CHAPTER SIX

FORMULATION OF EMPIRICAL EQUATIONS

6.1 General

In this chapter, regression analyses of the experimental results (compressive strength f'_{cf} ,splitting tensile strength f_{spf} , modulus of rupture f_{rf}) for the different NSRPC mixes as well as the ultimate shear capacity V_u of the tested NSRPC beams were all performed following the statistical method SPSS software version 18[81]. Accordingly, empirical equations for these parameters were established and presented in this chapter. The effect of important variables such as nanosilica content NS, silica fume content and volumetric ratio of steel fibers V_f , steel reinforcement ratio ρ and the shear span to effective depth ratio a/d were considered in establishing the proposed empirical equation for the shear capacity of a specified NSRPC beam .

6.2 Present Models

All the experimental results of the present investigation were used as data for performing the regression analyses to estimate the empirical equations for all the NSRPC beam properties.

6.2.1 Statistical Modeling

In statistical modeling, the overall objective is to develop a predictive equation relating a criterion variable to one or more predictor variables. In this research, the criterion variables include the compressive strength of concrete, modulus of rupture, splitting tensile strength as a function of additive materials such as nanosilica, silica fume and steel fibers. The terms are defined as follows:

NS =Nanosilica content .

SF = Silica Fume content.

VF =Steel Fibers volumetric ratio .

f'_{cf}=Compressive strength of NSRPC, MPa.

f'_{spf} = Splitting Tensile Strength of NSRPC, MPa.

f'_{rf} = Modulus of Rupture of NSRPC, MPa.

From the correlation matrices, it is evident that some variables have high inter - correlation and low correlation with the criterion variables. As the correlation coefficient provides only a single-valued index of the degree of near association between pairs of variables, it is used primarily as a data screening technique. Stepwise regression technique used for model development provides coefficients for a prediction equation and can also be used to assess the relative importance of the predictor variables. The measures of goodness of fit are aimed to quantify how well the proposed regression model obtained fits the data. The two measures usually presented are coefficients of multiple determinations (R^2) and standard error of regression SER.

The R^2 is the percent variation of the criterion variable explained by the suggested model and is calculated according to following equation:

$$R^2 = 1 - {}^{SSE}/_{SST} \tag{6-1}$$

Where, SSE is the measure of how much variation in (y) is left unexplained by the proposed model, and it is equal to the error sum of squares = $\sum (y_i - y_i')^2$.

y_i, is the actual value of criterion variable for the (i^{th}) case.

y_i', the regression prediction for the (i^{th}) case.

SST is the measure of the total amount of variation in the observed (y), and it is equal to the total sum of squares $\sum (y_i - \bar{y}_i)^2$.

$\bar{y}$= the mean observed (y).

R^2, is bounded between (0) and (1), the higher the value of (R^2), the more successful is the regression model in explaining (y) variation. If R^2 is small, the analyst will usually want to search for an alternative models (i.e., non-linear) that can more effectively explain (y) variation. Because R^2always increases when a new variable is added to the set of the predictor variables and in order to balance the cost of using more parameters against the gain in R^2, many statisticians use the adjusted coefficient of multiple determinations adjust, R^2, which is calculated as follows:

$$\text{adjust } R^2 = \left[\frac{(n-1)R^2-K}{n-1-K}\right] \qquad (6\text{-}2)$$

where:

n = the sample size.

k = the total number of the predictor variables.

The second measure, standard error of regression (*SER*), is calculated according to the following equation:

$$\text{SER}=\sqrt{\frac{SSE}{n-(k+1)}} \qquad (6\text{-}3)$$

In general, the smaller the (SER) value, the better the proposed regression model.

6.2.2 Proposed Equations for NSRPC

The current parametric study provides a database for the whole parameters considered in the present study. This database can be used to develop expressions for the compressive strength, splitting tensile strength and modulus of rupture of NSRPC. These empirical equations can be extended to develop software capable of predicting the compressive strength and all related mechanical properties of concrete for design purposes.

6.2.2.1 Compressive Strength

Regression models are proposed in the present work based on (62) experimental data points obtained from this investigation (presented in Table 6-1) and other investigations given in Appendix A , by using SPSS software version 18.The proposed expressions for predicting the compressive strength of NSRPC are given by Eq.(6-4) as follows:

$$f'_{cf} = f'_c + 11.957 V_f + 3.872\, NS \qquad (6\text{-}4)$$

R^2=0.97825

where

f'_{cf}=Compressive strength of NSRPC, MPa

f'_c= Compressive strength of RPC, MPa

V_f= Volume fraction of steel fibers

NS=Persentage of Nanosilica

To test equation(6-4), the relative compressive strengths (f'_{cf} *test/* f'_{cf} *proposed*) was found using the (62) experimental data points obtained from this investigation (presented in Table 6.1) and other investigations given in Appendix A. The values of the mean (μ), standard deviation (SD) and coefficient of variation (COV) are 1.02128, 0.107363 and 10.5% respectively. Fig.6.1 shows tests value versus proposed of f'_{cf} for (62) test results using Eq.(6-4)

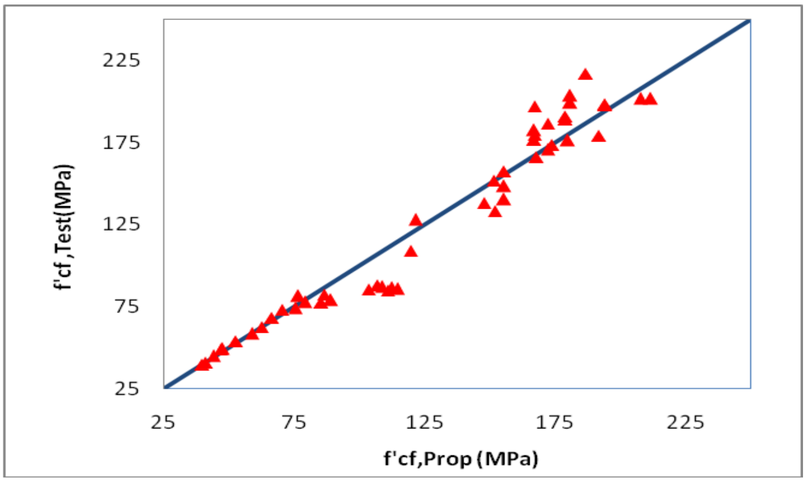

Fig. 6.1 :Test Values Versus Proposed Values of Compressive Strength NSRPC Using Eq. (6-4)

6.2.2.2 Splitting Tensile Strength

Regression models are proposed in the present work based on (30) experimental data points obtained from this investigation (presented in Table 6.2) and other investigations given in Appendix A, by using SPSS software version 18.The proposed expressions for predicting the splitting tensile strength of NSRPC are given by Eq. (6-5) as follows:

$$f'_{spf} = 0.125 f'_{cf} \qquad (6\text{-}5)$$

R^2=0.93265

where

f'_{spf}=Splitting tensile strength of NSRPC, MPa

f'_{cf}=Compressive strength of NSRPC, MPa

To test equation(6-5), the relative splitting tensile strength (*f_{spf} test/ f_{spf} proposed*) was found using the (30) experimental data points obtained from this investigation (presented in Table 6.2) and other investigations given in Appendix A . The values of the mean (μ), standard deviation (SD) and coefficient of variation (COV) are 0.97456, 0.2237 and 22.9% respectively. Fig.6.2 shows tests value versus proposed of f'_{spf} for (30) test results using Eq.(6-5)

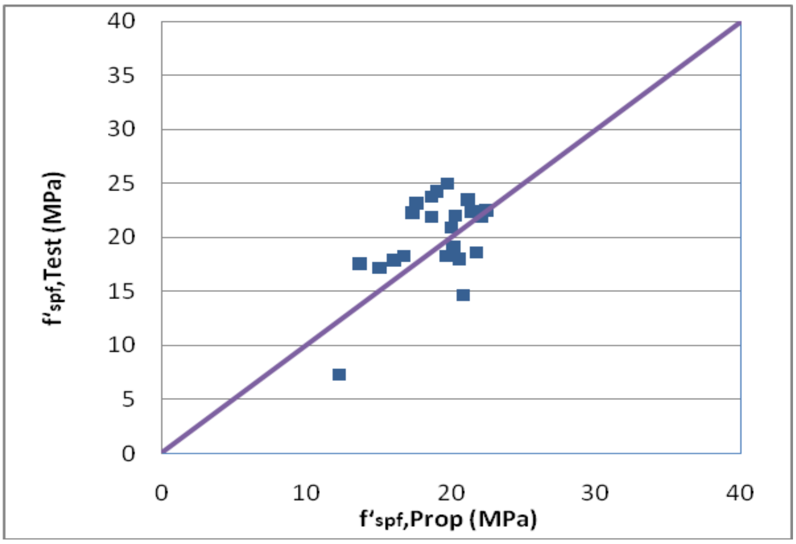

Fig. 6.2 :Test Values Versus Proposed Values of Splitting Tensile Strength NSRPC using Eq. (6-5)

6.2.2.3 Modulus of Rupture

Regression models are proposed in the present work based on (30) experimental data points obtained from this investigation (presented in Table 6.3) and other investigations given in Appendix A , by using SPSS software version 18.The proposed expressions for predicting the modulus of rupture of NSRPC are given by Eq.(6-6) as follows:

$$f'_{rf} = 0.154\, f'_{cf} \tag{6-6}$$

R^2=0.92476

where

f'_{rf}=Modulus of rupture of NSRPC, MPa

f'_{cf}=Compressive strength of NSRPC, MPa

To test equation(6-6), the relative modulus of rupture (*f_{rf} test/ f_{rf} proposed*) was found using the (30) experimental data points obtained from this investigation (presented in Table 6-3) and other investigations given in Appendix A .The values of the mean (μ), standard deviation (SD) and coefficient of variation (COV) are 1.07955, 0.2277 and 21% respectively.

Fig.6.3 shows tests value versus proposed of f'_{rf} for (30) test results using Eq.(6-6)

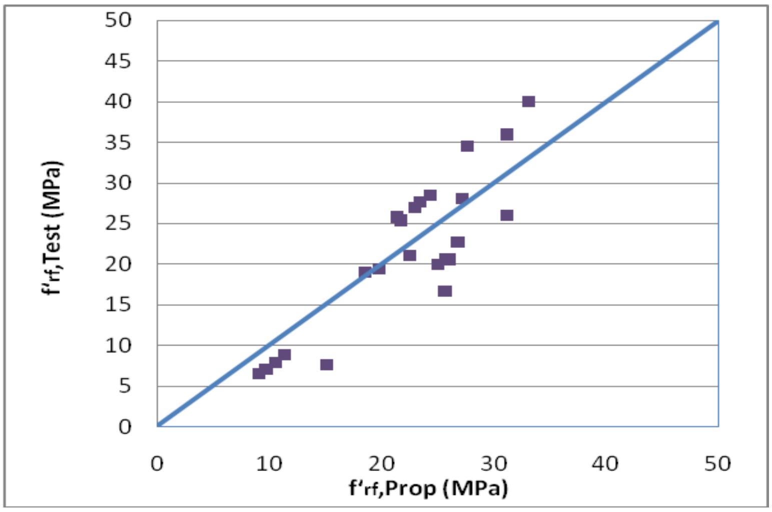

Fig. 6.3: Test Values versus Proposed Values of Modulus of Rupture NSRPC Using Eq. (6-6)

6.3 Shear Strength of Reinforced Concrete Beam

To estimate the shear resistance of beams, standard codes and researchers all over the world have specified different formulae considering different parameters into consideration. The shear failure of reinforced concrete beams without web reinforcement is a distinctive case of failure which depends on various parameters such as shear span to effective depth ratio (*a*/*d*), longitudinal tension steel ratio (ρ), aggregate type, strength of concrete, type of loading, and support conditions.

According to ACI Building Code 318-2011 [80], the shear strength of concrete members without transverse reinforcement subjected to shear and flexure is given by following equation;

$$V_c = (0.16\lambda\sqrt{f_c^{'}} + 17\rho_w \frac{V_u\, d}{M_u}) b_w\, d \qquad (6\text{-}7)$$

For most designs, it is convenient to assume that the second term of Eq. (6-7) equals to $0.01(f_c^{'})^{0.5}$, so that equation (6-7) becomes;

$$V_c = (0.17\lambda\sqrt{f_c^{'}}) b_w\, d \quad (kN) \qquad (6\text{-}8)$$

Equations (6-8) used for normal concrete and when the compressive strength of concrete is not greater than 70 MPa. In the present study the concrete behaved as high strength concrete because of the compressive strength of concrete is more than 70 MPa, thereforeEq.(6-8) cannot be used. Design of cross sections subject to shear shall be based on;

$$\phi V_n \geq V_u \qquad (6\text{-}9)$$

Where V_u is the factored shear force at the section considered and V_n is nominal shear strength computed by;

$$V_n = V_c + V_s \tag{6-10}$$

6.4 Shear Strength of NSRPC Beam

In the present study, shear reinforcement was not adopted so that the second term of equation (6-10) is to be ignored. Therefore the shear strength of a member depends on the shear resistance of the concrete and the final expression is;

$$V_u = \phi V_c \tag{6-11}$$

Applying regression analysis on the data from experimental tests, gave the following expression for high compressive strength;

$$V_c = 0.636 \sqrt{f'_{cf}}\ b_w\, d \tag{6-12}$$

A comparison between Eq.(6-8) for normal strength concrete and Eq.(6-12) for high strength concrete indicates a difference in the multiplying constant coefficient since both equations are empirical. It can be clearly seen that the shear strength of the high compressive strength concrete is 3.74 times higher than the shear strength of the normal compressive strength concrete. This means that in high strength concrete members , stirrups can be used with only minimum amount to control the width and length of the crack if occurred.

6.5 Proposed Equation for the Equation of Shear Capacity of NSRPC Beams

$$Vu = [(0.156 f'_{cf} + 2F)(d/a) + (19\rho(d/a))\ ]bw.d \qquad (6\text{-}13)$$

R^2=0.96658

where

V_u= ultimate shear force of slender beam without shear reinforcement (N)

f'_{cf}= Compressive strength of NSRPC, MPa

a/d= shear span to depth ratio

ρ= ratio of tension reinforcement

bw=width of beam (mm)

d=depth of beam (mm)

F= Fibers factor given by

$$F = \frac{l_f}{d_f} V_f \left(B_f\right) \qquad (6\text{-}14)$$

where

l_f / d_f = l_f and d_f are length and diameter of the steel fibers respectively.

V_f= Volume fraction of the steel fibers

(B_f) is the bond factor that accounts for bond characteristics of the steel fibers. Based on a large series of pullout tests by Narayanan and Palanjian[4] (B_f) was assigned a relative value of 0.5 for round fibers, 0.75 for crimped fibers, and 1.0 for indented fibers.

6.6 Evaluation of the Proposed Shear Capacity Equation

6.6.1 Comparison with experimental results

Equation (6-13) gives reasonable estimates of the actual shear capacity of NSRPC beams as determined from the present experimental tests with the mean experimental to theoretical ratios (μ) of 1.05478 with standard deviation (SD) of 0.285and coefficient of variation (COV) of 27%.

Fig. (6-4) shows the ultimate shear capacity obtained from the experimental work versus the corresponding theoretical value from Eq. (6-13), while Fig. 6.5 shows the ratio Vu-test/Vu-proposed versus different factors that influence *the* shear strength of NSRPC beams including f'_{cf}, ρ, a/d, NS and V_f.

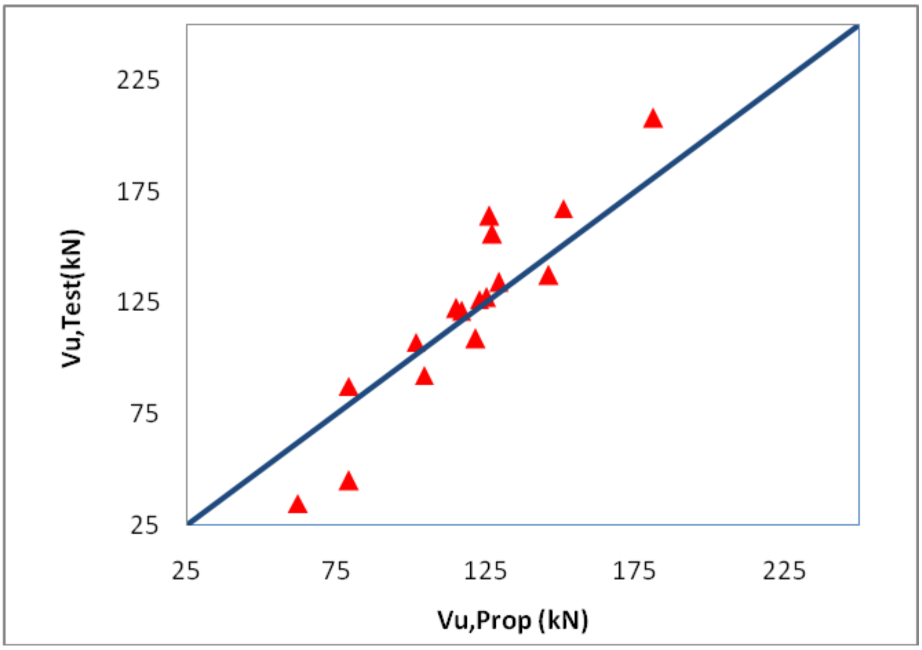

Fig. 6.4: Experimental Versus Proposed Values for The Shear Capacity of NSRPC Beams.

In Fig. 6.4 , the points above the 45 ° straight line represent conservative estimates of the shear capacity by the proposed equation, points lie at the line mean ideal solution and the points below the 45° line indicate non conservative solution . Fig. 6.4 shows that around 82% of the data points are conservative, therefore the proposed equation can be considered acceptable and gives solution fairly close to the experimental test results.

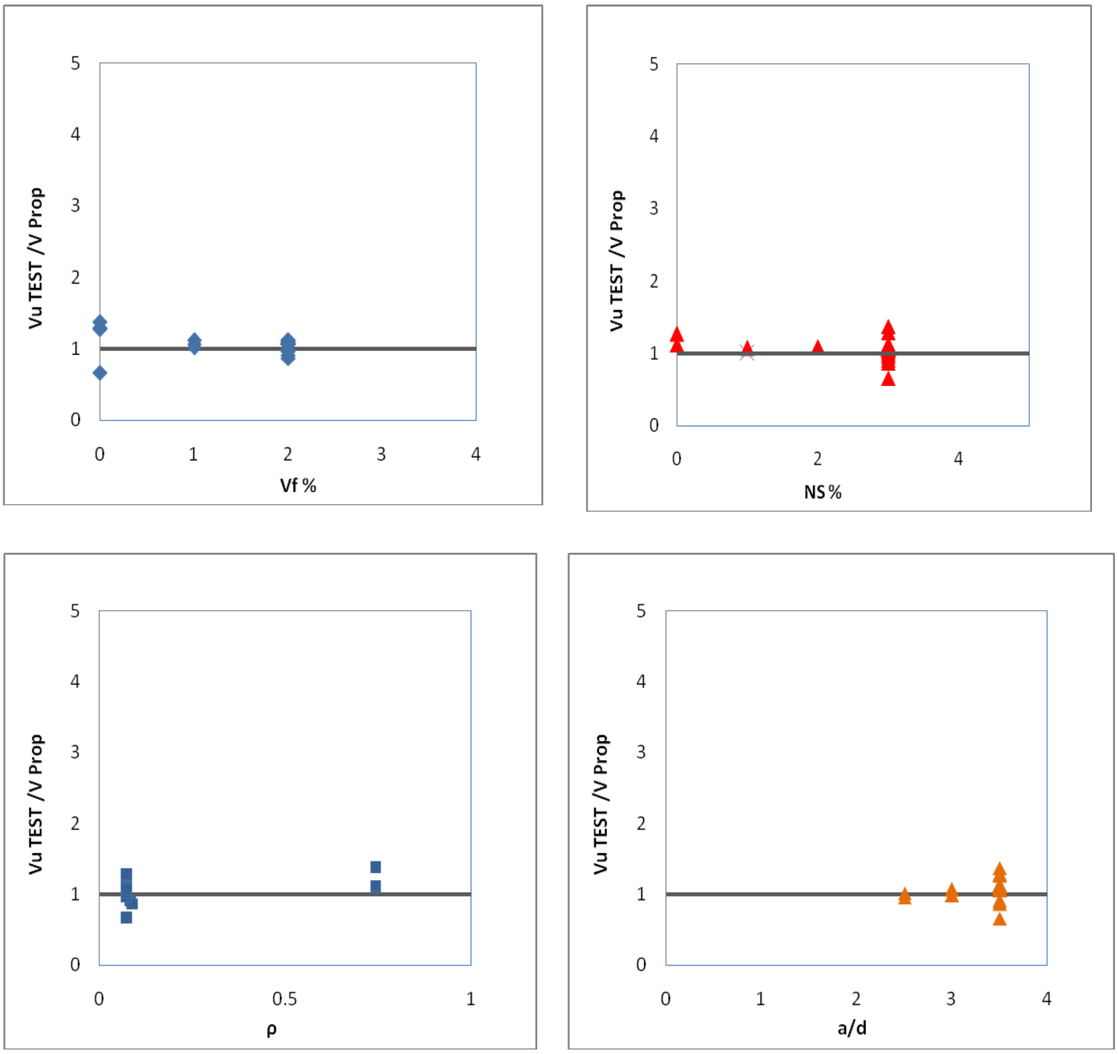

Fig. 6.5: The Ratio Vu-T/Vu-Proposed Versus Different Factors *(NS,Vf* ,ρ and *a/d Vf*) Respectively

In Fig. 6.5, all points are very close to the standard average line (mean). This is another sign of the applicability of the proposed shear capacity equation since it gives close conservative results in comparison with the experimental tests.

6.6.2 Comparison with various design approaches

6.6.2.1 Review of an existing design method for shear of RPC

In this section, one method of approach for the shear design of concrete members is presented. This was given by Ashour et. al [50] for describing the shear behavior of high strength fiber reinforced concrete beams .

The predictive expression for the shear capacity of HSFRC beams as given by Ashour et. al[50] is:

$$Vu = [(0.7\sqrt{f_{cf}} + 7F + 17.2\rho)(d/a).]bwd(N) \qquad (6\text{-}15)$$

The proposed shear capacity equation for NSRPC beams (Eq.6-13) can now be compared with the corresponding equations given by Ashour et.al[2](Eq.6-15 for HSFRC beams). For such comparison ,the practical results of compressive strength obtained by Beigi et al[40] considering certain values of NS and Vf in the mix (as given in Table 6.4) will be adopted and used.

The discrepancies between the mentioned shear capacity equations are shown in Fig. 6.6. The figure clearly shows that the use of NS in the cement matrix can certainly enhance the shear capacity of NSRPC beams and that such enhancement is greater in beams with higher NS. It is also clear that using an increased amount of Vf in the mix can result in an apparent increase in the shear capacity.

For the small values of Vf shown in Fig. 6.6,the proposed shear capacity equation(Eq. 6-13) appeared to give higher prediction for the shear capacity than the other two equations but with increasing Vf the discrepancies between the three equations become rather small.

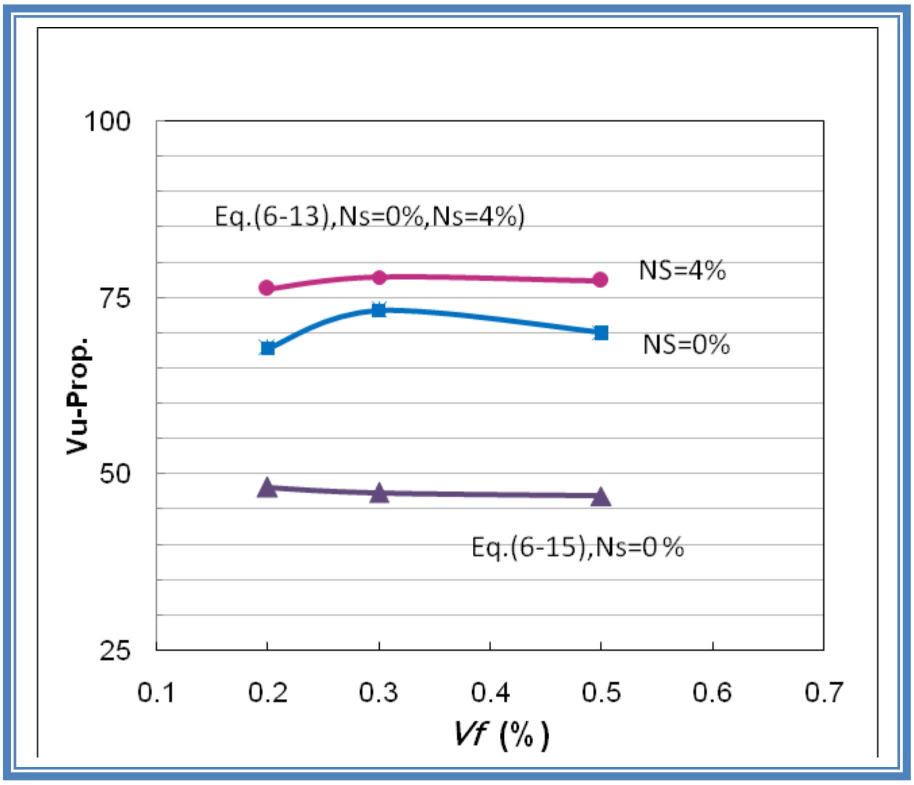

Fig. 6.6: Comparison between Eq.(6-13)and(6-15)

Table 6.1:Cylinder Compressive Strength of NSRPC in (MPa)

Control Specimens No.	Cylinder Compressive Strength (f_{cf})of NSRPC in (MPa)								
	Type of Mix								
	M0,15,2	M1,15,2	M2,15,2	M3,15,2	M3,15,1	M3,15,0	M3,10,2	M3,5,2	M0,15,0
1	122	138.5	156	157.2	127.9	97.5	150.1	142	77
2	120	138	147.5	156.4	128.1	99.3	148.7	140.6	73
3	118	138.6	152.5	159.1	128.4	97.2	147.3	139.2	74.5
4				157.5	128.3				
5				156.2	127.5				
6				154	127.6				
7				156.3	128.4				
8				157.1	127.8				
9				155.9	128				
10				158.9					
11				157.3					
12				160					
13				159					
14				162					
15				157.1					

Table 6.2: Splitting Tensile Strength of NSRPC in (MPa)

Control Specimens No.	Splitting Tensile Strength (f_{spf})of NSRPC in (MPa)								
	Type of Mix								
	M0,15,2	M1,15,2	M2,15,2	M3,15,2	M3,15,1	M3,15,0	M3,10,2	M3,5,2	M0,15,0
1	17	22.4	22.5	24.3	17.7	7	24	24.6	4.6
2	17.5	22	21.9	25.1	17.9	7.6	22.9	23.4	3.7
3	17.1	22.5	22.5	25.8	18.5	7.3	24.5	21.6	7.6
4				24.1	18.1				
5				24.9	17.5				
6				24.6	17.6				
7				24.6	17.8				
8				25.3	17.7				
9				24.6	18.3				
10				24.7					
11				25.6					
12				25.3					
13				25.7					
14				245					
15				25.4					

Table 6.3: Modulus of Rupture of NSRPC in (MPa)

Control Specimens No.	Modulus of Rupture (f_{rf})of NSRPC in (MPa)								
	Type of Mix								
	M0,15,2	M1,15,2	M2,15,2	M3,15,2	M3,15,1	M3,15,0	M3,10,2	M3,5,2	M0,15,0
1	19.3	25.6	27.9	28.4	19.5	7.9	28.3	25.8	6.1
2	18.8	26	27.5	28.8	19.6	7.2	26.5	24.3	4.8
3	18.9	25.8	27.7	28.5	19.7	7.7	26.2	26.1	5.3
4				28.6	19.6				
5				28.3	18.8				
6				28.7	18.9				
7				29	19.5				
8				28.6	19.3				
9				27.7	19.7				
10				27.9					
11				28.5					
12				28.8					
13				28.4					
14				29.1					
15				28.8					

Table 6.4: Cylinder Compressive Strength of the Hardened Concrete by Beigi,et al [40]

Mix No.	Series	Ns%	Vf%	Compressive Strength (MPa)
1	A	0	0.2	74.3
2			0.3	81.5
3			0.5	78
1	B	2	0.2	77.5
2			0.3	82.7
3			0.5	79.2
1	C	4	0.2	85.7
2			0.3	88
3			0.5	87.2