

ABSTRACT

The Global Positioning System (GPS) derived-heights have no relationship to the geopotential surface of the Earth's gravity field because it is referenced to a geocentric ellipsoid. There is a need to refer heights to the gravity field, in most cases, for the determination of the fluid flows. To convert GPS-derived ellipsoidal heights (h) to orthometric heights (H), it is necessary to use a geoid model (N).

This study describes the development of gravimetric geoid model referred to WGS84 reference surface for the study area. Three different approaches for geoid determination have been studied. In the first approach; a gravimetric geoid model is constructed using: the observed gravity anomaly, the Global Geopotential Models (GGM) gravity anomaly and the digital elevation models (DEM). In the second approach; the GPS-derived ellipsoidal heights (h) and precise levelling data are used together to obtain the geometric geoid. In the third approach a combination of the gravimetric method and the geometric method is used to establish the hybrid geoid.

A number of data files were compiled for this work, containing more than 25,500 gravity point data. Global geopotential models were used to determine the long wavelength effect of the geoid surface. The GGMs contributions were evaluated with GPS/levelling data to choose the best one to be used in the combined formula.

Gridding algorithms (*Kriging, Inverse Distance Weighting, Nearest Neighbor and Polynomial Regression*) are used to obtain a regular data grid. Corrector surface (CS) for conversion of GPS height (h) to orthometric height (H) is created. EGM96 global geopotential model up to degree and order 360 and EGM08 global geopotential model up to degree and order 360 were tested to choose the best one fitting the study area. EGM96 is chosen as a reference model for the gravimetric geoid. An additive correction is calculated by the summation of four corrections, the Down Ward Continues effect (N_{DWC}), the terrain correction (N_{TC}), the atmospheric correction (N_{ATM}) and the ellipsoidal correction (N_e). The new gravimetric geoid (SDG2011) has been tested using 18 GPS/levelling points. The overall accuracy is **26** cm.

From the available 128 GPS/Levelling data laid in the Northern part of the study area, 113 GPS/Levelling stations out of 128 were used to construct the *geometric geoid* model (N_{geom}) and the remaining points used for checking purpose. The gravimetric and the geometric geoid were combined to obtain the hybrid geoid model. The results show that the hybrid geoid is best of all since it gives standard deviation about **22** cm while the gravimetric and the geometric present standard deviations about **26** cm and **24** cm respectively.

Looking for more precision, the area of study is subdivided into three zones, according to the density of the data, to overcome the ill distribution of the GPS/Levelling data. The standard deviation in the North-West (empty zone) remains the same while the accuracy in the other two zones increased. The North-East zone presents standard deviation about **12** cm and Central zone shows about **19** cm.



الارتفاعات التي يتم الحصول عليها بنظام تحديد المواقع العالمي (GPS) ليست لها علاقة بسطح الجهد لقل جاذبية الأرض لأنها مرجعة إلى مجسم القطع الناقص من مركز الأرض. هناك حاجة لتحويل هذه الارتفاعات إلى قلة الجاذبية الأرضية في أكثر الحالات، لتصميم تدفق السوائل. لتحويل الارتفاع (h) المنسوب للسطح للاهليلجي إلى الارتفاع الاورثومتري (H) المنسوب الى متوسط سطح البحر (MSL) لابد من معرفة سطح الجيويد (N).

تصف هذه الدراسة تطوير نموذج جيويد محسوب من الجاذبية الارضية لمنطقة الدراسة المنسوب لسطح المرجعي WGS84. تم دراسة ثلاث طرق مختلفة لتصميم الجيويد. الطريقة الأولى هي طريقة الجاذبية وفيها يتم الحصول على نموذج الجيويد باستعمال شذوذ الجاذبية المرصودة، النماذج العالمية للجهد (GGM) ونماذج الارتفاع الرقمية (DEM). الطريقة الثانية هي الطريقة الهندسية وفيها يتم استخدام الارتفاع الاهليلجي (h) وبيانات الميزانية الدقيقة (H) للحصول على الجيويد الهندسي. في الطريقة الثالثة يتم فيها دمج الطريقتين الأولى والثانية للحصول على نموذج للجيويد جديد يعرف بالجيويد الهجين.

تم جمع عدد من الملفات والبيانات لهذا العمل، تحتوي على أكثر من 25,500 بيان نقطة جاذبية- نماذج الجهد العالمية (GGM)- بيانات نظام تحديد المواقع العالمي وبيانات الميزانية الدقيقة. تم استخدام الخوارزميات للحصول على شبكة بيانات منتظمة. تم اختيار EGM96 للحصول على جيويد منسوب للجاذبية لمنطقة الدراسة. تم حساب

تصحيح مضاف مكون من أربعة تصحيحات، أسفل الردهة المستمر، تصحيح التضاريس، التصحيح الجوي والتصحيح الاهليلجي. تم الحصول على الحيويد الجديد (SUDG2011) بدقة عامة تساوي 26 سنتيمتر

من البيانات المتوفرة من نظام تحديد المواقع العالمي في الجزء الشمالي لمنطقة الدراسة، تم بناء نموذج جيويد هندسي باستخدام 113 نقطة وتم استخدام عدد 12 نقطة لغرض التدقيق. ثم تم دمج الجيويد الهندسي مع جيويد الجاذبية للحصول على نموذج هجين. تظهر النتائج بان الجيويد الهجين أفضل حيث أنه يعطي إزخراف معياري حوالي 22 سنتيمتر بينما كل من جيويد الجاذبية و الجيويد الهندسي تعطى إزخرافات معيارية حوالي 26 سنتيمتر و 24 سنتيمتر على التوالي.

للبحث عن الدقة الأكثر، تم تقسيم منطقة الدراسة إلى ثلاث مناطق، طبقاً لكثافة البيانات وللتغلب على التوزيع الغير متساوي لبيانات نظام تحديد المواقع العالمي وبيانات الميزانية الدقيقة. بعد عادة حساب الجيويد لكل منطقة على حدا نجد ان الإزخراف المعياري في المنطقة الشمالية الغربية (منطقة فارغة) يبدون تغيير بينما الدقة في المنطقتين الأخرتين زادتاً. تقدم المنطقة الشمالية الشرقية إزخراف معياري حوالي 12 سنتيمتر و المنطقة المركزية حوالي 19 سنتيمتر.

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TABLE OF CONTENTS

Abstract	
.....	i
Acknowledgements	
.....	iv

Table	of	
contents		
.....v		
List	of	
Tables		
..... ix		
List	of	
Figures.....		
..... xi		
List	of	Abbreviations
.....		xiv
List	of	Symbols
.....xvi		
Chapter	One:	Introduction
1		
1.1	Back ground
.....	1	
1.2	Geoid model
.....	3	
1.3	Objectives of the thesis
.....	4	
1.4	Methodology
.....	5	
1.5	Thesis structure
.....	6	
Chapter Two:	Height Systems.....	
.....	8	
2.1	Introduction
.....	8	
2.2	Geopotential numbers.....	
.....	8	
2.3	Gravity	
9		
2.3.1	Gravity	
definition.....	9	

2.3.2 Corrections for gravity.....	11	for
2.4 Height	12	Systems
2.4.1 Dynamic heights.....	12	
2.4.2 Orthometric heights.....	13	
2.4.3 Helmert heights.....	14	orthometric
2.4.4 Neithammer heights.....	15	orthometric
2.4.5 Mader heights.....	16	orthometric
2.4.6 Normal heights.....	17	

Chapter Three: Algorithms and Methodology for Combined Height Adjustment.....
18

3.1 Introduction	18	
3.2 Combined adjustment.....	18	height
3.2.1 Introduction.....	18	
3.1.2 Role of the parametric model.....	18	
3.3 Geoid modeling technique.....	19	
3.3.1 Introduction.....	19	
3.3.2 Factors affecting modelling.....	20	

3.3.3	Interpolation	
methods.....		20
3.3.3.1	Inverse Distance Weighting (IDW).....	
.....		21
3.3.3.2	Geostatistical	
<i>Kriging</i>		23
3.3.3.3	Nearest	
Neighbor.....		23
3.3.3.4	Polynomial Regression.....	
.....		24
3.3.4	The mathematical	
models.....		25
Chapter Four	Gravimetric geoid computation	
	methods and Stokes's	
	integral.....	
	27	
4.1	Introduction.....	
		27
4.2	Vertical datum.....	
.....		28
4.3	Geoid and quasigeoid.....	
.....		29
4.4	Modification of Stokes' formula.....	
.....		31
4.5	Stokes integral and	
kernel.....		34
4.6	Truncation of Stokes's	
formula.....		35
4.7	Global Geopotential Models (GGMs).....	
.....		38
4.8	Selection of geoid modeling	
parameters.....		40

4.9 Gravimetric	geoid	computation
methods.....	40	
4.9.1		
Introduction.....		40
4.9.2 Remove-Compute-Restore		(RCR)
method.....	41	
4.9.2.1 Quasi-geoid by Residual Terrain Model		
(RTM).....	41	
4.9.2.2 Helmert's second condensation		
method.....	42	
4.10 The		additive
correction.....		43
4.10.1 Downward Continuation Effect (N_{dwc})		
.....	43	
4.10.2 The Combined Topographic		
Correction.....	44	
4.10.3 The Atmospheric Correction (N_{ATM})		
.....	45	
4.10.4 The ellipsoidal		
Correction.....	45	
4.11 Geometric geoid		
determination.....		46
4.11.1		
Introduction.....		46
4.11.2 Computation		
method.....	47	
4.12 The Hybrid geoid		
determination.....		48
Chapter	Five	Data
Acquisition.....		49
5.1 Gravity surveys in		
Sudan.....		49
5.2 Gravity		
anomaly.....		50
5.2.1 Normal		
gravity.....		50

5.2.2 Free	air
correction.....	51
5.2.3 Bouguer	
correction.....	53
5.3 Observed	terrestrial
gravity.....	54
5.4 Gravity data validation	and
gridding.....	54
5.4.1 Gravity	
validation.....	54
5.4.2 Gravity	data
gridding.....	56
5.5 Global Geopotential Models(GGM)	
.....	56
5.5.1	
Background.....	56
5.5.2	Types of
GGM.....	58
5.5.2.1 Satellite-only	
GGMs.....	58
5.5.2 .2 Combined	
GGMs.....	59
5.5.2.3 Tailored GGMs.....	
59	
5.6 Satellite Gravity	
Missions.....	60
5.6.1	
CHAMP.....	60
5.6.2	
GRACE.....	60
5.6.3	
GOCE.....	61
5.7 OSU91.....	
62	
5.8 EGM96.....	62
5.9	
EGM2008.....	63

5.10 GPS/Levelling data.....	63
Chapter six: Computations and Results Analysis.....	65
6.1	
Introduction.....	65
6.2 Gravimetric geoid determination.....	66
6.2.1 Gravity data.....	
66	
6.2.1.1 Observed terrestrial gravity.....	66
6.2.1.2 GGM's gravity data.....	67
6.2.2 Gravity data gridding.....	68
6.2.3 Evaluation of GGM's gravity data.....	70
6.2.4 Gravity data validation.....	72
6.2.5 The Digital Elevation Model (DEM).....	72
6.2.6 Evaluation of the integral (Stokes') formula.....	73
6.2.7 External accuracy of the gravimetric geoid model.....	82
6.3 Geometric geoid determination.....	89
6.3.1	
Introduction.....	89
6.3.2 GPS/Levelling data.....	89
6.4 The Hybrid geoid.....	97
6.5 Geoid by portioning the study area.....	100

6.5.1	Introduction.....	10
6.5.2	The central zone.....	100
6.5.3	The North-East zone.....	100
6.5.4	The North-West zone.....	101
Chapter Seven	Conclusion and Recommendations.....	105
7.1	Conclusion.....	105
7.2	Recommendations.....	109

LIST OF TABLES

Table (4.1):	Global geopotential models that are presented in this research (S: Satellite, G: Gravity, A: Altimetry, Tide-free: all tidal effects have been removed)	39
Table (5.1):	Physical and geometrical constants of GRS80 used in gravity reductions.....	52
Table (6.1):	Statistics of the four gridding methods applied to the observed gravity anomalies for Sudan, (Unit: mGal)	69
Table (6.2):	Statistics of the differences between the three GGM's gravity anomalies extracted from (EGM96), (OSU91a) and (GGP_Japan) and the observed gravity anomaly (dg_Obs) in Sudan, (Units: mGal).....	71
Table (6.3):	The coefficients of 10-Parameter model fitting the additive correction (N_{AC}) for the new geoid.	78

Table (6.4): The GPS/levelling data: Latitude, Longitude and derived geoid height used as external measure of the geoid accuracy.....83

Table (6.5): Differences between GPS/Levelling and the EGM96 geoid model.....84

Table (6.6): Residuals between GPS/Levelling and the new gravimetric geoid (N_{grv}) before fitting.....85

Table (6.7): Statistical analysis of absolute accuracy of new gravimetric geoid using 18 GPS/levelling stations.....86

Table (6.8): Residuals between GPS/Levelling and the new gravimetric geoid (SUDG2011) after 10_Parameter fitting.86

Table (6.9): Residuals between GPS/Levelling and the geoid (N_{96}), and gravimetric geoid (N_{grv}) after adding the correction.....87

Table (6.10): Statistical analysis of the accuracy of EGM96 and the new gravimetric geoid (N_{grAF}) compared to GPS/leveling model (N_{gps})..... 88

Table (6.11): Statistical analysis of the residuals derived from GPS/levelling data and GGM's.....91

Table (6.12): Statistical analysis of absolute accuracy of North Sudan geoid derived from GPS/levelling data using three different modelling.....92

Table (6.13): Latitude, Longitude in WGS84, GPS/levelling geoid height (N_{gps}) and EGM96 geoid (N_{96})93

Table (6.14): The coefficients used for interpolating the geometric correction ($N_{GPS/Levelling} - N_{96}$).94

Table (6.15): Differences between GPS/Levelling and the two geoid (N_{96}) and geometric geoid (N_{geom})94

Tabel (6.16): Latitude, Longitude in WGS84, GPS/levelling geoid height (N_{gps}) and EGM96 geoid (N_{96}) and the geometrical geoid (N_{geom}).....96

Tabel (6.17): Latitude, Longitude in WGS84, GPS/levelling geoid height (N_{gps}) and the difference ($N_{gps} - N_{geom}$)96

Table (6.18): Residuals between GPS/Levelling geoid and the three derived geoid for North Sudan:97

Table (6.19): Statistical analysis of absolute accuracy of the three derived geoid for North Sudan: The gravimetric, the geometric and the Hybrid geoid..... 99

Tabel (6.20): Latitude, Longitude in WGS84, GPS/levelling geoid height (N_{gps}) and EGM96 geoid (N_{96}) and the gravimetric geoid (N_{gr}) for the North-East part of Sudan.....103

Tabel (6.21): Latitude, Longitude in WGS84, GPS/levelling geoid height (N_{gps}) and EGM96 geoid (N_{96}) and the gravimetric geoid (N_{gr}) for the Central part of Sudan.....103

Table (6.22): Statistical analysis of absolute accuracy of the three derived geoid model after portioning.....104

LIST OF FIGURES

Figure (2.1): The orthometric height (H_{ortho}) of point P (adapted from Featherstone and Kuhn, 2006)

Figure (3.1) Interpolation point k of which geoid undulation value is going to be computed and neighbor reference points that contributes the computation of geoid undulation at point k

Figure (4.1): The normal height (H^N), height anomaly (ζ), ellipsoid (h), orthometric (H), and geoid heights (N)

Figure (4.2): The spherical cap σ_c with radius σ_0
Figure (5.1): Distribution of the Sudan gravity anomaly data (BGI) in mGal.....
Figure (5.2): The GPS satellites and the two GRACE satellites.....
Figure (5.3): Distribution of 128 GPS/Levelling Stations in the area of study.....
Figure (6.1): Contour map for Sudan observed bouguer gravity anomaly .Contour interval 10 mGal.....67
Figure (6.2): Gravity anomaly extracted from global geopotential models EGM96, OSU91a and Japan Global Gravity Project GGP.....68
Figure (6.3): Comparison Between Observed gravity anomalies (dg_Obs) ,and GGM gravity anomalies in Hegleee area.....71
Figure (6.4): Grid lines with equi-angular blocks 5' x 5'74
Figure (6.5): DTM for Sudan extracted from GGM's OSU91a, BGI and EGM96.....75
Figure (6.6): Downward continued correction (N_{dwc})76
Figure (6.7): Combined topographic corrections (N_{TC})76
Figure (6.8): Combined atmospheric correction(N_{ATM})77
Figure (6.9): Ellipsoidal correction (N_e).....77
Figure (6.10): The additive correction (N_{AC})78
Figure(6.11): The new gravimetric geoid (SUDG2011)79
Figure (6.12): Sudan geoid extracted from the global geoid models and new gravimetric geoid81

Figure (6.13): The gradian of the new gravimetric geoid in the direction from the South-East corner to the North-West.	81
Figure (6.14): The gradian of the new gravimetric geoid in the direction from the South-West corner to the North-East.	82
Figure (6.15): Distribution of 18 GPS/Levelling stations	83
Figure (6.16): Visual comparison between the differences before fitting and after fitting using the three models:	88
Figure (6.17): Differences between GPS/Levelling and EGM96 (N_{96}), and the new gravimetric geoid (N_{grAF}).	88
Figure (6.18): Comparison between the GPS/Levelling and (EGM96) in (a) and the additive correction in (b)	90
Figure (6.19): Combination of the two models in Figure (6.18)	91
Figure (6.20): Distribution of 15 GPS/Levelling stations	93
Figure (6.21): GPS/Levelling minus EGM96 ($N_{gps}-N_{96}$) and GPS/Levelling minus EGM2008 ($N_{gps}-N_{g08}$) interpolated using	95
Figure (6.22): GPS/Levelling minus EGM96 ($N_{gps}-N_{96}$) and GPS/Levelling minus geometric geoid ($N_{gps}-N_{geom}$)	95
Figure (6.23) : GPS/Levelling minus EGM96 ($N_{gps}-N_{96}$) compared to the GPS/Levelling minus gravimetric geoid	98
Figure (6.24): GPS/Levelling minus EGM96 ($N_{gps}-N_{96}$) and GPS/Levelling minus Hybrid geoid ($N_{gps}-N_{Hybrid}$).....	99
Figure (6.25): Differences between GPS/Levelling and three derived models.....	99
Figure (6.26): Distribution of GPS/Levelling in three zones.....	101

Figure (6.27): Distribution of 12 test points in the Central part of study area.....102

Figure (6.28): Distribution of 12 test points in the Central part of study area.....102

LIST OF ABBREVIATIONS

KTH	Kungliga Tekniska högskolan
CHAMP	Challenging Mini-satellite Payload
CI	Confidence interval
DC	Downward continuation
DEM	Digital elevation model
DOT	Dynamic ocean topography
DTE	Direct topographical effect
EDS	Earth deformation study
EGM96	Earth Gravity Model 1996
ESA	European Space Agency
FFT	Fast Fourier transformation

GBVP	Geodetic boundary-value problem
GEODAS	Geophysical data system
GGM	Global geopotential model
GNSS	Global Navigation Satellite System
GOCE	Gravity field and steady state Ocean Circulation Explorer
GPS	Global Positioning System
GPU	Geopotential units
GRACE	Gravity Recovery and Climate Experiment
GRAS	Geological Research Authority of Sudan
GRS80	Geodetic Reference System 1980
HG	Heck and Grüninger (1983) kernel modification
IAG	International Association of Geodesy
IGSN71	International Gravity Standardization Network 1971
ITRF96	International Terrestrial Reference Frame 1996
LSC	Least-squares collocation
LVD	Local vertical datum
ML	Meissl (1971) kernel modification
MSL	Mean Sea Level
MSST	Mean dynamic Sea Surface Topography,
NAVD88	North American Vertical Datum 1988
NOAA	National Oceanic and Atmospheric Administration

NOC	Normal-orthometric correction
PITE	Primary indirect topographical effect
RB	Refined Bouguer gravity anomaly
RCR	Remove-compute-restore
RMS	Root mean square
RTK	Real-time kinematic
SB	Simple Bouguer gravity anomaly
IS	International System
SITE	Secondary indirect topographical effect
STD	Standard deviation
TC	Terrain correction
UNB	University of New Brunswick
WG	Wong and Gore (1969) kernel modification
WGS72	World Geodetic System 1972
WGS84	World Geodetic System 1984

LIST OF SYMBOLS

a	Semi-major axis of the reference ellipsoid
dn	Height difference
$d\psi, d\alpha$	Surface spherical coordinates
$d\sigma$	Surface area element on the sphere
$d\gamma/dh$	Vertical free-air gradient
	Squared eccentricity of reference ellipsoid
f	Flattening of the reference ellipsoid

f^*	Gravity flattening constant
g	Gravity vector along the plumb line Constant gravity Gravity observation in the mean Earth-tide model Average surface gravity Surface gravity observation Helmert approximation of integral mean gravity along Plumb line
$g_{\text{Neithammer}}$ along	Neithammer approximation of integral mean gravity Plumb line
g_{Mader}	Mader approximation of integral mean gravity along plumb line
h	Ellipsoidal height
(r, ϕ, λ)	Geocentric polar coordinates
k	Normal gravity constant
l	Planar distance between two points
m	Order of spherical harmonics
n	Degree of spherical harmonics
r	Geocentric radius of the Earth
C	Geopotential number
G	Universal gravitational constant
GM	Product of Newtonian gravitational constant and mass of Earth Dynamic height Orthometric height Normal height Normal-orthometric height
K	Stokes error kernel function
L	Degree of Molodensky-type kernel modification

M	Degree of GGM expansion Maximum complete degree of harmonic expansion
N	Geoid height Geoid undulation from GGM
P	Degree of spheroidal kernel modification Position of P projected onto the geoid Legendre polynomial Associated Legendre function Truncation error coefficients
R	Mean radius of the Earth
S	Spherical Stokes's kernel function
U	Normal potential Normal potential on telluroid at point Q Global geopotential Geopotential at point P
α	Azimuth
δ	Atmospheric correction Planar Bouguer correction Free-air correction Terrain effect at topographic surface Terrain effect at geoid
σ	Fully normalised spherical harmonic coefficients
δ_ϕ	Difference in latitude
η	East-west vertical deflection
γ	Normal gravity of reference ellipsoid Normal gravity at 45°N/S Normal gravity along the ellipsoidal normal Normal gravity at equator
λ	Geodetic longitude
λ'	Longitude of variable surface element

ϕ'	Latitude of variable surface element $d\sigma$
ϕ	Geodetic latitude Average latitude
ψ	Angular distance between two points Angular integration cap radius
ρ	Topographic mass density
σ	Sphere of integration
σ_{average}	Weighted average standard deviation
ξ	North-south vertical deflection
ζ	Quasigeoid height/height anomaly
ΔC	Geopotential difference
Δg	Gravity anomalies reduced to the quasigeoid Simple Bouguer anomaly Free air gravity anomaly