Father, my Brothers, my Wife

and my Daughter, Samah,

and my Sons Amjed and Alaa Eldeen

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ملخص البحث

تعتبر تقانة الليزر واحدة من التقنية الواسعة الانتشار التي تستخدم في العديد من التطبيقات. حيث جذبت الإنتباه إلى تطوير آليات تؤثر في توليد الليزر, مثل ليزر الإلكترون الحر الذي يستند على النسبية الخاصة وشدة المجال المغناطيسي. ولكنه في الإطار النظري يعاني من القصور بسبب صيغة النسبية الخاصة التي تتجاهل تأثير المجال على الكتلة ومعامل إينشتاين. هذه التعقيدات حفزت للبحث عن نموذج جديد لتفسير هذا القصور النظري. في هذا البحث استخدمت النظرية النسبية الخاصة المعممة لإشتقاق صيغة الطاقة بإستخدام قانون حفظ الإندفاع (الزخم) بالإضافة إلى قانون حفظ الطاقة.

تم الحصول على تأثير شدة المجال المغناطيسي على شدة شعاع الليزر نظرياً بإستخدام النظرية النسبية الخاصة المعممة. هذه العلاقات النظرية كانت قريبة ومتطابقة مع النتائج التي تم الحصول عليها تجريبياً مثل ليزرات البلازما والغازات وأشباه الموصلات.

Abstract

Laser technology recently becomes one of the widespread technology in many applications. This attracts attention to generation of laser and the mechanisms affecting this generation. One of the recent developments in lasing is the so-called free-electron laser (FEL) which is based on special relativity (SR). FEL shows that lasing intensity is affected by magnetic fields. However, the theoretical framework of this effect is complex and cannot explain the effect of the magnetic fields on lasers produced by materials. These setbacks motivates us to search for a new model based on SR to account for these theoretical defects. In this work, generalized special relativistic mass expression is derived by using the momentum conservation law. Moreover, a new energy conservation law is also obtained. Within this generalized special relativistic framework, the effect of magnetic fields was found theoretically. These theoretical relations agree with the empirical ones for plasma, discharge gas and semiconductor lasers.

List of Figures

List of Tables

Contents

Chapter 1

Introduction

1.1 Laser and its Importance

Light is known as one of the most popular energy source. It stimulates human eye, therefore it is used for vision. It is also used to produce electricity by solar cells. Some electronic devices are used as light sensors for security and control reasons. Recently scientists discovered that light can be amplified to produce energetic concentrated beam of light known as laser. Laser is now a days used for a wide variety for applications. In the middle of the 1970s John Madey and colleagues constructed the first free electron laser operating in the infrared wavelength range [1, 2]. Lasers are devices that amplify or increase the intensity of light to produce a highly directional, high-intensity beam that typically has a very pure frequency or wavelength. They come in sizes ranging from approximately one-tenth the diameter of a human hair to that of a very large building. Lasers produce powers ranging from nanowatts to a billion trillion watts ($10^{21}W$) for very short bursts. They produce wavelengths or frequencies ranging from the microwave region and infrared to the visible, ultraviolet, vacuum ultraviolet, and into the soft-X-ray spectral regions. They generate the shortest bursts of light that man has yet produced, or approximately five million-billionths of a second ($5 \times 10^{15} sec$). Lasers are a primary component of some of our most modern communication systems and are the probes that generate the audio signals from our compact disk players. They are used for cutting, heat treating, cleaning, and removing materials in both the industrial and medical worlds. They are the targeting element of laser-guided bombs and are the optical source in both supermarket checkout scanners and tools that print our microchips.

The FEL is really different from conventional lasers, because, instead of exploiting the stimulated emission from atomic or molecular systems, it makes use of the radiation emitted from a relativistic accelerated electron beam to obtain radiation amplification, through the interaction of the electron-beam with a spatially periodic static magnetic field. [3]. Also it is different in many ways from other types of lasers. As the name suggests, the radiation from the FEL is produced by a beam of free or unbound electrons. One of the most attractive features of the FEL is its tunability, as well as the Other important features of the FEL are its high power capabilities, tunability, and its relatively high efficiency. Efficiencies of transferring this electron kinetic energy to FEL radiation have reached 40 percent at millimeter-to centimeter wavelengths. These unique features are desirable for a wide range of applications. For example, there is a need for tunable, efficient infrared to ultraviolet sources for biomedical and photochemical applications, laser isotope separation, materials processing, and physics research. High power sources at these wavelengths have a number of military applications. There are also a number of applications at longer wavelengths. Plasma heating at the electron cyclotron resonance in high-field fusion devices requires efficient millimeter to submillimeter sources at powers $> 10MW$. High-resolution, long-range radar also needs powerful millimeter to submillimeter sources. FELs are also being examined for various advanced particle accelerator concepts, such as high-frequency, high-accelerating-gradient rf accelerators.

The interaction between the electron beam and the output radiation field in an FEL is mediated by a periodic wiggler magnetic field. In conventional terminology, the periodic magnetic field in synchrotron light sources is referred to as an undulator while that used in FELs is called a wiggler, although there is no fundamental difference between them. As the electron beam traverses the wiggler field it emits incoherent radiation. It is necessary for the electron beam to form coherent bunches in order to give rise to the stimulated emission required for a free-electron laser [4]. This can occur when a light wave traverses an undulatory magnetic field such as a wiggler because the spatial variations of the wiggler and the electromagnetic wave combine to produce a beat wave, which exerts a slowly varying ponderomotive force on the electrons. It is the interaction between the electrons and this beat wave which gives rise to stimulated emission.

The special theory of relativity introduced by Albert Einstein who first put forward a coherent and comprehensive solution in his 1905 paper "On the electrodynamics of moving bodies", which introduced the special theory of relativity [5]. The special relativity, be able to derive its basic equations of Lorentz transformations, from the postulates of special relativity [5].

1.2 Research Problems

Despite the success of laser theories in explaining the amplification, but still there are some problems. A theoretical framework showing the effect of fields on the laser intensity is not well established. This may be related to the fact that the effect of fields on physical parameters. Controlling amplification is not recognized. This needs a theory showing effect of fields on physical parameters like, mass, length, and time.

1.3 Literature Review

Many attempts are made to see how laser amplification and intensity are affected by physical properties of matter as well as by fields. In a models developed by A. Hagar Abed Rahaman and others [15]. Matter polarization as well as friction affect amplification. In free electron laser models the magnetic field affect laser amplification. For example H. P. Freunda and W. H. Miner, Jr. have been studied the Efficiency enhancement in seeded and self-amplified spontaneous emission free-electron lasers by means of a tapered wiggler [6]. Luqi Yuan, Anatoly A. Svidzinsky and Marlan O. Scully, produced transient lasing without inversion: generation of high frequency light by driving low frequency transition[8]. This attempts which lead to generate free electron laser. The important features of the FEL are its high power capabilities and its relatively high efficiency. Because there is no physical lasing medium which must support the radiation field, problems of heating or breakdown which plague conventional solid or gaseous lasers are absent. Enormous powers can be deposited in relativistic electron beams propagating in vacuum. Some researchers shows that laser intensity is affected by the magnetic field [55]. This effect exists in lasers produced by a medium.

1.4 Aim of the Work

The generalized special relativity is one of the a new theory that shows the effect of fields on mass, length and time. The aim of the research is to utilize the generalized special relativity (GSR) to explain the effect of fields on lasing intensity and amplification factor.

1.5 Presentation of the Thesis

This thesis consists of sex chapters, chapter one represent the introduction, chapter two is concerned with laser amplification, chapter three is devoted for special relativity (SR). While in chapter four, free electron laser with electromagnetic field. Also chapter five, cover literature review . In chapter six the contribution, discussion and conclusion will be given.

Chapter 2

Laser Amplification

2.1 Introduction

In this chapter will be discussed the amplification of light which is affected by three processes, emission processes as well as absorption of electromagnetic waves in medium. Will be derived the equation of exponential growth or absorption of an incident light beam passing through medium based upon specific condition of both the beam and the medium. The specific condition which include the beam frequency, population inversion population densities of the upper and lower laser levels. The conditions associated with amplification or gain coefficient for making laser beam.

2.2 Transition and Absorption of light

Let us consider a two-level atomic system as shown in (Fig:2.1) in the presence of an electromagnetic field of frequency $\nu \sim \frac{E_2 - E_1}{h}$ an atom can undergo a transition from state 1 to 2, absorbing in the process a quantum of excitation (photon) with energy $h\nu$ from the field. If the atom happens to be in state 2 at the moment when it is first subjected to the electromagnetic field, it may make a downward transition to state 1, emitting a photon of energy $h\nu$. Let us now assume that the atom is initially at level 2. Since $E_2 > E_1$, the atom tends to decay to level 1. The corresponding energy difference $E_2 - E_1$ must therefore be released by the atom. When this energy is delivered in the form of an electromagnetic (em) wave, the process is called spontaneous (or radiative) emission. The frequency ν_0 of the radiated wave is then given by the well known expression [9, 10]:

$$\nu_0 = \frac{E_2 - E_1}{h} \tag{2.2.1}$$

Where h is Planck's constant Let the energy density per unit frequency be $\rho(\nu)$. We assumed that the induced transition rates per atom from $2 \rightarrow 1$ and $1 \rightarrow 2$ are both proportional [10].

$$\begin{aligned} (W'_{21})_{induced} &= B_{21}\rho(\nu) \\ (W'_{12})_{induced} &= B_{12}\rho(\nu), \end{aligned} \tag{2.2.2}$$

where B_{21} and B_{12} are constants. The total downward $2 \rightarrow 1$ transition rate is the sum of the induced and spontaneous contributions

$$W'_{21} = B_{21}\rho(\nu) + A_{21} \tag{2.2.3}$$

where A_{21} is the spontaneous transition rate. The total upward $(1 \rightarrow 2)$ transition rate is

$$W'_{12} = (W'_{12})_{induced} = B_{12}\rho(\nu) \tag{2.2.4}$$

Since the magnitudes of the coefficients B_{21} and B_{12} depend on the atoms and not on the radiation field [10].

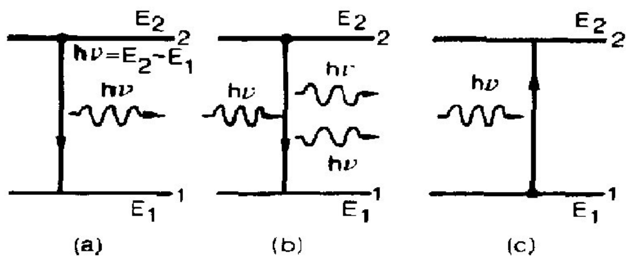

Figure 2.1: *Schematic illustration of the three processes: (a) spontaneous emission, (b) stimulated emission, (c) absorption*

Source: Orazio Svelto, David C. Hanna , Principles of Lasers , Springer P 2 (1998).

2.3 Spontaneous Emission(A_{21})

The atoms at energy level E_2 decay spontaneously to E_1 as shown in (fig:2.2) adding photon of energy $h\nu = E_2 - E_1$ to radiation field (photon population). at the same time, the population N_2 level E_2 decreases. the rate the decrease is proportional to the population at any time that is,

$$\left(\frac{dN_2}{dt}\right)_{spont} = -A_{21}N_2 \tag{2.3.5}$$

If spontaneous emission alone takes place, the solution to this equation yields

$$N_2(t) = N_t(0)e^{-A_{21}t} \tag{2.3.6}$$

The N_2 population decreases with a time constant $\tau = \frac{1}{A_{21}}$ depleting the number N_2 at level E_2 at a rate N_2/τ or $A_{21}N_2$ and increasing the number N_1 at level E_1 at the same rate. the constant τ is referred to as the spontaneous radiative lifetime of level E_2, the coefficient A_{21} is referred to as the radiative rate usually measured in unit of S^{-1} the coefficient A_{21} is constant characteristic of the atoms.

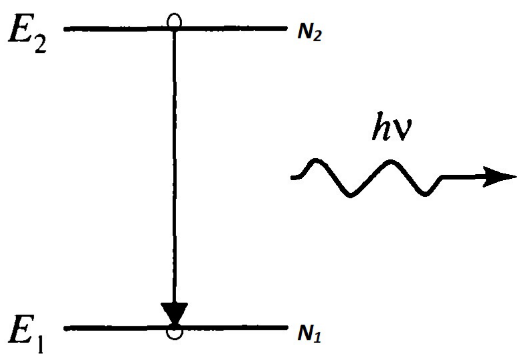

Figure 2.2: *Spontaneous Emission*

2.4 Stimulation Emission (B_{21})

In this process as shown in (fig:2.3) the rate at which the N_2 atoms are stimulated by the radiation field (photon) to drop from level E_2 to level

E_1 is proportional to both the number of atoms present and the density of radiation field.

$$\left(\frac{dN_2}{dt}\right)_{se} = -B_{21}N_2\rho(\nu) \tag{2.4.7}$$

Where the photon density is expressed as function of frequency by the factor $\rho(\nu)$

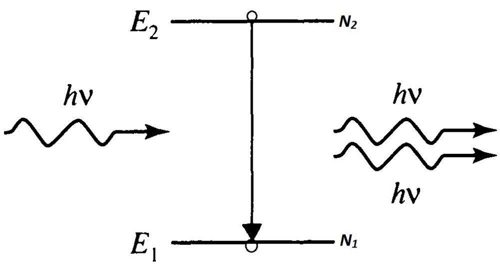

Figure 2.3: *Stimulation Emission*

2.5 Absorption (B_{12})

Absorption is also stimulated process, since it depends on the strength of the photon field. In effect, stimulation absorption and stimulated emission are inverse processes. The rate at which N_1 atoms are raised from energy level N_1 to N_2 is given by:

$$\left(\frac{dN_1}{dt}\right)_{abs} = -B_{12}N_1\rho(\nu) \tag{2.5.8}$$

The coefficient B_{12} is a constant characteristic of the atom. It turns out B_{12} and B_{21} are closely related. from (fig:2.4) The rate of change of atoms in level E_2 is by:

$$\frac{dN_2}{dt} = 0 = -N_2A_{21} - N_2B_{21}\rho(\nu) + N_1B_{12}\rho(\nu) \tag{2.5.9}$$

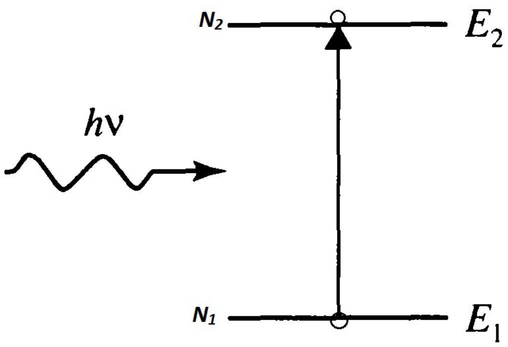

Figure 2.4: *Absorption*

The spectral energy density $\rho(\nu)$ of blackbody radiation [9, 10].

$$\rho(\nu) = \frac{8\pi h\nu^3}{c^3}\frac{1}{e^{\frac{h\nu}{kT}} - 1} \tag{2.5.10}$$

and for the boltzmann distribution of atoms between the two energy levels,

$$\frac{N_2}{N_1} = e^{-(E_2-E_1)/kT} = e^{-\frac{h\nu}{kT}} \tag{2.5.11}$$

In eqs (2.5.10) and (2.5.11) ν is frequency of radiation such that $h\nu = E_2 - E_1$, T is absolute temperature, and K is boltzmann constant. solving Eq:(2.5.9) for $\rho(\nu)$ and substituting for $\frac{N_1}{N_2}$ from Eq:(2.5.11) we obtain [11]

$$\rho(\nu) = \frac{A_{21}}{B_{12}(\frac{N_1}{N_2}) - B_{21}} = \frac{A_{21}}{B_{12}e^{\frac{h\nu}{kT}} - B_{21}} \tag{2.5.12}$$

Equating this expression for $\rho(\nu)$ to that given in Eq:(2.5.10)

$$\frac{A_{21}}{B_{12}e^{\frac{h\nu}{kT}} - B_{21}} = \frac{8\pi h\nu^3}{c^3}\frac{1}{e^{\frac{h\nu}{kT}} - 1} \tag{2.5.13}$$

Rearranging to isolate multipliers of the term $e^{\frac{h\nu}{kT}}$

$$\left(\frac{A_{21}}{B_{21}} - \frac{8\pi h\nu^3}{c^3}\frac{B_{12}}{B_{21}}\right)e^{\frac{h\nu}{kT}} - \left(\frac{A_{21}}{B_{21}} - \frac{8\pi h\nu^3}{c^3}\right) = 0 \tag{2.5.14}$$

Then,

$$\frac{A_{21}}{B_{21}} = \frac{8\pi h\nu^3}{c^3} \tag{2.5.15}$$

and

$$B_{12} = B_{21} \tag{2.5.16}$$

The last two equations were first derived by Einstein [12]. We can, using Equation (2.5.15), rewrite the induced transition rate (2.5.8) as

$$\frac{dN_1}{dt} = \frac{A_{21}c^3}{8\pi h\nu^3}\rho(\nu) = \frac{c^3}{8\pi h\nu^3 t_{spont}}\rho(\nu) \tag{2.5.17}$$

where $t_{spont} = 1/A_{21}$ is the spontaneous lifetime of the atom. Equation (2.5.17) gives the transition rate per atom due to a field with a uniform (white) spectrum with energy density per unit frequency $\rho(\nu)$.

2.6 Light Amplification and Population Inversion

Light amplification is achieved by stimulated emission. Ordinary optical materials donot amplify light. Instead, they tend to absorb or scatter the light so that the light intensity out of the medium is less than the intensity that went in. To get amplification you have to drive the material into a non-equilibrium state by pumping energy into it as shown in fig:(2.5). The amplification of the medium is determined by the gain coefficient γ which is defined by the following equation [13].

$$\begin{aligned} I(z+dz) &= I(z) + \gamma I(z)dz & (2.6.18) \\ &= I(z) + dI, & (2.6.19) \end{aligned}$$

where $I(z)$ represents the intensity at a point within the gain medium. The differential equation can be solved as follows

$$dI = \gamma I dz$$

$$\int_{I_0}^{I} \frac{dI}{I} = \int_0^z \gamma dz$$

$$\begin{aligned}\ln I - \ln I_0 &= \gamma z \\ \ln\left(\frac{I}{I_0}\right) &= \gamma z \\ \frac{I}{I_0} &= e^{\gamma z} \\ I &= I_0 e^{\gamma z} \end{aligned} \tag{2.6.20}$$

The amplification coefficient β also can be derived from the Lambert-beer law [9, 14]

$$I = I_0 e^{\beta z}, \tag{2.6.21}$$

where, I is the intensity transmitted through the sample, I_0 is the incident intensity and z stands for the distance traversed by radiation in the medium. Rate of atoms in level 1 and 2 can be given as

$$\begin{aligned}\frac{dN_1}{dt} &= (-B_{12}n_1 + B_{21}n_2)\rho(\nu) + A_{21}n_2 \\ \frac{dN_2}{dt} &= (B_{12}n_1 - B_{21}n_2)\rho(\nu) - A_{21}n_2 \end{aligned} \tag{2.6.22}$$

At equilibrium we will have a simple balancing, in which the net change in the number of any excited atoms is zero, being balanced by loss and gain due to all processes. With respect to bound-bound transitions, one will have detailed balancing as well, which states that the net exchange between any two levels will be balanced. This is because the probabilities of transition cannot be affected by the presence or absence of other excited atoms. Detailed balance requires that the change in time of the number of atoms in level 2 due to the above three processes be zero therefore; $\frac{dn_2}{dt} = 0$ thus Eq:(2.6.22) becomes: $(B_{12}n_1 - B_{21}n_2)\rho(\nu) - A_{21}n_2 = 0$. On the other hand the rate of electron transition $-\frac{dn_2}{dt}A\Delta z$ from level 2 is equal to the rate of photon emission $\frac{\Delta I}{hf}A$ through the rate A

$$-\frac{dN_2}{dt}A\Delta z = \frac{\Delta I}{hf}A \tag{2.6.23}$$

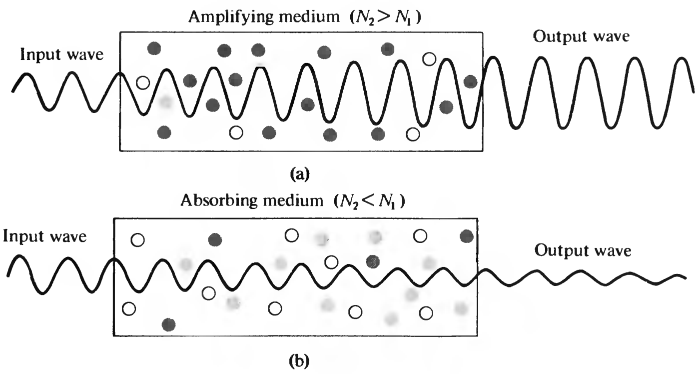

Figure 2.5: *Amplification of a traveling electromagnetic wave in (a) an inverted population* ($N_2 > N_1$) *and (b) attenuation in an absorbing* ($N_2 < N_1$) *medium.*

Source: Amnon Yariv and Pochi Yeh, Photonics Oxford University Press, P 28 (2007).

By reviewing of Eq:(2.6.23) and neglecting the condition of the spontaneous emission are getting

$$(B_{12}n_1 - B_{21}n_2)\rho(\nu)A\Delta z = \frac{\Delta I}{hf}A$$

But since $I = \rho c$, then,

$$(B_{12}n_1 - B_{21}n_2)\rho(\nu)I\frac{A\Delta z}{c} = \frac{\Delta I}{hf}A$$

Bearing in mind that

$$B = B_{21} = B_{12}$$

are getting

$$\frac{\Delta I}{\Delta z} = \frac{dI}{dz} = B(n_2 - n_1)\frac{hfI}{c} \tag{2.6.24}$$

Hence,

$$\int \frac{dI}{I} = B(n_2 - n_1)\frac{hf}{c}\int dz$$

$$\ln I = B(n_2 - n_1)\frac{hf}{c}z + c_0$$
$$I = I_0 e^{B(n_2-n_1)\frac{hf}{c}z} \tag{2.6.25}$$

Comparing Eqs: (2.6.21) with (2.6.25) one find that the amplification coefficient β is given by [15, 16].

$$\beta = B(n_2 - n_1)\frac{hf}{c}z \tag{2.6.26}$$

Chapter 3

Special Relativity

3.1 introduction

In this chapter will be discussed fundamental principles of the special theory of relativity, and deduce some of the consequences of the theory and postulates. The special theory of relativity gives the most basic foundation that underlies all physical laws in inertial systems of reference. Inertial systems of reference are those systems of reference for which the law of inertia holds. According to the law of inertia or Newton's first law, a material particle in an inertial system of reference, which is free from external influences, will stay at rest or continue to move in a straight line with constant velocity. Before we start to discuss the essential parts of relativistic electromagnetic theory, we briefly summarize the historical background from which the special theory of relativity emerged. The second law of the Newtonian mechanics, which had occupied the absolute position in physics before the latter half of the 19th century, is a basic law of motion which holds in an inertial system of reference. However, any system of reference moving with uniform velocity relative to an inertial system of reference is also another inertial system of reference, because the law of inertia holds in the latter system. Thus it follows that there are infinite sets of inertial systems of reference and Newton's second law holds in all these inertial systems. This principle is called the Galilean principle of relativity. In other words, the form of Newton's second law is kept invariant under the coordinate transformation referred to as the Galilean transformation. For the later discussion, it should be noted that the usual addition theorem of velocities holds for the Galilean transformation. On the other hand, the basic law to describe electromagnetic phenomena is the Maxwell equations established by Maxwell in 1864. With the aid of his equations, he predicted the existence of electromagnetic waves, showing that the light is a kind of electromagnetic wave. Special relativity (SR) or the 'special theory of relativity' was discovered by Albert Einstein and first published in 1905 in the article "On the Electrodynamics of Moving Bodies"[17, 18] . It replaced Newtonian notions of space and time and it incorporates Maxwell's theory of electromagnetism. The theory is called "special" because it applies the principle of relativity to the "restricted" or "special" case of inertial reference frames in 'flat' spacetime where the effects of gravity can be ignored. The special relativity is come from the published of the article entitled " on the electrodynamics of moving bodies written by Einstein. Where he was reformulated the notions of space and time starting from two postulates [19, 20].

3.2 Postulates of Special Relativity

Einstein published the article entitled On the electrodynamics of moving bodies (1905), where he reformulated the notions of space and time starting from two postulates [18].

1. The same laws of electrodynamics and optics will be valid for all frames of reference for which the equations of mechanics hold good. (Principle of relativity)

2. Light is always propagated in empty space with a definite velocity c which is independent of the state of motion of the emitting body.

The postulates refer to the validity of Maxwells laws in all inertial frames. In this way, Einstein includes Maxwells laws in the set of fundamental laws satisfying the Principle of relativity and eliminates any possibility of detecting the state of (absolute) motion of an inertial frame by means of electromagnetic experiments such as the phenomena of electrodynamics as well as of mechanics possess no properties corresponding to the idea of absolute rest.

3.2.1 Einstein Postulates

Einstein based the special theory of relativity on two postulates [18].

(A) The laws of nature are the same in all inertial reference frames.

(B) The speed of light is a finite constant that is the same in all inertial frames.

3.3 Galilean Transformations

The Galilean transformation is the transformation which connects one inertial frame with another. consider two inertial frames S and S' moving to positive x-axes with constant velocity. If S is an inertial frame then any

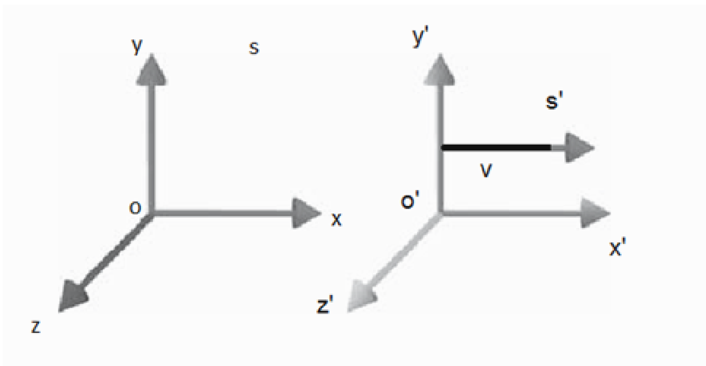

Figure 3.1: *Two inertial frames S and S' in standard configuration*
Source: M. P . Hobson, G . P . Efstathiou and A . N . Lasenby, General Relativity, Cambridge University Press, P 2 (2006).

other frame S' with parallel axes moving with constant velocity with respect to S is also an inertial frame. Suppose that S' moves along the x-axis with constant velocity v, and that the two frames coincide at time $t = 0$ if at time

t a given point P has coordinates (x, y, z) in S, then it should be clear from Fig.(3.1) that its coordinates $(x^{'}, y^{'}, z^{'})$ in $S^{'}$ are [21].

$$\begin{aligned} x^{'} &= x - \upsilon t \\ y^{'} &= y \\ z^{'} &= z \\ t^{'} &= t \end{aligned}$$

The formulas above are said to define a Galileo transformation (which is nothing more than a change of inertial frame). The inverse transformation is very simple [22].

$$\begin{aligned} t &= t^{'} \\ x &= x^{'} + \upsilon t \\ y &= y^{'} \\ z &= z^{'} \end{aligned}$$

In other words, we just have to change the sign of v. This is what one would expect, as S is moving with respect to $S^{'}$ with velocity $-v$

3.4 Velocity Addition Formula

A consequence of the Galileo transformations is the velocity addition formula is given by [23].

$$u = \frac{\Delta x}{\Delta t}$$

where $\Delta x = x_2 - x_1$ is the distance traveled by P between the positions x_1 and x_2 and $\Delta t = t_2 - t_1$ time interval between times t_1 and t_2 both measured in S. In $S^{'}$, P moves by $\Delta x^{'}$ between positions $x_1^{'}$ and $x_2^{'}$ in the time interval $\Delta t^{'}$ between times $t_1^{'}$ and $t_2^{'}$ of $t_1^{'}$, $t_2^{'}$, $x_1^{'}$, $x_2^{'}$ are related to: The values of t_1, t_2, x_1, x_2 by a Galileo transformation [23]

$$\begin{aligned} t_1^{'} &= t_1 \\ t_2^{'} &= t_2 \\ x_1^{'} &= x_1 - \upsilon t_1 \\ x_2^{'} &= x_2 - \upsilon t_2 \end{aligned}$$

Therefore

$$\begin{aligned}\Delta t^{'} &= \Delta t \\ \Delta x^{'} &= \Delta x - v\Delta t\end{aligned}$$

and the instantaneous velocity of P in $S^{'}$ is

$$u^{'} = \frac{\Delta x^{'}}{\Delta t^{'}} = \frac{\Delta x - v\Delta t}{\Delta t} = \frac{\Delta x}{\Delta t} - v = u - v$$

In other words, the velocity u of P in S is simply the sum of the velocity $u^{'}$ of P in $S^{'}$ with the velocity v of $S^{'}$ with respect to S.

3.5 Lorentz Transformations

The Principle of Special Relativity states that the laws of nature are invariant under a particular group of space-time coordinate transformations, called Lorentz transformations [21]. A lorentz transformation is a transformation from one system of space-time coordinates S to another system $S^{'}$. Let us derive a new coordinate transformation from one inertial system to another, which replaces the Galilean transformation, on the basis of Einstein's two postulates for the special theory of relativity. For this purpose, we consider two inertial systems $S(x, y, z, t)$ and $S^{'}(x', y', z', t')$. The inertial system $S^{'}$ is assumed to be uniformly moving in the z direction with constant velocity v relative to the inertial system S, keeping each coordinate axis parallel to the corresponding axis of the latter, as shown in Fig.(3.2). Now, let an event occur at the position (x, y, z) at the time t in the inertial system S, and let the same event occur at the corresponding position $(x^{'}, y^{'}, z^{'})$ and at the corresponding time $t^{'}$ in the inertial system $S^{'}$. Then, we try to find how the sets of space-time coordinates, (x, y, z, t) and $(x^{'}, y^{'}, z^{'}, t^{'})$ are transformed to each other under the two postulates for the special theory of relativity. From the law of inertia, it is apparent that a uniform rectilinear motion in one inertial system corresponds to another uniform rectilinear motion in the other inertial system. Hence, first of all, the transformation between two inertial systems S and $S^{'}$ must be given by linear equations in terms of space-time coordinates [24].

$$x^{'} = \gamma(x - vt), \qquad y^{'} = y, \qquad z^{'} = z,$$

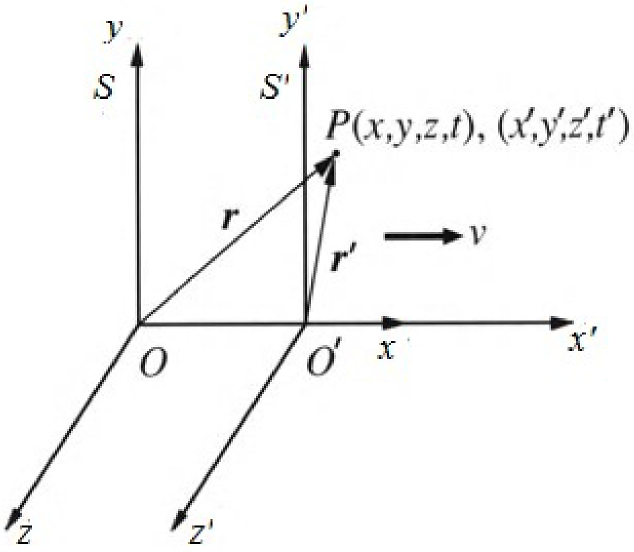

Figure 3.2: *Inertial systems S and S'*
Source: Toshiyuki Shiozawa, *Classical Relativistic Electrodynamics: Theory of Light Emission and Application to Free Electron Lasers*, Springer-Verlag Berlin Heidelberg, (2004).

$$t' = \gamma(t - \frac{vx}{c^2}),$$

where c represents the speed of light.
The special theory of relativity, developed by Einstein in 1905, boils down to analyzing the consequences of these transformations. The velocities with which we usually deal are much smaller than the speed of light, $|v| << c$. In this case c is almost equal to 1, and $\frac{vx}{c^2}$ is almost equal to zero. Therefore for most applications the Lorentz transformation formulas reduce to the Galileo transformation formulas. It is only when the velocities involved become comparable to the speed of light that the Lorentz transformations become important. It is easy to check that the inverse transformation formulas are obtained

$$t = \gamma(t' - \frac{vx'}{c^2})$$

$$x = \gamma(x' - vt')$$

The Lorentz transformation formulas require $|v| < c$ given two inertial frames, the velocity of one of them with respect to the other must be less than the speed of light. More generally, the Lorentz transformations imply that light moves with the same speed in all inertial frames. To check this fact we need the relativistic velocity addition formula.

$$\Delta t' = \gamma(\Delta t - \frac{v\Delta x}{c^2})$$
$$\Delta x' = \gamma(\Delta x - v\Delta t)$$

Consequently, the instantaneous velocity of P in S' is

$$u' = \frac{\Delta x'}{\Delta t'} = \frac{\Delta x - v\Delta t}{\Delta t - \frac{v\Delta x}{c^2}} = \frac{u - v}{1 - \frac{uv}{c^2}}$$

In the special case when $u = c$ we obtain:

$$u' = \frac{c - v}{1 - \frac{v}{c}} = c.\frac{c - v}{c - v} = c$$

If on the other hand $u = -c$ we get

$$u' = \frac{-c - v}{1 + \frac{v}{c}} = -c.\frac{c + v}{c + v} = -c$$

Therefore whenever P moves at the speed of light in S it also moves at the speed of light in S'.

3.6 Length contraction and time dilation

Two elementary consequences of the Lorentz transformations are length contraction and time dilation. Both these effects are easily derived from [24].

$$\begin{aligned} ct' &= \gamma(ct - \beta x) \\ x' &= \gamma(x - \beta ct) \\ y' &= y \\ z' &= z \end{aligned} \tag{3.6.1}$$

Where $\beta = \frac{v}{c}$ and $\gamma = (1-\beta^2)^{-1/2}$. This Lorentz transformation, also known as a boost in the x-direction, reduces to the Galilean transformation when $\beta << 1$.

3.6.1 Length contraction

Consider a rod fixed in $S^{'}$ with endpoints $x^{'}_{A}$ and $x^{'}_{B}$ as shown in Fig.(3.3). In S, the ends have coordinates x_A and x_B given by the Lorentz transformations

$$L_0 \;=\; x^{'}_{B} - x^{'}_{A}$$

We want to apply the Lorentz transformation formulae and so find what length an observer in frame S assigns to the rod. Applying the second formula in equation (3.6.1), we obtain

$$\begin{aligned} x^{'}_{A} &= \gamma(x_A - \upsilon t_A) \\ x^{'}_{B} &= \gamma(x_B) - \upsilon t_B \end{aligned}$$

relating the coordinates of the ends of the rod in $S^{'}$ to the coordinates in S. The observer in S measures the length of the rod at a fixed time $t = t_A = t_B$ as

$$L \;=\; x_B - x_A = \frac{1}{\gamma}(x^{'}_{B} - x^{'}_{A}) = \frac{L_0}{\gamma}$$

Hence in S the rod appears contracted to the length

$$L \;=\; L_0\gamma^{-1}$$

If a rod is moving relative to S in a direction perpendicular to its length, however, it is straightforward to show that it suffers no contraction.

3.6.2 Time dilation

Suppose we have a clock at rest in $S^{'}$, in which two successive clicks of the clock (events A and B) are separated by a time interval T_0 . The times of the clicks as recorded in S are

$$\begin{aligned} t_A &= \gamma(t^{'}_{A} + \frac{\upsilon x^{'}_{A}}{c^2}) \\ t_B &= \gamma(t^{'}_{A} + \frac{\upsilon x^{'}_{B}}{c^2}) \end{aligned}$$

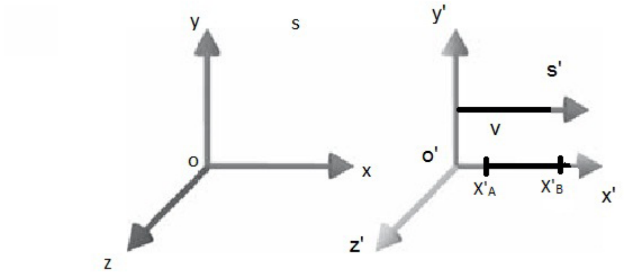

Figure 3.3: *Two inertial frames S and $S^{'}$ in standard configuration. A rod of proper length L_0 is at rest in $S^{'}$.*

Source: M. P . Hobson et al. General Relativity: An introduction for Physicist, Cambridge University Press, P 10 (2006).

Since the clock is at rest in $S^{'}$ we have $x_A = x_B$, and so on subtracting we obtain

$$T \;=\; t_B - t_A = \gamma T_0 = \frac{T_0}{(1 - \frac{v^2}{c^2})^{1/2}}$$

Hence, the moving clock ticks more slowly by a factor of $(1 - \frac{v^2}{c^2})^{1/2}$ (time dilation).

3.7 Derivation of the Lorentz Transformation Formulas

In what follows will discussed a derivation of the Lorentz transformation formulas, due to Einstein. Einstein started with two postulates [24]:

1. Relativity principle: Any two inertial frames are equivalent.

2. Invariance of the speed of light: The speed of light is the same in all inertial frames.

Since the Galileo transformations are not compatible with the second postulate, we cannot expect the obvious formula $x^{'} = x - \upsilon t$ to work. Suppose however that $x^{'}$ is proportional to $x - \upsilon t$, that is,

$$x^{'} = \gamma(x - \upsilon t)$$

for some constant γ (to be determined). Since S moves with respect to $S^{'}$ with velocity $-\upsilon$, the first postulate requires an analogous formula for the inverse transformation:

$$x = \gamma(x^{'} + \upsilon t^{'})$$

Solving for $t^{'}$ yields

$$t^{'} = \frac{x}{\upsilon\gamma} - \frac{x^{'}}{\upsilon}$$

Substituting in this formula the initial expression for $x^{'}$ gives

$$t^{'} = \left(\frac{1}{\gamma} - \gamma\right)\frac{x}{\upsilon} + \gamma t$$

We now use the second postulate. Consider a light signal propagating along the x-axis in S, passing through $x = 0$ at time $t = 0$. The position of the signal at time t will then be $x = ct$. On the other hand, the second postulate requires that the position of the signal in $S^{'}$ be $x^{'} = ct^{'}$. Therefore [24].

$$c = \frac{x^{'}}{t^{'}} = \frac{\gamma(x - \upsilon t)}{\left(\frac{1}{\gamma} - \gamma\right)\frac{x}{\upsilon} + \gamma t} = \frac{\frac{x}{t} - \upsilon}{\left(\frac{1}{\gamma^2} - 1\right)\frac{x}{\upsilon t} + 1} = \frac{c - \upsilon}{\left(\frac{1}{\gamma^2} - 1\right)\frac{c}{\upsilon} + 1}$$

$$c = \frac{c - \upsilon}{\left(\frac{1}{\gamma^2} - 1\right)\frac{c}{\upsilon} + 1}$$

$$c\left(\left(\frac{1}{\gamma^2} - 1\right)\frac{c}{\upsilon} + 1\right) = c - \upsilon$$

$$\left(\frac{1}{\gamma^2}-1\right)\frac{c^2}{\upsilon}+c \quad = \quad c-\upsilon$$

take c in right hand side become

$$\left(\frac{1}{\gamma^2}-1\right)\frac{c^2}{\upsilon} \quad = \quad -\upsilon$$

multiply above equation by $\frac{1}{\upsilon}$ we obtain

$$\left(\frac{1}{\gamma^2}-1\right)\frac{c^2}{\upsilon^2} \quad = \quad -1$$

also multiply above equation by $\frac{\upsilon^2}{c^2}$ yields

$$\left(\frac{1}{\gamma^2}-1\right) \quad = \quad -\frac{\upsilon^2}{c^2}$$

Now we isolate the desired term $\frac{1}{\gamma^2}$:

$$\frac{1}{\gamma^2} \quad = \quad 1-\frac{\upsilon^2}{c^2}$$

Inverting this, we get

$$\gamma \quad = \quad \pm\frac{1}{\sqrt{1-\frac{\upsilon^2}{c^2}}}$$

Taking square roots of both sides, we obtain

$$\gamma \quad = \quad \frac{1}{\sqrt{1-\frac{\upsilon^2}{c^2}}}$$

3.8 Lorentz transformations Important Formulas

By using the factor $\gamma = \frac{1}{\sqrt{1-\frac{\upsilon^2}{c^2}}}$ Lorentz transformations formulas gives [24].

$$t^{'} = \gamma(t - \frac{\upsilon x}{c^2})$$

$$x^{'} = \gamma(x - \upsilon t)$$

or

$$t = \gamma(t^{'} + \frac{\upsilon x^{'}}{c^2})$$

$$x = \gamma(x^{'} + \upsilon t^{'})$$

The formulas of velocity addition and time dilation gives by

$$u^{'} = \frac{u - \upsilon}{1 - \frac{u\upsilon}{c^2}}$$

or

$$u = \frac{u^{'} - \upsilon}{1 - \frac{u^{'}\upsilon}{c^2}}$$

$$\Delta t^{'} = \frac{\Delta t}{\gamma} = \Delta t\sqrt{1 - \frac{\upsilon^2}{c^2}}$$

Chapter 4

Free Electron laser with electromagnetic field

4.1 Introduction

The Free Electron Laser is essentially a device that transform the kinetic energy of an electron beam (e-beam) into electromagnetic waves radiation [25]. The relativistic e-beam passing through a periodic magnetic field oscillates in the transverse direction and emits radiation (synchrotron radiation) confined in a narrow cone along the propagation direction. The periodic magnetic field is provided by the so-called wiggler, an insertion device usually realized with two arrays of permanent magnets with alternating polarities or with two helical coils with current circulating in opposite directions. The wavelength of the emitted radiation depends on the wiggler period, on the strength of the magnetic field and on the electron energy. This means that the FEL can be continuously tuned in frequency, ranging from microwaves ($\lambda \simeq 1cm$) to X-rays ($\lambda \simeq 1A$); The physical mechanism in the free-electron laser depends upon the propagation of an electron beam through a periodic magnetic field. Both incoherent and coherent radiation result from the undulatory motion of the electron beam in the external fields which permits a wave-particle coupling to the output radiation. Coherent radiation depends upon the stimulated emission due to the ponderomotive wave formed by the beating of the radiation and wiggler fields. The wiggler field itself may be either magnetostatic or electromagnetic in nature.

The basic difference between magnetostatic and electromagnetic-wave wigglers lies in the frequency of the output radiation, which depends upon both the wiggler period and the beam energy in both cases. In the case of a magnetostatic wiggler, the wavelength of the output radiation scales as $\lambda = \frac{\lambda_u}{2\gamma^2}$ where λ_u denotes the wiggler period and γ is the bulk relativistic factor of the beam. In contrast, the wavelength of the output radiation for an electromagnetic-wave wiggler scales as $\lambda = \frac{\lambda_u}{4\gamma^2}$. As a result, for fixed wiggler periods and beam energies, the electromagnetic-wave wiggler will produce shorter output wavelengths. As a consequence, electromagnetic-wave wigglers become attractive alternatives to magnetostatic wigglers for the production of short wavelengths when the electron-beam energy is constrained. Several different configurations have been proposed, and analyzed, to make use of electromagnetic-wave wigglers. The electromagnetic wave acts as a wiggler which induces an undulatory motion on the beam. The electrons executing oscillations in the periodic magnetic field represent the active medium of a free-electron laser. The electric field of the light wave inside the undulator is written in the form, Amplification condition From Electromagnetic Theory, According to the Electromagnetic Theory, the electric field intensities is given by [28]:

$$E = E_0 e^{-k2x} e^{i(k_1 \omega t)} \tag{4.1.1}$$

Thus the light intensity is given by:

$$I = |E|^2 = I_0 e^{-2k2x} = I_0 e^{\beta x}, \tag{4.1.2}$$

where the factor β is called amplification factor. the static magnetic field of the undulator (that in the electron reference frame becomes an electromagnetic field). Let us now consider what happens when other EM modes are present during the interaction. We will observe the emission properties of such a system and the variations of the modes intensity during the process. The EM mode copropagating with the electrons inside the undulator, and Electrons oscillate inside the undulator in the transverse plane xz with period λ_u. In order to obtain energy exchange between the electrons and the EM field it is necessary to have synchronism between the transverse oscillations of the electrons and the oscillations of the Electric field of the copropagating EM wave. This will happens if the electron, after one undulator period, will find the electric field with the same phase. If we remind that the electron velocity $V_z < c$, it is evident that this condition can be fulfilled if the mean longitudinal electron velocity V_z is chosen in such a way that the electron performs a complete oscillation in the time needed for the light to cover an undulator period plus a wavelength. This condition can be expressed as follows [29]:

- Being t_e the time needed for the electron to cover an undulator period λ_u
- Being t_p the time needed for the EM wave to cover the distance $\lambda_u + \lambda$

the synchronism condition is expressed by the equation $t_e = t_p$; if we remind that: $t_e = \frac{\lambda_u}{V_z}$ and that $t_p = \frac{\lambda_u+\lambda}{V_f}$, where V_f is the phase velocity of the EM wave $v_f = \frac{\omega}{k}$, we obtain:

$$\frac{\lambda_u}{V_z} = \frac{\lambda_u + \lambda}{V_f}, \tag{4.1.3}$$

if we define $K_u = \frac{2\pi}{\lambda_u}$ eq.(4.1.3) can be rewritten as:

$$V_f \lambda_u = V_z(\lambda_u+\lambda) \Rightarrow \frac{V_f}{K_u} = V_z\left(\frac{1}{K_u} + \frac{1}{K}\right) = V_z\left(\frac{K+K_u}{KK_u}\right) \Rightarrow V_f K = V_z(K+K_u)$$

taking in mind that $V_f = \frac{\omega}{K}$ and $V_z = \beta_z c$ we obtain:

$$\frac{\omega}{c} = \beta_z(K + K_u) \tag{4.1.4}$$

This equation, that defines the so called beam line, describes the points of the $(k, \frac{w}{c})$ space where the synchronism condition is fulfilled. If the interaction occurs in vacuum, the dispersion relation is linear: $\frac{w}{c} = k$

$$\begin{cases} \frac{\omega}{c} = K \\ \\ \frac{\omega}{c} = \beta_z(K + K_u) \end{cases} \Rightarrow K(1-\beta_z) = K_u\beta_z$$

If we remind that $\gamma_z = \frac{1}{\sqrt{1-\beta_z^2}}$, then;

$$\frac{\omega}{c} = K = K_u\frac{\beta_z}{1-\beta_z} \tag{4.1.5}$$

$$
\begin{aligned}
&= K_u \frac{\beta_z}{(1-\beta_z)} \frac{(1+\beta_z)}{(1+\beta_z)} \\
&= K_u \frac{\beta_z(1+\beta_z)}{(1-\beta_z^2)} \\
&= K_u \beta_z \gamma_z^2 (1+\beta_z)
\end{aligned}
$$

Eq.(4.1.5) for relativistic electrons ($\beta \sim 1$) becomes [29]:

$$\frac{\omega}{c} = 2\gamma_z^2 K_u \tag{4.1.6}$$

In terms of wavelength we have [30]:

$$\lambda = \frac{\lambda_u}{2\gamma^2} \tag{4.1.7}$$

where λ_u is the undulator period, γ is the relativistic factor and k_u is the so called undulator parameter which is proportional to the magnetic field inside the undulator. The amplification is due to the energy transfer from the electrons to the previously emitted wave. The microbunching is caused by the interaction between the electrons oscillating in the transverse direction and the transverse B-field of the previously emitted waves. The microbunching Lorentz force is proportional to the transverse electron velocity and to the wave B-field strength B_W. This is because the force is given by

$$F = eBv$$

But

$$
\begin{aligned}
|E|^2 &= I \\
|E| &= I^{\frac{1}{2}}
\end{aligned}
$$

Since the magnetic flux density B is given by

$$B = cE$$

Thus

$$\frac{B}{c} = I^{\frac{1}{2}}$$

Hence

$$F = ecI^{\frac{1}{2}}v$$

Since B_W is proportional to the square root of the wave intensity, the microbunching force is proportional to $I^{1/2}$. Multiplied by the energy transfer rate for each electron, this factor gives $dI/dt = AI$ with $A =$ constant, corresponding indeed to an exponential intensity increase along the undulator. Assuming $A = u/L_G$, we obtain the commonly used form for the exponential intensity law [25].

$$I = I_0 exp\left(\frac{ut}{L_G}\right) = I_0 exp\left(\frac{x}{L_G}\right) \tag{4.1.8}$$

Where,

$$x = ut$$

The parameter L_G, called gain length, characterizes the amplification and the corresponding requirements to obtain lasing.
Since the lasing medium in free electron laser is unbounded free, relativistic electrons the generated laser is called free electron laser. FEL, is generated either in a undulator or a wiggler. Electrons traveling with almost the speed of light experience a fierce oscillating motion in the magnetic field of the undulator or wiggler which cause them radiating photons. The wavelength of the radiated photons can be estimated via: Wiggler magnets

$$\lambda = \frac{\lambda_w}{2\gamma^2}\left(1 + K^2\right)$$

Undulator magnets

$$\lambda = \frac{\lambda_u}{2\gamma^2}\left(1 + \frac{K^2}{2}\right),$$

where K is the magnetic deflection factor (also known as undulator or wiggler parameter) and is measure of the applied magnetic field's strength. Another important parameter is γ Lorentz contraction factor or relativistic Doppler shift. The wavelength of generated light denoted λ, λ_u and λ_u is measure of the undulator or wiggler period. The radiated photons can in their turn interact with the electrons energetically and force them to be divided into microbunches based on their energy. The bandwidth can be estimated by taking into account that each electron going through the undulator emits a wave train consisting of a number of wavelengths equal to the number of undulator periods, N_u. The time duration Δt of this pulse is the pulse length divided by the speed of light, $N_u\lambda/c$. According to the Fourier transforms, a pulse of duration Δt has a frequency bandwidth $\Delta\nu = 1/\Delta t$; thus, $\Delta\nu = c/(N_u\lambda)$. Wavelength and frequency are related as $\nu = c/\lambda$, which by differentiation gives $\Delta\nu = c\Delta\lambda/\lambda^2$, thus $\Delta\lambda = \Delta\nu\lambda^2/c = \lambda/N_u$ and since the duration of pulse is Δt and corresponding pulse length is $\Delta\lambda$. Thus the number of pulses for one wave is given by

$$N_u = \frac{\lambda}{\Delta\lambda}$$

$$\frac{\Delta\lambda}{\lambda} = \frac{1}{N_u}$$

a relative wavelength bandwidth decreasing as the number of undulator periods increases.

4.2 Factors influencing the gain length and the amplification

The exponential amplification is preceded by a preliminary phase with a slower intensity build-up, and is followed by the saturation phase. Remember that the rate of energy transfer from an individual electron to the pre-existing wave is proportional to $I^{1/2}v_T$. Thus, to find the amplification we must evaluate v_T. However, the total correlated emission intensity from all electrons also depends on microbunching; thus, to find the amplification we must also evaluate the degree of microbunching. For transverse-motion dynamics, the relevant equation is Newtons law with the relativistic mass [25]

$$\gamma m_0 \frac{dv_T}{dt} = transverse \quad force = -euB = -euB_0 \sin\frac{2\pi ut}{L}$$

which gives

$$v_T = \left(\frac{-euB_0}{\gamma m_0}\right)\left(\frac{L}{2\pi u}\right)\cos\left(\frac{2\pi ut}{L}\right) = -\frac{eB_0L}{2\pi\gamma m_0}\cos\left(\frac{2\pi ut}{L}\right),$$

which is proportional to (B_0L/γ). Thus, the energy transfer rate by a single electron is proportional to $I^{1/2}\,(B_0L/\gamma)$. This is since the power is given by

$$Power = Pr = \frac{dw}{dt} = F.\frac{dr}{dt} = F.\upsilon,$$

where

$$F \sim eE$$

The electric intensity is

$$|E|^2 = I$$

$$F = eE = eI^{\frac{1}{2}}$$

$$\upsilon = \frac{eB_0L}{2\pi m_0\gamma}$$

$$Pr = \frac{I^{\frac{1}{2}}e^2B_0L}{\pi m_0\gamma}$$

$$Pr \sim I^{\frac{1}{2}}\left(\frac{B_0L}{\gamma}\right)$$

and [see (4.1.8)] to $I_0^{1/2}exp[ut/\,(2L_G)]$. The longitudinal force is given from the relation

$$F = Be\upsilon$$

But:

$$B = c|B| = cI^{\frac{1}{2}},$$

υ_t (effective value) $= \frac{\upsilon_{max}}{\sqrt{2}}$,

$$\upsilon = \frac{\upsilon_{max}}{\sqrt{2}} = \frac{eB_0L}{2\sqrt{2}\pi m_0\gamma} = \frac{c_0B_0L}{\gamma},$$

where c_0 = constant.
Thus,

$$F = \frac{c_0B_0}{\gamma}ecLI^{\frac{1}{2}}$$

Hence

$$longitudinal force = constant \times \left(\frac{B_0L}{\gamma}\right) I_0^{1/2} exp\left(\frac{ut}{2L_G}\right)$$

$E_n =$ energy density $= \frac{I}{c}$,

$$I = I_0 exp\left(\frac{ut}{L_G}\right)$$

$$\frac{dI}{dt} = \frac{u}{L_G}I = \frac{c}{L_G}I$$

energy transfer rate

$$= \frac{dE_n}{dt} = \frac{1}{c}\frac{dI}{dt} = \frac{I}{L_G}$$

This force induces a small longitudinal electron displacement Δx superimposed on the average motion with speed u. For longitudinal dynamics the relevant relativistic equation is derived from the general law that the time derivative of the longitudinal momentum $\gamma m_0(d\Delta x/dt)$ equals the longitudinal force.
But

$$\frac{d\gamma}{dt} = \frac{d}{dt}\left(1 - \frac{\upsilon^2}{c^2}\right)^{-1/2} = \frac{d}{dt}\left(1 - \frac{\upsilon^2}{c^2}\right)^{-1/2}\frac{\upsilon}{dt}$$
$$= -\frac{1}{2}\left(1 - \frac{\upsilon^2}{c^2}\right)^{-3/2}\left(\frac{-2\upsilon}{c^2}\frac{d\upsilon}{dt}\right) = \gamma^3\frac{\upsilon}{c^2}\frac{d\upsilon}{dt}$$

Thus:

$$\frac{d}{dt}m\upsilon = \frac{d}{dt}\gamma m_0 \upsilon = \upsilon m_0\frac{d\gamma}{dt} + \gamma m_0\frac{d\upsilon}{dt}$$
$$= \gamma m_0\frac{d\upsilon}{dt} + \gamma^3\left(\frac{\upsilon^2}{c^2}\right)m_0\frac{d\upsilon}{dt} = \gamma^3 m_0\frac{d\upsilon}{dt}\left[\gamma^{-2} + \frac{\upsilon^2}{c^2}\right]$$
$$= \gamma^3 m_0\frac{d\upsilon}{dt}\left[1 - \frac{\upsilon^2}{c^2} + \frac{\upsilon^2}{c^2}\right] = \gamma^3 m_0\frac{d\upsilon}{dt}$$

Thus;

$$\begin{aligned} F &= \frac{d}{dt}m\upsilon = \gamma^3 m_0\frac{d\upsilon}{dt} \\ &= \gamma^3 m_0\frac{d^2\Delta x}{dt^2} \end{aligned}$$

The result is

$$\gamma^3 m_0 \frac{d^2\Delta x}{dt^2} = longitudinal force = constant \times \left(\frac{B_0 L}{\gamma}\right) I_0^{1/2} exp\left(\frac{ut}{2L_G}\right),$$

where the factor $\gamma^3 m_0$ is the so-called relativistic longitudinal mass. After integration, the above equation gives a longitudinal displacement towards microbunching,

$$\begin{aligned}\Delta x &= constant \times \frac{1}{\gamma^3}\left(\frac{B_0 L}{\gamma}\right) L_g^2 I_0^{1/2} exp\left(\frac{ut}{2L_G}\right) \\ &= \left(\frac{B_0 L L_G^2}{\gamma^4}\right) I^{1/2}\end{aligned}$$

For $\Delta x = 0$ where the amplification and motion towards microbunching start. The corresponding number of electrons is proportional to $N(\Delta x/\lambda)$. Their contribution to the wave intensity is proportional to $N\left(\frac{\Delta x}{\lambda}\right)$ where

$$\lambda = \frac{L}{2\gamma^2} \tag{4.2.9}$$

in turn proportional to

$$N\frac{\left[\left(\frac{B_0 L L_G^2}{\gamma^4}\right) I^{1/2}\right]}{L/\gamma^2} = N\left(\frac{B_0 L_G}{\gamma^2}\right) I^{1/2}$$

. These arguments justify our previous assumption that microbunching effects correspond to a factor proportional to the longitudinal microbunching force and therefore to $I^{1/2}$. In addition, they reveal other important elements in this factor. Multiplying the factor by the energy transfer rate for one electron, we see that the total transfer rate is proportional to

$$N\left(\frac{B_0 L_G^2}{\gamma^2}\right) I^{1/2}\left(I^{1/2}\frac{B_0 L}{\gamma}\right) = N\left(\frac{B_0^2 L L_G^2}{\gamma^3}\right) I,$$

and we can write

$$\frac{dI}{dt} = constant \times N\left(\frac{B_0^2 L L_G^2}{\gamma^3}\right) I$$

$$\frac{dI}{dt} = \frac{I}{L_G}$$
$$\frac{I}{L_G} = N\left(\frac{B_0^2 L L_G^2}{\gamma^3}\right) I$$
$$L_G^3 = \frac{\gamma^3}{B_0^2}\frac{1}{N}$$
$$L_G = N^{-1/3}\gamma B_0^{-2/3} L^{-1/3}$$

This is, indeed, an equation of the form

$$dI/dt = AI = \frac{1}{L_G} I$$

whose solution is (1) as long as $\frac{u}{L_G}(\simeq \frac{c}{L_G})$ is proportional to $N\left(\frac{B_0^2 L L_G^2}{\gamma^3}\right) I$, or

$$L_G = constant \times N^{-1/3} B_0^{-2/3} L^{-1/3}\gamma \qquad (4.2.10)$$

The gain coefficient is thus given by

$$\beta = \frac{1}{L_G} = L_G^{-1}$$

$$\beta = L_G^{-1} = constant \times N^{1/3} B_0^{2/3} L^{1/3}\gamma^{-1} \qquad (4.2.11)$$

4.3 Electron Motion in an Undulator

In an FEL a beam of relativistic electrons produced by an accelerator passess through a transverse periodic field produced by a magnet called undulator and exchanges energy with an electromagnetic field (Fig.4.1). As a result of energy exchange, the electrons that gain energy begin to move ahead of the average electron, while the electrons that lose energy begin to fall behind the average.The wavelength of the emitted radiation at the resonance depends on the electron energy and the magnitude and periodicity of the undulator field according to the relation [31, 32].

$$\lambda = \frac{\lambda_u}{2\gamma^2}(1 + k^2)$$

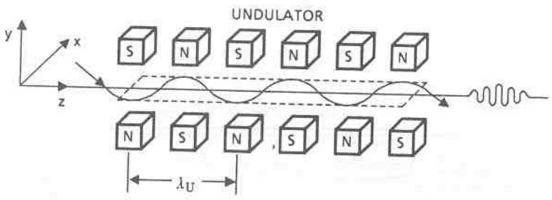

Figure 4.1: *Basic components of FEL*
Source: Dagnachew W. Workie, Basic Physical Processes and Principles of Free Electron Lasers (FELs), Cincinnati, Ohio 45221, P 2 (2001).

We call $W = E_{kin} + m_e c^2 = \gamma m_e c^2$ the total relativistic energy of the electron. The transverse acceleration by the Lorentz force is:

$$\gamma m_e \dot{\vec{v}} = -e\vec{v} \times \vec{B}. \tag{4.3.12}$$

This results in two coupled equations:

$$\ddot{x} = \frac{e}{\gamma m_e} B_y \dot{z} \qquad \ddot{z} = -\frac{e}{\gamma m_e} B_y \dot{x} \tag{4.3.13}$$

$$B_y = B_0 \cos(k_\mu Bct)$$

$$\begin{aligned} \ddot{x} &= \frac{e}{\gamma m_0} \beta c B_0 \cos(k_\mu Bct) \\ \dot{x} &= \frac{e\beta Bc}{\gamma m_0 k_\mu Bc} B_0 \sin(k_\mu Bct) \\ x &= -\frac{eB_0}{\gamma m_0 k^2 Bc} \cos(k_\mu Bct) \end{aligned}$$

$$\begin{aligned} x &= k\cos(k_z) \\ \frac{dx}{dz} &= -kk\sin(k_z), \end{aligned}$$

which are solved iteratively. To obtain the first-order solution we observe that $v_z = \dot{z} \approx v = \beta c = const$ and $v_x \ll v_z$. Then $\ddot{z} \approx 0$.

$$x(t) \approx -\frac{eB_0}{\gamma m_e \beta c k_u^2} \cos(k_u \beta ct), \qquad z(t) \approx \beta ct. \tag{4.3.14}$$

The electron travels on a cosine-like trajectory

$$x(z) = -A\cos(k_u z) \qquad \textit{with} \quad A = \frac{eB_0}{\gamma m_e \beta c k_u^2}\cos(k_u \beta c t). \tag{4.3.15}$$

The maximum divergence angle is:

$$\theta_{max} \approx \left[\frac{dx}{dz}\right]_{max} = \frac{eB_0}{\gamma m_e \beta c k_u^2}\cos(k_u \beta c t) = \frac{k}{\beta\gamma} \tag{4.3.16}$$

$$\begin{aligned} \theta_{max} &= \frac{eB_0}{\gamma m \beta c k} \\ &= \frac{eB_0 \lambda}{2\pi m \beta c} \end{aligned}$$

Here we have introduced the undulator parameter

$$K = \frac{eB_0}{\gamma m_e \beta c k_u^2} = \frac{eB_0 \lambda_u}{2\pi m_e c} \tag{4.3.17}$$

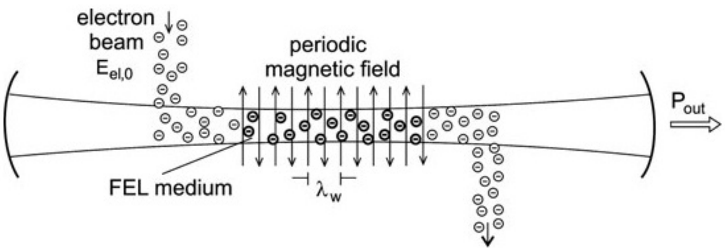

Figure 4.2: *Free-electron laser*
Source: Karl F. Renk, Basics of Laser Physics, Springer-Verlag Berlin Heidelberg, P335 (2012).

Synchrotron radiation of relativistic electrons is emitted inside a cone with opening angle $1/\gamma$. The relativistic energy of an electron, which enters a periodic magnetic field with the velocity $V_{z,0}$ is given by [33]:

$$E_{el,0} = \gamma m_0 c^2, \tag{4.3.18}$$

where, m_0 is the electron mass and

$$\gamma = \frac{1}{\sqrt{1 - \frac{V_{z,0}}{c^2}}} \tag{4.3.19}$$

is the Lorentz parameter; γ measures the relativistic energy of an electron in units of $m_0c^2; \gamma = E_{MeV}/0.51MeV$. The oscillation frequency of a free-electron oscillation is equal to

$$\nu_0 = K_{el}^2 \frac{2c\gamma^2}{\lambda_u} \tag{4.3.20}$$

$K_{el}^2 (< 1)$ is a measure of the deviation of the oscillation frequency from $2c\gamma^2/\lambda_w$. The quantity $\bar{\gamma} = K_{el}\gamma$ is an effective Lorentz parameter. It is smaller than γ according to a reduction of the initial energy of longitudinal motion due to conversion of energy of longitudinal motion into energy of transverse motion. We write

$$\frac{1}{K_{el}^2} = 1 + \frac{k_w^2}{2} \tag{4.3.21}$$

K_w is the dimensionless wiggler strength it is given by:

$$K_w = \frac{eB_w\lambda_w}{2\pi m_0 c^2} \tag{4.3.22}$$

B_w is the maximum strength of a magnetic field assumed to vary sinusoidally along the wiggler axis. Interaction of the free-electron oscillations with the high frequency field in the resonator results in conversion of a portion of power of the electron beam into power of laser radiation. The laser frequency has a value near the resonance frequency of the free-electron oscillations,

$$\nu \sim \nu_0$$

. But ν is slightly smaller than ν_0. It follows that the wavelength of the laser radiation is given by [34, 35]:

$$\lambda = \frac{\lambda_w}{2\gamma^2}\left(1 + \frac{K_w^2}{2}\right) \tag{4.3.23}$$

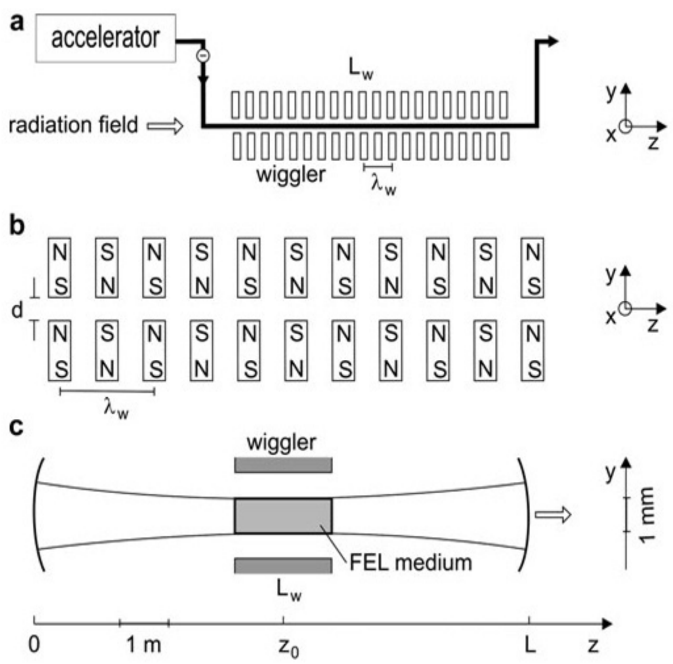

Figure 4.3: *Characterization of the free-electron laser. (a) Electron beam and wiggler. (b) Section of a wiggler. (c) Optical resonator with free-electron laser (FEL) medium*

Source: Karl F. Renk, Basics of Laser Physics, Springer-Verlag Berlin Heidelberg, P336 (2012).

The free-electron laser is a beam of relativistic electrons, produced by use of an accelerator (Fig.4.3a), traverses a spatially periodic magnetic field and excites a radiation field in the optical resonator. The electron beam, guided by a bending magnet into the resonator, passes through the periodic magnetic field, which is produced by use of a periodic magnet structure, the wiggler (= undulator). The electron beam then leaves the resonator by means of a second bending magnet. Along the resonator axis (z axis), the magnetic field direction assumes the $+y$ direction and the $-y$ direction in turn.

It varies in the simplest case sinusoidally,

$$B_y = B_w \sin \frac{2\pi}{\lambda_w} z \tag{4.3.24}$$

The length of the wiggler is $L_w = N_w \lambda_w$ and N_w is the number of wiggler periods. Due to the Lorentz force, the electrons execute oscillations perpendicular to the magnetic field direction (y direction) and perpendicular to the z direction. The electrons oscillate with displacements in $\pm x$ direction. The wiggler can consist of two rows of equal magnets, with north poles N and south poles S arranged periodically (Fig.4.3b). The magnetization of a magnet and the distance d between the rows determine the field strength B_w. Magnets prepared from a samarium-cobalt alloy, which has a high magnetization, are suitable as wiggler magnets. Alternatively, the wiggler is a superconducting magnet with a helical winding of the superconducting wires, leading to a circular sinusoidally varying transverse magnetic field. The electron beam in the range between the wiggler magnets constitutes the free electron laser medium (Fig.4.3c). A radiation field propagating in $+z$ direction is amplified. The length L of the optical resonator is larger than the length L_w of the active medium; z_0 is the center of both the optical resonator and the active medium. Thus, the optical beam is a parallel (Gaussian) beam within the active medium.

4.4 Frequency of Free-Electron Oscillations

Will be discussed the frequency of free-electron oscillations and the electron energy. An oscillating free electron moving at a relativistic velocity emits radiation at a frequency according to the relativistic Doppler effect. The oscillation frequency of the free-electron oscillations,is given by:

$$\nu_0 = \frac{2c\gamma^2}{\lambda_w} \tag{4.4.25}$$

The spectrum of spontaneously emitted dipole radiation, centered at ν_0. The motion of an electron through a wiggler takes the time (Fig.4.4).

$$t = \frac{N_w \lambda_w}{v} \tag{4.4.26}$$

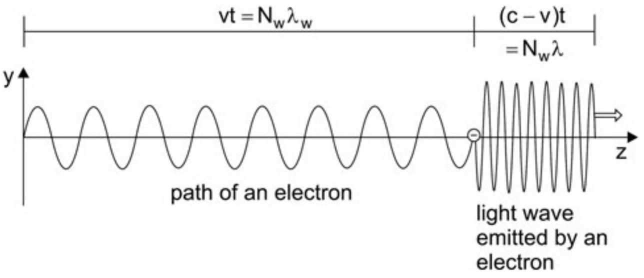

Figure 4.4: *Path of an electron through a wiggler field and dipole radiation*
Source: Karl F. Renk, Basics of Laser Physics, Springer-Verlag Berlin Heidelberg, P340 (2012).

N_w is the number of wiggler periods, λ_w the wiggler period and the electron velocity along the wiggler. In the same time in which an electron traverses the wiggler, the electron emits an electromagnetic wave packet with N_w oscillation cycles. A wave packet of radiation emitted in z direction has the spatial length gives by:

$$(c-v)t = N_w\lambda_w\frac{(1-\beta)}{\beta}, \tag{4.4.27}$$

$\beta = \frac{v}{c}$. Since $(c-v)t = N_w\lambda$ where λ is the wavelength of the radiation, it follows that

$$N_w\lambda = N_w\lambda_w\frac{(1-\beta)}{\beta} \tag{4.4.28}$$

$$\lambda = \lambda_w\frac{(1-\beta)}{\beta} \tag{4.4.29}$$

$$\lambda = \lambda_w\frac{(1-\beta)}{\beta}\frac{(1+\beta)}{(1+\beta)}$$

$$\lambda = \lambda_w\frac{(1-\beta^2)}{\beta(1+\beta)} \tag{4.4.30}$$

with $\frac{1}{\sqrt{1-\beta^2}} = \gamma$ and

$$\frac{1}{1-\beta^2} = \gamma^2 \tag{4.4.31}$$

$$\frac{1}{\gamma^2} = (1-\beta^2)$$

Substitute the value of $(1-\beta^2)$ in equation (4.4.30) yeileds

$$\lambda = \frac{\lambda_w}{\gamma^2 \beta(1+\beta)} \tag{4.4.32}$$

with $\beta \approx 1$, we find

$$\lambda = \frac{\lambda_w}{2\gamma^2} \tag{4.4.33}$$

The number of oscillation cycles in a wave packet is N_w. The rectangular envelope of the field has the temporal length.

$$\Delta t = \frac{N_w \lambda}{c}. \tag{4.4.34}$$

4.5 Energy-Level Description of a Free-Electron Laser Medium

We will discussed the characterize, in a simple picture, the energy levels of an electron in a spatially periodic magnetic field by an energy ladder system (Fig.4.5a),

$$E_\ell = \ell E_0, \tag{4.5.35}$$

where ℓ is an integer and

$$E_0 = h\nu_0 \tag{4.5.36}$$

is the transition energy, such as the energy distance between two next-near energy levels. Electromagnetic radiation interacts via spontaneous emission, absorption or stimulated emission according to the Einstein coefficients. However, absorption and stimulated emission processes have the same transition probability (Fig.4.5b). Therefore, the average rate of absorption processes is the same as the average rate Energy-Level Description of a Free-Electron Laser Medium of stimulated emission processes if the frequency of

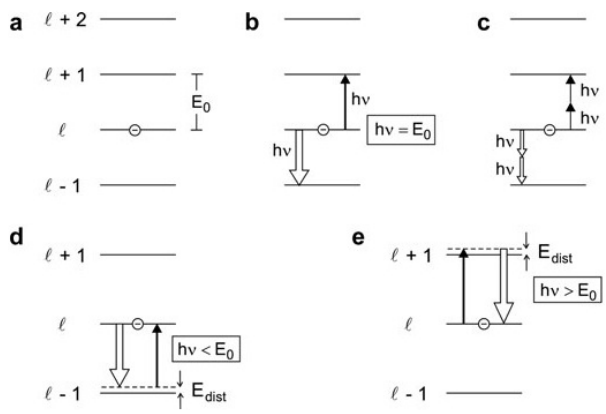

Figure 4.5: *Energy levels of an electron in a periodic magnetic field and transitions. (a) Energy ladder. (b) Absorption and stimulated emission. (c) Two-photon transitions. (d) Stimulated emission and absorption for $h\nu < E_0$. (e) Absorption and stimulated emission for $h\nu > E_0$*

Source: Karl F. Renk, Basics of Laser Physics, Springer-Verlag Berlin Heidelberg, P340 (2012).

the radiation is equal to the resonance frequency $\nu = \nu_0$. The description as a frequency modulation indicates that stimulated emission prevails $\nu < \nu_0$ and absorption if $\nu > \nu_0$. Accordingly, the gain coefficient curve is not a Lorentz resonance curve but a Lorentz dispersion curve. In a strong electromagnetic field, transitions between next-near levels are also allowed as multiphoton transitions (Fig.4.5c) corresponding to the condition.

$$nh\nu = h\nu_0; \qquad n = 1, 2, 3, ... \tag{4.5.37}$$

This corresponds to transverse velocity components of higher order. Whether a radiation field experiences a population inversion in the free-electron laser medium, depends on the frequency of the high frequency field [36]:

- A radiation field experiences a population inversion if $h\nu < E_0$ (Fig.4.5d). In a stimulated emission process by radiation at the frequency ν by an $\ell \to \ell - 1$ transition, the transition energy E_0 is converted to photon energy $h\nu$ and distortion energy E_{dist},

$$E_0 = h\nu + E_{dist} \tag{4.5.38}$$

A stimulated transition in an energy-ladder system leads to a distortion. Absorption does not occur as long as the states of distortion are not populated, i.e., the upper laser level has an occupation number of nearly unity, $f_2 \approx 1$, and the lower level of nearly zero, $f_1 \approx 0$, at small distortion. At large distortion, absorption processes compensate the stimulated emission processes: this corresponds to the saturation field amplitude A_∞ and to $.(f_2 - f_1)_\infty = 0$.

- A radiation field does not experience population inversion if $h\nu > E_0$ (Fig.4.5e). In an absorption process, a photon is converted into excitation energy E_0 and energy of distortion,

$$h\nu = E_0 + E_{dist} \tag{4.5.39}$$

The reverse process, namely stimulated emission by an $\ell + 1 \to \ell$ process, does not occur as long as the states of distortion are not populated, for instance the upper level has the occupation number of nearly zero, $f_2 \approx 0$, and the lower laser level of nearly unity, $f_1 \approx 1$. At large distortion, stimulated emission processes compensate the absorption processes at the saturation field amplitude A_∞, which corresponds to $.(f_1 - f_2)_\infty = 0$.

- If $h\nu = E_0$, upward and downward transitions are equally strong and there is no net energy transfer from the field to the electrons and vice versa. A high frequency field excites a transverse high frequency current that has a phase of $\frac{\pi}{2}$ relative to the field.
In the energy-level description, a limitation of the field amplitude A_1 is caused by a distortion of the energy-ladder systems and corresponds to a saturation of the average population difference at steady state oscillation, $.(f_2 - f_1)_\infty = 0$. We make use of the following quantities:

- E_0 = transition energy = resonance energy
- $\nu_0 = \frac{E_0}{h}$ transition frequency = resonance frequency.
- ν = laser frequency (slightly smaller than the resonance frequency).

- t = phase relaxation time.
- τ_{stim} = time between two subsequent stimulated emission processes.

In (Fig.4.6) illustrates, the energy-level description, the principle of the free electron laser. An electron of energy $E_{el,0}$ injected into the wiggler field forms an energy ladder system. A cascade of stimulated transitions in the energy-ladder system contributes to amplification of radiation. The electron leaves the wiggler field at an energy $E_{el,1}$. The energy difference $E_{el,0} -$

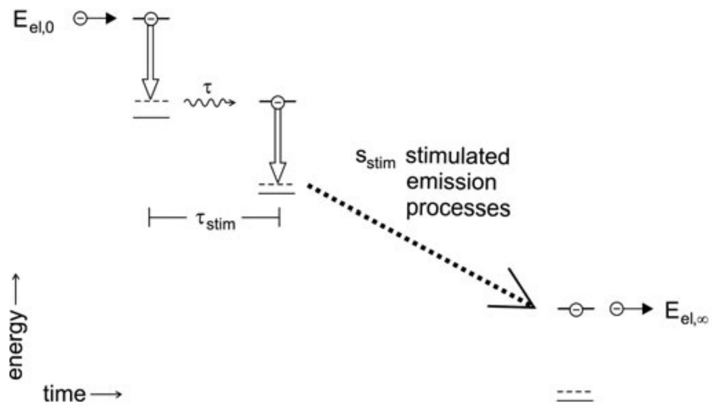

Figure 4.6: *Cascade of stimulated emission (and relaxation) processes in an energy-ladder system*

Source: Karl F. Renk, Basics of Laser Physics, Springer-Verlag Berlin Heidelberg, P362 (2012).

$E_{el,1}$ corresponds to the energy of the number S_{stim} of photons generated by stimulated emission. During the flight of an electron through the active medium, energy of longitudinal motion is converted to energy of the high frequency field; amplification occurs only for radiation propagating in $+z$ direction. A stimulated transition from a level ℓ of an energy-ladder system occurs to the high-energy wing of the level $\ell - 1$ of the same energy-ladder system. Phase relaxation, with the phase relaxation time, removes the energy of distortion from the energy-ladder system. The change from gain at $h\nu < E_0$ to absorption at $h\nu > E_0$ is due to a change of the phase between the high frequency current and the high frequency field, in accord with the frequency dependencies of the conductivities $\sigma_1(\nu)$ and $\sigma_2(\nu)$. If $h\nu = E_0$, upward and downward transitions are equally strong and there is no net energy transfer from the field to the electrons and vice versa. A high frequency field excites a transverse high frequency current that has a phase of $\frac{\pi}{2}$ relative to the field. The free-electron medium is transparent the resonance frequency ν_0 is a transparency frequency.

4.6 Einstein coefficients of a free-electron laser

We will be estimated the Einstein coefficients of stimulated emission and of absorption from the expression of the gain coefficient [36].

$$\alpha(\nu) = \alpha_p \bar{g}_{L,disp}(\nu) \tag{4.6.40}$$

by comparison with an expression given by:

$$\gamma(\nu) = \int_0^\infty h\nu g_{homo}(\nu - \nu_0) B_{21}(N_2 + N_1)(f_2 - f_1) g_{inh}(\nu_0) d(\nu_0) \tag{4.6.41}$$

is the growth coefficient.We used the relation $(N_2 - N_1) = (N_2 + N_1)(f_2 - f_1)$. The gain coefficient is $\alpha = (\frac{n}{c})\gamma$. Two-level atomic systems that have different resonance frequencies ν_0 contribute to the gain coefficient at frequency ν. The Einstein coefficient B_{21} can depend on frequency. The comparison yields

$$\alpha(\nu) = (\frac{1}{c}) h\nu B_{21} \frac{2}{\pi \Delta\nu_0} \bar{g}(\nu)(N_2 + N_1)(f_2 - f_1). \tag{4.6.42}$$

We replace the Lorentz dispersion function $g_{L,disp}$ by the Lorentz resonance function $g_{L,res}$ and obtain, by replacing $N_2 + N_1$ by N_0, and with $f_2 - f_1 = 1$, the Einstein coefficient of stimulated emission [36]

$$B_{21} = \frac{\pi c e k}{4\epsilon_0 h \nu_0 Q_0} \tag{4.6.43}$$

The Einstein coefficient of stimulated emission is proportional to the coupling strength k. And it is inversely proportional to the resonance frequency ν_0, and to the quality factor $Q_0 = N_w = \frac{\nu_0}{\Delta\nu_0}$ of the electron oscillation. The Einstein coefficient of absorption is equal to the Einstein coefficient of stimulated emission,

$$B_{12} = B_{21}$$

Spontaneous emission of radiation of electrons moving with a velocity near the speed of light occurs into a cone with a cone angle $\frac{1}{\bar{\gamma}}$. In comparison with emission into all spatial directions, the reduction of the density of states available for spontaneous emission is therefore reduced by the factor $\frac{1}{4\bar{\gamma}^2}$. We obtain the Einstein coefficient of spontaneous emission [36].

$$A_{21} = \frac{1}{4\bar{\gamma}^2}\frac{8\pi h\nu_0^3}{c^3}B_{21} \tag{4.6.44}$$

The spontaneous lifetime is given by [36]:

$$\tau_{sp} = \frac{1}{A_{21}} \tag{4.6.45}$$

Table (4.1) shows values of Einstein coefficients characterizing transitions

Table 4.1: Einstein coefficients of a free-electron laser medium

	value	
ν_0	6×10^{13}Hz	Resonance frequency
$\Delta\nu_0 = \frac{\nu_0}{N_w}$	1.2×10^{12}Hz	Width of resonance
A_∞	$4 \times 10^6 Vm^{-1}$	Amplitude of saturation
$k = \frac{3.6\pi\nu_0}{A_\infty}$	$2 \times 10^8 mV^{-1}s^{-1}$	Coupling strength
$Q_0 = N_w = \frac{\nu_0}{\Delta\nu_0}$	50	Quality factor
$B_{21} = \pi cek/4\epsilon_0 h\nu_0 Q_0$	$4 \times 10^{26} Vm^3J^{-1}s^{-2}$	Einstein coefficient
$\bar{\gamma}$	70	
$A_{21} = (1/4\bar{\gamma}^2) \times 8\pi h\nu_0^3 B_{21}/c^3$	$2 \times 10^8 s^{-1}$	Einstein coefficient
τ_{sp}	$5 \times 10^{-9}s$	Spontaneous lifetime

Source:Karl F.Renk, Basics of Laser Physics, Springer-Verlag Berlin Heidelberg, P 364 (2012).

between energy-ladder levels of a free-electron laser medium. The Einstein coefficient of stimulated emission is larger than that of active media of conventional lasers. The gain cross section $\sigma_{21}(\omega)$ has the same frequency dependence as the gain coefficient. The maximum gain cross section is $\sigma_{21,m} = \frac{\alpha_m}{N_0}(= 2 \times 10^{-18}m^2)$. In comparison, a naturally broadened two-level system propagating with a velocity corresponding to a Lorentz factor $\bar{\gamma}$

would have a gain cross section $4\bar{\gamma}^{2}\lambda^2/2\pi(\sim 10^{-7}m^2)$. The states belonging to an energy-ladder system are transient states according to the finite time of flight of an electron through the wiggler. However, the time of flight is by many orders of magnitude larger than the period of a free-electron oscillation. An illustration of a free-electron medium as an ensemble of energy-ladder systems may therefore be justified.
The electrons execute, under the action of a static field and the periodic potential, free-electron oscillations. An electromagnetic field modulates the free-electron oscillations, which leads to a synchronization of the free-electron oscillations to the field and to gain for the field. The gain coefficient curve is a Lorentz dispersion curve. The states of an electron subject to both a periodic potential and a static field are quantum mechanical describable as WannierStark states. The energy levels of an electron form a WannierStark ladder i.e., an energy-ladder with equidistant energy levels. An electron occupies one of the levels. A stimulated transition occurs from the occupied WannierStark level to an intermediate level that corresponds to the distorted level of the energetically next-near level of lower energy. The active medium of a free-electron laser consists of an ensemble of free electrons that execute, under the action of a periodic magnetic field, free-electron oscillations. An electromagnetic field modulates the free-electron oscillations, which leads to a synchronization of the free-electron oscillations to the field and to gain for the field. The gain coefficient curve is a Lorentz dispersion curve. We introduced on basis of the similarity of the formal description of the free electron oscillations in a free-electron laser and the free-electron oscillations in a Bloch laser an energy-ladder description.

4.7 Electron Energy Loss by Spontaneous Undulator Radiation

The large power of spontaneous radiation has a number of consequences. Firstly, it complicates the commissioning procedure of the X-ray FEL. Secondly, the energy loss by spontaneous radiation may drive the electrons out of resonance.The fractional electron energy change due to the emission of spontaneous undulator radiation in an undulator of length L_u can be computed using equation

$$P_{spont} = \frac{e^4\gamma^2 B_0^2}{12\pi\varepsilon_0 c m_e^2} \tag{4.7.46}$$

One finds

$$\frac{\Delta W}{W} = \frac{\Delta\gamma}{\gamma} = \frac{e^2\gamma K^2 K_u^2 L_u}{12\pi\varepsilon_0 c m_e c^2} \tag{4.7.47}$$

Chapter 5

Literature Review

5.1 Introduction

Many attempts to produce amplify free electron laser have been made[38, 40]. Some of these attempts devoted to study the gain of free electron laser[38]. Some authors devoted to time study amplification process of free electron laser due to effect of field and force[42]. Also there are some researches have been written in coherence and Growth rate enhancement of free electron laser[40]. while in other attempts studied lasing of free electron laser was studied[37, 45]. In this chapter some attempts to new fined amplification mechanism is presented.

5.2 Growth rate enhancement of free-electron laser by two consecutive wigglers with axial magnetic field

The study of free-electron laser (FEL) as a high-power tunable source of radiation has been the subject of many papers published by different groups all around the world. The radiation is generated by relativistic electron beam passing through a wiggler. In the conventional FEL configuration, the energy of a relativistic electron beam is transferred into high-frequency coherent radiation. Since the radiation wavelength varies with electron energy, it can be continuously tuned in frequency. The theory of conventional

FEL has been studied extensively. Not only can the existence of an axial magnetic field focus on the electron beam, against the self-field, but it can also exploit the resonance between the frequency of the focussing device and the frequency of the wiggler. As a result, the axial magnetic field greatly increases the gain and growth rate in an FEL . The purpose of using a wiggler in an FEL is to impart sufficient transverse oscillatory motion to the electrons of the beam to interact with the radiation that is amplified. Recently, considerable attention has been paid to the interaction of a relativistic electron beam and electromagnetic wave in an FEL having one wiggler. But the study of FEL with two wigglers and magnetized electron beam is comparatively limited. The motivation for this work is to present an analytic the expression for the dispersion relation in an FEL consisting of a uniform axial magnetic field and a two-sectioned helical wiggler having opposite circular polarization. An FEL device operating with two undulators can provide an output linearly polarized field. In optical-Klystron FEL, two undulators having opposite polarization are employed because the output light may be linearly polarized. The particular configuration one has employed is that of a relativistic electron beam propagating along the z direction through an ambient magnetic field composed of two consecutive magnetic wigglers having opposite polarization and a uniform guide field [46].

$$\vec{B} = \vec{B}_w B_0 \hat{e}_z \tag{5.2.1}$$

Where B_0 denotes the magnitude of the solenoidal guide field and the wiggler field may be written as

$$\begin{cases} B_{w1} = B_w(-\sin K_w z\hat{e}_x + \cos K_w z\hat{e}_y) & 0 < z < \frac{1}{2} \\ B_{w2} = B_w(\sin K_w z\hat{e}_x + \cos K_w z\hat{e}_y) & \frac{1}{2} < z < l, \end{cases} \tag{5.2.2}$$

where B_w, k_w and l denote the amplitude of the wiggler, the wave number and the wiggler length, respectively. In the coordinate frame that rotates with the wiggler field and is described by the basic vectors:

$$\hat{e}_1 = -\hat{e}_x(\cos K_w z\hat{e}_x + \sin K_w z) \tag{5.2.3}$$

$$\hat{e}_2 = \hat{e}_x(\sin K_w z\hat{e}_x + \cos K_w z \tag{5.2.4}$$

$$\hat{e}_3 = \hat{e}_z \tag{5.2.5}$$

the relativistic equations of motion for $0 < z < \frac{1}{2}$, first section, is obtained as

$$\frac{d\upsilon_1}{dt} = -\left(\frac{\Omega_0}{\gamma} - k_w\upsilon_3\right)\upsilon_2 + \frac{\Omega_\omega}{\gamma}\cos 2k_w z \quad (5.2.6)$$

$$\frac{d\upsilon_2}{dt} = -\left(\frac{-\Omega_0}{\gamma} - k_w\upsilon_3\right)\upsilon_1 + \frac{\Omega_\omega}{\gamma}\sin 2k_w z \quad (5.2.7)$$

$$\frac{d\upsilon_3}{dt} = -\frac{\Omega_\omega}{\gamma}\upsilon_1\cos 2k_w z + \frac{\Omega_\omega}{\gamma}\upsilon_2\sin 2k_w z, \quad (5.2.8)$$

where $\Omega_0, w = eB_{0w}/mc, -e$ and m are charge and mass of the electron, γ_0 denotes the relativistic factor and c is the speed of light in vacuum, respectively. The above equations can be solved by a first-order perturbation analysis. The axial oscillation is of second order in B_w; consequently, the quasi-steady-state trajectories of electron to first order in B_w can be found as

$$\upsilon_1 = \frac{(\Omega_w/\gamma_0)\upsilon_\parallel}{(\Omega_0/\gamma_0) - k_w\upsilon_\parallel}\sin k_w z. \quad (5.2.9)$$

$$\upsilon_2 = \frac{(\Omega_w/\gamma_0)\upsilon_\parallel}{(\Omega_0/\gamma_0) - k_w\upsilon_\parallel}\cos k_w z. \quad (5.2.10)$$

$$\upsilon_3 = \upsilon_\parallel \quad (5.2.11)$$

These orbits are employed as the initial electron velocities upon entering the second section. Therefore, the initial velocity of the electron for the second part of the wiggler in Cartesian coordinates can be written as

$$\upsilon_0 = \frac{(\Omega_w/\gamma_0)\upsilon_\parallel}{((\Omega_0/\gamma_0) - k_w\upsilon_\parallel)}(-\sin k_w z\hat{e}_x + \cos k_w z\hat{e}_y) + \upsilon_\parallel z \quad (5.2.12)$$

The axial velocity, υ_0, can be obtained by noting that the total energy is conserved, $(d\gamma_0/dt) = 0$. Hence we obtain [46].

$$\upsilon_\parallel^2\left[1 + \frac{\Omega_w^2/\gamma_0^2}{(\Omega_0/\gamma_0 - k_w\upsilon_\parallel)^2}\right] = (1-\gamma^{-2})c^2 \quad (5.2.13)$$

One defines a fourth-order equation for $\upsilon_\parallel$. One has illustrated the influence of two-sectioned wiggler in the growth rate clearly that the growth rate for

FEL with two-sectioned wiggler is much higher than that for the conventional FEL. Numerical calculations show that the growth rate in conventional FEL is less than for typical parameters, compared to the FEL with two-sectioned wiggler.

$$\bar{\omega} = \frac{\omega}{ck_\omega}, \qquad \bar{k} = \frac{k_n}{k_\omega}, \qquad \bar{\omega} = \left(\frac{4\pi n_2 e^2}{mc^2 k_\omega^2}\right)$$

and

$$\beta_{z0} = \frac{\upsilon_{z0}}{c}$$

Growth rate enhancement of free-electron laser are plots of growth rate, $Im\bar{k}$, versus. $\bar{\omega}$ for several values of $\bar{\Omega}_0$

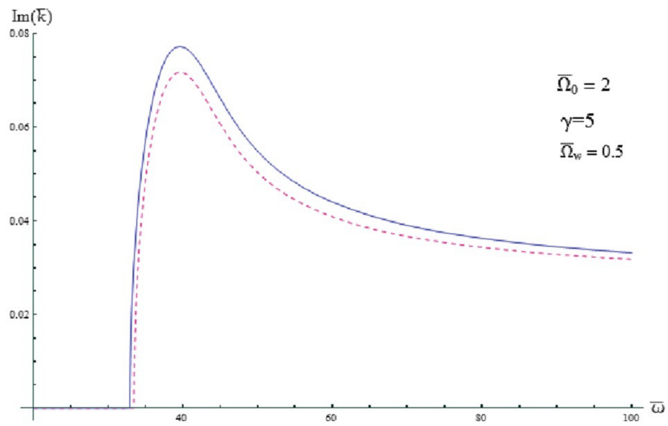

Figure 5.1: *Graph of the normalized spatial growth rate vs. the normalized frequency for two configurations, one-sectioned wiggler (dashed curve) and two-sectioned wiggler (solid curve).*

Source: A hasanbeigi, A farhadian and E Khademi Bidhendi, Growth rate enhancement of free-electron laser by two consecutive wigglers with axial magnetic field, Pramana journal of physics, Indian Academy of Sciences Vol. 82, No. 6 P1059 (2014).

5.3 Comparison of Growth Rate of Electromagnetic Waves in Pre-bunched Cerenkov Free Electron Laser and Free Electron Laser

Cerenkov free electron laser (CFEL) is the widely used source of broad- band, high power microwave generation at short wavelengths. In this device, an electron beam passing through wave structure resonantly interacts with wave whose phase velocity equals the drift velocity of electrons and the wave grows at the expense of energy of the beam. Since the electron velocity cannot exceed the velocity of light, a slow wave structure is needed to slow down the phase velocity of electromagnetic modes. In case of Cerenkov free electron laser (CFEL). which employs a slow wave medium to slow down the phase velocity of transverse electric (TE) or transverse magnetic (TM) modes to less than c, the velocity of light so that they can be excited by a moderately relativistic electron beam by the process of cerenkov emission. A Cerenkov free electron laser generally employs two kinds of slow wave structures: (i) A dielectric whose dielectric constant is $|\varepsilon| > 1$ reduces the phase velocity of the radiation below c. A moderately relativistic electron beam can excite the electromagnetic radiation by cerenkov emission, (ii) A plasma lining have a dielectric constant $\varepsilon = 1 - \frac{\omega_p^2}{\omega^2}$ can act as a slowing down medium for $\omega_p >> \omega$ so that $\varepsilon >> 1$ (where ω_p is the electron plasma frequency and ω is the radiation frequency). A CFEL consisting of two dielectrically lined parallel plates driven by dense moderately relativistic electron beam has been studied and reported to produce coherent high power radiation from 375 micrometer to $1mm$ wavelengths.

More recently, a lot of research work has been carried out in studying the free electron laser by pre-bunched electron beams. A high power microwave free electron laser experiment has been performed using pre-bunched electron beam of $35Mev$. Here when the electron beam is prebunched at a frequency close to an eigen frequency of the cavity, the oscillation build process is speed up and the radiation build time is shortened significantly. Free electron maser experiment with a pre-bunched electron beam has been demonstrated at Tel Aviv University. In this case, they utilize a $1.0A$ current pre-bunched electron beam obtained from a microwave tube. The electron beam is bunched at $4.87GHz$ frequency and is subsequently accelerated to $70KeV$.

The bunched beam is injected into a planar wiggler ($BW = 300$ gauss,$\lambda_W = 4.4cm$, where B_W is the wiggler field and λ_W is the wiggler wavelength) constructed in a Halbach configuration with 17 periods. A theoretical model for gain and efficiency enhancement in a FEL using pre-bunched electron beam has been developed and studied by Beniwal et al. Sharma and Bhasin have studied the gain and efficiency enhancement in a slow wave FEL using prebunched electron beam in a dielectric loaded waveguide. They have found that the growth rate and gain of a slow wave FEL increase with the increase in modulation index and is maximum when the pre-bunched beam velocity is comparable to the phase velocity of the radiation wave. In this section, we develop a theoretical model of a prebunched CFEL and present the analytical analysis for the excitation of electromagnetic waves by a pre-bunched electron beam in a CFEL. We compare the increase in growth rate with the increase in the modulation index for pre-bunched CFEL with a pre-bunched FEL. The growth rate has been calculated at experimentally known CFEL and FEL parameters. Consider a dielectric loaded waveguide of effective permittivity ε_0 . A pre-bunched relativistic electron beam of density n_{b0}, velocity υ_{bz}, relativistic gamma factor $\gamma = 1+\frac{e\upsilon_b}{mc^2}(1+\Delta\sin\omega_0\tau) \approx \gamma_0(1+\Delta\sin\omega_0\tau)$ [where Δ is the modulation index (its value lie from 0 to 1), mc^2 is the rest mass energy of the electrons, e is the electronic charge, $\omega_0(\approx k_{zo}\upsilon_b)$ and k_{zo}bare the modulation frequency and wave number of the pre-bunched electron beam], respectively propagates through the waveguide An electromagnetic signal E_1 is also present in the interaction region [47].

$$E_1 = E_0 e^{-(\omega_1 t - k_1 \bar{x})} \tag{5.3.14}$$

$$B_1 = \frac{c}{\omega_1} k_1 \times E_1, \tag{5.3.15}$$

where, E_0 and k_1 lie in the $x-z$ plane $\frac{\partial}{\partial y} = ik_y = 0$. The response of the beam electrons to the signal is governed by the relativistic equation of motion [47].

$$\frac{\partial}{\partial t}(\gamma\upsilon) + \upsilon.\nabla(\gamma\upsilon) = -\frac{e}{m}(E + -\upsilon \times B) \tag{5.3.16}$$

Velocity components in the x and z directions are given by

$$\upsilon_{x1} = \frac{e}{im\gamma(\omega_1 - k_z\upsilon_b)}\left[E_{x1} - \frac{k_z\upsilon_b E_{x1}}{\omega_1} + \frac{k_{x1}\upsilon_b E_{z1}}{\omega_1}\right] \tag{5.3.17}$$

$$v_{z1} = \frac{eE_{z1}}{im\gamma(\omega_1 - k_z v_b)\gamma^3} \tag{5.3.18}$$

On linearizing and solving equation of continuity, we obtain density perturbation

$$n_1 = n_{b0}\frac{k_1 v_1}{(\omega_1 - k_z v_b)} \tag{5.3.19}$$

One has determined the growth rate of the $CFEL$ instability, we use the first order perturbation techniques .In the presence of the right hand side terms, $(n_{b0} \neq 0)$, one has assumed that the eigenfunctions are not modified but their eigenvalue are. We expand ω_1 as

$$\omega_1 = \omega_{1r} + \delta = k_z v_b + \delta = k_{z0} v_b + \delta,$$

where δ is the small frequency mismatch and $\omega_{1r} = \frac{k_1 c}{\sqrt{\varepsilon}}$

$$\delta = \left[\frac{\omega_{pb}^2(\omega_{1r}^2 + k_{x1}^2 v_b^2 \gamma^2)}{2\omega_{1r}\gamma^3\varepsilon}\right]^{1/3} e^{i\frac{2n\pi}{3}}, \quad n = 1, 2, 3, ... \tag{5.3.20}$$

Hence the growth rate, i.e., the imaginary part of δ is given as

$$\Gamma = \left[\frac{\omega_{pb}^2(\omega_{1r}^2 + k_{x1}^2 v_b^2 \gamma_0^2)}{2\omega_{1r}\gamma^3\varepsilon}\right]^{1/3} \frac{\sqrt{3}}{2}, \tag{5.3.21}$$

Where,

$$\gamma = \gamma_0(1 + \Delta \sin \omega_0 \tau)$$

For maximum gain it is assumed that all electrons are bunched in the decelerating zone, $\omega_0 \tau = -\frac{\pi}{2}$. This gives $\gamma = \gamma_0(1 - \Delta)$ [47]. Where Δ is the modulation index, its value lies between 0 to 1 and $\Delta \neq 1$.

5.4 Efficiency enhancement in free-electron laser amplifier with one dimensional helical wiggler and ion-channel guiding

The free-electron laser (FEL) can produce high power and tunable electromagnetic radiation from microwave to x-ray. The development activities of FEL are in two directions. In one direction, high brightness coherent femtosecond x-ray emissions are produced and in the other, high powers of infrared radiation are generated, which require low energy and high current electron beam. This kind of beams usually requires quadrupole or solenoidal magnetic field or ion channel to focus the beam against its self-field. When an electron beam passes through a preionized plasma, it ejects the electrons and the positive ions will guide the electron beam. This scheme has been proposed for use in $FELs$, demonstrated experimentally and investigated by numerical simulations. Theoretical studies of the FEL with ion-channel guiding have shown considerable gain enhancement in the low and high gain regimes. There are several advantages for this type of focusing. It is less expensive compared to magnetic focusing. Other benefits of its use are suppression of the transverse beam break up instability and emittance growth due to scattering and to obtain current larger than vacuum limit. Many studies were reported for the enhancement of efficiency in FELs. By increasing the wave number of the wiggler, after the saturation point, in Ref. The efficiency of the FEL was increased. On the other hand, efficiency has been shown to increase when the wiggler and/or the axial magnetic field are tapered. In another technique in Ref., rf power was injected in order to accelerate the electron in the wiggler. In Ref. The electron beam energy was detuned from the resonance condition in order to increase the efficiency experimentally. The experimental demonstration of efficiency enhancement has been reported for both fundamental resonance wavelength and the third harmonic when the amplitude of wiggler was reduced after the saturation point. Efficiency enhancement of FEL in Raman regime by tapered wave number of the wiggler has been studied for both axial and ion-channel guiding by numerical simulation. On the other hand, analytical and nonlinear simulations of this problem in the Compton regime have been carried out in one and three dimensions, using tapered wiggler and/or axial magnetic field.

This type of tapering is more widely used in operational and proposed experiments compared to the tapering of the wiggler wave number. The purpose of the present study is to carry out a nonlinear simulation for the efficiency enhancement of FEL with tapered wiggler and ion-channel density. A set of coupled and nonlinear equations is derived that describe the evolution of the electromagnetic radiation. Parameters correspond to the high-gain Compton regime; therefore, spacecharge fields are neglected. In order to obtain a better understanding of the efficiency enhancement, an analytical treatment of the problem is also presented which involves the derivation of a modified pendulum equation based on the small signal theory. This theory will primarily provide the appropriate signs of the tapering that will be used in the simulation. A nonlinear simulation of FEL in amplifier mode is carried out, in one dimension, to find the saturated radiation amplitude and efficiency. The parameters are in the high gain Compton regime and the radiation wavelength corresponds to the microwave. Therefore, self-fields and the space-charge potential of the electron beam are neglected. Interaction of electrons with each other takes place indirectly by coupling to the electromagnetic field (radiation) through the source term in the wave equation. The steady-state simulation with no time dependency will be used which requires for the electron pulse to be long enough so that the slippage of radiation a) over the electron beam to be negligible. In the present investigation, Lorentz force equations are integrated over the complete equations of motion without using wiggler-averaged approximation. The configuration is composed of the combination of a tapered helical wiggler magnetic field in one dimension and the electrostatic field of an ion channel

$$B_w(z) = B_w \hat{e}_x \cos k_w z + \hat{e}_y \sin k_w z$$

$$E_1(x, y) = 2\pi e n_i(z)(x\hat{e}_x + y\hat{e}_y),$$

where $B_w(z)$ denotes the wiggler amplitude, k_w is the wiggler wave number, and $n_i(z)$ is the ion number density with positive charge e. It is assumed that both the wiggler amplitude and the ion number density vary adiabatically to model the injection of the beam electrons.

Here, using modified pendulum equation, the efficiency enhancement by tapered field configuration in FEL amplifier with the one dimensional helical wiggler and ion-channel guiding is investigated analytically. The time derivative of relativistic factor can be written as [48].

$$\frac{d\gamma}{dt} = -\frac{e}{mc^2}V.E = -\frac{\omega_i^2}{c^2}(\upsilon_1 x_1 + \upsilon_2 x_2) + \delta a\frac{\omega}{c}(\upsilon_1 \sin\psi + \upsilon_2 \cos\psi). \quad (5.4.22)$$

In the small signal domain, the radiation amplitude is assumed to be small compared to the wiggler, therefore, the velocity and position of electrons can be perturbed about the steady state orbits as $\upsilon = \upsilon_0 + \delta\upsilon$, $x = x_0 + \delta x$ and $y = y_0 + \delta y$.
The simulation consists of solution of a set of coupled nonlinear differential equations for FEL in the amplifier mode using the fourth order Runge Kutta method. To calculate the averages in the dynamical equations, Simpson technique is used. A nonlinear analysis and numerical simulation of the FEL amplifier with tapered wiggler and ion-channel density are presented in order to enhance the efficiency. Also to demonstrate the physical basis for the efficiency enhancement, the small signal theory for the gain in the case of tapering is used to derive a modified pendulum equation. The gain equation for the added gain gives the current sign of the tapering to be used in the simulation. Both theory and simulation show that there is always an asymptotic state in which the efficiency levels off as a function of axial length and no further growth is possible except for some oscillations.
The gain due to efficiency enhancement can be defined as [48]

$$G = \frac{[\partial a_{(z=\ell+z_0)} - \partial a_{(z=z_0)}]}{\partial a_{(z=z_0)}} \quad (5.4.23)$$

The gain will lead to

$$G \simeq -\frac{\bar{\omega}_b \qquad \bar{\ell}}{2\bar{k}_+(1+\bar{k}_+)\partial a^2_{z=z_0}}\gamma_0\beta_{10}\frac{1+\varphi}{\varphi}[\bar{\omega}_i^2\varepsilon_i - \varepsilon_\omega(\bar{\omega}_i^2 - \gamma_0\beta_\parallel^2)] \quad (5.4.24)$$

It is important to emphasize that $G < 1$ [48].

5.5 Feasibility of UV lasing without inversion in mercury vapor

Developing powerful, coherent light sources ranging from UV to X-ray is a major quest in laser development with relevant applications from spectroscopy, lithography to material science. Conventional lasing requires population inversion, which becomes increasingly difficult for shorter wavelengths since the threshold pumping power scales with the laser frequency ω^4 to ω^6. In the UV regime lasing without inversion (LWI) is a possible pathway to overcome this problem. To date, several experiments have been conducted showing that inversionless lasing is in fact feasible. However, the lasing wavelengths were not significantly shorter than the driving fields, wavelengths. Despite all commitment, a laser based on the LWI concept operating in the UV regime is yet to be built. The large majority of existing UV lasers are based on nonlinear harmonic frequency generation. Developing an alternative to this technique using LWI might allow for new applications. Doppler broadening is a major obstacle in UV lasing without inversion when driving frequencies are strongly disparate. One path to circumvent this problem is transient lasing without inversion. However, it is limited to pulsed lasing. Another path allowing for Doppler-free $cwLWI$ has been proposed by Fry et al. It is based on the concept of interacting dark resonances. The proposed experiment allows for lasing on the $6^3P_1 \longleftrightarrow 6^1S_0$ transition in mercury at a wavelength of $253.7nm$. This idea can also be applied to similar schemes for example in mercury and krypton at wavelengths of $185nm$ and $116.5nm$ respectively. In this section, we provide a realistic three-dimensional theoretical analysis of the experiment proposed by Fry et al. One has used the linear gain coefficient of the spatially inhomogeneous mercury vapor and evaluate the intracavity field modes of a four-mirror ring laser resonator self-consistently within Fourier optics. One has implemented LWI in a four-level scheme, with three allowed dipole transitions driven by a strong and a weak external electric field E_s and E_w respectively and a probe field E_p, one finds interacting dark resonances. For each of these fields, $j = s, w, p$, the positive frequency components are given by [49].

$$E_j^{(+)}(r,t) \quad = \quad \varepsilon_j(r,t)\epsilon_j e^{-i\omega_j t}, \qquad (5.5.25)$$

with the angular frequencies ω_j , polarization vectors ϵ_j , and slowly varying amplitudes ε_j . For each of the three dipole transitions we obtain the corresponding Rabi frequencies $\Omega_p = d_{ab}.\epsilon_p\varepsilon_p/\hbar, \Omega_s = d_{ca}.\epsilon_s\varepsilon_s/\hbar$ and $\Omega_\omega = d_{cd}.\epsilon_\omega\varepsilon_\omega/\hbar$ with the dipole matrix elements of the respective transitions $d_{ab}, d_{ca}, andd_{cd}$. Within the dipole and rotating wave approximation, one finds for the Hamiltonian matrix where the matrix elements are sorted in the order of the basis $\{|a_i\rangle, |b_i\rangle, |c_i\rangle, |d_i\rangle\}$. The detunings are defined as $\Delta_p = \omega_p - (\omega_a - \omega_b)$, $\Delta_s = \omega_s - (\omega_c - \omega_a)$ and $\Delta_\omega = \omega_\omega - (\omega_c - \omega_d)$, with $\hbar\omega_j$ being the energy of the respective atomic state. The origin of lasing without inversion can be understood best in the dressed state picture. These results reveal that even a small amount of incoherent pumping can invert the sharp absorption peak into a gain dip and lead to lasing on the probe transition. In the previous sections the gain mediums linear response to the external probe field was investigated, whereas in this section the field E_p will be treated as a dynamical quantity, the lasing field. By applying semiclassical laser theory this will lead to the stationary laser power. The dynamics of the lasing field is determined by the Maxwell equations from which the wave equation [49]

$$\left(\frac{1}{c^2}\partial_t^2 + \mu_0\sigma\partial_t - \Delta\right) E_p(r,t) \;=\; -\mu_0\partial_t^2 P_p(r,t) \qquad (5.5.26)$$

can be deduced under the assumptions that the charge density, the gradient of P_p, and the magnetization vanish and the current density is given by conductivity times the electric field E_p. The polarization density P_p couples this wave equation to the mediums Bloch equations. To solve this problem, one has started by expanding the laser field

$$\varepsilon_p(r,t) \;=\; \sum_n \varepsilon_n(t) u_n(r) e^{[i\phi_n(t)]}, \qquad (5.5.27)$$

in the resonators Hermite-Gauss modes $u_n(r)$ that are known to accurately describe modes in ordinary optical resonators. ϕ_n is the phase and ε_n is the real amplitude of the n-th mode. One has assumed that the spatial distribution of the laser field is that of a Gaussian mode. One has developed a realistic multi-level model for the UV lasing scheme in mercury vapor proposed, including technical noise of the driving field, Doppler broadening, the spatial inhomogeneous structure of the gain medium, and self-consistent eigenmodes of a four mirror ring cavity.

The linear gain parameter [49].

$$\alpha = -\frac{\sigma}{\varepsilon_0} - \frac{\omega_p V_0}{V_c}\chi_1^{''} \tag{5.5.28}$$

the nonlinear saturation parameters

$$\beta = \frac{\hbar\omega_p^2 V_1}{2\varepsilon_0 V_c^2}\chi_3^{''} \qquad\qquad \gamma = \frac{\hbar^2\omega_p^3 V_2}{4\varepsilon_0 V_c^3}\chi_5^{''} \tag{5.5.29}$$

and $\chi_m^{''} = \Im(\langle\chi^{(m)}\rangle)$. Thus, the stationary photon number is given

$$n_{st} = \begin{cases} -\frac{\beta}{2\gamma} + \sqrt{\frac{\beta^2}{4\gamma^2} + \frac{\alpha}{\gamma}} = \frac{\alpha}{\beta} + O(\gamma), & \alpha > 0 \\ 0, & \alpha \leq 0 \end{cases}$$

The sign of the linear gain parameter α indicates if the laser system is above ($\alpha > 0$) or below ($\alpha \leq 0$) threshold, while β and γ are saturation parameters that determine the stationary power when above threshold [49]. For $\gamma \to 0$, we obtain the standard form of the photon number equation for a laser.

5.6 High refractive index and lasing without inversion in an open four-level atomic system

The optical properties of atomic gases can be radically modified by quantum coherence and quantum interference. Quantum coherence and interference in an atomic medium can result in many appealing out comes. A marvellous consequence of preparing an atomic system in a coherent superposition of states is the absorption elimination that leads to the lasing without inversion, enhancement of there fraction index and electromagnetically induced transparency (*EIT*). Under the conditions of electromagnetically induced transparency (*EIT*) it is feasible to control the optical response and related absorption of weak laser light. This effect has been deeply studied in atomic physics. *EIT* has many noteworthy usages in quantum optics,such as the multi-wave mixing, enhancement of Kerr nonlinearity and optical bistability and multistability.

More interestingly,the EIT effect has been found applications in quantum information science,such as the photon in formations to ring and releasing in an atomic assemble, correlated photon pairs generation and even the entanglement of remote atomic assembles, which form the building blocks of the quantum communication and the quantum computation. In view of many proposals, the transient properties of the weak probe field via quantum interference such as transient-absorption,transient-dispersion, and transient-gain without inversion are widely investigated. Zhu presented the condition required for observing the inversion less gain in the transient requirement for V and Λ schemes. The effect of SGC on transient process in the three level system has also been investigated. It is shown that EIT medium can be used as an absorptive optical switch, in which the transmission of highly absorptive medium is controlled dynamically by an additional signal (switching) light. Transient two photon absorption property in a n-doped three-level semi conductor quantum well system is also investigated. It is shown that the intensities and detunings of the optical fields can affect the two-photon absorptions pectra dramatically,which can be used to suppress or enhance the two-photon absorption coefficient. Yang et al. studied the transient and steady state absorption of a weak probe beam by means of a coupled double quantum well structure. However,almost all of these studies are considered with a closed system. An ideal level structure atomic system with appreciate interference and coherence features will bring great help to achieve more bright results.To the best of our knowledge, the transient properties of four-level open atomic media is never investigated, which motivates us to carry out this section. The presented scheme is based On Refs., but our scheme is very different from those works. First,we investigate the transient evolution of the atomic response in stead of steady-state response. Second, transient behavior in our scheme is realized by atomic exit rate and atomic injection rates which are characteristics of open systems and thus, is very different from other conventional closed schemes. Finally, we show new convenient ways to obtaining the high refractive index with out absorption as well as lasing with and without absorption, which make our scheme much more practical than the other counterparts. denotes an open four-level atomic system coupled by a weak probe field,and two strong coupling fields.

Using the rotating-wave and the electric dipole approximations and in the interaction picture,the density matrix equations of motion of this system can be written as [50].

$$\begin{aligned}
\dot{\rho}_{11} &= \gamma_{31}\rho_{33} + \gamma_{41}\rho_{44} + i\Omega_p(\rho_{31} - \rho_{13}) + i\Omega_s(\rho_{41} - \rho_{14}) + J_1 - r_0\rho_{11}, \\
\dot{\rho}_{22} &= \gamma_{32}\rho_{33} + \gamma_{42}\rho_{44} + i\Omega_c(\rho_{32} - \rho_{23}) + J_2 - r_0\rho_{22}, \\
\dot{\rho}_{33} &= -(\gamma_{31} + \gamma_{32})\rho_{33} + i\Omega_p(\rho_{13} - \rho_{31}) + i\Omega_c(\rho_{23} - \rho_{32}) - r_0\rho_{33},
\end{aligned}$$

$$\begin{aligned}
\dot{\rho}_{12} &= i(\Delta_p - \Delta_c)\rho_{12} + i\Omega_p\rho_{32} + i\Omega_s\rho_{42} - i\Omega_c\rho_{13} \\
\dot{\rho}_{13} &= -\left[\frac{\gamma_{31} + \gamma_{32}i\Delta_p}{2}\right]\rho_{13} + i\Omega_s\rho_{43} - i\Omega_c\rho_{12} + i\Omega_p(\rho_{33} - \rho_{11}) \\
\dot{\rho}_{14} &= -\left[\frac{(\gamma_{41} + \gamma_{42})}{2}i\Delta_s\right]\rho_{14} + i\Omega_p\rho_{34} + i\Omega_s(\rho_{44} - \rho_{11}) \\
\dot{\rho}_{23} &= -\left[\frac{(\gamma_{31} + \gamma_{32})}{2}i\Delta_c\right]\rho_{23} - i\Omega_p\rho_{21} + i\Omega_c(\rho_{33} - \rho_{22}) \\
\dot{\rho}_{24} &= -\left[\frac{(\gamma_{41} + \gamma_{42})}{2}i(\Delta_p - \Delta_s - \Delta_c)\right]\rho_{24} + i\Omega_c\rho_{34} + i\Omega_s\rho_{21} \\
\dot{\rho}_{34} &= -\left[\frac{(\gamma_{31} + \gamma_{32}\gamma_{41} + \gamma_{42})}{2} + i(\Delta_p - \Delta_s)\right]\rho_{34} - i\Omega_p\rho_{14} + i\Omega_c\rho_{24} + i\Omega_s\rho_{31}
\end{aligned}$$

$$\rho_{11} + \rho_{22} + \rho_{33} + \rho_{44} = 1 \qquad (5.6.30)$$

The frequency detuning parameters are defined as $\Delta_p = \omega_{31} - \omega_p, \Delta_c = \omega_{32} - \omega_c\Delta_s = \omega_{41} - \omega_s$. In this set of equations, if $J_1 = J_2 = r_0 = 0$, Eq.(1) changes to those for a closed four-level atomic system interested in the effect of cavity parameters i.e.atomic injection rates and exit rate from cavity on transient properties of open four-level atomic scheme. As well known, gain-absorption and refractive index of the probe field on transition $|3\rangle \longrightarrow |1\rangle$ are proportional to imaginary and real part of ρ_{31} which can be obtained from Eq.(1). If Im $(\rho_{31}) > 0$ the system exhibits absorption for the probe field, while for $Im(\rho_{31}) < 0$, the probe laser will be amplified. When $\rho_{33} > \rho_{11}$ and Im $\rho_{31} < 0$, the lasing with inversion can be obtained, whereas when $\rho_{33} < \rho_{11}$ and $Im(\rho_{31}) < 0$, the lasing without inversion can be realized. lasing without population inversion is obtained in open four-level system via ratio of injection rates.

Therefore, one has showed that the open system changes from a state with population inversion to a state without population inversion. This Flexibility and controllability of open system to achieve lasing with and without inversion shows its superiority than corresponding closed system which may provide some new possibilities for technological applications. The probe gain is obtained,that is to say the weak probe field will be amplified in open system as shown (Fig. 8(a)) [50]. An investigation on Fig. 8(b) shows that the population distribution in level $|1\rangle$ reduces,while it increases in level $|2\rangle$. Accordingly, the population distribution in level $|2\rangle$ is more than level $|1\rangle$, i.e. $\rho_{22} > \rho_{11}$, thus population inversion occurs.It can be concluded that the gain is obtained in the presence of population inversion [50]. In other words,lasing with inversion is achieved in open system via atomic exit rate.

5.7 Nano wave guide and lasing

A seminal work of A. H. Abdlrahman and M. Dirar shows a possibility of lasing by a rectangular wave guide. The amplification factor γ is related to the dimension of guide according to the relation A. H. Abdlrahman and M. Dirar, the possibility of utilizing a nano and micro rectangular wave guide a solar cell and a lasing carry J. of science and technology, Vol. 13, No, 1, march (2012).

$$\gamma = \sqrt{2\left(\frac{n\pi^2}{b}\right)^2 - \mu\varepsilon\omega^2}, \tag{5.7.31}$$

where n is an integer called the mode of the wave, μ, ε, and ω are the electric, magnetic permeability and angular frequency respectively. By setting the mode to be

$$n = 2 \tag{5.7.32}$$

the suitable dimension b to amplify light satisfies [51].

$$b < \frac{2}{n_0}\lambda \tag{5.7.33}$$

n_0 is refractive index and λ is light wave length for glass visible light.

$$n_0 = 1.5, \qquad \lambda = 50nm \tag{5.7.34}$$

The suitable dimension of the guide is

$$b < 66.6nm \tag{5.7.35}$$

Thus one needs nano rectangular tube for lasing [51].

5.8 Quantum mechanical lasing mechanism

Quantum mechanical laws are used to find the wave function factor β in the form A. H. Abdlrahman and M. Dirar, quantum mechanical nano lasing mechanism, Africa University, journal of science, Vol. 2 (2012).

$$\beta = \frac{2}{c}\sqrt{\frac{p_0 - E_0}{x_0 m}} \tag{5.8.36}$$

x_0, m, p_0, E_0 are the vibration amplitude atomic mass, polarization and external field amplitude respectively. Lasing takes place when

$$p_0 > E_0 \qquad n_m > n_i, \tag{5.8.37}$$

where n_m, n_i are the number of emitted and incident photons respectively. Thus amplification exists when emitted photon exceed incident ones. The expression for amplification factor for population inversion shows lasing can takes place when lattice force F_l, which is related to collision and excitation rate, exceeds the external one F_l,[52].
Where

$$\beta \propto (\frac{F_l - F_e}{F_e}) \tag{5.8.38}$$

5.9 Enhanced beam characteristics of a discharge-pumped soft-x-ray amplifier by an axial magnetic field

The first demonstration of large soft-x-ray amplification in a discharge-driven plasma was recently realized using a fast capillary discharge to generate a hot and dense plasma column in which collisional electron excitation of Ne-like

Ar ions produced amplification in the $J = 01$ line of Ne-like Ar at $46.9nm$. In this excitation scheme, a fast current pulse rapidly compresses the plasma, creating a hot and narrow plasma column with length-to-diameter ratios approaching $1000 : 1$. During the final stage of the compression, plasma conditions for soft-x-ray amplification by collisional excitation are obtained [53]. In the initial experiments, a gain-length product of $gl \sim 7.2$ at $46.9nm$ was reported for a $12-cm$-long plasma column. The experiments were conducted in a capillary discharge excited, collisionally pumped $46.9 - nm$ Ne-like Ar amplifier. To conduct the study, we modified a fast capillary discharge setup previously described. To include an axial magnetic field. The discharge consists of a $3 - nF$ capacitor which is pulse-charged by a Marx generator and is discharged through a pressurized SF_6 switch into a capillary channel containing a selected pressure of preionized argon gas. In the experiments the generator was used to excite plasmas in polyacetal capillaries $4mm$ in diameter and $10cm$ in length with current pulses having a first half cycle duration of approximately $64ns$. The axial magnetic field was generated by a $9 - cm$ diameter, $15 - cm$ long coil positioned concentrically with the capillary channel. The coil, which was excited by a current pulse with a period of $200ms$ is obtained by discharging a $420 - \mu F$ capacitor through a spark gap, was used to produce magnetic fields up to $0.3T$. The intensity of the magnetic field was selected by varying either the capacitor charging voltage or the delay time between the triggering of the latter spark gap and the firing of the fast capillary discharge [53]. The discharge electrodes were made of stainless steel and were slotted to improve the uniformity of the magnetic field along the capillary axis, which was measured, using an axial Hall effect probe, to be better than 20% over the $10 - cm$-long region where the capillary is located. The dc calibration of Hall probe given by the manufacturer, $0.1146V/T$, was checked in the pulsed mode by placing it on the axis and in the center of the coil described above and comparing the measured value with the one calculated from the expression for a finite solenoid. The soft-x-ray radiation exited the capillary through the hollowed ground electrode. The laser radiation was collected by a cylindrical copper mirror of $13cm$ in radius and focused onto the slit of a $2.2 - m$ vacuum spectrometer provided with a $1200l/mm$ diffraction grating placed at 4.2 with respect to the incoming radiation. The detection system consisted of an intensified charge-coupled device (CCD) array detector that was gated by pulsing the gain on the multichannel plate intensifier with a high voltage pulse with a duration of about $25ns$. The variation of the measured integrated intensity of the Ar IX $46.9-nm$ laser line as a function of the magnetic field strength is shown in Fig (5.2). The laser intensity increases with the magnetic field and reaches a maximum at approximately$0.15T$ decreasing monotonically for higher field strengths.The same figure also shows the calculated variation of the intensity corresponding to two calculations which differ from each other in the inclusion of Zeeman splitting of the laser line [53]. In our case, the Zeeman splitting is mostly caused by the compressed

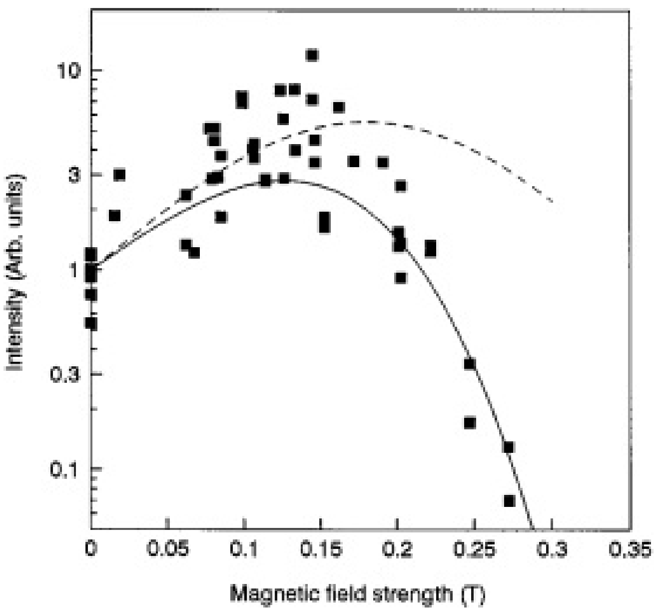

Figure 5.2: *Variation of the integrated intensity of the 46.9-nm Ar IX laser line as a function of the strength of the externally applied axial magnetic field*
Source: F. G. Tomasel, V. N. Shlyaptsev, and J. J. Rocca, Enhanced beam characteristics of a discharge-pumped soft-x-ray amplifier by an axial magnetic field, Physical Review A volume 54, No 3 SEPTEMBER, P2476 (1996).

axial magnetic field. The azimuthal magnetic field is comparatively small in the gain region, situated near the axis, due to the distribution of the current and the boundary condition $B_\theta = 0$ at $r = 0$. In both computations the laser intensity first increases to subsequently decrease at higher values of the magnetic field. The intensity increase is due to decreased refraction losses and to a larger electron temperature caused by a decrease in heat losses. Only the computation that considers broadening of the laser line due to the Zeeman effect shows good agreement with the experiment. In the condition of our experiments the line profile is largely determined by Doppler broadening, and collisional and natural broadening contributions are small. Moderate magnetic fields, of the order of $10T$, contribute to broaden the $2p^5 3s1_0^S - 2p^5 3p1_1^P$ transition of Ne-like argon by $\Delta\nu/\nu \sim 5 \times 10^{-5}$. This value is comparable to the Doppler linewidth of the order of $\Delta\nu/\nu \sim 1 \times 10^{-4}$, therefore causing a very significant decrease in the measured g_ℓ product. These results suggest that the Zeeman effect is likely to be a major cause for the observed decrease of the laser output intensity at higher magnetic fields. The laser intensity decrease observed at large magnetic fields in the computation that excludes the Zeeman effect is the result of a smaller gain caused by a decrease in the density, reduced transient effects associated with ionization and excitation, and an increase in the optical depth [53]. The larger optical depth at higher magnetic fields is due to a reduction of the very important radial motional Doppler effect, which is in turn caused by the previously discussed reduction of the density gradients.

5.10 The effect of a weak axial magnetic field on a He-Cd laser

In this work an internal cavity He-Cd laser which was $2.81cm$ internal diameter and $72cm$ in discharge lengths was used as is shown in fig (5.3). so that the polarization effects mentioned above could be observed [54]. The optical resonator, which was $136cm$ in length, was composed of a mirror with a curvature radius of $3m$ and a reflectivity of 99% and an output coupling plane mirror with reflectivity of 98%. Mounted on a non-magnetic support, the laser tube was placed in a coaxial glass tube of larger diameter so that electrical leakage from the discharge capillary could be prevented to ensure the accuracy of measurement. A coil was wound around the coaxial glass tube ($8.5C/cm$) and was supplied by an adjustable D.C. power set so that the strength of the $WAMF$ could be changed.

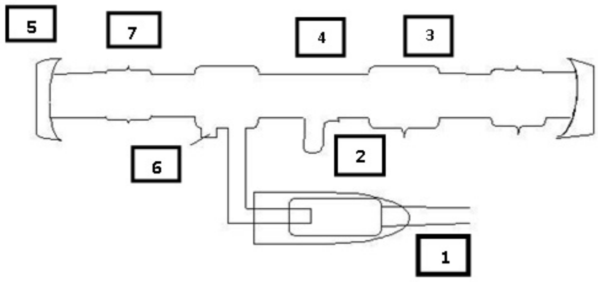

Figure 5.3: *structure of laser tube,*1-*cathode,*2-*cd oven,*3-*anode,*4-*active bore* 5-*mirror,*6-*auxiliary anode,*7-*bellows.*

Source: Gailan Asad Kazem, Diala , Journal , Volume , 39 , (2009). By experimenting on a He-Cd laser in a weak axial magnetic field (WAMF)

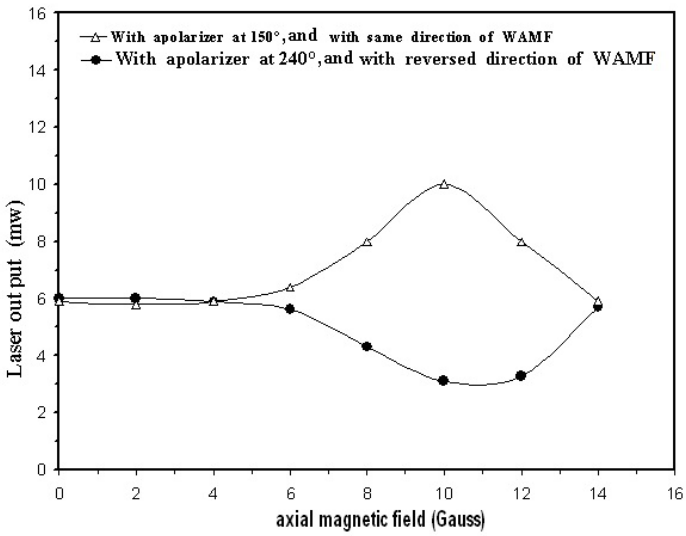

Figure 5.4: *laser output as a function of the WAMF strength,*(Δ) *with a polarizer at* 150 *with same direction of* $WAMF$, *and* ($\bullet$) *with a polarizer at* 240 *with reversed direction of* $WAMF$.

Source: Gailan Asad Kazem, Diala , Journal , Volume , 39 , (2009). By experimenting on a He-Cd laser in a weak axial magnetic field (WAMF)

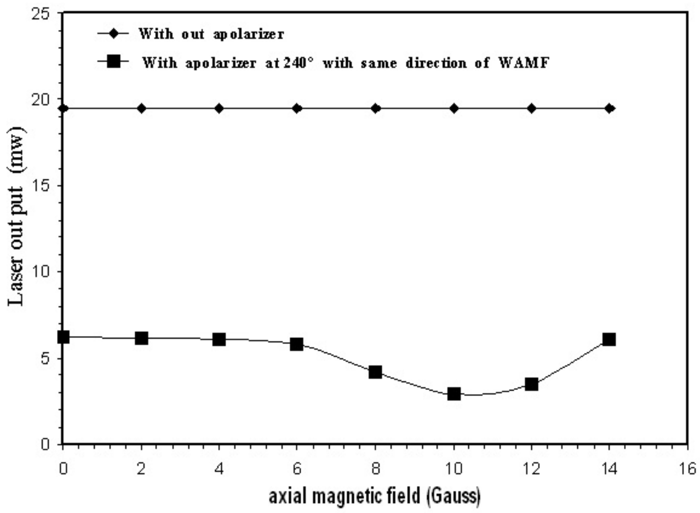

Figure 5.5: *laser output as a function of the* $WAMF$ *strength,*(■) *with a polarizer at* 240 *with same direction of* $WAMF$*,* (♦) *with out polarized . Table*(2) *laser output as a function of the* $WAMF$ *strength with a polarizer at* 150 , 240 *with same direction of* $WAMF$*, and* 240 *with reversed direction of* $WAMF$*,and also without polarizer..*

Source: Gailan Asad Kazem, Diala , Journal , Volume , 39 , (2009). By experimenting on a He-Cd laser in a weak axial magnetic field (WAMF)

The total output of the laser remained unchanged without the polarizer in front of the detector. It can be concluded that the weak axial magnetic field only changed the polarization of the laser rather than the gain of the laser. When a polarizer was inserted, however, the laser power varied with the weak axial magnetic field. the weak axial magnetic field causes The anisotropy of the laser gain, the rotation of the main polarization axis of the laser, and the change the laser output which is a function of the polarization direction of the polarizer. In figs. (5.4) and (5.5). the Lines marked with (Δ) and (■) refer to cases in which the polarization axis of the polarizer are at angles of 150 and 240 respectively with the abscissa, and there is difference of 90 between these two cases. The curves showing the laser output versus the weak axial magnetic field for these two cases in figs. (5.4) and (5.5) [54]. Vary in opposite direction. When the weak axial magnetic field was stronger than 10 Gauss, the laser output was modulated by more than 80% of its amplitude.

5.11 Influence of Magnetic Field on semiconductor laser

The work titled " Influence of magnetic field on laser " made by R. Abd Elgani (2003). Studied the effect of magnetic field on laser beam intensity and polarization. One has found that the magnetic field increases the polarization range and increases also the laser intensity by increasing the number of photons. The data as shown in table (5.1) was obtained empirically and it relate the intensity of laser beam $I(J/m^2s)$ to the applied magnetic field H/T

Table 5.1: Relation of intensity of laser beam $I(J/m^2s)$ versus of the applied magnetic field H/T

H / T	$I(J/m^2s)$
0.039	8.972236×10^{-3}
0.074	9.170857×10^{-3}
0.100	9.300988×10^{-3}
0.136	9.335233×10^{-3}
0.167	9.348931×10^{-3}
0.202	9.431119×10^{-3}
0.231	9.513307×10^{-3}
0.254	9.527005×10^{-3}
0.282	9.568099×10^{-3}
0.303	9.611560×10^{-3}

Source: R. Abd Elgani, Influence of magnetic field on laser, P. 60 (2003).

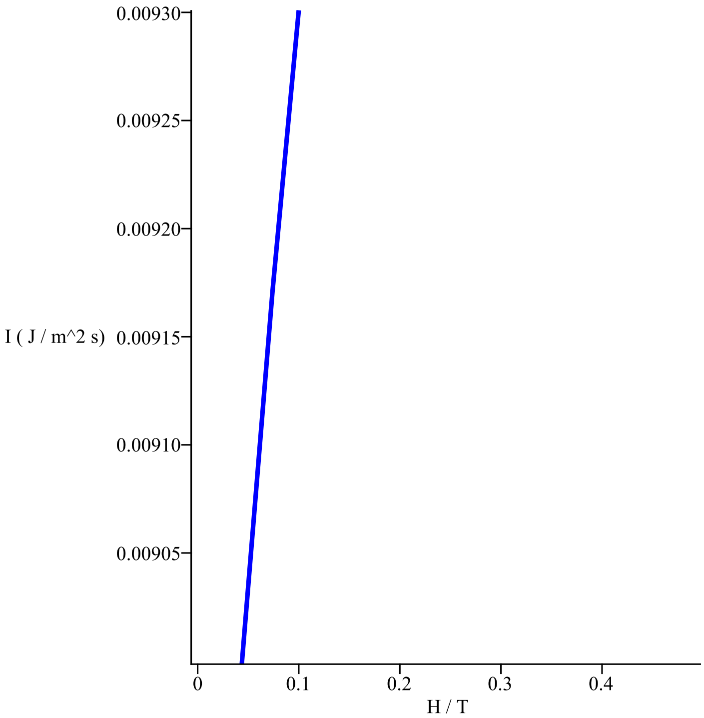

Figure 5.6: *Intensity of laser beam $I(J/m^2s)$ versus of the applied magnetic field H/T*

The intensity of laser beam as a function of the applied magnetic field is displayed in figure (5.6) according to table (5.1) [55]. Obtained that the laser intensity increases with increases of magnetic field [55].

Chapter 6

Generalized Relativistic Mass-energy Expression and Lasing

6.1 introduction

In this chapter one has to discuss how gain coefficient (amplification coefficient) is effected by the new theory called Generalized Special Relativity theory (GSR). furthermore, the study will discussed the effects of fields on light amplification with population inversion within the framework of (GSR), the effects of fields on amplification for free elements in the presence of static magnetic fields and electric fields, the effects of field on laser induced by oscillating electric field beside sound vibration, the effects of fields on lasing of thermally vibrating atoms electrons in the presence of electric and magnetic fields, effects of field on amplification due to dipole moment and the effects of fields on harmonic oscillator gain coefficient.

6.2 Expression of Mass in Generalized Special Relativity

In this work the expression of time in generalized special relativity together with the principle of conservation of momentum are utilized to find a useful expression for mass and energy of a moving particle in a field generating a constant acceleration. The energy and mass expression shows that they both depends on potential as well as velocity. The two expressions are in conformity with Savickas and generalized special relativity modes.

6.3 Generalized Special Relativity

The expression of the energy momentum tensor of the gravitation field, together with the space time interval in a curved space, were used by some authors to derive expressions for time, length mass and energy in the presence of weak fields in the form [56, 57, 58].

$$t = \frac{t_0}{\sqrt{g_{00} - \frac{v^2}{c^2}}} = \gamma t_0 \tag{6.3.1}$$

$$L = L_0\sqrt{g_{00} - \frac{v^2}{c^2}} = \gamma^{-1} L_0$$

$$m = \frac{g_{00} m_0}{\sqrt{g_{00} - \frac{v^2}{c^2}}} = g_{00}\gamma m_0$$

$$E = mc^2 \tag{6.3.2}$$

Where

$$g_{00} = 1 + \frac{2\phi}{c^2}, \qquad \gamma = \frac{1}{\sqrt{g_{00} - \frac{v^2}{c^2}}}, \tag{6.3.3}$$

with t_0, L_0, m_0, standing for the mass at rest in free space, while t, l, m, represents the corresponding values in the presence of field having a potential ϕ per unit for particles moving with speed v. The speed of light in vacuum is designated here by c.

6.4 The Relativistic Expression of Mass in the presence of fields by using momentum conservation

To find an expression of the mass m of particle moving with speed υ in potential per unit mass ϕ consider two particles colliding elastically. Let frame S' move with constant acceleration a. In a field with respect S which is in free space. Before collision let particle 1 having a mass m_1 is at rest in S, while particle 2 with mass m_2 in frame S'. At the same time m_1 was thrown

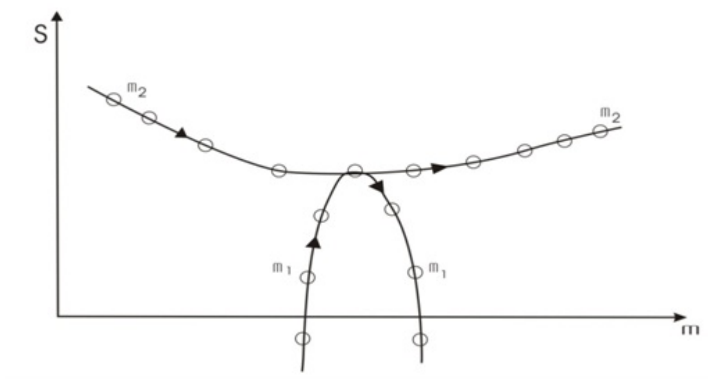

Figure 6.1: *The two masses as observed in S*

in the $+y$ direction at speed υ_1 while m_2 was thrown in the $-y$ direction at speed υ_2' such that

$$\upsilon_1 = \upsilon_2' \qquad (6.4.4)$$

After collision, m_1 rebounds in the $-y$ direction at the speed υ_1 while: m_2 rebounds in the $+y'$ direction at the speed υ_2' If the particles are thrown from positions L apart, the observer find collision occurs at

$$y = \frac{1}{2}L \qquad (6.4.5)$$

and that in $S^{'}$ find it occurs at

$$y^{'} = \frac{1}{2}L \tag{6.4.6}$$

The round trip for m_1 as measured in SI is

$$t_0 = \frac{L}{\upsilon_1} \tag{6.4.7}$$

Which is the same for m_2 in $S^{'}$

$$t_0 = \frac{L}{\upsilon_2^{'}} \tag{6.4.8}$$

The momentum conservation in S requires

$$m_1\upsilon_1 = m_2\upsilon_2, \tag{6.4.9}$$

where m_1, m_2, υ_1, υ_2 are the masses and velocities as measured in S. In the S frame the speed υ_2 is given by

$$\upsilon_2 = \frac{L}{t} \tag{6.4.10}$$

but

$$t = \gamma t_0 \tag{6.4.11}$$

thus

$$\upsilon_2 = \frac{L}{\gamma t_0} \tag{6.4.12}$$

Substitute Eqs.(6.4.7) , (6.4.10) in Eq.(6.4.9) yields

$$m_1\frac{L}{t_0} = m_2\frac{L}{\gamma t_0} \tag{6.4.13}$$

$$m_1 = \frac{m_2}{\gamma}$$

Since for S m_1 is at rest while m_2 moves , thus

$$m_1 = m_0, \qquad m_2 = m$$

hence

$$m = \gamma m_0 = \frac{m_0}{\sqrt{g_{00} - \frac{v^2}{c^2}}} \tag{6.4.14}$$

$$m = \frac{m_0}{\sqrt{1 + \frac{2\phi}{c^2} - \frac{v^2}{c^2}}} \tag{6.4.15}$$

This expression indicates that the mass is affected by field potential as well as on velocity v. To find energy expression consider a particle moving with initial velocity v with constant acceleration a. The velocity v_f at any time t is given by

$$v_f = v^2 - 2ax, = v^2 - 2\phi, \tag{6.4.16}$$

where the potential V is given by

$$V = m\phi = max \tag{6.4.17}$$

Thus Eq.(6.4.15) can be rewritten as

$$m = \frac{m_0}{\sqrt{1 - \frac{v_f}{c^2}}} \tag{6.4.18}$$

Thus the kinetic energy T is given by

$$\begin{aligned} T &= \int F dx = \int \frac{d(mv_f)}{dt} dx = \int \frac{dx}{dt} d(mv_f) \\ &= \int v d(mv_f) = [mv_f^2] - \int mv_f dv_f \\ &= mc^2 - m_0c^2 \end{aligned} \tag{6.4.19}$$

Thus the energy is given by

$$\begin{aligned} E &= mc^2 = \frac{m_0c^2}{\sqrt{1 - \frac{v_f}{c^2}}} \qquad (6.4.20) \\ &= \frac{m_0c^2}{\sqrt{1 + \frac{2\phi}{c^2} - \frac{v^2}{c^2}}} \end{aligned}$$

For low velocity

$$\begin{aligned}
E &= m_0c^2(1-\frac{v_f^2}{c^2})^{-1/2} = m_0c^2(1+\frac{1}{2}\frac{v_f^2}{c^2}) \\
&= m_0c^2+\frac{m_0v_f^2}{2} = m_0c^2+\frac{m_0}{2}(v^2-2\phi) \\
&= m_0c^2+\frac{1}{2}m_0v^2-m_0\phi \\
&= m_0c^2+T+V \qquad (6.4.21)
\end{aligned}$$

This is a new form of the energy expression. It reduces to the ordinary Newtonian energy expression, as shown also by same authors [59]. For low speed of the observer v and as far as the rest mass of photon is extremely small, thus;

$$T = \frac{1}{2}m_0v^2 \to 0$$

but since photon energy in vacuum and inside a field are given by

$$hf_0 = m_0c^2, \qquad hf = mc^2$$

thus equation (6.4.21) given by

$$hf = hf_0 + V$$

thus the energy expression can also explain the gravitational red shift phenomenon as shown also by the same authors [59].

6.5 Energy Conservation Law within framework of Generalized Special Relativity (GSR)

Using the Generalized Special Relativity (GSR) to proved energy conservation law. let us consider E_0 is initial energy at rest, $\phi = 0$, and E is final energy. Therefore, the initial energy E_0 is given by

$$E_0 = \frac{m_0c^2}{\sqrt{1-\frac{v_0^2}{c^2}}} \qquad (6.5.22)$$

The final energy E also given by

$$E = \frac{m_0c^2}{\sqrt{1+\frac{2\phi}{c^2}-\frac{\upsilon^2}{c^2}}} \tag{6.5.23}$$

What happen when $E = E_0$. The relativistic mass m is given by

$$m = \frac{m_0}{\sqrt{1+\frac{2\phi}{c^2}-\frac{\upsilon^2}{c^2}}} \tag{6.5.24}$$

Substitute equation (6.5.24) into (6.5.23) yields

$$E = \frac{m_0c^2}{\sqrt{1+\frac{2\phi}{c^2}-\frac{\upsilon^2}{c^2}}} \tag{6.5.25}$$

When we compare equation (6.5.22) and (6.5.25) we obtain

$$\frac{m_0c^2}{\sqrt{1-\frac{\upsilon_0^2}{c^2}}} = \frac{m_0c^2}{\sqrt{1+\frac{2\phi}{c^2}-\frac{\upsilon^2}{c^2}}} \tag{6.5.26}$$

We omit the $m_0, 1, c^2$ and multiply by $\frac{1}{2}m$ it gets

$$\frac{-\upsilon_0^2}{c^2} = \frac{-\upsilon^2}{c^2} + \frac{2\phi}{c^2} \tag{6.5.27}$$

Then;

$$\upsilon^2 = \upsilon_0^2 + 2\phi \tag{6.5.28}$$

$$\frac{1}{2}m\upsilon^2 = \frac{1}{2}m\upsilon_0^2 + \phi m$$

This is a new form of the energy conservation law.

$$T = T_0 + V,$$

where, T is total energy, T_0 kinetic energy and V is potential.
Consider two points 1 and 2, having velocities υ_1 and υ_2 and potentials ϕ_1 and ϕ_2, the energies are given by

$$E_1 = \frac{m_0c^2}{\sqrt{1+\frac{2\phi_1}{c^2}-\frac{\upsilon_1^2}{c^2}}}, \qquad E_2 = \frac{m_0c^2}{\sqrt{1+\frac{2\phi_2}{c^2}-\frac{\upsilon_2^2}{c^2}}} \tag{6.5.29}$$

for the two energies to be equal

$$E_1 = E_2$$

$$\frac{2\phi_1}{c^2} - \frac{\upsilon_1^2}{c^2} = \frac{2\phi_2}{c^2} - \frac{\upsilon_2^2}{c^2}$$

Multiply both sides by $\frac{mc^2}{2}$, one gets.

$$m\phi_1 - \frac{1}{2}m\upsilon_1^2 = m\phi_2 - \frac{1}{2}m\upsilon_2^2$$

$$V_1 - T_1 = V_2 - T_2$$
$$T_1 - V_1 = T_2 - V_2$$

But the lagrangian is defined to be

$$L = T - V \tag{6.5.30}$$

Thus conservation law requires

$$L_1 = L_2$$

The equation (6.5.30) is also a new form of conservation law in case of Lagrangian formula.

6.6 Free electron lasing within framework of Generalized Special Relativity (GSR)

Let us consider that the relativistic factor $\gamma = \left(1 - \frac{\upsilon_0^2}{c^2}\right)^{-1/2}$, in generalized special relativity $\gamma = \left(1 + \frac{2\phi}{c^2} - \frac{\upsilon^2}{c^2}\right)^{-1/2}$, we substitute in equation (4.2.11) as following:
Substitute the factor $\gamma = \left(1 - \frac{\upsilon_0^2}{c^2}\right)^{-1/2}$ we get

$$\beta = const \times N^{1/3} B_0^{2/3} L^{1/3} \left(\left(1 - \frac{\upsilon_0^2}{c^2}\right)^{-1/2}\right)^{-1} \tag{6.6.31}$$

When the relativistic factor γ used in equation (6.6.31) considered, the expression for gain coefficient or lasing in special relativity (SR) takes the form

$$\beta = const \times N^{1/3} B_0^{2/3} L^{1/3} \sqrt{1 - \frac{v_0^2}{c^2}} \tag{6.6.32}$$

while substitute $\gamma = \left(1 + \frac{2\phi}{c^2} - \frac{v^2}{c^2}\right)^{-1/2}$ of generalized special relativity we get

$$\beta = const \times N^{1/3} B_0^{2/3} L^{1/3} \left(\left(1 + \frac{2\phi}{c^2} - \frac{v^2}{c^2}\right)^{-1/2} \right)^{-1} \tag{6.6.33}$$

Then,

$$\beta = const \times N^{1/3} B_0^{2/3} L^{1/3} \sqrt{1 + \frac{2\phi}{c^2} - \frac{v^2}{c^2}} \tag{6.6.34}$$

The Generalized Special Relativity theory is a new form of the special relativity theory that adopts the gravitational potential.
Comparing equations (6.6.32) with (6.6.34), leads to observe that potential field does not affect in gain coefficient, when one dealing with the special relativity (SR). One has explained that the potential field does not appears. By using Generalized Special Relativity theory (GSR), gravitational potential, or the field in which the mass is measured appears. ϕ denotes the gravitational potential, which can be given by

$$\phi = \frac{-MG}{r}$$

It is important to note that ϕ here is the potential per unit mass for any field. the appearance of potential field leads to affect of magnetic field to gain coefficient or lasing.

6.7 Effects of Fields on Light Amplification with Population Inversion within the framework of GSR

The amplification coefficient β can be derived from the Lambert-Beer law [61].

$$I = I_0 e^{\beta z}, \tag{6.7.35}$$

where I is the intensity transmitted through the sample I_0 is incident intensity and z stands for the distance traversed by radiation in the medium. The amplification coefficient β is given by:

$$\beta = B(n_2 - n_1)\frac{hf}{c}z, \tag{6.7.36}$$

where β is amplification coefficient, B is Einstein Coefficient, n is the number of particle at a certain energy level, h Blank Constant, f frequency, c speed of light z axis. From equation (6.7.36) and referring to Einstein equation:

$$m_0 c^2 = hf \tag{6.7.37}$$

The equation inside medium become:

$$mc^2 = h\acute{f} \tag{6.7.38}$$

It is possible to use the new theory of Generalized Special Relativity (GSR) can be given:

$$m = \frac{g_{00} m_0}{\sqrt{g_{00} - \frac{v^2}{c^2}}}, \tag{6.7.39}$$

where $g_{00} = 1 + \frac{2\phi}{c^2}$
Since

$$m = \frac{m_0(1 + \frac{2\phi}{c^2})}{\sqrt{1 + \frac{2\phi}{c^2} - \frac{v^2}{c^2}}} \tag{6.7.40}$$

And referring to equation (6.7.36) it follows that $\acute{\beta}$ becomes:

$$\acute{\beta} = B(n_2 - n_1)\frac{h\acute{f}}{c} \tag{6.7.41}$$

$$\frac{c\acute{\beta}}{B(n_2 - n_1)} = h\acute{f} \tag{6.7.42}$$

$$\frac{m_0 g_{00}}{\sqrt{g_{00} - \frac{v^2}{c^2}}} c^2 = h\acute{f} \tag{6.7.43}$$

Comparing equation (6.7.42) with (6.7.43) yields:

$$\frac{c\acute{\beta}}{B(n_2 - n_1)} = \frac{m_0 g_{00}}{\sqrt{g_{00} - \frac{v^2}{c^2}}} c^2 \tag{6.7.44}$$

When $g_{00} = 1 + \frac{2\phi}{c^2}$, then the amplification factor in the presence of a field, of potential ϕ per unit mass, becomes:

$$\acute{\beta} = \frac{Bc(n_2 - n_1)m_0(1 + \frac{2\phi}{c^2})}{\sqrt{1 + \frac{2\phi}{c^2} - \frac{v^2}{c^2}}} \tag{6.7.45}$$

This expression represents the Gain coefficient within framework of Generalized Special Relativity (GSR). On other hand from equation (6.7.36) the gain coefficient is given by:

$$\acute{\beta} = B(n_2 - n_1)\frac{hf}{c} \tag{6.7.46}$$

But since

$$m_0 c^2 = hf \tag{6.7.47}$$

And

$$mc^2 = hf$$

$$\beta = B(n_2 - n_1)\frac{hf}{c}$$

$$c\beta = B(n_2 - n_1)hf$$

$$\frac{c\beta}{B(n_2 - n_1)} = hf \tag{6.7.48}$$

Comparing equation (6.7.47) with equation (6.7.48) yields:

$$\frac{c\beta}{B(n_2 - n_1)} = m_0 c^2$$

$$\beta = B(n_2 - n_1) m_0 c \tag{6.7.49}$$

When neglecting the field potential and when the υ is slow speed in equation (6.7.45).

$$\phi \to 0, \qquad \frac{\upsilon^2}{c^2} \to 0$$

Thus the amplification coefficient becomes:

$$\beta = Bc(n_2 - n_1) m_0 \tag{6.7.50}$$

Thus the amplification coefficient reduces to the conventional one for low speed particles in the absence of fields. When the crystal field is not neglected.

$$mc^2 = hf \tag{6.7.51}$$

The amplification coefficient becomes:

$$\acute{\beta} = B(n_2 - n_1)\frac{hf}{c} \tag{6.7.52}$$

$$\frac{c\acute{\beta}}{B(n_2 - n_1)} = hf \tag{6.7.53}$$

Comparing equation (6.7.51) with equation (6.7.53) yields:

$$\frac{c\acute{\beta}}{B(n_2 - n_1)} = mc^2$$

Then;

$$\acute{\beta} = B(n_2 - n_1) mc$$

When,

$$m = \frac{m_0 g_{00}}{\sqrt{g_{00} - \frac{\upsilon^2}{c^2}}} \qquad \Rightarrow \qquad \frac{m_0(1 + \frac{2\phi}{c^2})}{\sqrt{1 + \frac{2\phi}{c^2} - \frac{\upsilon^2}{c^2}}}$$

$$\acute{\beta} = B(n_2 - n_1)\frac{m_0 c g_{00}}{\sqrt{g_{00} - \frac{v^2}{c^2}}}$$

$$\acute{\beta} = B(n_2 - n_1)\frac{m_0 c(1 + \frac{2\phi}{c^2})}{\sqrt{1 + \frac{2\phi}{c^2} - \frac{v^2}{c^2}}} \tag{6.7.54}$$

$$\begin{aligned}
\acute{\beta} &= C_0\left(1 + \frac{2\phi}{c^2}\right)\left(1 + \frac{2\phi}{c^2}\right)^{-1/2} \\
&= C_0\left(1 + \frac{2\phi}{c^2}\right)\left(1 - \frac{\phi}{c^2}\right) \\
&= C_0\left(1 - \frac{\phi}{c^2} + \frac{2\phi}{c^2} - \frac{2\phi^2}{c^4}\right) \\
&= C_0\left[1 + \frac{\phi}{c^2} - \frac{2\phi^2}{c^4}\right] \\
&= Bm_0 c(n_2 - n_1)\left[1 + \frac{\phi}{c^2} - \frac{2\phi^2}{c^4}\right] \\
&= B(n_2 - n_1)\left[m_0 c + \frac{V}{c} - \frac{2V^2}{c^3}\right]
\end{aligned}$$

$$\acute{\beta} = \frac{(1 + \frac{2\phi}{c^2})}{\sqrt{1 + \frac{2\phi}{c^2} - \frac{v^2}{c^2}}}\beta$$

When the field is weak $\phi << c^2$, $\frac{\phi}{c^2} \simeq 0$ n = refractive index $= \frac{c}{v}$
Thus,

$$\acute{\beta} = \frac{\beta}{\sqrt{1 - \frac{1}{n}}} = \frac{\beta}{\sqrt{\frac{n-1}{n}}} \tag{6.7.55}$$

Thus amplification factor depends on the refractive index. Thus the amplification factor is affected by the crystal field as well as the refractive index as shown by equation (6.7.54) and (6.7.55).

6.8 Effects of Fields on Amplification for free electrons in the presence of static Magnetic and Electric Fields

If a resistive medium with characteristic relaxation time is subjected to an electric field in the x-direction and magnetic filed in the direction of z in such a way that it moves in the $x-y$ plain with velocity, magnetic field and electric field given by [62]:

$$\upsilon_T = \upsilon_x\vec{i} + \upsilon_y\vec{j} = \upsilon\vec{i} + \upsilon\vec{j} \tag{6.8.56}$$

$$B_T = B_z\vec{k} \tag{6.8.57}$$

Where one assumes that

$$\upsilon_x = \upsilon_y = \upsilon \tag{6.8.58}$$

In this case the equation of the motion reads

$$ma = eE + eB \times \upsilon - \frac{mT}{\tau} \tag{6.8.59}$$

$$\begin{pmatrix} \vec{i} & \vec{j} & \vec{k} \\ \upsilon_x & \upsilon_y & \upsilon_z \\ 0 & 0 & B_z \end{pmatrix} = (\upsilon B_z)\vec{i} - (\upsilon B_z)\vec{j} \tag{6.8.60}$$

$$ma_x = eE_x + e\upsilon_y B_z - \frac{m\upsilon_x}{\tau} \tag{6.8.61}$$

$$ma_y = eE_y + e\upsilon_x B_z - \frac{m\upsilon_y}{\tau} \tag{6.8.62}$$

$$m\frac{d\upsilon}{dt} = eE_x + e\upsilon B - \frac{m\upsilon}{\tau} \tag{6.8.63}$$

$$m\frac{d\upsilon}{dt} = eE_y + e\upsilon B - \frac{m\upsilon}{\tau} \tag{6.8.64}$$

Confining our selves to the x direction the current density J_x is given by [62]:

$$J_x = ne\upsilon_x = ne\upsilon = \sigma E_x \tag{6.8.65}$$

For motion with constant speed

$$\frac{d\upsilon}{dt} = 0 \tag{6.8.66}$$

And equation (6.8.63) reduced to $(\frac{m}{\tau} - Be)\upsilon = eE_x$, thus

$$\upsilon = \frac{e}{\frac{m}{\tau} - Be} E_x = \frac{\tau e}{m - Be\tau} E_x \tag{6.8.67}$$

Utilizing this result in (6.8.65) yields:

$$J_x = \frac{ne^2\tau}{m - Be\tau} E_x = \sigma E_x \tag{6.8.68}$$

As result the conductivity is given by:

$$\sigma = \frac{ne^2\tau}{m - Be\tau} \tag{6.8.69}$$

Since σ stands for the real part σ_1 it follows from relations $\sigma_1 = -\omega\epsilon_0\chi_2$, $\sigma_2 = \omega\epsilon_0\chi_1$ that,

$$\beta = \frac{\mu\sigma_1}{n_1} = \frac{\mu ne^2\tau}{n_1(m - Be\tau)} \tag{6.8.70}$$

According to equation:

$$mn(\frac{d\upsilon}{dt} + \upsilon\nabla\upsilon) = enE_e - \gamma K_B T\nabla n$$

Amplification takes place when $\beta > 0$ this requires:

$$m > Be\tau, \qquad \frac{m\upsilon}{\tau}, \qquad F_r > F_m \tag{6.8.71}$$

This indicates that amplification of electromagnetic waves is possible if the resistive force F_r exceed the magnetic field F_m. The resistive force which is measure of amount of energy lost in collision also measures the number of excited atoms which gain energy loosed by free electron during collision. Thus the larger number of excited atoms are emitting large number of photons responsible of inducing laser.
When one has applied the new theory of Generalized Special Relativity

(GSR) into equation (6.8.70) it produces the following equation, this can be done using the identity.

$$m = \frac{m_0 g_{00}}{\sqrt{g_{00} - \frac{v^2}{c^2}}}, \tag{6.8.72}$$

where $g_{00} = 1 + \frac{2\phi}{c^2}$
The gain coefficient can be written in terms of conductivity in the form

$$\beta = \frac{\mu\sigma_1}{n_1} = \frac{\mu n e^2 \tau}{n_1(m - Be\tau)} \tag{6.8.73}$$

When one substitutes the relativistic mass see equation (6.8.72) into equation (6.8.73) one gets

$$\beta = \frac{\mu n e^2 \tau}{n_1 \left(\frac{m_0 g_{00}}{\sqrt{g_{00} - \frac{v^2}{c^2}}} - Be\tau \right)} \tag{6.8.74}$$

$$\beta = \frac{\mu n e^2 \tau}{n_1 \left(\frac{m_0 g_{00} - Be\tau\sqrt{g_{00} - \frac{v^2}{c^2}}}{\sqrt{g_{00} - \frac{v^2}{c^2}}} \right)} \tag{6.8.75}$$

$$\beta = \frac{\mu n e^2 \tau}{n_1 \left(\frac{m_0(1+\frac{2\phi}{c^2}) - Be\tau\sqrt{1+\frac{2\phi}{c^2} - \frac{v^2}{c^2}}}{\sqrt{1+\frac{2\phi}{c^2} - \frac{v^2}{c^2}}} \right)}$$

$$\beta = \frac{\mu n e^2 \tau \sqrt{1 + \frac{2\phi}{c^2} - \frac{v^2}{c^2}}}{n_1 \left(m_0(1 + \frac{2\phi}{c^2}) - Be\tau\sqrt{1 + \frac{2\phi}{c^2} - \frac{v^2}{c^2}} \right)} \tag{6.8.76}$$

Equation (6.8.76) shows again that the field applied externally into the material or the internal fields affect amplification factor.

6.9 Effects of Fields on Laser Induced by Oscillating Electric field beside Sound Vibration

Consider free electrons in a conductor having resistive medium characterized by relaxation time τ. When these electrons are affected by an oscillating electric field beside a sound wave in the form.

$$E = E_0 e^{i\omega t}, \qquad F = F_0 e^{i\omega t} \tag{6.9.77}$$

The motion of free electrons become

$$m\frac{dv}{dt} = eE_0 e^{i\omega t} t + F_0 e^{i\omega t} - \frac{mv}{\tau} \tag{6.9.78}$$

The solution of this equation can be suggested to be,

$$v = v_0 e^{i\omega t} \tag{6.9.79}$$

Incorporating (6.9.79) in (6.9.78) one can write

$$\begin{aligned} im\omega v_0 &= eE_0 + F_0 - \frac{mv_0}{\tau} \\ \left(im\omega - \frac{F_0}{v_0} + \frac{m}{\tau}\right) v_0 &= eE_0 \end{aligned}$$

So we get.

$$v_0 = \frac{eE_0}{im\omega + \frac{m}{\tau} - \frac{F_0}{v_0}} \tag{6.9.80}$$

Thus;

$$\underline{v} = \frac{e}{im\omega + \frac{m}{\tau} - \frac{F_0}{v_0}} \underline{E} \tag{6.9.81}$$

According to the definition of current density J

$$J = \sigma E = ne\underline{v} = \frac{ne^2}{im\omega + \frac{m}{\tau} - \frac{F_0}{v_0}} E \tag{6.9.82}$$

Hence

$$\sigma = \frac{ne^2\left[\left(\frac{m}{\tau}-\frac{F_0}{v_0}\right)-im\omega\right]}{\left[\left(\frac{m}{\tau}-\frac{F_0}{v_0}\right)+m^2\omega^2\right]} = \sigma_1 + i\sigma_2$$

Hence the real and imaginary parts of the conductivity are given by:

$$\sigma_1 = \frac{ne^2\left[\frac{m}{\tau}-\frac{F_0}{v_0}\right]}{\left[\left(\frac{m}{\tau}-\frac{F_0}{v_0}\right)^2+m^2\omega^2\right]} \tag{6.9.83}$$

$$\sigma_2 = \frac{-m\omega ne^2}{\left[\left(\frac{m}{\tau}-\frac{F_0}{v_0}\right)^2+m^2\omega^2\right]}$$

With the aid of equation (6.8.73) the gain coefficient is given by:

$$\beta = \frac{\mu ne^2\left[\frac{m_0}{\tau}-\frac{F_0}{v_0}\right]}{n_1\left[\left(\frac{m_0}{\tau}-\frac{F_0}{v_0}\right)^2+m_0^2\omega^2\right]} \tag{6.9.84}$$

Amplification takes place if $\beta > 0$, $\frac{m}{\tau}-\frac{F_0}{v_0} > 0 \rightarrow \frac{mv_0}{\tau} > F_0$ then, $\frac{mv}{\tau} > F$ so we find that $F_r > F$

When we substitute relativistic mass and relative force within the frame work of Generalized Special Relativity (GSR) theory, into equation (6.9.84) one gets the relativistic mass is given by:

$$m = \frac{m_0 g_{00}}{\sqrt{g_{00}-\frac{v^2}{c^2}}} \qquad \Rightarrow \qquad \frac{m_0(1+\frac{2\phi}{c^2})}{\sqrt{1+\frac{2\phi}{c^2}-\frac{v^2}{c^2}}} \tag{6.9.85}$$

and the force is given by

$$F = ma$$

$$F = \frac{\left(1+\frac{2\phi}{c^2}\right)m_0 a}{\sqrt{1+\frac{2\phi}{c^2}-\frac{v^2}{c^2}}} \qquad \Rightarrow \qquad \frac{\left(1+\frac{2\phi}{c^2}\right)F_0}{\sqrt{1+\frac{2\phi}{c^2}-\frac{v^2}{c^2}}} \tag{6.9.86}$$

$$\acute{\beta} = \frac{\mu n e^2 \left[\frac{m}{\tau} - \frac{F}{v_0}\right]}{n_1 \left[\left(\frac{m}{\tau} - \frac{F}{v_0}\right)^2 + m^2\omega^2\right]} \tag{6.9.87}$$

Then inserting (6.9.85), (6.9.86) in (6.9.87) yields:

$$\acute{\beta} = \frac{\mu n e^2 \left[\frac{m_0 g_{00}}{\tau\sqrt{g_{00} - \frac{v^2}{c^2}}} - \frac{\sqrt{1+\frac{2\phi}{c^2}-\frac{v^2}{c^2}}}{v_0\left(1+\frac{2\phi}{c^2}\right)}F_0\right]}{n_1\left[\left(\frac{m_0 g_{00}}{\tau\sqrt{1+\frac{2\phi}{c^2}-\frac{v^2}{c^2}}} - \frac{\sqrt{g_{00}-\frac{v^2}{c^2}}}{v_0(g_{00})}F_0\right)^2 + \left(\frac{m_0 g_{00}}{\sqrt{g_{00}-\frac{v^2}{c^2}}}\right)^2\omega^2\right]}$$

$$\acute{\beta} = \frac{\mu n e^2 \left[\frac{m_0(1+\frac{2\phi}{c^2})}{\tau\sqrt{1+\frac{2\phi}{c^2}-\frac{v^2}{c^2}}} - \frac{\sqrt{1+\frac{2\phi}{c^2}-\frac{v^2}{c^2}}}{v_0\left(1+\frac{2\phi}{c^2}\right)}F_0\right]}{n_1\left[\left(\frac{m_0(1+\frac{2\phi}{c^2})}{\tau\sqrt{1+\frac{2\phi}{c^2}-\frac{v^2}{c^2}}} - \frac{\sqrt{1+\frac{2\phi}{c^2}-\frac{v^2}{c^2}}}{v_0(1+\frac{2\phi}{c^2})}F_0\right)^2 + \left(\frac{m_0(1+\frac{2\phi}{c^2})}{\sqrt{1+\frac{2\phi}{c^2}-\frac{v^2}{c^2}}}\right)^2\omega^2\right]} \tag{6.9.88}$$

$$\beta = \frac{\mu n e^2 \left[\frac{m_0(1+\frac{2\phi}{c^2})}{\tau\sqrt{1+\frac{2\phi}{c^2}-\frac{v^2}{c^2}}} - \frac{\sqrt{1+\frac{2\phi}{c^2}-\frac{v^2}{c^2}}}{v_0\sqrt{1+\frac{2\phi}{c^2}-\frac{v^2}{c^2}}}F_0\right]}{n_1\left[\left(\frac{m_0(1+\frac{2\phi}{c^2})}{\tau\sqrt{1+\frac{2\phi}{c^2}-\frac{v^2}{c^2}}} - \frac{\sqrt{1+\frac{2\phi}{c^2}-\frac{v^2}{c^2}}}{v_0\sqrt{1+\frac{2\phi}{c^2}-\frac{v^2}{c^2}}}F_0\right)^2 + \left(\frac{m_0(1+\frac{2\phi}{c^2})}{\sqrt{1+\frac{2\phi}{c^2}-\frac{v^2}{c^2}}}\right)^2\omega^2\right]}$$

Thus the amplification factor is affected by the crystal field as well as the velocity of the electrons.

6.10 Effects of Fields on lasing of Thermally Vibrating Atoms and electrons in the presence of electric and magnetic Fields

Ionized atoms emit radiation according to electromagnetic theory as for as accelerated and oscillating charged particle emit electromagnetic radiation. Two cases are considered here. The first case concerns with ionic crystals vibration while the second one concerns the electrons vibration. When ions in crystal is vibrating the equation of motion ions in the presence of oscillating electric field and constant magnetic field takes the form:

$$m\ddot{u_n} = -c(2u_n - u_{n-1} - u_{n+1}) + eE + Beu_n \quad (6.10.89)$$

$$m\ddot{u_{n+1}} = -c(2u_{n+1} - u_n - u_{n+2}) + eE + Beu_{n+1} \quad (6.10.90)$$

Where,

$$E = E_0 e^{i\omega t}, \qquad u_n = u_0 e^{i\omega t} e^{ikna} \quad (6.10.91)$$

The solution of this equation requires;

$$\chi_2 = \frac{\beta_1 n_0 e^2}{\alpha_1^2 + \beta_1^2} + \frac{\beta_2 n_0 e^2}{(\alpha_1^2 + \beta_1^2)} \quad (6.10.92)$$

With

$$\beta_1 = \beta_2 = e\omega B \quad (6.10.93)$$

The gain coefficient is given by:

$$\beta = -\frac{\mu\epsilon_0\omega}{n_1}\chi_2 \quad (6.10.94)$$

In view of equation (6.10.93) and (6.10.94) amplification takes place when

$$\beta_1 = e\omega B = -e\omega|B| \quad (6.10.95)$$

When the magnetic field B is oriented is such away that the force excreted by it apposes that excreted by the electric field. In this case

$$B = -|B|, \qquad \chi_2 = -, \qquad \beta = + \quad (6.10.96)$$

Laser can also induced by vibrating electrons in resistive medium with the aid of oscillating electric field E and constant magnetic field B. With the effect of all these forces the equation of motion of the electron is given by:

$$m\frac{dv}{dt} = eE + Bev - \frac{mv}{\tau} \quad (6.10.97)$$

Where

$$E = E_0 e^{i\omega t} \tag{6.10.98}$$

The solution of (6.10.97) requires

$$\chi_2 = \frac{-n_0 e^2 \beta_3}{(\alpha_3^2 + \beta_3^2)} \tag{6.10.99}$$

$$\alpha_3 = -\omega^\ell m \qquad \beta_3 = \omega(\frac{m}{\tau} - \beta e) \tag{6.10.100}$$

In view of equation (6.10.94), (6.10.99) and (6.10.90) lasing takes places when

$$\chi_2 = -, \qquad \beta_3 = +, \qquad \frac{m}{\tau} - Be > 0$$

$$\frac{mv}{\tau} > \frac{Bev}{\tau}, \qquad F_r > F_m \tag{6.10.101}$$

a gain the condition of amplification requires the resistive force to be large. Using the Generalized Special Relativity theory (GSR), the masses are given by:

$$m = \frac{m_0 g_{00}}{\sqrt{g_{00} - \frac{v^2}{c^2}}} \Rightarrow \frac{m_0(1 + \frac{2\phi}{c^2})}{\sqrt{1 + \frac{2\phi}{c^2} - \frac{v^2}{c^2}}} \tag{6.10.102}$$

The ordinary amplification factor is given by:

$$\beta = \omega\left(\frac{m_0}{\tau} - Be\right)$$

The corresponding are in the framework of Generalized Special Relativity theory (GSR) is given by:

$$\acute{\beta} = \omega\left(\frac{m}{\tau} - Be\right) \tag{6.10.103}$$

Inserting (6.10.102) in (6.10.103) yields:

$$\acute{\beta} = \omega\left(\frac{m_0(1 + \frac{2\phi}{c^2})}{\tau\sqrt{1 + \frac{2\phi}{c^2} - \frac{v^2}{c^2}}} - Be\right) \tag{6.10.104}$$

One can be find $\acute{\beta}$ and β from the equation (6.10.104) When the crystal field is neglected, and when the speed of the electrons are slow, such that;

$$\frac{\phi}{c^2} \quad \to 0, \qquad\qquad \frac{v^2}{c^2} \quad \to 0$$

Equation (6.10.104) becomes

$$\acute{\beta} \;\; = \;\; \omega\left(\frac{m_0}{\tau} - Be\right) = \beta$$

Which is the ordinary conventional amplification factor.

6.11 Effects of Fields on Amplification due to Photon perturbation

The photon wave function can be obtained by using the relativistic relation.

$$E^2 = p^2c^2 + m_0^2c^4 \tag{6.11.105}$$

Since the photon rest mass vanishes it follows that

$$m_0 = 0 \qquad\qquad E = Pc \tag{6.11.106}$$

The operator for E and P can be obtained again from the radiation.

$$\psi = Ae^{\frac{i}{\hbar}(p_x - Et)} \tag{6.11.107}$$

Where;

$$i\hbar\frac{d}{dt}\psi = E\psi, \qquad\qquad \frac{\hbar}{i}\nabla\psi = P\psi \tag{6.11.108}$$

To check that this equation describe the free photon consider the solution.

$$\psi = Ae^{\frac{i}{\hbar}(p_x - Et)} \tag{6.11.109}$$

With;

$$P = \frac{h}{\lambda} = \frac{h}{2\pi} \times \frac{2\pi}{\lambda} = \hbar K \tag{6.11.110}$$

Substituting (6.11.109) in (6.11.108) with the aid of (6.11.110) yields

$$i\hbar\left(-\frac{i}{\hbar}E\right)\psi = \frac{\hbar c}{i}\left(-\frac{i}{\hbar}P\psi\right), \qquad Pc\psi = \frac{hc}{\lambda} = hf\psi$$

Hence;

$$E = hf \tag{6.11.111}$$

This is the ordinary expression of the photon energy. Thus the expression (6.11.109) stands for the wave function of the photon. To see how the photon wave function looks like within a medium one can recall Doppler effect and compton effects which shows that the photon frequency is affected by the motion of radiation source beside the fields and collision. Equation (6.11.108) can be made time dependent by making the substitution.

$$\psi = e^{\frac{iE}{\hbar}tu} \tag{6.11.112}$$

In equation (6.11.108) to get.

$$Eu = \frac{\hbar c}{i}\nabla u \tag{6.11.113}$$

The solution of the equation in one dimensional space takes the form $u = Ae^{i\alpha x}$, is

$$\nabla u = i\alpha u \tag{6.11.114}$$

Hence, $\frac{iE}{\hbar c} = \nabla u = i\alpha u$ thus, $\alpha = \frac{E}{\hbar c}$ we obtain

$$u = Ae^{\frac{iE}{\hbar c}x} \tag{6.11.115}$$

Since $E = \hbar\omega$ it follows that the photon wave function becomes:

$$u = Ae^{\frac{i\omega}{c}x} \tag{6.11.116}$$

One can assume that the frequency of the oscillating polarized atoms is the same as the frequency of the photon, as suggested by harmonic oscillator solution of Schrodinger equation and proposed by classical electromagnetic theory. In this case the frequency of the photon can be found from the equation of the classical harmonic oscillator in which polarization fields and external fields acts on electrons.

$$\begin{aligned} F &= ec_0E - eE \\ &= e\left(c_0 - 1\right)E_0e^{i\omega t} \\ &= \frac{e\left(c_0 - 1\right)}{x_0}x_0Ee^{i\omega t} \\ &= \frac{e\left(c_0 - 1\right)}{x_0}E_0x = -Kx \end{aligned}$$

$$F = -m\omega^2 x \tag{6.11.117}$$

Thus from $K = \frac{(1-c_0)}{x_0}E_0 = m\omega^2$ the photon frequency is given by.

$$\omega = \pm\sqrt{\frac{(1-c_0)}{x_0m}E_0} \tag{6.11.118}$$

In the case when $c_0 > 1$

$$\omega = \pm\sqrt{\frac{(1-c_0)}{x_0m}E_0} = \pm i\omega_0, \qquad \omega_0 = \pm\sqrt{\frac{(c_0-1)}{x_0m}E_0} \tag{6.11.119}$$

As a result equation (6.11.116).

$$u = Ae^{\pm\frac{\omega_0}{c}x} \tag{6.11.120}$$

Since the intensity of radiation is $I = hfpc = c|u|^2 = cAe^{\pm\frac{2\omega_0}{c}x} = I_0e^{\beta}x$ One can take the plus sign for amplification to get.

$$I = I_0e^{\frac{2\omega_0}{c}x} = I_0e^{\beta x} \tag{6.11.121}$$

In view of (6.11.119) lasing is possible when,

$$\beta = \frac{2\omega_0}{c} = \frac{2}{c}\sqrt{\frac{1-c_0}{x_0m}E_0} \tag{6.11.122}$$

This requires the polarization field E to exceed the external field.

$$\omega_0 > 0, \qquad c_0E_0 > E_0, \qquad P_0 = c_0E_0 > E_0 \tag{6.11.123}$$

This not surprising as for as the polarizing field reflects the number of photons emitted by the medium

$$n_m = |c_0 E_0|^2$$

This should exceed the incident photons.

$$n_i = |E_0|^2$$

Where equation (6.11.123) requires $|c_0 E_0|^2 > |E_0|^2$ one find $n_m > n_i$. The force in classical theory can be written as follows

$$F = ma, \tag{6.11.124}$$

where m is the mass and a is acceleration. The relative mass in the Generalized Special Relativity theory (GSR), is given by:

$$m = \frac{m_0 g_{00}}{\sqrt{g_{00} - \frac{v^2}{c^2}}} \tag{6.11.125}$$

When we substitute the value $g_{00} = 1 + \frac{2\phi}{c^2}$, in equation (6.11.125) it gives.

$$m = \frac{m_0 \left(1 + \frac{2\phi}{c^2}\right)}{\sqrt{1 + \frac{2\phi}{c^2} - \frac{v^2}{c^2}}} \tag{6.11.126}$$

When one substitute in equation (6.11.124) yields:

$$F = \frac{\left(1 + \frac{2\phi}{c^2}\right) m_0 a}{\sqrt{1 + \frac{2\phi}{c^2} - \frac{v^2}{c^2}}} \tag{6.11.127}$$

Where

$$F_0 = m_0 a \tag{6.11.128}$$

Then one substitute equation (6.11.128) into equation (6.11.127) it gives.

$$F = \frac{\left(1 + \frac{2\phi}{c^2}\right) F_0}{\sqrt{1 + \frac{2\phi}{c^2} - \frac{v^2}{c^2}}} \tag{6.11.129}$$

The relation between the force and electric field is given by:

$$F_0 = eE_0 \tag{6.11.130}$$

$$E_0 = \frac{F_0}{e} \tag{6.11.131}$$

When one substitute the equation of relative mass into equation (6.11.127) and equation (6.11.131) yields:

$$\beta \quad = \frac{2}{c}\sqrt{\frac{(1-c_0)}{x_0 m_0}E_0} \qquad = \sqrt{\frac{(1-c_0)}{x_0 m_0}\frac{F_0}{e}} \tag{6.11.132}$$

The amplification factor within the framework of the Generalized Special Relativity theory GSR, is given by:

$$\acute{\beta} = \frac{2}{c}\sqrt{\frac{(1-c_0)}{x_0 m}\frac{F}{e}} \tag{6.11.133}$$

But since,

$$m = \frac{g_{00} m_0}{\sqrt{g_{00} - \frac{v^2}{c^2}}}$$

Then the equation become,

$$\begin{aligned} F &= \frac{g_{00} m_0 a}{\sqrt{g_{00} - \frac{v^2}{c^2}}} \\ &= \frac{g_{00} F_0}{\sqrt{g_{00} - \frac{v^2}{c^2}}} \\ &= \frac{\left(1 + \frac{2\phi}{c^2}\right) F_0}{\sqrt{\left(1 + \frac{2\phi}{c^2}\right) - \frac{v^2}{c^2}}} \end{aligned}$$

When one substitute the above relations in equation (6.11.132) one gets

$$\acute{\beta} = \frac{2}{c}\sqrt{\frac{\frac{1-c_0}{x_0\left(1+\frac{2\phi}{c^2}\right)}}{\sqrt{1 + \frac{2\phi}{c^2} - \frac{v^2}{c^2}}} \times \frac{\left(1 + \frac{2\phi}{c^2}\right) F_0}{e\sqrt{1 + \frac{2\phi}{c^2} - \frac{v^2}{c^2}}}}$$

Which is the amplification factor within the framework of Generalized Special Relativity theory (GSR). This equation can be simplified by canceling some terms to get,

$$\acute{\beta} = \frac{2}{c}\sqrt{\frac{1-c_0}{x_0 m_0 g_{00}}\sqrt{g_{00}-\frac{v^2}{c^2}}\frac{g_{00}F_0}{e\sqrt{g_{00}-\frac{v^2}{c^2}}}}$$

$$\acute{\beta} = \frac{2}{c}\sqrt{\frac{1-c_0}{x_0 m_0}\frac{F_0}{e}} = \beta$$

The relation between the ordinary amplification coefficient β and the real amplification coefficient $\acute{\beta}$

$$\acute{\beta} = \frac{2}{c}\sqrt{\frac{1-c_0}{x_0 m_0}\frac{F_0}{e}} = \beta$$

Thus the relativistic effect does not appear here. However the effect of the field manifests use it through the term;

$$\frac{F_0}{m} = -\nabla\phi$$

6.12 Effects of fields on Harmonic Oscillator Gain Coefficients

According to classical electromagnetic theory oscillating dipoles emit electromagnetic radiation, the quantum mechanical treatment of harmonic oscillator also indicates that the energy of oscillating body takes the form.

$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega \qquad (6.12.134)$$

This means that the charge in energy between successive levels is given by:

$$\Delta E = \hbar\omega \qquad (6.12.135)$$

This indicates that these oscillators either emit or absorb photons as for as the energy of photon is given by equation (6.12.135) and as for as the classical electromagnetic theory is concerted. This means that if these emitted photons are in phase and coherence they may produce laser beam. To see how this happen, one can utilize equation.

$$\beta = \frac{8\pi^3 f\,(N_2 - N_1)}{3hc}\mu^2$$

and equation

$$\mu = \int \bar{u_m}\hat{H}_1 u_n dr$$

By considering to the wave function of the harmonic oscillator.

$$\mu = \int \bar{u_m}\hat{X}u_n dx = \frac{(n+1)}{2\alpha}, \tag{6.12.136}$$

where $m = n + 1$.
Therefore,

$$\alpha = \left(\frac{mk}{h^2}\right)^{\frac{1}{4}} \qquad = \qquad \left(\frac{m\omega}{h}\right)^{\frac{1}{2}} \tag{6.12.137}$$

Where one make use of the fact that

$$\begin{aligned} F &= -kx \\ &= ma \\ &= -m\omega^2 \end{aligned}$$

However, one can be find

$$K = m\omega^2 \tag{6.12.138}$$

Hence the amplification coefficient is given by:

$$\begin{aligned} \beta &= \frac{8\pi^3 f}{3\hbar}\frac{(N_2 - N_1)}{c}\mu^2 \qquad (6.12.139) \\ &= \frac{8\pi^3 f}{3\hbar c}(N_2 - N_1)\frac{(n+1)^2}{4\alpha^2} \\ &= \frac{2\pi^3 f}{3\hbar c}(N_2 - N_1)(n+1)^2\left(\frac{\hbar}{m\omega}\right) \\ &= \frac{2\pi^3 f}{3c}(N_2 - N_1)(n+1)^2\left(\frac{\omega}{m\omega^2}\right) \end{aligned}$$

In view of equation of

$$K = \frac{\frac{mv_0}{\tau} - eE_0 - F_0}{x} \tag{6.12.140}$$

Amplification is possible if

$$\frac{mv_0}{\tau} - eE_0 - F_0 > 0 \qquad\qquad F_r > F_e + F_z \tag{6.12.141}$$

The resistive force dominate again in complete agreement with classical electromagnetic theory. Another lasing condition can be obtained. If one replaces the ordinary mass m in (6.12.141) by the effective mass m in which the effect of lattice field is incorporated via the term F_ℓ which represents the lattice force where given by:

$$m^* = \left(\frac{F_e}{F_e + F_\ell}\right) m \tag{6.12.142}$$

Which F_e is stands for the external field. This relation comes from the equation.

$$ma = F_e + F_\ell, \qquad\qquad m^* a = F_e \tag{6.12.143}$$

$$ma = |F_\ell| + |Fe|, \qquad = m^* a = |F_e|, \qquad = m^* = m\left(\frac{|F_e|}{|F_\ell| + |F_e|}\right) \tag{6.12.144}$$

Thus by replacing m^* by m in equation (6.12.144) lasing takes place when $|F_\ell| > |F_e|$ when the lattice force exceeds the external one. By using Generalized Special Relativity theory (GSR), and substituting relativistic mass in equation (6.12.139) yields:

$$m = \frac{m_0 g_{00}}{\sqrt{g_{00} - \frac{v^2}{c^2}}}$$

Where $g_{00} = 1 + \frac{2\phi}{c^2}$ then,

$$m = \frac{m_0\left(1 + \frac{2\phi}{c^2}\right)}{\sqrt{1 + \frac{2\phi}{c^2} - \frac{v^2}{c^2}}} \tag{6.12.145}$$

Therefore the amplification factor within the framework of Generalized Special Relativity theory (GSR) becomes

$$\acute{\beta} = \frac{8\pi^3}{3hf}\frac{(N_2 - N_1)(n-1)^2}{c}\left(\frac{\hbar}{m_0\omega}\right) \tag{6.12.146}$$

Substituting equation (6.12.145) in equation (6.12.146) yields:

$$\acute{\beta} = \frac{8\pi^3}{3hf}\frac{(N_2 - N_1)(n-1)^2}{c}\left(\frac{\hbar}{\frac{m_0 g_{00}}{\sqrt{g_{00}-\frac{v^2}{c^2}}}.\omega}\right)$$

Thus;

$$\acute{\beta} = \frac{8\pi^3}{3hf}\frac{(N_2 - N_1)(n-1)^2}{c}\left(\frac{\hbar}{\frac{m_0\left(1+\frac{2\phi}{c^2}\right)}{\sqrt{1+\frac{2\phi}{c^2}-\frac{v^2}{c^2}}}.\omega}\right)$$

$$\acute{\beta} = \frac{8\pi^3}{3hf}\frac{(N_2 - N_1)(n-1)^2}{c}\left(\frac{\hbar\sqrt{1+\frac{2\phi}{c^2}-\frac{v^2}{c^2}}}{m_0\left(1+\frac{2\phi}{c^2}\right).\omega}\right)$$

The amplification coefficient within the framework of Generalized Special Relativity theory (GSR) is given by:

$$\acute{\beta} = \frac{8\pi^3}{3hf}\frac{(N_2 - N_1)(n-1)^2}{c}\left(\frac{\hbar\sqrt{1+\frac{2\phi}{c^2}-\frac{v^2}{c^2}}}{m_0\left(1+\frac{2\phi}{c^2}\right).\omega}\right) \tag{6.12.147}$$

Which indicates that the fields and kinetic energy bath affects amplification factor. The ordinary of approximation amplification coefficient (β) and the real amplification coefficient ($\acute{\beta}$) are given by [63]:

$$\beta = \frac{8\pi^3}{3hf}\frac{(N_2 - N_1)(n-1)^2}{c}\left(\frac{\hbar}{m_0\omega}\right) \tag{6.12.148}$$

This is the ordinary amplification coefficient. The Generalized Special Relativity (GSR) are is given according to equation (6.12.147) by:

$$\acute{\beta} \quad = \quad \frac{8\pi^3}{3hf}\frac{(N_2 - N_1)(n-1)^2}{c}\left(\frac{\hbar\sqrt{1+\frac{2\phi}{c^2}-\frac{v^2}{c^2}}}{m_0\left(1+\frac{2\phi}{c^2}\right).\omega}\right)$$

In view of (6.12.148) and (6.12.147) are gets

$$\acute{\beta} \quad = \quad \frac{\sqrt{1+\frac{2\phi}{c^2}-\frac{v^2}{c^2}}}{\left(1+\frac{2\phi}{c^2}\right)}.\beta$$

This the real amplification coefficient.

6.13 The Effect of Magnetic field on Amplification factor and Intensity of Laser Beam

In Einstein general relativity (GR) the length, time, frequency and mass are not affected by fields. However in generalized special relativity (GSR) they are affected by any field. To see how can this happen, consider the amplification factor in equation (6.7.52). Where

$$\beta = B(n_2 - n_1)\frac{hf}{c} \tag{6.13.149}$$

If one takes into account the wave properties of light, the frequency f is related to the periodic time T according to the relation

$$f = \frac{1}{T} \tag{6.13.150}$$

But according to generalized special relativity GSR see equation (6.3.1)

$$T = \gamma T_0 = T_0\left(1+\frac{2\phi}{c^2}-\frac{v^2}{c^2}\right)^{-1/2}$$

Thus,

$$f \quad = \quad \frac{\left(1+\frac{2\phi}{c^2}-\frac{v^2}{c^2}\right)^{1/2}}{T_0}$$

Neglecting the velocity of frame of reference υ, i.e the atoms emitting photons, and for small ϕ

$$f = \frac{\left(1+\frac{2\phi}{c^2}\right)^{1/2}}{T_0} = \frac{\left(1+\frac{\phi}{c^2}\right)}{T_0}$$

$$f = \frac{\left(1+\frac{m_0\phi}{m_0c^2}\right)}{T_0} = \frac{\left(1+\frac{V}{m_0c^2}\right)}{T_0} \tag{6.13.151}$$

But the magnetic flux density B induces potential V

$$V = \frac{enB}{c}, \tag{6.13.152}$$

where n is the number of particles per unit volume. Thus inserting equation (6.13.152) and (6.13.151) in (6.13.149) yields

$$\beta = \frac{B_0(n_2-n_1)}{cT_0}\left(1+\frac{enB}{m_0c^3}\right)$$

This can be written as.

$$\beta = C_1[1+C_2B] \tag{6.13.153}$$

With

$$C_1 = \frac{B_0(n_2-n_1)}{cT_0}$$

$$C_2 = \frac{en}{m_0c^3} \sim \frac{10^{-19}n}{10^{-31}\times 10^{25}} \sim 10^{-13}n$$

For $n \sim 10^{12}$ then $C_2 = 10^{-1}$ plotting equation (6.13.153) for I, where

$$I = I_0e^{\beta L} \tag{6.13.154}$$

for small βL

$$I = I_0[1+\beta L] = I_0[1+C_1+C_1C_2B] \tag{6.13.155}$$

The relation between I and B becomes as in figure (6.2).

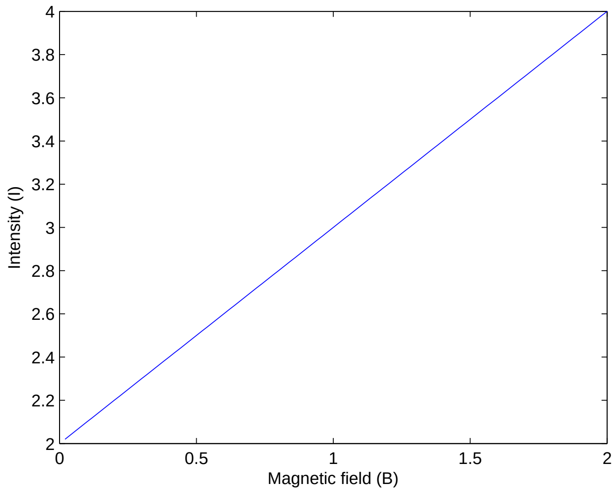

Figure 6.2: *Relation between I and B*

The relation between I and B can also be investigated in view of equation (6.13.156), by consider the particle nature of light, where

$$\beta = B(n_2 - n_1)\frac{g_{00}m_0}{\sqrt{g_{00} - \frac{\upsilon^2}{c^2}}} \tag{6.13.156}$$

$$\beta = B(n_2 - n_1)\left(1 + \frac{2\phi}{c^2}\right) m_0 \left(1 + \frac{2\phi}{c^2} - \frac{\upsilon^2}{c^2}\right)^{-1/2} \tag{6.13.157}$$

Neglecting again υ, for small ϕ, B becomes

$$\begin{aligned} \beta &= B(n_2 - n_1)m_0\left(1 + \frac{2\phi}{c^2}\right)\left(1 - \frac{\phi}{c^2}\right) \\ &= C_3(1 + \frac{\phi}{c^2} - \frac{2\phi^2}{c^4}) \end{aligned} \tag{6.13.158}$$

Using equation (6.13.149)

$$\beta = C_3(1 + \frac{enB}{m_0c^3} - \frac{2e^2n^2}{m_0^2c^5}B^2)$$

$$\beta = C_3(1 + C_4 B - C_5 B^2) \tag{6.13.159}$$

Where

$$\begin{aligned} C_3 &= B(n_2 - n_1)m_0 \\ C_4 &= \frac{en}{m_0 c^3} \\ C_5 &= \frac{2e^2 n^2}{m_0^2 c^5} \end{aligned} \tag{6.13.160}$$

$$I = I_0 e^{C_3(1 + C_4 B - C_5 B^2)} \tag{6.13.161}$$

Plotting I against B according to equation (6.13.161) one gets figure (6.11.127).

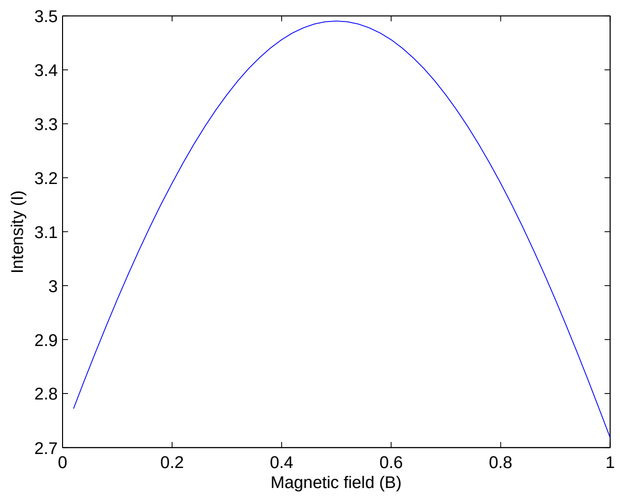

Figure 6.3: *Relation between I and B*

If one considers the electrons and atoms emitting laser photons, as harmonic oscillators, the situation becomes different. The amplification factor becomes according to equation (6.12.139).

$$\beta = \frac{2\pi^3 f}{3c}(n_2 - n_1)(n + 1)^2 \left(\frac{\omega}{m\omega^2}\right) \tag{6.13.162}$$

But according to harmonic oscillator model.

$$k = m\omega^2$$

Thus

$$\begin{aligned}
\beta &= \frac{2\pi^3 f}{3c}(n_2 - n_1)(n+1)^2\left(\frac{2\pi f}{k}\right) \qquad (6.13.163)\\
&= \frac{4\pi^4(n_2-n_1)(n+1)^2 f^2}{3ck}\\
&= C_5 f^2
\end{aligned}$$

But

$$f = \frac{1}{T} = \frac{\left(1+\frac{2\phi}{c^2}-\frac{\upsilon^2}{c^2}\right)^{1/2}}{T_0}$$

Neglecting υ and for small ϕ

$$f = \frac{(1+\frac{\phi}{c^2})}{T_0} = T_0^{-1}\left(1+\frac{\phi}{c^2}\right)$$

Thus

$$f^2 = T^{-2}(1+\frac{\phi}{c^2})^2 = T_0^{-2}\left(1+\frac{2\phi}{c^2}+\frac{\phi^2}{c^4}\right)$$

If the magnetic field apposes the motion

$$V = m_0\phi = \frac{-enB}{c}$$

Therefore

$$f^2 = T_0^{-2}\left(1-\frac{2enB}{m_0c^3}+\frac{e^2n^2B^2}{m_0^2c^5}\right) \qquad (6.13.164)$$

Inserting (6.13.164) in equation (6.13.163) yields

$$\beta = C_5T_0^{-2}(1-C_6B+C_7B^2)$$

$$\beta = C_8 - C_9B + C_{10}B^2, \qquad (6.13.165)$$

where C_8, C_9, and C_{10} are constants. Thus

$$I = I_0e^{(C_8-C_9B+C_{10}B^2)} \qquad (6.13.166)$$

plotting I versus B yields (6.4) below.

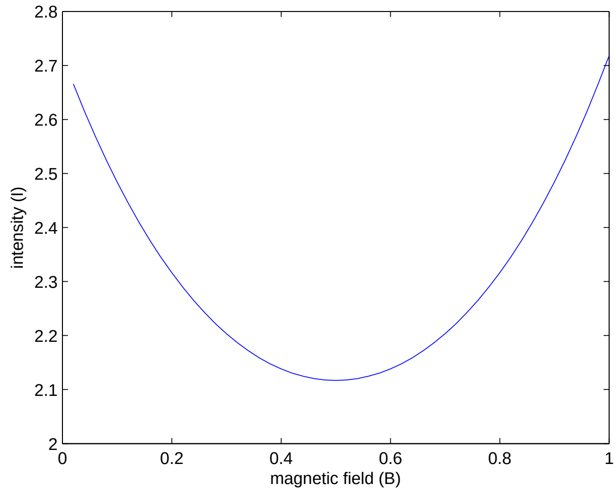

Figure 6.4: *Relation between I and B*

6.14 Discussion

The expression for the energy in Eq.(6.4.20) is obtained by using momentum conservation together with time dilation relation of the GSR theory. This expression shows that the mass and energy are affected by the kinetic energy per unit mass $\frac{1}{2}mv^2$ as well as the potential ϕ per unit mass. This expression is in conformity with common sence and with GSR [60], and Savickas model [59]. The expression of energy satisfies correspondence principle as it reduces to Newtonian energy relation it consists of potential term beside kinetic term. as shown by equation (6.4.21). The fact that the mass and energy expressions resembles that of savickas and the GSR indicates that these expressions rests on a solid ground. Such expressions can cure some of SR setbacks like Newtonian limit and red shift phenomena. The fact that the relativistic time and mass in equations (6.3.1), (6.3.3) and (6.4.15) can explain the effect of magnetic field on laser amplification factor as well as

laser intensity. Comparing the theoretical relation between I and B in figure (fig:6.2), with the empirical relation in figure (fig:5.2), it is clear that the two curves are similar. This is not surprising as well as the active lasing media is a plasma which consist of free atoms that oscillates and act as a harmonic oscillator. The effect of magnetic field on Helium cadmium (He-Cd) laser discharged gas is displayed in figures (fig:5.4) and (fig:5.5). These empirical relations are similar to the theoretical relations in figures (fig:6.3) and (fig:6.4) which are concerned with particle nature of light and harmonic oscillator. This is not surprising since the discharged gas which have free isolated ionized atoms treat light as particles called photons with mass m. The ionized atoms also oscillates and thus behaves as harmonic oscillator.
Finally the effect of magnetic field on semiconductor laser is linear as shown in figure (fig:5.6). This empirical relation conforms with the linear theoretical one in figure (fig:6.2) which was derived by considering the change of frequency by the magnetic field. This is not surprising, since for the semiconductor laser, which is a balk matter, light behaves as waves with a certain frequency.

6.15 Conclusion

The fact that generalized special relativity (GSR) predicts that the mass, time and length are effected by physical field open a new horizon for explaining a wide variety of physical phenomena. The (GSR) with this a new version succeeded in explaining the change of laser intensity with the magnetic field strength.

6.16 Recommendations

1. The (GSR) prediction need to be used to see how physical fields affect physical quantities and material properties

2. The effect of (GSR) on lasing process should also extended to include a wide variety of theoretical models

3. The theoretical framework of free electron laser need to be simplified.

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