

# Comparison of some Classical PID and Fuzzy Logic Controllers

Eisa Bashier M. Tayeb and A. Taifour Ali

**Abstract**— The proportional-integral-derivative (PID) controller is tuned to find its parameters values. Generally most of the tuning methods depend mainly on the experimental approach of open-loop unit step response. The controller parameters can be found if the system truly can be approximated by First Order Plus-Dead Time (FOPDT). The performance of most of them deteriorates as the ratio of approximated equivalent delay L to the overall time constant T changes. On the other hand fuzzy PID controller is not tuned through the same conventional tuning procedures. It's constructed as a set of control rules and the control signal is directly deduced from the knowledge base and the fuzzy inference. Fuzzy controller parameters tuned by starting from the equivalent values obtained for optimum controller. The performances of different PID tuning techniques are simulated for different order systems and compared with fuzzy-PD+I controller. MATLAB simulation results show that Fuzzy PD+I have better performances over other conventional PID controllers.

**Index Terms** — Fuzzy PD+I Controller, PID Controller Tuning, Controllers Comparison, Tuning Techniques.

## 1 INTRODUCTION

The PID controller has several important functions; it provides feedback, has the ability to eliminate steady state offsets through integral action, and it can anticipate the future through derivative action. PID controllers are the largest number of controllers found in industries sufficient for solving many control problems.

The determination of the controller parameters is called the controller tuning or design. Many approaches have been developed for tuning PID controller and getting its parameters for single input single output (SISO) systems. Among the well-known approaches are the Ziegler-Nichols (Z-N) method, the Cohen-Coon (C-C) method, integral of squared time weighted error rule (ISTE), integral of absolute error criteria (IAE), internal-model-control (IMC) based method, gain-phase margin method [1-9]. This paper focuses on studying the FPID and compares it with optimum tuning method, Z-N which has been explored since 1942 and is still used in industry and C-C. The PID control law is the sum of three types of control actions: a proportional, an integral and a derivative control actions. Mathematically PID controller in the time-domain is given by the following equation:

$$u(t) = K_p[e(t) + \frac{1}{T_i} \int_0^t e(\tau) d\tau + T_d \frac{de(t)}{dt}] \quad \dots(1)$$

Where  $u(t)$  is the controller output (input signal to the plant model), the error signal  $e(t)$  is defined as  $e(t) = r(t) - y(t)$ , and  $r(t)$  is the reference input signal while  $y(t)$  is the output. The controller parameters are proportional gain  $K_p$ , integral time  $T_i$ , and derivative time  $T_d$  [10].

If a mathematical model of the PID-controlled plant can be

derived, then various design techniques for determining the controller parameters can be applied. However, if the plant is so complex that its mathematical model cannot easily be obtained, then analytical approach to design PID controller is not possible [11]. Then we must resort to experimental approaches for tuning of PID controllers.

## 2 PID CONTROLLERS DESIGN

Optimum setting algorithms for a PID controller were proposed by Zhuang and Atherton [19] for various criteria. The methods involve searching for minimum of the cost function  $J_n(\varphi)$  in its general form:

$$j_n(\varphi) = \int_0^{\infty} [t^n e(\varphi, t)]^2 dt \quad \dots\dots (2)$$

Where  $e(\varphi, t)$  is the error signal, with  $\varphi$  as PID controller parameters. The optimum controller parameters are found when the partial derivative of  $J_n(\varphi)$  with respect to  $\varphi$  equals zero. The error signal used for optimization can be a result set-point or of load disturbance. Therefore, it is possible to obtain two sets of parameters: one for the set-point input and the other for the disturbance signal. In particular, three values of  $n$  ( $n = 0, 1, 2$ ) are discussed. These three cases correspond, respectively, to three different optimum criteria: the integral squared error (ISE) criterion, integral squared time weighted error (ISTE) criterion, and the integral squared time-squared weighted error (IST<sup>2</sup>E) criterion. The expressions given were obtained by fitting curves to the optimum theoretical results [19, 20].

A large number of industrial plants can approximately be modeled by the first order plus dead time (FOPDT) model with transfer function as follows [21]:

$$G(s) = \frac{K}{Ts + 1} e^{-Ls} \quad \dots\dots (3)$$

- Eisa Bashier M., PH-00249915458516. E-mail: [eisabashier@yahoo.com](mailto:eisabashier@yahoo.com);
- Co-Author A. Taifour Ali. E-mail: [ayha\\_114@yahoo.com](mailto:ayha_114@yahoo.com);
- Both authors are assistant professors at: School of Electrical and Nuclear Engineering; College of Engineering; Sudan University of Science & Technology

Sometimes one may want to design a controller having good rejection performances on the disturbance signal. The parameters equations to design controllers for disturbance rejection using the optimal method are different than the set point used here.

PID controller equation suggested by Z-N is:

$$G_{ZN}(s) = 1.2 \frac{T}{L} \left( 1 + \frac{1}{2Ls} + 0.5Ls \right) \quad \text{And} \quad \dots (4)$$

suggested gains setting as:

$$\left. \begin{aligned} K_p &= \frac{1.25}{a} \left( 1 + \frac{0.18\tau}{1-\tau} \right) \\ T_i &= \frac{2.5-2\tau}{1-0.39\tau} L \\ T_d &= \frac{0.37-0.37\tau}{1-0.81\tau} L \end{aligned} \right\} \dots (5)$$

Where;  $a = k_p L/T$  &  $\tau = L/(L+T)$

While the gains of optimal PID controller can be set as follows [19]:

$$\left. \begin{aligned} K_p &= \frac{a_1}{k} \left( \frac{L}{T} \right)^{b_1} \\ T_i &= \frac{T}{a_2 + b_2(L/T)} \\ T_d &= a_3 T \left( \frac{L}{T} \right)^{b_3} \end{aligned} \right\} \dots (6)$$

Where the parameters (a, b) should be determined according to Table A1 in appendix A. the selection of (a, b) depends mainly on the value of (L/T).

### 3 FUZZY CONTROLLER MODELING AND DESIGN

The three principal elements to a fuzzy logic controller are shown in figure (1).

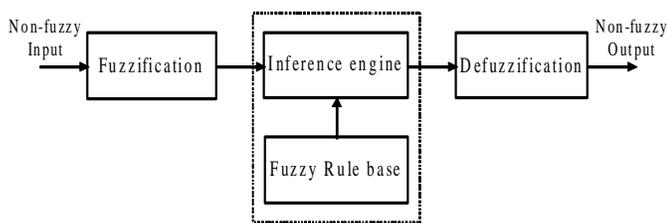


Fig (1): Basic Structure of Fuzzy Logic System

**The proposed Fuzzy Controller;** the input to the general FPD controller is the error "e", the derivative of the error "ce" and the output is the control signal "u", while the conventional I-controller is known to work on the integral of the error "ie". The controller output is a nonlinear function of the error, change in the error and integral of error.

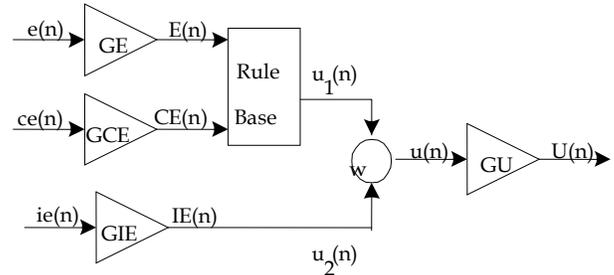


Figure (2): Fuzzy (Proportional-Derivative) plus Integral Controller (FPD+I)

$$U(n) = f_1(GE * e(n), GCE * ce(n)) * GU + f_2(GIE * ie(n)) * GU \quad (7)$$

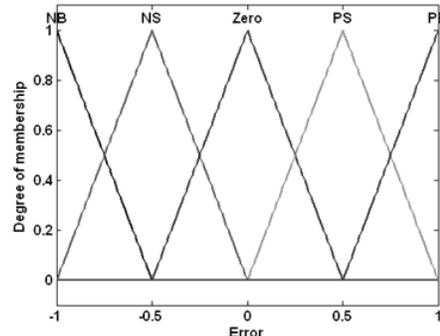
Where, the functions  $f_1, f_2$  are the input-output map of the fuzzy controller and G stands for gain.

One way of constructing such controller is achieved by summing the fuzzy PD controller output and the integral part. In many simple systems, Fuzzy logic integral control is able to work however it's slow. This slower response can be reduced by combining I-controller with another form, such as P or PD controller. Thus the proposed fuzzy logic controller is FPD+I controller [22, 23].

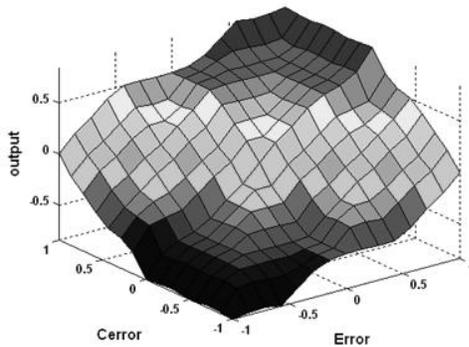
In this paper the inputs to fuzzification block are chosen to be the Error (E) and its derivative (d(E)/dt). The type of the FLC used is Mamdani-type which has fuzzy rules of the form IF-THEN rules. All membership functions for controller inputs (E and change of E) and controller output are defined on the common normalized domain [-1, 1]. Fig.(3) shows the Inputs/Output relations of the FPD Controller.

Table 1: Rules for Fuzzy PD-Controller Error

	NB	NS	Z	PS	PB
NB	NB	NB	NS	NS	Z
NS	NB	NS	NS	Z	PS
Z	NB	NS	Z	PS	PB
PS	NS	Z	PS	PS	PB
PB	Z	PS	PS	PB	PB



(a) Membership functions



(b) Output surf of the controller.

**Figure (3): FPD Controller Inputs/Output Relations**

The inputs and outputs are transformed to five fuzzy linguistic variables NB, NS, Z, PS, and PB which stand for Negative Big, Negative Small, Zero, Positive Small and Positive Big respectively.

#### 4 TUNING THE PID CONTROLLERS

The key feature in the optimum methods for PID controller tuning is to obtain the response of the plant to a unit step input. If it involves neither integrator nor dominant complex-conjugate poles, then such an open-loop unit step response curve may be characterized by three constants, gain K delay time L and time constant T [10].

The following is an example of PID-controlled systems and their responses for different ratios when tuned using Z-N and C-C methods.

These constants are either to be found experimentally or instead of experimental approaches, a simulation may be used to get these parameters. In the following are different systems examined to illustrate the method for tuning the controllers.

$$\text{System1}; G_1(s) = \frac{120}{(s^2 + 6s + 3)} \dots (8)$$

From the step response we obtained the parameters (K, L, and T) as (K = 40, L = 0.174, T = 1.826). The range of (L/T) from the given transfer function is equal to (0.174/1.826)=0.095.

**Table 2: The Controller Parameters of System1**

PID Controller Parameters			
Criterion	K <sub>p</sub>	T <sub>i</sub>	T <sub>d</sub>
Z-N	0.3143	0.3485	0.0871
C-C	0.3330	0.507	0.0633
IST <sup>2</sup> E	0.2024	1.92	0.071

$$\text{System2}; G_2(s) = \frac{200}{(s + 1)(s + 4)(s + 6)} \dots (9)$$

From the step response we obtained the parameters (K, L, and T) as (K = 8.333, L = 0.3725, T = 1.0442). The range of (L/T) from the given transfer function is equal to (0.3725/1.0442)=0.3557. The parameters of the controller are obtained as in Table (3):

**Table 3: The Controller Parameters of System2**

PID Controller Parameters			
Criterion	K <sub>p</sub>	T <sub>i</sub>	T <sub>d</sub>
Z-N	0.4036	0.7450	0.1863
C-C	0.4475	0.8600	0.1291
IST <sup>2</sup> E	0.2949	1.1775	0.1316

$$\text{System3}; G_3(s) = \frac{4}{(s + 1)^4} \dots (10)$$

From the step response we obtained the parameters (K, L, and T) as (K=2, L=2, T=2). The range of (L/T) from the given transfer function is equal to 1. Table (4) shows the parameters of the different controllers.

**Table 4: The Controller Parameters of System3**

PID Controller Parameters			
Criterion	K <sub>p</sub>	T <sub>i</sub>	T <sub>d</sub>
Z-N	0.3	4	1
C-C	0.3687	3.500	0.622
IST <sup>2</sup> E	0.2420	2.762	0.632

$$\text{System4}; G_4(s) = \frac{3}{(s + 2)(s + 1)^6} \dots (11)$$

From the step response we obtained the parameters (K, L, and T) as (K = 1.5, L = 4, T = 2.5), L/T=1.6; Therefore the parameters of the controller are obtained as in Table (5).

**Table 5: The Controller Parameters of System4**

PID Controller Parameters			
Criterion	K <sub>p</sub>	T <sub>i</sub>	T <sub>d</sub>
Z-N	0.500	8	2
C-C	0.671	6.177	1.135
IST <sup>2</sup> E	0.5378	3.981	1.164

#### 5 SIMULATION RESULTS AND DISCUSSIONS

In this section, a simulation for the four different systems is carried out according to the parameters values obtained in the tables 2 to 5. Fuzzy controller parameters start from values obtained for optimum controller. Then this FPD+I controller is finely tuned by adjusting GE, GCE, GIE and GU. A comparison is made based on the parameters characterizing transient response. Fig.(4) to Fig.(7) show the closed-loop step response for each method.

For the different tested systems, the FPD+I gives accepted rise time, the best settling time and perform very much better than others in the overshoot behavior. This controller gives very fast response especially for lower order systems. For higher order systems the responses of Z-N results in longer settling time and approaching critically damped systems which consi-

dered as its shortcoming.

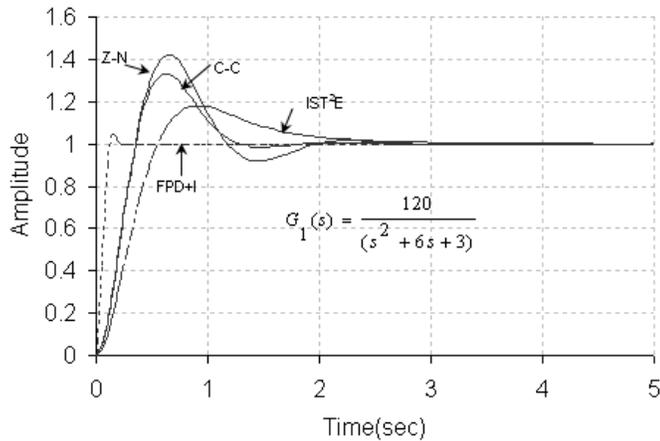


Fig. 4 System1 Step Response

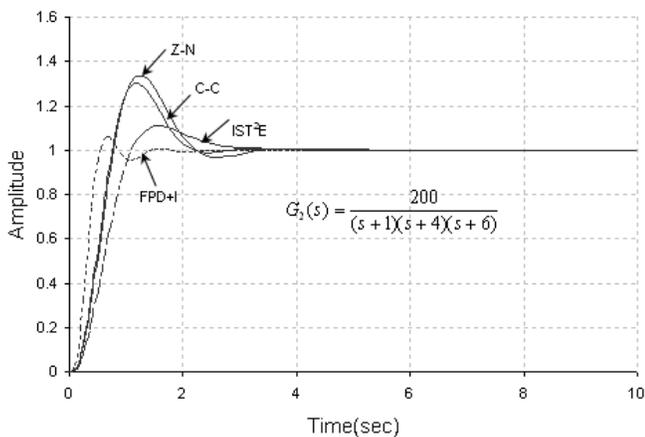


Fig. 5 System2 Step Response

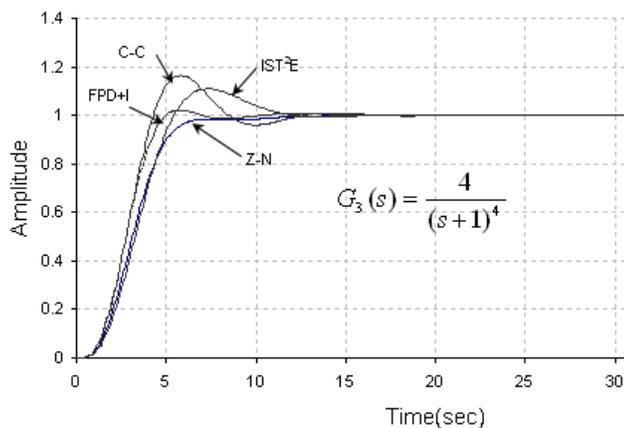


Fig. 6 System3 Step Response

## 6 CONCLUSIONS

The PID control is still of great interest, and is a promising control strategy that deserves further research and investigation. These tuning methods are only valid for open loop system which can be described by the first order plus dead-time model and for 'ideal' PID control structure case. FPD+I controllers show better results than optimally tuned

PID, C-C and Z-N. The response of the later deteriorates as the approximated equivalent delay L to the overall time constant T increases. Optimally tuned controller sustain for wide range of systems due to their consideration to L/T.

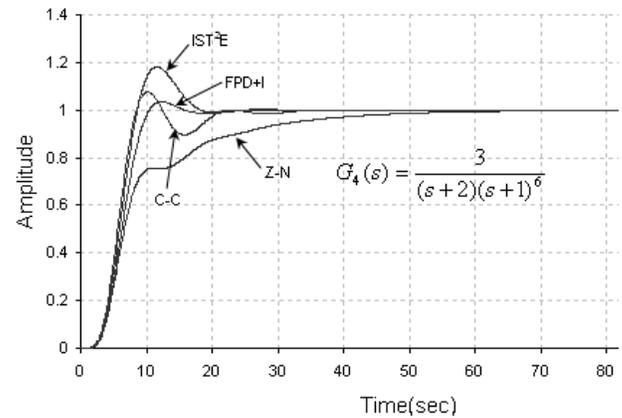


Fig. 7 System4 Step Response

## References:

- [1]. J. G. Ziegler and N. B. Nichols, (1942) "Optimum settings for automatic controllers," *Transactions of American Society of Mechanical Engineers*, Vol. 64.
- [2]. G. H. Cohen and G. A. Coon, (1953) "Theoretical investigation of retarded control," *Transactions of American Society of Mechanical Engineers*, Vol. 75.
- [3]. M. Zhuang and D. P. Atherton, (1993) "Automatic tuning of optimum PID controllers," *IEE Proceedings on Control and Applications*, Vol. 140.
- [4]. D. W. Pessen, (1994) "A new look at PID-controller tuning," *Journal of Dynamical Systems Measures and Control*, Vol. 116.
- [5]. M. Morari and E. Zafiriou (1989) "*Robust Process Control*" Prentice-Hall, Englewood Cliffs, New Jersey.
- [6]. W. K. Ho, C. C. Hang, and L. S. Cao, (1995) "Tuning of PID controllers based on gain and phase margin specifications," *Automatica*, Vol. 31.
- [7]. P. Cominos, N. Munro. (2002) *PID Controllers: Recent Tuning Methods and Design to Specification*. in: *IEE Proceedings, Control Theory and Applications*, vol. 149, 46-53.
- [8]. C. Lee. (2004) A Survey of PID Controller Design based on Gain and Phase Margins. *International Journal of Computational Cognition*, 2(3): 63-100.
- [9]. G. Mann, B. Hu, R. Gosine. (2001) Time Domain based Design and Analysis of New PID Tuning Rules. *IEE proceedings, Control Theory Applications*, 148(3): 251-261.
- [10]. Karl J. Astrom and Tore Hagglund, (1995) "PID Controllers: Theory, Design, and Tuning," *Instrument Society of America*.
- [11]. Katsuhiko Ogata, (1997) "*Modern Control Engineering 3rd Edition*". Prentice-Hall, Inc, Simon & Schuster/A Viacom Company.
- [12]. S. G. Tzafestas, (1994) "Fuzzy systems and fuzzy expert control: An overview," *Knowl. Eng. Rev.*, vol. 9, no. 3, pp. 229-

268.

[13] S. G. Tzafestas and N. P. Papanikolopoulos, (1990) "Incremental fuzzy expert PID control," *IEEE Trans. Ind. Electron.*, vol. 37, no. 5, pp. 365-371.

[14] S.-Z. He, S. Tan, and F.-L. Xu, (1993) "Fuzzy self-tuning of PID controllers," *Fuzzy Sets Syst.*, vol. 2, pp. 37-46.

[15] Z.-Y. Zhao, M. Tomizuka, and S. Isaka, (1993) "Fuzzy gain scheduling of PID controllers," *IEEE Trans. Syst., Man, Cybern.*, vol. 23, no. 5, pp. 1392-1398.

[16] S. Chopra, R. Mitra, V. Kumar. (2005) Fuzzy Controller: Choosing an Appropriate and Smallest Rule Set. *International Journal of Computational Cognition*, 3(4): 73-78.

[17] H. Ying. (2000) Fuzzy Control and Modeling: Analytical Foundations and Applications. IEEE Press series on Biomedical Engineering, New York.

[18] Stefan Preitl, Radu-Emil Precup, Zsuzsa Preitl (2010) "Aspect Concerning the Tuning of 2-DOF Fuzzy Controllers" *Automatic Control and Robotics* Vol. 9, No 1, pp. 1 - 18

[19]. Dingyu Xue, Yang Quan Chen, and Derek P. Atherton., (2007) "Linear Feedback Control". *The Society for Industrial and Applied Mathematics*.

[20] L. Eriksson, (2005) "A PID Tuning Tool for Networked Control Systems", *WSEAS Transactions on Systems*, Vol. 4, Issue 1, pp. 91-97.

[21]. Saeed Tavakoli and Mahdi Tavakoli (2003) "Optimal Tuning Of PID Controllers For First Order Plus Time Delay Models Using Dimensional Analysis" *International Conference on*

*Control and Automation (ICCA'03)*.

[22]. E. B. M. Tayeb, F. A. Hussein "Automatic Generation Control of an Interconnected Power System Using Fuzzy Logic Control" M.Sc. thesis, College of Engineering, SUST, Khar-toum (2010).

[23] Satish. R. Vaishnav , Zafar. J. Khan ((2010)) "Performance of tuned PID controller and a new hybrid fuzzy PD + I controller" *World Journal of Modelling and Simulation*; Vol.6 No.2, pp. 141-149

**APPENDIX A:**

Table A1: Set-point PID controller parameters

$L/T$	0.1-1			1.1-2		
	$ISE$	$ISTE$	$IST^2E$	$ISE$	$ISTE$	$IST^2E$
a1	1.048	1.042	0.968	1.154	1.142	1.061
b1	-0.897	-0.897	-0.904	-0.567	-0.579	-0.583
a2	1.195	0.987	0.977	1.047	0.919	0.892
b2	-0.368	-0.238	-0.253	-0.220	-0.172	-0.165
a3	0.489	0.385	0.316	0.490	0.384	0.315
b3	0.888	0.906	0.892	0.708	0.839	0.832