

# Dedication

To My Family

# Acknowledgments

First of all I would like to thank my supervisor Associate Professor Dr. Shawgy Hussein Abdallah Head Dept. of Math, for his continuous and guidance to carry out this research.

Special thanks are directed to the teaching staff members of math Dept.- faculty of science – SUST:-

Associate professor Dr. Bakri Mirghani Ahmed, Associate professor Mohammed Arafa and Assistant professor Dr. Mansour ElShiekh Hassan for their valuable suggestions and support.

# Abstract

We show the classes of Banach spaces that are finitely, strongly finitely or elementary equivalent. On sets of these classes topological properties are defined in such way that sets of defined classes become compact totally disconnected topological spaces.

Result are used in the problem of synthesis of Banach spaces, and to describe omittable spaces that have been defined.

We considered two problems on extension of operators whose range spaces for the first problem (or domain spaces for the second one) belongs to the fixed class of finite equivalence, which is generated by a given Banach space  $X$ .

Both problems are considered in two variables: isometric and isomorphic ones. The first problem was solved completely, while a solution was obtained for the second problem which is close to the exact solution.

## الخلاصة

أوضحنا عائلات فضاءات باناخ التي هي منتهية ، منتهية بقوة، أو متكافئة أولياً، على فئات من مثل هذه العائلات الطوبولوجيا تعرف على نفس الطريقة. إن الفئات التي لها عائلات معرفة تصبح فضاءات طوبولوجية غير متصلة كلياً متراسة. استخدمت النتائج في مسألة تركيب فضاءات باناخ و تم وصف الفضاءات المحذوفة التي تم تعريفها. نعتبر مسألتين على تمديد المؤثرات التي لها مدى لأجل المسألة الأولى أو فضاء المجال لأجل الثانية ، ينتمى إلى عائلة ثابتة متكافئة منتهية ، والتي تولد بفضاء باناخ المعطي  $X$  . المسألتين معاً تعتبران في متغيرين: أيزومترى وايزومورفى لكل. تم الوصول لحل كامل للمسألة الأولى وحل قريب للنهاية للمسألة الثانية.

# Contents

Dedication	I
Acknowledgments	II
Abstract	III
الخلاصة	IV
Contents	V
<b>Chapter 1 : Basic Results</b>	1
<b>Chapter 2 : Topological and Continuality of sets of classes of Banach space</b>	
<b>Section (2.1):</b>	
Classification of Banach spaces by dimension and Elementary equivalence	17
<b>Section (2.2):</b>	
Classification of Banach spaces by strong finite equivalence	37
<b>Chapter 3 : Operators in classes of finitely equivalent Banach spaces</b>	
<b>Section (3.1):</b>	
Algebraically closed and Universally complement Banach spaces	42
<b>Section (3.2):</b>	
Divisible classes and Quotient closed classes of finite equivalence	50
<b>Chapter 4 : Extension of operators on separable and non-separable Hilbert spaces</b>	
<b>Section (4.1):</b>	
Bounded extension of operators separable case	61
<b>Section (4.2):</b>	
Bounded extension of operators non-separable case	69
Symbols	78
References	80





# Chapter 4



TOPOLOGICAL PROPERTIES AND EXTENSION OF  
OPERATORS IN CLASSES OF FINITELY  
EQUIVALENT BANACH SPACES

By

Samar Samir Bayoumi Elshami

A thesis submitted in partial fulfillment for the  
degree of M. Sc. in mathematics

Sudan University of Science & Technology  
College of Science

2004

