ABSTRACT

The Global Positioning System (GPS) derived-heights have no relationship to the geopotential surface of the Earth's gravity field because it is referenced to a geocentric ellipsoid. There is a need to refer heights to the gravity field, in most cases, for the determination of the fluid flows. To convert GPS-derived ellipsoidal heights (h) to orthometric heights (H), it is necessary to use a geoid model (N).

This study describes the development of gravimetric geoid model referred to WGS84 reference surface for the study area. Three different approaches for geoid determination have been studied. In the first approach; a gravimetric geoid model is constructed using: the observed gravity anomaly, the Global Geopotential Models (GGM) gravity anomaly and the digital elevation models (DEM). In the second approach; the GPS-derived ellipsoidal heights (h) and precise levelling data are used together to obtain the geometric geoid. In the third approach a combination of the gravimetric method and the geometric method is used to establish the hybrid geoid.

A number of data files were compiled for this work, containing more than 25,500 gravity point data. Global geopotential models were used to determine the long wavelength effect of the geoid surface. The GGMs contributions were evaluated with GPS/levelling data to choose the best one to be used in the combined formula.

Gridding algorithms (*Kriging, Inverse Distance Weighting, Nearest Neighbor and Polynomial Regression*) are used to obtain a regular data grid. Corrector surface (CS) for conversion of GPS height (h) to orthometric height (H) is created. EGM96 global geopotential model up to degree and order 360 and EGM08 global geopotential model up to degree and order 360 were tested to choose the best one fitting the study area. EGM96 is chosen as a reference model for the gravimetric geoid. An additive correction is calculated by the summation of four corrections, the Down Ward Continues effect (N_{DWC}), the terrain correction (N_{TC}), the atmospheric correction (N_{ATM}) and the ellipsoidal correction (N_e). The new gravimetric geoid (SDG2011) has been tested usig 18 GPS/levelling points. The overall accuracy is **26** cm.

From the available 128 GPS/Levelling data laid in the Northern part of the study area, 113 GPS/Levelling stations out of 128 were used to construct the *geometric geoid* model (N_{geom}) and the remaining points used for checking purpose. The gravimetric and the geometric geoid were combined to obtain the hybrid geoid model. The results show that the hybrid geoid is best of all since it gives standard deviation about **22** cm while the gravimetric and the geometric present standard deviations about **26** cm and **24** cm respectively.

Looking for more precision, the area of study is subdivided into three zones, according to the density of the data, to overcome the ill distribution of the GPS/Levelling data. The standard deviation in the North-West (empty zone) remains the same while the accuracy in the other two zones increased. The North-East zone presents standard deviation about **12** cm and Central zone shows about **19** cm.

□□□□□□

الارتفاعات التي يتم الحصول عليها بنظام تحديد المواقع العالمي (GPS) لَيْسَت لَها علاقةُ بسطحِ الجهد لحقلِ جاذبية الأرضَ لأنها مُرجعةُ إلى مجسم القطع الناقص من مركز الأرض. هناك حاجة لتحويلِ هذه الارتفعات إلى حقلِ الجاذبيةَ الارضية في أكثر الحالات، لتصميمِ تدفقِ السوائلِ. لتحويل الارتفاع (h) المنسوبِ للسطح للاهليليجي إلى الارتفاعَ الاورثومترى (H) المنسوب الىمتوسط سطح البحر(MSL) لابد من معرفة سطح الجيويد(N).

تَصفُ هذه الدراسةِ تطويرَ نموذجِ جيويد محسوب من الجاذبية الارضية لمنطقة الدراسة المنَسوب للسطحَ المرجعي WGS84. تم دراسة ثلاث طرق مختلفة لتصميم الجيويد. الطريقة الأولى هي طريقة الجاذبية وفيها يتم الحصول على نموذج الجيويد بإستعمال شذوذ الجاذبية المرصودة، النماذج العالمية للجهد (GGM) ونماذج الإرتفاع الرقمية (DEM). الطريقَة الثانيةهى الطريقة الهندسية وفيها يتم استخدام الارتفاع الاهليلجى (h) وبيانات الميزانيةَ الدقيَقة H)) للحُصُول على الجيويد الهندسي. في الطريقة الثالثة يتم فيها دمج الطريقتين الاولى والثانية للحصول على نموذج للجيويد جديد يعرف بالجيويد الهجين.

تم جمع عدد مِنْ الملفات والبيانات لهذا العملِ، تحتوي على أكثر مِنْ 25,500 بيان نقطة جاذبيةَ- نماذجَ الجهد العالَمية (GGM)- بياناتِ نظام تحديد المواقع العالمي وبياناتَ الميزانيَة الدقيقة. تم استخدام الخوارزميات للحُصُول على شبكة بيانات منتظمة. تم اختيار EGM96 للحصول على جيويد منسوب للجاذبية لمنطقة الدراسةَ. تم حسَاب

تصحيح مضاف مكون من أربعة تصحيحات، أسفل الردهة المستمر، تصحيح التضاريس، التصحيح الجوي والتصحيح الاهليليجى. تم الحصول على الجيويد الجديد (SUDG2011) بدقة عامة تساوى 26 سنتميتر

من البيانات المتوفره من نظام تحديد المواقع العالمي في الجزء الشمالي لمنطقة الدراسة، تم بناء نموذج جيويد هندسي باستخدام 113 نقطة وتم استخدام عدد 12 نقطة لغرض التدقيق. ثم تم دمج الجيويد الهندسي مع جيويد الجاذبية للحصول على نموذج هجين. تظهر النتائج بأن الجيويد الهجين أفضل حيث أنه يعطي إنحراف معياري حوالي 22 سنتيمتر بينما كل من جيويد الجاذبية و الجيويد الهندسي تعطى إنحرافات معيارية حوالي 26 سنتيمتر و 24 سنتيمتر على التوالي.

للبحث عن الدقة الأكثر، تم تقسيم منطقة الدراسة إلى ثلاث مناطق، طبقاً لكثافة البيانات وللتغلب على التوزيع الغيرمتساوى لبيانات نظام تحديد المواقع العالمي وبيانات الميزانية الدقيقة. بعد اعادة حساب الجيويد لكل منطقة على حدا نجد ان الإنحراف المعياري في المنطقة الشمالية الغربية (منطقة فارغة) يبقى بدون تغيير بينما الدقة في المنطقتين الأخريتين زادتا. تُقدمُ المنطقةُ الشمالية الشرقيةُ إنحراف معياري حوالي 12 سنتيمتر و المنطقة المركزية حوالي 19 سنتيمتر.

ACKNOWLEDGEMENT

Firstly I am deeply indebted to my supervisor Dr. Khlafallah M. Badi for his invaluable intellectual guidance, professional discussions and advices, patience, flexibility and availability lead to the successful end of this study. I feel honored to have him as my supervisor. I am also very grateful to Dr. Ali H. Fagir and Dr. Mohammed Khalid for owning me the first knowledge in geodesy and physical geodesy.

I am deeply indebted to Mr. Abdalla Ahmed, who has been very helpful with many records and references. I sincerely thank all my former and present staff members and graduate students at the Survey Department, Division of Geodesy for the discussions we had and many other issues, which cannot be listed here all.

I am indebted to individuals and institutions which provided me either with data, computer software or information, especially *Bureau Gravimétrique International* (BGI) for providing 12,306 gravity observations , Feisal Mustafa , Potrodar company Lt. , for his assistance with gravity data in Western Sudan and Dam Implementation Unit (DIU) for providing GPS/levelling observations in Merowe in 2007. I sincerely thank my family and relatives for the inspirational moral support and encouragement, especially my dear wife and my children.

I greatly appreciate the support of my employer, Sudan University of Science and Technology, who provided me with the opportunity to undertake this research. Specific credit needs to be given to Dr. Nagy Zomrawe who initially persuaded me to start this project and gave me excellent suggestions.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF ABBREVIATIONS

KTH	Kungliga Tekniska högskolan
CHAMP	Challenging Mini-satellite Payload
CI	Confidence interval
DC	Downward continuation
DEM	Digital elevation model
DOT	Dynamic ocean topography
DTE	Direct topographical effect
EDS	Earth deformation study
EGM96	Earth Gravity Model 1996
ESA	European Space Agency
FFT	Fast Fourier transformation

GBVP	Geodetic boundary-value problem
GEODAS	Geophysical data system
GGM	Global geopotential model
GNSS	Global Navigation Satellite System
GOCE	Gravity field and steady state Ocean Circulation Explorer
GPS	Global Positioning System
GPU	Geopotential units
GRACE	Gravity Recovery and Climate Experiment
GRAS	Geological Research Authority of Sudan
GRS80	Geodetic Reference System 1980
HG	Heck and Grüninger (1983) kernel modification
IAG	International Association of Geodesy
IGSN71	International Gravity Standardization Network 1971
ITRF96	International Terrestrial Reference Frame 1996
LSC	Least-squares collocation
LVD	Local vertical datum
ML	Meissl (1971) kernel modification
MSL	Mean Sea Level
MSST	Mean dynamic Sea Surface Topography,
NAVD88	North American Vertical Datum 1988
NOAA	National Oceanic and Atmospheric Administration

NOC	Normal-orthometric correction
PITE	Primary indirect topographical effect
RB	Refined Bouguer gravity anomaly
RCR	Remove-compute-restore
RMS	Root mean square
RTK	Real-time kinematic
SB	Simple Bouguer gravity anomaly
IS	International System
SITE	Secondary indirect topographical effect
STD	Standard deviation
TC	Terrain correction
UNB	University of New Brunswick
WG	Wong and Gore (1969) kernel modification
WGS72	World Geodetic System 1972
WGS84	World Geodetic System 1984

LIST OF SYMBOLS

a	Semi-major axis of the reference ellipsoid
dn	Height difference
$d\psi$, $d\alpha$	Surface spherical coordinates
$d\sigma$	Surface area element on the sphere
$d\gamma/dh$	Vertical free-air gradient
	Squared eccentricity of reference ellipsoid
f	Flattening of the reference ellipsoid

f^*	Gravity flattening constant
g	Gravity vector along the plumb line
	Constant gravity
	Gravity observation in the mean Earth-tide model
	Average surface gravity
	Surface gravity observation
	Helmert approximation of integral mean gravity along Plump line
$g_{Neithammer}$ along	Neithammer approximation of integral mean gravity Plumb line
g_{Mader}	Mader approximation of integral mean gravity along plumb line
h	Ellipsoidal height
(r, ϕ, λ)	Geocentric polar coordinates
k	Normal gravity constant
l	Planar distance between two points
m	Order of spherical harmonics
n	Degree of spherical harmonics
r	Geocentric radius of the Earth
C	Geopotential number
G	Universal gravitational constant
GM	Product of Newtonian gravitational constant and mass of Earth
	Dynamic height
	Orthometric height
	Normal height
	Normal-orthometric height
K	Stokes error kernel function
L	Degree of Molodensky-type kernel modification

M	Degree of GGM expansion
	Maximum complete degree of harmonic expansion
N	Geoid height
	Geoid undulation from GGM
P	Degree of spheroidal kernel modification
	Position of P projected onto the geoid
	Legendre polynomial
	Associated Legendre function
	Truncation error coefficients
R	Mean radius of the Earth
S	Spherical Stokes’s kernel function
U	Normal potential
	Normal potential on telluroid at point Q
	Global geopotential
	Geopotential at point P
α	Azimuth
δ	Atmospheric correction
	Planar Bouguer correction
	Free-air correction
	Terrain effect at topographic surface
	Terrain effect at geoid
,	Fully normalised spherical harmonic coefficients
δ_{ϕ}	Difference in latitude
η	East-west vertical deflection
γ	Normal gravity of reference ellipsoid
	Normal gravity at 45ºN/S
	Normal gravity along the ellipsoidal normal
	Normal gravity at equator
λ	Geodetic longitude
λ'	Longitude of variable surface element

ϕ'	Latitude of variable surface element $d\sigma$
ϕ	Geodetic latitude
	Average latitude
ψ	Angular distance between two points
	Angular integration cap radius
ρ	Topographic mass density
σ	Sphere of integration
$\sigma_{average}$	Weighted average standard deviation
ξ	North-south vertical deflection
ζ	Quasigeoid height/height anomaly
ΔC	Geopotential difference
Δg	Gravity anomalies reduced to the quasigeoid
	Simple Bouguer anomaly
	Free air gravity anomaly