Sudan University of Science and Technology

College of Graduate Studies

An Optimized Type-2 Fuzzy Logic Based System for Detection of Financial Defaults in Sudanese Banking Sector

نظام محسن مبني على المنطق الضبابي من النوع الثاني لاكتشاف التعثر المالي في قطاع البنوك السودانية

A thesis submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Computer Science

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DECLARATION

I hereby declare that this thesis is the result of my investigation, except where otherwise stated. I also declare that it has not been previously or concurrently submitted as a whole for any other degrees at Sudan University of Science and Technology or other institutions.

Ahmed Salih Mohamed

Signature …………………………….. **date……………………………..**

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# ABSTRACT

The recent global financial-economic crisis has led to the collapse of several companies from all over the world. This has created the need for powerful models which can predict and reduce potential risks in financial applications. Such models help organizations to enhance the quality and productivity of their services as well as reduce financial risk. The widely used techniques to build predictive models in the financial sector are based on statistical regression, which is deployed in many financial applications such as risk forecasting, customers' loan default, and fraud detection. However, in the last few years, the use of Artificial Intelligence (AI) techniques has increased in many financial institutions because they can provide powerful predictive models. However, the vast majority of the existing AI techniques employ black box models like Support Vector Machine (SVMs) and Neural Network (NNs) which are not able to give clear and transparent reasoning to explain the extracted decision. However, nowadays transparent reasoning models are highly needed for financial applications.

In this thesis, the researcher will present an intelligent Type-2 Fuzzy Logic System for the prediction of financial default to assist the decision-makers in the financial sector to prevent and control the frequent default risk. The Type-2 Fuzzy Logic System would be clear to present a highly explainable and transparent model that is very appropriate to handle different types of uncertainties associated with the financial sector and convert the gathered data to linguistic formats which can be easily stored and analyzed. Fuzzy Logic Based Systems provide a transparent model which provides IF-Then rules that can be easily investigated and understood by the end-users. The proposed model was trained and tested using real data obtained from Albald Bank in Sudan dating back to the period 2007 – 2017.

In this study, the two components of Type-1 and Type-2 Fuzzy Logic Systems which are the rule base and fuzzy membership functions are learned dynamically from data. The

Fuzzy C-Means (FCM) clustering technique is used to cluster the parameters of the Type-1 and Type-2 Fuzzy Logic controller.

To optimize the Type-2 fuzzy logic System the researchers used the Big Bang-Big Crunch (BB-BC) algorithms to optimize the proposed model's parameters which are the membership functions and the rule base, furthermore, the rule base is optimized in two levels, the number of rules in the rule base and the number of antecedents in the rule itself. As a result of this optimization, the proposed Fuzzy Logic Based System allows achieving relatively high prediction accuracy with optimized membership functions and a small number of rules which increase the System readability and then allow prediction of default risk in the financial sector. The BB-BC optimized type-2 Fuzzy Logic prediction System gained 84% prediction accuracy in our testing dataset which is better than its counterpart's Type-1 and non-optimized Type-2 Fuzzy Logic prediction system.

It is known that the black-box model could not show the details when it provides the prediction, and it is not very good for people to recognize the reasoning behind the given decision, on the contrary, the fuzzy logic prediction system is a white box model which has the transparency that could explain the details behind the reasoning process in giving decision. The results showed that the optimized proposed Type-2 Fuzzy Logic System provided a more interpretable model which has a rational number of rules; only 400 rules which provided a more interpretable rule base that can be easily understood and analyzed by the decision-maker, furthermore the proposed optimized model provided good prediction model which can predict default and serve both sides of business stakeholders; the bank and the customers.

المستخلص

أدت الأزمة المالية الاقتصادية العالمية الأخيرة إلى انهيار العديد من الشركات من جميع أنحاء العالم. وقد خلق هذا حوجة ماسة إلى نماذج تنبؤ قوية يمكنها التنبؤ بالمخاطر المحتملة في التطبيقات المالية حتى يتم تفاديها. وبالتالي تساعد هذه النماذج المؤسسات على تحسين جودة وإنتاجية خدماتها بالإضافة إلى تقليل المخاطر المالية المحتملة .

تعتمد الأساليب المستخدمة على نطاق واسع لبناء نماذج تنبؤية في القطاع المالي على الانحدار الإحصائي ، والذي تم استخدامه في العديد من التطبيقات المالية مثل التنبؤ بالمخاطر ، والتخلف عن سداد قروض العملاء ، واكتشاف الاحتيال. في السنوات القليلة الماضية ، ازداد استخدام تقنيات الذكاء الاصطناعي في العديد من المؤسسات المالية لأنها يمكن أن توفر نماذج تنبؤية قوية. ومع ذلك ، فإن الغالبية العظمى من تقنيات الذكاء الاصطناعي الحالية تستخدم نماذج الصندوق الأسود مثل((Support Vector Machine (SVMs) ، والشبكة العصبية (NNs) التي لا تستطيع إعطاء تفسير واضح وشفاف لشرح أسباب اتخاذ القرار المستخرج من هذه النماذج. وعليه في الوقت الحاضر برزت الحوجة إلى نماذج تنبؤ تتمتع بالشفافية من ناحية تفسير القرار المتخذ من قبل النموذج لكي يتم استخدامها في التطبيقات المالية.

هذه الرسالة ، قامت بدارسة امكانية تطبيق نموذج ذكي مبني على المنطق الضبابي من النوع الثاني للتنبؤ بالتخلف عن سداد القروض المالية لمساعدة صانعي القرار في القطاع المالي على الحد من والسيطرة على خطر التخلف عن سداد القروض المالية والتي تحدث بصورة متكررة. من الواضح أن نظام المنطق الضبابي النوع الثاني يقدم نموذجًا واضحًا وشفافًا للغاية ومناسبًا جدًا للتعامل مع أنواع

مختلفة من حالات عدم اليقين المرتبطة بالقطاع المالي وتحويل البيانات المجمعة إلى تنسيقات لغوية يمكن تخزينها وتحليلها بسهولة. توفر الأنظمة المستندة إلى المنطق الضبابي نموذجًا شفافًا يوفر قواعد IF–Then التي يمكن التحقق منها وفهمها بسهولة من قبل المستخدمين النهائيين. تم تدريب واختبار النظام المقترح باستخدام بيانات حقيقية تم الحصول عليها من بنك البلد في السودان تعود إلى الفترة 2007–2017.

في هذه الدراسة ، يتم تعلم عنصري أنظمة المنطق الضبابي وهما قاعدة القاعدة ودالة العضوية الضبابية ديناميكيًا من البيانات. تُستخدم تقنية التجميع Fuzzy C–Means (FCM) لتجميع متغيرات التحكم.

لتحسين نموذج المنطق الضبابي من النوع 2 ، استخدم الباحثون خوارزمية الانفجار الكوني العظيم Big Bang–Big Crunch (BB–BC) لتحسين مكونات النموذج المقترح التي تمثل دالة العضوية وقاعدة القاعدة . تم تحسين قاعدة القاعدة في مستويين ، المستوى الأول من حيث عدد القواعد في قاعدة القاعدة والمستوى الثاني للتحسين كان في عدد السوابق في القاعدة الواحدة نفسها. نتيجة لهذا التحسين ، يسمح النظام المبني على المنطق الضبابي المقترح بتحقيق دقة تنبؤ عالية نسبيًا مع امتلاكه لدوال عضوية محسّنة وعدد صغير من القواعد التي تزيد من قابلية قراءة النموذج ثم تسمح بالتنبؤ بمخاطر التخلف عن السداد في القطاع المالي. اكتسب نظام التنبؤ المنطق الضبابي من النوع الثاني المحسن بواسطة خوارزمية الانفجار الكوني العظيم دقة تنبؤ بنسبة 84٪ في مجموعة بيانات الاختبار الخاصة بنا ، وتعتبر هذه الدقة أفضل من التي حصل عليها نظيره من النوع الأول الغير محسن .

من المعروف أن نموذج الصندوق الأسود لا يمكنه إظهار التفاصيل عندما يقدم التنبؤ ، وهذا ليس جيداً حيث يحتاج المستخدمين لأنظمة التنبؤ إلى تفسير القرار الذي تم اتخاذه بواسطة النموذج.

وعلى العكس من ذلك ، فإن نموذج التنبؤ المبني على المنطقي الضبابي يعتبر نموذج صندوق أبيض يتمتع بالشفافية التي يمكن أن تشرح التفاصيل الكامنة وراء عملية التفكير واسباب اتخاذ القرار. أظهرت النتائج أن النموذج المقترح المبني على المنطقي الضبابي من النوع الثاني المحسّن قدم نموذجًا أكثر قابلية للتفسير وله عدد منطقي من القواعد ؛ فقط 400 يمكن فهمها وتحليلها بسهولة من قبل صانع القرار ، علاوة على ذلك ، قدم النموذج المقترح المحسن نموذجًا جيدًا للتنبؤ يمكن أن يتنبأ بالتقصير ويخدم كلا الجانبين من أصحاب المصلحة في الأعمال ؛ البنك والعملاء.

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List of Abbreviations

ACI	American Concrete Institute
AI	Artificial Intelligent
AIT2FC	Aligned Interval Type-2 Fuzzy Clustering
BB–BC	Big Bang–Big Crunch
BPNN	Back Propagation Neural Network
CBR	Case Base Reasoning
C-GA	Compact Genetic Algorithm
DA	Discriminant Analysis
DEA	Data Envelopment Analysis
DHS	Discrete Harmony Search
DI	Discovery Investing
DT	Decision Tree
FA	Factor Analysis
FILM	Fuzzy Inductive Learning Method
FL	Fuzzy Logic
FLC	Fuzzy Logic Controller
FOU	Footprint Of Uncertainty
GA	Genetic Algorithm
HSM	Hot Strip Mill
IR	Information Retrieval
K-NN	K Nearest Neighbor
LP	Linear Programming
MF	Membership Function

MRRs	Modular and Reconfigurable Robots
MSE	Mean Square Error
N-FC	Neuro-Fuzzy Classifier
NN	Neural Network
OR	Operations Research
PSO	Particle Swarm Optimization
PSS	Power System Stabilizer
QP	Quadratic Programing
RAOFC	Reinforcement Ant Optimized Fuzzy Controller
RPA	Recursive Partitioning Algorithm
SC	Soft Computing
SR	Statistical Regression
STLF	Short Term Load Forecasting
STLF	Short Term Load Forecasting
SVM	Support Vector Machine
T1FLS	Type-1 Fuzzy Logic Systems
T2FLS	Type-2 Fuzzy Logic Systems
TCSC	Thyristor Controlled Series Capacitor
TSK	Takagi-Sugeno-Kang
WRED	Weighted Random Early Detection

Publications arising from this work

Journal Paper

a) A. Salih and H. Hagras, “A Type-2 Fuzzy Logic Based System for Decision Support to Minimize Financial Default in the Sudanese Banking Sector” , SUST Journal of Engineering and Computer Sciences (JECS), Vol. 20, No. 3, 2019.

b) A. Salih and H. Hagras,” A Big Bang –Big Crunch Optimized Type-2 Fuzzy Logic Based System for Default Prediction in Sudanese Banking Sector”, International Journal of Computer Science Trends and Technology (IJCST) –Vol. 8,No. 5, 2020.

Conference Papers

a) A. Salih and H. Hagras, "Towards a Type-2 Fuzzy Logic Based System for Decision Support to Minimize Financial Default in Banking Sector", In: Proceeding of 10th Computer Science and Electronic Engineering (CEEC), Colchester, United Kingdom, pp: 46-49,2018.

SECTION ONE INTRODUCTION

Chapter 1: Introduction

Chapter one Introduction

This chapter intends to present the fundamentals of the research and the key area of study, which is default prediction in financial applications. First, a brief introduction to the field of prediction models in the financial sector will be given in Section 1.1 followed by the motivation and problem statement of this study in Section 1.2. Section 1.3 is dedicated to an overview of the research methodology. Along the path to achieving the objectives of this research, which will be introduced in Section 1.4, major contributions to science have been made. These contributions are summarized in Section 1.5. Section 1.6 will deliver an ethical consideration related to the dataset which has been used in this research, followed by the structure of the thesis in section 1.7. And finally, chapter 1's summary will be given in Section 1.8

1.1 Introduction

The recent global economic crisis collapsed several companies around the world. As a result, millions of people lost their jobs. For example, "American families lost $360 billion in wages and salaries as a result of the weak economy following September 2008. This is a loss of $3,250 per household in 2008 and 2009" (Swagel, n.d.)and this happened with the existent predictive models which indicated the failure of existing models. Figure 1.1 shows the impact of the crisis on wages in the US (Swagel, n.d.)

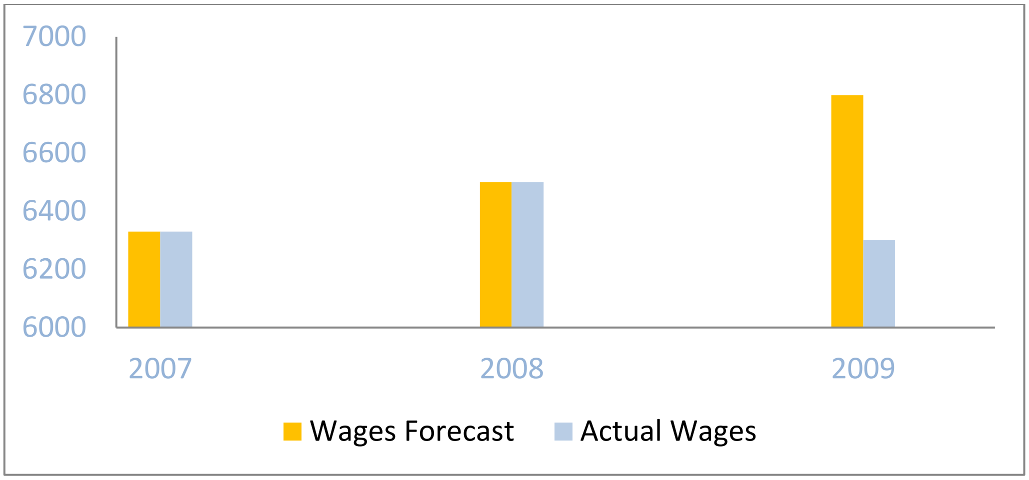


Figure 1-1Impact of the crisis on wages in the US with existence prediction techniques(Swagel, n.d.)

As organizations working in the economic sector began to recover slowly, they started focusing on finding new ways to reduce the potential risks related to their business.

Nowadays, there is a huge amount of online data available and accessible by organizations more than ever before. Hence, it became easy to store and process a huge amount of data (Bach et al., 2019). Thus, many financial organizations try to build predictive models that aim to mine historical data and try to extract indicators that can help the decision-makers to predict what will happen in the future, which can avoid potential risks (Onder et al., 2016).

Different techniques were used by different financial institutions to build predictive models and they can be classified into four categories: statistical-based, operation research-based, artificial intelligent based, and hybrid artificial intelligent based. **The statistical-based** predictive models contain many techniques like Discriminant Analysis (DA), Statistical Regression (SR), and Factor Analysis (FA)(Mai et al., 2019). These techniques are widely used because they are easy to develop. However, they assume a certain mathematical relationship between the input and the output which is not the case for the majority of the real-world data (Bernardo et al., 2013).

Operation research-based predictive models contain many techniques like Linear Programming (LP), Data Envelopment Analysis (DEA), and Quadratic Programming (QP) were also used widely due to simplicity of development but they are considered to be rather complicated and lead to complicated semi-black box models(Othman et al., 2016).

Artificial intelligent(AI) based predictive models can be subdivided into two approaches: **black-box approaches** which involve techniques like Neural network(NN)(Abiodun et al., 2018)(Kraus and Feuerriegel, 2017), Support Vector Machine(SVM), etc. which are applied on the spectrum of financial application domains, however as well as they achieved a good level of prediction accuracy they have the main drawback which is represented by providing black box models which are difficult to understand and analyze by the financial analysts to provide clear decisions that are supported by evidence which nowadays are highly required by the financial market due to strong competition in this domain and race to win customer confidence.

The second approach of AI techniques is the **white box approach** which contains techniques like Case Base Reasoning (CBR), Rough Set Theory, Decision Tree (DT) (Zięba et al., 2016), and Fuzzy Logic (FL). All techniques in this approach provide white box transparent models which are easy to understand by the financial analyst. However, they have also limitations, for example, a decision tree cannot handle uncertainty and the recursive partitioning operation is very difficult which leads to hard decision boundary extraction. Case-based reasoning is slow if the case is not already existing in the case base and is difficult to deal with noisy data(Han and Cao, 2015). Type-1 fuzzy logic cannot handle a high level of uncertainty and Type-2 fuzzy logic suffers from the curse of dimensionality problem (Erdem and Kumbasar, 2021).

Due to the extreme need for an accurate transparent white-box model, this research proposes to develop a novel type-2 fuzzy logic-based model for decision support to minimize financial default in the banking sector. The proposed model will be a hybrid model, it will have to use a type-2 fuzzy logic system to handle the high level of linguistic and numerical uncertainty associated with bank sector data. The proposed system will also employ an evolutionary computation approach namely Big Bang–Big Crunch (BB-BC) to overcome the curse of dimensionality problem by generating compact type-2 fuzzy logic rule bases which are capable to give good prediction accuracies. The proposed model will use the real data from Albalad Bank and the customer default as a case study and after testing and evaluating the proposed model it can be generalized to help a wide spectrum of financial institutions.

1.2 Motivation and Problem Statement

Due to a large amount of online information available nowadays, the decision-makers suffer a lot to analyze this historical information to predict potential risk by using traditional statistical methods. One of the most effective risks faced by financial institutions is defaulters (Serrano-Cinca et al., 2015). This trend motivates the leaders of the economic sector to direct the research to the data science field to develop new tools which can provide powerful frameworks to deal with the big data environments, and indeed there is a huge amount of investment in this track(Li et al., 2020). In recent years, there are few studies to apply Artificial Intelligence (AI) techniques to build prediction models in financial applications such as an innovative neural network (Pang et al., 2020) and Support Vector Machine (SVM) (Horak et al., 2020). However, such machine learning techniques have lacked to handle uncertainties and noises in the prediction process, even if they have reasonable accuracy. In addition, such machine learning techniques are black-box models which have not been represented in a more interpretable and transparent form the generated results for decision-makers.

Locally in our country –Sudan- After releasing Sudan from the list of countries supporting terrorism; this trend led to potential challenges likely to face the banking sector in Sudan which in turn should try to figure out the optimal solutions to be able to enter the world financial market and acquire the global capital and investments. In this regard, It should be noted that there is only one study related to the field of prediction in the Sudanese financial sector (Osman et al., 2013) and this study proposed a credit scoring model using a neural network and decision tree.

From the surveyed literature there are clear limitations on existing prediction techniques that are not able to satisfy recent financial market requirements(Bernardo et al., 2013). Thus, the main problem of this study is there is a real need to have a transparent white box decision support model to enable us to extract from the data simple If-Then rules which are easy to understand and analyze by the layman user (Bernardo et al., 2013). Such decision support systems should be able to handle the high levels of linguistic and numerical uncertainties available in the banking sector (Saeed, 2019).

1.3 Aim and Objectives

The essential aim of this research thesis is to develop a transparent white box decision support system which enables us to extract from the data simple If-Then rules which are easy to understand and analyze by layman users in the Sudanese banking sector. While studying the literature, the purpose is to find the appropriate components for the proposed model. With this motivation in mind, the objectives of the research can be listed as follows:

a) To Explore the works of literature associated with the prediction models in the financial sector.

b) To Recognize the weaknesses of existing prediction models to resolve the inadequacies of existing approaches.

c) To Develop white-box prediction optimized models based on the Type-1 and Type-2 FLS, to predict default in the Sudanese banking sector.

d) To use real masked data belonging to Albalad Bank in Sudan to construct the two different models mentioned in c) which can provide reliable results.

e) To compare the two proposed models, mention in c) their capabilities to handle uncertainties associated with default prediction in the Sudanese financial sector and evaluate their accuracy using average recall ratio as an evaluation metric.

f) To optimize the two components of the proposed Type-2 fuzzy logic model using evolutionary technique, specifically the BB-BC algorithm. The optimization in fuzzy sets component to acquire the optimal configuration of membership function to increase the accuracy of the model and the optimization in rule base component to reduce the size of the rule base and reduce the number of antecedents in the rule itself which can increase interpretability of the model.

g) To provide a finding and recommendations for future work based on the results extracted from the experiments.

1.4 Research Methodology

A comprehensive literature review has been done to better identify the gaps related to risk prediction in the financial sector. Then the acquisition for real datasets is done. Primary knowledge about fuzzy logic and Big Bang- Big Crunch have been studied. Developing the proposed method has been established by preprocessing on a real dataset acquired from the Sudanese banking sector manly from Albalad Bank. Then two different prediction models have been built using (type-1 and type-2 fuzzy logic). Furthermore, the type-2 fuzzy logic model has been optimized using the BB-BC optimization approach. All models were applied using the Java programming language. Then an evaluation of the optimized proposed model against a non-optimized method has been done. Finally, findings and recommendations for future work have been provided based on the results extracted from the experiments.

1.5 Contribution to Science:

The main contribution to science accomplished during addressing and exploring the objectives of this study can be summarized as follows:

a) This thesis provides work on real financial data in the Sudanese banking sector; which can provide more reliable results.
b) The implemented system provides a powerful tool that can reduce the risks for the two sides of business stakeholders – the bank and costumer- because it depends on the white-box model, unlike the black-box model.
c) Extensive literature review on prediction models.

d) This thesis offers a design of FLS mainly type-2 fuzzy logic systems to model problems with higher levels of uncertainty in financial data which are not addressed by previous work. The various methods applied before for prediction default in the financial sector are Statistical and Mathematical techniques and also black-box AI techniques.

e) The novelty of the developed model primarily comes from employing the combination of the capability of type-2 fuzzy reasoning to handle higher levels of uncertainty and the capability of BB-BC to optimize the fuzzy logic controller and this can provide a traceable rule base that can easily analyze by the decision-maker.

f) This study provides an unprecedented explainable type-2 fuzzy logic prediction systems to predict default in the Sudanese financial sector.

1.6 Structure of the Thesis

This thesis consists of eight chapters, and each chapter has sections and subsections, and they are organized as follows:

Chapter 2 provides a review of prediction models in a financial application; this chapter provides an overview of what has been done in this field and highlights the drawbacks of different prediction models to identify the gap in this field.

Chapter 3 gives a theoretical framework to fuzzy logic, starting with a brief history on its foundations, continuing with some terminology followed by the types of fuzzy logic as well as fuzzy logic systems.

Chapter 4 is dedicated to evolutionary optimization techniques namely Big Bang-Big Crunch (BB-BC) and genetic algorithm optimization methods.

Chapter 5 will present two different proposed models based on type-1 and type-2 fuzzy logic models to predict default in the Sudanese banking sector, and a comparison between the two proposed models will be conducted to address the shortcoming of the two models. Finally, the experiments and results of the proposed system will be presented and discussed.

Chapter 6 will provide a proposed optimized type-2 fuzzy system to predict default in the Sudanese banking sector. This chapter will illustrate how to use the Bang- Big Crunch (BB-BC) optimization method to optimize type-2 fuzzy logic components to produce a more interpretable model which can be easily analyzed by the layman user. Finally, the experiments and results of the proposed BB-BC-optimize- type-2 model will be presented and discussed.

Finally, chapter 7 offers the conclusion of this study based on the conducted results and its effects on the field of prediction in the Sudanese financial sector. Then, possible future work is recommended based on what was accomplished in this study.

1.7 Summary

In this chapter, a brief introduction to different prediction models has been explored. And the researcher has stated motivation and problem statement to this study, after that aims and objectives expected to be reached from this thesis were listed, the methodology followed in this study have been summarized, after that contribution to science achieved during investigating the objectives have been highlighted, the researcher presented ethical consideration related to the dataset which has been used in this research, and finally, the organization of this thesis presented.

SECTION TWO: THEORETICAL BACKGROUND

Chapter 2 : review of prediction models for the financial sector defaults

Chapter 3: introduction to fuzzy logic systems

Chapter 4: introduction to evolutionary optmization

Chapter Two: Review of Prediction Models for the Financial Sector Defaults

2.1 Introduction:

The recent financial crisis, the generality and spread of systemic risk in the supplementary international financial environment, and the high social costs of bank failures have drawn consideration to the mechanisms of control of financial policies followed by banks (López Iturriaga and Sanz, 2015). For example, Figure 2.1 shows the impact of the financial crisis on Non-performing Loans (NPL) in a different country with existing prediction techniques.

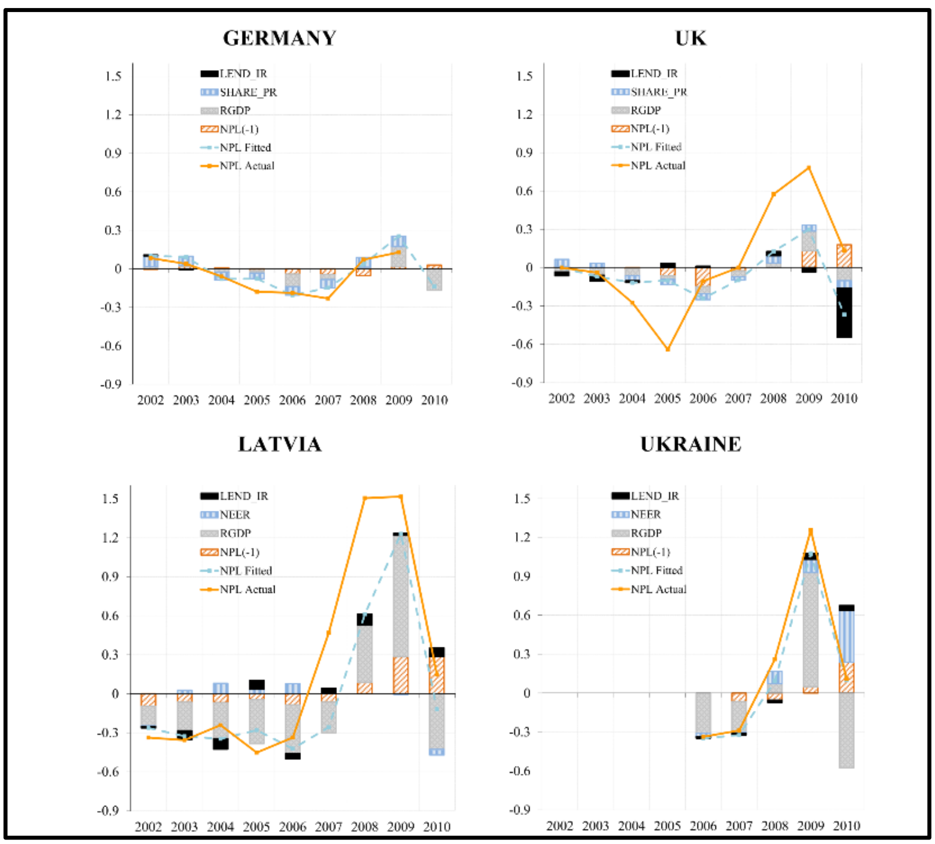


Figure 2-1 :Impact of the crisis on NPL withe existent of prediction models (Beck et al., 2015)

Recently and due to the high level of competition in the financial market and a huge amount of online accessible data, building a predictive model is highly required. There are many types of research and projects in this area (Alaka et al., 2018). Many different techniques were used to build predictive models and they can be classified into four categories: statistical based, operation research based, artificial intelligent based and hybrid artificial intelligent based (Bernardo et al., 2013),(Kumar and Ravi, 2007). Figure 2.2 provides classification of predictive techniques which were studied in surveyed literature.

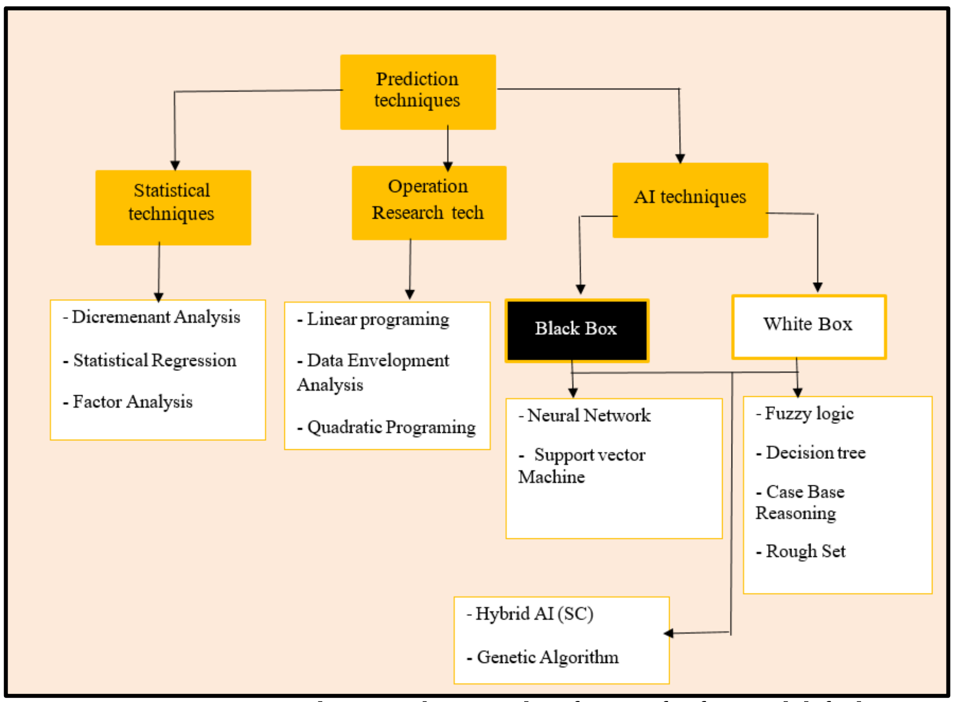


Figure 2-2. Prediction techniques classification for financial defaults

2.2 Types of financial defaults prediction model

This section discusses different types of financial defaults prediction techniques and examples of different works related to each of them. As shown in **Figure 2.2** the researcher classifies them into four different classes which are Statistical Base Predictive Models, Operations Research Predictive Models, artificial intelligence models and hyper prediction models. Thus, in the following subsection each class will be presented and a summarization of its advantages and shortcomings will be highlighted.

2.2.1 Statistical Base Predictive Models:

The statistical techniques like: Discriminant Analysis (DA), Statistical Regression (SR) and Factor Analysis (FA) are widely deployed in financial domain, especially in risk forecasting (Bernardo et al., 2013)in order to build predictive models. This section tries to survey what was done in this domain from a research scope point of view.

The authors of (Martin, 1977), (Laitinen and Laitinen, 2000), and (Premachandra et al., 2009)used logistic regression to predict banks failure and (Ohlson, 1980)to predict firms failure. A Simple linear model was developed by (Dietrich and Kaplan, 1982)and (Canbas et al., 2005)to classify loan risks and predict commercial bank failure in Turkey. In (West, 1985)they used factor analysis and logit estimation for evaluating bank condition.

Statistical techniques are widely used because they are easy to develop. However, they have disadvantages which make them less preferable by technologies producers to face the current challenges associated with financial applications. For example, statistical predictive models assume a certain mathematical relationship between the input and the output which is not the case for the majority in real-world data.

2.2.2 Operations Research Predictive Models:

Many decades ago, there were attempts made to use a scientific method in the organization's management and these attempts can be referred to as the origins of Operations Research (OR). However, the real beginning of operations research couples with military services early in World War II. Due to hard ware circumstances, there is greatly a need for better resources utilization. The big challenge at that time was how to allocate limited resources to the several military operations and to the tasks inside each operation effectively (Hillier, 2012). Therefore, the military management in Britain and U.S.A provided this mission to large numbers of scientists and they were asked to apply scientific methods in order to find suitable solutions. In reality, they were asked to do research on (military) operations and this team of scientists was considered to be the first OR team (Hillier, 2012).

There are many techniques used in this context such as Linear programming (LP), Data Envelopment Analysis (DEA) and Quadratic Programming (QP).

There are many financial applications which use operations research to build predictive models. For example, the authors of (Banks and Abad, 1994) proposed linear programming model to predict firm bankruptcy. And (Lam and Moy, 2002)presented a method that combines several discriminant methods to predict the classification of new observations. Authors of (Cielen et al., 2004)compared a linear programming model, data envelopment (DEA) model and a rule induction (C5.0) model. Advanced predictions of the performances of 24 commercial banks in Taiwan based on their financial forecasts were done by (Kao and Liu, 2004). In this work the forecasts based on uncertain financial data are represented in ranges, instead of as single values, and a (DEA) model for interval data is formulated to predict the efficiency.

Operation research based predictive models were used wildly due to simplicity of development but they are considered to be rather complicated and this can lead to complicated semi decision black box models.

2.2.3 Artificial Intelligence Predictive Models:

Many artificial Intelligence techniques were implemented successfully in financial application and provided good levels of performance in solving classification and optimization problems which are associated with applications in this context. From transparency point of view artificial intelligence techniques can be categorized into two main classes: black box techniques and white box techniques (Bernardo et al., 2013). The next two subsections provide explanation of two class and the surveyed works which used them.

2.2.3.1 : Black Box Artificial Intelligence Techniques:

The term ***Black Box*** is refers to the Artificial Intelligent Technique that cannot provide transparent reasoning behind the extracted decision which let such techniques considered difficult to understand and analyzed by the normal end users (Bernardo et al., 2013). For example, in financial application if a customer applied to get an investment loan from a bank, the investment employer in the bank must check the customer’s information in order to take decision either to accept or refuse the client's request. Suppose that the bank has decision support system which is responsible for classifying any new customer to be a good or a bad customer and this system produces a decision that this customer is a bad customer. At that time the investment employer will find himself in a bad situation because he will not be able to explain the reasons behind the refusal decision. And this can lead bank to lose their customers. **Figure 2.3** provides the simplest definition of the black box technique.

There are many black box artificial Intelligence techniques such as Neural Network (NN), Support victor machine (SVM), etc. which were applied successfully in financial domain the following three subsection provide simplest explanation about each black Box technique and their survey application in this domain.

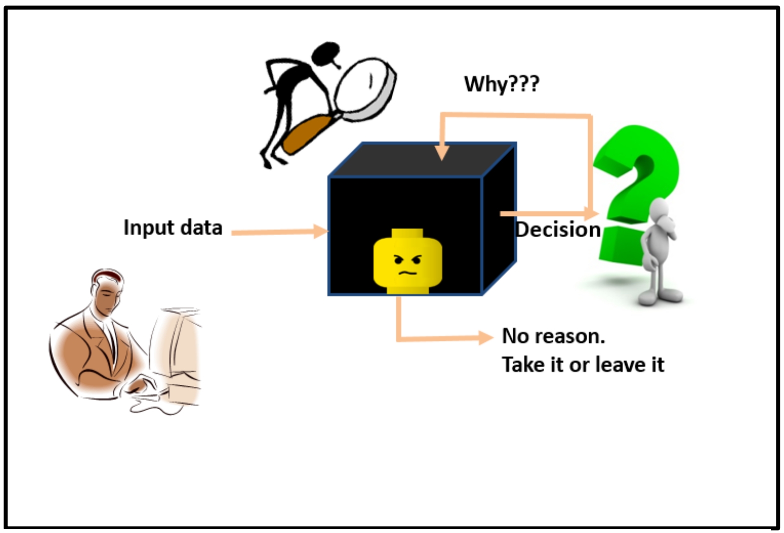


Figure 2-3 Black box explanation

A. Support Victor Machine (SVM):

Support vector machine (SVM) is one of the artificial intelligence techniques that are used extensively to solve classification problems. The idea behind this technique is trying to find line that can separate the data into two classes as long as maximizing the margin between the line and each class. If the data is not linearly separable these technique uses Kernels function in order transform the data from non-linearly separable to linearly separable. Figure 2.3 explains simple SVM classification example.

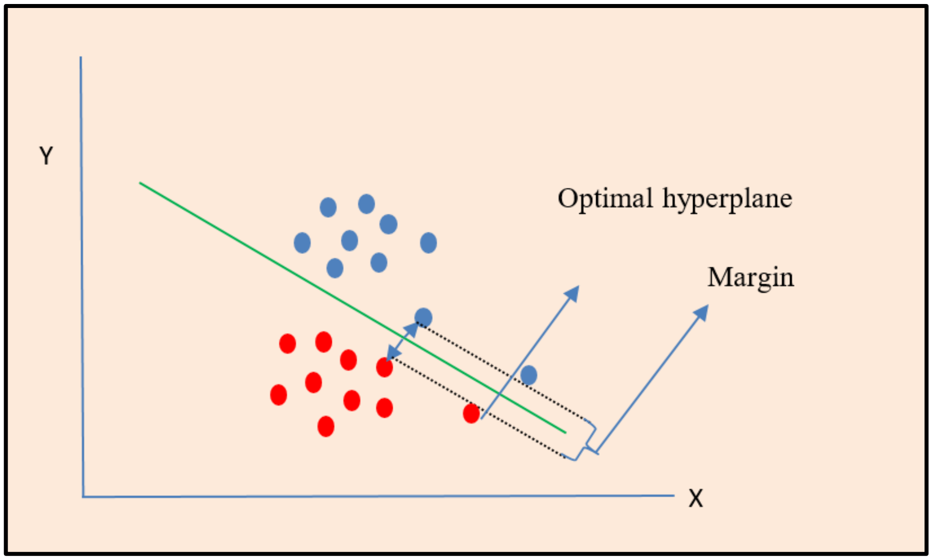


Figure 2-4 SVM Classification

There are many financial applications which use Support Victor machine to build predictive models. For example, (Min and Lee, 2005)proposed SVM for bankruptcy prediction. He proposed grid search technique using fivefold cross validation to find out the optimal parameter values of kernel function of SVM. And (Li et al., 2006)developed a loan evaluation model using SVM to identify potential applicants for consumer loans.

SVM is widely used in the financial domain due to its high level of accuracy; however, it has many limitations. For example, it is considered to be complex to implement. Furthermore, is classified as black box which is complex to understand and trace its behavior by the end users and suffers from overfitting problem.

B. Neural Networks (NN):

Neural Networks (NN) are artificial intelligence techniques that try to imitate the biological neural network in the human. The human's neural system in composed of big number of interconnected neurons. Any neuron in this system receives signals from many others neurons. The neuron's output path in the human brain is called (Axon) is connected to many others neurons through a junction called (synapse) which is connected to input paths for those neurons and those paths are called (Dendrites) (Bell, 1997).

Neurons in the human's neural system interact together by exchanges signals which have chemical nature. "The human's brain learns by modifying the strength of these chemical interconnections between neurons" (Bell, 1997). Figure 2.5 shows the human neuron cell.

An artificial neural network builds from many process elements which called (Nodes). Any node in this network receives inputs from others nodes each input is multiplied by the weight of connection from which input coming on and sums all weighted input to produce single output. Neural network has two phases: the learning phase in which NN use training data in order to tune the system and set important parameter like: weight vector, learning rate and network topology.

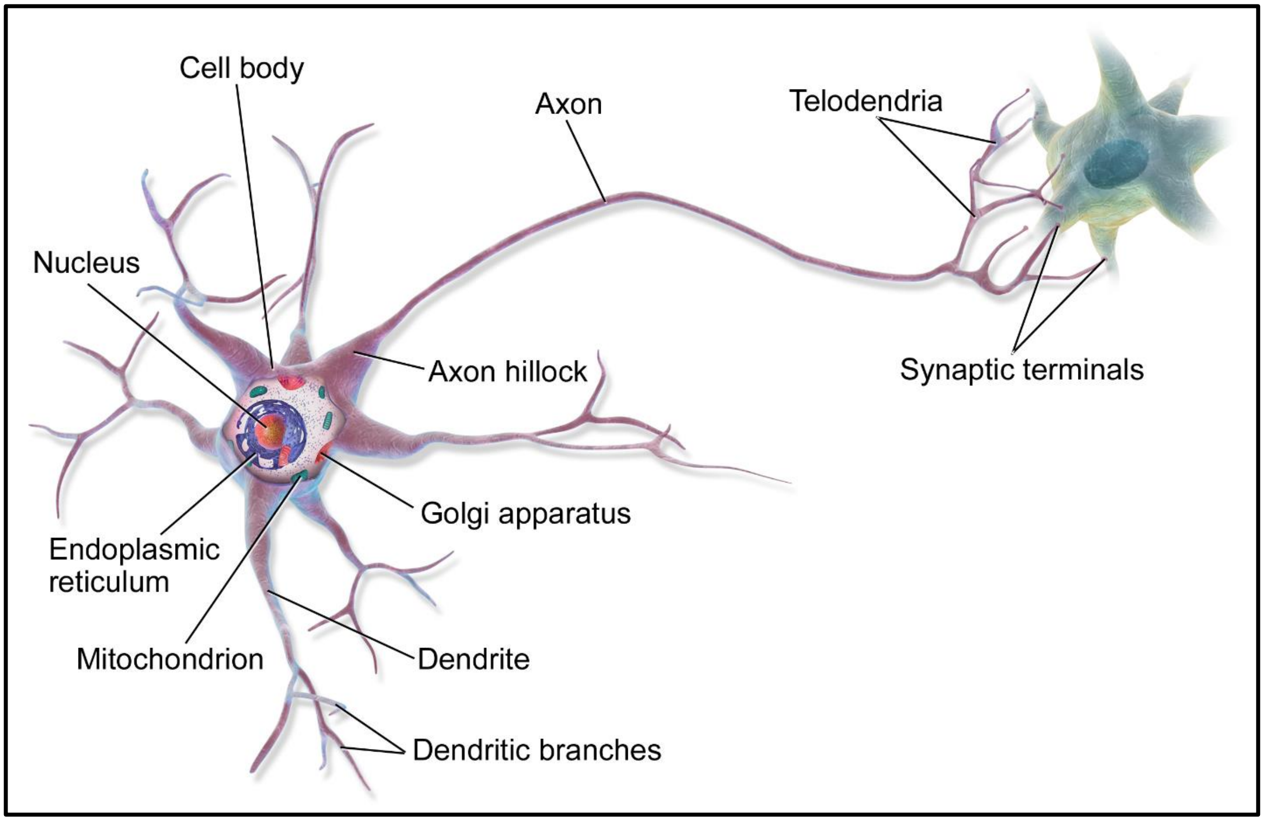


Figure 2-5 Human neuron cell (Herrup and Yang, 2007)

Neural networks were used wildly to solve, clustering, classification and pattern-recognition problems (Bell, 1997). Figure 2.6 explains a simplest classification problem and Figure 2.7provides the single layer NN that can solve such classification problem.

Class	Weight (gram)	Length(cm)
Fruit 1	121	16.8
	114	15.2
Fruit 2	210	9.4
	195	8.5

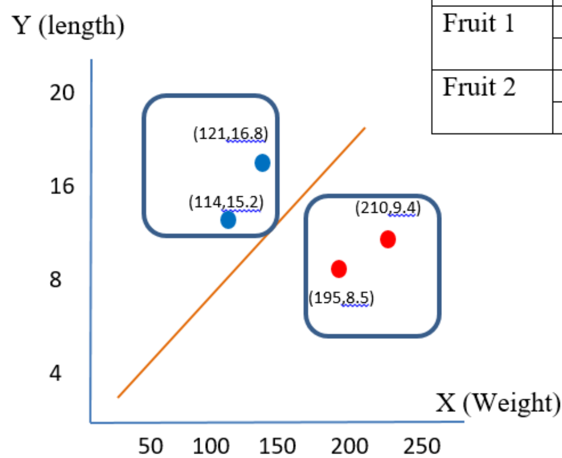


Figure 2-6 Simplest classification problem

There are many financial applications in the surveyed related works which used NN to build predictive model. For example, (Tam, 1991) proposed Back propagation NN (BPNN) based technique for bankruptcy prediction and compared it with factor logistic, K-NN and ID3. And they show that the (BPNN) performs better than other technique in terms of predictive accuracy. And (Salchenberger et al., 1992)present BPNN to predict thrift institutions failure and the proposed model were compared with the logistic model. In (Wilson and Sharda, 1994), they compared the predictive accuracy of BPNN with that of DA and their comparison results show that the BPNN outperformed the DA in all test. In (Tsukuda and Baba, 1994), they developed one hidden layer BPNN base system to predict bankruptcy in Japanese corporations and they compared the proposed BPNN with the DA and the result show that the NN proved to be superior to the DA. In (Bell, 1997), they compared the NN model and the logistic regression model in Predict Commercial Bank Failures context and they showed that the NN model and the logistic regression model perform equally well in this decision context. And (Piramuthu et al., 1998), they proposed NN based system to solve financial risk classification problem. The developed feature construction methodology improves the learning speed and classification accuracy of neural network algorithms.

Neural networks have adaptive learning ability from observation (Piramuthu et al., 1998)and high accuracy. However, the training phase takes considerable time to tune the NN parameters like Weight vector, training rate, learning method and network topology as well as needing a lot of training data and training cycles which let the NN being computationally expensive beside Complex to understand and trace its behavior by the normal end users (black box).

2.2.3.2 White Box Artificial Intelligent Techniques:

The term "***White Box"*** is referring to the artificial intelligent technique that can provide transparent reasoning behind the extracted decision which lets such technique considered to be easy to understand and analyze by the normal end users (Bernardo et al.,

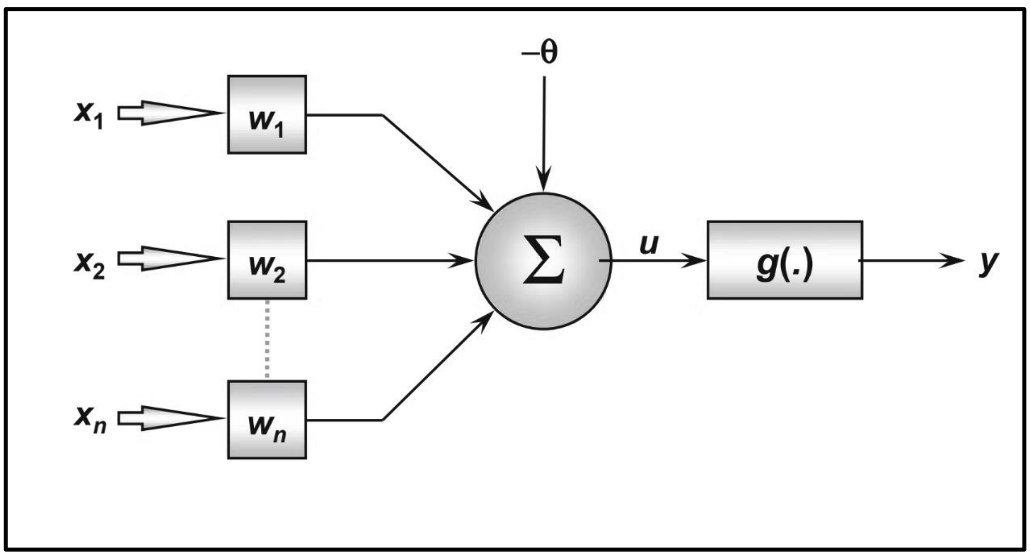


Figure 2-7 Single layer NN (Silva et al., 2017)

2013). For example, considering the same scenario in the above black box section, if the bank has decision support system that works based on white box technique such system can provide clear and transparent reasoning about the extracted decision. And at that time the investment employee can take confident decision about the customer's request because he has a transparent vision about his client. Figure 2.8 provides a simple explanation about white box technique.

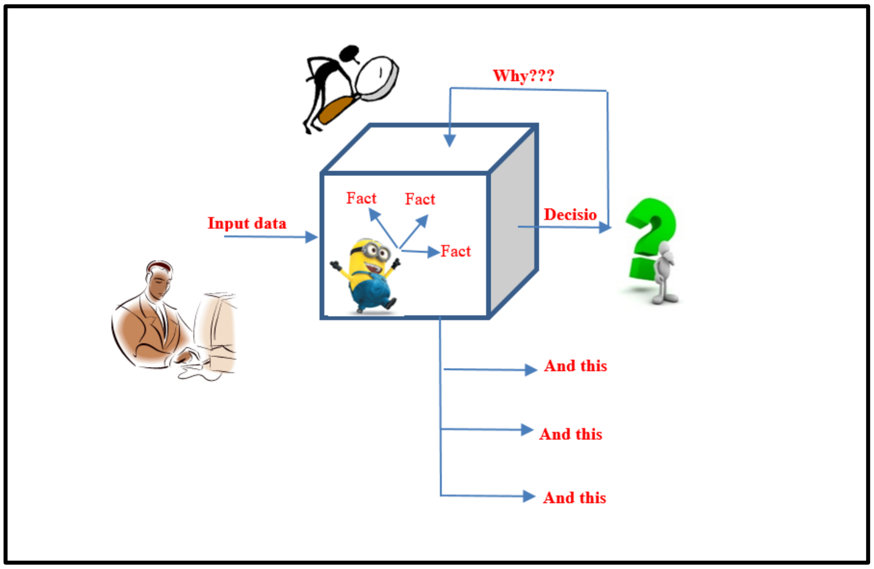


Figure 2-8 White Box Explanation

There are many white box artificial Intelligence techniques such as case-based reasoning (CBR), rough sets theory, decision trees (DT) and Fuzzy logic (FL) which have been applied successfully in the financial domain. The flowing three subsections provide simple explanation about each white Box technique and their survey application in this domain.

A. Case-Based Reasoning (CBR):

Case-based reasoning (CBR) is a method that attempts to study solutions that were used to solve problems in the past to solve, by analogy or association (Ketler, 1993). Figure 2.9 explains the case-based reasoning architecture.

There are many financial applications in the surveyed literature works used (CBR) to build predictive model. For example, (Park and Han, 2002)proposed framework called analytic hierarchy (AHP) using K-nearest neighbor (k -NN) method which is a CPR algorithm to predict bankruptcy. And (Yip, 2004)used CPR approach to predict business failure. They used (k-NN) algorithm in case retrieval phase with additional statistical evaluations to ensure the effective retrieval of similar cases.

As (CBR) based techniques is considered to provide transparent reasoning and fast model. However, it is difficult to deal with noisy data.

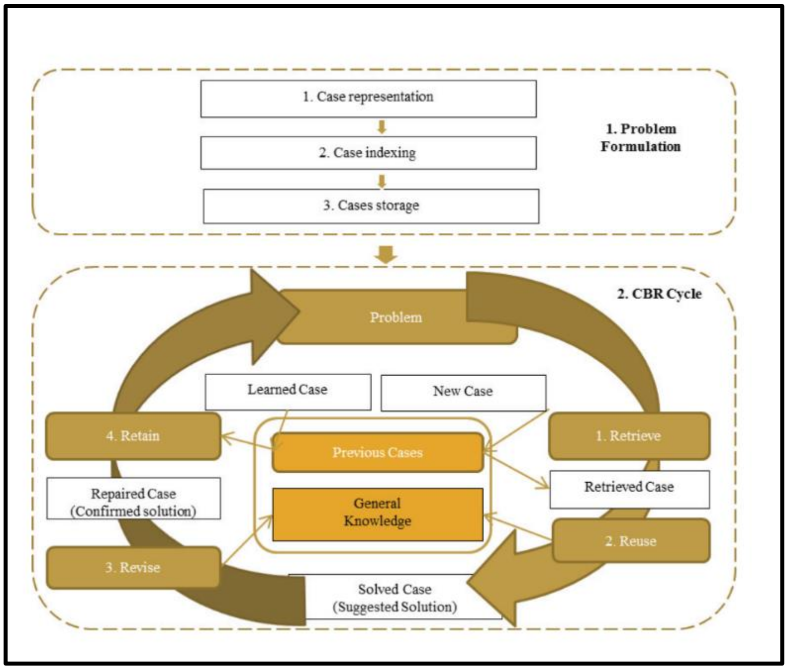


Figure 2-9:Case-Base reasoning architecture (Xiao et al., 2017)

B. Rough Sets Theory:

Rough sets theory was proposed by (Pawlak, 1982)and it is based on the assumption that for any item in the given universe there is hiding information belonging to this item, this information can be invisible until many items in the same universe are clustered together. It can be seen as complementary to fuzzy sets theory (Kumar and Ravi, 2007).

There are many financial applications in the surveyed literature which used rough sets theory to build predictive model. For example, (Greco et al., 1998)proposed an approach based on rough set theory to solve bankruptcy prediction problem. And (Dimitras et al., 1999)used rough set approach to provide a set of rules able to discriminate between healthy and failing firms in order to predict business failure.

Rough sets theory can provide transparent models which can be easily understood and analyzed by the normal end user beside having the ability to deal with uncertainty (Dimitras et al., 1999)and producing decision rules from data (Rissino and Lambert-Torres, n.d.). But the main drawbacks of the traditional rough set model is the inefficiency of rough set methods and algorithms in computing the core attributes and the reduction and identification of the dispensable attributes (Hu et al., 2004).

C. Decision Trees (DT):

Decision trees (DT) is artificial intelligence technique which use recursive partitioning algorithm to produce rules on a specific data set (Kumar and Ravi, 2007). DT algorithms take training data set and they try to extract the decision boundaries. These decision boundaries are used to build a decision tree and from this decision tree it will be able to extract a decision rule which can provide reasoning tools that can provide clear understanding about the extracted decisions. Figure 2.10 shows a simple DT classification example.

There are many financial applications in the surveyed literature using decision trees to build predictive model. For example, (FRYDMAN et al., 1985)proposed Recursive Partitioning Algorithm (RPA) and apply it in the context of firm financial distress, and compared it with Discriminate Analysis (DA) and concluded that the RPA it has outperformed the DA. In (Marais et al., 1984), they used recursive partitioning and bootstrapping techniques to proposed recursive partitioning algorithm to predict bankruptcy in firms.

Decision trees provide transparent reasoning model. However, they have many limitations such as: Cannot handle uncertainty and the recursive partitioning operation is very difficult which leads to hard decision boundary extraction.

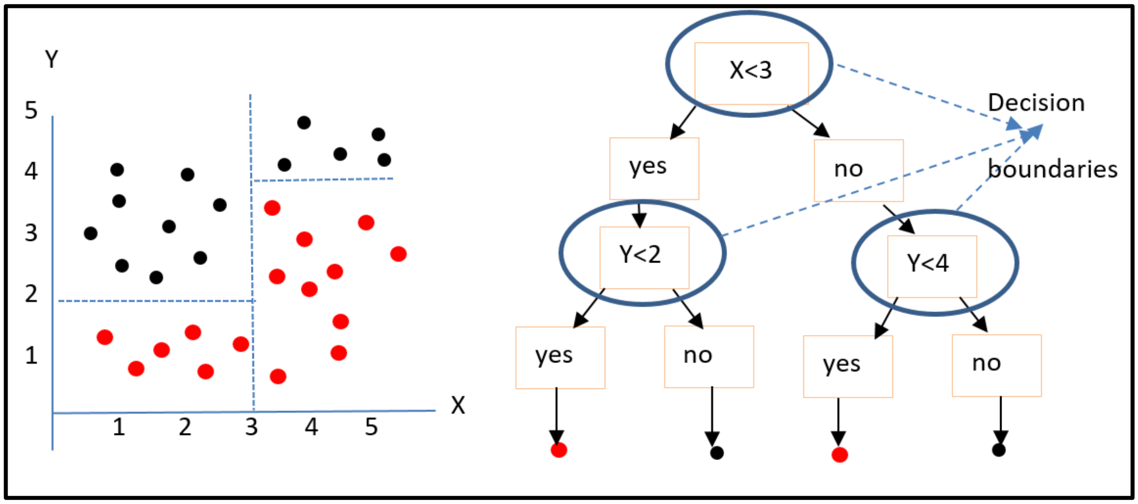


Figure 2-10 Decision Trees Classification Example ("Intro to Machine Learning - Udacity," n.d.)

D. Fuzzy Logic (FL):

Fuzzy Logic Theory (FL) was invented by Lotfi A. Zadeh in the 1960s (Zadeh, 1999)in order to provide a framework that is able to handle uncertainties associated with natural languages. Fuzzy logic controller tries to mimic a human's brain in order to think in approximate way rather than precise way. Fuzzy logic controller is built based on fuzzy set theory which provides a means of calculating intermediate values between absolute true and absolute false with resulting values ranging between 0 and 1 and this led to smooth transition between different set. The fuzzy logic controller is built from 4 components which are: fuzzifier, inference engine, rule base and defuzzifier.

There are many financial applications in the surveyed literature which used fuzzy logic to build predictive model. For example, (De Andrés et al., 2005) proposed fuzzy-rule-based classifiers for bankruptcy prediction problem and compared their classifier with LDA, logit and perceptron NN techniques and they concluded that MLP and fuzzy rule based classifier outperformed LDA and logistic regression.

Fuzzy logic can provide transparent reasoning model; however, type-1 fuzzy logic cannot handle high level of uncertainty. Type-2 fuzzy logic can handle the high level of uncertainty, but like type-1 fuzzy systems, it suffers from the curse of dimensionality problem.

2.2.3.3 Hybrid Intelligent Techniques (soft computing):

The term (Hybrid Inelegant Technique) is referred to the artificial intelligence technique that tries to combine more than one artificial intelligence techniques in order to get benefits from the advantages of any of them and to overcome the limitations of them.

There are many financial applications in the surveyed literature which used hybrid inelegant technique to build predictive model. For example, (Michael et al., 1999)present the combined use of a fuzzy rule generation method and a data mining technique for the assessment of financial risks. In (Michael et al., 1999), they compared their model with LDA, QDA, logit analysis and probability analysis and they concluded that fuzzy rule based classifier outperformed other methods. In (Back et al., 1996), they developed hybrid architecture for bankruptcy prediction. They used LDA, logit and Genetic Algorithms (GA) for feature extraction phase and the selected features were used as predictors for BPNN. They concluded that the model that used GA feature selection + BPNN used for prediction) got improved results than (DA + BPNN) and logit + BPNN. “Ontogenic NN” was proposed by (Ignizio and Soltys, 1996)which is GA-based model for the simultaneous design and training of neural networks for firm failure prediction. They show that the hybrid approaches give a smaller number of misclassified cases compared to other methods like (DA, Back-propagation Model, and Perceptron Model). In(Jeng et al., 1997) , they designed a hyper intelligent model that was built from three techniges (DA, BPNN and case-based forecasting (CBFS)) for bankruptcy prediction. They showed that the integrated model produces higher prediction accuracy than individual models.

In (Jeng et al., 1997), they presented a fuzzy inductive learning (FILM) method that converts a decision tree induced from a regular method into a fuzzy decision tree. And they compare FLIM with DA and ID3 and they conclude that the FILM outperforms the DA and ID3.

In (Gorzalczany and Piasta, 1999), they proposed two hybrid techniques neuro-fuzzy classifier (N-FC) and rough set based rule induction system with statistical techniques classifier (RC) to predict corporate bankruptcy. Their results show that N-FC outperforms RC.

In (Elhadi, 2000), they combined information retrieval (IR) and CBR in his proposed system for legal domain of bankruptcy law (BanXupport). In (Ahn et al., 2000), they proposed hybrid rough set approach and neural network system that predicts the failure of firms.

In (Lin and McClean, 2001), they proposed a hybrid model using DA, logistic regression, NN, and decision tree (C5.0) to predict corporate failure in UK. In (Bian and Mazlack, 2003), they proposed a new rough nearest-neighbor (NN) approach based on the fuzzy-rough sets theory to predict financial distress in Chain's company.

And (Bernardo et al., 2013)they presented Genetic Type-2 Fuzzy Logic System (FLS) for modeling and prediction of financial applications. In (Osman et al., 2013)the authors used Decision Tree (DT) and Artificial Neural Networks (ANN) to building Credit Scoring Models (CSMs) for the Sudanese banks. They used Principal Component Analysis (PCA) and Genetic Algorithms (GA) as feature selection techniques. They used two different datasets to evaluate their model and their experiment result show that the ANN models outperform DT models in most cases.

Hybrid intelligent are beneficial as they amplify the advantages and nullify the disadvantages of associated AI techniques, however they require a good amount of data (Kumar and Ravi, 2007).

2.3 Summary

A brief introduction of different prediction models has been given in this chapter. The researcher classified these models into four different classes with the presentation of their benefits and drawbacks.

Chapter Three: Introduction to Fuzzy Logic Systems

Fuzzy Logic-based algorithms are a type of computational paradigm that can solve problems in the same way as nature does. Fuzzy Logic-based techniques give a method of reasoning that allows the system to receive input values and analyze them using the reasoning method to produce the matching output values(Zadeh, 1965). This chapter will provide an overview of fuzzy logic, beginning with a brief history of its foundations and progressing to Fuzzy Logic Sets and Fuzzy Logic Systems.

3.1 Fuzzy logic a brief history

Around a century ago, American philosopher Charles Peirce was one of the first researchers in the modern era to recognize and regret that logicians have too much disregarded the study of vagueness, not suspecting the crucial role it plays in mathematical reasoning(Dubois et al., 2000). Bertrand Russell, a few years later in 1923, expressed a similar viewpoint. Discussions of the relationship between logic and ambiguity are common in philosophical literature from the first half of the twentieth century Copilowish , Hempel in 1939 and Even Wittgenstein in 1953 observed that concepts in common language do not have a fixed set of attributes that define them, but rather have extendable borders, and that a category might have central and less central members (Dubois et al., 2000).

Despite the fact that Jan Lukasiewicz and his school developed logics with intermediary truth value(s) in the 1930s, it was American philosopher Max Black (1937) who first proposed so-called "consistency profiles" (the ancestors of fuzzy membership functions) in order to characterize vague symbols(Dubois et al., 2000). H. Weyl in 1940 was the first to address the generalization of the classical characteristic function, which he explicitly replaces it with a continuous characteristic function. In 1951, Kaplan and Schott developed a similar type of generalization and they proposed calculi for generalized characteristic functions of vague predicates, and the fuzzy set connectives were already mentioned in these works(Dubois et al., 2000). While fuzzy set theory has been noticeably developed by Professor Zadeh the author in (Dubois et al., 2000) state that Surprisingly, it was Karl Menger, a probabilistic metric space mathematician, who was the first to use the phrase "ensemble flou" (the French equivalent of "fuzzy set") in the title of his work in 1951. Menger, on the other hand, couldn't get away from probability theory and statistics, whereas Zadeh created the concept of fuzzy sets as a technique to quantify fuzziness, and explained fuzzy concepts in relation with classes which do not possess sharply defined boundaries, in other words, classes with a continuum of grades of membership (JERRY, 2017). Professor Zadeh further stated that such imprecisely defined classes play an important role in human thinking, particularly in the domains of pattern recognition, communication of information, and abstraction (JERRY, 2017).

In scientific circles, and sometimes even in ordinary life, the term "***fuzzy set***" has become a trendy (and frequently derided) phrase. It is widely advertised, frequently ill-defined, and frequently misused and misinterpreted. M. M. Gupta in 1977 correctly characterized the concept of fuzzy-ism as "a corpus of concepts and practices aimed at providing a systematic framework for deal with the vagueness and imprecision inherent in human mental processes" (Dubois et al., 2000) .Thinking, ambiguity, and imprecision are the three essential terms, the three aspects of this concept that comprise its philosophical foundation.

Fuzzy logic has been extensively popular among philosophers, scientists, researchers, engineers, as well as companies and industrial institutions, over the last few decades. The next sections will outline the most often used types of fuzzy sets and related terminologies, and hence varieties of fuzzy logic, as well as their development timelines. Type-1 FL and Type-2 FL are the two most popular categories in the fuzzy logic literature. In addition, new Type-2 FL classifications such as Interval Type-2 FL and General Type-2 FL will be presented.

3.2 Linguistic variables:

The concept of a linguistic variable according to Zadeh (Zadeh, 2009)reflects the reality that most human thinking is approximate rather than accurate, and that variable values in human conversation are frequently stated in words rather than numbers. In this context, the linguistic approach's goal is to provide a systematic foundation for computing with words and/or numbers, as well as for inferring from facts stated as propositions in natural or synthetic languages(Zadeh, 2009).

In more concrete terms, a linguistic variable X is described by a *quintuple (X, T(X),U, G, M),* in which *X* is the variable's name, e.g., Pressure; *T(X)* is the variable's term-set, that is, the collection of its linguistic values e.g., *T(X) = {weak, low, okay, strong, high}, U* is a universe of discourse, e.g., in the case of *Pressure U=[100 psi, 2300 psi]*, the set *G* is a syntactic rule for generating linguistic terms, and *M* is a semantic rule that assigns each linguistic term which associates with each term x in *T(X)* its meaning, *M*(*x*), where *M(x)* denotes a possibility distribution in *U*. thus, the meaning of *x* is defined by a membership function or, equivalently, a possibility distribution function $\mu_x U \rightarrow [0,1]$ which associates with each *u* in U its possibility $\mu_x(u)$ given the proposition 'X is x'(Zadeh, 2009) . Figure 3.1 provides an example of membership function MFs for the variable Pressure.

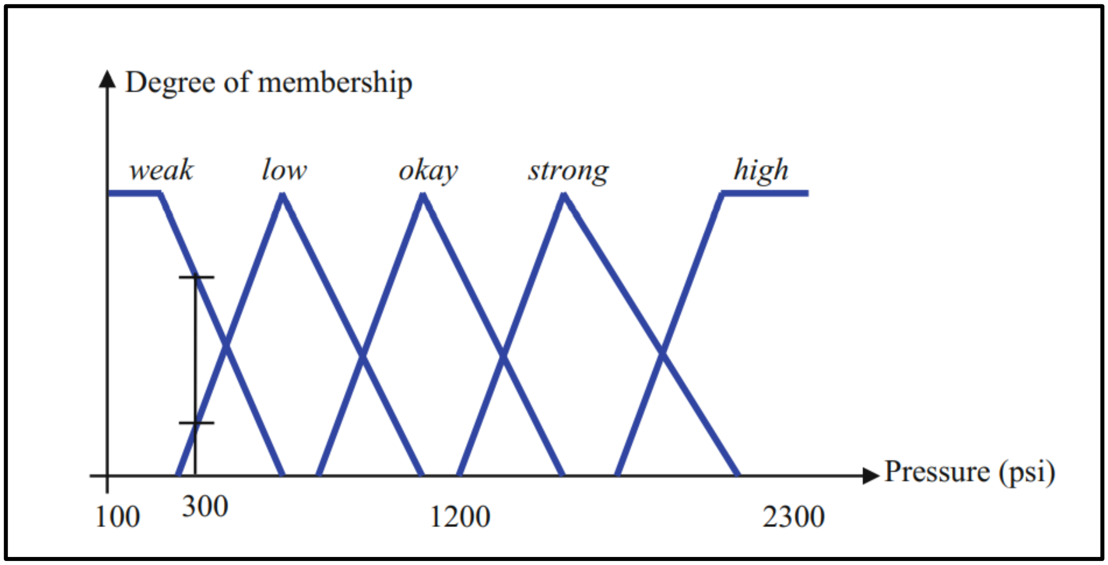


Figure 3-1 example of membership function for Pressure variable(source (JERRY, 2017))

3.3 Types of Fuzzy Logic Sets

In 1965, Zadeh developed the concept of fuzzy sets without any type of distinction (Zadeh, 1965) . As a result, in the 1970s, academics and scientists began investigating and learning about type-1 fuzzy sets. Zadeh created type-2 fuzzy sets in 1975, over a decade after the fuzzy set theory was published. Type-2 fuzzy sets, on the other hand, were not investigated in depth until the late 1990s(John and Coupland, 2007). People have a good understanding of what can be done with type-1 fuzzy sets, therefore the transition between the two types of fuzzy logic was natural. As a result, the researchers have begun to investigate more difficult problems involving type-2 fuzzy logic. These key categories will be presented in the following subsections, which have been exhaustively researched in the FL literature.

3.4 Type-1 Fuzzy Logic Set

Fuzzy logic is based on fuzzy set theory which is an extension of classical set theory. The strength that characterizes the fuzzy logic system to deal with uncertain information is inherited from the characteristics of the fuzzy set which provides it superiority over the classical crisp set (Zadeh, 1965). Classical crisp set theory expresses the knowledge in a very precise way and any item in a given universe of discourse can belong to only one crisp set and this is due to the sharp boundaries between crisp sets that lead to the ability to represent only absolute true and absolute false membership values in the crisp set. For example, any person cannot belong to the two crisp sets (TALL and SHORT) at the same time. These sharp boundaries between crisp sets can cause a sudden change in the control system which is lead to unexpected output from the overall control system(JERRY, 2017). Figure 3.2 provides an example of partitioning of the set of all automobiles in New York City into subsets by a color, b domestic or foreign, and c number of cylinders. And Figure 3.3 shows interpreting crisp sets as crisp partitions (JERRY, 2017).

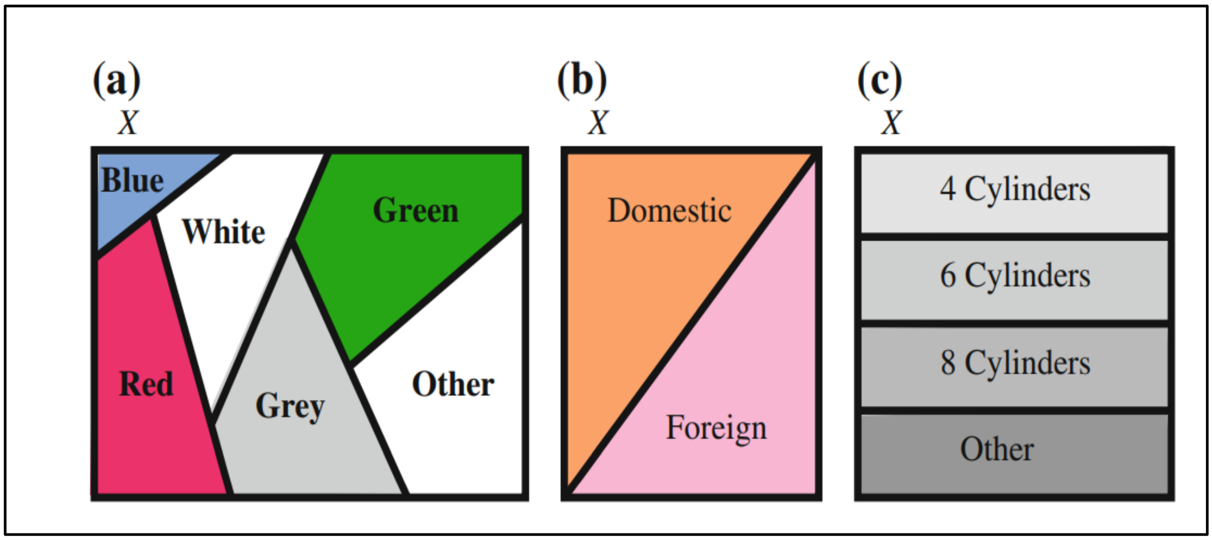


Figure 3-2example of Partitioning the set of all automobiles in New York City into subsets by a color, b domestic or foreign, and c number of cylinders (JERRY, 2017)

The most prevalent type of fuzzy sets (FSs) and thus type-1 fuzzy logic in the literature is type-1 fuzzy logic. In the traditional sense, they are also known as '***classical fuzzy sets***' or simply '***fuzzy sets***'. Based on Zadeh (Zadeh, 1965), the formal definition of a type-1 fuzzy set will be stated as follows:

Let X be a set of points, with x being a generic element of X. The membership function (MF) of the fuzzy set A in X associates each point in X with a real number in the interval [0,1], in other words, a value of the membership degree of x in A denoted as $\mu A(x)$. As a result, the closer $\mu A(x)$gets to unity, the more x belongs to A. Formally, the set of pairs completely characterizes A(Dubois, 1980):

$$A = \{(x, \mu A(x)), \forall x \in X\} \tag{3.1}$$

When the universe of discourse X is continuous, A is characterized as follows (JERRY, 2017):

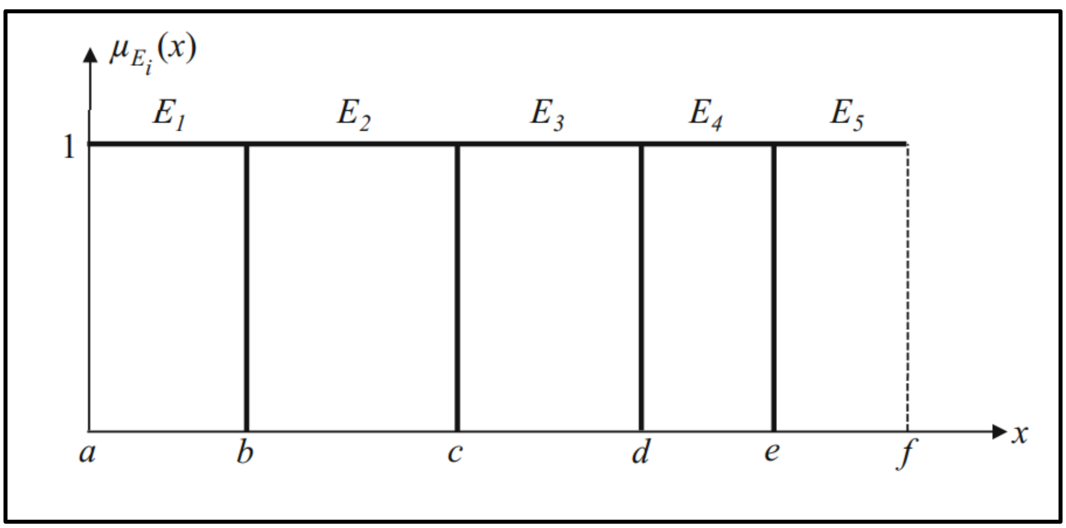


Figure 3-3Interpreting crisp sets as crisp partitions(JERRY, 2017)

$$A = \int \mu_A(x)/x \tag{3.2}$$

When the universe of discourse X is discrete, A is characterized as follows(JERRY, 2017):

$$\boldsymbol{A} = \sum \boldsymbol{\mu}_{\boldsymbol{A}}(\boldsymbol{x})/\boldsymbol{x} \tag{3.3}$$

In the other words, fuzzy sets overlap in certain areas, forming non-crisp or fuzzy boundaries which provide a means of calculating intermediate values between absolute true and absolute false with resulting values ranging between 0 and 1(JERRY, 2017). And this overlapping between fuzzy sets leads to a smooth transition between different sets rather than the sudden changes which cause problems in control. Using fuzzy sets allows us to incorporate the fact that no sharp boundaries between sets exist. Thus, not like a crisp set, any item in a universe of discourse can belong to more than one fuzzy set with different membership values.

Figure 3.4 shows fuzzy sets example. In this example, the x-axes represent the lengths of different football players. And there are two fuzzy sets [Short, Medium, Tall] any player can belong to the two different sets at the same time. So, the player hose length is 7 he belongs to Medium and Tal with different degrees of membership value.

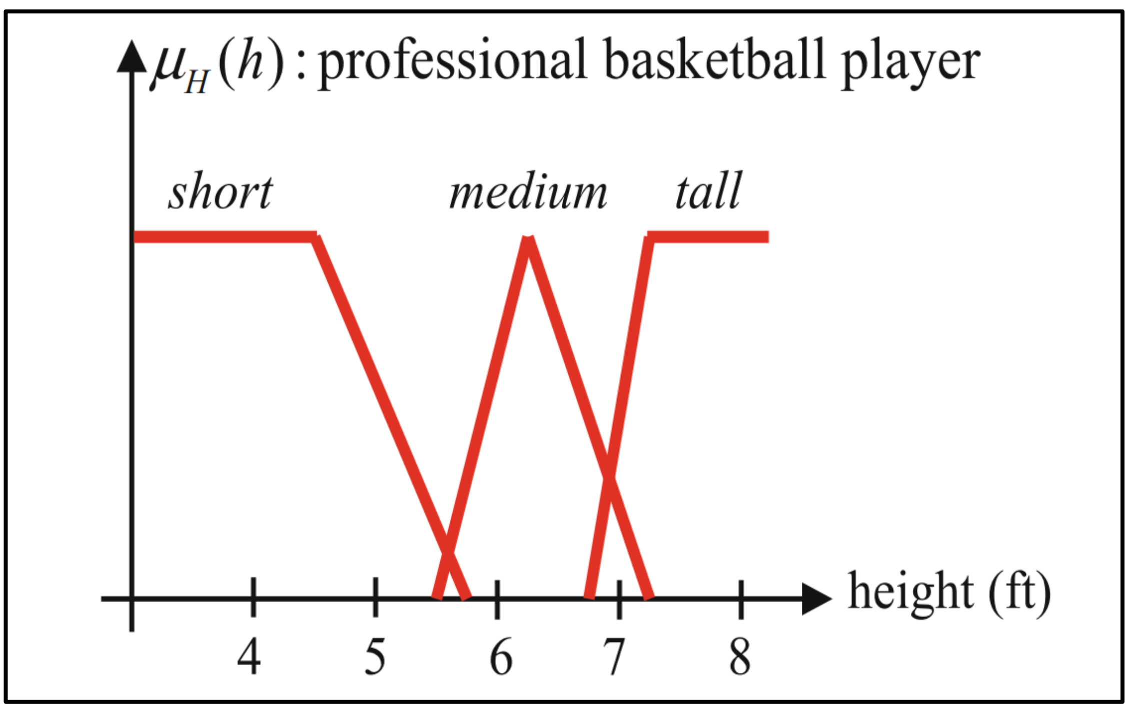


Figure 3-4 professional basketball player fuzzy set example (JERRY, 2017)

Triangular MF, trapezoidal MF, Gaussian MF, and singleton MF are some examples of regularly used MFs that are used to define type-1 fuzzy sets. Type-1 fuzzy sets, as can be seen, have clearly defined membership functions or degrees of membership Figure 3.5 shows an example of the four types of MF.

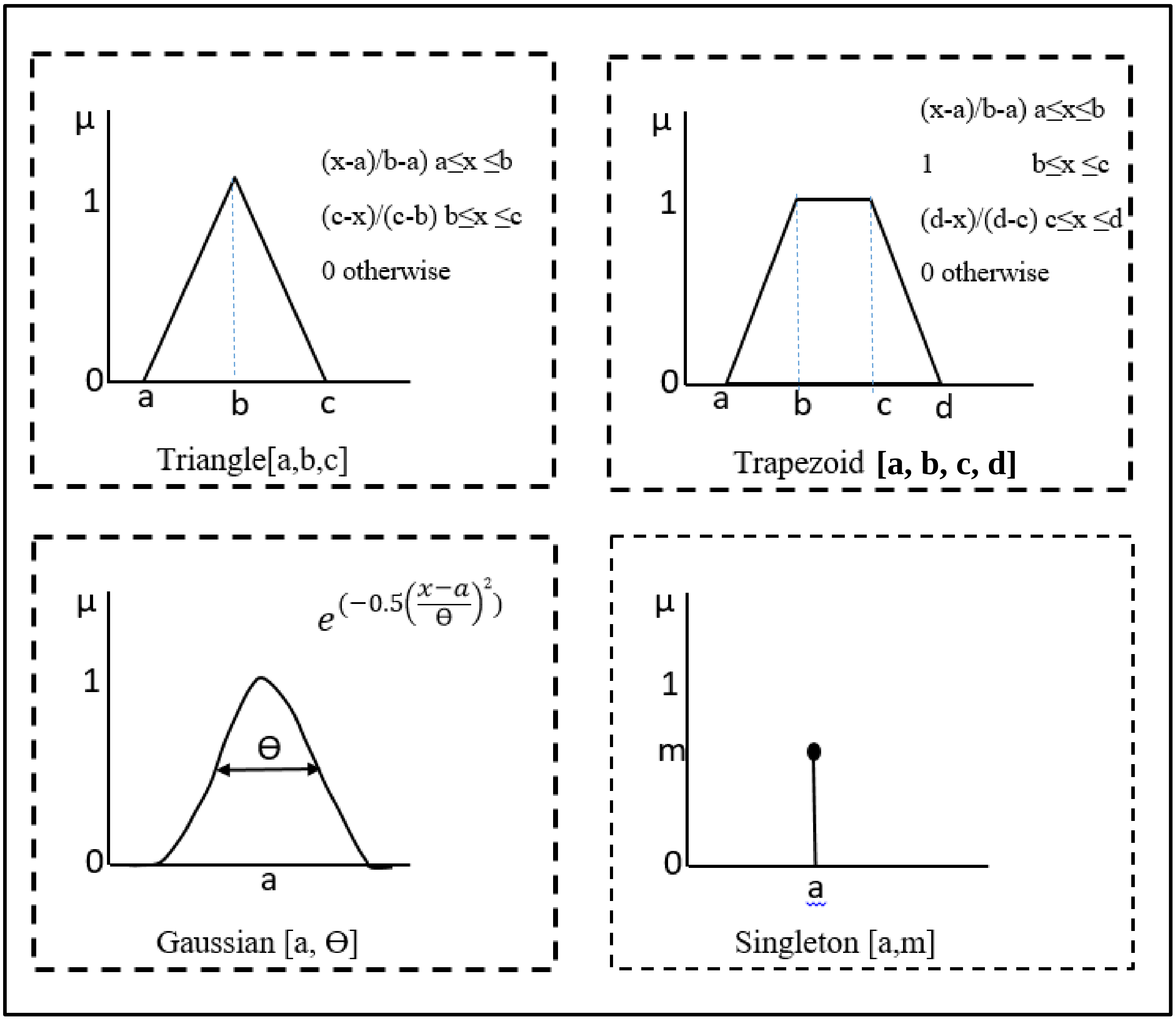


Figure 3-5 example of the four types of MF(Mendel, 2001)

3.4.1 Type-2 Fuzzy Logic sets:

Type-2 Fuzzy set originally invented by Zadeh (Zadeh, 1975) as an extension of type-1 fuzzy set such sets are fuzzy sets whose membership grades themselves are type-1 fuzzy sets; they are very useful in circumstances where it is difficult to determine an exact membership function for a fuzzy set. Furthermore, the correlation, which indicates that a type-n fuzzy set's membership function ranges over type-(n-1) fuzzy sets was used to coin the roots of not only type-2, but also type-n fuzzy sets.(N. Karnik and M. Mendel, 2001). Further to that, a higher-type fuzzy relation (e.g., type-2) has been regarded as one way to increase the fuzziness of a relation(N. Karnik and M. Mendel, 2001), and Hisdal (Hisdal, 1981) concluded that increased uncertainties in a description means increased ability to handle inexact information in a logically consistent manner.

Because of the complexity and degrees of freedom of a type-2 set, Type-2 Fuzzy Logic can be divided into two types: Interval Type-2 FL and General Type-2 FL. Within the two decades following the birth of type-2 conception, the growth of type-2 fuzzy sets is defined by the promotion of interval type-2 fuzzy sets, the following subsections will discuss Interval Type-2 FL and General Type-2 FL.

3.4.1.1 Interval Type-2 Fuzzy Logic Sets:

Figure 3.6 shows the generally used words and notations, as well as a graphical description of a triangular MF used to describe an interval type-2 fuzzy set, which can be

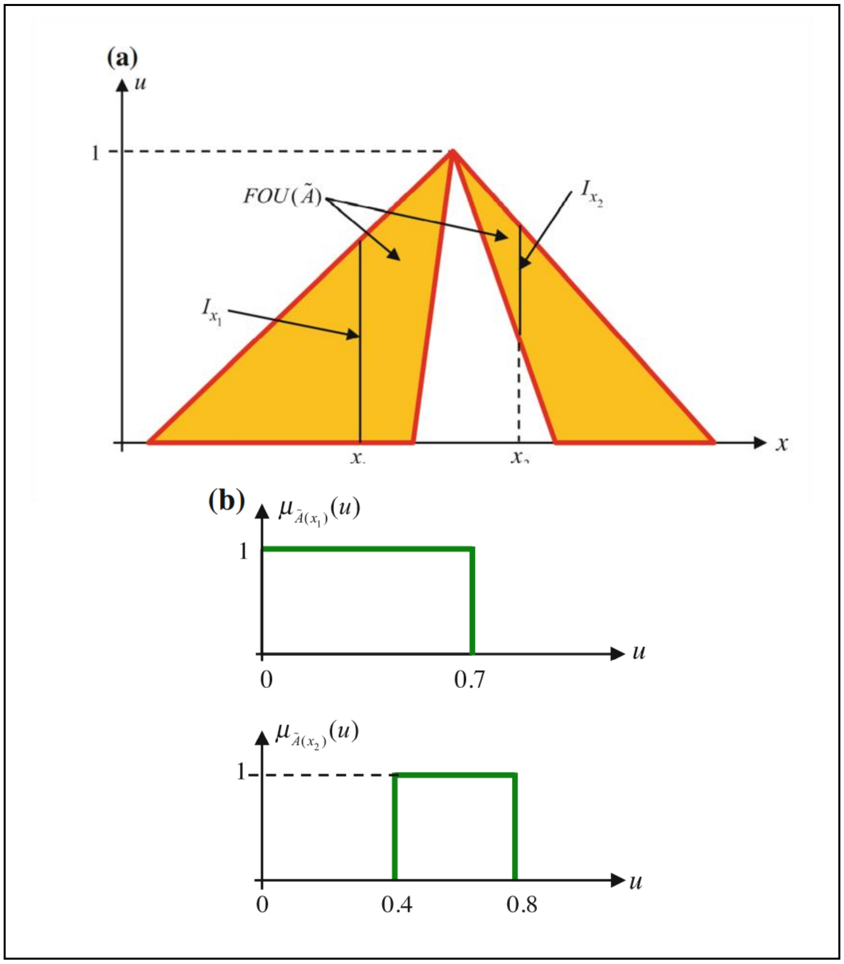


Figure 3-6 Interval Type-2 Fuzzy Logic Sets(source :(JERRY, 2017))

viewed as a set with an infinite number of embedded type-1 fuzzy sets, The shaded region in a is the FOU of a IT2 FS. The supports of the secondary MFs, I_{x1} and I_{x2}, are shown at x_1 and x_2 in **(a)**, and their associated secondary MFs, $\boldsymbol{\mu_{\tilde{A}\,(x1)}(u)}$ and $\boldsymbol{\mu_{\tilde{A}\,(x2)}(u)}$, are shown in **(b).**

Formally, let $\tilde{A}$ be an interval type-2 fuzzy set (IT2 FS) as shown in **Figure 3.6 a**, which is characterized as follows (Wu and Mendel, 2007):

$$\tilde{\boldsymbol{A}} = \int_{x\in X} \int_{u\in J_x} 1/(x,u) = \int_{x\in X} \left[\int_{u\in J_x} 1/u\right]/x \quad \textbf{(3.4)}$$

where x, the primary variable, has domain X; $u \in U$, the secondary variable, has domain Jx at each $x \in X$; Jx is called the primary membership of x and is defined in Equation (3.5); and the secondary grades of $\tilde{A}$ all equal to 1 (Wu and Mendel, 2007).

$$J_x = \left\{(x,u): u \in \left[\underline{\mu}_{\tilde{A}}(x), \bar{\mu}_{\tilde{A}}(x)\right]\right\} \quad \textbf{(3.5)}$$

Furthermore, uncertainty about $\tilde{A}$ is carried by the union of all the primary memberships, which is called the footprint of uncertainty (FOU) of $\tilde{A}$, and is formalized as (Wu and Mendel, 2007):

$$FOU(\tilde{A}) = \cup_{\forall x\in X} J_x = \{(x,u): u \in J_x \subseteq [0,1]\} \quad \textbf{(3.6)}$$

Interval type-2 fuzzy sets(IT2), as the name implies, are based on the concept of an interval for a type-2 fuzzy set's additional degree of freedom, which is the third dimension referred to as the secondary membership function, as shown in Figure 3.6b. Interval techniques became popular because the more general case, when the secondary membership function is non-interval, made calculation significantly more complex, according to(John and Coupland, 2007). As a result, IT2 fuzzy sets have received more attention and utilization in terms of applications and research until lately.

Fuzzy logic has numerous applications including rice cookers, subway control, air-conditioning, braking, automatic focusing for a compact camera, color adjustment of TV, lifts, shower temperature control, vehicle speed, and steering, and many heavy industries, etc. (Hagras and Wagner, 2012). The vast majority of the FLCs that have been used so far were based on the traditional type-1 FLCs. However, type-1 FLCs cannot fully handle or accommodate for the high levels of linguistic and numerical uncertainties and this is because type-1 FLCs use precise type-1 fuzzy sets and membership functions. For example, for a house environment, a "Moderate" temperature could be associated with the triangular type-1 fuzzy membership function. However, the center of this triangular membership function and its endpoints vary according to the user of the system, where different users will have different preferences. Even for the same user, his preference will vary according to the season of the year, his mode, country, context, and the room in the house where "Moderate" temperature in the kitchen will be different to "Moderate" temperature in the living room that is emphasized by the adage "*words mean different things to different people*" (Mendel, 1999). Different examples of IT2 fuzzy logic set will be presented in section 3.4.3.

3.4.1.2 General Type-2 Fuzzy Logic Sets

Because of the nature of General Type-2 Fuzzy Sets, the logic of the Generalized Type-2 Fuzzy Logic System (GT2FLS) is similar to that of T1FLS and T2FLS, but the operations are slightly different. The computation complexity of General Type-2 Fuzzy Sets grows as they get more complicated than Type-1 Fuzzy Sets or Interval Type-2 Fuzzy Sets. Especially in the defuzzification process, where the amount of calculations is basically insurmountable, because of its combinative nature, it becomes non-computable as the number of discretization increases(Sanchez et al., 2015).

The difficulty of general type-2 fuzzy logic has become more manageable as a result of tremendous advances in interval type-2 fuzzy logic. The invention of zSlices (Wagner and Hagras, 2010) and alpha-planes (Mendel et al., 2009) has recently helped to fill the gap formed by the complexity of designing and implementing general type-2 fuzzy sets, particularly for real-world applications.

Officially, a general type-2 fuzzy set $\tilde{G}$ is defined as follows(Mendel, 2001):

$$\tilde{G} = \int_{x \in X} \int_{u \in J_x} \mu_{\tilde{G}}(x,u)/(x,u) \quad \textbf{(3.7)}$$

where $0 \leq \mu_{\tilde{G}}(x, u) \leq 1$ and the integral denotes union over all admissible x and u.

A vertical slice of $\mu_{\tilde{G}}(x, u)$ at $x = x'$, which is a 2-D plane whose axes are u and $\mu_{\tilde{G}}(x', u)$, can be represented as follows (Mendel, 2001):

$$\mu_{\tilde{G}}(x = x', u) = \int_{u \in J_{x'}} f_{x'} u/u \quad J'_x \subseteq [0,1] \quad \textbf{(3.8)}$$

Where $0 \leq f x'(u) \leq 1$.

Hence, a general type-2 fuzzy set $\tilde{G}$ can be re-expressed in term of union of all slices as in Equation (3.9)(Mendel, 2001)

$$\tilde{G} = \int_{x \in X} \left[\int_{u \in J_x} f_x(u)/u\right]/x \quad J_x \subseteq [0,1] \quad \textbf{(3.9)}$$

As previously stated, type-2 fuzzy logic provides for better (than type-1 fuzzy logic) modeling of uncertainty because type-2 fuzzy sets include a FOU, which offers type-2 fuzzy sets additional degrees of freedom in association to their third dimension (Wagner and Hagras, 2010). Figure 3-7 depicts the secondary membership functions (third dimension) of type-1 Fuzzy Sets (Figure 3-7(A)), Interval Type-2 Fuzzy Sets (Figure 3-7(B)), and General Type-2 Fuzzy Sets (Figure 3-7(C)).

In type-1 fuzzy sets, the secondary MF has just one value in its domain, which corresponds to the primary membership value at which the second grade is 1, as shown in Figure 3-7 (A). As a result, in type-1 Fuzzy Sets, there is no uncertainty associated with the primary membership values for each x value(Wagner and Hagras, 2010). On the other hand, Figure 3-7 (B) shows that the secondary MF of an Interval Type-2 Fuzzy Sets contains highest uncertainty because the primary membership includes values in the interval [a, b], with each point in this interval having an associated secondary membership of 1(Wagner and Hagras, 2010). The uncertainty (expressed in the secondary MF) in General Type-2 Fuzzy

Sets can be modeled to any degree between type-1 and Interval Type-2 fuzzy sets, such as the triangular secondary MF illustrated in Figure 3.7 (C) (Wagner and Hagras, 2010). As a result, GT2 fuzzy sets can accurately depict third-dimensional uncertainty, ranging from almost no uncertainty (type-1) to maximum (IT2, where the uncertainty is evenly distributed in the third dimension)(Wagner and Hagras, 2010).

The following section introduces a fuzzy logic system (FLS) and its components, which combined provide a straightforward approach to map an input space to an output space, particularly in situations where uncertainty must be addressed.

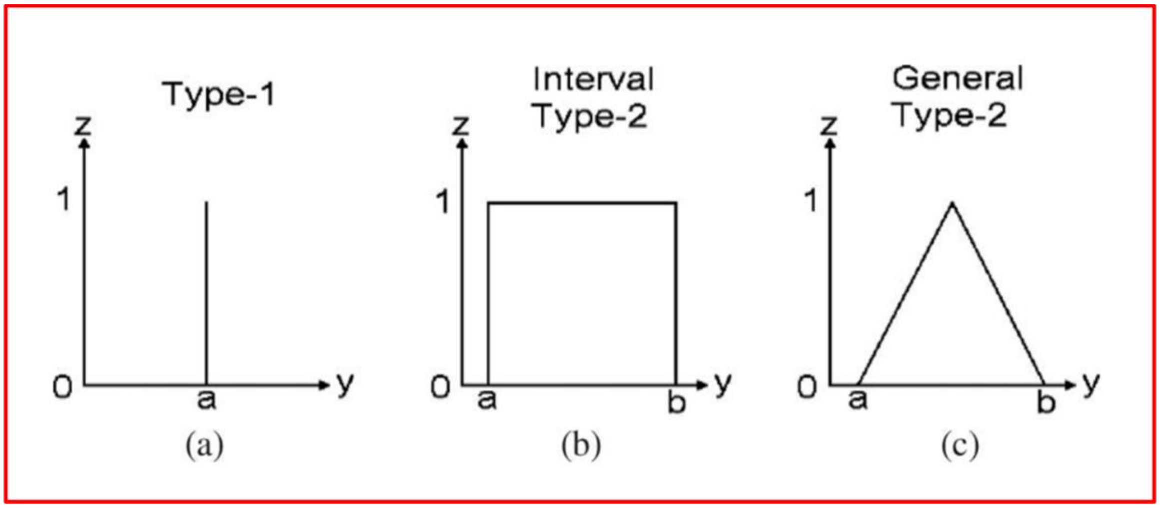


Figure 3-7 depicts the secondary membership functions (third dimension) of type-1 Fuzzy Sets (A), Interval Type-2 Fuzzy Sets (B), and General Type-2 Fuzzy Sets (C) (source :(Wagner and Hagras, 2010))

3.5 Types of Fuzzy Logic Systems

A system has a set of relationships between measurable properties such as inputs and outputs, according to (Dubois, 1980). Furthermore, whether inputs or outputs are treated as fuzzy sets, a system is considered fuzzy(Dubois, 1980). A nonlinear mapping of an input data vector into an output data vector is another general description of a fuzzy logic system (FLS), according to Mendel (Mendel, 2001).

The fuzzy sets that are being operated on became the reason to termed a fuzzy logic system. As a result, the literature contains several kinds of FLSs.Type-1 fuzzy sets, for example, are used to construct a type-1 fuzzy logic system. At least one general type-2 fuzzy set is used to construct a general type-2 fuzzy logic system. The type-1 FLS is presented first, as it is the foundation for higher-order FLSs, and then a type-2 FLS is presented in the following subsections.

3.5.1 Type-1 Fuzzy Logic Systems

Figure 3.8 depicts a fuzzy inference system, also known as a fuzzy rule-based system, fuzzy model, fuzzy associative memory, or a fuzzy controller. The goal of a FLS is to map inputs into outputs using fuzzy reasoning embedded in the rules. Type-1 FLSs have four components, as shown in Figure 3.8: fuzzifier, rules, inference engine, and defuzzifier(JERRY, 2017).

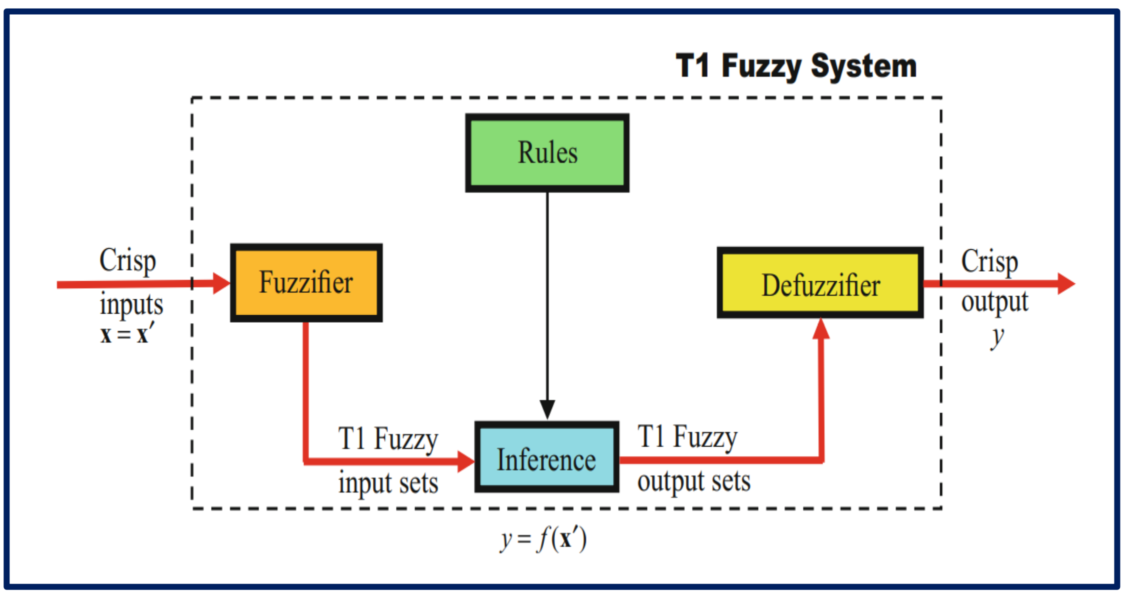


Figure 3-8: Type-1 Fuzzy logic System (Source: (JERRY, 2017))

Type-1 fuzzy logic control systems usually consist of four major parts: Fuzzification interface, Fuzzy rule-base, Fuzzy inference machine, and Defuzzification interface as in Figure 3.8.

To provide a simple idea about the role of each component before we introduce their formal definition, consider this example of the self-driving car is controlled by type-1 fuzzy logic control system. suppose that the car is in moving mode in this mode the front sensors measure the distance between the car and any object which appears in front of the car. If the front sensors in time T (n) measure the distance between the car and an in front object as 10m the **fuzzification interface** takes this crisp input for the distance parameter and converts it to linguistic control information like: NEAR or FAR and this conversion is doing base on pre-defined member ship function which can takes many shapes such as: triangular, trapezoidal and Gaussian membership function. And the output from the **fuzzification interface** is the membership values of the crisp input to any of the fuzzy sets. For example, the 10m crisp input can belong to NEAR Fuzzy set with membership value of 0.8 and to FAR fuzzy set with membership value 0.2. The fuzzy inference machine receives type-1 input fuzzy sets and triggers the corresponding rules from the rule base and calculates the firing strength of each fired rule. Figure 3.9 provides an example of rule base. The output from the inference machine is type-1 output fuzzy sets for example if the fired rules are R1and R3 -from Figure 3.9 - the output fuzzy sets it will be **LOW** and **MEDUM**. After that Defuzzifier converts these fuzzy sets to single crisp output which represents the overall output from the fuzzy control system. Defuzzification strategy is aimed at producing a non-fuzzy control action that best represents the possibility distribution of an inferred fuzzy control action.

There are different defuzzification strategies such as: Maximum Defuzzifier, Mean

R1:*If the distance is* **FAR** *and speed is* **LOW** *Then the pressing brake is* **LOW**

R2:*If the distance is* **FAR** *and speed is* **HIGH** *Then the pressing brake is* **MEDUM**

R3:*If the distance is* **NEAR** *and speed is* **LOW** *Then the pressing brake is* **MEDUM**

R4:*If the distance is* **NEAR** *and speed is* **HIGH** *Then the pressing brake is* **HIGH**

Figure 3-9 Rule Base Example

of Maxima Defuzzifier, Centroid Defuzzifier, Height Defuzzifier and Modified Height Defuzzifier. Unfortunately, there is no systematic procedure for choosing a defuzzification strategy.

The following subsection explains the formal definition of type-1 fuzzy logic controller components.

3.5.1.1 Fuzzifier

Fuzzifier converts crisp inputs to fuzzy sets by evaluating crisp inputs $x = (x_1, x_2, \dots, x_n)$ using the antecedent sections of the rules and assigning each crisp input a degree of membership $\mu_{A_i}(x_i)$in the input fuzzy set. There are two different kinds of fuzzifiers in the literature: singleton fuzzifiers and non-singleton fuzzifiers(Mendel, 2021). The singleton fuzzifier is the most extensively used fuzzifier, due to simplicity and lower processing requirements(Mouzouris and Mendel, 1997). However, this type of fuzzifier may not always be adequate, particularly when there is noise in the data, necessitating a separate strategy to account for the data's uncertainty. The following is the formula for the two types of fuzzifiers(Mouzouris and Mendel, 1997):

a. **Singleton fuzzifier** (Mouzouris and Mendel, 1997): The crisp input $x \in U$ is mapped into a fuzzy set X with support xi , where $\mu X(xi)$ = 1 for $xi = x$ and $\mu X(xi)$ = 0 for

$xi \neq x$. Hence, there is one and only one-point x in the support with a nonzero membership degree.

b. **Non-singleton fuzzifier**(Mouzouris and Mendel, 1997): The crisp point $x \in U$ is mapped into a fuzzy set X with support xi , where μX is maximum at $xi = x$ and is decreasing while moving away from $xi = x$.

Figure 3.10 provides example for the two different types of fuzzifier. Figure 3.10 (a) singleton fuzzifier and Figure 3.10 (b). non-singleton fuzzifier.

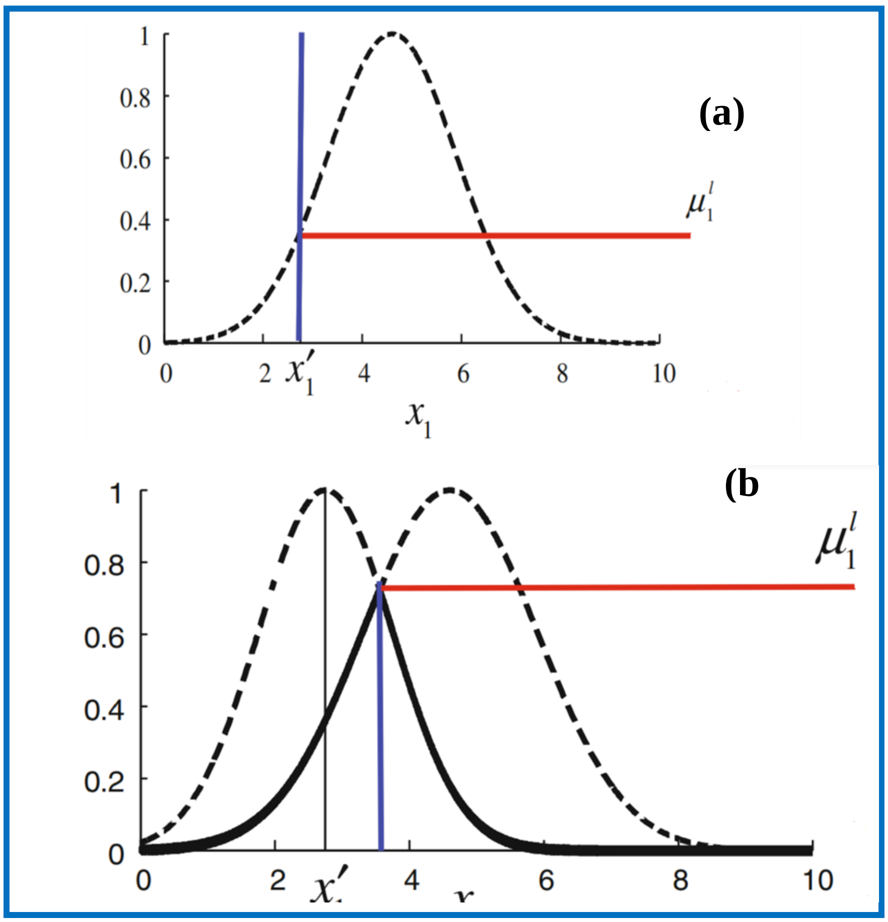


Figure 3-10:example for the two different types of fuzzifier, **(a)** singleton and **(b)** non-singleton (source: (JERRY, 2017)).

3.5.1.2 Rule Base

The rule base consists of the rules that are considered the heart of a FLS (Mendel, 2001)and characterize a fuzzy system's dynamic behavior(Lee, 1990). According to Wang and Mendel, most real-world control issues have two types of information for design, evaluation, and realization: numerical information derived from sensor data, and language information derived from human specialists(Wang and Mendel, 1992).

As a consequence, there are two basic approaches to construct rules in a FLS: utilizing expert linguistic information and numerical data from sampling data. The structure of the rules can be described in IF-THEN statements in both circumstances, with the IF part including the antecedents and the THEN section containing the consequents(JERRY, 2017). According to Mendel the formal for representing the l^{th} fuzzy rule in a FLS with M rules gives as the following (JERRY, 2017):

$$\boldsymbol{R\ l : IF\ x1 is\ F1\ l\ and\ x2\ is\ F2\ l\ and\ \dots\ and\ xp\ is\ Fp\ l\ , THEN\ y\ is\ G\ l\ ,}$$

$$\boldsymbol{l = 1, \dots, M} \qquad \textbf{(3.10)}$$

where xi s are inputs; $Fi\ l$ s are *antecedent* sets (i = 1, …, p); y is the output; and G l s are *consequent* sets.

3.5.1.3 Inference Engine

The fuzzy rules are applied in the component named *inference*, as shown in Figure 3.8**,** according to the fuzzy logic principles (Mendel, 2001). The fuzzy inference engine is in charge of merging fuzzy IF-THEN rules with a mapping from fuzzy input sets to fuzzy output sets. Each rule is viewed as a fuzzy implication(Mendel, 2001), and a fuzzy implication can be formulated in a variety of ways in fuzzy logic(Chuen, 1990). Mamdani type is the most prevalent type of implication utilized in engineering applications of fuzzy logic, as explored by Mendel(Mendel, 2001), and is formally expressed as follows(JERRY, 2017):

$$R^l: F_1^l \times \dots \times F_p^l \rightarrow G^l \ \ l = 1, \dots, M \qquad \textbf{(3.11)}$$

R^l l is defined by the membership function $\mu_{R^l}(x, y)$ which can be written as $\mu_{F_1^l} \times \ldots \times {}_{F_p^l}(x, y)$ where $x = (x_1 \ldots, x_p)^T$ (Mendel, 2001) As a result according to (Mendel, 2001):

$$\mu_{R^l}(x, y) = \mu_{F_1^l \times \ldots \times F_p^l}(x, y)$$

$$= \mu_{F_1^l \times \ldots \times F_p^l}(x) \star \mu_{G^l}(y)$$

$$= \mu_{F_1^l}(x_1) \star \ldots \star \mu_{F_p^l}\left(x_p\right) \star \mu_{G^l}(y) \qquad \textbf{(3.12)}$$

$$= \left[T_{i=1}^{p} \mu_{F_i^l}(x_i)\right] \star \mu_{G^l}(y)$$

where multiple antecedents are connected by t-norms and T is short for a t-norm, which is an operator for fuzzy intersection and denoted $\star$ (Mendel, 2001). It is noted that most engineering applications of fuzzy sets use the minimum or algebraic product t-norm for fuzzy intersection (JERRY, 2017).

Let B^l, be a fuzzy set and $\mu_{B^l}(y)$ be the scalar output fuzzy set for R^l, where the p-dimensional input to each rule R^l is given by the fuzzy set $A_x$whose membership function is $\mu_{A_x}(x) = T_{i=1}^{p} \mu_{X_i}(x_i)$. Then, the interpretation of a fuzzy inference engine as a system, one that maps fuzzy sets into fuzzy sets by means of the sup-star composition, can be further formalized as follows(JERRY, 2017):

$$\mu_{B^l}(y) = sup_{x \in X}\left[\mu_{A_x}(x) \star \mu_{R^l}(x, y)\right] \qquad \textbf{(3.13)}$$

$$= \ sup_{x \in X}\left[T_{i=1}^{p} \mu_{X_i}(x_i) \star \left[T_{i=1}^{p} \mu_{F_i^l}(x_i)\right] \star \mu_{R_l}(x\, , y)\right]$$

$$= sup_{x\in X}\left\{\left[T_{i=1}^{p}\mu_{X_i}(x_i)\star\mu_{F_i^l}(x_i)\right]\star\mu_{G^l}(y)\right\}$$

$$= \mu_{G^l}(y)\star\ sup_{x\in X}\left\{\left[T_{i=1}^{p}\mu_{X_i}(x_i)\star\mu_{F_i^l}(x_i)\right]\right\}$$

Because a t-norm is commutative and $\mu Xi\ (xi) \star \mu Fi\ l(xi)$ is simply a function of xi, each supremum is only over a scalar variable(JERRY, 2017). The final fuzzy set, B, is generated by all M rules and is obtained by combining $B\ l$ and its associated membership function $\mu B\ l(y)$ for all l = 1, …, M.

3.5.1.4 Defuzzifier

The goal of the defuzzifier interface in the fuzzy logic system is to create a non-fuzzy control action that best describes the distribution of possible outcomes for an inferred fuzzy control action. Thus, the defuzzifier simply transforms fuzzy sets into a single crisp output that represents the overall output of the fuzzy control system as shown in Figure 3.8. It is employed in the case when the crisp output from the fuzzy system is required as in the majority of real-world fuzzy logic applications(Chuen, 1990). Various methods of defuzzification have been presented in the literature, with computational simplicity being one of the criteria for selecting a defuzzifier. Maximum, mean-of-maxima, centroid, canter-of-sums, height, modified height, and canter-of-sets are some examples of defuzzification procedures (JERRY, 2017). In this regard, it should be noted that in this study the defuzzifier will not be used because the proposed model is a binary classifier which means the final output from the system is linguistic, either defaulted or not-defaulted.

3.5.2 Type-2 Fuzzy Logic Systems(T2FLS)

Type-2 Fuzzy set originally invented by (Zadeh, 1975) as an extension of type-1 fuzzy set such sets are fuzzy sets whose membership grades themselves are type-1 fuzzy sets; they are very useful in circumstances where it is difficult to determine an exact membership function for a fuzzy set (N. Karnik and M. Mendel, 2001). Unlike type-1 fuzzy logic system type-2 fuzzy logic system use Type-2 fuzzy set which is provide it ability to handle high levels of uncertainties (Bernardo et al., 2013). Figure 3-11 describes the architecture of a type-2 FLS, which shows the interaction of the five components (*fuzzifier*, *rules*, *inference engine*, ***type-reducer***, and *defuzzifier*). The structure of a type-2 FLS is extremely similar to that of a type-1 FLS. Type-2 FLSs, unlike type-1 FLSs as can be seen in Figure 3-8, contains a *type-reducer block*, which is an extension of a type-1 defuzzification operation and represents a mapping of a type-2 fuzzy set into a type-1 fuzzy set.

The processes used to create type-2 fuzzy output sets in a T2FLS are similar to those used to create type-1 fuzzy output sets, with the exception that one of the antecedents or subsequent fuzzy sets is of type-2. The following subsections provide a brief about T2FLS's components.

3.5.2.1 Fuzzifier

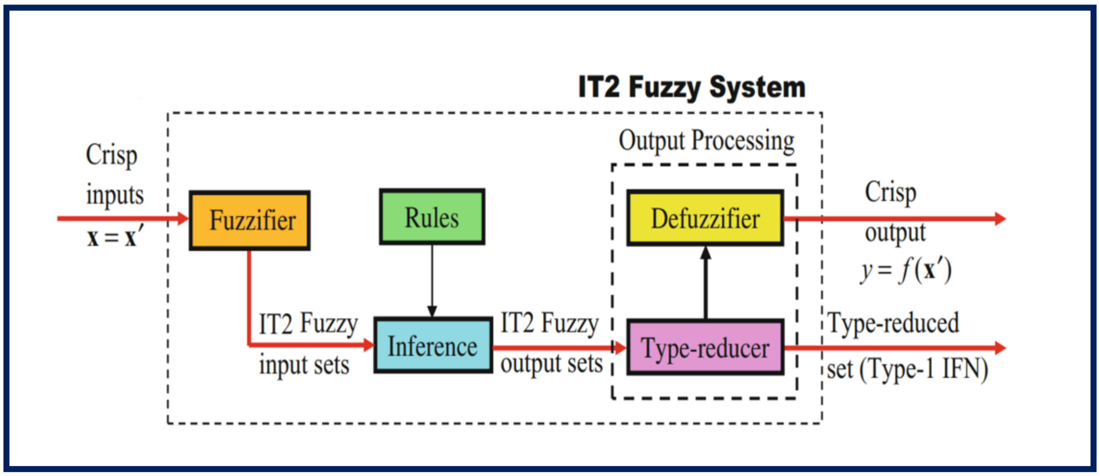
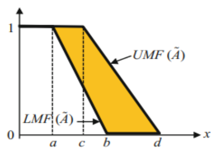
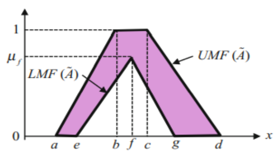
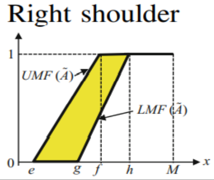


Figure 3-11:Type-2 fuzzy logic system structure (source: (JERRY, 2017))

Fuzzifier translates crisp inputs into type-2 fuzzy sets in the same way that type-1 FLS does by evaluating the crisp inputs x = ($x1$, $x2$, ..., xp) based on the antecedent's part of the rules and assigning each crisp input to its type-2 fuzzy set $A(x)$with its membership grade in each type-2 fuzzy set. Table 3.1 provides a formula for upper and lower MFs for piecewise liner left shoulder, interior and right shoulder FOUs.

There are two categories of fuzzification procedures (singleton and non-singleton), which were discussed in Section 3.4.1.1 and will not be discussed again in this context.

Table 3-1 provides formulas for upper and lowers MFs (source: (JERRY, 2017))

FOU	$\overline{\mu}_{\tilde{A}}(x)$	$\underline{\mu}_{\tilde{A}}(x)$
Left shoulder 	$\overline{\mu}_{\tilde{A}}(x) = \begin{cases} 1 & 0 \le x \le c \\ \frac{d-x}{d-c} & c < x \le d \\ 0 & x > d \end{cases}$	$\underline{\mu}_{\tilde{A}}(x) = \begin{cases} 1 & 0 \le x \le a \\ \frac{b-x}{b-a} & a < x \le b \\ 0 & x > b \end{cases}$
Interior 	$\overline{\mu}_{\tilde{A}}(x) = \begin{cases} 0 & x \le a \\ \frac{x-a}{b-a} & a < x \le b \\ 1 & b < x \le c \\ \frac{d-x}{d-c} & c < x \le d \\ 0 & x > d \end{cases}$	$\underline{\mu}_{\tilde{A}}(x) = \begin{cases} 0 & x \le e \\ \frac{x-e}{f-e}\mu_f & e < x \le f \\ \frac{g-x}{g-f}\mu_f & f < x \le g \\ 0 & x > g \end{cases}$
Right shoulder 	$\overline{\mu}_{\tilde{A}}(x) = \begin{cases} 0 & x \le e \\ \frac{x-e}{f-e} & e < x \le f \\ 1 & f < x \le M \end{cases}$	$\underline{\mu}_{\tilde{A}}(x) = \begin{cases} 0 & x \le g \\ \frac{x-g}{h-g} & g < x \le h \\ 1 & h < x \le M \end{cases}$

3.5.2.2 Rule Base

The difference between type-1 and type-2 FLS is due to the nature of the membership functions, which is unnecessary while creating rules; thus, the structure of the rules in type-2 FLS stays unchanged, with the exception that some or all of the sets involved are now of type-2(Karnik et al., 1999). However, in a type-2 FLS with M rules, the formal expression of the l^{th} fuzzy rule can be reformulated as follows(JERRY, 2017):

$$R^l: IF\ x_1 is\ \widetilde{F_1^l}\ and\ x_2 is\ \widetilde{F_2^l} and\ \ldots and\ x_p is\ \widetilde{F_p^l}\ THEN\ y\ is\ \widetilde{G^l}\quad l = 1, \ldots, M \tag{3.14}$$

Where x_is are inputs; $\widetilde{F_i^l}$s are antecedent sets (i = 1, …, p); y is the output; and $\widetilde{G^l}$s are consequent sets.

3.5.2.3 Inference Engine

Karnik (Karnik et al., 1999) states that the inference process in a type-2 FLS is same as that of a type-1 FLS. The inference engine formally combines rules and provides a mapping from input type-2 fuzzy sets to output type-2 fuzzy sets(Karnik et al., 1999). In detail, the t-norm (equivalent to set intersection) is used to connect many antecedents in rules. The sup-star composition is used to combine the membership grades in the input and output sets. Multiple rules can be concatenated using the t-conorm operation (which corresponds to set union) or weighted summation during defuzzification process (Karnik et al., 1999).

3.5.2.4 Type Reduction

The type reduction produces type-1 fuzzy set outputs, which are subsequently defuzzified to produce crisp overall fuzzy logic system outputs. In this situation, extended types of type-1 defuzzification procedures are used to produce a type-1 fuzzy set, also known as a type-reduced set(Karnik et al., 1999). Centroid, center-of-sums, height, modified height, and center-of-sets type reduction are among the methods. As Mendel (JERRY, 2017)points out, center-of-sets type reduction has a fair computing complexity that falls somewhere between the computationally expensive centroid type-reduction and the basic and modified height type-reductions, which have issues when just one rule fires. Furthermore, if the rulebase is small, center-of-sets type reduction enables for real-time operation(Hagras, 2004).

Type reduction for arbitrary type-2 fuzzy sets can be computationally expensive. Because the computation of meet and join for interval type-2 sets is comparably easier, the formalization for type-reduced sets using the center-of-sets type reduction for IT2 FLS is given below (JERRY, 2017):

$$Y_{cos}(x) = [y_l, y_r] = \int_{y^1\in[y_l^1,y_r^1]} \cdots \int_{y^M\in[y_l^M,y_r^M]} \int_{f^1\in[\underline{f}^1,\overline{f}^1]} \cdots \int_{f^M\in[\underline{f}^M,\overline{f}^M]} 1 \Big/ \frac{\sum_{i=1}^{M} f^i\, y^i}{\sum_{i=1}^{M} f^i} \qquad \textbf{(3.15)}$$

where $Y_{cos}(x)$is an interval set determined by its left most point y_l and its right most point y_r , i = 1 $\cdots$ M and M is the number of rules. y^i corresponds to the centroid of the type-

2 interval consequent set of the i th rule, and is a pre-computed type-1 interval fuzzy set determined by its left most point y_l^i and its right most point y_r^i (JERRY, 2017). f^i denotes the firing strength (degree of firing) of the ith rule which is an interval type-1 set determined by its left most point $\underline{f}^i$ and right most point $\overline{f}^i$ (Hagras, 2004).

3.5.2.5 Defuzzifier

The defuzzification can be done using the average of y_l^k and y_r^k to produce the crisp output for each output *k* based on the Y_{cos} returned from the type-reduction as follow(Hagras, 2004):

$$Y_k(x) = \frac{y_l^k + y_r^k}{2} \quad \textbf{(3.16)}$$

3.5.3 Type-2 Fuzzy Logic Systems in Real World Applications:

In the recent year's type-2 FLSs have grown in popularity due to their ability to handle high levels of uncertainties. Type-2 FLSs was deployed widely in many deferent applications and it provides promise performer. The following subsection provide an overview for applying of type-2 fuzzy logic in some world application in specific domains.

3.5.3.1 Robotic Control:

The author in (Hagras, 2004) presents reactive control architecture for autonomous mobile robots. He implements the basic navigation behaviors and the coordination between these behaviors using a type-2 FLC to produce a type-2 hierarchical FLC. And his study implements this architecture in unstructured environments. And the author show that the proposed system has ability to handle and control uncertainties facing mobile robots in such challenging environments with very good performance that outperformed the type-1-based control.

Researchers in (Biglarbegian et al., 2011) design interval type-2 Takagi-Sugeno-Kang fuzzy logic controllers (IT2 TSK FLCs) for modular and reconfigurable robots (MRRs) for tracking purposes which can effectively in real-time application. The study show that the proposed controller can outperform some well-known linear and nonlinear controllers for different configurations.

Type-2 fuzzy switching control system for biped robots was presented by(Liu et al., 2007) based on non-liner model to guarantee the gait stability and to achieve a robust control performance with a simplified control scheme.

The authors in (Juang and Hsu, 2009) develop reinforcement ant optimized fuzzy controller (RAOFC) to control mobile robot wall-follower. The researchers used aligned interval type-2 fuzzy clustering (AIT2FC) to generate the rule base automatically. And the experiment result shows the effectiveness and efficiency of their proposed methods.

In (Kumbasar and Hagras, 2014) present an application of Big Bang–Big Crunch optimization (BB–BC) algorithm to Interval Type-2 Fuzzy PID (IT2-FPID) controllers in a cascade control structure to solve the path tracking control problem of mobile robots. The research experiments used the PIONEER 3-DX mobile robot as platform to evaluate the proposed control systems. The experiments results found that the IT2-FPID controller was superior in the control performance in the presence of disturbances and uncertainties in comparison to PID, T1-PID and T1-FPID self-tuning controllers.

A type-2 fuzzy logic controller (FLC) is based on a sliding mode control strategy is proposed in(Chaoui and Gueaieb, 2008) for robot manipulators with joint elasticity and structured and unstructured dynamical uncertainties. The proposed model takes into account a trade-off between the link position and the actuators' internal stability, and it is based on sliding mode control. The experiments shown that the proposed controller was superior to T1 FLC in simulations.

The authors in (Lu et al., 2017) designed an optimization of interval type-2 fuzzy logic controller (IT2 FLC) for Delta robot trajectory control. The interval type-2 fuzzy logic controller was generated from type-1 fuzzy logic controller using three deferent types of methods to fuzziness the T1 fuzzy membership functions. The proposed model is compared against its type-1 counterpart in the presence of external and internal uncertainties. The simulation results illustrate that the optimized IT2 FLC can deliver better trajectory tracking performance.

In (Hassan and Kothapalli, 2012) an Interval Type-2 fuzzy position control of electro-hydraulic actuated robotic excavator was proposed. The study show that Type-2 Fuzzy Logic Controller (IT2FLC) has the ability to control the position of each of the three axes with minimum actuator position errors.

3.5.3.2 Power Management and Electrical Control

In (Khosravi et al., 2011), a type-2 FLS was employed for electricity short term load forecasting. Accurate Short Term Load Forecasting (STLF) is essential for a variety of decision-making processes in energy distribution. However, forecasting accuracy may drop due to the presence of uncertainty in the operation of energy systems or unexpected behavior of exogenous variables. Experiments conducted with real data sets showed that type-2 FLS models appropriately estimate future load demands to an acceptable degree of accuracy. Furthermore, they demonstrated an encouraging degree of accuracy superior to feed-forward neural networks used in this study.

In (Tripathy and Mishra, 2011), a type-2 FLS has been applied to Thyristor Controlled Series Capacitor (TCSC) for improving power system stability. The authors report that the type-2 FLS along with the Power System Stabilizer (PSS) in the system damp out the speed and power oscillations following different critical faults satisfactorily. It was shown that the damping performance of type-2 FLS is considerably better compared to its fixed gain Bacteria Swarm based tuned PSS and TCSC counterparts. Moreover, the performance of the type-2 FLS did not deteriorate even under uncertainty in the input signal to the controller. This shows the power of type-2 FLS in providing adequate performance even in conditions of increasing uncertainty (in the inputs).

In (Flores et al., 2009) type-2 FLSs were employed to support the calculation of the risk of power transformer failure. Specifically, the condition of the paper insulation (i.e., its degree of polymerization) and the probability of the given transformer withstanding the short circuit current flow were employed as inputs to a type-2 FLS to generate the risk level. The resulting system was deployed as a quantitative metric of equipment reliability in the electric power system of Honduras.

3.5.3.3 Applications to Real-World Automatic Control

In (Méndez et al., 2010), a type-2 FLS was applied to the modeling and control of coiling entry temperature of a Hot Strip Mill (HSM) in the steel industry. In an HSM, there is a major need to satisfy quality requirements such as the steel strip thickness; finishing temperature and coiler temperature (the latter determines the final strip mechanical properties). The most critical section of the coil is the head-end section. This is due to the uncertainties involved at the head-end of the incoming steel bar, and the varying conditions from bar to bar.

In (Zaheer and Kim, 2011) a type-2 FLS was applied to airplane altitude control for the propulsion based RC airplane, where the throttle is used to regulate the speed of the airplane by varying the rotational speed of the propeller. The elevator is used to control the airplane's ascent and descent, the aileron is used for airplane's lateral stabilization and midair turning, and the rudder is used for the on-ground taxiing of the airplane. The work compared type-1 and type-2 FLS in airplane control. It was found that under high uncertainty levels the type-2 FLS outperformed the type-1 FLS. Specifically, the type-1 FLS showed oscillatory behavior around the reference altitude set-points.

In (Ren et al., 2010), the authors present the use of a type-2 FLS for the estimation of the dynamic cutting forces. Ren et al. note: "The type-2 fuzzy estimation not only filters the noise and estimates the instantaneous cutting force in micro milling using observations acquired by sensors during cutting experiments, but also assesses the uncertainties associated with the prediction caused by the manufacturing errors and signal processing." Moreover, the interval output of the type-2 fuzzy system gives very useful information to machine tool controllers in order to maximize material removal while controlling tool wear or tool failure to maintain part quality specifications.

3.5.3.4 Applications in the Networks Domain

In (Jammeh et al., 2009) a type-2 FLS was presented for congestion control of video transmission across IP networks where the type-2 FLS provided a superior delivered video quality compared to existing traditional controllers and type-1 FLSs.

In (Shu et al., 2008) a type-2 FLS was applied to the analysis of the lifetime of a wireless sensor network. A type-2 FLS (referred to as FLLE2) was compared against a type-1 FLS (referred to as FLLE1) as well as against a non-fuzzy system (referred to as the first node die method). The results of the comparison show that the type-2 FLS based system (FLLE2) outperforms the non-fuzzy “first-node-die” method and the type-1 FLS based system FLLE1 in terms of better estimation [lower Mean Square Error (MSE)].

The author in (Alhassan and Hagras, n.d.) presents a methodology to improve the Weighted Random Early Detection (WRED) by combining it with type-2 fuzzy logic. The study shows that the proposed model outperformed type-1 counterpart interim of packet loss rate which has been reduced at the rate of 32%.

In (Linda et al., 2011), a previously developed type-1 fuzzy logic based anomaly detection network security cyber sensor was extended by relying on type-2 fuzzy logic. The resulting system was experimentally tested incorporating a TOFINO network security cyber sensor based on a network test bed schematically. The paper showed how the increased amount of information in terms of the uncertainty flow in the type-2 FLS (though the width of the type-2 reduced fuzzy output set) is highly relevant in informing the judgment about a specific state of cyber security and is an advantage not available when employing type-1 FLSs.

3.5.3.5 Applications in Heart Failure Tele-Treatment

In (Lee et al., 2010), researchers from Changhua Christian Hospital, National University of Tainan, Taiwan and University of Essex, UK, developed a type-2 fuzzy FLS ontology applied to personal diabetic diet recommendation.

In (Ceylan et al., 2010)a type-2 FLS web-based tele-cardiology system had been proposed for diagnosis consultation, and treatment of heart failures.

The author in (Abdelgader et al., 2013) present type-2 fuzzy-logic-based system to learn, from data a prediction model for Basal Metabolic Rate diabetes in Sudan. The proposed model can help patients to control the disease and achieve a healthy lifestyle.

3.6 Fuzzy C-mean Clustering (FCM)

The Fuzzy C-Means Clustering (FCM) approach was used to obtain the T1FLS membership function from data. FCM can cluster data into a predetermined number of overlapping groups/clusters, with each group being treated as a membership function or a fuzzy set. Fuzzy C-Means clustering algorithm (FCM) was originally invented by Dunn (Bezdek, 1984) , and generalized by Bezdek [107]. FCM as its name indicates, is a clustering method that allows each data point to belong to multiple clusters with varying degrees of membership. FCM is based on the minimization of the following objective function (Bezdek, 1984):

$$j_m = \sum_{i=1}^{D} \sum_{j=1}^{N} \mu_{ij}^{m} \left\| x_i - c_j \right\|^2 \tag{3.17}$$

Where:

D is the number of data points.

N is the number of clusters.

m is fuzzy partition matrix exponent for controlling the degree of fuzzy overlap, with m>1. Fuzzy overlap - which refers to how fuzzy the boundaries between clusters are - that is the number of data points that have significant membership in more than one cluster. The x_i is the i^{th}data point and $c_j$is the center of the j^{th}cluster. $\mu_{ij}$is the degree of membership of x_i in the j^{th} cluster. For a given data point, x_i, the sum of the membership values for all clusters is one.

The FCM operates as follows:

1. Randomly initialize the cluster membership values, μ_{ij}.

2. Calculate the cluster centers (Ghosh and Dubey, 2013):

$$c_j = \frac{\sum_{i=1}^{D} \mu_{ij}^{m} x_i}{\sum_{i=1}^{D} \mu_{ij}^{m}} \qquad \textbf{(3.18)}$$

3. Update μ_{ij} according to the following(Starkey et al., 2016):

$$\mu_{ij} = \frac{1}{\sum_{k=1}^{N} \left(\frac{\|x_i - c_j\|}{\|x_i - c_k\|} \right)^{\frac{2}{m-1}}} \qquad \textbf{(3.19)}$$

4. Calculate the objective function, J_m.

5. Repeat steps 2–4 until J_m improves by less than a specified minimum threshold or until after a specified maximum number of iterations.

3.7 Summary

In this chapter a theoretical background of fuzzy logic has been explored, beginning by providing a historical background of fuzzy logic and then traversing to different types of fuzzy logic sets (T1FS, IT2FS, and GT2FS) and the equivalent fuzzy logic systems which arise from different types of fuzzy sets which are type-1 and type-2 fuzzy logic sets and system. In this context, it's important to bear in mind that the fuzzy logic sets utilized throughout the rest of this thesis are T1, and IT2 fuzzy logic sets, which are will be used to build the proposed models.

From investigated literature, it has been clear that the type-1 fuzzy logic system's greatest shortcoming is that it is incapable of robustly representing the high levels of complex uncertainty found in real-world applications. The interval type-2 fuzzy logic system was proposed and frequently used to overcome this problem. When compared to type-1 fuzzy sets, the primary advanced extension of IT2FLS is that it uses universal type-2 fuzzy sets, which have proven to be very robust in handling uncertainty. Finally, the FCM algorithm has been presented on the way through to the end of the chapter, which will be used to cluster the proposed system's numerical parameters.

Chapter Four: Introduction to Evolutionary Optimization

4.1 Introduction

Real-world applications have become significantly more sophisticated in recent years. Robotics, operations research, decision-making, bioinformatics, machine learning, data mining, and a variety of other problems are extremely complicated and difficult to solve(Vikhar, 2017). Evolutionary Computations is a proposed method for dealing with such complicated challenges that are inspired by Darwinian natural evolution. Different algorithms, referred to as Evolutionary Algorithms (EAs). Simulated evolution is used by EA to investigate solutions to complicated real-world situations. The evolutionary method is best suited to applications when heuristic solutions are not possible and may result in insufficient results. EA is generating a lot of attention, especially because of the way it's being used to solve real-world problems. Evolutionary Algorithms have grown in popularity during the last two decades as a method for searching, optimizing, and solving complicated problems (Whitley, 2001).

The next subsections will provide a brief background on the most often used evolution algorithms for optimizing which are the genetic algorithm (GA), and Big Bang – Big Crunch algorithm (BB-BC).

4.2 Genetic Algorithms:

The Genetic algorithm is a population genetics-based adaptive heuristic search method. John Holland invented genetic algorithms in the early 1970s.(Kumar et al., 2010). The genetic algorithm is a probabilistic search method based on natural selection and genetics mechanics. A genetic algorithm is a computational search strategy for finding perfect or approximate answers to optimization and search issues. Genetic algorithms are classified as global search heuristics(Kumar et al., 2010). Genetic algorithms are a type of evolutionary algorithm that use processes like inheritance, mutation, selection, and crossover that are inspired by evolutionary biology. In a variety of fields, genetic algorithms have been utilized to identify optimal solutions to challenging issues such as biology, engineering, computer science, and social science(Kumar et al., 2010). In contrast to local search approaches, genetic algorithms are built on a series of independent computations governed by a probabilistic strategy. A solution for a problem under discussion is referred to as an individual in classical terminology. A population is a group of individuals that are being considered. Each individual has a single chromosomal string that encodes their data features. On the other hand, a chromosome is a collection of alleles that each represent one quantum of information, such as a bit, a digit, or a letter (Lemarchand et al., 1992). Formally, the genetic algorithm cycle contains four operations: initialization, selection, reproduction, and termination. The following is an explanation of what they are.

A. **Initialization**: Initially, many numbers of individual solutions are created at random to establish an initial population. The size of the population varies depending on the situation, but it usually encompasses several potential answers. The population is traditionally produced at random, spanning the whole range of possible solutions (the search space) (Kumar et al., 2010).
B. **Selection**: A part of the existing population is chosen to breed a new generation throughout each succeeding generation. Individual solutions are chosen based on their fitness, with fitter solutions (as assessed by a fitness function) being more likely to be chosen. Certain methods of selection rate the fitness of each solution and select the best solutions first(Kumar et al., 2010).
C. **Reproduction**: The following stage is to create a second-generation population of solutions using genetic operators such as crossover (also known as recombination) and/or mutation. A pair of "parent" solutions are chosen for breeding from the pool of previously selected solutions for each new solution to be generated. By employing the preceding crossover and mutation methods to construct a "child" solution, a new solution is created that generally has many of the traits of its "parents." For each new

child, new parents are chosen, and the process is repeated until a new population of solutions of the necessary size is formed. These activities eventually result in a chromosomal population that differs from the previous generation. Because only the best individuals from the first generation are picked for breeding, along with a small number of less fit solutions, the population's average fitness will have grown (Katoch et al., 2021).

D. **Termination:** Termination: This generational process is repeated until a termination condition has been reached such as a fixed number of generations reached, a solution is found that satisfies minimum criteria, or the highest-ranking solution's fitness is reaching (Kumar et al., 2010).

Figure 4.1 provides the procedure for a simple GA.

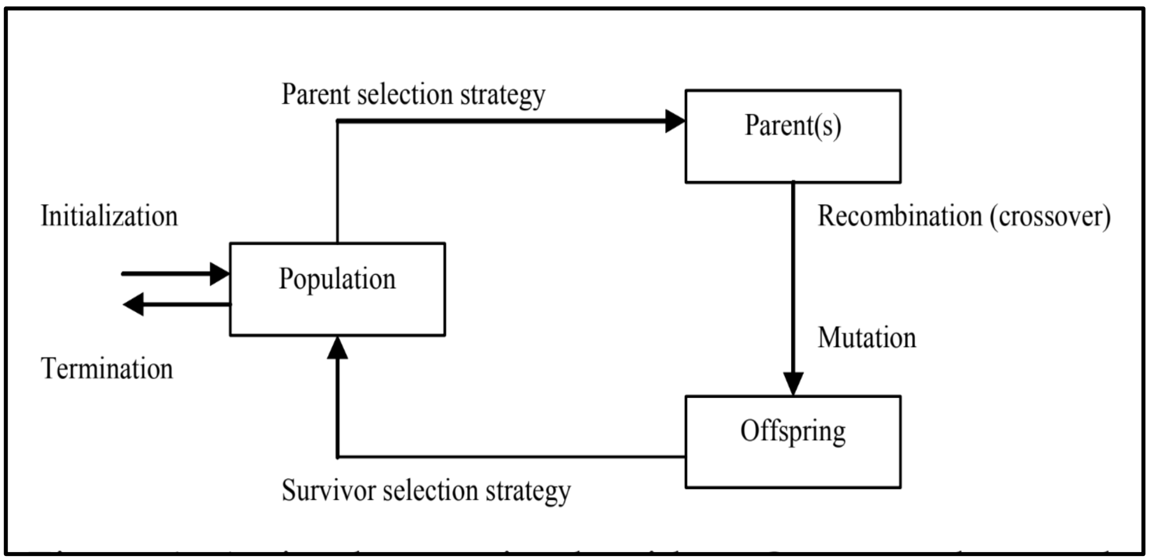


Figure 4-1:procedure for simple GA (source (Brintrup et al., 2006))

4.2.1 Application of GA:

Genetic algorithms have been used in a variety of hard optimization problems below are some examples of genetic algorithm applications.

A. **GA in Image processing**: Reprocessing, segmentation, object detection, denoising, and recognition are the most common image processing tasks. Image segmentation is a crucial step in resolving image processing issues. Decomposing/partitioning a picture takes a long time to compute. GA is employed to overcome this problem because of its superior search power(Chouhan et al., 2018) (Khan et al., 2019). Enhancement is a method of enhancing an image's quality and contrast. To analyze the supplied image, higher image quality is necessary. GAs have been employed to enhance images and increase natural contrast (Kaur et al., 2018).

B. GA in Medical Application GA was applied in themedical field extensively. For example, the author in (Reddy et al., 2020) applied a hybrid genetic algorithm with afuzzy logic classifier to propose a model for heart disease diagnosis. As known that the medical information regarded to the patients is very sensitive introduced a medical record managing and securing blockchain-based system supported by a Genetic Algorithm and Discrete Wavelet Transform to protect the medical records.

4.3 Overview of BB- BC

The Big Bang–Big Crunch (BB-BC) is an optimization method originally invented by Erol and Eksin (Erol and Eksin, 2006) this method was inspired from the beginning of the universe in astrophysics, namely the Big Bang – Big Crunch Theory. This Algorithm is considered to be a global search method and it works as a transformation of a convergent solution to a disordered state and then back to a single tentative solution point(Kumbasar and Hagras, 2014). The researchers in (Erol and Eksin, 2006) compare their algorithm with the Compact Genetic Algorithm(C-GA) on benchmark problems. The study confirmed that the BB-BC provided a superior performance than (C-GA) in terms of computational time and convergence speed. Thus, this superior performance provides good chance for BB-BC to be preferable in numerous engineering applications because in such applications the computational time and convergence time are important factors(Kumbasar and Hagras, 2014). The BB-BC optimization method as presented in (Erol and Eksin, 2006) involves two key stages, namely the ‘‘Big Bang’’ and ‘‘Big Crunch’’ phases. The first phase is the ‘‘Big Bang’’ phase where candidate solutions are arbitrarily scattered over the search space. The initial ‘‘Big Bang’’ population is arbitrarily produced over the whole search space as in other evolutionary search methods. The next phase is the ‘‘Big Crunch’’ phase in which a shrinkage process calculates the center of mass for the population. All subsequent ‘‘Big Bang’’ phases are arbitrarily distributed about the center of mass or the best-fit individual in a similar manner(Erol and Eksin, 2006). After the ‘‘Big Bang’’ phase, a shrinkage process is applied such as the ‘‘Big Crunch’’ phase to form a center or a representative point for further ‘‘Big Bang’’ operations. In this phase, the shrinkage operator takes the current positions of each candidate solution in the population and its related cost function value and calculates a center of mass. The center of mass can be calculated as:

$$\boldsymbol{x_c} = \frac{\sum_{i=1}^{N} \frac{x^i}{f^i}}{\sum_{i=1}^{N} \frac{1}{f^i}} \qquad \textbf{(4.1)}$$

Where $x_c$the position of the center of mass, x^i is the position of the candidate within a p- dimensional search space, f^i is the cost function value of the i^{th} candidate, p is the number of parameters to be optimized and N is the population size.

Instead of the center of mas best-fit individual can also be chosen as the starting point in the ‘‘Big Bang’’ phase. The new generation for the next iteration ‘‘Big Bang’’ phase is normally distributed around x_c. The new candidates around the center of mass are calculated by subtracting or adding a normal random number whose value decreases as the iterations elapse. This can be formalized as:(Kumbasar and Hagras, 2014).

$$x^{new} = x_c + \frac{yp(x_{max} - x_{min})}{k} \qquad \textbf{(4.2)}$$

Where γ is a random number, ρ is a parameter limiting search space, x_{min} and x_{max} are lower and upper limits, and k is the iteration step. The following steps demonstrates the BB-BC algorithm (Erol and Eksin, 2006):

Step 1: (Big Bang Phase): In this step, an initial generation of N candidates is generated like other evolutionary search algorithms.

Step 2: After generating the initial generation computes the cost function values of all the candidate solutions.

Step 3: (Big Crunch Phase): this phase can be considered as a convergence operator and in this step, you can compute either the best-fit individual or the center of mass. The center of mass can be computed using equation (4.1).

Step 4: This step generates new candidates around the new point calculated in Step 3 and this is achieved by adding or subtracting a random number whose value decreases as the iterations elapse as in equation (4.2).

Step 5: Continuously return to step 2 until stopping criteria have been met.

Note that, in this study, we will use the maximum number of iterations as the stopping criteria. More detailed information about the BB–BC optimization method can be found in (Erol and Eksin, 2006).

4.3.1 Some application of BB-BC Optimization algorithm:

Due to its simplicity of deployment and superior performance than other evolutionary optimization algorithms in term of computational time and convergence speed; the BB-BC optimization method is widely deployed in real world application. For example, Authors in (Ahmadi and Abdi, 2016), present a method for optimal sizing of a stand-alone hybrid power system including photovoltaic panel, wind turbine and battery bank using Hybrid Big Bang–Big Crunch (HBB–BC) algorithm. And this study applied to a village in Qazvin, Iran. And this study confirm that HBB–BC algorithm with high accuracy can find the optimal solution and it has the best performance in comparison with Particle Swarm Optimization (PSO) and Discrete Harmony Search (DHS) algorithms.

In (Tang et al., 2010), a BB–BC algorithm is implemented for a multi-modal optimization problem with high dimension and its performance was compared with the (PSO) and Genetic Algorithms (GAs). The offered results of this study display that the BB–BC provide superior results than the PSO and GA.

The effectiveness of BB-BC evolutionary algorithm has been also verified when the optimization problem must be solved in comparatively small sampling times such as in inverse fuzzy model control and fuzzy model adaptation(Kumbasar et al., 2012, 2011) . Furthermore, in the optimization of highly nonlinear engineering problems such as in controller design(Iplikci, 2010), rescheduling power systems (Kucuktezcan and Genc, 2012)and size decrease of space trusses (Kaveh and Talatahari, 2009), the BB–BC algorithm has been preferred because of its high convergence speed.

In (Camp and Huq, 2013) the BB-BC used to the design of reinforced concrete frames in order to reduce the total cost or the CO2 emissions related with construction of reinforced concrete frames subjected based on the specifications and guidelines recommended by the American Concrete Institute (ACI).The researchers compared their study with GA and their results show that the BB-BC algorithm generated designs that reduced the cost and the CO2 emissions of construction.

The authors in (Jaradat and Ayob, 2010) applied the BB-BC optimization algorithm to solve post-enrolment course timetabling problem and their study confirmed that BB-BC can provide good solutions comparable with some of the approaches used in the literature.

4.4 Summary:

In this chapter, a brief introduction about evolution optimization was provided, and the two extensively used optimization algorithms in the literature (Genetic algorithm, and Big Bang- Big Crunch algorithm) have been discussed. And some examples of the application of the two algorithms in the real world have been introduced. The surveyed literature concluded that the leading advantage of BB–BC over genetic algorithms is the high convergence speed and the low computation time. Thereby, the BB-BC algorithm will be used to optimize our proposed model.

SECTION THREE PROPOSED METHODOLOGIES

Chapter 5:

The proposed Type-2 Fuzzy Logic model to Predict Default in Sudanese Financial Sector.

Chapter 6:

A proposed bb-bc optimized type-2 fuzzy logic model to predict default in the Sudanese banking sector

Chapter 7:

conclusions and future work

Chapter Five the Proposed Type-2 Fuzzy Logic Model to Predict Default in Sudanese Financial Sector

5.1 Introduction

The motivations for developing a prediction paradigm in the financial sector, particularly in our country Sudan, as discussed in section 1.2, prompted us to present a model that incorporates a solution to the problem of loan default in the Sudanese banking industry.

This chapter demonstrates the detailed description of our proposed type-2 fuzzy logic-based system for predicting **default** risk in the Sudanese banking sector(T2F-DRP).

The proposed system, built on a well-trained fuzzy logic-based system, aims to analyze real financial costumers data obtained from Albalad Bank in Sudan to forecast future default cases that can serve the two sides of the financial industry stakeholders.

The proposed model is an implementation of a system this system takes the customer's information as an input and provides a classification of this customer (default/ not default). Figure 5.1 shows the structure for the proposed (T2F-DRP). The proposed model is divided into two phases: the modeling and the prediction phases. The details of these phases will be discussed in the following subsections.

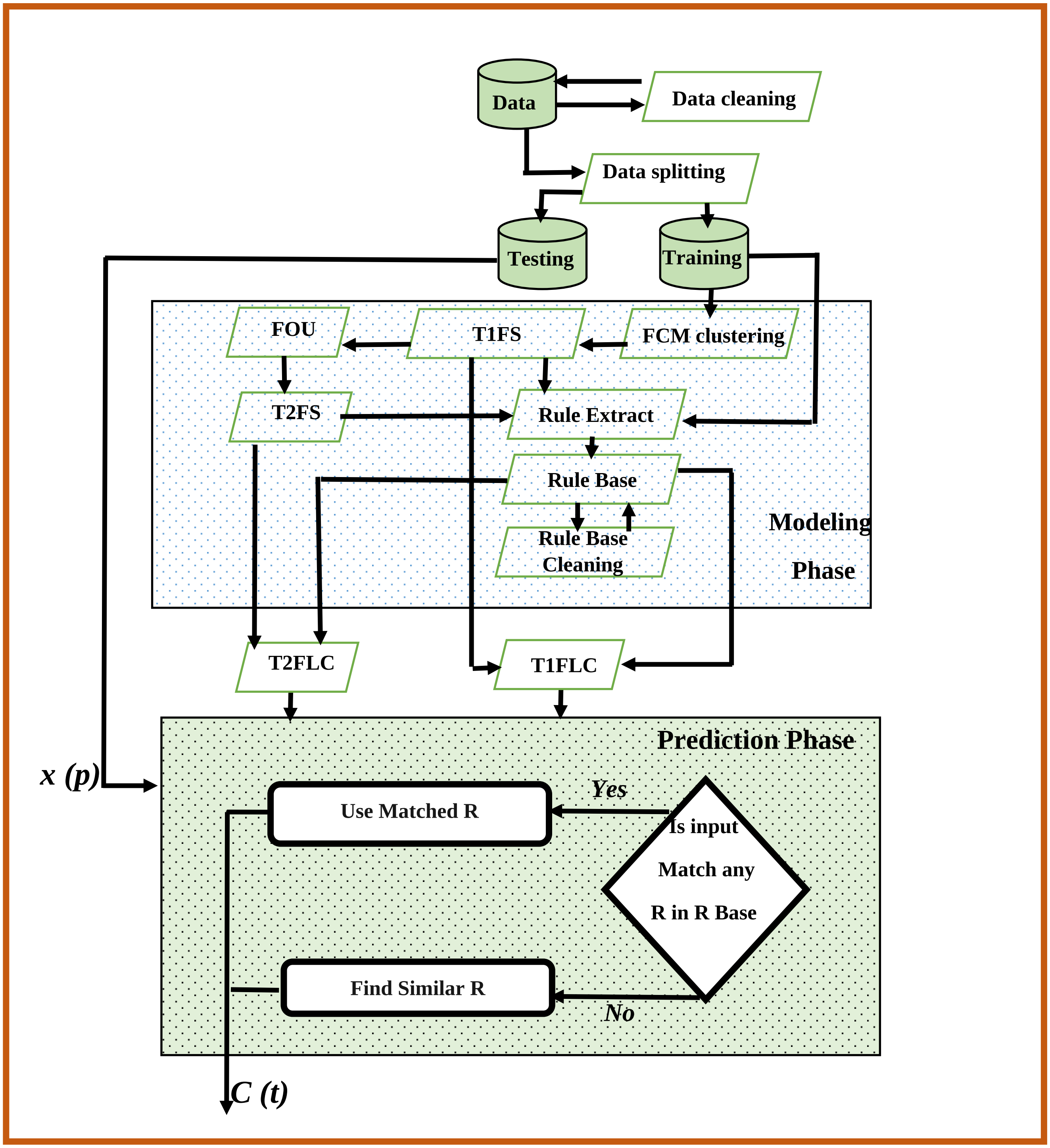


Figure 5-1 the structure for the proposed (T2F-DRP)

5.2 Data description and cleaning:

The data set which was used to build and test the proposed model is real data collected from AlBalad Bank in Sudan. The researcher got authorization from the administration of the bank to use the data as mentioned in **section 1.6**. Because the customer financial data is confidential, all personal information fields were omitted. The data set is characterized as follows:

a. Contains records dating back to the period between 2007 and 2017.

b. Contains 101,257 records.

c. Contains 1,120 records categorized as defaulted.

d. Contains 100,137 categorized as non- defaulted

e. Collected from 23 bank branches distributed across Sudan.

Figure 5.2 shows a snapshot of the original dataset obtained from Albalad Bank

125	01/01/58	Male	Married									19968	TerminatedAccordingTheContract
126	05/09/58	Male	Married	8	5500	1	Academicl	10000	SD	Khartoum	الخرطوم	2991500	TerminatedAccordingTheContract
127	05/09/58	Male	Married	8	5500	1	Academicl	10000	SD	Khartoum	الخرطوم	188750	TerminatedAccordingTheContract
128	05/09/58	Male	Married	8	5500	1	Academicl	10000	SD	Khartoum	الخرطوم	130550	TerminatedAccordingTheContract
129	05/09/58	Male	Married	8	5500	1	Academicl	10000	SD	Khartoum	الخرطوم	43084	TerminatedInAdvanceCorrectlyBecauseOfsubjectsWish
130	05/09/58	Male	Married	8	5500	1	Academicl	10000	SD	Khartoum	الخرطوم	178500	TerminatedAccordingTheContract
131	05/09/58	Male	Married	8	5500	1	Academicl	10000	SD	Khartoum	الخرطوم	218225	TerminatedAccordingTheContract
132	05/09/58	Male	Married	8	5500	1	Academicl	10000	SD	Khartoum	الخرطوم	93776.7	TerminatedAccordingTheContract
133	05/09/58	Male	Married	8	5500	1	Academicl	10000	SD	Khartoum	الخرطوم	986971	TerminatedAccordingTheContract
134	05/09/58	Male	Married	8	5500	1	Academicl	10000	SD	Khartoum	الخرطوم	92860	TerminatedAccordingTheContract
135	05/09/58	Male	Married	8	5500	1	Academicl	10000	SD	Khartoum	الخرطوم	2991500	TerminatedAccordingTheContract
136	05/09/58	Male	Married	8	5500	1	Academicl	10000	SD	Khartoum	الخرطوم	2991500	TerminatedAccordingTheContract
137	05/09/58	Male	Married	8	5500	1	Academicl	10000	SD	Khartoum	الخرطوم	46500	TerminatedAccordingTheContract
138	05/09/58	Male	Married	8	5500	1	Academicl	10000	SD	Khartoum	الخرطوم	11289.6	TerminatedAccordingTheContract
139	05/09/58	Male	Married	8	5500	1	Academicl	10000	SD	Khartoum	الخرطوم	70000	TerminatedAccordingTheContract
140	05/09/58	Male	Married	8	5500	1	Academicl	10000	SD	Khartoum	الخرطوم	45900	TerminatedAccordingTheContract
141	05/09/58	Male	Married	8	5500	1	Academicl	10000	SD	Khartoum	الخرطوم	140325	TerminatedAccordingTheContract
142	05/09/58	Male	Married	8	5500	1	Academicl	10000	SD	Khartoum	الخرطوم	5623.5	TerminatedAccordingTheContract
143	05/09/58	Male	Married	8	5500	1	Academicl	10000	SD	Khartoum	الخرطوم	19800	TerminatedAccordingTheContract
144	05/09/58	Male	Married	8	5500	1	Academicl	10000	SD	Khartoum	الخرطوم	24000	TerminatedAccordingTheContract

Figure 5-2snapshot of the original dataset

The original dataset as shown in Figure 5.2 contains 13 features divided into two types of features categorical(linguistic) and numerical features. Table 5.1 provides a discerption of dataset features. To prepare the dataset to be more convenient for building the proposed system. The following points explain what has been done to each of the original dataset features:

A. **B_Date feature**: the B_Date feature was transformed into age using the following equation:

$$Age = \boldsymbol{Date}_{\mathbf{current}} - \boldsymbol{Date}_{\mathbf{Birth}} \quad \mathbf{(5.1)}$$

Where $Date_{\text{current}}$ represent date of obtained dataset and $Date_{\text{Birth}}$ represents age of costumer. **Figure 5.3** shows a snapshot of **(A)** Birthdate in the original dataset and **(B)** the age after transformation.

Table 5-1description of dataset features

#	Parameter Name	Data type	Description
1	B_Date	Numerical	costumer's Birthdate
2	SEX	Linguistic	costumer's gender
3	M_STATUS	Linguistic	costumer's marital status
4	DEP_CHILDREN	Numerical	number of costumer's dependent children
5	INCOME	Numerical	costumer's income per month
6	DEP_SPOUSES	Numerical	number of costumer's dependent spouses
7	OCCUPATION	Linguistic	costumer's occupation
8	MONTH_EXP	Numerical	costumer's average monthly expenditure
9	LIVE_COUN	Linguistic	costumer's live country
10	LIVE_STATE	Linguistic	costumer's live state
11	LIVE_CITY	Linguistic	costumer's live city
12	TOT_AMOUNT	Numerical	total costumer's loan amount
13	FINAL_ EVALUATION	linguistic	Overall customer's deal evaluation

B. **SEX:** is considered to be a linguistic variable that can take one of the two values {Male, Female} and there is no needed change in this feature**.**

C. **M_STATUS:** is considered to be a linguistic variable that can take one of these values {Divorced, Married, Single, widowed} and there is no needed change in this feature**.**

D. **DEP_CHILDREN:** this feature is numerical and there is no needed change in this feature.

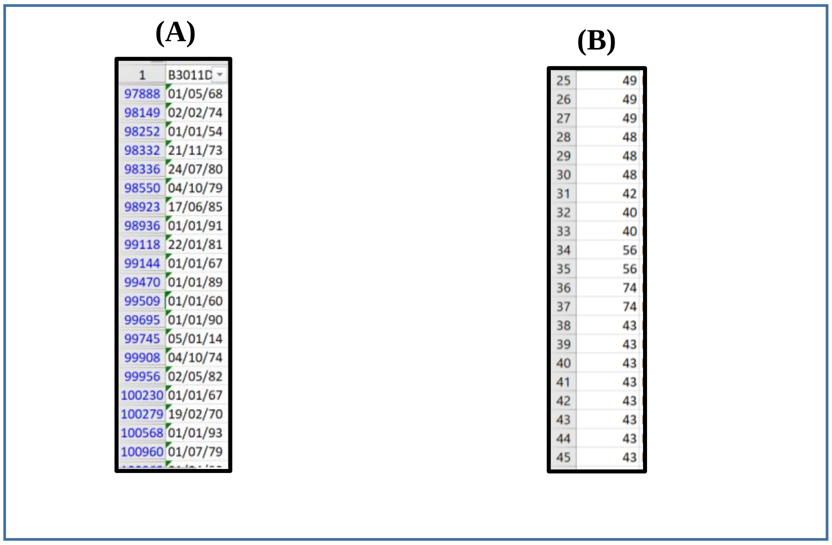
E. **INCOME :** this feature is numerical and there is no needed change in this feature.

F. **DEP_SPOUSES**: this feature is numerical and there is no needed change in this feature.

G. **OCCUPATION:** is considered to be a linguistic variable that can take one of these values {Academic Degree, Basic, Higher Education, Other, Secondary, Unfinished} and there is no needed change in this feature**.**

H. **MONTH_EXP:** this feature is numerical and there is no needed change in this feature.

I. **LIVE_COUN:** is considered to be a linguistic variable that can take one of the different countries around the world and there is no needed change in this feature**.**


(A)

1	B3011D
97888	01/05/68
98149	02/02/74
98252	01/01/54
98332	21/11/73
98336	24/07/80
98550	04/10/79
98923	17/06/85
98936	01/01/91
99118	22/01/81
99144	01/01/67
99470	01/01/89
99509	01/01/60
99695	01/01/90
99745	05/01/14
99908	04/10/74
99956	02/05/82
100230	01/01/67
100279	19/02/70
100568	01/01/93
100960	01/07/79

(B)

25	49
26	49
27	49
28	48
29	48
30	48
31	42
32	40
33	40
34	56
35	56
36	74
37	74
38	43
39	43
40	43
41	43
42	43
43	43
44	43
45	43

Figure 5-3 a snapshot of birthdate parameter

J. **LIVE_STATE:** is considered to be a linguistic variable that can take one of the different states in Sudan and there is no needed change in this feature**.**

K. **LIVE_CITY:** this feature is discarded because there is no need to identify the customer location with 3 different features besides can lead to no-needed computation overhead.

L. **TOT_AMOUNT:** this feature is numerical and there is no needed change in this feature.

M. **FINAL_ EVALUATION:** is considered to be a linguistic variable that can take one of these values {TerminatedAccordingTheContract, TerminatedInAdvanceCorrectlyBecauseOfsubjectsWish, TerminatedInAdvanceIncorrectlyBecauseOfsubjectsNegativeBehaviour} and they wear transformed into 0,0,and 1 respectively. Note that “0” to represent non-defaulted costumer and “1” to represent defaulted costumer.

Figure 5.4 provides a snapshot of the data after cleaning and preprocessing.

	A	B	C	D	E	F	G	H	I	J	K	L	M
127	59	Male	Married	8	5500	1	Academicl	10000	SD	Khartoum	188750	0	
128	59	Male	Married	8	5500	1	Academicl	10000	SD	Khartoum	130550	0	
129	59	Male	Married	8	5500	1	Academicl	10000	SD	Khartoum	43084	0	
130	59	Male	Married	8	5500	1	Academicl	10000	SD	Khartoum	178500	0	
131	59	Male	Married	8	5500	1	Academicl	10000	SD	Khartoum	218225	0	
132	59	Male	Married	8	5500	1	Academicl	10000	SD	Khartoum	93776.68	0	
133	59	Male	Married	8	5500	1	Academicl	10000	SD	Khartoum	986971	0	
134	59	Male	Married	8	5500	1	Academicl	10000	SD	Khartoum	92860	0	
135	59	Male	Married	8	5500	1	Academicl	10000	SD	Khartoum	2991500	0	
136	59	Male	Married	8	5500	1	Academicl	10000	SD	Khartoum	2991500	0	
137	59	Male	Married	8	5500	1	Academicl	10000	SD	Khartoum	46500	0	
138	59	Male	Married	8	5500	1	Academicl	10000	SD	Khartoum	11289.64	0	
139	59	Male	Married	8	5500	1	Academicl	10000	SD	Khartoum	70000	0	
140	59	Male	Married	8	5500	1	Academicl	10000	SD	Khartoum	45900	0	
141	59	Male	Married	8	5500	1	Academicl	10000	SD	Khartoum	140325	0	
142	59	Male	Married	8	5500	1	Academicl	10000	SD	Khartoum	5623.5	0	
143	59	Male	Married	8	5500	1	Academicl	10000	SD	Khartoum	19800	0	
144	59	Male	Married	8	5500	1	Academicl	10000	SD	Khartoum	24000	0	
145	37	Male	Single		3000		HigherEdu	2000	SD	Khartoum	20000	0	
146	37	Male	Single		3000		HigherEdu	2000	SD	Khartoum	20000	0	

Figure 5-4a snapshot of the data after cleaning and preprocessing

5.3 Data splitting:

To learn and test the proposed model the data should be divided into two parts, one part to train the proposed model and another part to test the proposed model. From the served literature the researcher found that the most suitable way to divide the dataset is 70% for learning and 30% for testing. Thus, consequently, the dataset is divided using the same schema as in literature. In this context, it’s important to note that the dataset used in this study is highly unbalanced, which is mean the number of defaulted customers is very small in comparison to non-defaulted customers. Thus, taking into account this note, the researcher carefully divided the dataset into training part contains (70% of defaulted customers and 70% of non-defaulted customers) and the testing part contains (30% of defaulted customers and 30% of non-defaulted customers).

5.4 Modeling phase:

In this phase, two components must be constructed. These are the fuzzy sets Membership Function (MFs) and the rule base. As mentioned in chapter 3 there are two

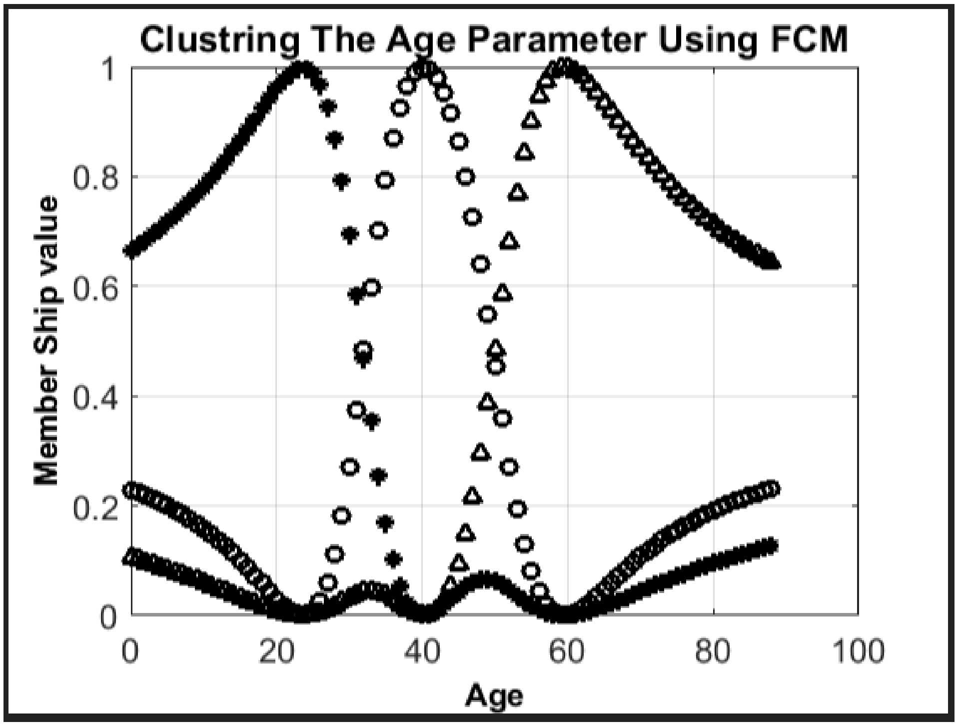


Figure 5-5 an example of extracted clusters for the age parameter

different ways to build the components of fuzzy logic controller either by learning them from expertise in the specific domain or by learning them from the data. The proposed model will be fully learned from training data. The following two subsections will explain how the proposed model components have been constructed.

5.4.1 Constructing the Fuzzy Membership Functions (MFs):

To build the MFs for each numerical continuous input parameter in the dataset **see** Table 5.1, we used the fuzzy C-Means clustering algorithm (FCM) (Bezdek, 1984; Ghosh and Dubey, 2013) see section 3.5. FCM can cluster data into a predefined number of overlapping groups, with each group acting as a membership function. In this study, each numerical input is represented by three trapezoidal membership functions such as LOW, MEDIUM, and HIGH by determining the membership parameters (i.e. a,b,c, and d see Figure

3.5) dynamically from data using FCM. The generated fuzzy sets of antecedents have overlapping domain interval ranges. Figure 5.5 shows an example of extracted clusters for the age parameter. After that these clusters are approximated to generate type-1 MFs. Figure 5.6 provides an example of approximated type-1 MFs for the age parameter. which could be written in the following equation(JERRY, 2017)

$$\mu_A(x) = \left\{ \begin{array}{cc} \frac{x-a}{b-a} & a \le x \le b \\ 1 & b \le x \le c \\ \frac{d-x}{x-c} & c \le x \le d \\ 0 & otherwise \end{array} \right\} \tag{5.2}$$

Where $\mu_A(x)$ represent the membership degree for the value x to the fuzzy set A, and [a,b,c, and d] is parameter for trapezoidal MFs. See Figure 3.5.

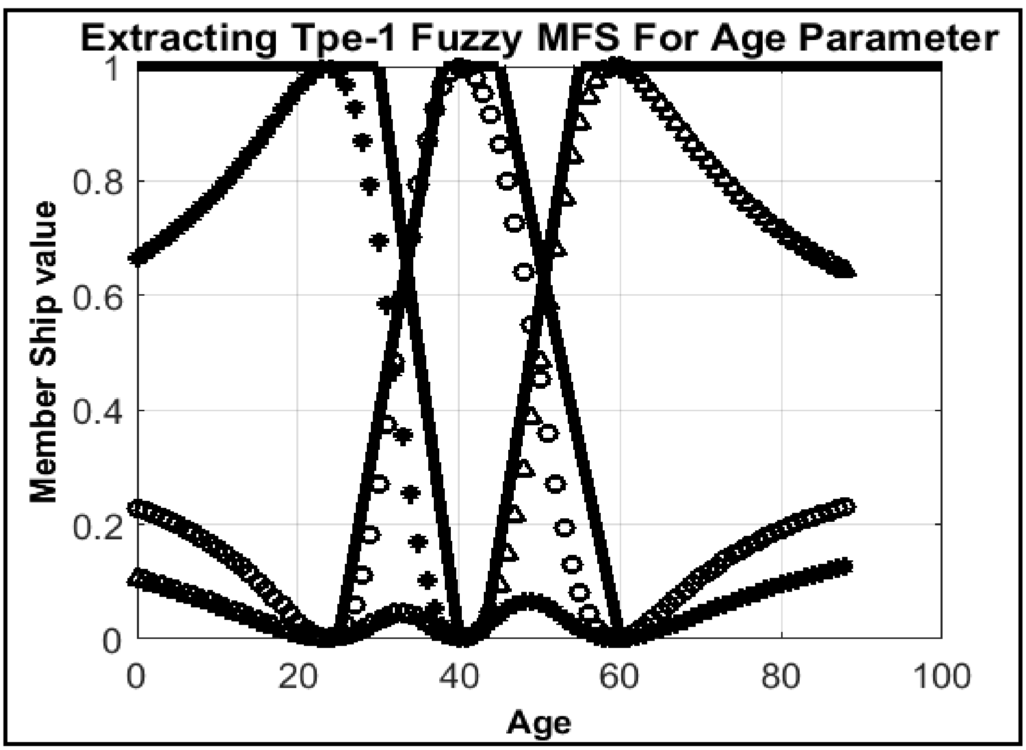


Figure 5-6 example of approximated type-1 MFs

To construct the type-2 MFs in our proposed model, we add Footprint of uncertainty (FOU) to the given type-1 fuzzy sets (extracted by FCM) by adding 10%,20% and 30% FOU for each MFs parameter

$$\beta_{T2} = \beta_{T1} \pm \frac{\beta_{T1} \times \delta}{100} \qquad \textbf{(5.3)}$$

Where β_{T2} is T2 MFs parameter, β_{T1} is T1 MFs parameter and δ represent the percentage (i.e 10%, and 20%).

By applying equation (5.2) to all extracted T1 fuzzy sets, an example of obtained T2 MFs can be as shown in Figure 5.7

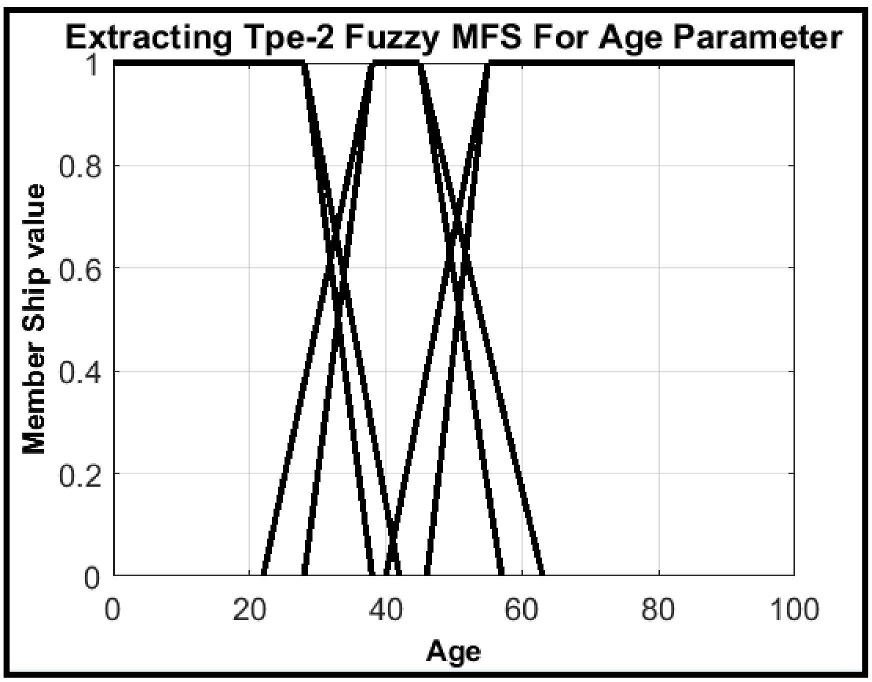


Figure 5-7example of obtained T2 MFs

5.4.2 Rule Base Extraction

To extract the final rule base that will be used in the proposed model two operations should be done which are (Raw rule extraction, and rule base cleaning) the following subsections will explain each of them in detail.

5.4.2.1 Raw Rule Extraction

From the training data set which contains numbers of records, each one represents input-output pair (x(t), C(t)), t =1,…, T (T is the total number of training dataset records available for the training phase) as shown in Figure 5.8.

A. Calculate the upper and lower membership values ($\bar{\mu}_{A_s^q}$, $\underline{\mu}_{A_s^q}$) for any antecedent

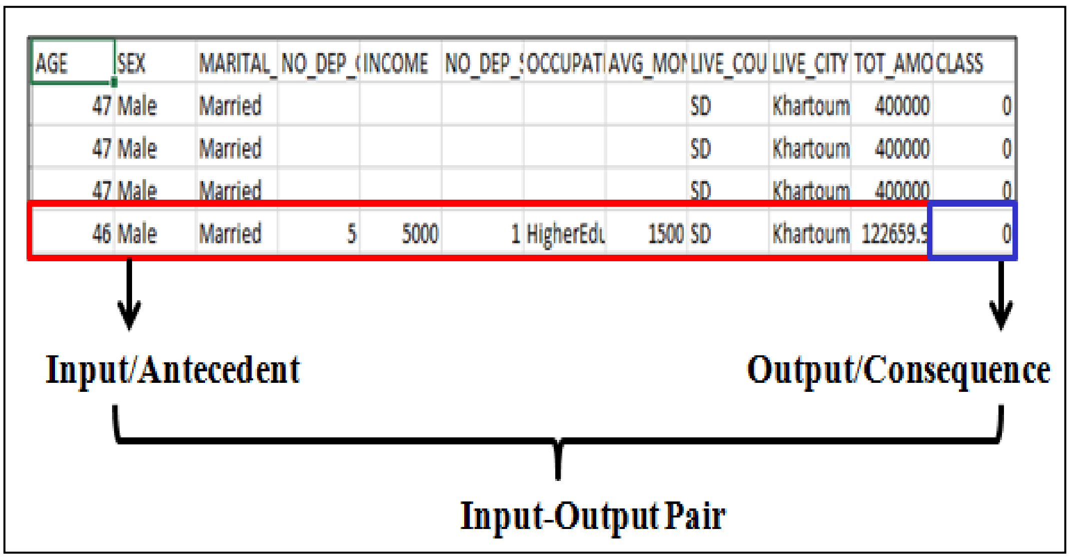


Figure 5-8 input-output pair

fuzzy set q=1…K (K is the total number of fuzzy sets representing the input pattern s where s=1…n.) by using the suitable formula from table 3.1.

B. Generate all possible rules that can be extracted from each input-output pair (x(t), C(t)) as a result number of rules will be generated from the single input-output pair (x(t), C(t)) with same consequence C(t) but different antecedences each of them could be written as follows (Bernardo et al., 2013):

$$R^j: If\ x_1 is\ \tilde{A}_1^{\mathrm{qjt}}\ and\ \dots and\ x_n\ is\ \tilde{A}_{\mathrm{n}}^{\mathrm{qjt}} then\ class\ C_t\ ,t = 1,2 \dots T \quad \textbf{(5.4)}$$

C. To measure the strength of the point $x^{(t)}$ belonging to the fuzzy area that covered by the generated rule we need to calculate the firing strength F^t which is defined with its lower and upper bound ($\overline{f^{(t)}}$, $\underline{f^{(t)}}$) These bounds can be calculated as follows (Bernardo et al., 2013):

$$\overline{f^{Jt}}(x^{(t)}) = \overline{\mu_{A_1^{qJt}}}(x_1) * \dots .* \overline{\mu_{A_n^{qJt}}}(x_n) \tag{5.5}$$

$$\underline{f^{jt}}(x^{(t)}) = \underline{\mu_{A_1^{qjt}}}(x_1) * \dots .* \underline{\mu_{A_n^{qjt}}}(x_n) \tag{5.6}$$

Where (*) represents the minimum t-norm.

D. Because financial data is imbalanced by nature which means the majority class is a good customer and the minority class is a bad customer, in this context, it is important to note that there is an issue that needs to be considered, which is the competition fairness between the two consequences classes. Thus to handle imbalanced data by trying to give minority classes a fair chance when competing with the majority class we use the "weighted scaled dominance" approach introduced by (Bernardo et al., 2013) and "weighted confidence" which is presented by (Ishibuchi and Yamamoto, 2005). To calculate the scaled dominance for a given rule with a consequent Class C_j, divide its firing strength by the sum of the firing strengths of all the rules with C_j as the consequent class. This allows for the handling of data imbalances toward a specific class. We scale the firing strength by scaling the firing strength's upper and lower bounds as follows (Ishibuchi and Yamamoto, 2005):

$$\overline{\boldsymbol{fs^{Jt}}} = \frac{\overline{\boldsymbol{f^{Jt}}}}{\boldsymbol{\Sigma_{J\epsilon\, class\, j}\overline{f^{Jt}}}} \tag{5.7}$$

$$\underline{\boldsymbol{fs^{jt}}} = \frac{\underline{\boldsymbol{f^{jt}}}}{\boldsymbol{\Sigma_{J\epsilon\, class\, j}\underline{f^{jt}}}} \tag{5.8}$$

steps A to D are repeated for all the t data points in the training data from 1 to T to obtain generated rules in the form of Equation (5.4). Table 5.2 provides algorithm 1 that is used to extracts the raw rules.

Table 5-2 Pseudocode for Raw rule extraction

Algorithm1: *Pseudocode for Raw rule extraction*

For each input-output pair (x(t), C(t)) in the training data

  Calculate the upper and lower membership values antecedent fuzzy set

  Generate all possible rules that can be extracted from each input-output pair (x(t), C(t))

  Calculate upper and lower firing strength using Equ. (5.4) and (5.5)

  Scall the upper and lower firing strength using Equ. (5.6) and (5.7)

5.4.2.2 Rule base cleaning:

After the rule base has been extracted using algorithm 1 as in **Table (5.2)** it is observed that the obtained rule base contains some conflicting rules. This conflict happens when some of the extracted rules have the same antecedent but different consequence classes which can cause conflicts for the fuzzy logic system in the prediction phase. For example, consider that we have two rules:

R1: if x1 is low and x2 is mid then class is good

R2: if x1 is low and x2 is mid then class is bad

The problem here is which one of these rules will need to be used to predict customer status. The customer cannot be good and bad at the same time. To resolve this conflict, we need to replace any group of rules which share the same antecedent with one single rule. We will calculate the scaled confidence and scaled support which are introduced by (Ishibuchi and Yamamoto, 2005)by grouping the rules that have the same antecedents and conflicting classes. Therefore, to resolve the conflict issue the following steps should be followed:

A. For a set of m rules with the same antecedents but conflicting classes calculate the scaled confidence ($\tilde{\boldsymbol{A}}_{\boldsymbol{q}} \rightarrow \boldsymbol{C}_{\boldsymbol{q}}$) defined by its upper bound $\bar{c}$ and lower bound $\underline{c}$ as follows(Ishibuchi and Yamamoto, 2005):

$$\bar{\boldsymbol{c}}\left(\tilde{\boldsymbol{A}}_{\boldsymbol{q}} \rightarrow \boldsymbol{C}_{\boldsymbol{q}}\right) = \frac{\Sigma_{x_s\,\epsilon\,class\ Cq}\overline{fs^{jt}(x_s)}}{\sum_{j=1}^{m}\overline{fs^{jt}(x_s)}} \tag{5.9}$$

$$\underline{\boldsymbol{c}}(\tilde{\boldsymbol{A}}_{\boldsymbol{q}} \rightarrow \boldsymbol{C}_{\boldsymbol{q}}) = \frac{\Sigma_{x_s\,\epsilon\,class\ Cq}\underline{\underline{fs^{jt}x(x_s)}}}{\sum_{j=1}^{m}\underline{fs^{jt}(x_s)}} \tag{5.10}$$

Where $\boldsymbol{C}_{\boldsymbol{q}}$ is the consequence class for the antecedents $\tilde{\boldsymbol{A}}_{\boldsymbol{q}}$, $\boldsymbol{fs^{jt}}$ are the upper and lower scaled firing strength calculated by equation (5.7), and (5.8) respectively.

The scaled confidence can be viewed as measuring the validity of ($\tilde{\boldsymbol{A}}_{\boldsymbol{q}} \rightarrow \boldsymbol{C}_{\boldsymbol{q}}$).

B. Calculate the scaled support (defined by its upper bound $\bar{s}$ and lower bound $\underline{s}$ as follows(Ishibuchi and Yamamoto, 2005)

$$\bar{\boldsymbol{s}}(\tilde{\boldsymbol{A}}_{\boldsymbol{q}} \rightarrow \boldsymbol{C}_{\boldsymbol{q}}) = \frac{\Sigma_{x_s\,\epsilon\,class\ Cq}\overline{fs^{jt}(x_s)}}{\boldsymbol{m}} \tag{5.11}$$

$$\underline{\boldsymbol{s}}(\tilde{\boldsymbol{A}}_{\boldsymbol{q}} \rightarrow \boldsymbol{C}_{\boldsymbol{q}}) = \frac{\Sigma_{x_s\,\epsilon\,class\ Cq}\underline{\underline{fs^{jt}(x_s)}}}{\boldsymbol{m}} \tag{5.12}$$

Where $\boldsymbol{C}_{\boldsymbol{q}}$ is the consequence class for the antecedents $\tilde{\boldsymbol{A}}_{\boldsymbol{q}}$, $\boldsymbol{fs^{jt}}$ are the upper and lower scaled firing strength calculated by equation (5.7), and (5.8) respectively, m is number of conflicting rules in the same group.

The support can be viewed as measuring the coverage of training patterns by ($\tilde{\boldsymbol{A}}_{\boldsymbol{q}} \rightarrow \boldsymbol{C}_{\boldsymbol{q}}$).

C. Calculate the scaled dominance, (defined by its upper bound $\overline{\boldsymbol{d}}$ and lower bound $\underline{\boldsymbol{d}}$) by multiplying the scaled support from Equ. (5.11) and (5.12) times scaled confidence from Equ. (5.9) and (5.10) of the rule as follows(Bernardo et al., 2013):

$$\overline{\boldsymbol{d}}(\widetilde{\boldsymbol{A}}_{\boldsymbol{q}} \rightarrow \boldsymbol{C}_{\boldsymbol{q}}) = \overline{\boldsymbol{s}}(\widetilde{\boldsymbol{A}}_{\boldsymbol{q}} \rightarrow \boldsymbol{C}_{\boldsymbol{q}}) * \overline{\boldsymbol{c}}(\widetilde{\boldsymbol{A}}_{\boldsymbol{q}} \rightarrow \boldsymbol{C}_{\boldsymbol{q}}) \qquad \textbf{(5.13)}$$

$$\underline{\boldsymbol{d}}(\widetilde{\boldsymbol{A}}_{\boldsymbol{q}} \rightarrow \boldsymbol{C}_{\boldsymbol{q}}) = \underline{\boldsymbol{s}}(\widetilde{\boldsymbol{A}}_{\boldsymbol{q}} \rightarrow \boldsymbol{C}_{\boldsymbol{q}}) * \underline{\boldsymbol{c}}(\widetilde{\boldsymbol{A}}_{\boldsymbol{q}} \rightarrow \boldsymbol{C}_{\boldsymbol{q}}) \qquad \textbf{(5.14)}$$

D. Calculate the average dominance (defined in terms of $\overline{d_{ave}}$ and $\underline{d_{ave}}$) over fuzzy rules with the same antecedent $\tilde{A}_q$ but different consequent classes C_{q_i}, where i = (1,2) as follows:

$$\overline{\boldsymbol{d}_{\boldsymbol{ave}}} = \frac{\overline{\boldsymbol{d}}(\widetilde{\boldsymbol{A}}_{\boldsymbol{q}} \rightarrow \boldsymbol{C}_{\boldsymbol{q}_1}) + \overline{\boldsymbol{d}}(\widetilde{\boldsymbol{A}}_{\boldsymbol{q}} \rightarrow \boldsymbol{C}_{\boldsymbol{q}_2})}{\boldsymbol{2}} \qquad \textbf{(5.15)}$$

$$\underline{\boldsymbol{d}_{\boldsymbol{ave}}} = \frac{\underline{\boldsymbol{d}}(\widetilde{\boldsymbol{A}}_{\boldsymbol{q}} \rightarrow \boldsymbol{C}_{\boldsymbol{q}_1}) + \underline{\boldsymbol{d}}(\widetilde{\boldsymbol{A}}_{\boldsymbol{q}} \rightarrow \boldsymbol{C}_{\boldsymbol{q}_2})}{\boldsymbol{2}} \qquad \textbf{(5.16)}$$

E. Calculate the "***weighted scaled dominance***" (which is defined by its upper bound $\overline{\boldsymbol{wd}}$ and lower bound $\underline{\boldsymbol{wd}}$) as follows(Bernardo et al., 2013)

$$\overline{\boldsymbol{wd}}(\widetilde{\boldsymbol{A}}_{\boldsymbol{q}} \rightarrow \boldsymbol{C}_{\boldsymbol{q}}) = \overline{\boldsymbol{d}}(\widetilde{\boldsymbol{A}}_{\boldsymbol{q}} \rightarrow \boldsymbol{C}_{\boldsymbol{q}}) * \overline{\boldsymbol{d}_{\boldsymbol{ave}}} \qquad \textbf{(5.17)}$$

$$\underline{\boldsymbol{wd}}(\widetilde{\boldsymbol{A}}_{\boldsymbol{q}} \rightarrow \boldsymbol{C}_{\boldsymbol{q}}) = \underline{\boldsymbol{d}}(\widetilde{\boldsymbol{A}}_{\boldsymbol{q}} \rightarrow \boldsymbol{C}_{\boldsymbol{q}}) * \underline{\boldsymbol{d}_{\boldsymbol{ave}}} \quad \textbf{(5.18)}$$

F. Calculate "***average weighted scaled dominance"*** $\boldsymbol{wd}_{\boldsymbol{Avg}}(\widetilde{\boldsymbol{A}}_{\boldsymbol{q}} \rightarrow \boldsymbol{C}_{\boldsymbol{q}})$ for each of the consequence class $\boldsymbol{C}_{\boldsymbol{q}}$ as follows(Bernardo et al., 2013):

$$\boldsymbol{wd}_{\boldsymbol{Avg}}(\widetilde{\boldsymbol{A}}_{\boldsymbol{q}} \rightarrow \boldsymbol{C}_{\boldsymbol{q}}) = \frac{\overline{\boldsymbol{wd}}+\underline{\underline{\boldsymbol{wd}}}}{\boldsymbol{2}} \quad \textbf{(5.19)}$$

G. Finally replace the group of rules which share the same antecedence but different class with one rule that have antecedence that shared by this group and the consequent class which will be corresponding to the rule that gives the highest average weighted scaled dominance that which have been calculated by Equ. (5.19).

By the end of step (G) the rule base has been cleaned and its ready to be tested by the prediction phase. Table 5.3 provides algorithm 2 that is used to clean the rule base.

Table 5-3Pseudocode for Rule base cleaning

Algorithm2: *Pseudocode for Rule base cleaning*

1 For each group of m rules in the rule base that share the same antecedence but have different consequences do

2 Calculate the upper and lower confidence using Equs. (5.9) and (5.10) respectively.

3 Calculate the upper and lower scaled support using Equs. (5.11) and (5.12) respectively.

4 Calculate the upper and lower scaled dominance using Equs. (5.13) and (5.14) respectively.

5 Calculate the upper and lower average dominance using Equs. (5.15) and (5.16) respectively.

6 Calculate the upper and lower weighted scaled dominance using Equs. (5.17) and (5.18) respectively.

7 For the two conflicting class calculate the average weighted scaled dominance using Eq. (5.19)

9 Replace the m rules which shared with one rule that have antecedence that shared by m and the consequent class corresponding to class that gives the highest $\boldsymbol{wd_{Avg}}$ calculated in step 8

5.5 Prediction phase

So far, we have a full Type-2 fuzzy logic classifier that is ready to accept new input patterns with unknown classes and predict which classes this input belongs to. When a new input pattern x(p) is presented to the proposed model, then one of two possible cases can happen **First Case:** The input x(p) matches any of the X rules in the rule base, and in this situation, the system follows the steps explained in Case 1. **Second Case:** If the input x(p) does not match any of the X rules in the rule base, then in this situation, the system follows the steps explained in Case 2.

A. First Case (The input matches one of the existing rules)**:** In this case, the input pattern x(p) can generate one or more than one rule. If the input pattern x(p) generates one rule and this rule matches any rule in the rule base then only the predicted class for this input pattern x(p) is the consequence class for the matched rule. However, if the input pattern x(p)generates more than one rule, then in this situation there is a possibility for conflict in the consequences of these rules. Any of the generated rules by the input pattern x(p) can match different rules in the rule base, and any of these rules in the rule base may have a different consequence class. In this case, we will need to choose one of these classes to be the predicted class for the input pattern x(p). To do this we need to calculate a vote for each class as follows (Bernardo et al., 2013):

$$\bar{z}Class_h\left(x^{(p)}\right) = \frac{\Sigma_{j \in h}\overline{f^j}(x^{(p)}) * \overline{wd}(\tilde{A}_q \rightarrow C_q)}{max_{j \in h}\overline{f^j}((x^{(p)}) * \overline{wd}(\tilde{A}_q \rightarrow C_q))} \quad \textbf{(5.20)}$$

$$\underline{z}Class_h\left(x^{(p)}\right) = \frac{\Sigma_{j \in h}\underline{f^j}(x^{(p)}) * \underline{wd}(\tilde{A}_q \rightarrow C_q)}{max_{j \in h}\underline{f^j}((x^{(p)}) * \underline{wd}(\tilde{A}_q \rightarrow C_q))} \quad \textbf{(5.21)}$$

In the above equations $max_{j \in h}\overline{f^j}((x^{(p)}) * \overline{wd}(\tilde{A}_q \rightarrow C_q))$ and $max_{j \in h}\underline{f^j}((x^{(p)}) * \underline{wd}(\tilde{A}_q \rightarrow C_q))$ represent taking the maximum of the product of the upper and lower firing strengths and the weighted scaled dominance respectively among the 'K' rules selected for each class.

After we calculate the upper $\bar{z}Class_h\left(x^{(p)}\right)$ and $\underline{z}Class_h\left(x^{(p)}\right)$ now we calculate the total vote strength for each of competitors classes as follows(Bernardo et al., 2013):

$$\boldsymbol{zClass_h} = \frac{\bar{z}\boldsymbol{Class_h}\,(\boldsymbol{x^{(p)}}) + \underline{z}\boldsymbol{Class_h}\,(\boldsymbol{x^{(p)}})}{\boldsymbol{2}} \qquad \textbf{(5.22)}$$

Now the class with the highest $zClass_h$ will be the winner class and it will be taken as predict class for the input pattern x (p).

B. Second Case (the input does not match any of the existing rules): In this case, the input pattern $x^{(p)}$ can generate more than one rule which could not match any existing rule in the rule base. In this situation, we need to decide which one of the two classes could be the predicted class for the input pattern $x^{(p)}$. In order to resolve this conflict, let MR ($x^{(p)}$) be the set of rules generated by the input pattern $x^{(p)}$. For any rule in MR ($x^{(p)}$) we find the closest rule in the rule base and then use the same steps as in Case 1. To find the closest rule for any rule in MR ($x^{(p)}$) in the rule base, we need to find the similarity (or distance) between any rule generated by the input pattern $x^{(p)}$ and each rule stored in the rule base. Then the rule with the highest similarity is selected to be the most similar rule. To calculate the similarity between one rule generated by the input pattern $x^{(p)}$and the other rule stored in the rule base we use the following equation(Bernardo et al., 2013):

$$Similarity_{\ input\ r\leftrightarrow j} = ((1 - \left|\frac{vinput1r - vj1}{v1}\right|) * (1 - \left|\frac{vinput2r - vj2}{v2}\right|) * \ldots .* (1 - \left|\frac{vinputnr - vjn}{vn}\right|) \qquad \textbf{(5.23)}$$

Where $vinput\ r = (vinput\ 1r, vinput\ 2r, \ldots vinput\ nr)$ represent the linguistic labels that correspond to the rule generated by the input pattern x(p). vj = (vj1, vj2 … vjn) represent the linguistic labels that correspond to the rule stored in the rule base. Each of these linguistic labels could be decoded into an integer. v1,v2 … vn represent the number of linguistic labels representing each variable.

Sometimes the input pattern can generate more than one rule, for all of them, there is no matched rule in the rule base. In this situation, we need to find for each generated rule by the input pattern the most smiler rule from the rule base using Equ. (5.23), as a result each smiler rule from the rule base will have a similarity factor. Finally, to identify the final predicted class, we use the same steps as Case 1 with multiplying each rule's "weighted scaled dominance" using (5.20) and (5.21) by its corresponding similarity factor.

5.6 Evaluation:

To evaluate the proposed model, The AVG-Recall is used as an evaluation metric. And the AVG-Recall could be calculated in a confusion matrix which displays information about predicted and actual classification done by a classifier (Kohavi, 1998). This information is used to measure the classifier's performance. For example, if there exists an input item and two classes (positive and negative), then there would be four possible cases that can occur as follows:

A. The input item is positive and the classifier classifies it truly as positive and this case is known as **True Positive (TP)**.

B. The input item is negative and the classifier classifies it as positive and this case is known as **False Positive (FP)**.

C. The input item is positive and the classifier classifies it as negative and this case is known as **False Negative (FN)**.

D. The input item is Negative and the classifier classifies it truly as negative and this case is known as **True Negative (TN)**.

From the information provided by the confusion matrix, we can calculate Recall which is called sensitivity for both classes (positive and negative) as follow (Ishibuchi and Yamamoto, 2005)

$$Recall\ Positive\ Rate = \frac{TP}{TP+FN} \quad \textbf{(5.24)}$$

$$Recall\ Negaitive\ Rate = \frac{TN}{TN+FP} \quad \textbf{(5.25)}$$

We can calculate the AVG-Recall which is used as thecost function of the proposed model as follows (Bernardo et al., 2013):

$$Avg\ Recall = \frac{Recall_{Positive}+Recall_{Negaitive}}{2} \quad \textbf{(5.26)}$$

To test the proposed model 30% of the dataset which is dedicated to the testing phase is used (see section 5.2). and to evaluate the performance of the proposed model 4 different experiments have been done as follows:

1. Type-1 FLs using Type-1 fuzzy sets generated by FCM
2. Type-2 FLC using Type-2 fuzzy sets with equal space FOU
3. Type-2 FLC using Type-2 fuzzy sets generated by FCM with 10% FOU
4. Type-2 FLC using Type-2 fuzzy sets generated by FCM with 20% FOU

As mentioned previously the Avg-Recall is used as evaluation metric Table 5.4 provides brief description for all conducted experiments. And summarizes the results extracted using testing data.

Table 5-4 Extracted Results

Exp #	Model Type	FCM	FOU	Avg-Recall
1	T1	Yes	0	0.69
2	T2	No	equal	0.79
3	T2	Yes	10%	0.83
4	T2	Yes	20%	0.82

From Table 5.4, it can be extracted that the Type-2 fuzzy-based system using FCM with 10% FOU (Experiment #3) achieved 0.83 Avg-Recall compared to 0.79 Avg-Recall achieved by Type-1 fuzzy-based system using FCM counterpart. And that means the proposed Type-2 model outperforms Type-1 model in term of prediction accuracy. This result interprets the most important advantage of Type-2 fuzzy logic system over Type-1 fuzzy logic which is that Type-2 is more capable than type-1 to represent uncertainty associated with the financial domain that is being studied.

Another observation that is important to note in this context is that from the conducted experiments, Type-1 is simpler to implement and provides less computation overhead, on the contrary Type- 2 is a little bit complex to implement and gives more computation overhead. And this explains the frequent use of the type-1 over type-2 in the real-world applications.

The inference engine of the proposed model is built on IF-THEN rules as shown in Table 5.5 which is provides an example of extracted rules by the proposed model. The proposed model gives you an inside look at how they work and this is the main advantage provided by white-box models. By analyzing these rules, the decision-maker can reduce the potential risks that can face the organization as well as protect customers from defaulting by advising him according to the analyzed information.

Another factor that needs to be focused on is the rule base interpretability. The rule base could be more interpretable by the decision-maker if it contains a small number of readable rules as much as possible. Table (5.6) shows a comparison between different models applied in the experiments. The comparison in this table focuses on number of rules in the rule base.

Table 5-5example of extracted rules by the proposed model

R1	*if age is* ***Young*** *& sex is* ***Male*** *& marital_Status is* ***Married*** *& no_Dep_Child is* ***Mid*** *& income is* ***Low*** *& no_Dep_Spouses is* ***Low*** *& accupation is* ***Basic*** *& Avg_Month_Exp is* ***High*** *& live_Country is* ***SD*** *& live_City is* ***Khartoum*** *& tot_Amoount is* ***Low*** *Then class is* ***default***
R2	*if age is* ***Young*** *& sex is* ***Male*** *& marital_Status is* ***Married*** *& no_Dep_Child is* ***High*** *& income is* ***Low*** *& no_Dep_Spouses is* ***Low*** *& accupation is* ***HigherEducation*** *& Avg_Month_Exp is* ***Low*** *& live_Country is* ***SD*** *& live_City is* ***Khartoum*** *& tot_Amoount is* ***High*** *Then class is* ***default***

Table 5-6comparison between different models applied in the experiments

Exp #	Model Type	FCM	FOU	N of Rules
1	T1	Yes	0	5512
2	T2	No	equal	8321
3	T2	Yes	10%	8214
4	T2	Yes	20%	8416

From Table 5.5 can be observed that the type-1 model gives 5512 rules which is a smaller number of rules compared to type-2 models which have 8214 rules in its best case (i.e., experiment #3). However, the size of the rule-bases achieved by all experiments still can be considered as untraceable rule-bases because this huge number of rules is difficult to be traced by the end-user. The next chapter will propose an optimization method base on BB-BC to reduce the number of rules in the rule-base which can improves the interpretability of our proposed model.

5.7 Summary:

This chapter displays our fuzzy logic model to predict defaults in the Sudanese financial sector. To improve the prediction performance at high levels of uncertainty associated with the financial sector, the proposed prediction approach was built based on an interval type-2 fuzzy logic system.

Firstly, the real data used to build the proposed model has been described, and all issues related to data cleaning and preprocessing wear clarified. The proposed model is divided into two phases modeling phase and prediction phase, each of these phases was explained in detail.

Then to evaluate the proposed model, four different models wear conducted. One of them is based on type-1 fuzzy logic and the rest is based on type-2 fuzzy logic with different levels of the footprint of uncertainty (equal space, 10%, and 20%). The average recall was used as an evaluation metric to measure the accuracy of prediction. The results of all conducted experiments showed that the proposed type-2 model achieved 83% prediction accuracy and outperformed its type-1 counterpart which achieved 69% prediction accuracy. From another point of view, another comparison was done based on the number of rules in the rule-base, all conducted models achieved a huge number of rules (5512, and 8214) for type-1 and type-2 in its best case respectively. Thus, the next chapter will discuss an optimization method based on the BB-BC algorithm to optimize our proposed model components.

Finally, it is important to note that by the end of this chapter many objectives of this study wear achieved mainly those labeled (**c**, **d**, and **e**) in the research objectives section.

Chapter Six: A Proposed BB-BC Optimized Type-2 Fuzzy Logic Model to Predict Default in the Sudanese Banking Sector

6.1 Introduction

Vagueness exists everywhere in human life communications and this explains the subjectivity involved in interpreting the linguistic variables. As Mendel's adage said, "*words mean different things to different people*"(JERRY, 2017). As a result, to draw subjective membership functions in an environment with a high level of uncertainties, it's a very complicated problem. Furthermore, the rule-base in the fuzzy logic system has a high impact on the system's accuracy and interpretability. And therefore, the two components of the fuzzy logic system must be configured very carefully.

In comparison to the Type-1 Fuzzy Logic System, the presented Type-2 fuzzy logic system in the previous chapter provided a higher accuracy prediction result and reasonable interpretability. But on the other hand, the proposed Type-2 prediction fuzzy system has a higher computation cost for real-time prediction than the Type-1 fuzzy logic system. This computational cost is inherited from the curse of dimensionality problems associated with the type-2 fuzzy logic system. Also, from the perspective of the end-user needs transparency of the proposed type-2 fuzzy logic system needs to be optimized because no one can trace a rule-base with 8214 rules.

This chapter proposes a BB-BC optimized type-2 fuzzy logic model to predict default in the Sudanese financial sector.

6.2 The Proposed BB-BC Optimized Type-2 Fuzzy Logic Model:

To optimize the proposed type-2 fuzzy logic system the components of the system (MFs, Rule-base) that need to optimize must be determined first. Then they will be passed to the BB-BC algorithm as an input. And the optimized parameter of the FMs and optimized rule-base are considered to be the output from the optimization stage. In this context there is an important point that must be taken into consideration that is the optimization must go in the same direction of having a higher prediction accuracy as the previously proposed model in chapter 5.

As shown in Figure 6.1 the optimized proposed model is divided into two main phases, the modeling phase, and the prediction phase. The following subsections will describe those two phases in detail.

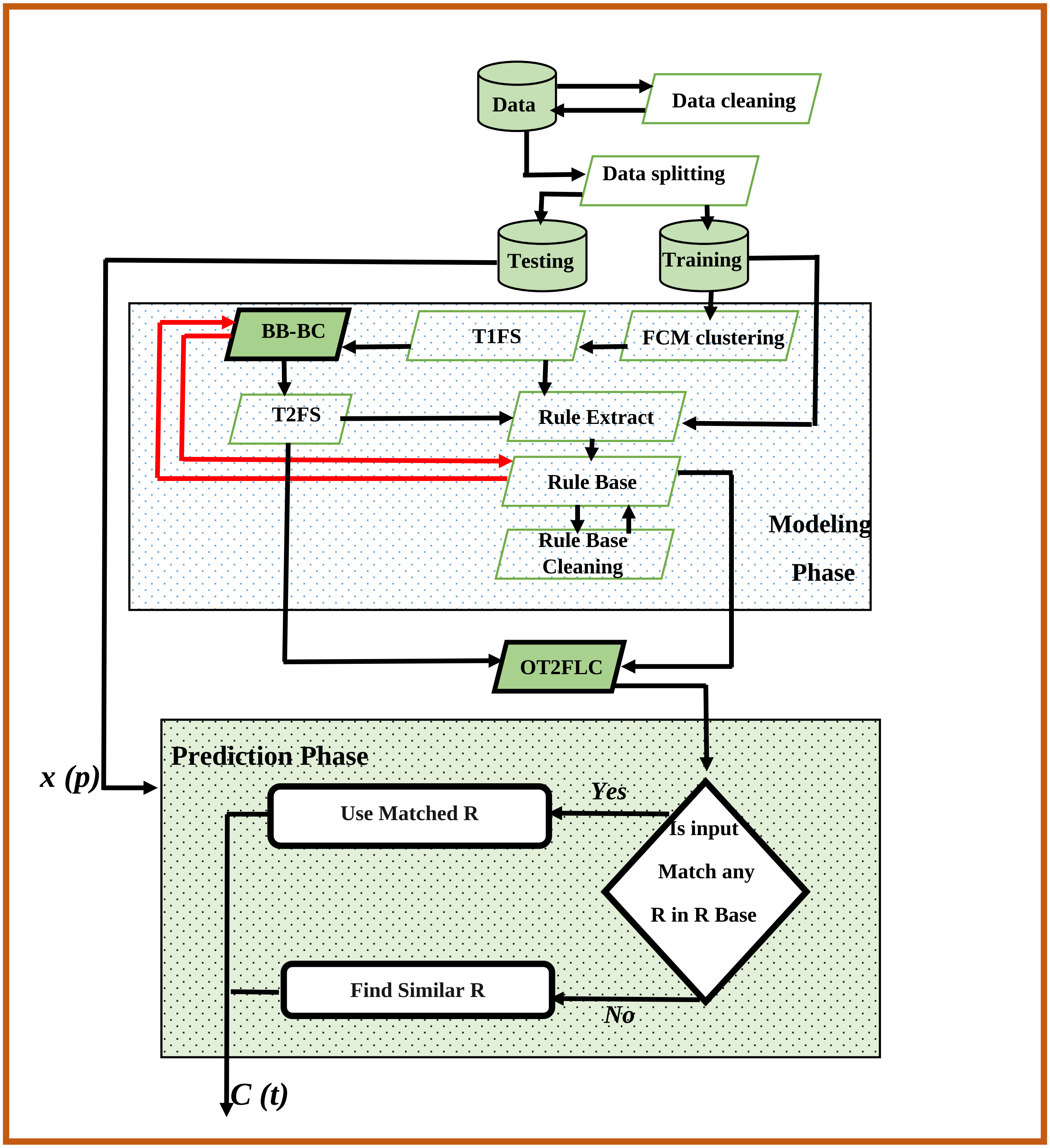
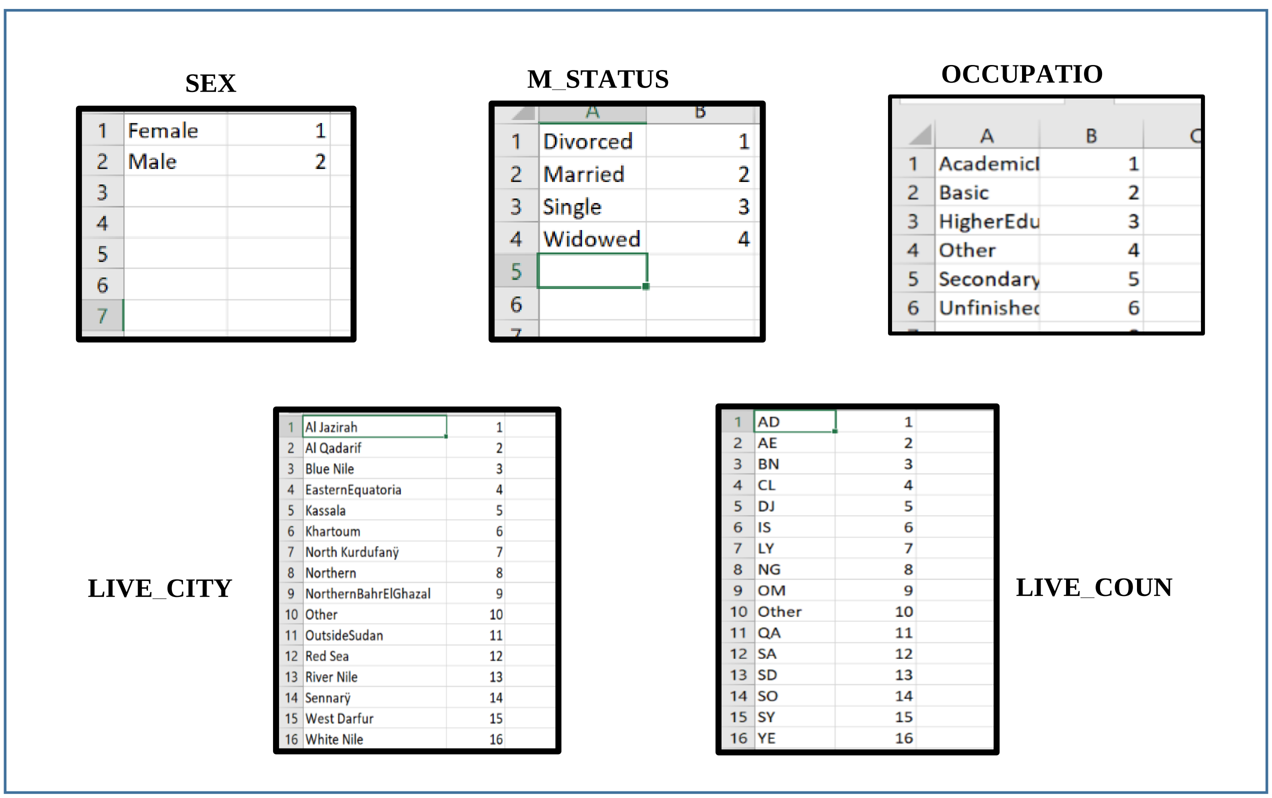


Figure 6-1structure for the proposed Optimized Type-2 Fuzzy logic Model

6.2.1 Data preparation:

The same data set that was used in the previously proposed type-2 model in chapter 5 is used to learn and test the proposed optimized model and the same percentage of data splitting is kept. But on the other hand, a small change in the data set should be done to apply the BB-BC optimization method. As we mentioned in chapter 4 all input to the BB-BC algorithm must be numerical data. Thus, all linguistic parameters are shown in Table 5.1 must be transformed into numerical values. To do such transformation for each linguistic feature the possible linguistic values have been mapped into integer values (starting from 0 up to n) where n represents the number of linguistic values that each linguistic variable could have. Figure 6.2 shows the different lookup tables for all linguistic features.

By the end of this transformation of linguistic parameters, all the parameters are now



SEX

1	Female	1
2	Male	2

M_STATUS

	A	B
1	Divorced	1
2	Married	2
3	Single	3
4	Widowed	4

OCCUPATIO

	A	B
1	Academicl	1
2	Basic	2
3	HigherEdu	3
4	Other	4
5	Secondary	5
6	Unfinishec	6

LIVE_CITY

1	Al Jazirah	1
2	Al Qadarif	2
3	Blue Nile	3
4	EasternEquatoria	4
5	Kassala	5
6	Khartoum	6
7	North Kurdufanÿ	7
8	Northern	8
9	NorthernBahrElGhazal	9
10	Other	10
11	OutsideSudan	11
12	Red Sea	12
13	River Nile	13
14	Sennarÿ	14
15	West Darfur	15
16	White Nile	16

LIVE_COUN

1	AD	1
2	AE	2
3	BN	3
4	CL	4
5	DJ	5
6	IS	6
7	LY	7
8	NG	8
9	OM	9
10	Other	10
11	QA	11
12	SA	12
13	SD	13
14	SO	14
15	SY	15
16	YE	16

Figure 6-2different lookup tables for all linguistic features.

represented in numerical form. Figure 6.3 shows a snapshot of the prepared data that will be entered into the optimization algorithm.

124	59	2	2	8	5500	1	1	10000	13	6	188750	0
125	59	2	2	8	5500	1	1	10000	13	6	130550	0
126	59	2	2	8	5500	1	1	10000	13	6	43084	0
127	59	2	2	8	5500	1	1	10000	13	6	178500	0
128	59	2	2	8	5500	1	1	10000	13	6	218225	0
129	59	2	2	8	5500	1	1	10000	13	6	93776.68	0
130	59	2	2	8	5500	1	1	10000	13	6	986971	0
131	59	2	2	8	5500	1	1	10000	13	6	92860	0
132	59	2	2	8	5500	1	1	10000	13	6	2991500	0
133	59	2	2	8	5500	1	1	10000	13	6	2991500	0
134	59	2	2	8	5500	1	1	10000	13	6	46500	0
135	59	2	2	8	5500	1	1	10000	13	6	11289.64	0
136	59	2	2	8	5500	1	1	10000	13	6	70000	0
137	59	2	2	8	5500	1	1	10000	13	6	45900	0
138	59	2	2	8	5500	1	1	10000	13	6	140325	0
139	59	2	2	8	5500	1	1	10000	13	6	5623.5	0
140	59	2	2	8	5500	1	1	10000	13	6	19800	0
141	59	2	2	8	5500	1	1	10000	13	6	24000	0
142	37	2	3		3000		3	2000	13	6	20000	0
143	37	2	3		3000		3	2000	13	6	20000	0
144	33	1	2				0		0	0	12880	0

Figure 6-3a snapshot of the BB-BC input data

6.2.2 Modeling Phase:

As mentioned previously the MFs and the rule base needed to be identified before they could be optimized. And also, the proposed model is fully learned from the data. To extract the type-2 MFs again firstly type-1 fuzzy set should be generated from the training data using FCM. After that type-1 fuzzy set can be generalized into a type-2 fuzzy set as mentioned in chapter 5. However, in type-2 fuzzy sets, establishing the boundary between upper and lower membership functions is still difficult. The extracted type-2 fuzzy MFs then passed to the BB-BC module to optimize. The next section will illustrate how type-2 MFs have been optimized.

On the other hand, the second component of the proposed model, the rule base, also needs to be generated before it can be optimized. To extract the rule base also we have followed the same steps as in chapter 5, beginning with raw rule extraction and then rule base cleaning. Then the final rule base after cleaning is passed to the BB-BC to optimize. **Section 6.2.2.2** will explain how the rule base has been optimized.

6.2.2.1 Optimizing the Type-2 MFs with BBBC

In the previously proposed model in chapter 5, the uncertainty factor $\boldsymbol{\alpha}$ was chosen to enhance the achieved accuracy. From the previously proposed method in chapter 5 the best IT2FLS results in the highest average accuracy which is obtained by using the uncertainty factor of $\boldsymbol{\alpha}$ =10%. as shown in Figure 6.4 when the FOU percentage is $\boldsymbol{\alpha}$ =0, the type-2 system is degraded to the type-1 system. Note that the $\boldsymbol{\alpha}$ =10% is the same for all the fuzzy sets for the various inputs. We will employ the type-2 fuzzy sets $\boldsymbol{\alpha}$ =10% as the initial population for the BB-BC algorithm which will optimize the values of $\boldsymbol{\alpha}$.

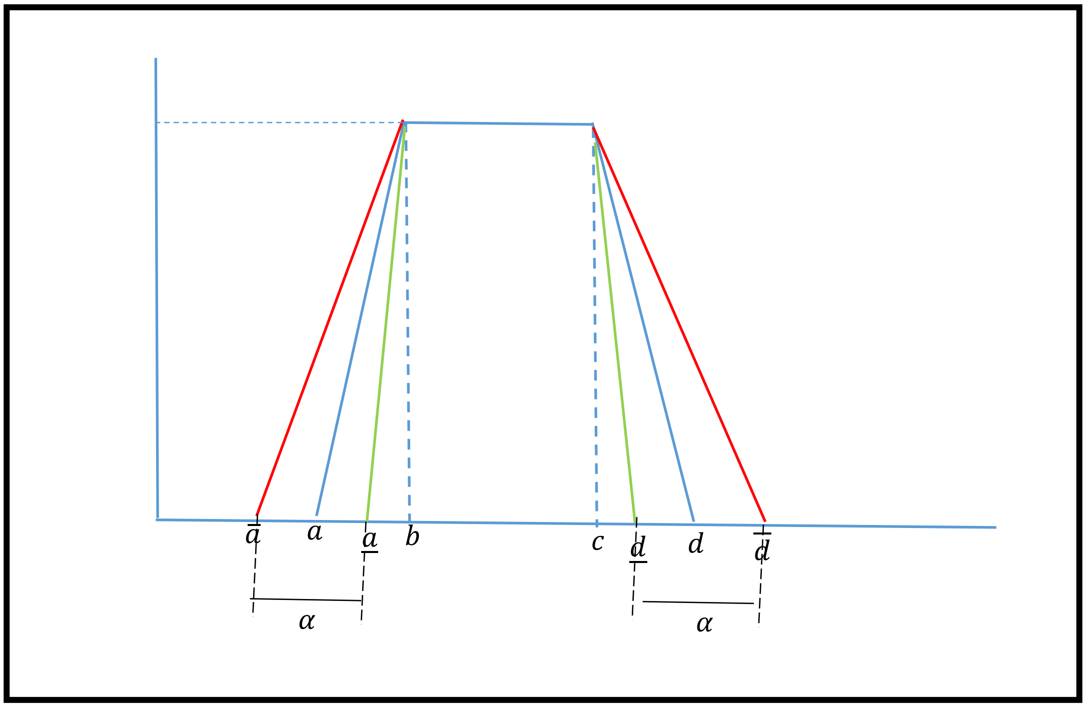


Figure 6-4 Transform T1Fs Into T2Fs

The feature parameters of type-2 MFs must be encoded into a population form in order to use BB-BC. So, we need to obtain the FOU, depicted as $\boldsymbol{\alpha_k^j}$ for each fuzzy set of the MFs. Where $j = 1, \ldots, q$, q is the number of inputs, and $k = 1\ldots, p$, p is the number of fuzzy sets per MF. For illustration, as in the MFs of the proposed system, three type-2 fuzzy sets are utilized for modeling each of the 6 features linguistic terms including, *LOW, MEDIUM and HIGH*, the total number of the parameters for the input type-2 MFs is 3×6=18. Therefore, the structure of the population is built as displayed in Figure 6.5.

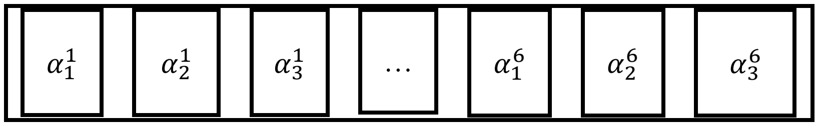


Figure 6-5 the MFs population structure

Figure 6-5 The population representation for the parameters of type-2 MFs

The Average recall defined in equation (5.26) is used as a cost function for the BB-BC algorithm.

6.2.2.2 Optimizing the Rule Base Using BB-BC:

To optimize the rule base using BB-BC the proposed model optimizes the rule base in two different levels the first one is in term of the rule base by reducing the number of rules in the rule base into a rational number of rules. The second level is in terms of the rule itself which is need to be shrinking by reducing the number of antecedents for the rule. To implement the BB-BC for the two levels; firstly the rule base parameters should be encoded in form of the population (Yao et al., 2016). The rule base can be represented as shown in Figure 5.3.

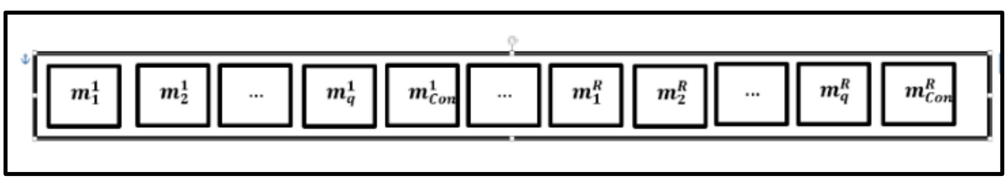


Figure 6-6encoding the rule base parameter in form of population

Figure 6-7encoding the rule base parameter in form of population

As appear in Figure 6.6 m_j^r are the antecedents and O_{Con}^R is the consequent of each rule respectively, where j = 1…, q, q is the number of antecedents; r = 1…, R, and R is the number of the rules to be tuned. However, the values describing the rule base are discrete integers while the BB-BC supports continuous values. Thus, as an alternative to equation (4.2) the following equation is used in the BB-BC model to round off the continuous values to the nearest discrete integer values modeling the indexes of the fuzzy set of the antecedents or consequents (Yao et al., 2016).

$$D^{new} = D_c + round\left[\frac{y\ p(D_{max} - D_{min})}{K}\right] \tag{6.1}$$

Where Dc is the fittest individual, r is a random number, ρ is a parameter limiting search space, D_{min} and D_{max} are lower and upper bounds, and k is the iteration step.

The rule base constructed from the modeling phase as shown in Figure 6.1 is used as the initial generation of candidates. After that the rule base can be tuned with BB-BC using the average recall defined in equation (5.26) is used as a cost function for the BB-BC since the proposed model is a binary classifier.

6.2.3 Prediction phase:

By the end of the modeling phase now the two components of the proposed model are fully optimized and the model is ready to be tested. 30% of the testing data is used in this phase. The prediction phase is similar to the prediction phase in the previously proposed model in chapter 5. See section 5.4 and no need to be repeated here again.

6.3 Experiments and Results:

We used the BBBC algorithm to reduce the size of rule bases and discover the best configuration of membership parameters in type-2 fuzzy sets to optimize the FLS. The deployment of BB-BC increases system performance by increasing prediction accuracy and system interpretability.

To test the proposed model 30% of the dataset which is dedicated to the testing phase is used. and to evaluate the performance of the proposed model 2 different experiments have been done as follows:

A. Type-1 model optimized with BB-BC.
B. Type-2 model optimized with BB-BC

As mentioned previously the Avg-Recall is used as evaluation metric Table 6.1 provides brief description for all conducted experiments. And summarizes the results extracted using testing data.

Table 6-1 the proposed optimized T2FL model extracted results

Exp #	Model Type	Avg-Recall
1	**BB-BC Optimized T1**	**0.75**
2	**BB-BC Optimized T2**	**0.84**

From Table 6.1, it can be extracted that the proposed optimized Type-2 fuzzy-based system (Experiment #2) achieved 0.84 Avg-Recall compared to 0.75 Avg-Recall achieved by optimized Type-1 fuzzy-based system counterpart. And that means the proposed optimized Type-2 model outperform optimized Type-1 model in terms of prediction accuracy. This result interprets the superiority capability of Type-2 fuzzy logic system to represent uncertainty associated with the financial domain.

Another observation that is important to note in this context is that from the conducted experiments, the computation overhead for all optimized versions is reasonable compared to non-optimized versions and this is due to rational number of rules in optimized rule base Table 6.2 shows a comparison between the optimized and non-optimized models.

Table 6-2comparison between the optimized and non-optimized models

#	Model Type	N of Rules
1	**BB-BC Optimized Type-1**	**350**
2	**Non-Optimized Type-2**	**8214**
3	**BB-BC Optimized Type-2**	**400**

From Table 6.2 The key main observation is that; the rule base generated by the non-optimized type-2 model contains 8214 rules which increased the computational cost. However, the optimized proposed model extract rule base contains only 400 IF... Then rules; furthermore, each rule contains only three antecedents compared with 11 antecedents in non-optimized counterpart and this can decrease the computational cost.

As the main advantage provided by white-box models, the optimized version of the rule base provides a transparent view of what is happening inside the model. Table 6.3 shows examples of extracted rules by the proposed optimized type-2 fuzzy logic model.

Table 6-3examples of extracted rules by the proposed optimized T2FL model

N	***Rule***
R1	*If age is **Young** &no_Dep_Child is **Mid** & income is **Low** Then class is **default***
R2	*If income is **Low** & Avg_Month_Exp is **High** & tot_Amoount is **High** Then class is **default***

If we compare the extracted rule base by the optimized type-2 model as shown in Table 6.3 with a non-optimized counterpart in Table 5.5 we can show that the optimized rule base is very short contains only 3 antecedents per single rule which is easy to read and analyze by human decision-maker. By analyzing these rules, the decision-maker can reduce the potential risks that can face the organization as well as protect customers from defaulting through advising following the analyzed information.

6.4 Summary

This chapter introduces the proposed optimized type-2 model to predict financial default risk in the sundaes banking sector. The proposed optimized model is an improvement to our previous non-optimized model discussed in chapter 5. The BB-BC has been employed to optimize the proposed model. The Optimization has been done to the two components of the proposed model. In the MFS component, the optimization targeted the parameters of the MFS to obtain a better configuration to handle a high level of uncertainty associated with the financial sector thereby achieving a good prediction accuracy. In the rule base component, the optimization has been done to the rule base in two levels, in the first level the number of rules in the rule base was reduced, and in the second level the rule itself was shrinked into only three antecedents to improve the model interpretability.

The results demonstrate the superior performance of the proposed BB-BC based T2FLS system against its counterpart T1FLS and non-optimized T2FLS regarding prediction accuracy and model transparency. Thereby one of the main objectives of this study (is labeled as (f) in this study objectives section) has been achieved.

Chapter Seven: Conclusions and Recommendation

This research looked at how artificial intelligence approaches, notably fuzzy logic, can be applied to solve predicting challenges in the banking industry. Because the banking industry is the backbone of a country's economy, thereby it has a significant impact on people's lives. This thesis mainly proposed an optimized type-2 fuzzy logic model to predict defaulting in Sudanese banking sector.

This chapter summarizes and concludes this study and based on the achieved results the findings and suggestions for further research will be given.

7.1 Conclusions:

The conclusion of this thesis provided chapter wise as follows:

In chapter1, a brief introduction to different prediction models was given. And motivation and problem statement of this study was stated after that aims and objectives of the thesis are listed, the methodology followed in this study was presented, after that contribution of the research to science was highlighted, the researcher presented ethical considerations related to the dataset used in this research, and finally, the organization of this thesis presented.

In chapter2, A brief introduction of different prediction models was given. And the researcher classified the prediction model into four different classes. Different works in any class were presented. Strengths and weaknesses of each category wear highlighted in order to state the gap in this field.

In chapter3, A theoretical background of fuzzy logic was explored, historical background of fuzzy logic was presented, then different types of fuzzy logic sets (T1FS, IT2FS, and GT2FS) wear discussed, and the equivalent fuzzy logic systems which arise from different types of fuzzy sets (type-1 and type-2 fuzzy logic sets and system) are explained. Also, the weakness of type-1 fuzzy logic in dealing with the significant level of uncertainty associated with the real-world application was discussed. As a result, we determined that a system capable of reliably addressing these uncertainties automatically and adaptively in order to adjust to variations and changes in financial application is required. Finally, the FCM algorithm was presented as parameter clustering mechanism to cluster the proposed system's numerical parameters.

In chapter4, the concept of evolution is discussed and the main evolution algorithm Big Bang- Big Crunch (BB-BC) is studied, and her superiority over genetic algorithm was presented, and finally, some applications of using the BB-BC and genetic algorithm as an optimization method in different fields were introduced.

In chapter5, the proposed type-2 fuzzy logic model to predict default in the Sudanese financial sector was introduced. To improve the prediction performance at high levels of uncertainty associated with the financial sector, the proposed prediction approach was built based on an interval type-2 fuzzy logic system. Firstly, the real data used to build the proposed model has been described, and all issues related to data cleaning and pre-processing wear clarified. The proposed model is divided into two phases modelling phase and prediction phase, each of these phases was explained in detail. Then to evaluate the proposed model, four different models wear conducted. One of them is based on type-1 fuzzy logic and the rest is based on type-2 fuzzy logic with different levels of the footprint of uncertainty (equal space, 10%, and 20%). The results of all conducted experiments showed that the proposed type-2 model achieved 83% prediction accuracy and outperformed its type-1 counterpart which achieved 69% prediction accuracy. From another point of view, another comparison was done base on the number of rules in the rule-base, all conducted models achieved a huge number of rules (5512, and 8214) for type-1 and type-2 in its best case respectively.

In chapter6, the proposed optimized type-2 model to predict financial default risk in the Sundanese banking sector was introduced. The proposed optimized model is an improvement to the previous non-optimized model discussed in chapter 5. the BB-BC was employed to optimize the proposed model. The Optimization was done to the two components of the proposed model. In the MFS component, the optimization targeted the parameters of the MFS to obtain a better configuration to handle a high level of uncertainty associated with the financial sector thereby achieving a good prediction accuracy. In the rule base component, the optimization was done to the rule base in two levels, in the first level the number of rules in the rule base was reduced, and in the second level, the rule itself was shrunk into only three antecedents to improve the model interpretability. The results demonstrated the superior performance of the proposed BB-BC-based T2FLS system against its counterpart T1FLS and non-optimized T2FLS regarding prediction accuracy and model transparency.

7.2 Recommendations:

The banking sector in Sudan suffers from many problems in the recent period especially after the revolution in December 2018, for example due to instability in the economic policies; the inflation rate is increasing reputedly. In our future works we will try to generalize our proposed optimized model in order to predict the inflation rate which can provide very valuable information to Sudanese banking sector.

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