Sudan University of Science and Technology
College of Graduate Studies

Symmetries of Differential Equations and Some Applications

تماثلات المعادلات التفاضليه وبعض التطبيقات

Thesis Submitted in Partial Fulfillment for the Degree of M.Sc. in Mathematics
By :
Shiraz Altom Ahmed Elhussin

Supervisor:
Dr : Emad Eldeen Abdallah Abdel Rahim

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Dedication

To my Mother, Father, Brothers and Sisters

Acknowledgments

MY grate fully thanked to:

Dr : Emad Aldeen Abdallah Abdelrahim

To my teachers

Abstract

We consider one of the ingenious techniques for obtaining

exact solutions of Differential equations, which exploit

symmetries of differential equations. For this we introduce

the concept of point symmetries of differential. Also we use

point symmetries to solve some ordinary differential equations

by finding a suitable transformation of independent and

dependent variables. And then we briefly indicate some

applications.

الخلاصه

اعتبرنا واحده من التقنيات المبتكره للحصول على الحلول التامة للمعادلات التفاضليه, التي تستغل تماثلات المعادلات التفاضليه.لهذا قدمنا مفهوم نقطة تماثلات المعادلات التفاضليه. أيضاً استخدمنا نقطة التماثلات لحل بعض المعادلات التفاضليه الاعتيادية بايجاد تحويل مناسب من المتغيرات المستقلة والغير مستقلة "تابعة" وبعد ذلك أشرنا بايجاز لبعض التطبيقات.

The Contents

Chapter (1)

Lie Symmetries of First-Order ODEs

Chapter (2)

Lie Point Symmetries Of ODEs and One-Parameter Lie Group

Chapter (3)

Lie Point Symmetries and Obtaining Exact Solutions of PDEs

Chapter (4)

Lie algebra and Symmetries of Differential Equations

INTRODUCTION

One of the biggest challenges to mathematics today is the symmetry, which is the key to solving differential equations. There are many well known techniques for obtaining exact solutions, but most of them are merely special cases of a few powerful symmetry methods .These methods can be applied to differential equations of an unfamiliar type. This Dissertation contains several new methods that will interest those whose research involves symmetries. In particular, methods for obtaining discrete symmetries and first integrals, and it is organized as follows :

Firstly we begin with a brief introduction to Lie symmetry condition, and we introduce the techniques to find symmetry of a complicated first-order ODEs and PDEs, and then we discuss the action of Lie symmetry on the plane and how to solve ODEs with Lie symmetries. Also we study the linearized symmetry condition, and canonical coordinates which are associated with a particular Lie group. Also we study the standard methods of symmetries and infinitesimal generator with some application

In chapter 2 we illustrate the symmetry condition and examining some of its consequen As applications of this notion, we present the Determining equations for Lie point symmetries, inparticular we discuss the Linear ODEs .Also we discuss the justification of the symmetry condition. Also we present the important notion of Reduction of order by using canonical coordinates. Also the technique of variational symmetries to derive ODEs from a variational principle is described ,and then we discuss the variant and invariant solutions.

In chapter 3 we investigate the techniques of the group-invariant solutions of PDEs and as example of it we present a methods of finding a

new solutions from known ones for classical and non classical symmetries, and as an application we discuss the Huxley equation. Finally we discuss the Lie group and its Lie algebra and the action of Lie group .

Also we illustrate the symmetries of algebraic equations and differential equations with some examples, and as applications of these we present the theorem of the infinitesimal generators of symmetries and two theorems of the generators of symmetries of ODEs and PDE_s . We then discuss as an application the Reduction of the order of a given ODEs and others.