Sudan University of Science and Technology
College of Postgraduate Studies

Optimized Detection Of Co-located Targets using MIMO RADAR System

تحسين تقنية كشف الاهداف المتقاربة باستخدام أنظمة الرادار متعدد المدخلات والمخرجات

A thesis Submitted in Partial Fulfillment of the Requirement for the Degree of M.Sc. in Electronics Engineering (Communications Engineering)

Prepared By:
Omer Ibrahim Mohammed Eisa

Supervisor:
Dr. Ebtihal Haider Gismalla Yousif

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Name of Candidate: OMER IBRAHIM MOHAMMED EISA

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Approved by:

1. External Examiner

Name: Dr. Elsadig Saad Gebreel

Signature: .... Date: 3-2-2020

2. Internal Examiner

Name: Dr. Fath Elrahman Ismael Khalifa

Signature: .... Date: 3/2/2020

3. Supervisor

Name: Dr. Ebtihal Haider Gismalla Yousif

Signature: .... Date: 3.2.2020

cgs @ sustech edu. البريد الالكتروني فاكس/ 83 769363 ص.ب 407

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Optimization of Detection and Localization of Colocated Targets in MIMO Radar Systems

أمثلة كشف وتحديد الأهداف القريبة فى أنظمة الرادار متعدد المدخلات والمخرجات

الإستهلال

بِسْمِ ٱللَّهِ ٱلرَّحْمَٰنِ ٱلرَّحِيمِ

وَإِذْ قالَ موسىٰ لِفَتىٰهُ لا أَبرَحُ حَتّىٰ أَبْلُغَ مَجْمَعَ ٱلْبَحْرَيْنِ أَوْ أَمْضِىَ حُقُبًا ﴿٦٠﴾

فَلَمّا بَلَغا مَجْمَعَ بَيْنِهِما نَسِيا حوتَهُما فَٱتَّخَذَ سَبيلَهُ فِى ٱلْبَحْرِ سَرَبًا ﴿٦١﴾

فَلَمّا جاوَزا قالَ لِفَتىٰهُ ءاتِنا غَداءَنا لَقَدْ لَقينا مِنْ سَفَرِنا هٰذا نَصَبًا ﴿٦٢﴾

قالَ أَرَءَيْتَ إِذْ أَوَيْنا إِلَى ٱلصَّخْرَةِ فَإِنّى نَسيتُ ٱلْحوتَ وَما أَنْسىٰنيهُ إِلَّا ٱلشَّيْطٰنُ أَنْ أَذْكُرَهُ وَٱتَّخَذَ سَبيلَهُ فِى ٱلْبَحْرِ عَجَبًا ﴿٦٣﴾

قالَ ذٰلِكَ ما كُنّا نَبْغِ فَٱرْتَدّا عَلىٰ ءاثارِهِما قَصَصًا ﴿٦٤﴾

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Abstract

This thesis concerns with the techniques for detection of co-located targets in the Multiple-Input Multiple-Output (MIMO) radar systems. MIMO radars achieve improved performance by the simultaneous transmission and reception of waveforms from different locations. This thesis provide an overview of MIMO radar and the advantages it offers as compared to its phased array counterpart. Also discuss detection of closer target (co-located) in MIMO radar and develop the signal model for it. Algorithms for transmit beam forming are discussed.

This thesis is proposed to enhance MIMO radar performance of the receiving signals and evaluate the probability of detection at the co-located targets situation and compared with phased array in same situation. signal processing algorithm namely Estimation of Signal Parameters via Rotational Invariance Technique (ESPRIT) based on MATLAB was used to meet the main objectives of this project. where we use convex optimization to optimize the signal covariance matrix. by using DOA estimator detector. We illustrate that the closer target parameter and transmit signal have an influence on the detector performance through target extent and the SNR respectively. The simulation results show that MIMO radar is much butter in probability of detection than phased array radar in co-located targets situation.

المستخلص

تناقش هذه الأطروحة تقنيات الكشف عن الأهداف تشاركيّة التواجد بإستخدام أنظمة الرادار متعددة المدخلات والمخرجات (ميمو). في هذا البحث تحقق أنظمة رادارات (ميمو) تحسّنا ملحوظا في الأداء عن طريق البث والإستقبال المتزامن للموجات الصوتية من عدة اتجاهات مختلفه. تقدم كذلك هذه الأطروحة لمحة عامة عن أنظمة رادارات (ميمو) ومزاياها ومقارنة مع نظيرها التقليدي ذي المصفوفات المرحليّه. أيضا تناقش الأطروحة عمليات الكشف عن الأهداف القريبة (تشاركيّة التواجد) في نظام رادار (ميمو) ومراحل تطويرها ونموذج الإشارة الخاص بها بالإضافة لخوارزميّات بث الإشعاع.

بنيت هذه الدراسة بغرض تحسين أداء أنظمة رادارات (ميمو) لإستقبال الإشارات، وأيضا لتقييم قابلية هذه الأنظمة في الكشف عن الأهداف تشاركيّة التواجد مقارنة بنظيرها التقليدي ذو المصفوفات المرحليه مع الأخذ في الإعتبار نفس ظروف الإختبار لكلا النظامين. في هذا البحث تم استخدام خوارزميات معالجة الإشارة وبصورة أخص خوارزميات تقدير معاملات الإشارة بإستخدام تقنية الثبات الدوراني (إيسبرت)، هذه الخوارزميات تم تطويرها بإستخدام برنامج (ماتلاب) الحاسوبي وإستخدمت لتلبّي أهداف هذا البحث. أخيرا، وبإستخدام كاشف محدد (DOA) تمكنا من إستنتاج ان معايير الأهداف قريبة المدى وإشارات البث لها تأثير على جودة أداء الكاشف بالنسبة لمدى الهدف و SNR على السواء. تشير نتائج المحاكاة الحاسوبية الى أن أنظمة رادارات (ميمو) أفضل في الكشف عن الأهداف تشاركيّة التواجد مقارنة بنظيرها التقليدي.

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List of Abbreviations

ESPRIT	Estimation of Signal Parameters via Rotational Invariance Technique
i.i.d.	independent and identically distributed
AWGN	Additive White Gaussian Noise
BER	Bit Error Rate
MIMO	Multiple Input Multiple Output
MISO	Multi-Input Single-Output
MUD	Multi-User Detection
MUSIC	MUltiple SIgnal Classification
PDF	Probability Density Function
RCS	Radar Cross Section
RIP	rotational invariance property
RMS	root mean square
RMSE	root mean square error
SDD	Spatial-Division Based Design
SIMO	Single-Input Multi-Output
SISO	Single-Input Single-Output
SINR	Signal to Interference plus Noise Ratio
SM	Spatial Multiplexing
SNR	Signal to Noise Ratio

List of Symbols

$\lvert\cdot\rvert$	Magnitude operator
$\lVert\cdot\rVert$	Euclidean norm operator
$(\cdot)^T$	Transposition operator
$(\cdot)^{\mathrm{H}}$	Hermitian operator
$(\cdot)^H$	hermitian operator (conjugate transpose)
M	number of array elements
$\mu(t)$	waveform emitted by Radar
r	range of target
c	speed of wave propagation
$\nu(t)$	additive noise variable
α	the amplitude response of the target
$y(\tau)$	matched filter output
(t)	additive noise
θ	direction angle
$\mathbf{a}(\theta^{'})$	transmitted vectors
$\mathbf{H}$	channel matrix
H_0	null hypothesis
H_1	alternate hypothesis

Chapter one
Introduction

1.1 Overview

Radar is an electromagnetic sensor for the detection and location of reflecting objects. Radar was originally an acronym for “RAdio Detection And Ranging”.Fig 1.1 bellow explain radar operation,Radar operation can be summarized as Radar radiates electromagnetic energy from an antenna to propagate in space , Some of radiated energy is intercepted by a reflecting object (target),The energy intercepted by target reradiated in many directions,Some of the reradiated (echo) energy is turned to and received by the radar antenna , After amplification and processing the target location and possibly other information about the target is acquired [1].

Radar systems can be categorized into monostatic and bistatic. The transmitter and the receiver of monostatic radar are located in the same location while the transmitter and receiver of the bistatic radar are far apart relative to the wavelength used in the radar. According to the characteristics of the transmitted signals, the radar systems can be further categorized into continuous waveform radar and pulse radar. The continuous waveform radar

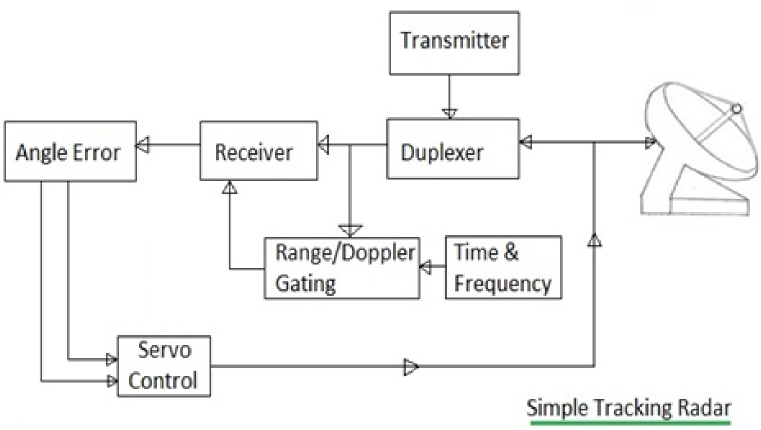

Figure 1.1: Radar Operation [2]

transmits a single continuous waveform while the pulse radar transmits multiple short pulses [3]. Detection is the most fundamental function of a radar system. After emitting the electromagnetic waveform, the radar receives the reflected signal. To detect the target, it is necessary to distinguish the signal reflected from the target, from the signal containing only noise. After detecting the target, one can further calculate the range. In the radar community the word range is used to indicate the distance between the radar system and the target [4].

Consider a monostatic radar system with one antenna as shown in Fig 1.2 .The radar emits a waveform $\mu(t)$ into the space. The waveform hits the target located in range r and comes back to the antenna. After demodulation, the received signal can be expressed as:

$$R(t) = \alpha\mu\left(t - \frac{2r}{c}\right) + v(t) \tag{1.1}$$

Where c is the speed of wave propagation, $r$is the range of the target, (t) is the additive noise, and α denotes the amplitude response of the target. The amplitude response μ is determined by the radar cross section (RCS) of the target, the range r of the target, the beam pattern of the antenna, and the angle of the target. In the receiver, a matched filter is usually applied to enhance the signal-to-noise ratio (SINR). The matched filter output can be expressed as

$$y(\tau) = \int_{-\infty}^{\infty} \alpha\mu^*(t - \frac{2r}{c})dt + \int_{-\infty}^{\infty} v(t)\mu^*(t-\tau)dt \tag{1.2}$$

$$y(\tau) = \alpha r_{uu}\mu^*(t - \frac{2r}{c}) + \int_{-\infty}^{\infty} v(t)\mu^*(t-\tau)dt \tag{1.3}$$

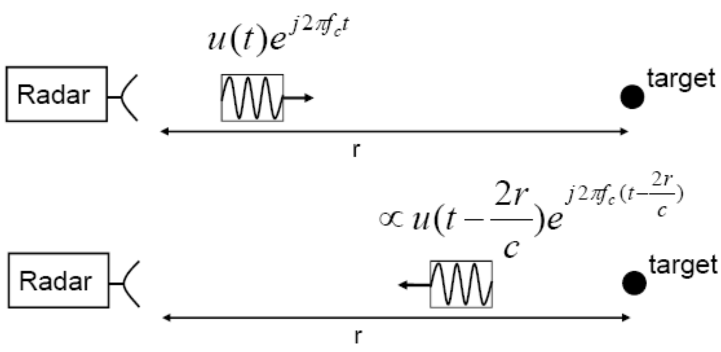

Figure 1.2: Detection and ranging

Multiple Input Multiple Output (MIMO) radar define as a radar system deploying multiple antennas to simultaneously transmit arbitrary waveforms and utilizing multiple antennas to receive signals which are then processed jointly [3].As shown in Fig 1.3.

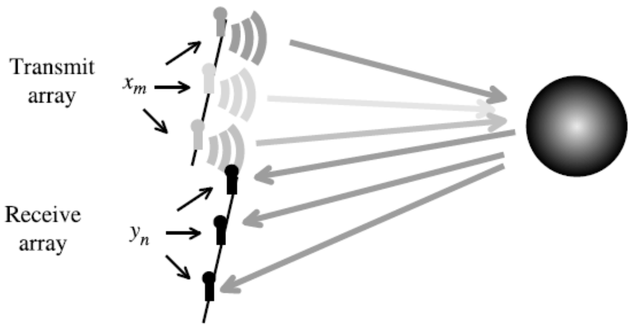

Figure 1.3: MIMO Radar

Every antenna element in a MIMO radar system transmits different waveforms. These may be orthogonal, mutually uncorrelated or simply linearly independent. This is called waveform diversity and it is a distinguishing property of MIMO radar [3]. To benefit from this diversity, in every MIMO radar receiver, there are as many matched filters as the number of transmitted signals. The target returns are passed through these filters matched to every transmitted signal [4].

1.2 Problem Statement

MIMO radar is growth rapidly and the needs for services which are different according to the development of applications today.For this reason the researches concentrate to finding solves of those problems facing the development, the spectrum efficiency and detection with minimum SINR is hot issue for all Radar communication system [5]. Localization of detected moving targets is main function of MIMO radar. However there are a big challenge when a closer targets (co-located) condition. In this case the hazard of incident (or may be accident) will increase. When the targets are relative close to each other appeared as one target. The safe air space with less aircraft collision probability.

1.3 Proposed Solution

The study is focused to detect closer target (co-located) of the receiving signal and evaluated the probability of detection at the receiver end of MIMO Radar.

1.4 Motivation

MIMO radar has provided a new paradigm for signal processing research. The promising capabilities of a MIMO radar has drawn the attention of engineers and researchers throughout the globe. The waveform diversity in MIMO radar offers superior capabilities as compared to a standard phased array radar [6]. Some of these are:

- Improved target detection capability Enhanced accuracy in angle estimation
- Lower minimum detectible velocity
- Direct applicability of adaptive algorithms
- Enhanced spatial diversity gain
- High degree of flexibility in designing beampattern

One of the most important aspect of MIMO radar is the flexibility it offers in designing the transmit beampattern. The transmit beamforming methods which are based on optimizing the signal covariance matrix can use different methods for optimization. Thus it very interesting research area to look for more accurate and faster optimization algorithms which give closed form solutions. Developing methods for real time beampattern synthesis for tracking targets and generalizing the beampattern synthesis algorithms for both narrow as well as wide band signals are areas in which much work remains to be done. Another interesting and highly worthy area to be explored is design of fixed cross-correlation constant modulus signals. Thus from both mathematical as well as theoretical perspective, MIMO radar offers a highly interesting area of research and in the present thesis work we shall explore some of the methods and algorithms related to transmit beampattern synthesis.. We shall be concerned only with the narrow band probing signals.

1.5 Objective

The main objective of this thesis is to:

- Decrease the aircraft accident probability and this will lead to increase air space safety.
- To obtain better performance, reduce interference effects to acceptable level and give a minimum bit error rate.

1.6 Methodology

After extensive literature review, mathematical models for MIMO radar will be derived and then these equations are combined with co-located targets by using estimate Direction of Arrival (DOA) algorithm by using ESPRIT Techniques, The system is simulated in MATLAB program and the results will compared with linear array radar localization method.

1.7 Thesis Outlines

The rest of this thesis is organized as follows. Chapter two provide a literature review of MIMO radar beside give an idea of detection of targets and co-located target, Chapter three presents the employed methods of using MIMO Radar, Chapter four presents the simulation and discuss the results. Finally, chapter five concludes the thesis and provides recommendations of the future work.

Chapter Two
Background and Literature Review

2.1 Introduction

In this chapter we briefly introduce the basic concepts of MIMO , radar and MIMO radar and also introduce the target types detected by radar with a brief introduce concept of closer target (Co-located) . We give an idea of the state of current MIMO radar research through a brief survey of relevant literature.

Since Christian Hulsmeyer patented and demonstrated a spark-gap transmitter and. parabolic reflecting receive antenna in 1902, intended to help ships avoid colliding with objects, the field of radar has been long and well studied. Remarkably, with the advances of computational power and the resulting improved processing capabilities, the field continues to evolve. An example of this evolution is multiple-input multiple-output (MIMO) radar, which is considered here. While many of the approaches exemplified by MlMO radar have been long employed without need of the "MIMO nomenclature, recognizing this connection can help motivate new radar concepts. MlMO radars can increase the number of available degrees of freedom. These degrees of freedom can be exploited to improve resolution, clutter mitigation, and classification performance [4].

2.2 Multiple Antennas Techniques

Multiple inputs multiple output (MIMO) systems. Is the systems with multiple antennas at the transmitter and receiver, the multiple antennas can be used to increase data rates through multiplexing or to improve performance through diversity.Fig 2.1 shown bellow explain SISO,SIMO,MISO and MIMO. In MIMO systems the transmitting and receiving antennas can both be used for diversity gain. This also increases spectral efficiency of the electromagnetic waves. In addition to spectral efficiency gains, ISI and interference from other

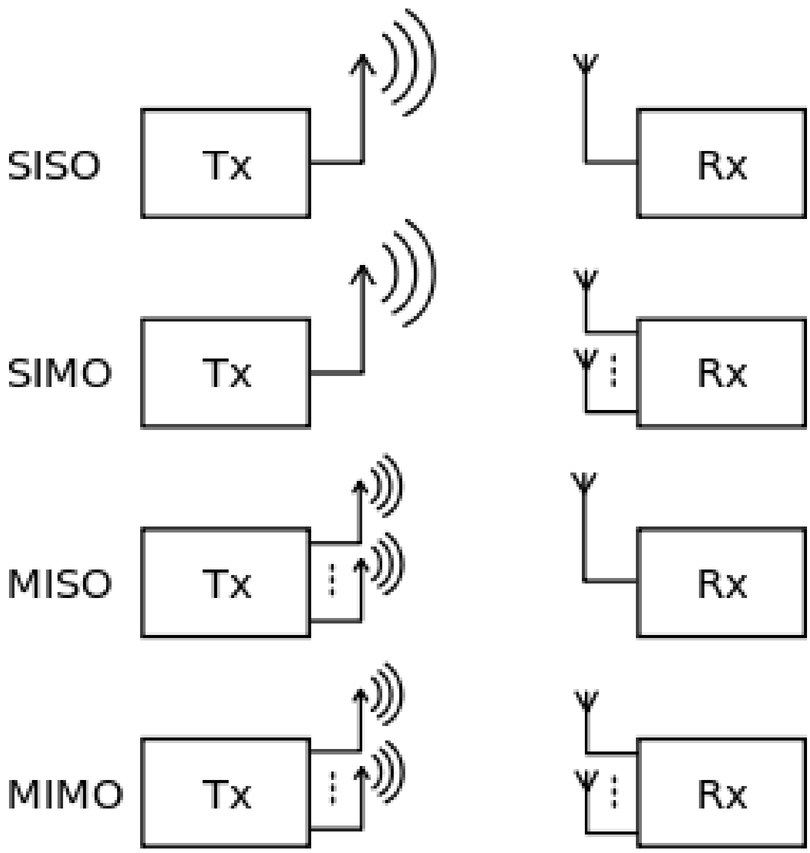

Figure 2.1: SISO, SIMO, MISO and MIMO

users can be reduced using smart antenna techniques. The cost of the Performance enhancements obtained through MIMO techniques is the added cost of deploying multiple antennas, the space and power requirements of these extra antennas, and the added complexity required for multi-dimensional signal processing [7].

2.3 Phased Array Radar

Phased Array Radar uses uniform linear arrays for transmitting and receiving signals. In case of phased array radar each antenna is allowed to transmit only scaled versions of the same waveform .

a MIMO radar is a natural extension of the phased array antenna that has been used by radar systems for decades . A phased array consists of a number of radiating elements. Each element transmits the same signal except that a phase shift (or time delay) is applied so that a beam is steered in a particular direction. In a phased array,the signals transmitted from each element are perfectly correlated. A MIMO radar is a generalization of a phased array in that the signals need not be correlated from element to element [8] .

2.3.1 DOA Estimation

The DOA algorithms are classified as quadratic type and subspace type The Barltett and Capon (Minimum Variance Distortionless Response) are quadratic type algorithms. The both methods are highly dependent on physical size of array aperture, which results in poor resolution and accuracy. Subspace based DOA estimation method is based on the eigende composition. The subspace based DOA estimation algorithms MUSIC and ESPRIT provide high resolution, they are more accurate and not limited to physical size of array aperture. The various DOA algorithm performance is analysed based on number of snapshots, number of users, user space distribution, number of array elements, SNR and MSE [9].

2.3.1.1 MUSIC Estimation

MUSIC is an acronym which stands for Multiple Signal classification. It is high resolution technique based on exploiting the eigenstructure of input covariance matrix as shown in Fig 2.2. MUSIC makes assumption that the noise in each channel is uncorrelated making correlation matrix diagonal. The incident signals are somewhat correlated creating non diagonal signal correlation matrix [10].

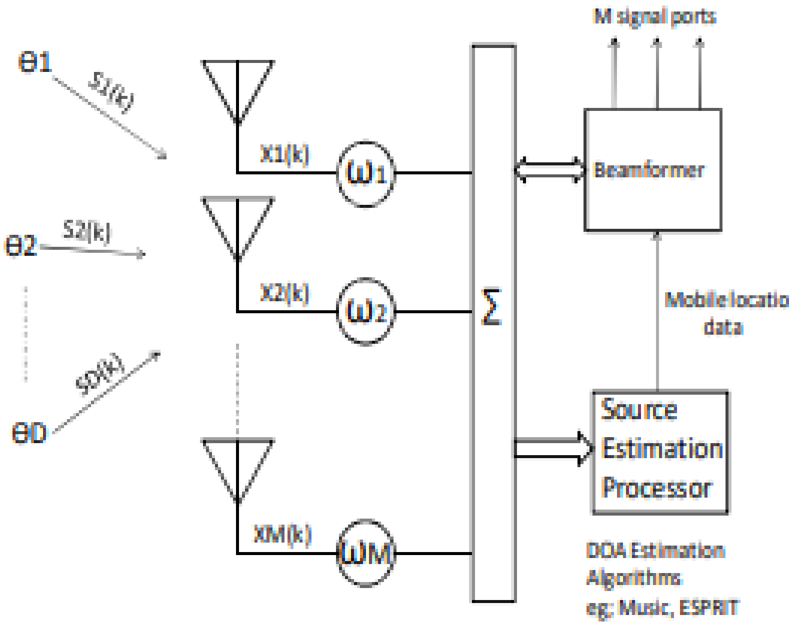

Figure 2.2: M element antenna array with D arriving signals

If the number of signals impinging on M element array is D, the number of signal eigenvalues and eigenvectors is D and number of noise eigenvalues and eigenvectors is M-D.

2.3.1.2 ESPRIT Estimation

ESPRIT achieves a reduction in computational complexity by imposing a constraint on the structure of an array. The ESPRIT algorithm assumes that an antenna array is composed of two identical subarrays (see Figure). The subarrays may overlap, that is, an array element may be a member of both subarrays [11]. If there are a total of M elements in an array and m elements in each subarray, the overlap implies that $M \leq 2m$. For subarrays that do not overlap, $M = 2m$. The individual elements of each subarray can have arbitrary polarization, directional gain, and phase response, provided that each has an identical twin in its companion subarray. Elements of each pair of identical sensors, or doublet, are assumed to be separated physically by a fixed displacement (translational) vector. The array thus, displacement vectors). This property leads to the rotational invariance of signal subspaces spanned by the data vectors associated with the spatially displaced subarrays the invariance is then utilized by ESPRIT to find DOAs [12].

2.4 MIMO Radar

The notion of MIMO radar is simply that there is multiple radiating and receiving sites,Collected information is then processed together. In some sense, MIMO radars as shown in Fig 2.3 are a generalization of multistatic radar concepts. The underlying concepts have most likely been discovered independently numerous times [3]. In the traditional phase array radar, the system can only transmit scaled versions of a single waveform. Because only a single waveform is used, the phase array radar is also called SIMO (single input multiple output) radar in contrast to the MIMO radar. We will use "SIMO radar" or "phase array radar" alternatively throughout the thesis.

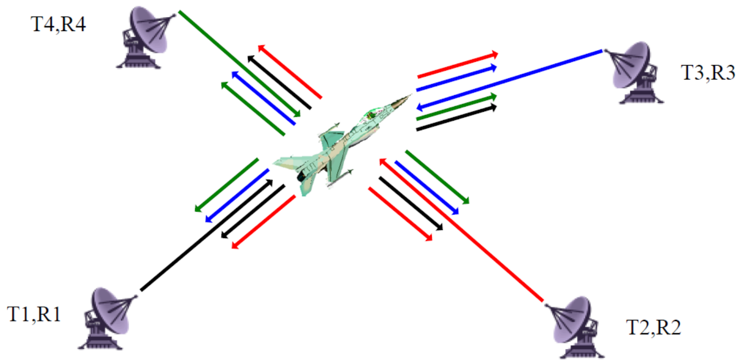

Figure 2.3: MIMO radar

The MIMO (multiple-input multiple-output) radar system allows transmitting orthogonal (or incoherent) waveforms in each of the transmitting antennas [13] . These waveforms can be extracted by a set of matched filters in the receiver. Each of the extracted components contains the information of an individual transmitting path. There are two different kinds of approaches for using this information. First, the spatial diversity can be increased. In this scenario, the transmitting antenna elements are widely separated such that each views a different aspect of the target. Consequently the target radar cross sections (RCS) are independent random variables for different transmitting paths. Therefore, each of the components extracted by the matched filters in the receiver contains independent information about the target. Since we can obtain multiple independent measurements about the target, a better detection performance can be obtained. Second, a better spatial resolution can be obtained. In this scenario, the transmitting antennas are co-located such that the RCS observed by each transmitting path are identical. The components extracted by the matched filters in each receiving antenna contain the information of a transmitting path from one of the transmitting antenna elements to one of the receiving antenna elements. By using the information about all of the transmitting paths, a better spatial resolution can be obtained. The phase differences caused by different transmitting antennas along with the phase differences caused by different receiving antennas can form a new virtual array steering vector. With judiciously designed antenna positions, one can create a very long array steering vector with a small number of antennas. Thus the spatial resolution for clutter can be dramatically increased at a small cost [13] [7].

2.5 Radar Targets Techniques

The objective of every radar to detect desired targets for example weather radar interesting in change in weather and dense cloud also ATC radar interesting in aircraft , even radar classified depend on detected targets such as fixed or moving target radar or ground moving target indication (GMTI).

2.5.1 Parameters of The Target Position

There are three main parameters of the target position. For a better explanation, these parameters are illustrated on the Fig 2.4. The angle, which is formed by the horizontal projection of the line R–T and the North direction is called azimuth. Where R represents the radar and the T represents the target and N direction is the direction of the north. The angle , which is formed by the horizontal plane and the line R–T is called elevation. The line R–T is the distance from the target. Using this three parameters we locate the position of the target in the space [14] .

2.5.2 Co-located Targets

When there are targets near each other's or their locations similar to be one location as shown in Fig 2.5 . The outstanding features of the signal returned from complex targets (its fluctuations and its wavelength dependence) can be studied by considering a simple model consisting of two equal isotropic targets a distance l apart. This distance is assumed to be smaller than $cr/2$, where r

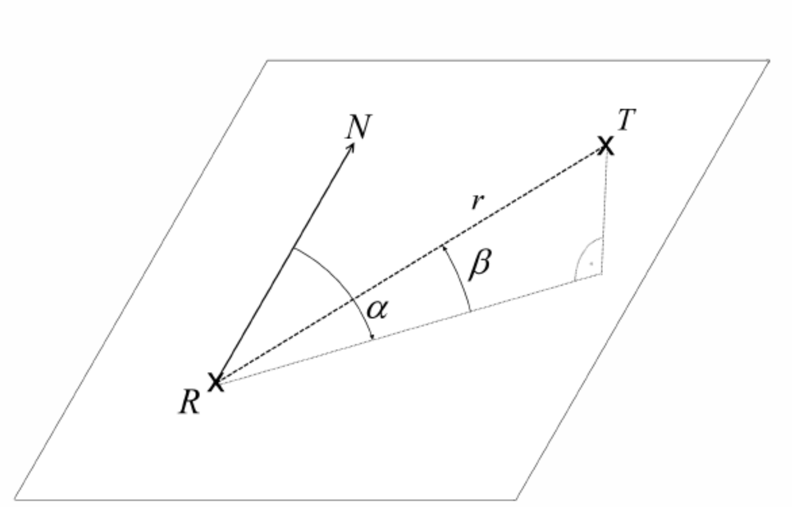

Figure 2.4: Parameters of the Target Position [14]

is the pulse duration, so that the signals overlap . The ratio of the received power from the two targets to that which would be received from one of the targets alone.

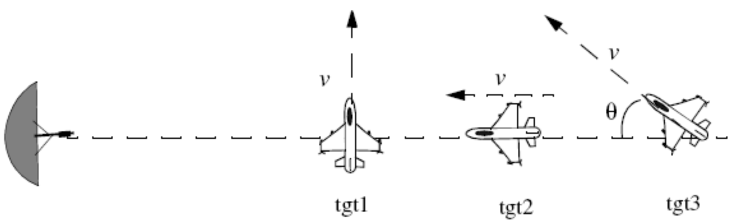

Figure 2.5: Co-located targets

2.6 Related Studies

Stoica et. al, [15], study beam pattern design problems for MIMO radar systems. They Focus on transmit power around the locations of the targets of interest while minimize the cross-correlations of the signals reflected back to the radar which give good results. However the authors proved that the MIMO radar is much butter transmit beam pattern than the phase array radar, but not consider the moving target detection .

Q.He et. al, [16], study the performance of MIMO radar for moving target detection problems which give good results. However the author compare in target detection between the MIMO radar and the phase array radar. Also, they only consider the results for homogeneous clutter. Gorji, [17], did compare targets localization techniques between the MIMO radar and the multistatic radar. The author approved that at low target SNR, MIMO radar provides better localization results than multistatic radars.

Nion and Sidiropouls, [18], study PARAFAC-BASED technique for detection and localization of multiple targets in a MIMO radar system, which give good results. But the paper not discusses any other techniques. Furthermore, Gorji et.al, [19], study MIMO radar waveform optimization with prior Information of the extended target and clutter developed new model to estimate DOA for multi-target tracking and Localization, which give good results. But the authors not discuss co-locating targets condition. From the above reviews none of the authors suggest to work in co-located targets condition.

Chapter Three
System Model and Formulation Approach

3.1 Introduction

In this Chapter, we explain the system model of MIMO Radar of this research , then evaluating the probability of detection in MIMO Radar and evaluating the Beamspace of MIMO Radar.

3.2 System Model

The radar we consider in this research is a colocated MIMO radar with M transmit and receive antennas and is mounted on a ship.with a transmitter being uniform phased array (UPA) of M collocated antennas with inter-element spacing measured in wavelength as show in Fig 3.1, and a receive array of antennas configured in a random shape. The colocated MIMO radar has antennas that have spacing on the order of half the wavelength. also we consider a point target model which is defined for targets having a scatterer with infinitesimal spatial extent.

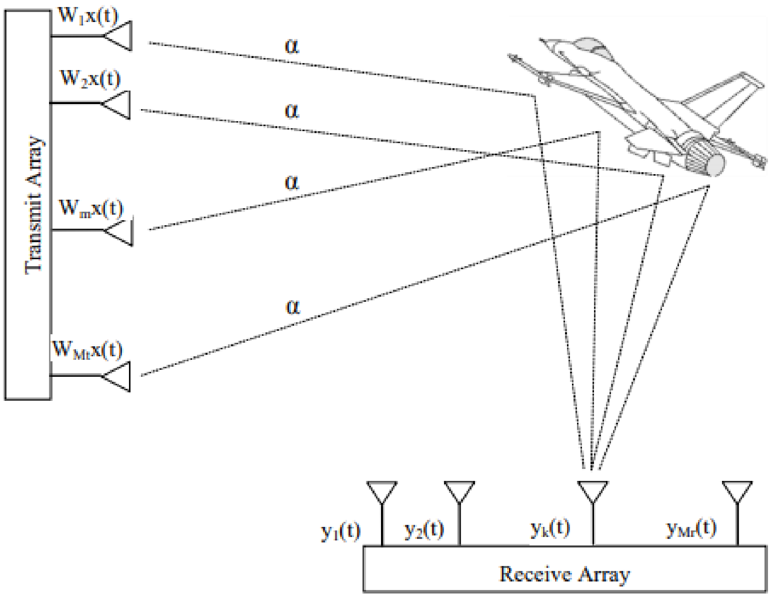

Figure 3.1: Phased Array Radar Configuration [20]

3.3 Detection of coherent MIMO Radar

as previous section we assumption that MIMO radar system, that has transmitter and receiver array consisting of M and N elements. Let $\sqrt{(E_t/M)}X_m(t)$ be transmitted by the m-th transmit antenna. $x(t)$ is receive signals. If the transmit array performs transmit beamforming in the direction of $\theta^{'}$ the transmitted signal can be written in the vector form as

$$x(t) = a(\theta^{'})\sqrt{(\frac{E_t}{M})X_m(t))} \tag{3.1}$$

where $a(\theta^{'})$ is a transmitted vectors. When there is a stationary target at the far field of the arrays in the direction of θ the propagation is nondispersive the signal at the target location [21],$x^t(t)$, can be written as

$$x^t(t) = a^H(\theta)x(t-\tau_t) = a^H(\theta)a(\theta^{'})\sqrt{(\frac{E_t}{M})X_m(t\tau_t))}. \tag{3.2}$$

Since the antenna elements of the phased array radar are closely spaced, all transmit receive pairs see the same bi-static RCS

$$y^r = \alpha b(\theta^{'})a^H(\theta)a(\theta^{'})\sqrt{(\frac{E_t}{M})X_m(t\tau_t)) + n(t)} \tag{3.3}$$

where τ represents the total time delay between transmitter and receiver,$n(t)$is a zero mean vector of complex random processes. If the received signal is applied to a filter matched to $x(t)$, and the output is sampled at τ time instant, the output of the matched filter becomes

$$y = \alpha b^H(\theta^{'})b^{(}\theta^{'})a^H(\theta)a(\theta^{'})\sqrt{\frac{E_t}{M}X_m(t\tau_t)) + n(t)} \tag{3.4}$$

For the case of phased array radar $M \times N$ a channel matrix $\mathbf{H}$ can defined as

$$\mathbf{H} = \mathbf{b}^H(\theta^{'})\mathbf{a}^H(\theta)\mathbf{a}(\theta^{'})\alpha \tag{3.5}$$

If the direction of the target with respect to arrays is known and transmit and receive beams,resulting a coherent processing gain [22] of $M \times N$. Then the received signal model becomes

$$y = \sqrt{(\frac{E_t}{M})M \times N\alpha + n} \tag{3.6}$$

3.3.1 Detection In Phased Array Radar

The detection problem in phased array radar can be formulated as binary hypothesis testing problem:

$$H_0: \quad y = w \tag{3.7}$$

$$H_1: \quad y = \sqrt{\frac{E_t}{M}} M \times N\alpha + n \tag{3.8}$$

The probability density of under hypothesis can be written as

$$p\left(y \mid H_0, \sigma_w^2\right) = \frac{1}{\pi N \sigma_w^2} \exp\left(-\frac{|y|^2}{N\sigma_w^2}\right) \tag{3.9}$$

$$P_{fa} = \text{Prob}\left\{\exp\left(-\frac{1}{N\sigma_w^2}\right) > T^{'}\right\} \tag{3.10}$$

The P_d namely the probability of detection can be calculated in terms of threshold

$$P_d = \exp\left(\frac{\ln(P_{fa})}{\text{SNR}MN + 1}\right) \tag{3.11}$$

3.3.2 Algorithm of Detection In Phased Array Radar

Algorithm 1 show that the probability of detection P_d of MIMO Radar function of SNR, for a given value of the probability of false alarm and number of receiver N .

Algorithm 1 Search algorithm used in computing the probability of Detection in Coherent MIMO Radar

procedure $(\{M, P_{fa}, \mathbf{H}_0, \mathbf{H}_1\})$
  for $i \leftarrow 1, 4, \text{length}(N)$ **do**
    Obtain the probability of Detection Eq(3.11);
  end for
  return Output=P_d
end procedure

3.4 Transmit Beamspace Design

In this section we design transmit Beamspace.The transmit energy focusing can be achieved using an $M \times K$ transmit Beamspace matrix W where $k \leq M$

is the number of orthonormal basis waveforms [23]. Then the $M \times 1$ vector of transmitted signals can be expressed as

$$s(t) = \mathbf{W}\phi(t) \quad 0 \leq t \leq T \tag{3.12}$$

where $\phi(t) = [\phi_1(t), ..., \phi_k(t)]$ is the set of orthonormal basis waveforms such that $\int_0^T \phi(t)\phi^H(t) = \mathbf{I}_K$. The transmit power distribution pattern can be expressed

$$G(\theta) = d^H Rd(\theta) \tag{3.13}$$

$$\mathbf{R} = \int_0^{T_P} s(t)s^H(t)dt, -pi/2 \leq \theta \leq \pi/2 \tag{3.14}$$

the main goal is transmission power on a desired sector, i.e., the sector where the targets are located, results in significant improvement in the DOA estimation performance [23] [24].to design a transmit beamspace matrix which achieves a spatial beampattern that is as close as possible to a certain desired one.the spatial beampattern can be rewritten as

$$G(\theta) = \sum_{i=1}^{k} w_i^H d(\theta)d^H(\theta)w_i \tag{3.15}$$

Now we use the minimax criterion , to design the transmit beamspace matrix based on minimizing the difference between the desired beampattern given in Eq(3.11) and the actual beampattern,which satisfies the aforementioned requirements can be formulated as the following optimization problem

$$\min_{w} \max_{\theta} \qquad \left| G_\theta(t) - \sum_{i=1}^{k} w_i^H d(\theta)d^H(\theta)w_i \right| \tag{3.16a}$$

$$s.t. \qquad \sum_{i=1}^{k} |w_j|^2 = \frac{P_t}{M} \qquad j = 1, ..., k \tag{3.16b}$$

where $G_\theta(t), \theta$ from $-\pi/2$ to $\pi/2$ is the desired beampattern and P_t is the total transmit power. To develop a simple practical DOA estimation algorithm based on transmit beamspace preprocessing, we consider in this research the special case of two orthonormal waveforms. Thus, the dimension $\mathbf{W}$ is $M \times 2$ Then under the aforementioned assumption of ULA at the MIMO radar transmitter, the RIP can be satisfied by the matrix

$$\mathbf{W} = [\mathbf{W}, \tilde{\mathbf{W}}^*] \tag{3.17}$$

where $\mathbf{W}$ is is the flipped version of vector $\mathbf{W}$.

3.4.1 Two Transmit Waveforms

Assuming that the number of the transmit antenna elements M is even, the optimization problem Eq(3.16a) and Eq(3.16b) can be equivalently rewritten as

$$\min_{w} \max_{\theta} \quad |G_{\theta}(t) - \sum_{i=1}^{k} w_i^H d(\theta) d^H(\theta) w_i| \tag{3.18a}$$

$$s.t. \quad \sum_{i=1}^{k} |w_j|^2 = \frac{P_t}{M} j = 1, ..., k \tag{3.18b}$$

$$|W_k^H d(\theta)| = |W_{\frac{k}{2}+k}^H d(\theta)| \tag{3.18c}$$

Substituting (3.16a) in (3.16b)–(3.17) and introducing the auxiliary variable , we can optimization problem (3.16b)–(3.17) can be reformulated as follows for the case of two transmit waveforms and when the number of transmit antennas is even

$$\min_{\delta,\theta} \quad \delta \frac{G(\theta_q)}{2} - \left|W^H d(\theta_q)\right| \leq \delta, \qquad q = 1, ..., Q \tag{3.19a}$$

$$\frac{G(\theta_q)}{2} - |W^H d(\theta_q)| \geq -\delta, \qquad q = 1, ..., Q \tag{3.19b}$$

$$|w(i)|^2 + |w(M-1+i)|^2 = \frac{P_t}{M}, \qquad i = 1, ..., \frac{k}{2} \tag{3.19c}$$

3.4.2 Even Number of Transmit Waveforms

Let us consider now the $M \times K$ transmit beamspace matrix $\mathbf{w} = [\mathbf{w}_1, ..., \mathbf{w}_k]$ where $k \leq M$ and k is even number In the previous subsection, we saw that by considering the specific structure $[\mathbf{w}, \mathbf{w}^*]$ for the transmit beamspace matrix with only two waveforms, the RIP is guaranteed at the receive antenna array. In this part, we obtain the RIP for the more general case of more than two waveforms. It provides more degrees of freedom for obtaining a better performance. For this goal, we first show that if for some $k^{'}$ the following relation holds

$$\left|\sum_{i=1}^{k^{'}} d^H(\theta) w_i(i)\right| = \left|\sum_{i=1+k^{'}}^{k} d^H(\theta) w_i(t)\right| \tag{3.20}$$

then the two new sets of vectors defined as the summation of the first $k^{'}$ data vectors specifically, by defining the following vectors

$$g_1(\tau) = \sum_{i=1}^{k^{'}} y_i(\tau) = \sum_{l=1}^{L} \beta_l(\tau) \sum_{i=1}^{k^{'}} d^H(\theta) w_i(i) b(\theta_l) + \sum_{i=1}^{k^{'}} z_i(\tau) \tag{3.21}$$

$$g_2(\tau) = \sum_{i=1}^{k'} y_i(\tau) = \sum_{l=1+k'}^{L} \beta_l(\tau) \sum_{i=1+k'}^{k'} d^H(\theta) w_i(i) b(\theta_l) + \sum_{i=1+k'}^{k'} z_i(\tau) \tag{3.22}$$

Based on the latter fact, a simple structure on the beamspace matrix **w** which guarantees the satisfaction of the (3.24) for any arbitrary is as follows:

- K is an even number.
- k' is equal to $k/2$.
- $\mathbf{W} = \tilde{\mathbf{W}}^*_{k'+i}, i = 1, ..., k/2$

if the transmit beamspace matrix has the following structure

$$\mathbf{w} = \begin{bmatrix} \mathbf{w}_1 & ... & \mathbf{w}_{k/2} & \tilde{\mathbf{W}}^*_1 & ... & \tilde{\mathbf{W}}^*_{k/2} \end{bmatrix} \tag{3.23}$$

Substituting Eq(3.27) in Eq(17)–(19), the optimization problem of transmit beamspace matrix design can be reformulated as

$$\min_{w} \max_{\theta} \quad \left| G_\theta(t) - \sum_{i=1}^{k/2} \|[w_k \tilde{\mathbf{W}}^*_k]^H\|^2 d(\theta_q) \right| \tag{3.24}$$

$$\sum_{k=1}^{k/2} |w(i)_k|^2 + |w_k(i)|^2 = \frac{P_t}{M}, \qquad i = 1, ..., \frac{k}{2} \tag{3.25}$$

where $|.|$ denotes the Euclidean norm. the following similar steps as in the case of two transmit waveforms, the problem Eq(3.28)–Eq(3.30) can be equivalently rewritten as

$$\min_{\mathbf{X}_k} \max_{\theta_q} \quad \left| \frac{G_{\theta_q}}{2} - \sum_{i=1}^{k/2} d^H(\theta) d^H(\theta_q) \mathbf{X}_k \right| \tag{3.26}$$

$$\sum_{k=1}^{k/2} \mathbf{X}_k \mathbf{A}_i = \frac{P_t}{M}, \qquad i = 1, ..., \frac{M}{2} \tag{3.27}$$

$$\text{rank}\left\{\mathbf{X}_k\right\} = 1, \qquad k = 1, ...k/2 \tag{3.28}$$

3.4.3 Spatial-Division Based Design (SDD)

It is worth noting that instead of designing all transmit beams jointly, an easy alternative for designing **W** is to design different pairs of beamforming vectors $[\mathbf{W}, \tilde{\mathbf{W}}^*]$ in order to avoid the incoherent summation of the terms in $\sum_{k=1}^{k/2} d^H(\theta_l) w_i$ This alternative design is referred to as the SDD method [25].

3.4.4 Algorithm of transmit beampattern of MIMO Radar

In algorithm 2 to calculate transmit beampattern of MIMO Radar and The probability of resolve two closely located targets versus SNR with parameter as given table 4.1

Table 3.1: Parameter values

Parameters	Values
Number of transmit antenna (M)	10
Number of receive antenna (N)	10
Transmitter spacing in wavelength (d)	0.5m
Number of radar pulses (T)	50
Signal to noise ratio SNR Rang	[-30:30]
DOA Estimator	ESPRIT
Montcarlo	500

Algorithm 2 Search algorithm used in computing Beamspace for MIMO Radar

procedure $(\{M, N, d, T)$
    Initialize Directional of targets to broadside of the array.
    Initialize uplink steering vectors.
    Initialize downlink steering vectors.
    Initialize Mean Square Error(MSE) according to given rang SNR.
    for $i \leftarrow 1, 500$ **do**
        Generate Additive white Gaussian Noise (AWGN).
        for $J \leftarrow 1, length(T)$ **do**
            Generate number of radar pulses (slow-time) and add AWGN.
        end for
        for $SNR \leftarrow 1, length(SNR)$ **do**
            Compute signals observed at targets
            Compute Transmit beampatterns of the MIMO Radar, according to Eq(3.19).
            Compute Transmit beampatterns of the MIMO Radar, according to Eq(3.26).
            Compute Transmit beampatterns of the MIMO Radar, according to **SDD** .
        end for
        Calculates **cross-correlation matrix** according to Eq(3.14).
        Calculates the **ESPRIT** according to given beamspace in previous calculated beampatterns.
        Estimates DOA and Resolution calculation according to **ESPRIT** techniques.
    end for
end procedure

Chapter Four
Results and Discussion

4.1 Detection in Coherent MIMO Radar

Figure 4.1 shows the probability of detection versus the signal-to-noise ratio. The simulation is conducted for three cases of changing value of $M = \{1, 6, 11\}$ for a fixed value of $N = 5$. It can be seen that for a fixed value of SNR, the probability of detection increases as the value of M is increased. In general for any given value of M, increasing the SNR will increase the probability of detection.

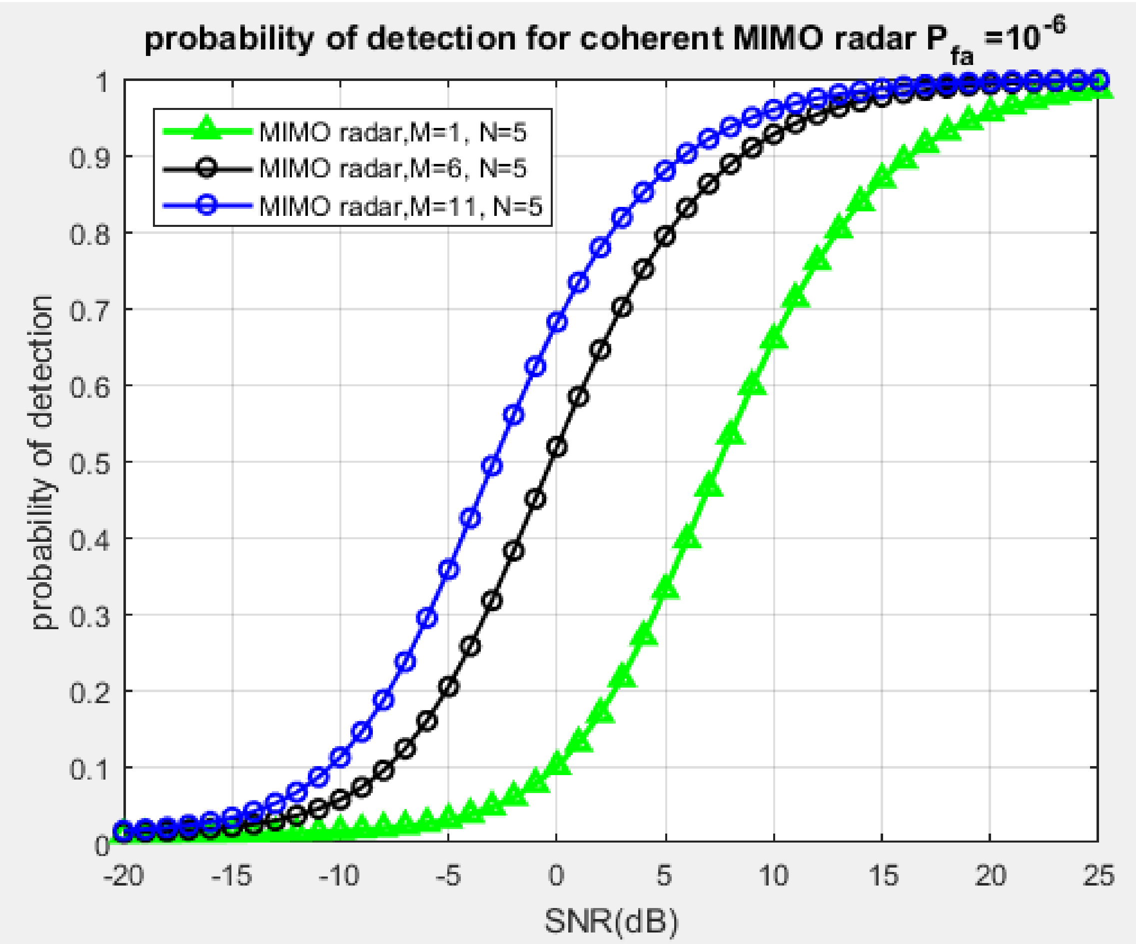

Figure 4.1: Coherent MIMO Radar: The performance of Changing N

4.2 Transmit beamspace design

Consider a MIMO radar which consists of a ULA of $M = 10$ omni-directional transmit antennas spaced half a wavelength apart from each other and an arbitrary array of $N = 10$ receive antennas whose locations are drawn uniformly within the interval $[0, 4.5]$ measured in wavelength. Temporally and spatially independent white Gaussian noise with zero mean and unit variance is assumed. Two targets are located in the directions -5^o and 5^o. The total transmit energy $E = 1$ is taken, the total number of 50 snapshots are used to compute the sample covariance matrix for all methods tested, and the RMSEs for all methods are computed based on 500 independent runs.

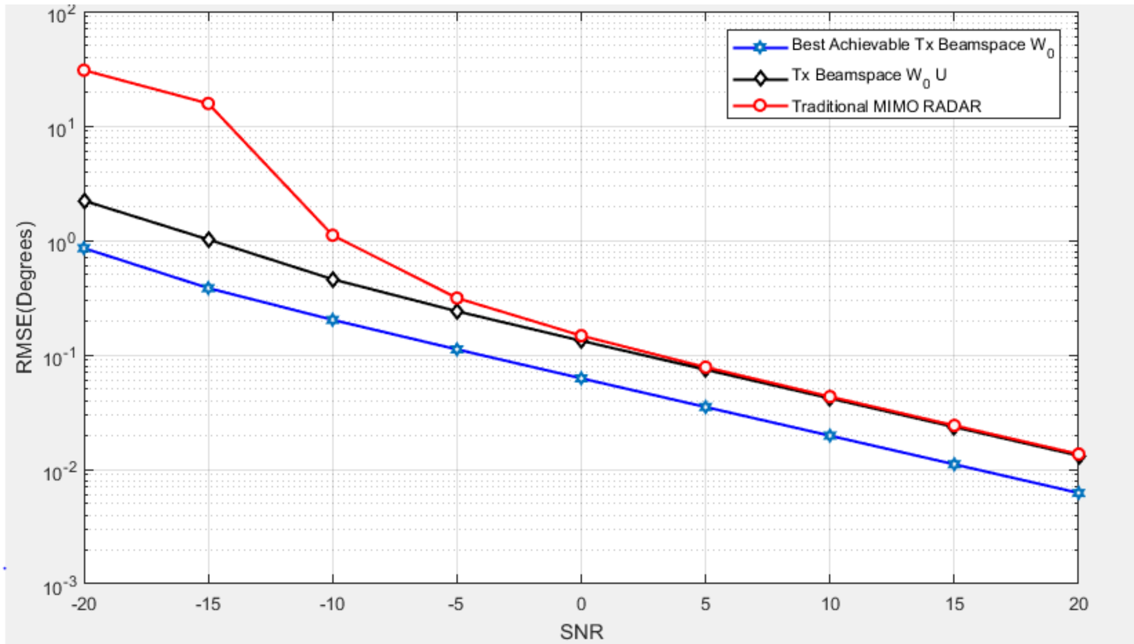

Figure 4.2: The root mean square error RMSE Versus SNR

Figure 4.3 show the root mean square error (RMSE) curves as a function of the signal-to-noise ratio. The results compare the proposed DOA estimation method $\mathbf{W}_0$ with the case of using $\mathbf{W}_O\mathbf{U}$ and conventional MIMO Radar. It can be seen from this figure that the performance is enhanced as using the proposed method reduces the root mean square error.

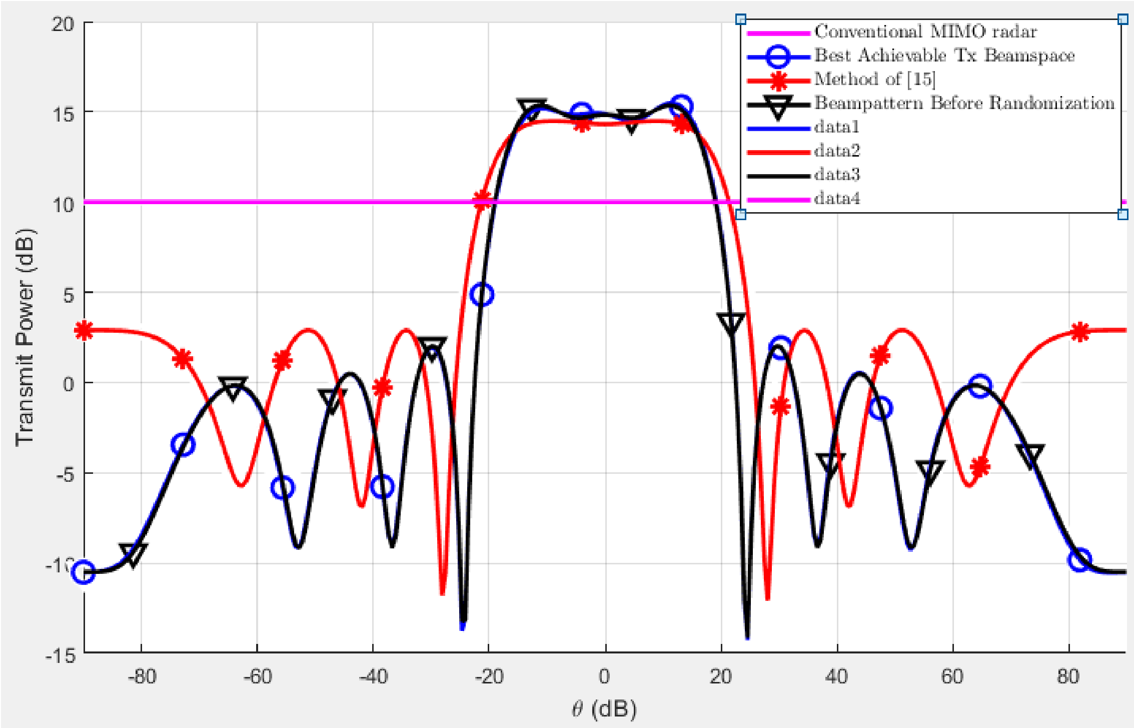

Figure 4.3: Transmit beampatterns of the traditional MIMO

Figure 4.3 shows the normalized transmit beampattern corresponding to the so obtained transmit beamspace matrix, the beampattern before randomization and for comparison with the best achievable beamspace. It also show an almost exact correspondence between the pre-randomization and post-randomization beampatterns. In order to test the RMSE performance of the methods tested, two targets are assumed to be located at $\theta_1 = -9^o$ and $\theta_2 = -8^o$.

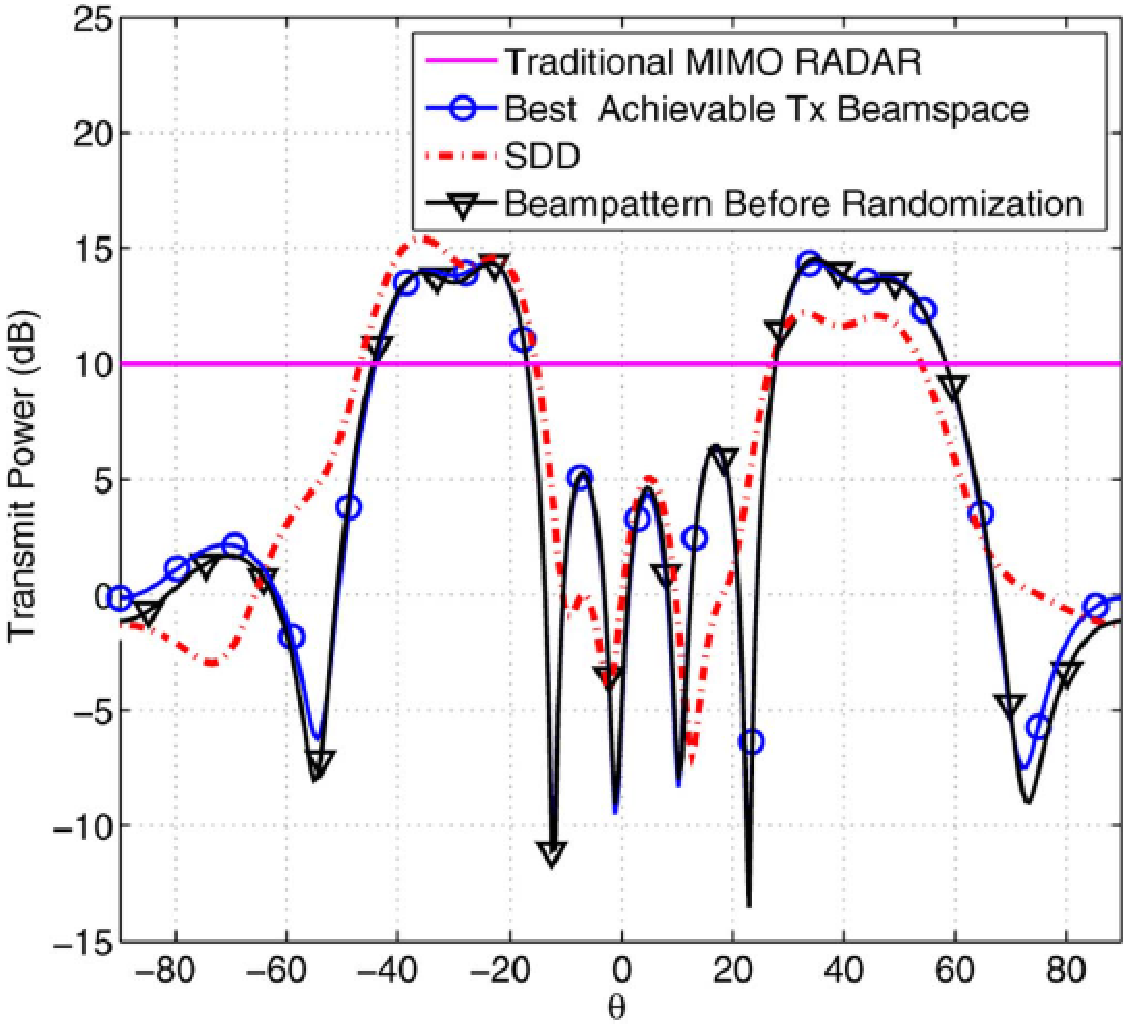

Figure 4.4: Transmit beampatterns of the traditional MIMO,Two Separated Sectors of Width 20 Degrees Each.

Figure 4.4 shows transmit beampatterns of the traditional MIMO,Two Separated Sectors of Width 20 Degrees Each of the traditional MIMO with uniform transmit power distribution, both the best achievable and SDD designs for, and the optimal beampattern obtained through the relaxed version of the optimization problem Eq(3.30)–(3.32) before randomization.

Finally, in figure 4.5 it can be observed that probability of resolving two closely located targets which is the proposed methods outperforms the method of Eq[3.17], and traditional MIMO radar as expected.

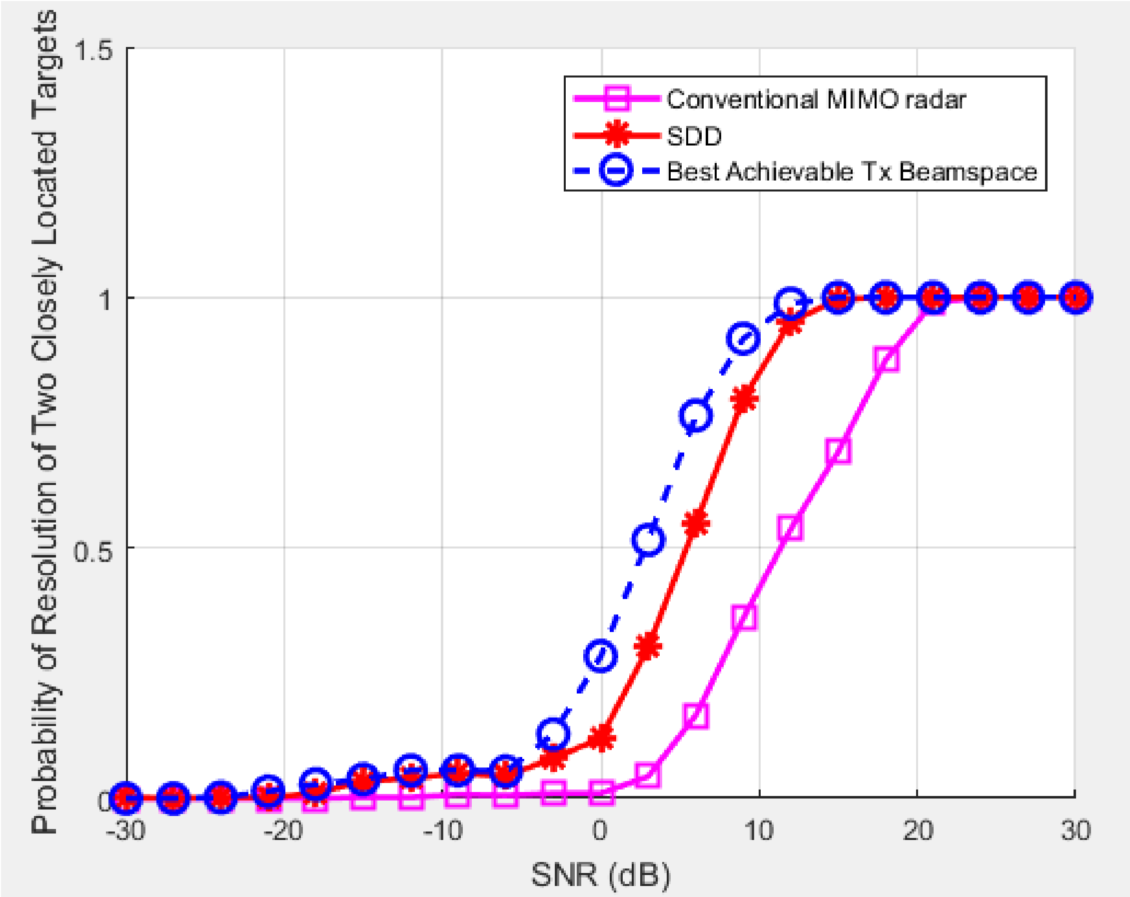

Figure 4.5: The probability of resolve two closely located targets versus SNR

Chapter Five
Conclusions and Recommendations

5.1 Conclusion

The problem of transmit beamspace design for MIMO radar with colocated antennas with application to DOA estimation has been considered. A new method for designing the transmit beamspace matrix that enables the use of search-free DOA estimation techniques at the receiver has been introduced. The essence of the proposed method is to design the transmit beamspace matrix based on minimizing the difference between a desired transmit beam-pattern and the actual one. The case of even but otherwise arbitrary number of transmit waveforms has been considered. MIMO radar is much better in probability of detection and the accuracy of locating targets than phase array radar in closer target (Co-located) situation .

5.2 Recommendations

The open points for future works are as follows.

- use the MUltiple SIgnal Classification (MUSIC) algorithm in MIMO Radar and compare it with the Estimation of Signal Parameters via Rotational Invariance Technique (ESPRIT).
- use amassive MIMO to increase performance of MIMO Radar to detect co-local targets

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