APPENDIX (1)

EXAMPLES OF THE NEWMARK

Mumerical Method

The Newmark (29) numerical method for computing moments, deflections, and buckling loads is the procedure used to analyze the steel poles in this study. Examples are presented below to demonstrate the main features of the method as it applies tosteel transimision poles. The six examples are:

1. Prismatic, transversely loaded cantilever,
2. Tapered, transversely loaded cantilever,
3. tapered, transversely loaded cantilever with discontinuous El and concentrated moment.
4. prismatic, axially loaded antilever (to determine buckling load),
5. prismatic cantilever with P-delta effects included, and
6. prismatic cantilever with P-Delta and large deflection effects included.

Because the structures considered in this study are statically determinate, moments are easily computed in terms of known structural geometry and assurfied deflections. With moments known, the purpose of Newmark's method is simply to integrate curvature (the ratio of moment of flexural rigidity, MIEI) to obtain deflections. The deflections computed may not be the same as

those assumed to calculate the moments which began the process. Thus, the method is iterative, and continues until assumed and computed deflections agree to within about one percent. (If moments can be computed without having to assume a deflected shape, the Newmark method does not require iteration, see example 1.)

Appendix (2)

PROBABILITY DISTRIBUTION BACKGROUND

A.1 From Data to Distributions:

Information about the probable values of quantities such as structure loads and material resistance is determined from the collection and evaluation of data.

Examples:

1. To estimate the probability that the maximum wind speed in the next year at a specified location will be within certain limits, one would examine the records of past yearly maximum speeds measured by an anemometer in the same area.
2. To estimate the probability that the strength of a structural element is within stated limits, one would examine results from past strength measurements of similar elements.

Generally, the data must be processed in some manner to make them useful. Common summary statistics for a set of data are the mean, m, variance, G standard deviation, G, and coefficient of variation, V.

Example:

Nowhere City Annual Fastest-Mile Wind Speeds

Year	Wind Speed, W, mph	Year	Wind Speed, W, mph
1960	55.3	1970	38.6
1961	42.1	1971	49.8
1962	40.6	1972	65.0
1963	25.1	1973	52.3
1964	70.3	1974	41.9
1965	60.2	1975	36.2
1966	49.8	1976	38.5
1967	52.1	1977	43.8
1968	45.3	1978	29.1
1969	32.9	1979	42.7

Summary Statistics

$$\text{mean } m = \frac{1}{n}\sum_{i=1}^{n} x_i$$

where n = number of data points,

x_i = data point i,

$$m = \frac{1}{20}(55.3+42.1+\ldots+42.7)$$

$$= \frac{1}{20}(911.6), \text{ and}$$

$$m = 45.58 \text{ mph}$$

Variance

$$\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - m)^2$$

$$= \frac{1}{20}[(55.3-45.58)^2 + (42.10-45.58)^2 + \ldots + (42.7-45.58)^2]$$

$$\sigma^2 = \frac{1}{20}\quad[2520]$$

$$= 126.0 \text{ (mph)}^2$$

Standard Deviation

$$\sigma = \sqrt{\sigma^2}$$

$$= \sqrt{126.0\ (\text{mph})^2}$$

$$\sigma = 11.23\ \text{mph}$$

Coefficient of Variation

$$V = \frac{\sigma}{m}$$

$$= 11.23./45.58$$

$$V = .\,246$$

Availability of these summary statistiscs does not suffice for a calculation such as: P [W<100 mph]. To make this calculation the cumulative distribution function (cdf) must be used. A numerical cdf may be constructed rom a set of data by ranking the data from lowest (j=1), to highest (j=n), and assigning the cdf value $\frac{j}{n+1}$ to each data point.

Example:

Nowhere city fastest-Miles

j	W_j	$P[W<W_j] = \frac{j}{n+1}$
1	25.1	1/21 = 0.0476
2	29.1	2/21 = 0.0952
3	32.9	0.1428
4	36.2	0.1905
5	38.5	0.2381
6	38.6	0.2857
7	40.6	0.3333
8	41.9	0.3810
9	42.1	0.4286
10	42.7	0.4762
11	43.8	0.5238
12	45.9	0.5714
13	49.8	0.6190
14	49.8	0.6667
15	52.1	0.7143
16	52.3	0.7619
17	55.3	0.8095
18	60.2	0.8571
19	65.0	0.9048
20	70.3	0.9524

By using the table in the example above one could estimate:

P 43.8 mph] = 0.5238 and use interpolation for intermediate values of W. However, it would be more convenient to use and equation relating W and its cdf. While a curve could be fit to the (Wi,) P points, this is not the common procedure. There are several convenient functional forms which meet the requirement for being a cdf, namely:

1. the function is 0 at -x and 1.0 at + and the function is always increasing toward + and therefore is always positive.

Among these are the normal, lognormal, Gumbel Type 1, Gumbel Type 2, and three parameter Weibull (described in A.2). The first four of these distributions are two parameter distributions whose parameters are obtained from the mean and standard deviation of the quantities they represent. One might hypothesize that the fastest-mile values in Nowhere City are normally distributed with mean equal to 45.6 mph and standard deviation equal to 11.2 mph, or Gumble Type 1 with that m and rn The three parameter Weibull distribution also requires specification of the location parameter, which is the lowest abscissa at which the pdf is defined. If the numerical cdf is available, the hypothesis can be tested by comparing the numerical cdf is available, the hypothesis can be tested by comparing the numerical cdf value at each data point to the cdf value predicted by the functional form. If the two are nearly equal or each data point, the hypothesized distribution fits the data well. The interested reader is referred to the text by Benjamin and Cornell (5) for further information on evaluating hypotheses.

A.2. Common Probability Distributions

A.2.1 The Normal Distribution

Random Variable: X

Parameters: Mean, m_x, and standard deviatin, σ_x

Cumulative Distribution Function (cdf), $F_x(x)$

$$F(x) = \int_{-\infty}^{x} \frac{1}{\sigma_x\sqrt{2\pi}} \quad \exp\left[-\tfrac{1}{2}\left(\frac{\tau - m_x}{\sigma_x}\right)^2\right] d\tau \qquad -\infty < \tau < \infty$$

Evaluation

The normal cdf is generally evaluated by using standard normal probability tables. The standard normal variate, u, is defined as

$$U = \frac{x - m_x}{\sigma_x} \ .$$

$F_x(x)$ is equal to $F_u(u)$, which is tabulated for values of u. give a polynomial approximation to $F_u(u)$ which may be used instead of tables (see A.3.6).

Inverse cdf

Often if is necessary to find the variable value which has a given cdf value (in Monte Carlo simulation, for example). For the normal distribution, a search of t he standard normal probability table yields the U value for a given cdf. Alternatively, give a polynomial approximation, $u = F_u^{-1}$ (given cdf value) see A.3.6). The value x is calculated as

$$x = \sigma_x u + m_x$$

A.2.2. The Lognormal Distribution

A random variable, Y, is lognormally distributed if x = Ln Y is normally distributed. The cdf of Y is evaluted by converting to the x distribution, since $F_y(y) = F_x(x)$, where x = Lny. The mean and standard deviation of the normal x distribution are calculated as follows :

$$\sigma x = \left[\mathrm{Ln}\left(\left(\frac{\sigma_y}{m_y}\right)^2 + 1\right)\right]^{1/2}$$

$$m_x = \mathrm{Ln}\left[my \exp\left(-\tfrac{1}{2}\sigma_x^{2}\right)\right]$$

Inverse

For a given cdf value, the corresponding value, y, is obtained by first using the inverse normal relationship to find x, then y is equal to exp (x).

A.2.3 The Gumbel Type 1 Distribution

Random Variable : x

Parameters : mode, U, and dispersion, α

Cdf, $F_X(x)$

$$F_X(x) = \exp\left[-e^{-\alpha(x-U)}\right] \qquad -\infty < x < \infty$$

Calculation of parameters from Mean and Standard Deviation

$$U \approx m_X - \frac{0.577}{\alpha}$$

$$\alpha \approx 1.282/\alpha_x$$

Inverse cdf

$$x = U + \frac{1}{\alpha} * (-L_n(-L_n(F_X(x))))$$

Calculation of Parametrs for n Period Curve from Those for 1 Period Curve

$$\alpha_n = \alpha_1$$

$$U_n = U_1 + \frac{Ln\,(n)}{\alpha}$$

(n period curve is Gumbel Type 1 also)

A.2.4 The Gumbel Type 2 Distribution

Rondom Variable: x

Parameters: mode, u, disperision, k

cdf, $F_X(x)$

$$F_X(x) = e^{-\left(\frac{U}{x}\right)^K} \qquad x > o$$

Calculation of Parameters from Mean and Standard Deviation

Approximate relation from Peyrot and Aznavour (31):

$$v = \frac{\sigma_x}{m_x}$$

K = (1.+(1.+3.8993*V*(1.+1.45565*V*(1.+1.31168*v)))
/ (3.5604*(1.+1.18672*V*(1.+0.94635))))/V

U = m_x/ Γ (1.-1/V)

Γ(x) = the gamma integral (see A.3.6)

Inverse cdf

x = U/ (-(L$_n$(F$_X$(x)))**(1./k)

Calculation of Parameters for n Period Curve from Those for 1 Period Curve

$$K_n = K_1$$

$$U_n = n^{1/K} U_1$$

(n period curve is Gumbel Type 2 also)

A.2.5 The three parameter Weibull Distributions

Random Variable : x

Parameters : Location, ε, scale, ω, and shape, K

Cdf, F_X (x)

$$F_x(x) = 1. - \exp\left[-\left(\frac{x-\varepsilon}{\omega}\right)^K\right] \qquad x \geq \varepsilon$$

Calculation of Scale and Shape Parameters from the the Mean, Standard Deviation, and Location Parameter

Approximate relation from Peyrot and Aznavour (31).

$$V = \frac{\sigma_x}{m_x - \varepsilon}$$

K = (1. + (1.-4 92*V* (1.-1.965059*V* (1.-0.74199*V)))

/ (3.5697* (1.-2.48368*V* (1.-1.02186*V)))/V

$\omega = (m_x - \varepsilon) / \Gamma(1.+1./K)$

Inverse cdf

$$x = \varepsilon + \omega\ [-\mathrm{Ln}\,(1.-F_x(x))]^{1/k}$$

A.3.6. Miscellaneous

Polynomial Approximation to Normal cdf (4)

Required: $F_u(u)$ where U is a standard normal distribution

Solution : V = 1./ (1.+0 2316419+ABS(U)

$F_U(u)$ = 1. – $f_u(u)$ * V * (0.3193815+V*(-0.3565638

+ V* (1. 781478+V*(-1.82156+V*(1.330274))))

If (U .LT. 0.) $F_U(u)$ = 1. – $F_U(u)$

$F_U(u)$ = standard normal pdf = $\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}u^2}$

Polynomial Approximation to the NOrmal Inverse cdf (33)

Required : The abscissa, u, which has a cdf value $F_U(u)$, where U is a standard normal distribution.

Solution : If ($F_U(u)$. GT.0.5) C = 1. – $F_U(u)$

If ($F_U(u)$. LE 0.5) C = $F_U(u)$

V = SQRT (-2. * ALOG (C))

U = V – (2.515517+0.802853*V+0.010328*V**2)

/ (1. + 1.432788*V+0.189269*V**2+0.001308*V**3)

If ($F_U(u)$. LT.0.5) u = -u

Polynomial Approximation to the Gamma Integral

Pequired : Γ (Y) = GAM (Y), Y > 0

Solution : A FORTRAN subroutine for this purpose is given below.

SUBROUTINE GAMMA (Y, GAM (Y))

GAMM (X) = 5.0774936 + X * (-12.896777+x

(18. 476555+x)-15.954062+x*(8.7247756+x* (-2.944098

+x*)0.5625838-x*0.04647091))))))

```
If ( Y. GE.1.) Go To 20
      X = Y + 1.
GAM (Y) = GAMM (X)/Y
GO TO 60

   20 If (Y.GT.2.) Go To 30
      GAM (Y) = GAMM (Y)
      GO TO 60

   30  N = IFIX (Y) -1
       X = Y –FLOAT (N)
       GAM (Y) = GAMM (X)
       DO 40 1 = 1,N
   40  GAM (Y) = GAM (Y) * (X-FLOAT (1))

   60 RETURN
      END
```