

DEDICATION

I dedicate this thesis with love and respect to

- My father and mother.....
- My brothers and sisters....
- My teachers
- My relatives and friends

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First of all fore most, I should offer my thanks, Obedience and gratitude to Allah the greatest from whom I receive guidance and help.

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Abstract

Working with general relativity particularly with Einstein's field equation, requires some understanding of differential geometry. In this thesis we develop the essential mathematics needed to describe physics in curved space time.

We related the geometrical concept of curvature to the physical concept of energy – momentum tensor, which leads to Einstein's equation of gravitation field.

الخلاصه

التعامل مع النسبيه العامه , خاصه معادله حقل انشتاين يتطلب بعض مفاهيم الهندسه التفاضيله . في هذا البحث تم تطوير الأساسيات الرياضيه المطلوبه لوصف الفيزياء في منحني الزمان والمكان .
ربطنا مفاهيم الانحناء الهندسيه بالمفاهيم الفيزيائيه – ممتده طاقه كميه الحركه
هذا يقود الي معادله انشتاين لحقل الجاذبيه .

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