Chapter one
Introduction

1.1 concept of vacuum

Vacuum plays an important role in physics it is not devoted from any physical meaning as people think. vacuum has it is own energy. At the early universe vacuum is responsible for generating inflation, which is responsible of solving some long standing problems, like horizon flatness and entropy problem . vacuum energy is also proposed to generate elementary particles , beside permitting the propagation of electro magnetic waves[1] .

1.2 Problems of vacuum energy:

The role of vacuum energy in generating masses of elementary particles through higgs field faces some problems. Till now the higgs field is not discovered . the generation of field from photon field which constitutes vacuum is not well also established [2].

1.3 Aim of the work:

The aim of this work is to utilize generalized general relativity (GGR) and photon theory to relate vacuum energy to the photons and to see how vacuum and photon energy can generate fields and elementary particles .the expression for the vacuum energy can be treated also as a cosmological constant to solve some cosmological problems like horizon, flatness and entropy problem.

1.4 Methodology:

The GGR energy momentum tensor is used to constract an expression for the vacuum energy by minimizing the energy. the coordinate condition is also used to simplify the problem .

The photon theory which relates photon energy to the field potentials is also used to see how the fields can be generated.
These models are used to solve some cosmological problems and to explain how the fields and masses of elementary particles are generated .

The results obtained are compared with experimental and empirical results. They are also compared with the previous work made by others .

Chapter 2
The Big Bang Model

2.1 introduction

The observed expansion of the universe is a natural (almost inevitable) result of any homogeneous and isotropic cosmological model based on general relativity . however, by itself, the Hubble expansion does not provide sufficient evidence for what we generally refer to as the Big- Bang Model of cosmology. While general relativity is in principle capable of describing the cosmology of any given distribution of matter, it is extremely fortunate that our universe appears to be homogeneous and isotropic on large scales .

The formulation of Big- Bang model began in the 1940s with the work of George Gamow and his collaborators. in order to account for the possibility that the abundances of the elements had a cosmological origin, they proposed that the early universe which was once very hot and dense and has expanded and cooled to its present state [3].

It was a Big Bang some 15 billion years ago, when the size of universe was zero and temperature was infinite. The universe then started expanding at near light speed [4].

2.2 Basic Concepts

2.2.1 The Friedmann- Robertson-Walker Metric

by the cosmological law as a physical constraint we can rite the metric to the form [5]

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2(\theta)d\phi^2\right)\right] \qquad (2.2.1)$$

where $a(t)$ is function of time, k is a constant take the values $+1,0,-1$

only on choosing suitable units for r. the values $+1, 0, -1$ stands for a closed , spatially flat and open universes respectively within the frame work of SBB as will be shown later. the above metric is known in cosmology as the Robert-Walker metric. The spatial polar coordinates r, θ and ϕ form a co-moving system in the sense that typical galaxies have constant spatial coordinates r, θ and ϕ.

2.2.2 The Hubble Constant and velocities [6,7]

In the $1920's$ Hubble measured the velocities of 18 spiral galaxies with a well-known distance. His fundamental discovery was that recession velocities $\vec{v}$ increased linearly with distance $\vec{r}$ between galaxies :

$$\vec{v} = \frac{\dot{a}}{a} r \qquad (3.2.2.1)$$

The above relation is known as Hubble s law and the combination

$$\frac{\dot{a}}{a} \equiv H_0 \qquad (3.2.2.2)$$

is known as the Hubble parameter. This law has been verified by the observation of some 3000 galaxies out to red shifts of $z \approx 0.5$.

The present values from [8] $50 to 85 km \sec^{-1} Mpc^{-1}$ (Mpc stands for mega parsec where one par sec (1pc)=3.26 light years). H_0 is found by plotting galactic velocities versus distance and finding the average slope. The galactic velocities are determined by the Doppler shifting of the observed light.

Thus Hubble s law predicts that the universe is expanding galaxies are receding from us.

2.2.3 The Red shifting of Light [9,10]

Our most important information about the scale factor $a(t)$ comes from the observation of shifts in frequency of light emitted by distant sources. To calculate such frequency shifts, we shall place ourselves at the origin r =0 of coordinates and consider an electromagnetic wave traveling to us along the –r direction with θ and ϕ fixed. The equation of motion of a given wave crest is

$$0 = dt^2 = dt^2 - a^2(t)\frac{dr^2}{1-kr^2} \qquad (2.2.3.1)$$

Hence if the wave leaves a typical galaxy, located at r_1, θ_1, ϕ_1 at time t, then it will reach us at a time t given by

$$\int_{t_1}^{t_0} \frac{dt}{a(t)} = f(r_1) \qquad (2.2.3.2)$$

Where,

$$f(r_1) = \int_0^{r_1} \frac{dr}{\sqrt{1-kr^2}} = \begin{cases} \sin^{-1} r_1 & k=1 \\ r_1 & k=0 \\ \sinh^{-1} r_1 & k=-1 \end{cases} \qquad \to (2.2.2.3)$$

If the next wave crest leaves r_1 at time $t_1 + \delta t_1$, it will arrive here at a time $t_0 + \delta t_0$, which is again given by

$$\int_{t_1+\delta t_1}^{t_0+\delta t_0} \frac{dt}{a(t)} = f(r_1) \qquad \to (2.2.3.4)$$

Taking the difference between (2.2.3.2) and (2.2.3.4) and noting that a(t) does not change much during the periods $\delta t_0 + \delta t_1$, one obtains

$$\frac{\delta t_0}{a(t_0)} = \frac{\delta t_1}{a(t_1)}$$

i.e.

$$\frac{\lambda_0}{a(t_0)} = \frac{\lambda_0}{a(t_1)} \tag{2.2.3.5}$$

hence the wavelength of light is actually stretched by the expansion of the universe. The frequency v_0 observed here is thus related to the frequency v_1 when emitted by the relation.

$$\frac{v_0}{v_1} = \frac{\delta t_1}{\delta t_0} = \frac{a(t_1)}{a(t_0)} \tag{2.2.3.6}$$

This can be expressed in terms of the red shift z, defined as the fractional increase in wavelength

$$z \equiv \frac{\lambda_0 - \lambda_1}{\lambda_1} \tag{2.2.3.7}$$

since $\frac{\lambda_0}{\lambda_1} = \frac{v_1}{v_0}$, equation (2.2.3.6) gives

$$z = \frac{a(t_0)}{a(t_1)} - 1 \qquad (2.2.3.8)$$

For nearly galaxies the value of z [11] is very small and the corresponding velocity is tiny with respect to that of light . the shift may be reasonably interpreted as due to a classical Doppler effect. by contrast for the most distant galaxies the value of z may exceed 1, which necessitates a complete theory of gravitation. Conversely the amount of expansion can be expressed in terms of the red shift

$$z\frac{a(t_0)}{a(t_1)} = 1 + z \qquad (2.2.3.9)$$

We derive these relations because of the central role the red shift of light plays in cosmology and to explicitly show that 1+z is equivalent to the ratio of scale factors.

To avoid confusion, it should be kept in mind that ν_1 and λ_1 are the frequency and wavelength of light if observed near the place and time of emission, while ν_0 and λ_0 are the frequency and wavelength of the light observed after its long journey to us. It $z > 0$ in (2.2.3.7) then $\lambda_0 > \lambda_1$ thus red shift occurs while if $z < 0$ then $\lambda_0 < \lambda_1$ and blue shift occurs.

If the universe is expanding then $a(t_0) > a(t_1)$ and (2.2.3.8) gives a red shift while if the universe is contracting then $a(t_0) < a(t_1)$ and (2.2.3.8) gives a blues shift. Such frequency shifts can be explained in terms of the Doppler effect which results from the relative motion of the source and the observed.

The first evidence for a systematic red shift of spectral lines from distant object was discovered by Vesto Melvin Slipher [12].in1922 he gave data for 41 spiral nebulae of which 36 showed red shifts and only five showed blue shift. these frequency shift were interpreted as due to the Doppler effect. However , Writz and K.

Lund mark showed that Slipher s red shifts increased with the distance of the spiral nebulae and therefore could be understood in terms of a general recession of distant galaxies, the furthest being those moving fastest. Thus the announcement by Hubble of a roughly linear relation between velocities and distances , equation (3.2.3.9) established the interpretation of the red shift as a cosmological Doppler effect.

2.2.4 The Concept of Horizon [13]

The horizon demarcates events that are observable at a certain instant in the life time of the universe ; these events belong to the past light cone . it the observer has coordinates (0,t) the coordinate v_1 of a point emitting a light signal at a time t_1 is given by the relation

$$\int_0^{r_1} \frac{dr}{\sqrt{1-kr^2}} = \int_{t_1}^{t} \frac{dt}{a(t)} \qquad (2.2.4.1)$$

If the integral on the right diverges when t tends towards 0, one can in principle receive a signal from all points in space (because the system of the coordinates cover the entire group of points). If , on the other hand, this integral converges , there exists a maximum finite value r_h for the coordinate r_1. The observer cannot receive information from points situated at $r > r_h, r_h$ being defined by

$$\int_0^{r_h} \frac{dr}{\sqrt{1-kr^2}} = \int_{t_1}^{t} \frac{dt}{a(t)} \qquad (2.2.4.2)$$

2.2.5 The Einstein Equations for the Robertson-Walker Metric

In this section we derive the Einstein equation for the Robertson-Walker Metric [14,15] , in which the matter is in the form of a perfect fluid of mass-energy density ρ and pressure p so tat the energy momentum tensor is given by

$$T^{\mu\nu} = (\rho + p)U^{\mu}U^{\nu} - pg^{\mu\nu} \qquad (3.2.5.1)$$

Where $U^{\mu} = (1,0,0,0)$ is the 4-velocity tensor as we are in commoving coordinates.

$$T^{tt} = \rho(t), \qquad T^{it} = 0 \text{ and } T^{ij} = -g^{ij}p$$

and

$$i, j = r, \theta, \phi$$

also

$$g_{\mu\nu}U^{\mu}U^{\nu} = +1 \qquad (2.2.5.2)$$

The non-zero metric components are given as follows (recall that $(x^0, x^1, x^2, x^3) = (ct, r, \theta, \phi)$)

$$g_{tt} = 1, \qquad g_{rr} = \frac{-a^2}{1-kr^2}, \qquad g_{\theta\theta} = -r^2a^2$$

$$g_{\phi\phi} = -a^2r^2\sin^2\theta$$

$$g^{tt} = 1, \qquad g^{rr} = -\frac{(1-kr^2)}{a^2}, \qquad g^{\theta\theta} = -(ra)^{-2}$$

$$g^{\phi\phi} = -(ar\sin\theta)^{-2} \qquad (2.2.5.3)$$

we put the non-vanishing Christoffel symbols $\Gamma^{\mu}_{\lambda\nu}$ in four groups according to the values t, r, θ, ϕ of the index μ, as follows

$$\Gamma^{t}_{rr} = \frac{aa^{\bullet}}{1-kr^2} \qquad \Gamma^{t}_{\theta\theta} = r^2 aa^{\bullet} \qquad \Gamma^{t}_{\phi\phi} = r^2(\sin^2\theta)aa^{\bullet}$$
$$\Gamma^{t}_{tr} = \frac{a^{\bullet}}{a} \qquad \Gamma^{r}_{rr} = \frac{kr}{1-kr^2}$$
$$\Gamma^{r}_{\theta\theta} = -r(1-kr^2) \qquad \Gamma^{r}_{\phi\phi} = -r(1-kr^2)\sin^2\theta$$
$$\Gamma^{\theta}_{t\theta} = \frac{a^{\bullet}}{a} = \Gamma^{\phi}_{t\phi} \qquad \Gamma^{\theta}_{r\theta} = \frac{1}{r} = \Gamma^{\phi}_{r\phi}$$
$$\Gamma^{\theta}_{\phi\phi} = -\sin\theta\cos\theta \qquad \Gamma^{\theta}_{\theta\phi} = \cot\theta \qquad (2.2.5.4)$$

Then

$$\Gamma^{\mu}_{\lambda\nu} = \frac{1}{2} g^{\mu\sigma}\left(g_{\sigma\nu,\lambda} + g_{\sigma\lambda,\nu} - g_{\nu\lambda,\sigma}\right) \qquad (2.2.5.5)$$

$$R_{\mu\nu} = \Gamma^{\lambda}_{\mu\nu,\lambda} - \Gamma^{\lambda}_{\mu\lambda,\nu} + \Gamma^{\lambda}_{\mu\nu}\Gamma^{\sigma}_{\lambda\sigma} - \Gamma^{\sigma}_{\mu\lambda}\Gamma^{\lambda}_{\nu\sigma} \qquad (2.2.5.6)$$

to get the following non-zero components of the Ricci tensor $R_{\mu\nu}$ (note that r is dimensionless while a(t) has the dimension of length)

$$R_{tt} = -\frac{3a^{\bullet\bullet}}{a}$$
$$R_{rr} = \left(aa^{\bullet\bullet} + 2a^{\bullet^2} + 2k\right)\left(1-kr^2\right)$$
$$R_{\theta\theta} = r^2\left(aa^{\bullet\bullet} + 2a^{\bullet^2} + 2k\right)$$
$$R_{\phi\phi} = r^2\sin^2\theta\left(aa^{\bullet\bullet} + 2a^{\bullet^2} + 2k\right) \qquad (2.2.5.7)$$

Thus the Ricci scalar can be evaluated using (2.2.5.7) as follows

$$R = g^{\mu\nu} R_{\mu\nu} = \frac{-6\left(aa^{\bullet\bullet} + a^{\bullet^2} + k\right)}{a^2} \qquad (2.2.5.8)$$

we are now in a position to write down the Einstein equation

$$R_{\mu\nu} = -\frac{1}{2} g_{\mu\nu} R = -8\pi G T_{\mu\nu} \qquad (2.2.5.9)$$

noting that the covariant components of the four-velocity are the same as contra variant ones: $U_\mu = (1,0,0,0)$ so that the non-zero components of $T_{\mu\nu}$ are

$$\begin{aligned} T_{tt} &= \rho \\ T_{rr} &= \frac{pa^2}{1-kr^2} \\ T_{\theta\theta} &= pr^2 a^2 \\ T_{\phi\phi} &= pr^2\left(\sin^2\theta\right)a^2 \end{aligned} \qquad (2.2.5.10)$$

the time –time component of (2.2.5.9) can be written using (2.2.5.3) and(2.2.5.7) as follows

$$R_{tt} - \frac{1}{2} g_{tt} R = 8\pi G T_{tt} \qquad (2.2.5.11)$$

i.e.

$$3\left(a^{\bullet^2} + k\right) = 8\pi G \rho a^2$$

dividing by 3

$$\left(a^{\bullet^2} + k\right) = \frac{8}{3}\pi G \rho a^2 \qquad (2.2.5.12)$$

the space-space components are given by

$$R_{ii}-\frac{1}{2}g_{ii}R=8\pi GT_{ii} \qquad (2.2.5.13)$$

$$2aa^{\bullet\bullet}+a^{\bullet^2}+k=-8\pi G\rho a^2 \qquad (2.2.5.14)$$

the term $a^{\bullet^2}$ can be eliminated by subtracting (2.2.5.12) from (2.2.5.14) to get

$$3a^{\bullet\bullet}=-4\pi G(\rho+3p)a \qquad (2.2.5.15)$$

Multiplying (2.2.5.12) by 3 and adding (2.2.5.14) yields

$$aa^{\bullet\bullet}+2a^{\bullet^2}+2k=4\pi G(\rho-p)a^2 \qquad (2.2.5.16)$$

In addition we have the equation of energy conservation

$$p^{\bullet}a^3=\frac{d}{dt}\left[a^3(\rho+p)\right] \qquad (2.2.5.17)$$

Or equivalently

$$\frac{d}{da}\left(\rho a^3\right)=-3pa^2 \qquad (2.2.5.18)$$

Thus given an equation of state $p=p(\rho)$, we can use this equation to determine ρ as a function of a. knowing ρ as a function of a, we can determine $a(t)$ for all time by solving (2.2.5.12) . thus the fundamental equation of dynamical cosmology are Einstein equation (2.2.5.12) , the energy-conservation equation (2.2.5.17) and the equation of state.

It is possible to learn [16] a good deal about the past and future expansion of the universe by inspecting the field equations. Equation (2.2.5.13) shows that as long as $\rho+3p$ remains positive, the acceleration $\frac{a^{\bullet\bullet}}{a}$ is negative . since at present $a > 0$ (by definition) it follows that the curve of a(t) versus t must be concave downwards and must have reached a(t) =0 at some finite in the past t=0

so that

$$a(0) = 0 \qquad (2.2.5.19)$$

In the future, we see from equation (2.2.5.18) that as long as the pressure p does not become negative, the density ρ must decrease with increasing a, at least as fast as a^{-3}, so that far $a \to \infty$, the right hand side of equation (2.2.5.10) vanishes at least as fast as a^{-1}. For $k = -1,\ a^{\bullet^2}(t)$ remains positive-definite, so that a(t) goes on increasing with $a(t) \to t$ as $t \to \infty$. For $k = 0, a^{\bullet^2} = \frac{8\pi G\rho a^2}{3}, a^{\bullet^2}(t)$ remains positive-definite so a(t) goes on increasing, but more slowly than t. for $k = +1, a^{\bullet^2} = -1 + \frac{8\pi G\rho a^2}{3}, a^{\bullet^2}(t)$ will reach zero when ρa^2 drops to the value $\frac{3}{8\pi G}$ Since $a^{\bullet\bullet}$ is negative-define, a(t) will then begin to decrease again and must eventually again reach a=0 at some finite time in the future. Hence the cosmic history of the universe is determined by sign of the spatial curvature: k=-1 or k=0, then the universe will go on expanding forever, whereas if k=+1, then the expansion will eventually cease and be followed by a contraction back to a singular state with a(t)=0. An alternative derivation [17] of the dynamical equation for expanding universe is as follows: if we consider a galaxy of gravitating mass m_G located at a radius r from the center of a sphere of mean density and mass $M = \frac{4\pi\rho r^3}{3}$. the gravitational potential arising from the matter is

$$U = \frac{-GMm_G}{r} = -\frac{4\pi G m_G \rho r^2}{3} \qquad (2.2.5.20)$$

where G is the Newtonian constant expressing the strength of the gravitational interaction. Thus the galaxy falls towards the center of gravitation, acquiring a radial acceleration

$$r^{\bullet\bullet} = \frac{-GM}{r^2} = \frac{-4\pi G\rho r}{3} \qquad (2.2.5.21)$$

This is Newton s law of gravitation, usually written as

$$F = \frac{-GMm_G}{r^2} \qquad (2.2.5.22)$$

where F (in old fashioned parlance) is the force exerted by the mass M on the mass m_G the negative signs in the (2.2.5.20)and (2.2.5.22) express the attractive nature of gravitation: bodies are forced to move the direction of decreasing r .

in an expanding Hubble universe the kinetic energy T of a galaxy receding with velocity v is

$$T = \frac{1}{2}mv^2 = \frac{1}{2}mH_0^2 r^2 \qquad (2.2.5.23)$$

Where m is the inert mass of the galaxy . setting $m_G = m,$ the total energy E is given by

$$
\begin{aligned}
E &= T + U \\
&= \frac{1}{2} m H_0^2 r^2 - \frac{4\pi G m \rho r^2}{3} \\
&= m r^2 \left(\frac{1}{2} H_0^2 - \frac{4\pi G \rho}{3} \right) \\
&= \frac{1}{2} \frac{m r^2}{a^2} \left(\dot{a}^2 - \frac{8\pi G \rho a^2}{3} \right)
\end{aligned} \qquad (2.2.5.24)
$$

With E constant (2.2.5.24) is the same as (2.2.5.12) provided that we identify the energy of a particle as

$$
E = \frac{-\frac{1}{2} m |r(t_0)|^2 k}{a^2(t_0)} \qquad (2.2.5.25)
$$

For k =-1, E is positive-definite [18] so gravitation cannot prevent the galaxies from dispersing to infinity, with a finite asymptotic velocity. For k=0, E vanishes and the galaxies are barely able to expand indefinitely. for k=+1, E is negative and the explosion must ultimately cease and be followed by an implosion.

If the mass density ρ of the universe is large enough, the expansion will halt .the condition for this to occur is E=0 or from (2.2.5.24) this critical density [19] is

$$
\rho_c = \frac{3H_0^2}{8\pi G} = 1.1 \times 10^{-29} \left(\frac{H_0}{75 km \sec^{-1} / MPc} \right)^2 g / cm^3 \qquad (2.2.5.26)
$$

A universe with density $\rho > \rho_c$ is called closed; and that with density $\rho < \rho_c$ is called open.

The density parameter Ω_0 can be introduced as

$$
\Omega_0 = \frac{\rho_0}{\rho_c} = \frac{8\pi G \rho_0}{3H_0^2} = 2q_0 \qquad (2.2.5.27)
$$

Where q_0 is the deceleration parameter which can be defined by

$$q_0 = \frac{a_0 \ddot{a}_0}{\dot{a}_0^2} = \frac{\ddot{a}_0}{a_0 H_0^2} \qquad (2.2.5.28)$$

Hence if $q_0 > \frac{1}{2}$ in (3.2.5.27), then $\rho_0 > \rho_c$ and k =+1, while if $q_0 < \frac{1}{2}$ then $\rho_0 < \rho_c$ and k=-1,while if $q_0 = \frac{1}{2}$ then $\rho_0 = \rho_c$ and the space is flat i.e. k=0.

2.3 Types of Matter

use the field equations and the equation of state to find the time dependence of three different types of matter energy.

2.3.1 Matter-Dominated Era [20]

If the particles are non relativistic the pressure p is negligible compared to the energy density ρ i.e. $p \ll \rho$, thus the equation of energy conservation

$$\frac{d\rho}{dt} = -3\frac{\dot{a}}{a}(\rho + p) \qquad (2.3.1.1)$$

Reduces to

$$\dot{\rho} = -3\frac{\dot{a}}{a}\rho \qquad (2.3.1.2)$$

Which is solved with

$$\rho = a^{-3} \qquad (2.3.1.3)$$

This solution has a simple physical interpretation. If there are N particles each with mass m in co-moving volume L^3, then the energy density in physical space must be

$$\rho = \frac{mN}{L^3 a^3} \propto \frac{1}{a^3} \qquad (2.3.1.4)$$

Insert this form into the field equation (2.2.5.12)

$$\left(\frac{a^{\bullet}}{a}\right)^{2}=\frac{8\pi G\rho}{3}-\frac{k}{a^{2}} \qquad (2.3.1.5)$$

Also into (2.2.5.15)

$$\frac{a^{\bullet\bullet}}{a}=\frac{-4\pi G}{3}(\rho+3p) \qquad (2.3.1.6)$$

$$a^{\bullet}=\pm\sqrt{\frac{2mG}{a}-k} \qquad (2.3.1.7)$$

Where

$$m=\frac{4\pi a^{3}\rho}{3}=\frac{4\pi}{\rho}C_{2} \qquad (2.3.1.8)$$

Hence

$$da^{\bullet}=\pm\frac{mGda}{a\sqrt{2mGa-ka^{2}}} \qquad (2.3.1.9)$$

On the other hand since p=0 in a matter era, hence by (2.3.1.3), equation (2.2.5.15) becomes

$$\frac{da^{\bullet}}{dt}=\frac{-4\pi G\rho a}{3}=\frac{-mG}{a^{2}} \qquad (2.3.1.10)$$

By using (2.3.1.9) one gets

$$da^{\bullet} = \frac{-mG}{a^2}dt = \frac{mGda}{a\sqrt{2mGa - ka^2}}$$

$$t = \int \frac{ada}{\sqrt{2mGa - ka^2}} \qquad (2.3.1.11)$$

If k=0, this equation yields

$$t = \pm\sqrt{\frac{2}{9mG}a^{\frac{3}{2}}}$$

$$\therefore a = \left(\frac{9mG}{2}\right)^{\frac{1}{3}} t^{\frac{2}{3}} \qquad (2.3.1.12)$$

Thus the universe expands for ever. If k=+1 then (2.3.1.11) gives

$$t = mG\sqrt{\frac{a}{2mg}\left(1 - \frac{a}{2mG}\right)} - \sin^{-1}\left(1 - \frac{a}{2mG}\right) \qquad (2.3.1.13)$$

and this indicates that the expansion will eventually cease and be followed by a contraction, while for k=-1

$$t = mG\sqrt{\frac{a}{2mg}\left(1 + \frac{a}{2mG}\right)} + \frac{1}{2}\ln\left[\frac{\sqrt{1 + \frac{a}{2mG}} - \sqrt{\frac{a}{2mG}}}{\sqrt{1 + \frac{a}{2mG}} + \sqrt{\frac{a}{2mG}}}\right] \qquad (2.3.1.14)$$

And the universe will expand for ever.

2.3.2 Radiation conquered Universe

In a universe where the energy density is dominated by relativistic hot particles, the pressure cannot be neglected. The pressure is one-third of the energy density, hence $p = \frac{1}{3}\rho$. from the conservation equation (2.3.1.1)

$$\frac{d\rho}{dt} = -3\frac{a^{\bullet}}{a}(\rho + p) \tag{2.3.2.1}$$

$$\rho^{\bullet} = -3\frac{a^{\bullet}}{a}(\frac{4}{3}\rho) = -4\frac{a^{\bullet}}{a}\rho \tag{2.3.2.2}$$

which is solved if $\rho \propto a^{-4}$. just as with cold particles, the number density of quanta decrease as a^{-3}. The extra power of a for relativistic particles comes from the fact that the particles energy E red shifts as

$$\langle E \rangle = \frac{1}{\langle wavelength \rangle} = \frac{1}{\langle \lambda \rangle a} \tag{2.3.2.3}$$

Where λ is the co-moving wavelength of the radiation. In analogy with equation (2.3.1.4), we can understand the energy density of radiation as a number N of mass less particles in a volume $(La)^3$ with present energy $\langle E \rangle / a$ thus

$$\rho = \frac{N}{l^3 a^3}\frac{\langle E \rangle}{a} \propto \frac{1}{a^4} = \frac{C_2}{a^4} \tag{2.3.2.4}$$

According to equation (2.2.5.17) [21] $a^{\bullet}$ is large in early universe hence the curvature term can be dropped from (2.2.5.12) which becomes

$$a^{\bullet^2} = \frac{8\pi G\rho a^2}{3} \tag{2.3.2.5}$$

Using (2.3.2.4) yields

$$\overset{\bullet}{a}^2 = \frac{8\pi G C_2 a^{-2}}{3} \qquad (2.3.2.5)$$

$$\therefore a = \left[\left(\frac{32\pi G C_2}{3} \right)^{\frac{1}{2}} t + C_3 \right]^{\frac{1}{2}} \qquad (2.3.2.6)$$

Since at t =0 , a=0, thus $C_3 = 0$

$$\therefore a = \left(\frac{32\pi G C_2}{3} \right)^{\frac{1}{4}} t^{\frac{1}{2}} \qquad (2.3.2.7)$$

One can see from comparing the solutions for the matter and radiation energy densities that if both types are present in an expanding universe, then eventually the matter will dominate.

The present value of radiation density is about $10^{-12} erg/cm^3$ [22] predominantly in the form of microwaves and infra-red light . the present matter is not known because we can only observe luminous matter (l um) and there may be other matter (dark) as well in the form of invisible particles.

2.3.3 The vacuum Energy [23]

The final type of matter that will consider is vacuum energy ρ_v usually the energy in the vacuum is of no dynamical interest. However, in GR, all forms of energy feel the force of gravitation and are important. Thus we can have the odd situation that the universe can be dynamically dominate by vacuum energy. This can happen, for example, when a symmetry-breaking occurs and the universe undergoes a phase transition.

This vacuum energy has odd properties when viewed in the frame work of a fluid. If we use the perfect fluid from $T^{\mu\gamma}$ as in equation (2.2.5.1), we find

$$p = -\rho_v \qquad (2.3.3.1)$$

So vacuum energy has a negative pressure. The conservation equation (2.3.2.1) confirms that the vacuum energy density is constant with this negative pressure [24]i.e.

$$\rho_v \propto constant \qquad (2.3.3.2)$$

In a universe with a mixture of matter and vacuum energy, the vacuum energy will quickly dominate any matter energy density, the same result follows if we replace matter energy with radiation energy.
Thus

$$\left(\frac{a^{\bullet}}{a}\right)^2 \approx \frac{8\pi G\rho_v}{3} = constant \qquad (2.3.3.3)$$

Which has the solution

$$a = e^{Ht} \qquad (2.3.3.4)$$

2.4 Cosmological Problems of the Standard big bang Model

We have spent the time discussing the observational support for the cosmological (SBB) model. Despite the successes of this model there are still a few puzzling features one would like to explain. In the following sections we will discuss some of these defects termed as cosmological problems.

2.4.1 The Horizon Problem

In the SBB the initial universe is assumed to be isotropic and homogeneous, yet it consists of a huge number of separate regions which are causally disconnected [25](i.e., these regions have not yet had time to communicate with each other via light signals). This homogeneity is predicted by measurements of the CMBR which give temperatures in different regions differing by less than O(10^{-4}) [26]

How did these disconnected patches come to have the same temperature? This is the horizon problem. the physical horizon distance $L(t)$ traveled by alight pulse beginning at t =0 is given by[27]

$$L(t) = a(t)\int_0^t \frac{dt'}{a(t')} = 2t \qquad (2.4.1.1)$$

thus whatever physical process operated at this epoch were limited in range by $L(t)$. on the other hand, the physical $L(t)$ of the observed part of the universe is given by [28]

$$L(t) = \frac{a(t)}{a(t_p)} L(t_p) = \frac{T(t_p)}{T(t)} L(t_p) \qquad (2.4.1.2)$$

Where t_p the present time, $T(t_p)$ is the present microwave background temperature. Hence

$$\frac{l(t)}{L(t)} = \left[\alpha T(t) T(t_p) L(t_p)\right]^{-1} \qquad (2.4.1.3)$$

Where $\alpha = \sqrt{\frac{8\pi^3 G(T)}{90}}$ and N is the effective number of spin degrees of freedom. With $N \approx 10^2, T(t_p) = 2.7k, L(t_p) = 10^{10}$ years and evaluating (2.4.1.3) at $10^{17} GEV$, we obtain

$$\frac{1}{L} \approx 10^{-28} \qquad (2.4.2.4)$$

which is the ratio of the horizon radius to the radius universe.

Thus

$$\frac{V_{univ}}{V_{horiz}} = \left(\frac{L}{l}\right)^3 \approx 10^{84} \qquad (2.4.2.5)$$

Which means that the initial universe consist of at least $\approx 10^{84}$ causally disconnected region indicating that our universe is neither homogeneous nor isotropic. This is in contrast with the observed isotropy and homogeneity.

2.4.2 the Entropy problem

The observed homogeneity and isotropy of the detected microwave background and the observed value of the mass density suggests that there was an enormous amount of entropy [29]mainly in the form of blackbody radiation [30]. further the entropy contained within the horizon at early times was 10^{63} . this is much less than the entropy today which is about 10^{88} , this means that entropy is increasing.

On the other hand, the classical Einstein equations are purely adiabatic and reversible. consequently, they predict constant entropy. Therefore, they can hardly provide any explanation related to the origin of cosmological entropy.

In the expanding universe, the second law of thermodynamics, applied to a commoving volume element V, implies that

$$Tds = dpV + pdV = d[(\rho + p)V] - Vdp \qquad (2.4.2.1)$$

Where ρ and p are the equilibrium energy density and pressure. The integrality condition

$$\frac{\partial^2 S}{\partial T \partial V} = \frac{\partial^2 S}{\partial V \partial T} \qquad (2.4.2.2)$$

Relates the energy density and pressure

$$\frac{TdP}{dT} = \rho + p \qquad (2.4.2.3)$$

or equivalently

$$dP = \frac{\rho + p}{T} dT \qquad (2.4.2.4)$$

substituting (2.4.2.4) into (2.4.2.1), it follows that

$$\begin{aligned} dS &= \frac{1}{T} d[(\rho + p)V] - (\rho + p)V \frac{dT}{T^2} \\ &= d\left[\frac{(\rho + p)V}{T} + cons\tan t\right] \end{aligned} \qquad (2.4.2.5)$$

That is the entropy per comoving volume is

$$S = a^3 \frac{(\rho + p)}{T} \qquad (2.4.2.6)$$

Recall that the first law (energy conservation) can be written as

$$d[(\rho + p)V] = Vdp \qquad (2.4.2.7)$$

Substituting (2.4.2.4) into (2.4.2.6) it follows that

$$d\left[\frac{(\rho + p)V}{T}\right] = 0 \qquad \frac{dS}{T} = 0 \qquad (2.4.2.8)$$

This result implies that the entropy S per comoving volume is conserved, which is in contradiction with the fact that the entropy of the universe should increase.

2.4.3 The Flatness Problem

We have from (2.2.5.12) that

$$\left(\frac{a^{\bullet}}{a}\right)^2+\frac{k}{a^2}=\frac{8\pi G}{3}\rho \qquad (2.4.3.1)$$

i.e.

$$H^2+\frac{k}{a^2}=\frac{8\pi G}{3}\rho \qquad (2.4.3.2)$$

where $H=\frac{a^{\bullet}}{a}$ is the Hubble parameter. The above equation can also be written as

$$\rho-\rho_c=\frac{3k}{8\pi G a^2} \qquad (2.4.3.3)$$

where $\rho_c=\frac{3H^2}{8\pi G}$ is the critical density. Dividing (2.4.3.3)by ρ_c one gets

$$\frac{\rho-\rho_c}{\rho_c}=\frac{3k}{8\pi G a^2\rho_c}\propto a \qquad (2.4.3.4)$$

Since the matter density ρ_c scales as a^{-3} . thus if there is a small deviation from the critical density at early times, this difference will grow at the same rate as expansion of the universe. Hence in the early universe the relative energy difference between ρ and ρ_c Must have been much smaller. Specifically [31]

$$\left|\frac{\rho}{\rho_c}-1\right| = \left|\Omega\left(10^{-43}\sec\right)-1\right| \le O(10^{-60})$$

whereas

$$\left|\Omega\left(1\sec\right)-1\right| \le O(10^{-16})$$

Hence if the universe is closed it would have collapsed millions of years ago, while if the universe is open the present energy density ρ would have dwindled to a value much less than the critical density ρ_c these two predictions are in conflict with the present observations which predicts that universe is expanding and its energy density ρ is in the range of

$$0.01 < \frac{\rho_0}{\rho_c} < 2$$

where ρ_0 is the present energy density.

The flatness problem can be related to the entropy problem as follows. Using the entropy conservation equation (2.4.2.6) and the fact that radiation density ρ and its pressure are given by

$$\rho = \sigma T^4 \qquad (2.4.3.5)$$

$$p = \frac{\rho}{3} = \frac{\pi^2}{90} N(T) T^4 \qquad (2.4.3.6)$$

where $\sigma = \frac{\pi^2 N}{30}$ and N is the effective number of spin degrees of freedom, yields

$$aT = cons\tan t \qquad (2.4.3.7)$$

Let us define a quantity f by

$$f = \frac{k}{8\pi G(at)^2} = constant \qquad (2.4.3.8)$$

In this case the Friedmann equation is given by

$$\rho - \rho_c = 3fT^2 \qquad (2.4.3.9)$$

Dividing both sides by ρ and using (2.4.3.5), one gets

$$\frac{|\rho - \rho_c|}{\rho} = \frac{3|f|}{\sigma T^2}$$

The entropy S, given by (2.4.2.6), together with equation (2.4.3.6) yields

$$S = \frac{a^3}{T}\left(\frac{1}{3}\rho + \rho\right) = \frac{4}{3}\frac{a^3}{T}\rho = \frac{4}{3}\sigma a^3 T^3 \qquad (2.4.3.10)$$

Combining equation (2.4.3.8) and equation(2.4.3.10),the resulting equation is

$$f = \frac{k}{8\pi G}\left(\frac{4}{3}\sigma\right)^{\frac{2}{3}} S^{\frac{-2}{3}} \qquad (2.4.3.11)$$

The equation (2.4.3.9) becomes

$$\frac{|\rho - \rho_c|}{\rho} = \frac{|k|}{4\pi G}\left(\frac{6}{\sigma}\right)^{\frac{1}{3}} \frac{1}{S^{2/3}T^2} \qquad (2.4.3.12)$$

today $T_0 = 2.7K, a_0 = 10^{28} cm$ and the entropy $S_0 = 10^{87}$ and since it is conserved it follows that

$$S = S_0 = 10^{87} \qquad (2.4.3.13)$$

Excluding the case k=0, then setting $/k/ = 1$, equation(2.4.3.12) gives

$$\frac{|\rho - \rho_c|}{\rho} \leq \frac{10^{-58}}{T^2}$$

At Planck time one finds

$$\frac{|\rho - \rho_c|}{\rho} \leq 10^{-58} \qquad (2.4.3.14)$$

If the universe density at Planck time ρ_{pl} was slightly greater than ρ_c, i.e. $\rho_0 \geq \rho_c(1+10^{-58})$ then the universe would be closed and it would have collapsed millions of years ago. This contradicts the fact that the universe exists and is expanding. On the contrary if $\rho_0 \geq \rho_c(1-10^{-58})$ then the universe would be open and the present energy density would be negligibly small and much less than the critical density ρ_c this again contradicts the present observation where $0.01 < \frac{\rho_0}{\rho_c} < 2$

Chapter 3
The Generalized field Equation

3.1 introduction

Einstein's general theory of relativity is a beautiful piece of art which connects gravitational fields with geometry of space and time and thus provides a scheme in which our universe can be discussed [32].

3.2 The Generalized field Equation Model

A homogeneous universe is described by Robertson walker(RW) metric[33]

$$g_{tt} = -1 \qquad g_{it} = 0 \qquad g_{ij} = a(t)^2 \tilde{g}_{ij}$$
$$\tilde{g}_{rr} = \left(1 - kr^2\right)^{-1} \qquad \tilde{g}_{\theta\theta} = r^2 \qquad \tilde{g}_{\phi\phi} = r^2 \sin^2\theta$$
$$\tilde{g}_{ij} = 0, \qquad for \quad i \neq j, \qquad i, j = r, \theta, \phi \tag{3.2.1}$$

and the affine connection is takes the form

$$\Gamma^t_{tt} = 0, \qquad \Gamma^t_{ti} = 0, \qquad \Gamma^t_{ij} = \frac{a^\bullet}{a} g_{ij},$$
$$\Gamma^i_{tj} = \frac{a^\bullet}{a}\delta^i_j, \qquad \Gamma^\lambda_{tt} = 0 \tag{3.2.2}$$

where a dot stands for a differentiation with respect to time. The Ricci from (2.2.5.7) tensor thus becomes

$$R_{tt} = \frac{3a^{\bullet\bullet}}{a}, \qquad R_{ti} = 0, \qquad R_{ij} = -\left(aa^{\bullet\bullet} + 2a^{\bullet^2} + 2k\right)\tilde{g}_{ij}, \tag{3.2.3}$$

the scalar curvature R is thus given by

$$R = g^{\mu\nu} R_{\mu\nu} = g^{tt} R_{tt} = g^{ii} R_{ii} = -6\left(\frac{a^{\bullet\bullet}}{a} + \frac{a^{\bullet^2}}{a^2} + \frac{k}{a^2}\right) \tag{3.2.4}$$

the covariant derivatives of R take the following form

$$R_{;t} = \frac{\partial R}{\partial t} = R^{\bullet}, \qquad R_{;r} = \frac{\partial R}{\partial r} = 0$$

$$and$$

$$R_{;\theta} = \frac{\partial R}{\partial \theta} = 0 \qquad R_{;\varphi} = \frac{\partial R}{\partial \varphi} = 0 \qquad (3.2.5)$$

Using the definition

$$V_{\mu;\nu} = \partial_\nu V_\mu - \Gamma^{\lambda}_{\mu\nu} V_\lambda$$

One obtains

$$R_{;\mu;\nu} = \frac{\partial R_{;\mu}}{\partial x^{\nu}} - \Gamma^{\lambda}_{\mu\nu} R^{\bullet}_{;\lambda}$$
$$= \frac{\partial R_{;\mu}}{\partial x^{\nu}} - \Gamma^{t}_{\mu\nu} R^{\bullet} \qquad (3.2.6)$$

Hence one can Find the following covariant derivatives

$$R_{;t;t} = \frac{\partial R^{\bullet}}{\partial t} - \Gamma^{t}_{tt} R^{\bullet} = R^{\bullet\bullet}$$
$$R_{;t;i} = \frac{\partial R^{\bullet}}{\partial x^{i}} - \Gamma^{t}_{ti} R^{\bullet} = 0$$
$$R_{;i;t} = \frac{\partial R_{;i}}{\partial x^{t}} - \Gamma^{t}_{it} R^{\bullet} = 0 \qquad (3.2.7)$$

$$R_{;i;i} = \frac{\partial R_{;i}}{\partial x^{i}} - \Gamma^{t}_{ii} R^{\bullet} = -\frac{a^{\bullet}}{a} g_{ii} R^{\bullet} \qquad (3.2.8)$$

$$for \qquad i \neq j, \qquad g_{ii} = 0$$

and

$$R_{;i;j} = \frac{\partial R_{;i}}{\partial x^{j}} - \Gamma^{t}_{ij} R^{\bullet} = 0 - \frac{a^{\bullet}}{a} g_{ii} R^{\bullet} = 0 \qquad (3.2.9)$$

the only non-vanishing components of R read

$$R_{;t} = R^{\bullet}, \qquad R_{;t;t} = R^{\bullet\bullet}, \qquad R_{;r;r} = -\frac{a^{\bullet}}{a} R^{\bullet} g_{rr}$$

$$R_{;\theta;\theta} = -\frac{a^{\bullet}}{a} R^{\bullet} g_{\theta\theta} \qquad R_{;\varphi;\varphi} = -\frac{a^{\bullet}}{a} R^{\bullet} g_{\varphi\varphi} \tag{3.2.10}$$

The Einstein generalized field equation (GFE) of motion is given by

$$L'''\left(R_{;\mu} R_{;\nu} - g_{\mu\nu} g^{\rho\sigma} R_{;\sigma} R_{;\nu}\right) + L''\left(R_{;\mu;\nu} - g_{\mu\nu} R\right) + L' R_{\mu\nu} - \frac{1}{2} R_{\mu\nu} L = 0 \tag{3.2.11}$$

If the lagrangian L is split into pure gravitational part L and non-geometrical part $\gamma = \rho + \rho_{v}$ representing matter and vacuum energy density respectively the above equation yields

$$-\frac{3a^{\bullet}}{a} R^{\bullet} L'' + \frac{3a^{\bullet\bullet}}{a} L' + \frac{1}{2} l + \frac{\rho + \rho_{v}}{2} = 0 \tag{3.2.12}$$

If one substitute a simple non-linear Lagrangian

$$L = -\alpha R^{2} + \beta R \tag{3.2.13}$$

In the above equation we get

$$-\frac{a^{\bullet}}{a} R^{\bullet} + \frac{a^{\bullet\bullet}}{a} R + \frac{R^{2}}{12} - \frac{\beta R}{12\alpha} - \frac{a^{\bullet\bullet}\beta}{2a\alpha} - \frac{\rho + \rho_{v}}{12\alpha} = 0 \tag{3.2.14}$$

where

$L' = -2\alpha R + \beta$ *and* $L'' = -2\alpha$ was used

3.3 The mainly appropriate lagrangian form

To find cosmological solutions during vacuum, radiation and matter eras it is important to determine the structure of the lagrangian. When the lagranian is linear i.e.[34]

$$L = \beta R + \gamma \tag{3.3.1}$$

$$\rho_v = \gamma$$

the vacuum energy which was found from the gravitational energy momentum takes the form

$$\rho_v = L - L'R \tag{3.3.2}$$

Vanishes and this situation can be avoided by adding extra terms to the Lagranian. The field equations of GR which correspond to

$$L = \beta R + \gamma$$

read

$$G^{\mu\nu} = -8\pi G T^{(m)\mu\nu} \tag{3.3.3}$$

where $T^{(m)\mu\nu}$ is matter energy-momentum tensor. This implies that matter energy – momentum tensor is conserved and this contradicts the fact that the total energy momentum tensor of matter and gravity should be conserved. One of the possible ways to remove this controversy is to add other terms to the Lagrangian. Also it should be noted that when the Lagrangian is linear the total energy-momentum tensor

$$T_{\mu\nu} = T_{\mu\nu}^{(m)} + T_{\mu\nu}^{(v)} \tag{3.3.4}$$

where $T_{\mu\nu}^{(m)}, T_{\mu\nu}^{(v)}$ are the matter and vacuum tensors respectively, is purely non-geometrical and have no geometrical component that would represent the gravitational field. All these pathological features associated with the linear lagrangian can be cured by adding higher order terms to the lagrangian. the suitable lagrangian structure can be deduced from the contracted form of the GFE where the expression

$$\Box^2 R = \frac{L'R - 2L}{3L''} - \frac{L'''}{L''} R_{;\rho} R^{\rho}_{;} \qquad (3.3.5)$$

represents the ordinary wave equation [34] and by setting

$$\begin{aligned} &L = -\alpha R^2 + \beta R + \gamma \\ &L' = -2\alpha R + \beta \quad , \quad L'' - 2\alpha, \quad L''' = 0 \end{aligned} \qquad (3.3.6)$$

equation(3.3.5) takes the form

$$\Box^2 R = \frac{(-2\alpha R + \beta)R - 2(-\alpha R^2 + \beta R + \gamma)}{3(-2\alpha)} \qquad (3.3.7)$$

$$= \frac{-\beta R - 2\gamma}{-6\alpha}$$

$$\Box^2 R == \frac{+\beta R + 2\gamma}{6\alpha} \qquad (3.3.8)$$

Giving an ordinary wave equation with a source term . thus we conclude that the suitable lagrangian which can describe the gravitational phenomena is that which consists of the quadratic term besides the linear one and the non-geometrical part.

3.4 Pure Radiation Era

According to SBB model radiation energy is dominant in the early universe, where its energy density exceeds that of matter[35]. if we consider the matter energy momentum tensor to be conserved as in GR, then the energy density of radiation would be given by

$$\rho = Ca^{-4} \tag{3.4.1}$$

Where C is a constant. The field equation in this era can be obtained by inserting and (3.2.4) and (3.4.1) in (3.2.14) to get

$$2\dddot{a}\dot{a}a^2 - \ddot{a}^2 a^2 + 2\ddot{a}\dot{a}^2 a \ - 3\dot{a}^4 - 2ka\dot{}^2 + k^2 + \frac{\beta a^2}{6\alpha}\left(\dot{a}^2 + k\right) - \frac{C}{36\alpha} = 0 \tag{3.4.2}$$

where we used

$$\dot{R} = \frac{-6}{a^3}\left[\dddot{a}\,a^2 - \dot{a}\,\ddot{a}\,a - 2\dot{a}^3 - 2k\dot{a}\right] \tag{3.4.3}$$

$$R^2 = \frac{36}{a^4}\left[\ddot{a}^2 a^2 + 2\ddot{a}\dot{a}^2 a \ + \dot{a}^4 + 2k\ddot{a}a + 2k\dot{a}^2 + k^4\right] \tag{3.4.4}$$

3.5 pure vacuum state of universe

in a pure vacuum state the lagrangian L and the vacuum energy ρ_v are given to be

$$L = -\alpha R^2 + \beta R \tag{3.5.1}$$

Since at vacuum stage k=0 and inflation is assumed to take place [33], i.e

$$a = a_0 e^{\mu t} \qquad \dot{a} = \mu a \qquad \ddot{a} = \mu^2 a$$

Thus from equation (3.2.4)

$$R = -12\mu^2 = const = R_0$$

Hence

$$\rho_v = L - L'R = \alpha R_0^2 = cons\tan t \qquad (3.5.2)$$

using (3.5.2) and taking into account that $\rho = 0$ in a pure vacuum state equation (3.2.12) can be rewritten as

$$-\frac{3a^{\bullet}}{a}R^{\bullet}L'' + \frac{3a^{\bullet\bullet}}{a}L' + \frac{1}{2}L + \frac{\rho_v}{2\alpha} = 0 \qquad (3.5.3)$$

$$-\frac{a^{\bullet}}{a}R^{\bullet} + \frac{a^{\bullet\bullet}}{a}R + \frac{R^2}{12} + \frac{\beta R}{12\alpha} - \frac{a^{\bullet\bullet}\beta}{2a\alpha} - \frac{\rho_v}{12\alpha} = 0 \qquad (3.5.4)$$

using equation (3.2.4), (3.4.3)and (3.4.4) the GFE(3.5.4) become

$$\frac{6}{a^4}\left(a^{\bullet\bullet\bullet}a^{\bullet}a^2 + a^{\bullet\bullet}a^{\bullet^2}a - 2a^{\bullet^4} - 2ka^{\bullet^2}\right) - \frac{6}{a^4}\left(a^{\bullet\bullet^2}a^2 + a^{\bullet\bullet}a^{\bullet^2}a - ka^{\bullet\bullet}a\right)$$
$$\frac{3}{a^4}\left(a^2a^{\bullet\bullet^2} + 2a^{\bullet\bullet}a^{\bullet^2}a + 2ka^{\bullet\bullet}a + a^{\bullet^4} + k^2\right) + \frac{\beta}{2\alpha a^4}$$
$$\left(a^{\bullet\bullet}a^3 + a^{\bullet^2}a^2 + ka^2\right) - \frac{\beta a^{\bullet\bullet}a^3}{2\alpha a^4} - \frac{\rho_v}{12\alpha} = 0 \qquad (3.5.5)$$

or

$$2a^{\bullet\bullet\bullet}a^{\bullet}a^2 - a^{\bullet\bullet^2}a^2 + 2a^{\bullet\bullet}a^{\bullet^2}a - 3a^{\bullet^4} - 2ka^{\bullet^2} + k^2 + \frac{\beta a^2}{6\alpha}\left(a^{\bullet^2} + k\right) - \frac{\rho_v a^4}{36\alpha} = 0 \qquad (3.5.6)$$

3.6 Radiation and matter in the presence of vacuum

At present, the universe consists of matter, radiation and vacuum energy. Thus we have to consider the presence of all these kinds of Matter simultaneously. To find vacuum energy we compare the equation of motion of the classical field [36]

$$\Box^2\varphi = \frac{\partial V}{\partial \varphi} \qquad (3.6.1)$$

with the contracted form of GFE (3.2.13) when

$$L = -\alpha R^2 + \beta R + \gamma$$

$$\Box^2 R = \frac{\beta}{6\alpha} R + \gamma_0 \qquad (3.6.2)$$

to get the following expression for the potential

$$V = \frac{\beta}{12\alpha} R^2 + \gamma_0 R \qquad (3.6.3)$$

Where

$$\gamma_0 = \frac{\gamma}{3\alpha}$$

the vacuum energy can be obtained by minimizing v [37]

$$\frac{dV}{dt} = \left(\frac{\beta}{6\alpha} R + \gamma_0 \right) R^{\bullet} = 0 \qquad (3.6.4)$$

This can be satisfied if $\dot{R} = 0$. which means that $R = R_0 = const$, which conforms with the fact that v is minimum during vacuum stage as shown by equation (3.5.2) but the energy density of the scalar field is given by[37]

$$\rho = R^{\bullet^2} + (\nabla R)^2 + V \qquad (3.6.5)$$

since R depends on t only, then the vacuum energy is given by

$$\rho_v = \rho_{\min} = R^{\bullet^2} + V = \frac{\beta}{12\alpha} R^2 + \gamma_0 R \qquad (3.6.6)$$

following the Ozer-Taha model[38], the functional dependence of radiation energy ρ_r and matter energy ρ_m on the cosmic scale factor are given by

$$\rho_r = C_0 a^{-4} \qquad \rho_m = Ca^{-3} \tag{3.6.7}$$

with the aid of equation (3.6.7) together with equations (3.2.4) and (3.6.6) the field equation (3.2.14) reads

$$2a^{\bullet\bullet\bullet}a^{\bullet}a^2 - a^{\bullet\bullet 2}a^2 + 2a^{\bullet\bullet}a^{\bullet 2}a - 3a^{\bullet 4} - 2ka^{\bullet 2} + k^2 + \frac{\beta a^2}{6\alpha}\left(a^{\bullet 2} + k\right)$$
$$-\frac{a^4}{36\alpha}\left(\frac{\beta}{12\alpha}R^2 + \gamma_0 R\right) - C_0 - Ca = 0 \tag{3.6.8}$$

3.7 Cosmological Solutions During Radiation, Matter and Vacuum State of the Universe

First we consider the pure radiation era, if we assume a solution of the form

$$a = D_0 t + D_1 \qquad a^{\bullet} = D_0 \qquad a^{\bullet\bullet} = 0 \tag{3.7.1}$$

The Lagrangian is assumed by some outhers to be [33]

$$L = -\alpha R^2 + \beta R + \gamma$$

At the early universe, the universe is dense , as far as the radius of the universe is small at radiation and pre radiation stage .thus the same authers assume that [33] the curvature R is large , thus one can neglect the linear term , thus the Lagrangian become

$$L = -\alpha R^2 + \gamma$$

The choose of this quadratic term can forms also with the electromagnetic Lagrangian [33]. Thus upon substitution (3.7.1) in (3.4.2), one gets

$$-3D_0^4 - 2kD_0^2 + k^2 - \frac{C}{36\alpha} = 0$$

i.e

$$\left(3D_0^2 + 2k\right)D_0^2 = k^2 - \frac{C}{36\alpha} \qquad (3.7.2)$$

which gives

$$D_0 = \sqrt{-\frac{k}{3} \pm \frac{1}{6}\sqrt{16k^2 - \frac{C}{3\alpha}}} \qquad (3.7.3)$$

The value of α is proposed by some researchers to be [33]

$$\alpha = -\sqrt{\frac{\beta}{24}} = -\sqrt{\frac{1}{24 \times 16\pi G}}$$

$$G = 6.67 \times 10^{-11} \qquad \alpha \approx 10^3$$

Thus since α is large, one can thus neglect the term consisting of α, to get from (3.7.3)

$$D_0 = \sqrt{\frac{k}{3}} \quad or \quad \sqrt{-k} \qquad (3.7.4)$$

Second , we consider the case in which only vacuum is present i.e. we try to solve equation (3.5.6) assuming

$$a = e^{\mu t} \qquad a^{\bullet} = \mu e^{\mu t} = \mu a, \qquad a^{\bullet\bullet} = \mu^2 a \qquad and \qquad a^{\bullet\bullet\bullet} = \mu^3 a \qquad (3.7.5)$$

The field equation gives

$$-2k\mu^2 a^2 + k^2 + \frac{\beta a^2}{6\alpha}\left(\mu^2 a^2 + k\right) - \frac{\rho_v a^4}{36\alpha} = 0 \tag{3.7.6}$$

$$\left(\frac{\beta\mu^2}{6\alpha} - \frac{\rho_v}{36\alpha}\right)a^4 + \left(\frac{\beta k}{6\alpha} - 2k\mu^2\right)a^2 + k^2 = 0 \tag{3.7.7}$$

to express μ in terms of known physical quantities, we equate the coefficients of a^4, a^2, a^0 with zero, i.e.

$$\frac{\beta\mu^2}{6\alpha} - \frac{\rho_v}{36\alpha} = 0 \tag{3.7.8}$$

which gives

$$\rho_v = 6\beta\mu^2 \tag{3.7.9}$$

and

$$k\left(\frac{\beta}{6\alpha} - 2\mu^2\right) = 0 \tag{3.7.10}$$

thus

$$k = 0 \qquad or \qquad \mu^2 = \frac{\beta}{12\alpha} \tag{3.7.11}$$

i.e.

$$\rho_v = \frac{6\beta^2}{12\alpha} = \frac{\beta^2}{2\alpha} \tag{3.7.12}$$

to secure a correct Newtonian limit,[39] give

$$\beta \approx 10^{19}\,Gev \qquad and \qquad \alpha \approx 10^{9}\,Gev \tag{3.7.13}$$

substituting (3.7.11) and (3.7.13)in equation(3.7.5), one gets

$$a = e^{\sqrt{\frac{\beta}{12\alpha}}t} \approx e^{10000\,t} \tag{3.7.14}$$

Now, let us consider the case of radiation and matter in the presence of vacuum energy, i.e. we consider equation(3.6.8) . Assuming the solution

$$a = Dt^2, \qquad a^{\bullet} = 2Dt, \qquad a^{\bullet\bullet} = 2D \tag{3.7.15}$$

Substituting in equation (3.6.8),one gets

$$\left[\frac{2\beta}{3\alpha} + \frac{\gamma_0}{\alpha}\right]D^4 t^6 + \left[-36D^4 + \frac{\beta k}{6\alpha}D^2 - \frac{3\beta}{\alpha^2}D^4 + \frac{\gamma_0 k D^2}{6\alpha}\right]t^4$$
$$+\left[-8kD^2 - CD - \frac{\beta k}{\alpha^2}D^2\right]t^2 + \left[k^2 - C_0 - \frac{k^2\beta}{12\alpha^2}\right] = 0 \tag{3.7.16}$$

One of the possible solution is to equate the coefficients of different powers of t with zero

$$\gamma_0 = -\frac{2}{3}\beta, \qquad D = \sqrt{\frac{k\beta\alpha}{36\alpha^2 + 3\beta}}$$
$$C = -k\left[8 + \frac{\beta}{\alpha^2}\right], \qquad C_0 = K^2\left[1 - \frac{\beta}{12\alpha^2}\right] \tag{3.7.17}$$

3.8 Solving the Cosmological Problems

First, we consider the horizon problem. In the case of

$$a = Dt^2 \qquad (3.8.1)$$

The horizon radius is given by [40]

$$d_H(t) = a(t)\int_0^t \frac{dt}{a(t)} = \frac{Dt^2}{D}\int_0^t t^{-2}dt = \infty \qquad (3.8.2)$$

and in the case of

$$a = D_0 t + D_1 \qquad (3.8.3)$$

the horizon radius is given by

$$d_H(t) = \frac{(D_0 t + D_1)^t}{D_0}\int_0^t \frac{D_0 dt'}{D_0 t' + D_1} \qquad (3.8.2)$$

$$= \frac{D_0 t + D_1}{D_0}\ln\left[D_0 t' + D_1\right]_0^t \qquad (3.8.3)$$

$$= \frac{D_0 t + D_1}{D_0}\ln\left[\frac{D_0 t}{D_1} + 1\right] \qquad (3.8.4)$$

If the universe is dominated by ultra-relativistic particles, $D_1 \to 0$ and

$$d_H(t) = \frac{D_0 t + D_1}{D_0}\ln\infty \to \infty \qquad (3.8.5)$$

and hence different parts of the universe become causally connected in the early times. Further to solve the entropy problem, equation(2.4.2.1) can be used to give

$$ds = \frac{1}{T}\left(d\rho V + pdV\right) \tag{3.8.6}$$

$$= \frac{1}{T}\left(d\rho a^3 + pda^3\right) \tag{3.8.7}$$

But the expression for the total energy-momentum conservation is given by

$$T^{\mu\nu}_{;\mu} = \frac{\partial\rho}{\partial t} + 3\frac{a^{\bullet}}{a}\left(\rho + p\right) + \frac{\partial p_v}{\partial t} - L''RR^{\bullet} - 3\frac{a^{\bullet\bullet}}{a}R^{\bullet}L'' = 0 \tag{3.8.8}$$

$$a^3\frac{d\rho}{dt} + p\frac{da^3}{dt} = a^3\left(L''RR^{\bullet} + 3\frac{a^{\bullet\bullet}}{a}R^{\bullet}L''\right) - a^3\frac{d\rho_v}{dt} \tag{3.8.9}$$

i.e.

$$\frac{ds}{dt} = a^3\left(L''RR^{\bullet} + 3\frac{a^{\bullet\bullet}}{a}R^{\bullet}L''\right) - \frac{a^3 d\rho_v}{dt} \tag{3.8.10}$$

using

$$a = D_0 t + D_1, \qquad a^{\bullet} = D_0, \qquad a^{\bullet\bullet} = 0 \tag{3.8.11}$$

Also

$$\rho_v^{\bullet} = -4\frac{a^{\bullet}}{a}\rho_v = \frac{48\alpha D_0\left(D_0^2 + k\right)}{a^5}\left[-9D_0^2 + \frac{8D_0^2}{w+1} - 3k\right] \tag{3.8.12}$$

where w is the constant which relates the pressure p to the energy density ρ in the general state equation for the perfect fluid and[41]

$$\rho_v = \frac{36\alpha k^2 (3w-1)^2}{(1+9w)(1+w)a^4} \qquad (3.8.13)$$

In equation (3.8.10), yields

$$\rho_v = \frac{-4D_0\rho_v}{a} = \frac{-144\alpha D_0 k^2 (3w-1)^2}{(1+9w)(1+w)a^4} \qquad (3.8.14)$$

Hence

$$\begin{aligned} \frac{ds}{dt} &= -2\alpha R\dot{R}a^3 + 4a^2 D_0 \rho_v \\ &= \frac{144\alpha D_0 \left(D_0^2 + k\right)^2}{a^2} + \frac{144\alpha D_0 k^2 (3w-1)^2}{(1+9w)(1+w)a^4} \end{aligned} \qquad (3.8.15)$$

since $D_0 > 0 \quad and \quad a(t) > 0,$ it follows that

$$\frac{ds}{dt} > 0 \qquad (3.8.16)$$

Hence the entropy of the universe increases in complete agreement with the second law of thermodynamics [42,43].

To solve the flatness problem, let us consider the field equation

$$\frac{36\alpha}{a^4}\left[2\dddot{a}\dot{a}a^2 - \ddot{a}^2 a^2 + 2\ddot{a}\dot{a}^2 a - 3\dot{a}^4 - 2k\dot{a}^2 + k^2 + \frac{\beta a^2}{6\alpha}\left(\dot{a}^2 + k\right)\right] - \rho - \rho_v = 0 \qquad (3.8.16)$$

if we denote

$$\rho_c = \frac{3\dot{a}^2}{8\pi G a^2}$$

$$F(t) = \frac{36\alpha}{a^4}\left[2\dddot{a}\dot{a}a^2 - \ddot{a}^2 a^2 + 2\ddot{a}\dot{a}^2 a - 3\dot{a}^4 - 2k\dot{a}^2 + k^2\right] \qquad (3.8.17)$$

Using equation (3.8.17),the field equation reads

$$F(t) + \frac{36\alpha}{a^4}\frac{\beta\alpha^2}{6\alpha}\left(\dot{a}^2 + k\right) - \rho - \rho_v = 0 \qquad (3.8.18)$$

$$F(t) + \frac{6\beta\dot{\alpha}^2}{a^2} + \frac{6\beta k}{a^2} - \rho - \rho_v = 0 \qquad (3.8.19)$$

Again ,if we use $\beta = \frac{1}{16\pi G}$ equation (3.8.19) yields

$$F(t) + \frac{3\dot{a}^2}{8\pi G a^2} + \frac{3k}{8\pi G a^2} - \rho - \rho_v = 0 \qquad (3.8.20)$$

$$F(t) + \rho_c - \rho - \rho_v + \frac{3k}{8\pi G a^2} = 0 \qquad (3.8.21)$$

Thus

$$\rho - \rho_c = F(t) - \rho_v + \frac{3k}{8\pi G a^2} \qquad (3.8.22)$$

This expression reduces to the ordinary GR expression when

$$F(t) = 0 \qquad \rho_v = 0$$

To get [15]

$$\rho - \rho_c = \frac{3k}{8\pi G a^2} \qquad (3.8.23)$$

Thus in the present model there is no direct relation between the critical density ρ_c and the intrinsic curvature of space even if vacuum energy does not exist. This is because the General Field Equation consist of two additional terms the first term $F(t)$ arising from the quadratic Lagrangian i.e. when the gravity is strong, and which when $\alpha = 0$, and the second term representing the vacuum energy.

Chapter 4

The Generalized General Relativity and Quantum Model

4.1 introduction

One of the most disastrous problem which false GR is to describe the early universe specially vacuum energy and radiation these stage, the universe has high energy and dominated by elementary particles which can be described by the lows of quantum mechanics .

In this chapter we write The Schrödinger equation for the gravitational field and vacuum energy and we solution of quantum gravity equation in the early blank and vacuum era [44].

4.2 The Schrödinger equation for the gravitational field

The Schrödinger equation for the gravitational field is given by [45]:

$$i\hbar\frac{\partial}{\partial t}\psi = E\psi = \frac{\hbar}{i\sqrt{3}}\psi + 4\alpha\left(-\frac{\ddot{a}}{a}+\frac{\dot{a}^2}{a^2}+\frac{k}{a^2}\right)\psi$$

$$i\hbar\frac{\partial}{\partial t}\psi = E\psi = \frac{\hbar}{i\sqrt{3}}\psi + f(t)\psi \qquad (4.2.1)$$

Where

$$f(t) = 4\alpha\left(-\frac{\ddot{a}}{a}+\frac{\dot{a}^2}{a^2}+\frac{k}{a^2}\right) \qquad (4.2.2)$$

This gravitational equation represent Schrödinger generalized solution from the Hamiltonian equation

$$H = \alpha R^2 - 6\alpha \frac{\ddot{a}^2}{a} \qquad (4.2.3)$$

Then by using separation of variable by setting

$$\psi(t,r,\theta,\phi) = \phi(t)Y(r,\theta,\phi) \qquad (4.2.4)$$

And from equation (4.2.1) & equation (4.2.4) one has

$$Yi\hbar\frac{\partial\phi}{\partial t} = \phi\frac{\hbar}{i\sqrt{3}}\nabla Y + f(t)Y\phi \qquad (4.2.5)$$

when one divides both sides by ($Y\phi$) it becomes

$$\frac{i\hbar}{\phi}\frac{\partial\phi}{\partial t} = \frac{1}{Y}\frac{\hbar}{i\sqrt{3}}\nabla Y + f(t) \qquad (4.2.6)$$

$$\frac{i\hbar}{\phi}\frac{\partial\phi}{\partial t} - f(t) = \frac{1}{Y}\frac{\hbar}{i\sqrt{3}}\nabla Y = C_0 \qquad (4.2.7)$$

Where $C_0 = constatnt.$

The time part of Schrödinger equation in view of equation (4.2.2) & (4.2.7)

$$\frac{i\hbar}{\phi}\frac{\partial\phi}{\partial t}+4\alpha\left(\frac{-\ddot{a}}{a}+\frac{\dot{a}^2}{a^2}+\frac{k}{a^2}\right)=C_0$$

$$i\hbar\frac{\partial\phi}{\partial t}+4\alpha\left(\frac{-\ddot{a}}{a}+\frac{\dot{a}^2}{a^2}+\frac{k}{a^2}\right)\phi=C_0\phi \qquad (4.2.8)$$

The spatial part can be obtained by considering the spherical coordinate, because the momentum components in the spherical coordinate takes the form

$$p_r=\frac{\hbar}{i}\frac{\partial}{\partial r}=\frac{\hbar}{i}\nabla_r$$

$$p_\theta=\frac{\hbar}{i}\nabla_\theta=\frac{\hbar}{i}\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}$$

$$p_\phi=\frac{\hbar}{i}\nabla_\phi=\frac{\hbar}{i}\frac{1}{r\sin\phi}\frac{\partial}{\partial\phi} \qquad (4.2.9)$$

Using equation (4.2.7),one gets

$$\frac{\hbar}{i\sqrt{3}}\frac{\partial y}{\partial r}=C_0 y$$

$$\int\frac{\partial y}{y}=\int C_0\frac{i\sqrt{3}}{\hbar}\partial r+C_0$$

$$y=A_1\frac{i\sqrt{3}C_0 r}{\hbar} \qquad (4.2.10)$$

but the mass density is given in terms of the mass of one particle m and the number of particles per unit volume n to be

$$\rho=mn=m\sqrt{|y|^2}$$

Since the density n is related to the wave function y , it follows that

$n=\sqrt{|y|^2}$ hence

$$\rho=m\sqrt{A_1 e^{\frac{i\sqrt{3}c_0 r}{\hbar}}A_1 e^{\frac{-i\sqrt{3}c_0 r}{\hbar}}}$$

$$\rho=mA_1 \qquad (4.2.11)$$

But in the case that C_0 is complex

$$C_0 = a_0 i$$

One gets:

$$y = A_1 e^{-\frac{\sqrt{3}a_0 r}{\hbar}}$$

$$\rho = mn = mA_1^{\,2} e^{\frac{-\sqrt{3}a_0 r}{\hbar}} \quad (4.2.12)$$

And to find (T)the temperatures we use the relation:

$$E = \rho c^2 = \sigma T^4$$

This implies that:

$$T = \sqrt[4]{\frac{\rho c^2}{\sigma}} \quad (4.2.13)$$

Where

$$\rho = m\phi\bar{\phi} \quad (4.2.14)$$

4.3 vacuum energy[46]:

To find the cosmological constant one minimizes the Hamiltonian with respect to the radius a By setting

$$g_{ii} = a^2 = x$$

The scalar curvature R which is given by

$$R = -6\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right)$$

Can take the form

$$R = -6\left(\frac{\ddot{a}}{x^{\frac{1}{2}}} + \frac{\dot{a}^2}{x} + \frac{k}{x} \right)$$

$$\frac{\ddot{a}}{a} = \ddot{a}\, x^{\frac{-1}{2}} \qquad (4.3.1)$$

From (4.2.3) Hamiltonian minimization yields

$$E = pc^2 = H = \alpha R^2 - 6\alpha R \frac{a^{\bullet\bullet}}{a}$$

Since $x^{\frac{1}{2}} = a \qquad , a^2 = x$

$$H = \alpha R^2 - 6\alpha R a^{\bullet\bullet} x^{-\frac{1}{2}}$$

$$\frac{dH}{da} = \frac{dH}{dx}\frac{dH}{da} = 0$$

Which yields

$$\frac{dH}{dx} = 0 \qquad (3.4.2)$$

$$\frac{dH}{dx} = 2\alpha R \frac{\partial R}{\partial x} - 6\alpha \frac{a^{\bullet\bullet}}{a} \frac{\partial x^{\frac{1}{2}}}{\partial x} = 0$$

$$\frac{dR}{dx} = -6\frac{\partial}{x}\left[a^{\bullet\bullet} x^{\frac{1}{2}} + a^{\bullet 2} x^{-1} + kx^{-1} \right] = 6\left[\frac{a^{\bullet\bullet}}{2x^{\frac{3}{2}}} + \frac{a^2}{x^2} + \frac{k}{x^2} \right] \qquad (3.4.3)$$

$$\frac{dH}{dx} = 2\alpha R(6)\left[\frac{a^{\bullet\bullet}}{2x^{\frac{3}{2}}} + \frac{a^{\bullet 2}}{x^2} + \frac{k}{x^2}\right] + 3\alpha a^{\bullet\bullet} x^{\frac{1}{2}} = 0$$

$$\frac{dH}{dx} = 12\alpha R\left[\frac{a^{\bullet\bullet}}{2} x^{\frac{1}{2}} + a^{\bullet 2} + k\right] + 3\alpha a^{\bullet\bullet} x^{\frac{1}{2}} = 0$$

$$\frac{dH}{dx} = 4\alpha R\left[\frac{a^{\bullet\bullet} a}{2} + a^{\bullet 2} + k\right] + \alpha a^{\bullet\bullet} x^{\frac{1}{2}} = 0$$

$$4\alpha R\left[\frac{a^{\bullet\bullet} a}{2} + a^{\bullet 2} + k\right] + \alpha a^{\bullet\bullet} x^{\frac{1}{2}} = 0$$

$$2\alpha R a^{\bullet\bullet} a + 4\alpha R a^{\bullet 2} + 4\alpha R k + a^{\bullet\bullet} a = 0$$

$$2\alpha R a^{\bullet\bullet} a + a^{\bullet\bullet} a = -4\alpha R\left[a^{\bullet 2} + k\right]$$

$$\ddot{a}[2\alpha R + 1]a = -4\alpha R\left[\dot{a}^2 + k\right] \qquad (4.3.4)$$

Substituting the values of R in terms of a with the aid of (4.3.1) and the fact that $a^2 = x$ yields

$$\ddot{a}\left[-12\alpha\frac{\ddot{a}}{a} - 12\alpha\frac{\dot{a}^2}{a^2} + 1\right]a = +4\alpha[6]\left[\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a}\right]\dot{a}^2 a \qquad (4.3.5)$$

This Can be solved by setting:

$$a = A = const \; \& \qquad a^{\bullet} = 0 \qquad \& \; \ddot{a} = 0$$

Thus the equation (4.3.5) is satisfied.

In vacuum k=0, thus equation (4.3.1) reduces to

$$R = -6\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}\right) \tag{4.3.6}$$

Now suggest the solution:

$$a = Ae^{\mu t} \qquad a^{\bullet} = \mu a \qquad \ddot{a} = \mu^2 a \tag{4.3.7}$$

Substituting (4.3.7)in (4.3.6) yields:

$$R = -12\mu^2 \tag{4.3.8}$$

Inserting (4.3.7) and (4.3.8) in (4.3.3) yields:(k=0 in vacuum)

$$\mu^2 a^2(-24\alpha\mu^2 + 1) = 48\alpha\mu^4\alpha^2$$
$$\mu^2 a^2(-72\alpha\mu^2 + 1) = 0$$

Thus

$$\mu^2 = \frac{1}{72\alpha} \qquad \mu = \frac{1}{\sqrt{72\alpha}} \tag{4.3.9}$$

Thus equation (4.3.3) which result from the minimization of the Hamiltonian can be satisfied by:

$$a = Ae^{\mu t} \tag{4.3.10}$$

Provided that:

$$\mu = \frac{1}{\sqrt{72\alpha}} \tag{4.3.11}$$

This means that during vacuum stage inflation takes place.
It is important to see how the wave function of the universe looks like at vacuum stage.
To do this substitute (4.3.10) in (4.2.2) for k=0, to get :

$$f(t) = 4\alpha\left[-\mu^2 + \mu^2\right] = 0 \tag{4.3.12}$$

Inserting (4.3.12) in (4.2.7) to get

$$i\hbar\frac{\partial\phi}{\partial t} = C_0\phi \tag{4.3.13}$$

But according to lows of quantum mechanic this equation represents the energy Eigen equation:

$$\hat{H}\varphi = i\hbar\frac{\partial}{\partial t}\varphi = E\varphi \tag{4.3.15}$$

with E standing for the energy of the system. But since during vacuum stage elementary particles to be produced, thus the energy E consists of complex potential to describe this situation. as a result one can write E a sum of real and imaginary part, in the form

$$E = -E_1 - iE_2 \tag{4.3.16}$$

The fact that the energy consist of real and imaginary part is utilize to describe inelastic scattering, where absorption and production of particles was will as loss of energy takes place. It is also tacked by Haronn and Dirar in their modified Schrödinger equation which is bored on the photon wave function inside the medium which is consist of damping term corresponding to energy loss. The minus sin in (4.3.16) result from the fact that the participles are bounded by attractive gravitational field. By substituting (4.3.16) in (4.3.15) gets:

$$i\hbar\frac{\partial}{\partial t}\varphi = -(E_1 + iE_2)\varphi \qquad (4.3.17)$$

$$\int\frac{\partial\varphi}{\varphi} = \int\left(\frac{E_1}{\hbar} - \frac{iE_2}{\hbar}\right)dt + C_1$$

$$lin\varphi = \left(\frac{E_1 t}{\hbar} - \frac{iE_2 t}{\hbar}\right) + C_1$$

$$\varphi = Ae^{\frac{iE_1}{\hbar}t}e^{\frac{-E_2}{\hbar}t} \qquad (4.3.18)$$

where

$$A = e^{C_1}$$

Thus the density of the universe is given according to (4.3.18)

$$\rho = mn = m|\varphi|^2 = A^2 e^{\frac{-2E_2}{\hbar}t} \qquad (4.3.19)$$

and exponential growth, the density of the universe decrease, as the universe expands exponentially. This expansion as far as which decreases the density of the universe.

4.4 Solution of quantum gravity equation in the early plank and vacuum era.

For cosmological application the state of the universe is time dependent as shown by standard model. Thus the required quantum equation is that which is time dependant as shown by (4.2.7) and (4.3.17) where:

$$i\hbar\frac{\partial}{\partial t}\varphi - f(t)\varphi = C_0\varphi = -(E_1 \pm iE_2)\varphi$$

$$i\hbar\dot{\varphi} - f(t)\varphi = C_0\varphi = -(E_1 \pm iE_2)\varphi \qquad (4.4.1)$$

$$f(t) = 4\alpha\left(-\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right) \qquad (4.4.2)$$

Since quantum gravity is suitable for describing the early universe, thus it is quite natural to see whether the quantum equation can predict to this consider inflation.

$$a = Ae^{\mu t} \qquad \dot{a} = \mu a \qquad \ddot{a} = \mu^2 a \qquad (4.4.3)$$

Since the vacuum state is characterized by the zero scale factor K.I.e.

$$K = 0 \qquad (4.4.4)$$

It follows that equation (4.4.2) becomes:

$$f(t) = 4\alpha\left[-\mu^2 + \mu^2 + 0\right] = 0 \qquad (4.4.5)$$

thus equation(4.4.1) takes the form

$$i\hbar\dot{\varphi} = (E_1 \pm iE_2)\varphi \qquad (4.4.6)$$

$$\int \frac{\partial\varphi}{\varphi} = \int\left(\frac{E_1}{\hbar} - \frac{iE_2}{\hbar}\right)dt + C_1$$

$$lin\varphi = \left(\frac{iE_1 t}{\hbar} - \frac{E_2 t}{\hbar}\right) + C_1$$

$$\varphi = e^{C_1} e^{\frac{iE_1}{\hbar}t} e^{\frac{\pm E_2}{\hbar}t} = De^{\frac{iE_1}{\hbar}t} e^{\frac{\pm E_2}{\hbar}t} \qquad (4.4.7)$$

for inflation to take place, the minus sign is suitable in this case.

$$i.e.\varphi = De^{\frac{iE_1}{\hbar}t} e^{\frac{-E_2}{\hbar}t} = De^{\frac{iE_1}{\hbar}t} e^{-\mu t} \qquad (4.4.8)$$

$$\mu = \frac{E_2}{\hbar}$$

Thus the density of the universe takes the form:

$$\rho = m[\varphi]^2 = m\varphi\varphi^* = D^2 e^{-2\mu t} \qquad (4.4.9)$$

this is quite reasonable since as the volume of the universe increases according to equation (4.4.3),i.e.

$$V \approx a^3 \approx A^3 e^{3\mu t}$$

The density decreases.

The average energy in this case takes the form

$$< E >= \int_0^\infty \varphi^* \hat{H}\varphi dt$$

$$= \int_0^\infty \varphi^*\left(i\hbar\frac{\partial}{\partial t}\varphi\right)dt$$

$$= \int_0^\infty \varphi^*(-E - i\hbar\mu)\varphi^* dt \qquad (4.4.10)$$

in view of (4.4.7) one gets:

$$(E_1 + iE_2)\int_0^\infty e^{-2\mu t} = \left(\frac{E_1}{2\mu} + \frac{iE_2}{2\mu}\right)[-1]$$

$$< E >= \left(\frac{E_1 + iE_2}{2\mu}\right) \qquad (4.4.11)$$

4.5 Non Singular Solution at the early universe:[47]

At the early universe when vacuum or elementary particles dominates, i.e. during plank or pre plank era:

$$K = 0 \qquad (4.5.1)$$

Thus

$$f(t) = 4\alpha\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}\right) \qquad (4.5.2)$$

Consider now anon singular solution:

$$a = c_1 t + c_2 \qquad a^\bullet = c_1 \qquad \ddot{a} = 0 \qquad (4.5.3)$$

$$f(t) = \frac{4\alpha c_1^2}{a^2} \qquad (4.5.4)$$

According to equation (4.4.1):

$$\frac{i\hbar}{\varphi}\frac{\partial}{\partial t}\varphi = \frac{4\alpha c_1^2}{a^2} = -(E_1 \pm iE_2)\varphi \qquad (4.5.5)$$

$$da = c_1 d$$

Thus equation (4.5.5) was red to

$$\frac{i\hbar c_1}{\varphi}\frac{\partial}{\partial a}=\frac{4\alpha c_1^2}{a^2}=-(E_1\pm iE_2)\varphi$$

$$\int\frac{d\varphi}{da}=-\int\frac{4i\alpha da}{\hbar a^2}+\int\frac{iE_1}{\hbar c_1}da-\int\frac{E_2}{\hbar c_2}da+c_3$$

$$lin\varphi=+\frac{4i\alpha da}{\hbar}+\frac{iE_1}{\hbar c_1}a+\frac{E_2}{\hbar c_2}a=\varphi_0 e^{i\theta}e^{\frac{E_2}{\hbar c_1}a} \qquad (4.5.6)$$

where

$$\theta=\frac{4\alpha}{\hbar}a+\frac{E_1}{\hbar c_1}a \qquad (4.5.7)$$

Thus the universe density is:

$$\rho=m|\varphi|^2=\varphi_0 e^{\frac{12E_2}{\hbar c_1}a}=\varphi_0^2 e^{\frac{12E_2}{\hbar c_1}(c_1t+c_2)} \qquad (4.5.8)$$

Thus according to (4.5.3) and (4.5.8) one has nonsingular expanding universe with exponential decreasing density.

Chapter 5
Literature Review

5.1 The 2ppi Expansion: Dynamical Mass Generation and vacuum energy

Lately, there was growing evidence for the existence of a condensate of mass dimension two in Yang-Mills (YM) theories in the landau gauge . an obvious candidate for such a condensate is $\langle A_\mu^a A_\mu^a \rangle$. the phenomenon-logical back ground of this type of condensate can be found in 1,2,3. also lattice simulations indicated anon –zero condensate $\langle A_\mu^a A_\mu^a \rangle^4$.

Thinking of simpler models like mass less $\lambda\phi^4$ or gross-Neveu and the role played by quartic interaction in the formation of a (local) composite (in particular, containing two fields) condensate and the consequent dynamical mass generation for the originally mass less field , it is clear the possibility exists that the quartic gluon interaction gives rise to a two field composite operator condensate in YM(QCD) and mass generation for the gluons too.

The U(N) invariant gross-Neveu Lagrangian in 2D Euclidean space time reads

$$L = \bar{\psi}\phi\psi - \frac{1}{2} g^2 (\bar{\psi}\psi)^2 \qquad (5.1.1)$$

This model possess a discrete chiral symmetry $\psi \to \gamma_5 \psi$, imposing $\langle \bar{\psi}\psi \rangle = 0$ perturbatively. We focus on the topology of vacuum diagrams. We can divide them into 2 disjoin classes : those diagrams falling apart in 2 separate pieces when 2 lines meeting at the same point are cut. We call those 2-point-particible or 2PPR All 2PPR sum building up the vacuum energy by summing them in an effective mass m. defining $\nabla = (\bar{\psi}\psi)$, it can be shown that

$$m = -g^2(1-\frac{1}{2N})\nabla \qquad (5.1.2)$$

Then the 2PPI vacuum energy E_{2PPI} given by the sum of all 2PPI , now with a mass m running in the loops. It is important to notice that E_{2PPI} is not the vacuum energy due to a double counting ambiguity, this can be solved by considering $\frac{dE}{dg^2}$ instead of E. the g^2 derivative can hit 2PPR or 2PPI vertex.

$$\frac{dE}{dg^2} = (1-\frac{1}{2N})\nabla^2 + \frac{dE_{2PPI(M)}}{dg^2} \qquad (5.1.3)$$

This can be integrated using the ansatz

$$E = E_{2PPI} + cg^2\Delta^2 \qquad (5.1.4)$$

It remains to determine the unknown constant c .it is easy to show that one has the following gap equation

$$\frac{\partial E_{2PPI}}{\partial m} = \Delta. \qquad (5.1.5)$$

Combination of the above formulae finally gives

$$E = E_{2PPI} + \frac{1}{2}g^2(1-\frac{1}{2N})\nabla^2 \qquad (5.1.6)$$

An important point is the renormalizability of the 2PPI expansion. Two possible problems could be mass renormalization and vacuum energy renormalization , since originally there was no external mass scale present. The proof is quite technical, but all formulate remain correct and are finite when the conventional counter terms of the cross-Neveu model are included. Essentially, the proof is based on coupling constant renormalization and the separation of 2PPI and 2PPR contributions.
It can be shown that

$$\frac{\partial E_{2PPI}}{\partial \Delta} = m \Leftrightarrow \frac{\partial E}{\partial m} 0 \qquad (5.1.7)$$

However , this does not mean that (ΔE) is meaning less if the gap equation(5.1.7) is not fulfilled.

In table1,we list the numerical deviations in terms of percentage between our optimized 2-loop result for the mass gap M and the square of minus the vacuum energy $\sqrt{-E}$ and the exact known values. We conclude that the 2PPI results are in relative good agreement with the exact values and converge to the exact $N \to \infty$ Limit.

N	Derivation M(%)	deviation $\sqrt{-E}$(%)
2	?	?
3	-4.5	47.7
4	-6.5	27
5	-6.1	19
10	-3.5	8.4
∞	0	0

3. SU(N) Yang-Mills theory in the Landau gauge

Next, we consider the Euclidean Yang –Mills action in the landau gauge where A_μ^a denotes the gauge field. Repeating the analysis of section 2 leads to

$$\Delta = \left\langle A_\mu^a A_\mu^a \right\rangle$$

$$m^2 = g^2 \frac{N}{N^2-1} \frac{d-1}{d} \Delta,$$

$$E = E_{2PPI} - \frac{g^2}{4} \frac{N}{N^2-1} \frac{d-1}{d} \Delta^2$$

$$\frac{\partial E_{2PPI}}{\partial m^2} = \frac{\Delta}{2} \Leftrightarrow \frac{\partial E}{\partial m} = 0 \qquad (5.1.8)$$

After some manipulation the 2-loop results became

$$\frac{g^2N}{16\pi} \approx 0.131 \qquad \sqrt{\Delta} \approx 536Mev \qquad E \approx -0.002Gev \qquad (5.1.9)$$

We notice that the relevant expansion parameter, $\frac{g^2N}{16\pi^2}$ is relatively small. As such, our results should be qualitatively trustworthy.
As a comparison, the value found by Boucaud et al from lattice simulations and an OPE treatment was $\langle A^2 \rangle_{\mu=10GeV} \approx 1.64GeV$. To find a value for the gluon mass m_g it self (as the pole of the gluon propagator) within the 2PPI frame work, the diagrams relevant for mass renormalization of m should be calculated [48].

5.2 mass quantization in quantum and susy cosmological models with matter content

The quantum solution of the FRW cosmological model has been calculated in many works , but not related to mass quantization[49].

The main purpose of this work is to consider a time independent Schrödinger equation and SUSY generalization to obtain a mass spectrum for the closed FRW model, in which dust matter is filling the universe, as well as the wave function of FRW cosmological model in both formalisms. It was made following the canonical quantization procedure .

Starting with the FRW model we consider the classical lagrangian for a pure gravitating system and the corresponding terms of matter content, perfect fluid with barotropic state equation $p = \gamma\rho$,and cosmological term

$$L = \frac{c^2R}{2NG}\left(\frac{dR}{dt}\right)^2 + N\frac{kc^4}{2G}R + N\frac{c^4\Lambda}{6G}R^3 - NM_\gamma c^2R^{-3\lambda} \qquad (5.2.1)$$

In particular, we will consider the dust case $\gamma = 0,$ with $k = 1$ and $\Lambda = 0$, the action for this system has the form

$$s = \int\left[-\frac{c^2}{2GN}RR^{\bullet^2} + \frac{c^4}{2G}NR - NE_s\right]dt, \qquad (5.2.2)$$

With $E_s = Mc^2,$ where M corresponds to the mass parameter of the closed universe and dust scenario.

Not that if we take the lapse function as

$$N(t) = \widetilde{N}(t)R(t)\frac{c^2}{M_{p1}G} \qquad (5.2.3)$$

We have an invariant action, obtaining the following canonical Hamiltonian using the usual scheme

$$H = \widetilde{N}\left[-\frac{P_R^2}{2M_{p1}}R\frac{M_{p1}}{2}w_0^2\left(R - \frac{MG}{c^2}\right)^2 + \frac{M}{2M_{p1}}Mc^2\right] \qquad (5.2.4)$$

With the fundamental frequency of the system $w_0 = \frac{c^2}{M_{p1}G}$. The lapse function $\widetilde{N}(t)$ is a Lagrange multiplier, which enforces the first class constrain H=0. We trans form Eq.(4) by defining $\xi = R - \frac{MG}{c^2}$, thus its momentum conjugate becomes $p_\xi = p_R$ and the constraint at the classical level reads as follows

$$H_{can} = \widetilde{N}H = \widetilde{N}\left[-\frac{P_\xi^2}{2M_{p1}} - \frac{M_{p1}}{2}w_0^2\xi^2 + \frac{M}{2M_{p1}}Mc^2\right] \qquad (5.2.5)$$

Making the usual realization of operator $\frac{P_\xi^2}{2M_{p1}} = -\frac{\hbar^2}{2M_{p1}}\frac{d^2}{d\xi^2}$ and applying it to the wave function ψ ,we get the following linear harmonic oscillator equation

$$\left[-\frac{\hbar^2}{2M_{p1}}\frac{d^2}{d\xi^2}+\frac{M_{PI}w_0^2}{2}\xi^2\right]\psi=-\frac{M}{M_{p1}}\frac{E_s}{2}\psi \qquad (5.2.6)$$

In this point we make the transformation

$$E_s=\frac{c^4}{2G}R_{\sup} \qquad (5.2.7)$$

Considering the form of E_s given in (5.2.2) we obtain that $R_{\sup}=\frac{2MG}{c^2}$, being the radius for the closed universe. Making the transformation $\frac{\xi}{l_{pl}}=x$, one can rewrite (5.2.6) as

$$\frac{1}{2}\left[x^2-\frac{d^2}{dx^2}\right]\psi=\frac{1}{4}\frac{R_{SUP}}{l_{PI}E_{PI}}\psi \qquad (5.2.8)$$

Using the creation-annihilation representation, with the usual algebra between them, $a,a^{\tau}=1$, we can rewrite eq.(5.2.8) as

$$a^{\Gamma}a\psi=\frac{1}{2}\left[x^2-\frac{d^2}{dx^2}\right]\psi-\frac{1}{2}\psi=\left(-\frac{1}{2}+\frac{1}{4}\frac{R_{SUP}E_S}{l_{PI}E_{PI}}\right)\psi=n\psi, \qquad n=0,1,2,...(5.2.9)$$

In this way, we have the following useful relations

$$R_{SUP}E_S=4\left(n+\frac{1}{2}\right)l_{pl}E_{pl}$$

$$=4\left(n+\frac{1}{2}\right)\hbar c, \qquad (5.2.10)$$

$$E_S^2=2\left(n+\frac{1}{2}\right)E_{PL,}^2 \qquad (5.2.11)$$

$$\frac{E_S}{2}=\left(n+\frac{1}{2}\right)\hbar w_0 \qquad (5.2.12)$$

One can see that when n is big, we find

$$\frac{R_{SUP}}{l_{PI}} = 2\sqrt{2n+1} \qquad (5.2.13)$$

Such that, when $n \to \infty, R_{SUP}$ coincide with the maximum expansion of the scale R.

Let us write the equation (5.2.8) in the following form

$$\frac{d^2\psi}{dx^2} + \left(\alpha_n^2 - x^2\right)\psi = 0, \qquad \alpha_n = \frac{E_S}{E_{PI}} \qquad (5.2.14)$$

Where a_n is parameter associated with the energy of the nth Eigen state, the quantum solution is similar to the harmonic oscillator case

$$\psi_n(x) = \left(\frac{1}{\sqrt{\pi}\, l 2^n}\right)^{\frac{1}{2}} H_n(x) e^{-\frac{1}{2}x^2}, \qquad (5.2.15)$$

With $H_n(x)$ the Hermite polynomials.

Now , it is clear that the system, even in its lowest energy state n=0, has a finite, minimal energy. Eq.(5.2.10) implies the following quantization mass rule

$$M_n = \sqrt{2n+1} M_{PI} \qquad (5.2.16)$$

The introduce the condition on the M_n parameter when $n \to \infty$ this parameter may be the classical mass parameter M_{SUP} for the closed universe, filled with dust matter, in the maximum expansion.

These results are similar to those obtained by other methods in the black hole .

The difference in mass between any two consecutive Eigen values is given by

$$\Delta M_{n+1} \equiv M_{n+1} - M_n = \left[\sqrt{1+\frac{2}{2n+1}} - 1\right] M_n \qquad n \qquad finine \quad (5.2.17)$$
$$= 0 \qquad\qquad n \to \infty$$

The result when $n \to \infty$ is in agreement with the correspondence principle.[49]

5. 3 Point Charge Self Energy In The General Relativity

A major general relativity principle is the equality of inert and gravitating masses. However, the classical solutions to the Einstein equations (Schwarzschild, Kerr, Reissner–Nordstrom and Kerr-Newman solutions) do not satisfy that principle at first sight. For the Schwarzschild and Kerr solutions the energy-momentum tensor and, hence, self- energy are zero, for the Reissner–Nordstrom and Kerr–Newman solutions the self-energy is infinite, whereas the gravitating mass is finite for all these solutions. A reason for this unconformity can be that the above solutions satisfy the Einstein equations not in the entire space.

The paper demonstrates that the Einstein tensor for the afore mentioned solutions in fact contains the generalized functions, which can be of a more complex nature than the Dirac δ-function. If were quire validity of the Einstein equations in the entire space, including $r = 0$, then an appropriate singular term must be added to the energy-momentum tensor.

It is simplest to elucidate the method determining if the generalized function appears in the singular function differentiation by the example of electrostatics. The point charge potential

$$\phi = \frac{e}{r} \qquad (5.3.1)$$

is singular at point r = 0 and satisfies Poisson equation

$$\Delta\varphi = -4\pi\rho \qquad (5.3.2)$$

where $\rho = e\delta(r)$. A method to ascertain this is the following. Replace potential (5.3.1) by the nonsingular function of form

$$\phi = \frac{e}{r}\theta(r - r_0) + \left(\frac{3e}{2r_0} - \frac{er^2}{2r_0^{\ 3}}\right)\theta(r_0 - r) \qquad (5.3.3)$$

where $\theta(x)$ is Heaviside function ($\theta(x)=1$ for $x>1$ and $\theta(x)=0$ for $x<1$). Having substituted this potential into (5.3.2) , we find that ϕ will be a solution to the Poisson equation for charge density

$$\rho = \frac{3e}{4\pi r_0}\theta(r_0 - r) \qquad (5.3.4)$$

The integral of (5.3.4) over volume is independent of r_0 and equal to e. In the limit

$r_0 \to 0$, $\phi \to e/r$, $\rho \to e\delta(r)$, i.e. the limit of solution (5.3.3) corresponds to the presence of a point source with charge e in the origin of coordinates and is a solution to equation (5.3.2). It is easy to show that this result is independent of the choice of the potential in range $r < r_0$, with the smooth behavior of the potential at point $r = r_0$ being not necessary .The result is always single :in the limit $r_0 \to 0$, the potential is $\phi = e/r$ and the charge density is $\rho = e\delta(r)$. Below we apply a similar procedure to the classical solutions of the Einstein equations.What should be meant by the self-energy in the general relativity is not a trivial question. This question is typically solved using the energy-momentum pseudo tensor .

A demerit of the approach is that the a For example, in electrostatics for the point charge potential we have

$$\Delta\varphi = -4\pi e\delta(r)$$

while the direct differentiation yields

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r^2\frac{\partial\varphi}{\partial r}\right) = 0$$

system self-energy definition is related to a special (Cartesian) system of coordinates and is not invariant under the coordinate transformations. The energy-momentum pseudo tensor allows energy density to be assigned to the gravitational field; the energy density, however, cannot be localized. A self-energy definition can be suggested based on the energy-momentum tensor of fields and material only. For stationary and static solutions there is Killing vector $\xi_0 - \partial/\partial t$ generating conserved current z, where $\xi_0{}^{\nu} = (1,0)$ are the contra variant vector components. As $\nabla_{\mu}J^{\mu} = 0$, conservation law

$$\frac{d}{dt} = \int d^3x\sqrt{-g}J^0 = \int dS_k\sqrt{-g}J_0{}^k \qquad (5.3.5)$$

is satisfied. If the energy density is defined as a zero component of the current,
then total energy

$$E = \int d^3x\sqrt{-g}J^0 = \int d^3x\sqrt{-g}T_0{}^0 \qquad (5.3.6)$$

will be independent of a choice of the system of coordinates.

The Reissner–Nordstrom solution is of the form

$$dS = \frac{\phi}{r^2}dt^2 - \frac{r^2dr^2}{\phi} - r^2\left(\sin^2\vartheta d\phi^2 + d\vartheta^2\right) \qquad (5.3.7)$$

where $\phi = r^2 - 2mr + Q$ (m and Q are the mass and charge, respectively).This solution satisfies Einstein equations

$$G^{\mu\nu} = 8\pi T^{\mu\nu} \tag{5.3.8}$$

Where $T^{\mu\nu} = \frac{1}{4}\left(F^{\mu}{}_{\alpha} F^{\alpha\nu} + \frac{1}{4} g^{\mu\nu} F_{\alpha\beta} F^{\alpha\beta} \right)$ is the electromagnetic field energy-momentum tensor every where, except for point $r = 0$, at which the solution is singular. The singularity structure of the tensor $G^{\mu\nu}$ and nature of the appearing generalized function can be found out using a procedure similar to that described in Section 1.

Consider the metric of form (5.3.7), having substituted the following function for ϕ in it:

$$\tilde{\phi} = \left(r^2 - 2mr + Q^2\right)\theta(r - r_0) + \frac{r^2}{r_0^{\ 2}}\left(r_0^{\ 2} - 2mr_0 + Q^2\right)\theta(r_0 - r) \tag{5.3.9}$$

In so doing the metric becomes non-singular and in the limit $r_0 = 0$ transfers to metric (5.3.7). The energy-momentum tensor corresponding to the metric can be derived from the Einstein equations. The (0,0) component of the tensor is

$$T_0^{\ 0} = \frac{1}{8\pi} G_0^{\ 0} = \frac{Q^2}{8\pi r^4}\theta(r - r_0) + \left(-\frac{Q^2}{8\pi r^2 r_0^{\ 2}} + \frac{m}{4\pi r^2 r_0} \right)\theta(r_0 - r). \tag{5.3.10}$$

In this expression the first term is the electrostatic field energy confined in range $r > r_0$. The second term appearing from the metric smoothing does not disappear in the limit $r_0 \to 0$. The self-energy in the solution constructed is

$$E = \int d^3x \sqrt{-g} T_0^{\ 0} = \frac{Q^2}{2r_0} + \left(-\frac{Q^2}{2r_0} + m \right) = m \tag{5.3.11}$$

Result (5.3.11) can be shown to be independent of the metric smoothing method.
In the limit $r_0 \to 0$ relation (5.3.10) can be written as

$$T_0^{\ 0} = \frac{1}{\sqrt{-g}}\left(m\delta(r) + \frac{1}{2}Q^2\varpi(r)\right) \qquad (5.3.12)$$

Here $\varpi(r)$ is the generalized function determined by the following integration
rule:

$$\int f(r)\varpi(r)d^3x = \int \frac{f(r) - f(0)}{r^4}d^3x \qquad (5.3.13)$$

where $f(r)$ is a bounded smooth function. For the Schwarzschild metric ($Q = 0$ in (5.3.7)) the term $m\delta(r)$ in $T_0^{\ 0}$
0 that corresponds to a point source can be obtained straightforwardly when the presence of term $\Delta\left(1/r\right)$ in $G_0^{\ 0}$ is considered. A more complicated generalized function $\varpi(r)$ appears as a source when $Q \neq 0$. It owes its origin to the presence of term $\Delta\left(1/r^2\right)$ in $G_0^{\ 0}$. Thus, the Schwarzschild and Reissner–Nordstrom solutions can be extended to the entire space, if the point source is added to the energy-momentum tensor[50].

5.4 On Problem of Mass Origin and Self-Energy Divergence in Relativistic Mechanics and Gravitational Physics

The problem of mass is central in Gravitational and Particle Physics, Astrophysics, Cosmology and field theories. In Kinematics of Special Relativity Theory (SRT), the total mass m_{tot} of a point-like particle is related to the total energy by $E_{tot} = m_{tot}c^2$. A constant proper mass m_0 and kinetic mass m_{kin} are relativistic components of the total mass

$$m_{tot} = \gamma m_0 \quad , \quad m_{kin} = (\gamma - 1) m_0 \qquad (5.4.1)$$

where $\gamma = 1/\sqrt{1-\beta^2}$ is the Lorentz factor for a relative speed $\beta = u/c$ in a given inertial reference frame. As concerns dynamical mass properties, we found that the proper mass depends on the potential[51] of force field, on the gravitational and the Coulomb potential, in particular. Consequently, potential and kinetic energy become defined in SRT as strictly as in Newtonian Physics but at a new (relativistic) level of understanding. In the Lagrangian formulation of Relativistic Mechanics of a single particle, the rate of 4-momentum change equals the Minkowski force. A variation of the proper mass follows from the corresponding SRT dynamical equations (using Synge's denotations.

$$\frac{d}{ds}\left[m(s) \frac{dx_i}{ds} \right] = K_i \qquad (5.4.2)$$

They describe a particle motion on a world line $x_i(s)$, $\vec{x} = \{x_1, x_2, x_3, ict\}$, with a 4-velocity dx_i/ds, where $K_i(s)$ is a Minkowski 4-force vector, and s is a line arc-length. By definition of a time-like world line of a massive particle, we have the fifth equation:

$$\sum_i \frac{dx_i}{ds}\frac{dx_i}{ds} = -1 \qquad (5.4.3)$$

that makes the problem definite with respect to five unknown functions: $x_i(s)$ and $m(s)$. The proper mass variation along the world line is explicitly seen from the next equation obtained from (5.4.2) and (5.4.3):

$$\frac{dm}{ds}\frac{dx_i}{ds} + m\frac{d^2 x_i}{ds^2} = K_i \qquad (5.4.4)$$

For the sake of convenience, one may consider the description of motion in 3- space $(\alpha = 1,2,3)$ and time t $(i = 4)$ rather than in space time using the relation $ds = cdt/\gamma$ and formulas for relative ("ordinary") forces F_α [51]

$$F_\alpha = c^2 K_\alpha / \gamma \qquad (5.4.5)$$

Now the equations of motion take the form:

$$\frac{d}{dt}\left(m\gamma u_0\right) = F_\alpha \qquad (5.4.6)$$

$$c^2\frac{d}{dt}\left(\gamma m\right) = F\,.u + \frac{c^2}{\gamma}\frac{dm}{dt} \qquad (5.4.7)$$

where $u_\alpha(t) = dx_\alpha / dt$ $(\alpha = 1,2,3)$ is the 3-velocity, and the proper mass m is dependent on space and time coordinates in a given inertial reference frame. On the right-hand side of (5.4.7) the term $\left(\frac{c^2}{\gamma}\frac{dm}{dt}\right)$ is recovered to account for the proper mass variation in a force field. The effect of the proper mass variation was noted in [4, 5] but was never paid attention in literature. For example, the fact that a particle speed cannot exceed the speed of light is often illustrated by the expression of motion of the particle driven by a constant inertial force $f_0 = const$:

$$cp(t) = f_0 t \quad , \quad \beta(t) = f_0 t \Big/ \sqrt{m^2 c^2 + {f_0}^2 t^2} \qquad (5.4.8)$$

where the momentum $p(t)$ is proportional to the time elapsed. The mass in (5.4.8) is supposed to be a constant proper mass m_0. To check it, one has to consider a general problem on acceleration of the particle by a pulse of force with transients specified. It follows from (5.4.6) and (5.4.7) that the proper mass varies during transients. When the force reaches a plateau it becomes constant but different from m_0, the difference being a binding energy of a particle in the system "particle-accelerator". In the end of the pulse the proper mass acquires the initial value m_0 in a new state of free motion with kinetic mass-energy (5.4.1) taken from the accelerator. A dynamical change of the proper mass is a manifestation of a potential difference developed between the particle and the accelerator; consequently, the interaction should be characterized by the corresponding mass- energy current .A general relativistic mass-energy formula following from (5.4.6) and (5.4.7) holds:

$$E(t)^2 = p(t)^2 c^2 + m(t)^2 c^4 \qquad (5.4.9)$$

It describes the instantaneous state of a single particle in a force field and leads to (5.4.8) under a constant force condition. For a free motion, the equation (5.4.9) is reduced to (5.4.1) and the known SRT Kinematics formula

$$E^2 = {p_0}^2 c^2 + {m_0}^2 c^4 = const \qquad (5.4.10)$$

We shall see further that for a particle in free fall in a static gravitational field the expression (5.4.9) takes the form of the total energy conservation law with the proper mass being variable

$$E^2 = p(t)^2 c^2 + m(t)^2 c^4 = {m_0}^2 c^4 \qquad (5.4.11)$$

To understand a physical meaning of equation (5.4.11), let us consider a point-like particle of proper mass m_0 in a spherical symmetric gravitational field; the latter is characterized by a classical potential $\phi(r)$ due to a uniform sphere of mass $M >> m_0$ and a radius R:

$$\phi(r) = -c^2 \left(r_g / r\right) \ , \ r \geq R \qquad (5.4.12)$$

Where $r_g = GM/c^2$ is a “gravitational radius”(G is the universal gravitational constant), r is a distance from the center of the sphere. So far, we assume that $R \geq r_g$. The potential (5.4.12) is defined per unit mass which could be a rest mass of a test particle in the Newtonian Mechanics. In SRT Mechanics the proper mass m must be field dependent. Imagine that the particle can be slowly moved with a constant speed along the radial direction with the use of an ideal transporting device supplied with a recuperating battery. Thus, the particle will exchange energy with the battery in a process of mass-energy transformation prescribed by the SRT mass-energy concept. The change of potential energy of the particle is related to the change of the proper mass[49]:

$$dm = -m(r) d\left(r_g / r\right) \ , \ r \geq R \qquad (5.4.13)$$

Thus, the proper mass of the particle is a function of the distance r:

$$m(r) = m_0 \exp\left(r_g / r\right) \ , \ r \geq R \qquad (5.4.14)$$

where m_0 is a proper mass at infinity. In a weak field approximation $r >> R$
we have

$$m(r) \cong m_0 \left(1 - r_g / r\right) \qquad (5.4.15)$$

with a Newtonian limit $m(r) = m_0$ at $r_g << R$. At $(r_g / r) \to \infty$ the proper mass tends to exhaust.
Once the proper mass variation is taken into account, a gravitational force takes a kinematical form:

$$F(r) = m_0 c^2 \left(r_g / r^2\right) \exp\left(-r_g / r\right) \qquad (5.4.16)$$

The same result follows from (5.4.6) and (5.4.7) when the interaction of the particle with the battery is taken into account. One can find a relativistic generalization of the static potential function (12):

$$m_0 \phi(r) = \int_{\tau}^{\infty} F\left(r, m(r)\right) dr = -m_0 c^2 \left[1 - \exp\left(-r_g / r\right)\right] \ , \quad r \geq R \qquad (5.4.17)$$

The expressions (5.4.16) and (5.4.17) have a point-like particle limit. In general, the proper mass of a test particle at a point r in 3-space uniquely characterizes a static gravitational field $\phi(r)$:

$$m(r) / \ m_0 = \left[1 + \frac{1}{c^2} \phi(r)\right] \qquad (5.4.18)$$

The potential changes within the range $-c^2 \leq \phi(r) \leq 0$; therefore, it is limited by the factor c^2. This is a result of fundamental importance. It shows that a singularity is absent in the relativistic form of gravitational potential.

Conservative field properties are embedded in equations (5.4.6) and (5.4.7). Consequently, for a particle in free fall in the spherical symmetric gravitational field (5.4.12) a total mass is constant.

$$m_{tot}(x) = \gamma_r m(r) = m_0 \qquad (5.4.19)$$

Putting the expression for a gravitational force

$$F(r)dr = c^2 m(r) d\left(r_g / r\right)$$

into equations (5.4.6) and (5.4.7), we have for

$$r \geq R \geq r_g :$$

$$m_0^{\ 2}(x) = m_0^{\ 2}\beta(r)^2 + m(r)^2 \qquad (5.4.21)$$

$$m(r)/m = 1/\gamma_r = \left(1 - r_g/r\right) = \sqrt{1-\beta(r)} \qquad (5.4.21)$$

$$\beta(r) = u(r)/c = \frac{1}{c}\frac{dr}{dt} = \sqrt{1-\left(1-r_g/r\right)^2} \qquad (5.4.22)$$

The total energy conservation law is given in (5.4.20) as a relativistic relationship between a varying proper mass and a momentum[49].
The expression (5.4.21) shows that a kinetic energy gain is equal to the corresponding potential energy change.
It is worth noting that in these dynamical relations the γ_r factor looks like a linear approximation of the corresponding exponential factor in kinematic expression (5.4.15); this is because the equations of motion account for relativistic rescaling of space-time coordinates under dynamical conditions, when the gravitational force acquires Minkowski force properties. Finally, the expression (5.4.22) describes a radial speed of a particle falling from rest at infinity. If the particle has an initial radial momentum $\gamma_0\beta_0 m_0 c(\gamma_0 > 1)$, then, taking into account the total mass conservation

$\gamma_0^2 m_0^2 = \gamma_0^2 m_0^2 \beta(r)^2 + m(r)^2$ and the mass dependence on field $m(r)/m_0 = 1 - r_g/r$, we have $\gamma = \gamma_0 \gamma_r$, and the expression (5.4.22) is modified:

$$\beta(r) = \sqrt{1-(1-r_g/r)^2/\gamma_0^2} \tag{5.4.23}$$

The solution formally shows that the proper mass vanishes at $r = r_g$. Because a baryon charge of a single particle cannot be destroyed, we have to conclude that the above case cannot be physically realized: the results are valid at r ≥ R > r_g.They show that a particle carrying a non-zero proper mass in free fall can never reach the ultimate speed of light, though it constantly accelerates (the condition $\beta(r) < 1, d\beta/dr > 0$ always takes place).
Next, let us consider a radial motion of a photon in gravitational field. Unlike the particle, the photon does not have a proper mass. From equations (5.4.6) and (5.4.7) one can note that any force changes a momentum through the action on a total mass. Because the total mass is constant, the only way the photon can change the momentum is by changing the speed. In other words, the speed should be influenced by the potential φ(r). The following expression is consistent with SRT Mechanics:

$$\beta_{ph}(r) = c(r)/c_0 = 1 - r_g/r \tag{5.4.24}$$

Actually, this is the relative speed of wave propagation c(r) = λ(r) with a constant frequency f = Const. The speed is constant on an equipotential surface $r = r_0$; in this case, it may be termed a tangential, or arc speed. Henceforth the speed of light at infinity will be denoted c_0. In addition to (5.4.24), one can define the radial "coordinate" speed $c^*(r) = \beta^*(r)c_0$. It is measured by the differential time-of-flight method by an observer at infinity with the use of the so-called standard clock. If a unit length were found from circumference measurements, the radial scale would be determined by the field-dependent length unit proportional to the wavelength of the standard photon emitted from infinity $dr/d\lambda = (1 - r_g/r)$. Therefore:

$$\beta^*(r) = \beta(r)\frac{dr}{d\lambda} = (1 - r_g/r)^2 \tag{5.4.25}$$

As is seen, the photon while approaching the sphere slows down and tends to stop at $r \to R \to r_g < R$. Our analysis of the phenomenon led us to the conclusion that the photon propagates in space of a gravitational field as in a refracting medium. The variation of the proper mass and the speed of light in a gravitational field is a consequence of the SRT mass-energy concept. Both phenomena should be considered as a result of interaction of the particle or the photon with the field; they are crucial for a metric determination in Relativistic Mechanics and should be verified in experiments[51].

Chapter 6
Vacuum Energy and mass generation

6.1 introduction

This chapter is concerned with the contribution which is concerned with finding a useful expression for vacuum energy and its relation to the process of mass generation.

6.2 vacuum and energy minimization

The form of the vacuum energy ρ_v can be found form the Hamiltonian of the GFE proposed by Ali Eltahir [52] ,which is given by

$$H = \alpha R^2 - \frac{1}{3}\beta R + \frac{1}{3}\gamma + \frac{\alpha B^{\bullet}}{AB} R^{\bullet} \qquad (6.2.1)$$

Where

$$\overset{\bullet}{B} = \frac{dB}{dt} \qquad \overset{\bullet}{R} = \frac{dR}{dt}$$

minimizing H with respect of R yields

$$\frac{dH}{dR} \approx 2\alpha R - \frac{1}{3}\beta = 0 \qquad (6.2.2)$$

the vacuum energy can also be obtained when the energy is minimized with respect to the field potential ϕ as already utilized in the electro weak model [53]

$$\frac{dH}{d\phi} = \frac{dH}{dR}\frac{dR}{d\phi} = \left(2\alpha R - \frac{1}{3}\beta\right)\frac{dR}{d\phi} = 0 \qquad (6.2.3)$$

Which can be satisfied, when

$$2\alpha R - \frac{1}{3}\beta = 0$$

$$2\alpha R = \frac{1}{3}\beta$$

Since α and β are constants ,hence

$$R = \frac{\beta}{6\alpha} = const \qquad R^{\bullet} = 0 \tag{6.2.4}$$

At early universe , the big bang model assumes that matter does not exist and vacuum energy dominates. Thus matter density γ vanishes, and the Hamiltonian H equals the vacuum energy ρ_v ,i.e

$$\begin{aligned} &\gamma = 0 \\ &H = \rho_v \end{aligned} \tag{6.2.5}$$

substituting equations (6.2.4) and (6.2.5) in (6.2.1),the vacuum energy is given by:

$$\rho_v = \alpha R^2 - \frac{1}{3}\beta R + 0 + 0 \tag{6.2.6}$$

$$\rho_v = \alpha\left(\frac{1}{6}\frac{\beta}{\alpha}\right)^2 - \frac{1}{3}\beta\left(\frac{1}{6}\frac{\beta}{\alpha}\right)$$

There for , the vacuum is given by

$$\rho_v = \frac{\beta^2}{36\alpha} - \frac{1}{18}\frac{\beta^2}{\alpha} = \frac{-\beta^2}{36\alpha} \tag{6.2.7}$$

The relation between α and β is proposed by M.dirar and others[54] to be

$$\alpha = -\sqrt{\frac{\beta}{24}} \tag{6.2.8}$$

substituting (6.2.8) in (6.2.7), yields

$$\rho_v = \frac{\beta^2}{36\sqrt{\frac{\beta}{24}}} = \frac{\beta^2 \cdot 2\sqrt{6}}{36\sqrt{\beta}} = \frac{\sqrt{6}\beta^2}{18\sqrt{\beta}} = \frac{\sqrt{6}\beta^{\frac{3}{2}}}{18}$$

$$\rho_v = \frac{\sqrt{6}\beta^{\frac{3}{2}}}{18} \qquad (6.2.9)$$

But the value of β is proposed by Ali Eltahir [55] to be related to the gravitational constant G, according to the relation

$$\beta = \frac{1}{16\pi G} \qquad (6.2.10)$$

Thus substituting the numerical value of G , in the expression for ρ_v one gets

$$\rho_v = \frac{\sqrt{6}}{18}\left(\frac{1}{16\pi}\left(\frac{n^2\pi^2}{x_0^2} + w^2\right)^{-2}\right)^{\frac{3}{2}} \qquad (6.2.11)$$

The value of ρ_v is large , which agrees with what proposed in inflation scenario [56]

6.3 mass generation

According to the red shift phenomena the potential increases the energy of the photon according to the relation

$$hf' = hf + V \qquad (6.3.1)$$

Where hf is the initial photon energy in vacuum and hf' is the photon energy in afield having potential V .

$$hf' - hf = V \qquad (6.3.2)$$

The mass m can be found from the [57] rest mass term m_0 , by using Einstein generalized special relativity (EGSR) to be

$$mc^2 = \frac{g_{00} m_0 c^2}{\sqrt{g_{00} - \frac{v^2}{c^2}}} \qquad (6.3.3)$$

In the Newtonian limit [45] g_{00} can be expressed in terms of the potential per unit mass ϕ

$$g_{00} = \frac{1+2\phi}{c^2} = \frac{1+2m_0\phi}{m_0c^2} = 1 + \frac{2V}{m_0c^2} \qquad (6.3.4)$$

Where the potential v for mass m_0 is given by

$$V = m_0\phi$$

if the particle is at rest:

$$v = 0$$

Then from (6.3.3)

$$m = \frac{g_{00} m_0}{\sqrt{g_{00}}} = g_{00} m_0 g_{00}^{\frac{-1}{2}} = \sqrt{g_{00}}\, m_0$$

$$m = \sqrt{g_{00}}\, m_0 \qquad (6.3.5)$$

Substitute (6.3.4) in (6.3.5) to get

$$m = \left(1 + \frac{2V}{m_0c^2}\right)^{\frac{1}{2}} m_0 \qquad (6.3.6)$$

One can simplify equation (6.3.6) by assuming the potential to be much less than the rest mass energy, this is quite natural since it is assumed that vacuum by it self generate negligible field .

The fact that at vacuum stage, no matter exist confirms also the smallness of v.

$$V << m_0c^2$$

Hence

$$m \approx \left(1+\frac{1}{2}\left(\frac{2V}{m_0c^2}\right)\right)m_0 \tag{6.3.7}$$

$$m \approx m_0 + \frac{V}{c^2} \tag{6.3.8}$$

Since one is concerned with mass generation by vacuum . thus it is obvious that the rest mass m_0 does not exist the frozen vacuum field energy is thus assumed to generate mass

$$m = \frac{V}{c^2} \tag{6.3.9}$$

Thus vacuum generates the mass and increases it.
If one follows GSR [58], Then vacuum energy according to the energy relation by

$$\rho_v = mc^2 = m_0c^2\left(1+\frac{2\varphi}{c^2}-\frac{v^2}{c^2}\right)^{\frac{-1}{2}} \tag{6.3.10}$$

Where the potential should be of the vacuum

$$for \quad v = 0 \quad \& \; for \quad 2\varphi << c^2$$

Using the identity $(1+x)^n = 1+nx$ For $x = \frac{2\varphi}{c^2} << 1$

$$\rho_v = m_0c^2\left(1+\frac{2\varphi}{c^2}\right)^{\frac{-1}{2}} = m_0c^2 - \frac{2\varphi}{2c^2}m_0c^2 \tag{6.3.11}$$

$$\rho_v = m_0c^2 - \frac{V}{c^2} \tag{6.3.12}$$

if one replace φ by $-\varphi$ by assuming repulsive energy (at vacuum inflation stage φ is assumed to be repulsive)

$$\rho_v = m_0 c^2 + V \tag{6.3.13}$$
$$for \quad m_0 = 0$$

$$\rho_v = V \tag{6.3.14}$$

This equations agrees with (6.3.9) as for as $\rho_v = mc^2 = V$

from equation (6.2.9)since vacuum energy is given by

$$\rho_v = \frac{\sqrt{6}\beta^{\frac{3}{2}}}{18} \tag{6.3.15}$$

it follows from (6.3.13) that

$$\frac{\sqrt{6}\beta^{\frac{3}{2}}}{18} = V \tag{6.3.16}$$

Thus the vacuum potential which responsible for generating mass is given by

$$V = \frac{\sqrt{6}\beta^{\frac{3}{2}}}{18} \tag{6.3.17}$$

Substitute (6.3.16) in (6.3.9) to get

$$m = \frac{\sqrt{6}}{18c^2}\beta^{\frac{3}{2}} \tag{6.3.18}$$

6.4 **vacuum energy During inflation and vacuum stage**

The above expression for vacuum and mass are derived from the general Hamiltonian of the gravitianal system .

In this section one needs to utilize the Hamiltonian of the universe which depends on RW metric and EGGR. This Hamiltonian takes the form [see(4.3.4)]

$$H = \rho c^2 \qquad (6.4.1)$$
$$= \alpha R^2 - \frac{1}{3}\beta R + \frac{\alpha B^{\bullet}}{AB} R^{\bullet}$$

For vacuum which is characterized by k=0 the cosmic scale factor a, and the scalar curvature R takes the forms [see (4.3.7),(4.3.8)]

$$a = Ae^{\mu t} \qquad (6.4.2)$$

$$R = 12\mu^2 \qquad (6.4.3)$$

Where μ is given, according to the equation of motion constraint from equation (4.3.9)

$$\mu = \frac{1}{\sqrt{72\alpha}} \qquad (6.4.4)$$

Thus

$$R = \frac{1}{6\alpha} \qquad (6.4.5)$$

Substituting (6.4.5) in (6.4.1) yields

$$\rho_v = \alpha R^2 - \beta R$$
$$= \frac{1}{36\alpha} - \frac{\beta}{6\alpha}$$

But according to (6.2.8)

$$\alpha = -\sqrt{\frac{\beta}{24}} \qquad (6.4.6)$$

Thus

$$\rho_v = \frac{-2\sqrt{6}}{36\sqrt{\beta}} + \frac{\beta \times 2\sqrt{6}}{6\sqrt{\beta}}$$
$$= -\frac{1}{12}\frac{1}{\sqrt{\beta}} + \frac{\sqrt{6}\sqrt{\beta}}{3} \qquad (6.4.7)$$

Hence

$$m_0 = \left(-\frac{1}{12}\frac{1}{\sqrt{\beta}} + \frac{\sqrt{6}\sqrt{\beta}}{3}\right) / c^2 \qquad (6.4.8)$$

But if one utilizes the expression of the vacuum energy by using the expression of the gravitinal energy-momentum tensor in equation (3.3.2)

$$\rho_v = L - L'R \qquad (6.4.9)$$

For

$$L = \alpha R^2 + \beta R, \qquad L' = 2\alpha R + \beta$$

$$\rho_V = \alpha R^2 + \beta R - 2\alpha R^2 - \beta R = -\alpha R^2 \qquad (6.4.10)$$

Sub (6.4.5) and (6.4.7) in(6.4.9) yields

$$\rho_v = -\frac{1}{36\alpha} = \frac{\sqrt{6}}{18\sqrt{\beta}} \qquad (6.4.11)$$

$$m_0 = -\frac{1}{36\alpha c^2} = \frac{\sqrt{6}}{18\sqrt{\beta}c^2} \qquad (6.4.12)$$

The vacuum energy can also be found by using the GFE equation by taking energy tensor to be that of vacuum and substituting

$$k = 0 \qquad a = e^{\mu t}$$

To get

$$R = -12\mu^2 \qquad (6.4.13)$$

And

$$\rho_v = 6\beta\mu^2$$

Substituting (6.4.4) and (6.4.6) yields

$$\rho_v = \frac{\beta}{12\alpha} = \frac{\sqrt{6}\beta^{\frac{1}{2}}}{6} = \frac{\beta^{\frac{1}{2}}}{\sqrt{6}} \qquad (6.4.14)$$

$$m_0 == \frac{\sqrt{\beta}}{\sqrt{6}c^2} \qquad (6.4.15)$$

However if one utilizes the quantum energy expression in equation (4.4.8) together with (6.4.4) , one gets

$$E = mc^2 = \hbar\mu = \frac{\hbar}{3\sqrt{8\alpha}} \qquad (6.4.16)$$

6.5 Mass generation at vacuum stage:

The vacuum energy form is given from equation (6.3.14) to be

$$\rho_v = V = m_0 c^2 \qquad (6.5.1)$$

But the vacuum energy ρ_v from equation (6.3.15) takes the form

$$\rho_v = \frac{\sqrt{6}\beta^{\frac{3}{2}}}{18} \qquad (6.5.2)$$

Then

$$\rho_v = m_0 c^2 = \frac{\sqrt{6}\beta^{\frac{3}{2}}}{18} \qquad (6.5.3)$$

Hence

$$m_0 = \frac{\sqrt{6}}{18}\frac{\beta^{\frac{3}{2}}}{c^2} \qquad (6.5.4)$$

From equation (6.2.10)

$$\beta = \frac{1}{16\pi G} \qquad (6.5.5)$$

According to Dirar and Isa paper [59] the gravitational constant G can be expressed in terms of the quantum number n , radius of the universe x_0 and angular frequency of the gravitational waves w to be

$$G = \frac{1}{\left(\frac{n^2\pi^2}{x_0^2} + w^2\right)^2} \qquad (6.5.6)$$

Where n=0,1,2,3,….

Inserting (6.5.6) and (6.5.5) in (6.5.4) gives

$$m_0 = \frac{\sqrt{6}}{18} \times \left(\frac{1}{16\pi G}\right)^{\frac{3}{2}} / c^2 \qquad (6.5.7)$$

$$m_0 = \frac{\sqrt{6}}{18} \times \left(\frac{1}{\left(\frac{1}{16\pi}\frac{n^2\pi^2}{x_0^2} + w^2\right)^2}\right)^{\frac{3}{2}} / c^2 \qquad (6.5.8)$$

6.6 mass quantization

According to equation (6.2.10), one have

$$\beta = \frac{1}{16\pi G} \tag{6.6.1}$$

Inserting (6.6.1) and (6.5.6) in (6.3.18) yields

$$m \approx \left(\frac{1}{\left(\frac{n^2 \pi^2}{x_0^2} + w^2 \right)^2} \right)^{\frac{3}{2}} \tag{6.6.2}$$

$$n = 0,1,2,3,\ldots.. \tag{6.6.3}$$

At present , when $x_0 \to \infty$ no mass quantization exists. This agrees with the fact that at present , where macroscopic large objects dominate no mass quantization exists. According to the work done Ibrahim Hassan Hassan[60] the universe radius is quantized and is given by

$$x_0 = r = r_0 n_0 = n_0 \pi \left[\frac{6|\alpha|}{\beta} \right]^{\frac{1}{2}} \tag{6.6.4}$$

Where: $n_0 = 1,2,3,\ldots\ldots\ldots..$

Thus the mass term according to equation (6.6.2) and (6.6.4) is given by

$$m = \left(\left(\frac{n^2 \pi^2}{r_0^2 n_0^2} + w^2 \right)^{-2} \right)^{\frac{3}{2}} \tag{6.6.5}$$

Rearranging this relation yields

$$\left(\frac{n}{n_0}\right)^2 = \left(m^{-\frac{1}{3}} - w^2\right)\frac{r_0^2}{\pi^2}$$

$$\frac{n}{n_0} = \left(m^{\frac{-1}{3}} - w^2\right)^{\frac{1}{2}} \frac{r_0}{\pi} = c_n \tag{6.6.6}$$

No matter, what the value of the mass is , finally the right hand side (r.h.s) of (6.6.6) is a number of the form

$$c_n = a_1 a_2 . a_3 a_4 \times 10^{-n_1} = a_1 a_2 . a_3 a_4 \times 10^{-n_1 - 2} \tag{6.6.7}$$

Where $a_1 a_2 . a_3 a_4$, 10^{-n_1-2} are natural numbers thus one can choose

$$n = a_1 a_2 . a_3 a_4$$

$$n_0 = 10^{n_1 + 2} \tag{6.6.8}$$

Which are both natural numbers .For example if

$$c_n = 37.25 \times 10^{-31} = 3725 \times 10^{-33}$$

$$a_1 = 3, \qquad a_2 = 7, \qquad a_3 = 2, \qquad a_4 = 5$$

According to equations(6.6.6) (6.6.8) one can select

$$\begin{aligned} n &= 3725 \\ n_0 &= 10^{n_1+2} = 10^{33} \\ n_1 &= 31 \end{aligned} \tag{6.6.9}$$

Which are both natural numbers .

At the very early universe where the masses are assumed to be produced the radius x_0 is minimum . Thus according to equation (6.6.4)

$$n_0 = 1$$

$$x_0 = r_0 = \pi \left[\frac{6|\alpha|}{\beta} \right]^{\frac{1}{2}}$$

Using equation (6.2.8)

$$|\alpha| = \sqrt{\frac{\beta}{24}}$$

Hence

$$r_0 = \pi \left(\frac{3}{\sqrt{6}} \beta^{\frac{-1}{2}} \right)^{\frac{1}{2}}$$

But from (6.2.10)

$$\beta = \frac{1}{16\pi G}$$

Where

$$G = 6.67 \times 10^{-11}$$

Thus the numerical value of x_0 is given by

$$r_0 = 26.635 \times 10^{-3} \qquad (6.6.10)$$

In the table below one can find the appropriate masses for some elementary particles by assuming the absence of gravitational waves , thus ignoring the graviton angular frequency w , and setting the minimum universe radius r_0 to be equal to be equal [31,60] [see (6.6.10)]

$$w = 0 \qquad r_0 = 26.635 \times 10^{-3} \tag{6.6.11}$$

Thus form (6.6.6) and (6.6.11)

$$\frac{n}{n_0} = \frac{\left(m^{\frac{-1}{3}}\right)^{\frac{1}{2}} r_0}{\pi} = c_n \tag{6.6.12}$$

Table (6.6.1) [61]

		Quantum number	
Particle name	Particle mass(in kg)	n	n_0
Electron	9.11×10^{-31}	19×10^{27}	1
Proton	1.672×10^{-27}	78×10^{23}	1
Quark	3.333×10^{-31}	66×10^{27}	1

The mass quantization can also be obtain by using GFE, and assuming the mass is generated during vacuum stage where inflation takes place as discussed in section (6.4).As for vacuum represents the minimum matter stage , it is quit natural to assume that masses of elementary particles to be generated during vacuum stage . in this case the masses are given by (6.4.15) together with equations (6.6.1), (6.5.6) and (6.6.4) to be

$$m_0 = \frac{\sqrt{\beta}}{\sqrt{6}c^2} = \frac{1}{4c^2\sqrt{6\pi}\sqrt{G}} = \frac{1}{4c^2\sqrt{6\pi}}\frac{1}{\sqrt{G}}$$

$$4\sqrt{6}\pi m_0 c^2 = \frac{1}{\sqrt{G}}$$

$$\frac{1}{4\sqrt{6}\pi m_0 c^2} = \left(\frac{n^2\pi^2}{n_0^2 r_0^2} + w^2\right) \qquad (6.6.13)$$

Rearranging (6.6.13) yields

$$\frac{n}{n_0} = \left[\frac{1}{4c^2\sqrt{6\pi}m_0} - w^2\right]^{\frac{1}{2}}\left[\frac{r_0}{\pi}\right] = c_n \qquad (6.6.14)$$

Following the same procedures as in equations (6.6.7) and (6.6.8) , one can choose an appropriate quantum numbers for any elementary particle mass .The table below shows the appropriate quantum numbers for some masses of elementary particles by assuming

$$w = 0 \qquad r_0 = 26.635 \times 10^{-3}$$

$$\frac{n}{n_0} = \left[\frac{1}{4c^2\sqrt{6\pi}m_0}\right]^{\frac{1}{2}}\left[\frac{r_0}{\pi}\right] = c_n$$

Table (6.6.2) [61]

		Quantum number	
Particle name	Particle mass(in kg)	n	n_0
Electron	9.11×10^{-31}	17×10^{8}	1
Proton	$1.67262178 \times 10^{-27}$	13×10^{5}	1
Quark	3.3332×10^{-31}	9×10^{12}	1

6.7 Neutrino and its properties

Neutrino is one of the most mysterious particle some a others thought that neutrino constitute part of the vacuum energy , as scientists still perplexed about its mass [62]. Some Scientists believe that the rest mass is zero while others believe that the rest mass is very small [63], According to the theory of special relativity, the mass m can be found in terms of the rest mass speed v according to the formula

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} \qquad (6.7.1)$$

Here one assumes the rest m_0 to be very small, i.e

$$m_0 \to 0$$

For m to be finite (as experimentally observed) m limited, this requires

$$\sqrt{1-\frac{v^2}{c^2}} \to 0$$

i.e

thus the neutrino speed v becomes

$$v \to c \qquad (6.7.2)$$

According to the big bang model , the neutrino in an expanding universe moves against gravity force F [64], thus equation of motion becomes

$$\frac{d(mv)}{dt} = \frac{d(mc)}{dt} = c\frac{dm}{dt} = F = \nabla\phi \tag{6.7.3}$$

Where ϕ is the potential and this potential depends on matter density ρ according to the relation

$$\nabla^2\phi = 4\pi G\rho$$

Considering the neutrino to have uniform density and spherical shape with radius r_0 .The neutrino density equals

$$\rho = \frac{3m}{4\pi r_0^3} \tag{6.7.4}$$

Then the equation (6.7.4) becomes

$$\nabla^2\phi = \frac{4\pi G(3m)}{4\pi r_0^3} = \frac{3G}{r_0^3}m \tag{6.7.5}$$

using equation (6.7.3) in (x) axes yields

$$\frac{\partial\phi}{\partial x} = \nabla\phi = +c\frac{dm}{dt}$$

$$\nabla^2\phi = \frac{\partial^2\phi}{\partial x^2} = \frac{d^2\phi}{dx^2} = \frac{\partial}{\partial x}\left(\frac{\partial\phi}{\partial x}\right) = \frac{\partial}{\partial x}\left(c\frac{dm}{dt}\right) = c\frac{d}{dx}\left(\frac{dm}{dt}\right)$$

$$= c\frac{d}{dt}\left(\frac{dm}{dt}\right)\times\frac{dt}{dx}$$

$$\nabla^2\phi = +c\frac{d^2m}{dt^2}\times\frac{1}{c} = +\frac{d^2m}{dt^2}$$

Where

$$x = ct$$

And using equation (6.7.5) the resulting

$$\frac{d^2m}{dt^2} = \frac{3G}{r_0^3}m \tag{6.7.6}$$

This equation describes the neutrino mass equation variable with time and the solution is becomes

$$m = m_0 e^{-\gamma_0 t} \qquad (6.7.7)$$

Substituting equation (6.7.7) in equation (6.7.6) to gives

$$\gamma_0^2 = \frac{3G}{r_0^3}$$

$$\gamma_0 = \sqrt{\frac{3G}{r_0^3}} \qquad (6.7.8)$$

thus equation (6.7.7) is consistent with the fact that the neutrino mass is very small now

where

$$m \to 0 \qquad\qquad t \to \infty$$

6.8 The Big Bang Theory equations

the Big Bang theory equations can be derived from Einstein's field equation of the gravitational field which takes the form [65]

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -8\pi G T_{\mu\nu} \qquad (6.8.1)$$

Where $R_{\mu\nu}$ is the Ricci tensor, $g_{\mu\nu}$ is the metric tensor, $T_{\mu\nu}$ is the energy momentum tensor . the Roberson Walker metric is given by

$$g_{tt} = -1 \qquad g_{rr} = \frac{a^2}{(1-kr^2)} \qquad g_{\theta\theta} = a^2 r$$

$$g_{\phi\phi} = a^2 r^2 \sin^2\theta \tag{6.8.2}$$

Thus Einstein's equations for the universe is found by subbing (6.8.2) in (6.8.1) to get

$$\dot{a}^2 + k = \frac{8\pi G}{3}\rho a^2 \tag{6.8.3}$$

$$\dot{\rho} + \frac{4\dot{a}}{a}\rho = 0 \tag{6.8.4}$$

Where ρ represents the matter density of the universe .The constant k take the following values

$$k = 0, \pm 1 \tag{6.8.5}$$

6.9 The role of the neutrino in the amplification of the universe

the equation of the theory of the Big Bang can be used to describe the early universe , Assuming that the neutrino particles have high density in the early universe, thus the density of the universe takes the form [66]

$$\rho = \frac{3N}{4\pi}\frac{m}{r_0^3} \tag{6.9.1}$$

Where N represents to the total number for Neutrinos. If the equation (6.9.1) together with (6.7.7)has been used in the equation (6.8.4), one gets

$$\frac{-3N}{4\pi r_0^3}\frac{\gamma_0}{4}m = \frac{-3N}{4\pi r_0^3}m\frac{\dot{a}}{a}$$

$$\frac{\gamma_0}{4} = \frac{\dot{a}}{a} \tag{6.9.2}$$

Then the standard cosmological factor a is becomes :

$$\int \frac{da}{a} = \frac{\gamma_0}{4} \int dt + c_1$$

$$\ln a = \frac{\gamma_0}{4} t + c_1$$

Where

$$a = e^{c_1} e^{\frac{\gamma_0 t}{4}}$$

when

$$a = a_0 \qquad t = 0$$

$$a_0 = e^{c_1}$$

Then

$$a = a_0 e^{\frac{\gamma_0}{4}} \qquad (6.9.3)$$

this equation predicts universe inflation and one can obtain the same solution of equation (6.7.7)and (6.8.3) for inflationary universe by assuming the universe to be flat (i.e k=0) in eqn (6.8.3) and assuming m to be constant at early stage, i.e

$$m \approx m_0 e^{-0} = m_0 \qquad t \to 0 \qquad (6.9.4)$$

Then equation (6.8.3) it becomes

$$a^{\bullet^2} = \frac{24\pi G}{3}\frac{m_0}{4\pi r_0^3}a^2$$

$$a^{\bullet^2} = \frac{2Gm_0}{r_0^3}a^2$$

$$a^{\bullet} = \sqrt{\frac{2Gm_0}{r_0^3}}a \tag{6.9.5}$$

Hence the solution is in the formula

$$a = a_0 e^{\beta_0 t} \tag{6.9.6}$$

where

$$\beta_0 = \sqrt{\frac{2Gm_0}{r_0^3}} \tag{6.9.7}$$

For consistency of the solutions of equations (6.9.3) and equation(6.9.6) and (6.9.7) one suggests

$$\gamma_0 = 4\sqrt{\frac{2Gm_0}{r_0^3}} \tag{6.9.8}$$

6.10 Inertial and Gravitation Mass

The dependence of particle masses by fields and motion was tackled by many scientists. Before Einstein SR theory, the particle masses are considered as universal constants. But SR theory, shows that the mass m increases according to the relation

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} \qquad (6.10.1)$$

Where m_0 is the rest mass.

Later on Mubarak Dirar and others find that the potential and the field affect the mass according to the relation (6.3.5)

$$m = \frac{g_{00} m_0}{\sqrt{g_{00}-\frac{v^2}{c^2}}} \qquad (6.10.2)$$

Where g_{00} is given by (6.3.4) to be

$$g_{00} = 1+\frac{2\varphi}{c^2} \qquad (6.10.3)$$

This model is called EGSR .

The dependence of mass on the field in a curved space-time is also proposed by Savickas to be

$$m = \frac{m_0}{\sqrt{g_{00}-\frac{v^2}{c^2}}} \qquad (6.10.4)$$

In his general relativity theory, Einstein proposed the equality of inertial mass m_i and the gravitational mass m_g, according to the so called equivalence principle . one can see the compatibility of this principle with EGSR and Savickas model [67]

consider now a particle at rest in an elevator falling freely with the particle . According to the observer on the earth the particle speed is

$$v^2 = v_0^2 + 2ax = 2ax = 2\varphi \qquad (6.10.5)$$

He see the particle moving with speed v in the field φ. Thus according to him equations (6.10.2) (6.10.3) and (6.10.5) the gravitational mass is given by

$$m_g = \frac{\left(1 + \frac{2\varphi}{c^2}\right) m_0}{\sqrt{1 + \left(\frac{2\varphi - v^2}{c^2}\right)}} = \left(1 + \frac{2\varphi}{c^2}\right)^{\frac{1}{2}} m_0 \qquad (6.10.6)$$

For the elevator the particle is at rest and no acceleration is observed . Thus

$$v = 0 \qquad a = 0 \qquad ax = \varphi = 0 \qquad (6.10.7)$$

Hence , equation (6.10.2) reads

$$m_i = \frac{m_0}{\sqrt{1}} = m_0 \qquad (6.10.8)$$

Thus according to EGSR model

$$m_i \neq m_g \qquad (6.10.9)$$

i.e the inertial mass is not equal to the gravitational mass. However, the situation is different for Savickas model: for the earth observer, equations (6.10.5) and (6.10.6) gives

$$m_g = \frac{m_0}{\sqrt{1+\left(\frac{2\varphi - v^2}{c^2}\right)}} = m_0 \qquad (6.10.10)$$

While for the elevator observer

$$m_i = m_0 \qquad (6.10.11)$$

Thus the inertial and gravitational masses are equal.

One can tackle the problem in another way by considering particle falling feely in gravity and another one in elevator is moving in free space with acceleration g with respect to a particle of mass m_0. For the particle moving in gravity

$$v^2 = 2ax = 2\varphi \qquad \varphi \neq 0 \qquad (6.10.12)$$

The mass expression in EGSR [see equation (6.10.2)] reads

$$m_g = \frac{\left(1+\frac{2\varphi}{c^2}\right)m_0}{\sqrt{1+\left(\frac{2\varphi - v^2}{c^2}\right)}} = \left(1+\frac{2\varphi}{c^2}\right)^{\frac{1}{2}} m_0 \qquad (6.10.13)$$

While for elevator the same equation gives

$$v^2 = 2gx = 2\varphi \qquad (6.10.14)$$

$$m_i = \frac{\left(1+\frac{2\varphi}{c^2}\right)m_0}{\sqrt{1+\left(\frac{2\varphi - v^2}{c^2}\right)}} = \left(1+\frac{2\varphi}{c^2}\right)^{\frac{1}{2}} m_0 \qquad (6.10.15)$$

The gravity and inertial mass are equal according to EGSR. However Savickas model for earth [see (6.10.4) and (6.10.12)] gives

$$m = \frac{m_0}{\sqrt{1+\left(\frac{2\varphi - v^2}{c^2}\right)}} = m_0 \qquad (6.10.16)$$

While for elevator [see (6.10.2) and (6.10.14)]

$$m_i = \frac{m_0}{\sqrt{1-\left(\frac{2\varphi - v^2}{c^2}\right)}} = \frac{m_0}{\sqrt{1}} = m_0 \qquad (6.10.17)$$

Again the gravity and inertial mass are equal in Savickas model

$$m_g = m_i \qquad (6.10.18)$$

6.11 Discussion

The vacuum energy in equation (6.2.11) is found by minimizing the Hamiltonian of the EGGR of the gravitational field. it was found to be constant and dependent on the gravitational constant G. the value of vacuum energy is large which agrees with previous works [68]. By using EGSR the mass in (6.3.9) is generated by potential. It is well known in physics that the mass is generated by vacuum. Thus the potential which generates mass should be that of vacuum.

Using the EGSR again a useful expression for vacuum potential related to vacuum energy is obtained.

This expression is compared to the expression of vacuum in (6.2.11) to find a useful expression for vacuum potential dependent on G in (6.3.16) . This expression is used to find the mass of elementary particles in (6.3.18).

Following the paper [69] which quantize the gravitational constant G, equation (6.6.3) shows that the mass is quantized as far as it depends on the quantum number n.

This means that vacuum can generate masses of elementary particles by changing the quantum number n .

Another expressions for vacuum energy and elementary particle masses are obtained by using the EGGR equation of motion of the universe at vacuum stage. The vacuum energies in (6.4.7), (6.4.11) and (6.4.14) are more realistic since they predict large vacuum energy which is in conformity with previous studies.

Again the masses of elementary particles are quantized as shown in equations (6.4.8),(6.4.12) and (6.4.15). and tables (6.6.1) and (6.6.2) the equality of inertial and gravitational masses confirm by Savickas model, while they are different according to EGSR [see section 6.10].

By considering vacuum is constituted by Neutrino a time decaying mass is found in (6.7.7) by assuming the Neutrino to move very fast in a gravitational field .

The decaying mass indicates that the Neutrino mass is very small at present which agrees with observations [70]
It also suggests that vacuum energy is large at early universe and very small at present.
This is in agreement with that proposed by particle physic's .

6.12 Conclusion

This model shows that vacuum energy obtained from EGSR and EGGR can generate masses of elementary particles.
The mass expression is quantized ,which indicates that vacuum can generate different masses. Each mass is characterized by a certain quantum number.

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