DEDICATION

To

My parents and wife

ACKNOWLEDGEMENTS

In the Name of Allah the Merciful, the Most Compassionate, all Praises be to Allah, the Lord of the Worlds.

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ABSTRACT

In this thesis we studied one of the most important properties of a submanifold, namely the curvature that describes different geometrical and topological properties of a submanifold. We have three classes of submanifolds in a manifold M, and these classes are: $immersed$, $imbedded$, $and\ regular$. We discussed the basic equations for submanifold and fundamental theorems of submanifolds. We also investigated some special submanifolds, namely, minimal, totally geodesic, totally umbilical, totally real, CR (Cauchy Riemann) and parallel. In particular submanifolds of finite type in Euclidean space and Chen's inequality have been treated and some results have been obtained.

المستخلص

في هذه الرسالة درسنا إحدى الخصائص المهمة لمتعدد الطيات الجزئي تلك هي الانحناء الذي يصف السمات الهندسية و التبولوجية لمتعدد الطيات الجزئي . لدينا ثلاث عائلات لمتعددات الطيات الجزئية داخل المتعدد الطية وهي: المغمور، المطمور و المنتظم . ناقشنا المعادلات الاساسية لمتعدد الطيات الجزئي وكذلك مبرهناته الاصلية. كذلك فحصنا بعضاً من متعددات الطيات الجزئية الخاصة ، علي وجه الخصوص متعددات الطيات الجزئية الأصغرية ، الجيوديسيكية كلياً ، السرية كلياً ، الحقيقية كلياً ، كوشي ريمان و المتوازية. بشكل خاص ، تمت معالجة متعددات الطيات الجزئية من النوع المحدود في الفضاء الاقليدي ومتباينات چين وتم الحصول علي بعض النتائج.

INTRODUCTION

The research presented in this thesis, is situated in one of the most important topics of differential geometry, namely the study of submanifolds. This is the generalization of the study of curves and surfaces to higher dimensional Euclidean space of arbitrary dimensions and codimensions and arbitrary ambient space.

The simplest space is the Euclidean space. This is a space such that at every point all sectional curvatures are zero.

There are two aspects of geometry of submanifolds, namely, intrinsic geometry and extrinsic geometry of submanifolds.

Intrinsic differential geometry of submanifolds describes the geometry inside the submanifolds. Extrinsic geometry of submanifolds deals with the shape of submanifolds as subsets of the ambient space.

An important result connecting intrinsic and extrinsic geometry of submanifolds is the1956 J. F. Nash embedding theorem which states that every Riemannain manifold can be isometrically embedded in a Euclidean space with sufficient high codimension.

According to B. Y. Chen, one of the basic problems in submanifold theory is to find simple relationships between the main intrinsic invariants and the main extrinsic invariants of submanifolds.

Problems in submanifold theory have been studied since the invention of calculus and it was started with differential geometry of plane curves. In the modern theory of submanifolds, the study of the relations between local and global properties has also attracted the interest of much geometry.

The theory of submanifolds as a field of differential geometry is as old as differential geometry itself. A study of submanifolds of a manifold is a very interesting field of differential geometry.

B. Y. Chen (1981), D. E. Blair (1976), A. Bejancu (1986), M. H. Sahid (1994–1995), and some others have studied different properties of submanifolds.

The objective of this thesis is to devote a self-contained study of Chen's inequalities for submanifolds. Submanifolds are solutions of differential equations where a submanifold is a constraint reflected geometrically curvature condition whereas algebraically it is given by equations.

The whole thesis is divided into five chapters. In chapter one we study the differentiable manifolds, Riemannian manifolds. Chapter two provides a study of the theory of submanifolds. In chapter three we study some special submanifolds while in chapter four we study submanifolds of finite type. Lastly in chapter five we study Chen's Inequality and some invariants.

LIST OF SYMBOLS AND ABBREVIATION

$\mathbb{R}$	real numbers
$\mathbb{R}^n$	Euclidean space of dimension n
$p = (p^1, \dots, p^n)$	point in $\mathbb{R}^n$
C^∞	smooth or infinitely differentiable
$C^\infty(M)$	space of smooth functions on the manifold
$T_p M$	tangent space at p of a smooth manifold
TM	tangent bundle of a smooth manifold
$T_p^* M$	cotangent space at p of a smooth manifold
$T^* M$	cotangent bundle of a smooth manifold
$\langle X, Y \rangle$	inner product of two tangent vectors X and Y
$X\langle Y, Z \rangle$	derivative of a function $\langle Y, Z \rangle$ along a vector field X
$\nabla_X Y$	covariant derivative of vector field Y along the vector field X
$[X, Y]$	Lie bracket for vector fields X and Y
∂_i	the basis vector fields
$\frac{\partial f}{\partial x^i}$	partial derivative with respect to x^i
∇_X	covariant derivation with respect to X
f^*	pullback via the map f.
∇f	gradient of the function f.

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