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Using Generalized General Relativity to Study the Emission of Gravity Waves and Generation of Elementary Particles by Black Hole and Astronomical Objects

استخدام النظرية النسبية العامة المعممة لدراسة اشعاع موجات الجاذبية وتوليد الجسيمات الأولية في الثقوب السوداء والأجسام الفلكية

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الاية

قال تعالى :

وَالشَّمْسُ تَجْرِي لِمُسْتَقَرٍّ لَهَا ذَلِكَ تَقْدِيرُ الْعَزِيزِ الْعَلِيمِ ﴿ 38 ﴾ وَالْقَمَرَ قَدَّرْنَاهُ مَنَازِلَ حَتَّى عَادَ كَالْعُرْجُونِ الْقَدِيمِ ﴿ 39 ﴾ لَا الشَّمْسُ يَنبَغِي لَهَا أَن تُدْرِكَ الْقَمَرَ وَلَا اللَّيْلُ سَابِقُ النَّهَارِ وَكُلٌّ فِي فَلَكٍ يَسْبَحُونَ ﴿ 40 ﴾ . صدق الله العظيم

سورة يس الايات (38 – 40)

Dedication

To my father who had implemented the love of knowledge in my mind,

To my mother who had successfully implemented the methods of discovering in my life,

To all my brothers and sisters waiting for graduation,

To all my friends who were supply me with moral help and support.

Acknowledgment

My great render to Sudan University of science and Technology college of graduate study which give me a chance to complete my higher studies

Also I wish to express sincere gratitude to my supervisor, Prof. Mubarak Dirar Abdalla, for his abundantly helpful and offered invaluable assistance, support and guidance.

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I wish to express my love and gratitude to my beloved family; for their understanding & endless love, through the duration of this study.

Abstract

In this study the generalized gravitational field equation has been adopted to see how gravitational waves can be generated. First one assumes spherically symmetric body solution. At the beginning of the work a useful expression for the time metric has been also obtained in terms of the potential for all fields including strong fields when the body is spherically symmetric. In this case standing, time oscillating, and radially decaying gravitational waves can be generated. However, if the black hole a quires a mass that exceeds a certain critical value a travelling gravitational waves can be emitted with wave length shorter than a certain critical wave length.

The singularity black hole problem and the infinite self-mass of elementary particles are one of the long-standing problems in physics. Using the expression of energy in a potential dependent special relativity an advanced model has been constructed. According to this model the energy at which both radius and mass give minimum energy, determine the self-mass and minimum radius. The self-mass was found to be finite, dependent on vacuum energy and coupling constant. The radius was found to be dependent on the short-range field coupling constant.

المستخلص

في هذه الدراسة تم استخدام معادلة مجال الجاذبية المعممة لمعرفة كيف يمكن توليد موجات الجاذبية. أولاُ يفترض حل الجسم المتماثل كرويًا. في بداية العمل وتم الحصول على تعبير مفيد لمقياس الوقت من حيث الإمكانات لجميع المجالات بما في ذلك الحقول القوية عندما يكون الجسم متماثلًا كرويًا. في هذه الحالة يمكن إنشاء موجة جاذبية ثابتة تتأرجح مع الوقت وتتحلل شعاعيًا، ومع ذلك إذا كان الثقب الأسود يتطلب كتلة تتجاوز قيمة حرجة معينة فيمكن أن تنبعث موجات جاذبية متنقلة بطول موجي أقصر من الطول الموجي الحرج المعين.

تعد مشكلة الثقب الأسود المتفرّد والكتلة الذاتية اللانهائية للجسيمات الأولية إحدى المشاكل طويلة الأمد في الفيزياء. باستخدام التعبير عن الطاقة في النسبية الخاصة المعتمدة على الإمكانات تم بناء نموذج متقدم وفقًا لهذا النموذج الطاقة التي يعطي عندها نصف القطر والكتلة الحد الأدنى من الطاقة حدد الكتلة الذاتية ونصف القطر الأدنى، وُجد أن الكتلة الذاتية محدودة وتعتمد على طاقة الفراغ وثابت الاقتران، ووجد أن نصف القطر يعتمد على ثابت اقتران المجال قصير المدى.

Content

CHAPTER ONE

Introduction

1.1 Preface

The relativity theory is one of the biggest achievements in physics. It changes radically our thinking about the nature of the space and time.

Albert Einstein proposed the theory of special relativity in 1905, deriving many theoretical results and empirical finding obtained by Albert, A. Michelson, Hendrik Lorentz, Henri point care and other, Maxplanck, Herman Minkowaski and others. Einstein developed general relativity (GR) between 1907 and 1915 so as to describe the gravitational field and the behavior of the universe with contributions made by others, general relativity was formulated in 1916 [1].

The theory of gravitation proposed by Newton was developed by Einstein. His theory is known as general relativity (GR). The development of general relativity began with the equivalence principal which shows that gravitational motion is the freely falling particle equivalent to the motion of accelerated particle in free space. For an observer freely falling, object in free fall is falling because there is no force being exerted on them [2, 3].

The theory of GR have many predictions, It shows that o'clock run slower in deeper gravitational wells compared to the one at the earth surface. This is called gravitational time dilation. It also predicts that orbits process in a way un expected in Newton's theory of gravity. The space deformation is caused by gravity since rays of light bend in the presence of gravitational field. A cording to GR rotating. Masses “drag along” the space time ground them a phenomenon termed “frame dragging”.

The Big Bang model shows that the universe is expanding and the far parts of it are moving away from us faster than the speed of light [2, 3]. In Einstein's general theory of relativity, gravity is treated as phenomenon resulting from the curvature of space time. This curvature is caused by the presence of mass. Generally, the move mass that is contained within a given volume of space. The greater the curvature of space time will be at the boundary of it's volume. As objects with mass move a round in space time, the curvature changes to reflect the changed locations of these objects. In certain circumstances accelerating objects generate changes in this curvature which propagate out words at the speed of light in a wavelike manner. These propagating were known as gravitational waves [4].

Despite the success of GR, it fails in explaining many phenomena at early universe [4, 5] these include flatness, Harizen, entropy and singularity problems [6,7). The behavior of exotic objects like black hole and pulsars is difficult to be explained by GR [8, 9]. This motivates Ali Eltahir and other to propose generalize general relativity (GGR). This GGR succeeded in solution some of these problem [10, 11, 12].

General relativity (GR) is one of the big achievements which enables scientists to understand the nature of our universe much [1]. It's theoretical big bang (BB) cosmological model which is designed to describe the universe evolution [2]. The BB model succeeded in describing a large number of astronomical observations including light red shift, relic microwave back ground radiation, deflection of light by the sun and gravitational waves [3 , 4].

According to the big bang model the light red shift indicates the universe expansion, which decreases matter density and temperature [5].

The BB model also explains galaxy formation and evolution of stars. The evolution of stars results from the nuclear energy consumption and the effect of pressure and attractive gravitational force. This evolution of neutron stars, pulsars, while duarfs, red giant stars and black holes. The behavior of these exotic astronomical objects needs promoting the cosmological models [6,7]. Many physical phenomena and long-standing problems are associated with these exotic objects. One of these main problems are associated with the singularity problem, infinite self-mass and generation of gravitational waves [8,9,10].

The attractive shiny glittering attracts people attention at very early times. This leads scientists to study the behavior of these beautiful objects. These astronomical objects are now well classified. The so-called stars, are very large radiation generators. They generate energy due to the nuclear fusion process [11,12]. The so-called planets revolve around the star. Satellites are known to revolve around each planet. A large number of star systems gather together to form the galaxy. Large number of galaxies accumulate themselves to form a cluster. The universe consists of large number of clusters. The recent widely accepted model to describe our universe is the known as the big bang model (BB) [13,14].

The observed red shift of the light coming from remote stars confirms the BB suggestion that the universe is expanding. The observed cold relic microwave back ground agrees with the suggestion that after the big bang. The universe enter the so called radiation era [15,16]. The theory of general relativity predicts many phenomena like the deflection of light by the sun, the existence of exotic objects which emits radiation with very large red shift. These exotic objects include neutron stars, pulsars and black holes. All these observations were confirmed experimentally [17,18].

Among these the black hole is one of the most important exotic objects which pay attention of large number of researchers. The researches concentrate to explain their peculur properties. These include the large red shift, light trapping, and the emission of gravitational waves [19,20].

1.2 Research problem

The GR theory succeeded in explaining many astronomical observations, but unfortunately it suffers from many problem in its cosmological model. First of all, the behavior of black holes neutron stars and binary pulsars are still incomplete. The behavior at the early universe at Plank time also needs Quantum Gravity Model.

1.3 Aim of the work

The aim of this study is to use generalized general relativity to explain the behavior of the cosmos at early stages, beside the behavior of exotic objects and the nature of gravitational waves.

1.4 Thesis Layout

The thesis consists of 4 chapters. Chapters 1 and 2 are concerned with the introduction and the theoretical background Chapters 3 and 4 are concerned with the literature review and the contributions.

CHAPTER TWO

General Relativity

2.1 Introduction

This chapter includes the conceptual framework of the equivalence principle and also mathematical basis GR using curvature tensors and geodesics. Important test of the theory is also presented.

The mathematical description in the form of the Schwarzschild metric with horizons and singularities will be derived and analyzed.

2.2 The principle of equivalence and space-time curvature

The principle of special relativity applies only to bodies moving with constant velocity. This principle is no longer valid in case of accelerated bodies, i.e when the velocity is not constant[21].

Einstein has tried to extend the scope of the principle of relativity to accelerated bodies. The starting point of his reflection was equivalence principle, this principle referred to two different categories phenomenon gravitation and inertia:

- The gravitational mass determines the intensity of the gravitational attraction force, that is mean the more massive bodies are the stronger.
- Inertia of an object could be interpreted as its resistance to any modification to its motion state. If the following condition satisfies for masses, one can then postulate that they are equal.
- These masses are equivalent that is mean they are actually proportional.
- One can change the ratio of the gravitational and inertia masses without affecting the physical phenomenon [22].

We can put a definition for the principle of equivalence as the following:

If the original observer O who uses coordinates Xt, his freely falling friend O who uses XE will detect no difference in the laws of mechanics, except that O will say that he feels a gravitational field and O will say that he does not. This principle reveals that space-time is curved by the presence of matter, but they do not indicate how much space-time curvature matter actually produces. [23. 24, 25]

To determine this curvature requires a specific metric theory of gravity, Einstein put ten field equations and in 1960 C-H brans & Robert Dicke. Developed a metric theory for additional gravitational field.

Gauss first conceived a metric space that includes abroad class of ordinary and non-ordinary curved spaces and which allows in an infinitesimally small region, the possibility of finding a locally Euclidian coordinate system.

The axiom made by Gauss to be the basis of a non-Euclidian geometry resembles the equivalence principle which admits the possibility of finding a locally inertial system at any point in space. The two-dimensional space of Gauss used in determing metric. Functions were expanded to n-dimensions and the complicated problem related to it was solved by Bernhard Riemann who established a complete geometry of space[26].

Einstein proceeded to combine the strong principle of equivalence with the general covariance. The laws of nature have to be described by generally covariant tensor equations, thus the law of gravitation has to be a covariant relation between mass density and curvature.

Because Einstein's principle of equivalence is in a deep analogy with Gauss and Riemann geometry we wanted conceive of gravitation as a manifestation of geometry or equally of the curvature of space as an indication of matter distribution in this space.

Equivalence principle teaches us, the laws of physics showed by equivalent not only among inertial systems of coordinates but also among accelerated ones which means that the proper line interval is invariant under transformation and it says that the equation of physics is called generally covarianty, if it holds the absence of gravitation as well as in its presence[27,28].

2.3 Mathematical Basis of Curvature

2.3.1 Contraverint vectors

Suppose we have two points and consider vector X^r and the component of the vector dx^r, if we want to connect these component with another component in different dx^s coordinates system then we must use the following formula[29]:

$$d\bar{x}^r = \frac{d\bar{x}^r}{d\bar{x}^s} dx^s \tag{2.3.1}$$

Consider p is fixed and Q vary then $d\bar{x}^r / dx^s$ remain constant and the transformation is linear homogenous. So we can define the contravariant vector as follow:

A set quantities T^r associated with a point p, are said to be the components of a contravariant vector if they transform on change of coordinates, according to the equation.

$$\bar{T}^r = T^s \frac{\partial \bar{x}^r}{\partial x^s} \tag{2.3.2}$$

The infinitesimal displacement is a particular example. We can put the definition of a contravariant tensor as follow. A set of quantities T^{rs} are said to be the components of a contravariant tensor of the second order if they transform according to the equation.

$$T^{rs} = T^{mn} \frac{\partial x^r}{\partial x^m} \frac{\partial \bar{x}^s}{\partial x^n} \qquad (2.3.3)$$

2.3.2 Covariant vectors and tensors:

Partial derivative of an invariant is a prototype of the general covariant vector. We define it as follows.

A set of quantities Tr are said to be the components of a covariant vector if they transform according to the equation[30].

$$T_r = Ts \frac{\partial x^s}{\partial \bar{x}^r} \qquad (2.3.4)$$

Let Φ be an invariant function of the coordinates, then

$$\frac{\partial \varphi}{\partial \bar{x}^r} = \frac{\partial \varphi}{\partial x^s} \frac{\partial x^s}{\partial \bar{x}^r} \qquad (2.3.5)$$

2.3.3 Kronecker data (δ^r_s)

It has mixed tensor character:

$$\bar{\delta}^r_s = \delta^m_n \frac{\partial \bar{x}^r}{\partial x^m} \frac{\partial x^n}{\partial \bar{x}^s} \qquad (2.3.6)$$

Where $\bar{\delta}^r_s$ = 1 if r = s

= 0 if $r \neq s$

If m = n

The right hand side reduces to

$$\frac{\partial \bar{x}^r}{\partial x^m} \frac{\partial x^m}{\partial \bar{x}^s} \qquad (2.3.7)$$

A tensor equation is true in all coordinate system, if true in one. Tensor transformation is linear and homogeneous. [31]

$$T_{rs} = 0 \qquad \text{also} \quad T_{sr} = 0 \tag{2.3.8}$$

$$A_{rs} = B_{rs} \tag{2.3.9}$$

$$\bar{A}_{rs} = \bar{B}_{rs} \tag{2.3.10}$$

Transitivity of tensor character (Tensorial) when we have 3 coordinates

$$T_{rs} = T_{mn} \frac{\partial x^{m}}{\partial \bar{x}^{r}} \frac{\partial x^{n}}{\partial \bar{x}^{s}} \tag{2.3.11}$$

2.3.4 Addition, multiplication, contraction of tensor:

$$C^{r}_{st} = A^{r}_{st} + B^{r}_{st} \tag{2.3.12}$$

$$\text{Symmetric } A_{rs} = A_{sr} \tag{2.3.13}$$

Anti symmetric or Kew – Symmetric

$$A_{rs} = -A_{rs} \tag{2.3.14}$$

$$A_{rs} - A_{sr}, A_{rs} + A_{sr} \tag{2.3.15}$$

This applies only for contravariant tensor not for mixed tensor.

Not for mixed tensor

i.e., $A^{r}_{s} = A^{s}_{r}$ (not correct)

2.4 Laws of gravitation

Newton could not answer the question of how to identify the inertial frame which at rest relative to this absolute space. Riemann realized that Euclidean geometry was just a particular choice suited to flat space but not correct in the space which is filled with fields. Einstein finally drew the conclusion and replaced the flat Euclidean three – dimensional space with curved Mikowskian four–dimensional space. [32]

2.5 Riemannian geometry

Gauss made a great emphasis on the inner properties of surface these properties for a cylinder is the same as that for a plane but it is different for spherical surfaces (metric function is different).

Gauss first conceived a metric space includes curved space this admits the possibility of finding a locally inertial system at any point in space , this expand to N-dimensions and the complicated problem related to it was solved by Riemann who established a complete geometry of space.

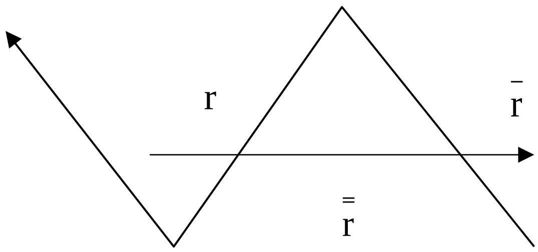
The three dimensional space are not invariant under Lorentz transformation, their values being different to observer in different frames so to find invariant replace them by the four dimensional space time of Hermann minkowski (x, y, z, ct). the distance between two events in three dimensional space dL is generalized to four dimensional space time distance ds

$$ds^2 = c^2 d\tau^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2 = c^2 dt^2 - |dr|^2 \qquad (2.5.1)$$

dt = proper time

$$|dL/dt| \leq c \qquad (2.5.2)$$

World line is inside the light cone consider the case as shown in fig :



$\bar{\bar{r}} + \bar{r} = r$

$\bar{r} - \bar{\bar{r}} = \bar{r}$

$$V_A(\bar{r}, t) = V_A(r, t) - V_A(\bar{\bar{r}}, t) \qquad (2.5.3)$$

Equation (2.5.3) is true if $V_A(r, t)$ has specific form

$$V_A(r, t) = f(t)\, r \qquad (2.5.4)$$

$f(t)_r$ proportional to r and is arbitrary function .

If f(t)= 0 the universe would be seen expand or if f(t) < 0, the universe limited gravitational contraction–expansion and contraction, are natural system consequences of cosmological principle. To study how signals are exchanged between inertial frames, Einstein considered physical measurements is all frames moving with respect to each other with constant velocity and all frames must be equivalent, the result of measurement identical, this possible if frames independent invariants and light travels by constant speed C.

2.6 The Newtonian limit

To make contact with Newton's theory let us consider the case of a particle moving slowly in a weak stationary gravitational field. According to equation (2.5.4) the time component is given by:

$$\frac{d^2x^\mu}{d\tau^2}+\Gamma^\mu_{00}\left(\frac{dt}{d\tau}\right)^2=0 \qquad (2.6.1)$$

The field is stationary, then all-time derivatives of $g_{\mu}\nu$ vanish and therefore

$$\Gamma^\mu_{00}=-\frac{1}{2}g^{\mu\nu}\frac{\partial g_{00}}{\partial x^\nu} \qquad (2.6.2)$$

Since the field is weak, we may adopt nearly Cartesian coordinate system which

$$g_{\alpha\beta}=y_{\alpha\beta}+h_{\alpha\beta} \qquad |h_{\alpha\beta}|<<1 \qquad (2.6.3)$$

To first order in $h_{\alpha\beta}$ (2.6.4)

$$\frac{d^2x}{d\tau^2}=\frac{1}{2}\left(\frac{dt}{d\tau}\right)^2\nabla h_{00} \qquad (2.6.5)$$

So dividing the equation for d^2x/dt^2 by $(dt / d\tau)^2$, we find $\frac{d^2t}{d\tau^2}=0$

$$\frac{d^2x}{d\tau^2} = \frac{1}{2}\nabla h_{00} \qquad (2.6.6)$$

The corresponding Newtonian result is

$$\frac{d^2x}{d\tau^2} = -\nabla\varphi \qquad (2.6.7)$$

Where φ is the gravitational potential, which at a distance r from the center of a spherical body of mass M takes the form

$$\varphi = -\frac{GM}{r} \qquad (2.6.8)$$

Comparing (2.6.6) with (2.6.7) we conclude that

$$h_{00} = -2\varphi + \text{constant} \qquad (2.6.9)$$

Furthermore, the coordinate system must become Minkowskian at great distances so h_{00} vanishes at infinity, and if we define φ to vanish at infinity as in (2.6.8) we find that the constant here is zero, so $h_{00} = -2\varphi$ and returning to the metric (2.6.3)

$$g00 = -(1 + 2\varphi) \qquad (2.6.10)$$

The gravitational potential φ is of the order 10^{-39} at the surface of a proton, 10^{-9} at the surface of the earth, 10^{-6} at the surface of the sun and 10^{-4} at the surface of a white dwarf star, so evidently the distortion in $g_{\mu\nu}$ produced by gravitation is generally very slight.

2.7 General relativity and the principle of covariance

The strongest force of nature on large scales is gravity, so the most important part of a physical description of the universe is a theory of gravity.

We therefore begin this chapter with a brief introduction to the basic of this theory.

In Euclidian space the invariant interval between two events at coordinates (t, x, y, z) and (tτdt, xtdx), yτdy, ztdz) is defined by

$$ds^2 = c^2dt^2 – (dx^2 + dy^2 + dz^2)$$

Where ds invariant under a change of coordinate system

The path of light Ray is given by ds = 0

The paths of particles between any two events as forgive stationary values of

$I = \int ds$ This corresponds to the shortest distance between two points being a straight line. (Motion of particle under no external forces). Gravitation and electromagnetism cause particle tracks to deviate from the straight line.

Einstein's theory is to transform it from being a force to being a property of space time. In this theory, the space time is not necessarily flat as it is in Minicowski space-time but may be curved the interval between two events.

$$ds^2 = g_{\mu v} dx^m dx^v$$

$$\mu, v = 0,1,2,3$$

$$x^0 = ct, \qquad x' = x, \qquad x2 = y, \quad x^3 = z$$

to x^1, x^2, x^3 space coordinates

Tensor $g_{\mu v}$ is metric tensor that describes the space time geometry the integral along path.

$$\delta \int_{path} ds = 0$$

From equation the path of a free particle which is called a geodesic can be show to be described by

$$\frac{d^2x^i}{ds^2}+\Gamma^i_{kl}\frac{dx^k}{ds}\frac{dx^L}{ds}=0$$

When Γ_s are called Christopher symbols

$$\Gamma^I_{KL}=\frac{1}{2}g^{l\nu}\left[\frac{\partial g_{\mu k}}{\partial xL}+\frac{\partial g_{mL}}{\partial xk}-\frac{\partial gkl}{\partial x\mu}\right]$$

And

$$g^{im}\, g_{mk} = \delta^i_k$$

Is the kronecker delta, which satisfies

$$\delta^i_k = 1 \quad i = k$$
$$i \neq k$$

g_{ij} determine by the matter

Einstein equation is the relationship between the distribution of matter and the metric describing the space-time geometry in general relativity and equations are tensor equation are tensor equation.

General tensor is a quantity which transforms as follows when coordinates are changed from x^i to x^{-i}

$$A'^{kl}_{pq}=\frac{\partial x^{-k}}{\partial x^m}\frac{\partial x^{-L}}{\partial x^n}\cdots\frac{\partial x^r}{\partial x^{-p}}\frac{\partial x^s}{\partial x^{-q}}$$

Where upper indices A^{mn}_{rs}

Are contravariant and the lower are covariant.

The difference can be illustrated by considering a tensor of Rank (1) which simply a vector (the Rank is the no of indices it carries). A vector will transformation according to some rules suppose we have original coordinate x^i and we transform it to a new system x^{-k} $\bar{A} = \frac{\partial x^{-k}}{\partial x^i} A$, this vector A is a covariant

$A = A^i$ contravariant

$\bar{A} = \frac{\partial x^i}{\partial x^{-k}}$ A is covariant

$A = A_i$ covariant

The tangent vector to a curve is an example of contra variant vector, the normal to a surface in a covariant vector.

The rule is a generalization of these concepts to tensors of arbitrary rank and to tensor of mixed character.

2.8 Einstein's gravitational field equation

To get similar laws for general relativity as in special relativistic physics, with equivalence of mass and energy, one can define the energy-momentum tensor T to be

$$T_{\mu v} = (\rho + \rho) V_m V_v + g_{\mu v} \rho \qquad (2.7.1)$$

The energy momentum tensor Tin describe the mass distribution with pressure p and energy density ρ for fluid. Where

$$T_{ik} = (p + pc^2)\ u;\ u_k - pg_{ik};$$

u_i = fluid for velocity

$$u_i = g_{ik}\ u^u = g_{ik}\ \frac{dx^k}{ds} \qquad (2.7.2)$$

x^k is the world line of a fluid element i.e the trajectory in space-time followed by the particle. The conservation of energy-momentum requires:

$$T_{ik;k}=0 \qquad (2.7.3)$$

Einstein wished to find a relation between matter and metric. Because in the appropriate limit, equation must reduce to Poisson equation describing Newtonian gravity

$$\nabla^2\varphi = 4\prod G\rho \qquad (2.7.4)$$

But according to equation (2.6.10)

$$g_{00} = -(1+2\phi)$$

Thus

$$\nabla^2\phi = -\frac{1}{2}\nabla^2 g_{00}$$

According to equation (2.7.1)

$$T_{00} = \rho$$

Thus one can be rewrite

Position equation

$$\nabla^2 g_{00} = -8\pi G T_{00}$$

The L.H.S represents the geometrical part which can be denoted by G,

More generally

$$G_{ik} = -8\pi G T_{00}$$

$$G_{ik} = -8\pi G T_{ik}$$

g_{ik} and their derivatives. The energy conservation requires

$$G_{ik;k} = -8\pi G T_{ik;k} = 0$$

but according to Bianchi identity

$$\left(R_{ik} - \frac{1}{2} g_{ik}\right)_{;k} = 0$$

thus one can propose that Einstein's field equation takes the form

$$G_{ik} = R_{ik} - \frac{1}{2} g_{ik} R = -8\pi G T_{ik}$$

when a cosmological constant A is taken into account, one gets

$$G_{ik} = R_{ik} \frac{1}{2} g_{ik} R - A g_{ik} = \frac{8\pi G}{c^4} T_{ik}$$

2.8 Solution of Einstein's free-field equations

Since the Einstein equation built from the metric tensor, therefore we will look for the solution in terms of space coordinate for this purpose we choose the simplest form with space symmetry and independence of time, i-e static metric, in this metric the gravitational field will depend on the rotational invariants $\bar{x}^2, d\bar{x}^2, \bar{x} - d\bar{x}$ and the invariant proper time interval should be the same for all points in symmetrical positions.

By using the spherical coordinates, that one can come to the following:

$$ds^2 = -g_{\mu v}\, dx^{\mu}\, dx^{v}$$

$$ds^2 = -\left[A(r)\, dr^2 + r^2 d\,\theta^2 + r^2 \sin^2\theta\, d^2\varphi - B(r)\, dt^2\right] \qquad (2.8.1)$$

the function A and B can be determined from the solution of the field equation. Non vanishing components are.

$$g_{rr} = A(r) \qquad g_{\theta\theta} = r^2 \qquad g_{\varphi\varphi} = r^2 \sin^2\theta$$

$$g_{tt} = -B(r) \qquad g^{rr} = A^{-1}(r) \qquad g^{\theta\theta} = r^{-2}$$

$$g^{\varphi\varphi} = r^{-2} (\sin^2\theta)^{-2} \qquad g^{tt} = -B^{-1}(r)$$

$$g = r^4 A(r) B(r) \sin^2\theta \tag{2.8.2}$$

so the invariant volume element is

$\sqrt{g}\, d^3x\, dt$

$$\sqrt{g}\, dr\, d\theta\, d\varphi = r^2 \sqrt{A(r) B(r) \mathrm{Sin}^2\theta\, dr\, d\theta\, d\varphi} \tag{2.8.3}$$

Using the usual formula

$$\Gamma^{\lambda}_{\mu\nu} = \frac{1}{2} g^{\lambda\rho}\left(\frac{\partial g_{\rho\mu}}{dx^{\nu}} + \frac{\partial g_{\rho\nu}}{\partial x^{\mu}} - \frac{\partial g_{\mu\nu}}{\partial x\rho}\right) \tag{2.8.4}$$

Non vanishing components are :

$$\Gamma^{\lambda}_{\mu\nu} = \frac{1}{2A(r)} \frac{dA(r)}{dr} \qquad \Gamma^{r}_{\theta\theta} = -\frac{r}{A(r)}$$

$$\Gamma^{r}_{\theta\theta} = \frac{-r\sin^2\theta}{A(r)} \qquad \Gamma^{r}_{tt} = -\frac{1}{2A(r)} \frac{dB(r)}{dr}$$

$$\Gamma^{\theta}_{r\theta} = \Gamma^{\theta}_{\theta r} = \frac{1}{r} \qquad \Gamma^{\varphi}_{\varphi\varphi} = -\sin\theta \cos\theta$$

$$\Gamma^{\varphi}_{\varphi r} = \Gamma^{\varphi}_{r\varphi} = \frac{1}{r} \qquad \Gamma^{\varphi}_{\varphi\theta} = \Gamma^{\varphi}_{\theta\varphi} = \cot\theta$$

$$\Gamma^{t}_{tr} = \Gamma^{t}_{rt} = \frac{1}{2B(r)} \frac{\partial B(r)}{\partial r}$$

Ricci tensor

$$R_{\mu K} = \frac{\partial \Gamma^{\lambda}_{\mu\lambda}}{\partial x^{k}} - \frac{\partial \Gamma^{\lambda}_{\mu K}}{\partial \chi\lambda} + \Gamma^{\lambda}_{\mu\lambda}\Gamma^{\lambda}_{k\eta} - \Gamma^{\eta}_{\mu K}\Gamma^{\lambda}_{\lambda\eta} \tag{2.8.5}$$

The Ricci tensor components will have the following terms

$$R_{rr} = \frac{\ddot{B}}{2B} - \frac{1}{4}\left[\frac{A'}{A} + \frac{B'}{B}\right]\frac{\hat{B}}{B} - \frac{1}{r}\frac{\hat{A}}{A}$$

$$R_{\theta\theta} = \frac{R_{\varphi\varphi}}{\sin^2\theta} = -1 + \frac{r}{2A}\left[-\frac{A'}{A} + \frac{B'}{B}\right] + \frac{1}{A}$$

$$R_{tt} = \frac{\ddot{B}}{2A} + \frac{1}{4}\left[-\frac{A'}{A} + \frac{B'}{B}\right]\frac{B'}{A} - \frac{\hat{B}}{rA} \quad (2.8.6)$$

$R_{\mu\nu} = 0$ for $\mu \neq \nu$

That is mean the scalar curvature in static isotropic metric does not depend on either the time or θ and φ, it is a function only on r.

$R = R(r)$

In the vacuum Einstein's equation give

$$R_{rr} = R_{tt} = R_{\theta\theta} = 0 \quad (2.8.7)$$

$$\frac{dAB}{dr} = 0$$

AB = constant

$$A(\infty) = B(\infty) = 1 \quad (2.8.8)$$

The flatness condition at large r

$$A(r) = B^{-1}(r) \quad (2.8.9)$$

This yield $\frac{d}{dr}\left[\frac{r}{A}\right] = 1$

$$B' = C_1\gamma^{-2} \quad (2.8.10)$$

By integration (1-60) the following solutions for A and B will be obtain :

$$A(r) = \left[1 + \frac{C1}{r}\right]^{-1}$$

$$B(r) = 1 + \frac{C1}{r} \quad (2.8.11)$$

The gravitational Newtonian potential of source mass M.

$$\varphi = -\frac{MG}{r}$$

Which is related to g_{tt} by

$$-g_{tt} = B^{r} \rightarrow \text{large } 1 + 2\varphi \quad (2.8.12)$$

The following metric space due to Schwarzschild (1916) result ds^2 = $\left[1-\frac{2M\varsigma}{r}\right]dt^2-\left[1-\frac{2M\varsigma}{r}\right]^{-1}dr^2$

$$-r^2\,\theta^2 - r^2\sin^2\theta\, d\varphi^2 \qquad (2.8.13)$$

By transforming the time coordinate in the form

$$\tau = t + 2M\varsigma \ln |1-r|2M|\,| \qquad (2.8.14)$$

And accordingly the metric tensor component we obtained the following expression.

$$ds^2 = d\tau^2 - dr^2 - r^2\,(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

$$-\frac{2M\varsigma}{r}(d\theta + dr)^2 \qquad (2.8.15)$$

Where the last term represent the non-flatness of space. This equation called the Schwarzschild metric which having singularities at r = 0 and r = 2Mς

The Schwarzschild solution has its limitation in strong field region, i.e it gives singularity and gravitational collapse and black holes.

The singularity at the origin is real since physically there is no point mass whose gravitational field is infinite and its coordinate un removable by the use of any coordinate transformation. One can prove the nonzero curvature invariant at r = 2Mς so this singularity is not real and infinite force crushes the collapsing body to infinite density. This imply the laws of physics break down near the singular point including the general relativistic law. That mean GTR is not the right model for strong gravity.

2.9 Application of general relativity

2.9.1 Time dilation:

Consider a clock in an arbitrary gravitational field moving with arbitrary velocity not necessarily in free fall. The equivalence principle tells us that its rate is unaffected by the gravitational field if we observe the clock from a locally inertial coordinate system ξ^α , the space – time interval $d\tau^\alpha$ between sticks is governed in this system by

$$\Delta\tau = \left(-\eta_{\alpha\beta} d\zeta^{\beta}\right)^{\frac{1}{2}} \tag{2.9.1}$$

Where $\Delta\tau$ is the period between ticks when the clock is at rest in the absence of gravitation. Hence in any arbitrary coordinate system the space-time interval between ticks will be governed by

$$\Delta\tau = \left(-y_{\alpha\beta}\frac{\partial\zeta^{\alpha}}{\partial x^{\mu}}dx^{\mu}\frac{\partial\zeta\beta}{\partial x^{v}}dx^{v}\right)^{\frac{1}{2}} \tag{2.9.2}$$

Or introducing the metric tensor

$$\Delta t = (-g_{\mu v}\, dx\, dx^{v})^{\frac{1}{2}} \tag{2.9.3}$$

If the clock has velocity dx^μ/dt then the time interval dt between ticks will be given by

$$\frac{dt}{\Delta\tau} = \left(-g_{\mu v}\frac{dx^{\mu}}{dt}\frac{dx^{v}}{dt}\right)^{-\frac{1}{2}} \tag{2.9.4}$$

In particular if the clock is at rest this becomes

$$\frac{dt}{\Delta\tau} = \left(-g_{00}\right)^{-\frac{1}{2}} \tag{2.9.5}$$

We can not observe the time dilation factors appearing in (2.6.10), (2.9.5) by merely measuring the time interval at between ticks and comparing with the value Δt specified by the manufacture because the gravitational field affects our time standards in exactly the same way as it affects the clock being studied.

That is if our standard clock says that a certain physical process takes 1 sec at rest in the absence of gravitational, that it will also tell us that it takes 1 sec in the presence of gravitation booth standard clock and process being affected by the field in the same way.

However, we can compare the time dilation factors at two different points in a field. For instance, suppose that at point 1 we observe the light coming from a particular atomic transition at point 2. if point 1 and 2 are at rest in a stationary gravitational field, then the time taken for a wave crest to travel from 2 to 1 will be a constant.

$$dt_2 = \Delta\tau\,(-g_{00}\,(x_2))^{-\frac{1}{2}} \qquad (2.9.6)$$

Hence for a given atomic the ratio frequency (observed at point 1) of the light from point 2 to that of the light from point 1 will be.

$$\frac{V_2}{V_1} = \left(\frac{g_{00}(x_2)}{g_{00}(x_1)}\right)^{\frac{1}{2}} \qquad (2.9.7)$$

In the weak field limit $g_{00} \approx -1 - 2\varphi$ and $\varphi << 1$

So $V_2 / V_1 = 1 + \Delta v / v$ where

$$\frac{\Delta v}{v} = \phi\,(x_2) - \varphi\,(x_1) \qquad (2.9.8)$$

From uniform gravitational field, this result could be derived directly from the principle of equivalence without introducing a metric or affine connection. Let us apply Eq (2.7.5) to case of light from the sum's surface observed on the earth. The sun's gravitational potential can be calculated as

$$\varphi_0 = \frac{-\varsigma Mo}{Ro} \qquad (2.9.9)$$

Where Mo and Ro are the sum's mass and radius

$Mo = 1.97 * 10^{33}$ g

$Ro = 0.695\ 10^{6}$ km

And ς is the gravitational constant

$\varsigma = 6.67\ 10^{-8}$ erg cm/gm^2

We find that the potential on the surface of the sum is

$\varphi_0 = -2-/2\ 10^{-6}$

The gravitational potential of the earth is negligible in comparison so ideally the frequency of light from the sun should be shifted to the red by 2-12 parts per million as compared with light emitted by terrestrial atoms.

2.9.2 The Red shift

The interesting effect of the gravitational field:

The slowing down of time in the field and the consequent red shift of spectral lines emitted by atoms located on massive bodies. The effect has been tested by experiment and been rather well verified; we thus have some experimental justification for the basic theoretical concepts we have set forth.

Consider for example a light wave emitted on the sun and received on the earth. Let the gravitational potential at the surface of the sun be φ_s. using the approximate g_{00} and proper-time intervals are related to coordinate-time intervals by the equation.

$$d\tau_s = \left(1 + \frac{2\varphi s}{c^2}\right)^{\frac{1}{2}} dt \qquad (2.9.10)$$

Similarly on the earth proper-time intervals are related to coordinate time intervals by

$$dt_\varepsilon = \left(1 + \frac{2\varphi\varepsilon}{c^2}\right)^{\frac{1}{2}} dt \qquad (2.9.11)$$

Where ε is the value of the gravitational potential on the earth? Suppose now n waves frequency v_0 are emitted in proper time $\Delta\tau_s$ from an atom on the sun, then

$$n = v_0 \, \Delta\tau_s \qquad (2.9.12)$$

On the earth are certainly receives n waves, but the frequency and time duration of the wave train have changed. Using a frequency – duration.

$$N = \nu_e \, \Delta\tau_e \qquad (2\text{-}13)$$

Since n is a constant

$\nu_o \, \Delta\tau_s = \nu_e \, \Delta\tau_e$

$$\nu_e = \nu_o \, \frac{\Delta\tau_s}{\Delta\tau_e} \qquad (2.9.14)$$

The coordinate – time duration of the wave corresponding to $\Delta\tau_s$ is

$$\Delta\tau = \frac{\Delta\tau_s}{\sqrt{1 + 2\varphi_s / c^2}} \qquad (2.9.15)$$

$$\Delta\tau = \frac{\Delta\tau_e}{\sqrt{1 + 2\varphi_e / c^2}} \qquad (2.9.16)$$

$$\frac{\Delta\tau_s}{\Delta\tau_e} = \left(\frac{1 + 2\varphi_s / c^2}{1 + 2\varphi_e / c^2} \right)^{\frac{1}{2}} \qquad (2.9.17)$$

$$\nu_e = \nu_o \, \frac{\Delta\tau_s}{\Delta\tau_e} = \nu_o \left(\frac{!+2\varphi_s / c^2}{1 + 2\varphi_e / c^2} \right)^{\frac{1}{2}}$$

Expanding to first order in the small quantities ϕ_s/c^2 and φ_e/c^2 we obtain.

$$\frac{\nu_e - \nu_o}{\nu_o} = \frac{\varphi_s - \varphi_e}{c^2} \qquad (2.9.18)$$

Or in briefer notation

$$\frac{\Delta\nu}{\nu_o} = \frac{\Delta\varphi}{c^2} \tag{2.9.19}$$

Since the sun is at a large negative potential relative to the earth, we see that $\Delta\varphi$ is negative. Thus, the frequency of light decreases as it leaves the sun, and when it is received on earth, we see a shift toward the red end of the spectrum. It is as if the atoms of the sun vibrated in slow motion when we viewed them from the earth. Of course, there is nothing special about using the earth and sun as the two points at different heights on the earth if our measurement is precise enough to detect the correspondingly small shift.

CHAPTER THREE

Literature Review

3.1 Introduction:

In this chapter many attempts to describe the behavior of black holes and exotic objects are presented. The papers which are directly related to our model were fully derived, while the others are summarized.

3.2 Equilibrium of Stars within the Framework of Generalized Special Relativity Theory

In this work the generalized special relativity energy relation was used for minimization of energy. For minimum energy the radius the critical value is typical to that of general relativity for black holes. At equilibrium the pressure and centrifugal force balance the attractive gravity. [33,34,35]

Equilibrium Conditions:

Consider first the Generalized Special Relativity GSR energy E equilibrium condition by minimizing E w.r.t. r

$$E = m_0 C^2 \left(1 + \frac{2\varphi}{C^2}\right)\left(1 + \frac{2\varphi}{C^2} - \frac{v^2}{C^2}\right)^{-1/2} \qquad (3.2.1)$$

$$\varphi = -\frac{GM}{r} \quad , m_0 = M \qquad (3.2.2)$$

$$\frac{v^2}{C^2} = \frac{m^2 v^2}{m^2 C^2} = \frac{p^2}{M^2 C^2} \qquad (3.2.3)$$

For simplicity consider the average momentum p is equal to the maximum momentump_F, by ignoring $\sqrt{2}$, where

$$p = \frac{p_F}{\sqrt{2}}$$

Thus

$p = p_F = \wedge \left(\frac{N}{V}\right)^{\frac{1}{3}} = \wedge\, n_0$

Where

$$\wedge = (3\pi^2)^{\frac{1}{3}}\hbar \qquad (3.2.4)$$

Therefore, with the aid of equations (2) – (4), equation (1) reads

$$E = E_F = Mc^2\left(1 - \frac{2MG}{r}\right)\left(1 - \frac{2MG}{r} - \frac{p_F^2}{M^2C^2}\right)^{-\frac{1}{2}} \qquad (3.2.5)$$

The radius r which makes the energy E minimum is given when

$$\frac{dE_r}{dr} = \frac{Mc^2\left(\frac{2MG}{r^2}\right)}{\left(1-\frac{2MG}{r}-\frac{p_F^2}{M^2C^2}\right)^{1/2}} + \frac{Mc^2\left(1-\frac{2MG}{r}\right)\left(-\frac{1}{2}\right)\left(\frac{2MG}{r^2}\right)}{\left(1-\frac{2MG}{r}-\frac{p_F^2}{M^2C^2}\right)} = 0$$

$$\frac{\mathrm{Mc}^2\left[\left(\frac{2MG}{r^2}\right)\left(1-\frac{2MG}{r}-\frac{p_F^2}{M^2C^2}\right)-\left(\frac{MG}{r^2}\right)\left(1-\frac{2MG}{r^2}\right)\right]}{\left(1-\frac{2MG}{r}-\frac{p_F^2}{M^2C^2}\right)^{3/2}} = 0$$

$$\frac{M^2c^2G}{r^2}\left(-1 + 2 + \frac{4MG}{r} - \frac{p_F^2}{M^2C^2}\right) = 0 \qquad (3.2.6)$$

This is satisfied when

$$\frac{4MG}{r} = \frac{p_F^2}{M^2C^2} - 1 \qquad (3.2.7)$$

Thus the minimum radius is given by

$r =$

$$\frac{4M^2c^2G}{p_F^2 - M^2c^2} \qquad (3.2.8)$$

Where

$$p_F^2 = (3\pi^2)^{1/3}\hbar n^{1/3} = \wedge\left(\frac{N}{V}\right)^{1/3} = \left(\frac{9\pi}{4}\right)^{1/3}\frac{N^{1/3}}{r_F}\hbar \qquad (3.2.9)$$

The equilibrium takes place when r is non negative, i.e when

$p_F^2 > M^2C^2$

$$p_F > Mc \qquad (3.2.10)$$

The critical mass is given by

$$M_c = \frac{p_F}{C} \qquad (3.2.11)$$

Thus for star to be at equilibrium one requires

$$\frac{p_F}{c} > M$$

$$M_C > M \tag{3.2.12}$$

$$M < M_C$$

Thus the maximum mass for stable star is

$$M_C = \frac{P_F}{c} = \frac{(3\pi^2)^{1/3}\hbar}{c}\left(\frac{N}{V}\right)^{1/3} \tag{3.2.13}$$

This condition resembles Chandrasekhar limit for stable white dwarf. i.e the star mass need to be less than the critical value in equation (3.2.11). The equilibrium condition can also be found by using generalized special relativity energy momentum relation [36,37]

$$g_{00}E^2 - p^2c^2 + g_{00}m_0^2c^4$$

$$E^2 = (g_{00})^{-1}p^2c^2 + g_{00}m_0^2c^4 \tag{3.2.14}$$

One can rewrite equation (14) to be

$$E = (a_1 - a_2p^2)^{1/2} \tag{3.2.15}$$

Where

$$a_1 = g_{00}m_0^2c^4 = \left(1 - \frac{2MG}{rc^2}\right)m_0^2c^4 \ , a_2 = (g_{00})^{-1}c^2$$

$$a_2p^2 = a_1\cos^2\theta \tag{3.2.16}$$

$$E = \int_0^{p_F}(a_1 - a_1\cos^2\theta)^{1/2}dp \tag{3.2.17}$$

Where

$$-dp = \sqrt{\frac{a_1}{a_2}}\sin\theta d\theta \tag{3.2.18}$$

$$E = \sqrt{a_1}\int(1-\cos^2\theta)^{1/2}\left(-\sqrt{\frac{a_1}{a_2}}\right)\sin\theta d\theta \tag{3.2.19}$$

$$= \sqrt{a_1}\left(-\sqrt{\frac{a_1}{a_2}}\right)\int\sin^2\theta d\theta \tag{3.2.20}$$

$$cos\,2\,\theta = \cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta$$

$$\sin^2\theta = \frac{1}{2}(1 - cos\, 2\theta)$$

$$E = \frac{\sqrt{a_1}}{2}\left(-\sqrt{\frac{a_1}{a_2}}\right)\left(\theta - \frac{sin2\theta}{2}\right) \qquad (3.2.21)$$

$$sin2\theta = 2\, sin\theta cos\theta \;\; , cos\theta = \sqrt{\frac{a_2}{a_1}}p$$

$$sin\theta = (1 - \cos^2\theta)^{1/2} = \left(1 - \frac{a_2}{a_1}p^2\right)^{1/2}$$

$$E = \sqrt{a_1}\sqrt{\frac{a_1}{a_2}}\sqrt{\frac{a_2}{a_1}}p_F(1 - a_3 p_F^2)^{1/2} + cos^{-1}\sqrt{\frac{a_2}{a_1}}p_F - \frac{\pi}{2}$$

$$= \sqrt{a_1}p_F(1 - a_3 p_F^2)^{1/2} + cos^{-1}\sqrt{\frac{a_2}{a_1}}p_F - \frac{\pi}{2} \qquad (3.2.22)$$

Where

$$a_3 = \frac{a_2}{a_1} = \frac{g_{00}^{-1}c^2}{g_{00}m_0^2c^4} = \frac{g_{00}^{-2}}{m_0^2c^4}$$

$$E = \left(1 - \frac{2MG}{rc^2}\right)p_F\left[1 - \frac{p_F^2c^2}{m_0^2c^4\left(1 - \frac{2MG}{rc^2}\right)^2}\right]^{1/2} + cos^{-1}\left(\frac{p_F c}{m_0c^2\left(1 - \frac{2MG}{rc^2}\right)}\right) - \frac{\pi}{2} \qquad (3.2.23)$$

It is clear from equation (3.2.23) that stability requires E to be real.

This can be satisfied when

$$1 - \frac{2MG}{rc^2} > 0$$

$$rc^2 > 2MG$$

$$r > \frac{2MG}{c^2}$$

The critical radius is given by

$$r_c = \frac{2MG}{c^2} \qquad (3.2.24)$$

Thus the radius should be greater than the black hole radius. Also

$$1-\frac{p_F^2c^2}{m_0^2c^4\left(1-\frac{2MG}{rc^2}\right)^2}>0$$

$$m_0^2c^4\left(1-\frac{2MG}{rc^2}\right)^2>p_F^2c^2$$

Thus

$$m_0c^2\left(1-\frac{2MG}{rc^2}\right)>\pm p_Fc$$

$$\left(1-\frac{2MG}{rc^2}\right)>\pm\frac{p_Fc}{m_0c^2}$$

$$rc^2-2MG>\pm\left(\frac{p_Fc}{m_0c^2}\right)rc^2 \qquad (3.2.25)$$

$$\left(1\pm\frac{p_Fc}{m_0c^2}\right)rc^2>2MG$$

$$r>\frac{2MGm_0}{m_0c^2\pm p_F} \qquad (3.2.26)$$

Thus the critical radius is given by

$$r_c=\frac{2Mm_0G}{(m_0c^2\pm p_Fc)} \qquad (3.2.27)$$

The equilibrium mass also satisfies

$$2MG>-rc^2\pm\left(\frac{p_Fc}{m_0c^2}\right)rc^2$$

$$M<\frac{rc^2}{2G}\pm\frac{p_Frc}{m_0} \qquad (3.2.28)$$

Hence the critical maximum mass is given by

$$M_c=\frac{rc^2}{2G}\pm\frac{p_Frc}{m_0} \qquad (3.2.29)$$

The equilibrium condition can also be found by minimizing$E$, where

$$E=mc^2=m_0c^2\left(1+\frac{2\varphi}{c^2}\right)\left(1+\frac{2\varphi}{c^2}-\frac{v^2}{c^2}\right)^{-1/2} \qquad (3.2.30)$$

Assuming the mass to be equal to the rest mass, and the potential to be the Newtonian, one gets

$$m_0 = M \quad , \quad \varphi = -\frac{GM}{R} \tag{3.2.31}$$

Therefore

$$E = Mc^2\left(1 - \frac{2GM}{Rc^2}\right)\left(1 - \frac{2GM}{Rc^2} - \frac{v^2}{c^2}\right)^{-1/2} \tag{3.2.32}$$

For small φ and velocity v compared to speed of light c, i.e

$$\frac{GM}{R} < 1 \quad , \quad \frac{v^2}{c^2} < 1$$

One gets

$$E = Mc^2\left(1 - \frac{2GM}{Rc^2}\right)\left(1 - \frac{2GM}{Rc^2} - \frac{1}{2}\frac{v^2}{c^2}\right)$$

$$E = \left(Mc^2 - \frac{2GM^2}{R}\right)\left(1 - \frac{2GM}{Rc^2} - \frac{1}{2}\frac{v^2}{c^2}\right)$$

$$E = Mc^2 + \frac{GM^2}{R} + \frac{1}{2}Mv^2 - \frac{2GM^2}{R} - \frac{2G^2M^2}{R^2c^2} - \frac{GM^2v^2}{Rc^2} \tag{3.2.33}$$

The mass which make the energy minimum for constant radius is given by

$$\frac{dE}{dM} = \frac{c^2\left(\frac{2GM}{Rc^2}\right)}{\sqrt{1-\frac{2MG}{Rc^2}-\frac{v^2}{c^2}}} + \frac{Mc^2\left(\frac{-2G}{Rc^2}\right)}{\sqrt{1-\frac{2GM}{Rc^2}-\frac{v^2}{c^2}}} + \frac{\frac{1}{2}Mc^2\left(1-\frac{2GM}{Rc^2}\right)\left(\frac{2G}{Rc^2}\right)}{\left(1-\frac{2GM}{Rc^2}-\frac{v^2}{c^2}\right)} \tag{3.2.34}$$

Neglecting the kinetic term yields

$$\frac{dE}{dM} = \frac{\left(c^2 - \frac{4MG}{R}\right)\left(1 - \frac{2MG}{Rc^2}\right) + \frac{MG}{R} - \frac{M^2G^2}{Rc^2}}{\left(1 - \frac{GM}{Rc^2} + \frac{1}{2}\frac{v^2}{c^2}\right)^{3/2}} = 0 \tag{3.2.35}$$

This requires

$$c^2 - \frac{2MG}{R} - \frac{4MG}{R} + \frac{8M^2G^2}{R^2c^2} + \frac{MG}{R} - \frac{2M^2G^2}{R^2c^2} = 0$$

$$c^2 - \frac{5MG}{R} + \frac{6M^2G^2}{R^2c^2} = 0$$

$$\frac{6G^2}{R^2c^2}M^2 - \frac{5G}{R}M + c^2 = 0$$

$$ax^2 + bx + c = 0\,, \qquad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$M = \frac{\frac{5G}{R} \pm \sqrt{\left(\frac{5G}{R}\right)^2 - \frac{24G^2c^2}{R^2c^2}}}{\frac{12G^2}{R^2c^2}} = \frac{R^2c^2}{12G^2}\left(\frac{5G}{R} \pm \sqrt{\frac{G^2}{R^2}}\right)$$

$$M = \frac{R^2c^2}{12G^2}\left(\frac{G}{R}\right)(5 \pm 1) = \frac{Rc^2}{12G}(5 \pm 1)$$

$$M = \frac{1}{2}\frac{Rc^2}{G} \quad , \quad \frac{1}{3}\frac{Rc^2}{G} \tag{3.2.36}$$

For stars one have two forces, pressure force which counter balance the gravity force, thus

$$p = \frac{NKT}{V} = \frac{1}{3}\frac{mv^2}{V} \tag{3.2.37}$$

Thus the pressure force is given by

$$F_p = PA = \frac{\frac{1}{3}mv^2(4\pi r^2)}{\frac{4\pi}{3}r^3} = \frac{mv^2}{r} \tag{3.2.38}$$

The gravity force is given by

$$F_g = \frac{GmM}{r^2} \tag{3.2.39}$$

At equilibrium the two forces counter balances themselves thus:

$$F_p = F_g$$

$$\frac{mv^2}{r} = \frac{GmM}{r^2} \quad , \quad mv^2 = \frac{GmM}{r} \tag{3.2.40}$$

If particles are considered as strings with v representing max speed. thus the average value is given by

$$v_a = \frac{v_m}{\sqrt{2}} \quad , \quad mv_a^2 = \frac{mv_m^2}{2} \tag{3.2.41}$$

Thus

$$mv_a^2 = \frac{mv_a^2}{2} = \frac{1}{2}mv^2 \tag{3.2.42}$$

One thus gets

$$\frac{1}{2}mv^2 = \frac{GmM}{r} = m\varphi \tag{3.2.43}$$

Hence

$$v^2 = 2\varphi \tag{3.2.44}$$

Hence

$$E = \frac{m_0c^2\left(1 + \frac{2\varphi}{c^2}\right)}{\left(1 + \frac{2\varphi - v^2}{c^2}\right)^{1/2}} = m_0c^2\left(1 + \frac{2\varphi}{c^2}\right) \tag{3.2.45}$$

But:

$$m_0 = M\,, \qquad \varphi = -\frac{GM}{R} \tag{3.2.46}$$

For attractive force

$$E = M\left(c^2 - \frac{2GM}{R}\right) + M\left(\frac{-2G}{R}\right) = 0 \tag{3.2.47}$$

$$-\frac{4GM}{R} + c^2 = 0$$

$$\frac{4GM}{R} = c^2 \tag{3.2.48}$$

$$M = \frac{Rc^2}{4G} \tag{3.2.49}$$

$$M = \frac{R}{2G}c_a^2 = \frac{R}{2G}\left(\frac{c_m}{\sqrt{2}}\right)^2 = \frac{R}{2G}c^2 \tag{3.2.50}$$

For

$c \rightarrow c_a = \frac{c_m}{\sqrt{2}} = \frac{c}{\sqrt{2}}$

The radius which make E minimum in (3.2.8) requires maximum mass given by (3.2.13). The condition for maximum mass resembles Chandrasekhar Limit. The equilibrium condition requires, here, E to be real. This makes the critical radius to be dependent on G and h as shown by equations (3.2.27) and (3.2.9). Equation (3.2.26) shows that this is the minimum equilibrium radius. But according to equations (3.2.28) and (3.2.29) the maximum critical mass depends two on G and h. The pence of these tow parameters reflects the quantum gravitational nature of the steller mass. The equilibrium condition is also studied by considering the effect of pressure force in relation to centrifugal force.

Equations (3.2.38) According to pressure force act as a centrifugal force which counter balance the gravity force. considering particles as strings it was shown by equation (3.2.37) that equilibrium takes place when kinetic and potential energy equal each other. The mass which makes E minimum also tackled in equations (3.2.34, 3.2.35 and 3.2.36). The mass at which E is minimum is given by equation conforms with that of black hole as shown by eqn (3.2.50)

3.3 Flat rotation curve without dark matter: the eneralized Newton's law of gravitation.

This paper is devoted to solve universe density problems: the flat rotation curve using generalized Newton's law of gravitation need not require dark matter. The potential is logarithmic at large distances from the center of the galaxy. The gravitomagnetic force generates constant velocity observed in flat rotation curve at very large distances from the center of the galaxy. Dynamical matter arising from moving stars far away from the center of the galaxy has large contribution to the mass of the galaxy. [38]

Gravitomagnetic force:

In the generalized Newton's law of gravitation developed by Arbab (2010, 2012), one has

$$F = -\frac{GMm}{r^2} - \frac{\pi m v^4}{v_c^2 r}, \qquad (3.3.1)$$

where v_c is some characteristic velocity. This force is the gravitational analogue of Lorentz force of electromagnetism. The second term in Eq. (3-3-1) accounts for the gravitomagnetic force arising from the motion of the orbiting mass, *m*. For ordinary velocities Eq. (3-3-1) reduces to the ordinary Newton's law of gravitation. However, since we are interested in the behavior of matter at very large distances where the object (star) speed is so big, the situation will be different, as we will describe below. For a circular motion, one has

$$\frac{mv^2}{r} = \frac{GMm}{r^2} + \frac{\pi m v^4}{v_c^2 r} \qquad (3.3.2)$$

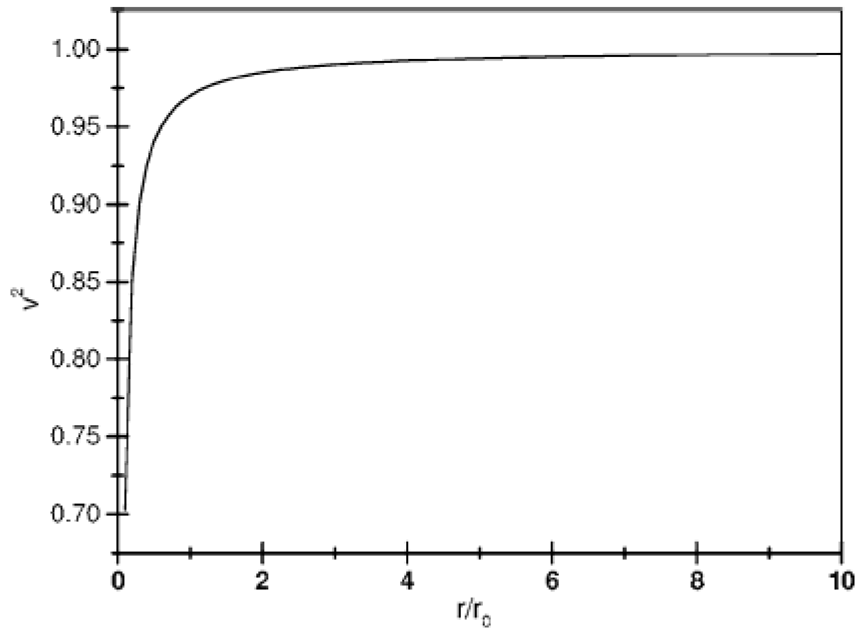


Fig. A flat rotation curve without dark matter: v^2/v_c^2 versus r/r_0

Equation (3-3-2) is solved to give

$$v^2 = \frac{v_c^2}{2\pi}\left(1 \pm \sqrt{1 - \frac{4\pi GM}{v_c^2 r}}\right) \qquad (3.3.3)$$

Let us now consider the case when, $r > \frac{4\pi GM}{v_c^2} = r_0$ By Taylor expanding the square root in Eq. (3-3-3) and neglecting the higher orders terms, we obtain the two solutions

$$v_-^2 \approx \frac{GM}{r}, \tag{3.3.4}$$

and

$$v_+^2 \approx \frac{v_c^2}{\pi}\left(1 - \frac{\pi GM}{v_c^2 r}\right). \tag{3.3.5}$$

Equation (5) yields

$$v_+ \approx \frac{v_c}{\sqrt{\pi}}\left(1 - \frac{\pi GM}{2v_c^2 r}\right). \tag{3.3.6}$$

Equation (3-3-4) gives the ordinary Kepler velocity. Eq. (3.3.5) gives a velocity distribution that in agreement with the observed flat rotation curve. The flattening of velocity curve becomes predominant at distances much bigger than a few parsecs. The gravitomagnetic force resolves the dark matter problem associated with Kepler law.

Inserting Eq. (3.3.3) in Eq. (3.3.1), we will get

$$F = -\frac{mv_c^2}{2\pi r}\left(1 \pm \sqrt{1 - \frac{4\pi GM}{v_c^2 r}}\right). \tag{3.3.7}$$

Taylor expansion of the square root in Eq. (3-3-7), neglecting the higher orders terms, yields, for $r > r_0$, the two forces

$$F_- \approx \frac{GMm}{r^2}, \tag{3.3.8}$$

and

$$F_+ \approx \frac{mv_c^2}{\pi r} + \frac{GMm}{r^2} \tag{3.3.9}$$

The first term in Eq. (3.3.9) may suggest that matter at large distances may not be gravitationally coupled to the central galaxy, and another law holds instead $\left(\frac{mv_c^2}{\pi r}\right)$ Recently, Kuhn and Kruglyak (1987) assumed a modified attractive force between two mass of a similar law (except for a sign) to that in Eq. (3.3.9) claiming to explain the mass discrepancy at many distance scales. While the first term in Eq. (3.3.9) gives an attractive

force that dominates at large distances, the second parts is related to some kind or repulsive force Equation (3.3.9) can be written as

$$F_+ = -\frac{GMm}{r^2}\left(\frac{v_c^2 r}{\pi GM} - 1\right) \tag{3.3.10}$$

Equation (3-3-10) can be written in the informative form

$$F_+ = -\frac{GM_{\text{eff.}}m}{r^2} \tag{3.3.11}$$

where

$$M_{\text{eff.}} = \left(\frac{v_c^2 r}{\pi G} - M\right) \tag{3.3.12}$$

and M is the visible mass, and we call $M_{\text{eff.}}$ the active (effective) mass. The term $\left(\frac{v_c^2 r}{\pi GM} - 1\right)$ Eq. (3-3-10) can be seen as a scaling of the visible mass of the galaxy. One can treat the term $\frac{v_c^2 r}{\pi G}$ as a dynamical mass present at a distance r (if one prefers, we may call it dark matter and M as passive) static (matter). In the relativistic hydrodynamics, the pressure equally contributes to the mass density of the fluid. As demonstrated by Ehlers et al. (2005), pressure like mass is a source of gravity too. Hence, the first term in Eq. (3-3-12) could reflect this contribution. This extra term was absent in the Newtonian theory. The absence of this term in Newtonian theory could reflect the fact that it is related to the curvature of the space that wasn't considered in Newtonian formulation.

This gravitomagnetic model is consistent with the prediction of the general theory of relativity. Equation (3-3-12) suggests that the effective mass increases linearly with r. This situation mimics the effect of dark matter at large distances. It seems that the gravity force at large scales becomes repulsive giving rise to the observed cosmic acceleration. If we now write $dM_{\text{eff.}} = \rho_{\text{eff.}} dV$, where $\rho_{\text{eff.}}$ is the galaxy density, and V is the volume, then Eq. (3-3-12) yields

$$\rho_{\text{eff.}} = \frac{v_c^2}{4\pi^2 G}\frac{1}{r^2}. \qquad (3.3.13)$$

This equation can be used to estimate the present density of the universe if we use $r_p = 2.7 \times 10^{26}$m. This amounts to $\rho_p = 3.1\times10^{-27}$ kg/m^3.

Equation (3-3-11) can be inverted and equally suggests that the gravitational constant increases with distance at large scale. This is evident if we write Eq. (3-3-10) as

$$F_+ = -\frac{G_{\text{eff.}} Mm}{r^{22}}, \quad G_{\text{eff.}} = \left(\frac{v_c^2 r}{\pi M} - G\right). \qquad (3.3.14)$$

Recently, Ehlers et al. (2005) have investigated the effect of a running gravitational constant on the flat rotation curve of galaxies and came up with similar conclusion. The dynamical mass of the universe can be estimated from $\frac{v_c^2 rp}{\pi G} \sim 10^{53}$ kg. taking $r_p \sim 10^{26}$m and $v_c = c$.

This estimate is of the same order of the mass of the present universe deduced by McCulloch (2014). We can associate a potential energy with the force in Eq. (3-3-9) of the form

$$U_+ = \frac{mv_c^2}{\pi}\, In\left(\frac{r}{r_S}\right) + \frac{GMm}{r} \qquad (3.3.15)$$

Where r_S is some reference distance from the galaxy. It is interesting to see that when $r < r_S$, the potential in U_+ is positive. However, when $r > r_S$, the first term in Eq. (3-3-15) becomes negative. At $r = r_S$ the potential is a pure repulsive Newtonian. These different situations correspond to the behavior of matter under these outlined regions. The potential energy in Eq. (3-3-15) is that one of a star at large distances from the central galaxy. It is interesting to note that such potential is recently suggested by Fabris and Campos (2009) to account for the spiral galaxies rotation curves. They attributed such a potential to arise from string theories or effective models of gravity due to quantum effects. Recently, Kinney and Brisudova (2000) introduce such a potential as representing a non-gravitational force component coupled to baryon number as a charge. They further assumed that this force becomes dominant on very large scales. The potential energy in Eq. (3-3-15) at very large distances reduces to

$$U_+ = \frac{mv_c^2}{\pi} \, In \left(\frac{r}{r_S}\right). \qquad (3.3.16)$$

A logarithmic potential energy usually arises from a singular isothermal sphere with a density profile given by $\rho = A\left(\frac{r_a}{r}\right)^2$, where ρ_0 and r_a are constants. This yields the potential, $V = 4\pi G\rho_0 r_a^2 \, \mathrm{In}\left(\frac{r}{r_a}\right)$ and a circular velocity independent of r. Moreover, the density ρ follows the same pattern as the one in Eq. (3-3-13).

Modified gravity model (MOND)

In this model it is assumed that the acceleration (a) of a mass having an acceleration beyond a characteristic acceleration (a_0), one has (Milgrom 1983)

$$a = \frac{\sqrt{GMa_0}}{r}, \qquad (3.3.17)$$

where a0 = 1.2 × 10−8 m/s−2. In a recent work, Arbab (2004) has shown that such an acceleration reflects the quantum nature of the present universe. Now equating the acceleration in Eq. (3-3-17) to the centripetal acceleration yields

$$v = \sqrt[4]{GMa_0} \qquad (3.3.18)$$

We thus see that the velocity v is independent of the distance r. This agrees with the observed flat rotation curve for spiral galaxies. Using Eq. (3-3-9), the acceleration of the mass m can be written as

$$a \approx -\frac{v_c^2}{\pi r} + \frac{GM}{r^2} \qquad (3.3.19)$$

At sufficiently large distances, this gives an attractive acceleration

$$a \approx -\frac{v_c^2}{\pi r} \qquad (3.3.20)$$

Comparing this with Eq. (3-3-17) yields

$$a \approx -\frac{v_c^4}{\pi^2 GM}. \qquad (3.3.21)$$

Apparently, the MOND acceleration is tantamount to gravitomagnetic acceleration. MOND as well as our generalized Newton's law of gravitation are capable of explaining the flat rotation curve exhibited by spiral galaxies. The solution for the flat rotation curve shows that gravitation should be govern by the generalized Newton's law of gravitation that works well at very large distances (cosmic). It is the nature of gravity that makes the rotation curve of the spiral galaxies flat. This new law is applicable at cosmological level.

Concluding remarks

The presence of the gravitomagnetic force leads to a logarithmic gravitational potential. This gravitational potential produces a velocity profile, for stars in a galaxy, in agreement with the presently observed one. The mass density of a galaxy is that of a singular isothermal sphere.

3.4 Quantum and Generalized Special Relativistic Model for Electron Charge Quantization.

This work was done by Hassabo. The electron and elementary particles charges are quantized using Hamiltonian in a curved space-time at vacuum stage of the universe, using quantum spin angular momentum and Klein-Gordon equation beside generalized special relatively. Electron charge is found to be quantized and the electron self-energy is finite. The electron radius is found to be extremely small than atomic particles. According to GR, the time – component of the metric is given by [39]

$$g_{00} = -\left(1 + \frac{2\emptyset_{\mathrm{g}}}{c^2}\right) \tag{3.4.1}$$

Where $\emptyset_{\mathrm{g}}$ is the gravity potential per unit mass and is related to electric potential $\emptyset$ and electron charge e through the relation

$$\emptyset_{\mathrm{g}} = \frac{U}{m} = \frac{e\emptyset}{m} \tag{3.4.2}$$

This comes from the fact that any energy form including electric can generate gravity field

Thus equation (3.4.1) becomes

$$g_{00} = -\left(1 + \frac{2e\emptyset}{mc^2}\right) \tag{3.4.3}$$

At early stages of the universe electric charge is generated due to the electromagnetic (e.m) field at vacuum stage. This requires minimizing the Hamiltonian (H) w.r.t electric potential $\emptyset$ to find the electric charge and see how it is generated. Since the Hamiltonian part representing charge itself can be neglected as for as they are independent of $\emptyset$. The charge field interactions are neglected for simplicity. One also assumes electric charge to be at rest. This means that the magnetic field is not generated. Therefore

$$A_0 = \emptyset \ , \qquad A_i = 0\, , i = 1,2,3, \ldots \tag{3.4.4}$$

To find the Hamiltonian in curved space, one can generalized the linear space form

$$H = \eta^{002}\varepsilon_i(\,\partial_i A_0 - \,\partial_0 A_i)^2 \tag{3.4.5}$$

to be written in a curved space in the form

$$H = g_{00}^2 \varepsilon_i (\partial_i A_0 - \partial_0 A_i)^2 \tag{3.4.6}$$

From Equation (3.4.3) one gets

$$H = \left(1 + \frac{2e\emptyset}{mc^2}\right)^2 (\nabla\emptyset)^2 \tag{3.4.7}$$

Thus minimization condition requires

$$\frac{dH}{d\emptyset} = \left(1 + \frac{2e\emptyset}{mc^2}\right)\left(\frac{2e}{mc^2}\right)(\nabla\emptyset)^2 = 0$$

$$1 + \frac{2e\emptyset}{mc^2} = 0, \quad \emptyset = -\frac{mc^2}{2e} \tag{3.4.8}$$

Assuming the mass energy to be resulting from electric field energy density E_d where $E_d = \varepsilon_0 E^2$ Inside electron of radius r_0, one gets

$$mc^2 = E_d V = \varepsilon_0 E^2 \frac{4}{3}\pi r_0^3 = \frac{\varepsilon_0 e^2}{16\pi^2 \varepsilon_0^2 r_0^4}\frac{4}{3}\pi r_0^3 = \frac{e^2}{12\pi\varepsilon_0 r_0} \tag{3.4.9}$$

The vacuum energy potential which results from electric charge becomes

$$U_v = -e\emptyset = \frac{e^2}{12\pi\varepsilon_0 r_0} \tag{3.4.10}$$

according to a vacuum energy potential which takes the form

$$U_v = \rho_v \left[\left(\frac{\pi^2 n^2}{x_0^2 n_0^2}\right) + \omega^2\right]^{-3} \tag{3.4.11}$$

Thus combining Equations (3-4-10) and (3-4-11) yields

$$\frac{e^2}{12\pi\varepsilon_0 r_0} = \left[\left(\frac{\pi^2 n^2}{x_0^2 n_0^2}\right) + \omega^2\right]^{-3}$$

Thus the electric charge is given by

$$e = \left[\left(\frac{\pi^2 n^2}{x_0^2 n_0^2}\right) + \omega^2\right]^{-3/2} (12\pi\varepsilon_0 r_0)^{1/2} \tag{3.4.12}$$

Setting to be equal to zero, for simplification. The electric charge is given by

$$e = (12\pi\varepsilon_0 r_0)^{1/2} (x_0 n_0 / n\pi)^2 \tag{3.4.13}$$

r_0 is the electron radius and x_0 is the universe radius. Thus, the electron radius can be found by assuming that the electron energy results from its spinning, where the spin angular momentum is given by

$$L_s = \hbar[s(s+1)]^{1/2} = \frac{\sqrt{3}\hbar}{2} \tag{3.4.14}$$

Where for electron

$$s = \mp\frac{1}{2} \tag{3.4.15}$$

At vacuum stage we choose minimums lower value.

$$L_s = \frac{1}{2}\hbar \tag{3.4.16}$$

Assume that rest mass is neglected in relativistic expression to get

$$mc^2 = E = cp \tag{3.4.17}$$

$$mc = p \tag{3.4.18}$$

The same relation can hold for Newtonian mechanics by considering wave nature of electrons, where the maximum velocity v_m is related to the effective value v through the relations

$$v = \frac{v_m}{\sqrt{2}} \tag{3.4.19}$$

By assuming

$$p = mv$$

Thus the Newtonian expression for free particle takes the form

$$E = \frac{1}{2}mv_m^2 = mv^2 = \frac{m^2v^2}{m} = \frac{p^2}{m} \tag{3.4.20}$$

If one believes in relativistic energy mass relation, one gets

$$mc^2 = E = \frac{p^2}{m}$$

Thus one gets:

$$m^2c^2 = p^2 \ , \ mc = p \tag{3.4.21}$$

Since the momentum p is related to L according to the

$$p = mv = \frac{mvr_0}{r_0} = \frac{L_s}{r_0} \quad (3.4.22)$$

It follows from equation (3-4-21) that

$$\frac{L_s}{r_0} = mc$$

Using equation (3-4-16) one gets

$$r_0 = \frac{L_s}{mc} = \frac{\hbar}{2mc} \quad (3.4.23)$$

Substituting the values of h, m and c, the electron radius can be calculated. The electric charge is assumed to be born at very early stages of the universe where vacuum exist and the minimum radius is x_0 where [40]

$$x_0 = 26.635 \times 10^{-3} m$$

The electric charge is numerically given by $e = 1.6 \times 10^{-19} C$. It can be obtained by adjusting the quantum numbers n and n_0 to be

$$\frac{n}{n_0} = \frac{\pi}{x_0} [e/(12\pi\varepsilon_0 r)^2]^{-1/3} \quad (3.4.24)$$

Similarly, the charges of quarks and charged leptons can be found by adjusting the quantum numbers n and n_0

Equation (3-4-10) shows that vacuum energy is repulsive due to the existence of positive sign. This can form with cosmological models, which suggests repulsive vacuum energy. Inflation models suggest also very large vacuum energy. If one believes in this model, such that

$$\emptyset = \frac{U_v}{m_0} \rightarrow \frac{c^2}{2} \quad (3.4.25)$$

in this case according to generalized special relativity model the electron mass is given by

$$m = m_0\left(1 - 2\emptyset_g/c^2\right) \rightarrow \text{large} \quad (3.4.26)$$

Assume for simplicity

$$m = 10^{13} m_0 = 10^{13} \times 9 \times 10^{-31} = 9 \times 10^{-18} kg \quad (3.4.27)$$

From Equations (3-4-16), (3-4-21) and (3-4-22) the electron radius can be given to be

$$r_0 = \frac{\hbar}{2mc} = \frac{h}{4\pi mc} = \frac{6.63 \times 10^{-34}}{4\pi \times 9 \times 10^{-18} \times 3 \times 10^{8}}$$

$$r_0 = 1954 \times 10^{-26} m \qquad (3-4-28)$$

Which is quite reasonable as far as nucleus or proton radius for very light atoms are

$$r_0 = 10^{-14} \qquad r_p = 10^{-16}$$

Vacuum energy is obtained by minimizing $\emptyset$ and is equated with that obtained from electric energy density according to equations (3-4-9),(3-4-10). The expression for classical angular momentum and quantum spin angular momentum are used to find electron radius. The electron charge is shown to be quantized according to equation (3-4-13) due to the existence of two quantum numbers n and $n_0$which can be adjusted early to find the value of e. The radius of the electron can be found by using equations (3-4-23),(3-4-28). The values obtained are very small compared to proton and nuclear radius which is quite reasonable. According to equations (3-4-9),(3-4-28), the electron self-energy is finite.

3.5 The Generalized Newton's Law of Gravitation versus the General Theory of Relativity.

The precession of binary pulsars can be accounted for as due to the existence of gravitomagnetism only, thus gravitomagnetism is equivalent to a curved space-time. The preces-sion of the perihelion of planets and binary pulsars may be interpreted as due to the spin of the orbiting planet (*m*) about the Sun (*M*). The spin (*S*) of planets is found to be related to their orbital angular momentum (*L*) by a simple formula. [41]

The General Theory of Relativity (GTR)

Einstein assumed the gravitational field, to be result from curvature of space-time induced by a massive object [42]. The effective gravitational potential of the object of mass m moving around a massive object of mass M takes the form [43]

$$U(r) = \frac{GMm}{r} + \frac{L^2}{2mr^2} - \frac{GML^2}{c^3mr^3} \qquad (3.5.1)$$

And the force, $F = \frac{\partial U}{\partial r}$, can be written as

$$F(r) = \frac{GMm}{r^2} + \frac{L^2}{mr^3} - \frac{3GML^2}{c^2mr^4} \qquad (3.5.2)$$

where L is the orbital angular momentum of the mass m. This inverse-cubic energy term in Equation (3-5-1) causes elliptical orbits to precess gradually by an angle $\delta\varphi$ per revolution [44]

$$\delta\varphi = \frac{6\pi GM}{c^2a(1-e^2)} \qquad (3.5.3)$$

where e and a are the eccentricity and semi-major axis of the elliptical orbit, respectively. This is known as the anomalous precession of the planet Mercury.

Another prediction famously used as evidence for GTR, is the bending of light in a gravitational field. The deflection angle is given by [45]

$$\delta\varphi = \frac{4GM}{c^2b} \qquad (3.5.4)$$

where b is the distance of closest approach of light ray to the massive object. Therefore, the gravitomagnetic force is equal to $\frac{\pi}{3}$ of the GTR force. Whether, the gravitational phenomena are in full agreement with our gravitomagnetic model or with GTR is a subject of the present and future observations.

The Generalized Newton Law of Gravitation

Newton law of gravitation can be written, as a Lorentz-like law, as [46]

$$F(r) = mE_g + mv \times B_g\ , \qquad E_g = a = \frac{v^2}{r} \qquad (3.5.5)$$

Where

$$B_g = \frac{v \times E_g}{c^2} \tag{3.5.6}$$

Thomas introduced a factor $\frac{1}{2}$ to account for the spin-orbit interaction in hydrogen atom [47]. Here B_g is measured in S^{-1} . to convert it to rad/sec, we multiply it by 2π. Hence, the gravitomagnetic force becomes

$$F_m(r) = \frac{\pi m v^4}{c^2 r}, a = \frac{v^2}{r}, v^2 = \frac{GM}{r} \tag{3.5.7}$$

The gravitomagnetic field is divergenceless, since

$$\nabla . B_g = \frac{1}{c^2} \nabla . (v \times E_g)$$

$$\nabla . B_g = \frac{1}{c^2} E_g . (\nabla \times v) - \frac{1}{c^2} \nabla . (v \times E_g)$$

$$\nabla . B_g = \frac{1}{c^2} v . \frac{\partial B_g}{\partial t} = -\frac{1}{c^2} \frac{\partial}{\partial t} \left(v . B_g \right) = 0$$

This implies that the gravitomagnetic lines curl around the moving mass (gravitational current) creating it. This may also rule out the existence of negative mass. Therefore, as no magnetic monopole exits; no gravitomagnetic monopole (antigravity) exits.

The angular momentum is defined by $L = mvr$, so that Equation (3-5-7) becomes

$$F_m(r) = \frac{\pi G M L^4}{m c^2 r^4} \tag{3.5.8}$$

The second term in Equation (3-5-2) is due to the centrifugal term arising from a central force field. In polar coordinates the force is written as

$$ma = m\left(\ddot{r} - r\dot{\theta}^2\right)\hat{e}_r + m\left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\hat{e}_\theta . \tag{3.5.9}$$

For a central force the second term vanishes. It yields $\dot{\theta} = \frac{L^2}{mr^2}$, so that the first term becomes

$$ma_r = m\ddot{r} \frac{L^2}{mr^3} \tag{3.5.10}$$

Substituting Equation (3-5-10) in Equation (3-5-5) yields the full effective central force, owing to gravitomagnetism, as

$$F(r) = -\frac{GMm}{r^2} + \frac{L^2}{mr^3} - \frac{\pi GML^2}{mc^2r^4} \tag{3.5.11}$$

The corresponding potential will be

$$U(r) = -\frac{GMm}{r} + \frac{L^2}{2mr^3} - \frac{\pi GML^2}{mc^2r^4}$$

Comparison of Equations (3-5-2) and (3-5-11) reveals that the gravitomagnetic force is equal to $\frac{\pi}{3}$ of the curvature force. Consequently, the generalized Newton law of gravitation and the general theory of relativity produce the same gravitational phenomena.

The gravitomagnetic force term, the last term in Equation (3-5-11), can be written as

$$\frac{\pi GML^2}{mc^2r^4} = \frac{\pi G^2M^2m}{c^2r^3}, \qquad where, \quad v^2 = \frac{GM}{r} \tag{3.5.12}$$

Finally, Equation (3-5-11) can be written as

$$F(r) = -\frac{GMm}{r^2} + \frac{J_{eff}^2}{mr^3} \tag{3.5.13}$$

Where

$$J_{eff}^2 = L^2 - \left(\frac{\sqrt{\pi}GMm}{c}\right)^2 \tag{3.5.14}$$

Precession of Planets and Binary Pulsars

Owing to the above equivalence between gravitomag- netism and GTR, we interpret the precession of the peri- helion of planets and binary pulsars as a Larmor-like precession, and not due to the GTR interpretation as due to the curvature of space-time. We may attribute this pre- cession as due to the precession of gravitational moment (mass) in a gravitomagnetic field induced by the massive objects (Sun). In electromagnetism, the Larmor prece- ssion is defined by [48]

$$\omega = \frac{e}{2m} B. \quad (3.5.15)$$

While in gravitation (since B_g $is\ in\ S^{-1}\ and\ e \Leftrightarrow m$) it is defined as [49]

$$\omega_g = 2\pi = \left(\frac{B_g}{2}\right) = \frac{\pi v^3}{rc^2}, \qquad B_g = \frac{va}{c^2} = \frac{v^3}{rc^2}, \quad (3.5.16)$$

Where ω_g $is\ in\ rad/seec)\ and$

$$\frac{v^2}{r} \quad (3.5.17)$$

The precession rate in Equation (3-5-16) can be written as

$$\omega_g = \pi \left(\frac{2\pi GM}{Tc^2 r}\right) = \frac{S\emptyset_g}{T}, \quad (3.5.18)$$

Where $T = \frac{2\pi r}{v}$ is the period of revolution. This corresponds to a precession angle of

$$S\emptyset_g = \pi \left(\frac{2\pi GM}{c^2 r}\right) rad/s, \quad (3.5.19)$$

That is equal to $\frac{\pi}{3}$ of the curvature effect, and for elliptical orbit $r = a(1 - e^2)$.

Deflection of α-Particles by the Nucleus

We would like here to interpret the deflection of light by the Sun gravity in an analogous way to the deflection of $\alpha -$ particles by the nucleus, without resorting to the GTR calculation. The deflection angle of α-particles by a nucleus is given by [50]

$$\Delta\theta_e = \frac{4keQ}{mbv^2} \quad (3.5.20)$$

where Q is the nucleus charge, v the α -particle speed, k Coulomb constant, and b the impact factor. The corre- sponding gravitational analog for the deflection of light will be.

$v \rightarrow c, e \rightarrow m\ , Q \rightarrow M\ , k \rightarrow G$ [51]

$$\Delta\theta_e = \frac{4kGM}{bc^2} \quad (3.5.21)$$

without resorting to GTR calculation. Recall that, accord- ing to Equivalence Principle, all particles in gravity acce- lerate without reference to their mass (whether massive or massless). Therefore, it doesn't matter whether light has a mass or not. The relation in Equation (3-5-21) is the same as the relation obtained by GTR as in Equation (3-5-4). The minimum distance α particles can approach the nucleus is given by equating the kinetic energy and the Coulomb potential energy that yields the relation

$$b_e = \frac{2kq_1q_2}{mv^2} \tag{3.5.22}$$

In gravitation and for light scattered by the Sun gravity, the above relation gives ($q_1 \to m\ ,\ q_2 \to M\ and\ k \to G$)

$$b_g = \frac{2GM}{c^2} \tag{3.5.23}$$

This is nothing but the Schwarzschild distance that no particle can exceed. Therefore, the complete analogy be- tween gravitation and electricity is thus realized. In this context, we have shown recently that the Larmor dipole radiation has a gravitational analogue [20]. Similarly, the same analogy exists between hydrodynamics and electro- magnetism [52].

The spin of planets had been known since long time (1851) that was demonstrated by Foucault's pendu- lum. According to our model one assumes the gravito- magnetism to be produced by moving planets as the magnetic field produced by moving charge. We then obtained the gravitational Ampere's and Faraday's laws of gravi- tomagnetism. The gravitomagnetic moment of a planet due to its orbital motion is given by [53]

$$\mu_L = \frac{v^3r^2}{2G} \tag{3.5.24}$$

For circular orbit, Equation (24) yields

$$\mu_L = \left(\frac{M}{2m}\right) L \tag{3.5.25}$$

In a similar manner the gravitomagnetic moment due to spin will be twice the above value (analogous to electromagnetism)

$$\mu_S = g_S \left(\frac{M}{2m}\right) S \tag{3.5.26}$$

where g_S defines some gyro-gravitomagnetic ratio that is independent of the planet's mass. If we assume the precession of planets is a spin-orbit interaction, then we can equate $\mu_S B_g$ (assuming the angle to be zero) to the potential term arising from the gravitomagnetic force in Equation (3.5.11). This yields, for circular orbit,

$$S = \left(\frac{4\pi}{3g_S}\frac{m}{M}\right) L. \quad S = \left(\frac{4\pi}{3g_S}\frac{Gm^2}{v}\right) \tag{3.5.27}$$

3.6 Generation of Elementary Particles inside Black Holes at Planck Time

Using string theory by treating particles as quantum strings and using generalized special relativity a useful expression for self-energy was found. The critical radius of a star when particles are created is that of the black hole. The elementary particles formation should also take place at Planck time which also conforms with that proposed by big bang model. [16]

Model Universe with A cosmology Constant:

Generalized special relativistic energy (GSR) energy, relation is given by

$$E = m_0 c^2 \left(1 + \frac{2\varphi}{c^2}\right)\left(1 + \frac{2\varphi}{c^2} - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} \tag{3.6.1}$$

Where the Newtonian potential takes the form

$$\varphi = -\frac{MG}{R} \tag{3.6.2}$$

$$E = m_0 c^2 \left(1 + \frac{2MG}{Rc^2}\right)\left(1 - \frac{2MG}{Rc^2} - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} \tag{3.6.3}$$

Minimizing E w.r.t M yields

$$\frac{dE}{dM} = m_0 c^2 \left[\frac{-\frac{2G}{Rc^2}}{\left(1 - \frac{2MG}{Rc^2} - \frac{v^2}{c^2}\right)^{\frac{1}{2}}} + \frac{\left(1 - \frac{2MG}{Rc^2}\right)\left(-\frac{1}{2}\right)\left(\frac{-2G}{Rc^2}\right)}{\left(1 - \frac{2MG}{Rc^2} - \frac{v^2}{c^2}\right)^{\frac{2}{3}}}\right] = 0$$

Thus

$$\frac{-\frac{2G}{Rc^2}\left(1 - \frac{2MG}{Rc^2} - \frac{v^2}{c^2}\right) + \frac{2G}{Rc^2}\left(1 - \frac{2MG}{Rc^2}\right)}{\left(1 - \frac{2MG}{Rc^2} - \frac{v^2}{c^2}\right)^{\frac{2}{3}}} = 0$$

If one consider

$$v^2 \ll c^2$$

$$-\frac{2G}{Rc^2}\left(1 - \frac{2MG}{Rc^2} - \frac{v^2}{c^2}\right) + \frac{2G}{Rc^2}\left(1 - \frac{2MG}{Rc^2}\right) = 0$$

$$-\frac{G}{Rc^2}\left(1 - \frac{2MG}{Rc^2}\right) = 0$$

The requires

$$\frac{2MG}{Rc^2} = 1$$

$$2MG = Rc^2 \tag{3.6.4}$$

Thus the mass which makes E minimum is

$$M = \frac{Rc^2}{2G} \tag{3.6.5}$$

Consider also the generalized special relativity energy E equilibrium condition by minimizing $E$with respect to radius r from equation (3-6-3), when the star particles speed are small compared to speed of light

$$\frac{v^2}{c^2} \ll 1$$

Thus

$$E = m_0 c^2 \left(1 - \frac{2MG}{Rc^2}\right)^{\frac{1}{2}} \tag{3.6.6}$$

$$\frac{dE_r}{dM} = m_0 c^2 \left(\frac{2MG}{r^2 c^2}\right)\left(\frac{1}{2}\right)\left(1 - \frac{2MG}{rc^2}\right)^{-\frac{1}{2}}$$

$$\frac{dE_r}{dM} = \frac{m_0 c^2 \left(\frac{2MG}{r^2 c^2}\right)\left(\frac{1}{2}\right)\left(1 - \frac{2MG}{rc^2}\right)}{\left(1 - \frac{2MG}{rc^2}\right)^{\frac{3}{2}}} = 0$$

Thus the radius which makes E minimum is given by

$$1 - \frac{2MG}{rc^2} = 0$$

The critical radius is thus given by

$$r_c = \frac{2MG}{c^2} \tag{3.6.7}$$

(This is the black hole radius)

But the critical mass is given by equation (3-6-7), i.e.

$$M = m_c = \frac{c^2 r_c}{2G} \tag{3.6.8}$$

Hence from (3-6-8)

$$2m_c G = c^2 r_c \tag{3.6.9}$$

The condition governing the equilibrium of the universe, from (3-6-9) and (3-9-4) we get

$$\frac{m_c R}{M r_c} = 1 \tag{3.6.10}$$

Where M and R are the mass and radius of the universe respectively. The mass of the universe (M = 2.2 × 1056g) and the radius (R = 1.6 × 1028cm) According to generalized general relativity (GGR) there is a short range repulsive gravitational force beside long range attractive gravity force given by [54]:

$$\varphi_s = \frac{c_1}{r} e^{-\frac{r}{r_c}} \qquad (3.6.11)$$

$$\varphi_L = -\frac{GM}{r} \qquad (3.6.12)$$

$$\varphi = \varphi_s + \varphi_L = \frac{c_1}{r} e^{-\frac{r}{r_c}} - \frac{GM}{r}$$

$$\varphi = \frac{1}{r}\left[c_1 e^{-\frac{r}{r_c}} - \frac{GM}{r}\right] \qquad (3.6.13)$$

For small radius r or strictly speaking small $\frac{r}{r_c}$

$$e^{-\frac{r}{r_c}} = 1 - \frac{r}{r_c} \qquad (3.6.14)$$

Hence

$$\varphi = \frac{1}{r}\left[c_1 - c_1 \frac{r}{r_c} - GM\right] \qquad (3.6.15)$$

To secure finite self-energy φ at small r, one requires

$$c_1 = GM \qquad (3.6.16)$$

Thus the star self-energy is given by

$$\varphi = -\frac{c_1}{r_c} = \frac{GM}{r_c} \qquad (3.6.17)$$

Since the star is a particle at rest thus the minimization of E requires (see equation (3-6-2), (3-6-4) and (3-6-17))

$$\varphi = -\frac{c_1}{r_c} = -\frac{c^2}{2} \qquad (3.6.18)$$

For photon (v = c) thus one gets

$$\varphi = \frac{c^2}{2} \qquad (3.6.19)$$

From equation (3 -6 -17) and (3 - 6 -18)

$$\varphi = -\frac{GM}{r_c} = -\frac{c^2}{2} \qquad (3.6.20)$$

Thus the critical radius is given by

$$r_c = \frac{2GM}{c^2} \qquad (3.6.21)$$

(This is the black hole radius)

Since rc should be small as shown by equation (3-6-14), thus requires

$$r_c < 1 \quad , \qquad \frac{2GM}{c^2} < 1$$

$$M < \frac{c^2}{2G} \qquad (3.6.22)$$

Thus there is a critical mass

$$M_c = \frac{c^2}{2G} \qquad (3.6.23)$$

Above it the particle rest mass energy cannot be formed form potential. We see from equation (3-6-4) that the present radius of the universe should be

$$R_0 = \frac{2GM}{c^2} \sim 10^{28} cm \qquad (3.6.24)$$

Which conforms to observations. Consider a star as consisting of photons gas, such that the critical radius is related to the wave number according to the relation

$$p = m_0 c = \hbar k = \frac{\hbar}{r_c} \quad , \qquad k = \frac{1}{r_c} \qquad (3.6.25)$$

For oscillating string the energy takes the form

$$E_{r_c} = m_0 c = \frac{\hbar c}{r_c} \qquad (3.6.26)$$

Hence

$$r_c = \frac{\hbar}{m_0 c} \qquad (3.6.27)$$

The photon which obeys quantum laws equations (3-6-19) and (3-6-1) gives

$$E = \frac{2m_0c^2}{\sqrt{2-1}} = 2m_0c^2 \qquad (3.6.28)$$

This conforms with the fact that photons can produce particle pairs. Newton's law of potential gives

$$E_{r_c} = U(r) = -G\frac{m_1m_2}{r} \qquad (3.6.29)$$

Gravity force is also given by

$$F = -G\frac{m_1m_2}{r^2}.\frac{r}{r} \qquad (3.6.30)$$

If

$$m_1 = m_2 = m_c$$

Thus (3-6-26) and (3-6-29) given

$$E_{r_c} = \frac{Gm_c^2}{r_c} = \frac{\hbar c}{r_c} \qquad (3.6.31)$$

Therefore

$$\hbar c = Gm_c^2 \qquad (3.6.32)$$

Hence

$$m_c = \left(\frac{\hbar c}{G}\right)^{\frac{1}{2}} \qquad (3.6.33)$$

Where

$$\hbar = 1.05 \times 10^{-27} erg.s, c = 3 \times 10^{10} cm.S^{-1}, G = 6.67 \times 10^{-8} erg.cm.g^{-1}$$

$$m_c = \left(\frac{\hbar c}{G}\right)^{\frac{1}{2}} \sim 2.2 \times 10^{-5} g \qquad (3.6.34)$$

(Equivalent Planck's mass)

Which matches the proposed value. The same equation applies to Planck's length, namely

$$R_P = \frac{G_PM_P}{c^2} \sim 10^{-33} cm \qquad (3.6.35)$$

(Planck's length)At distances smaller than this scale the gravitational interaction should be stronger than the quantum effects [2].Also the critical distance rc is equal

$$r_c = \frac{\hbar}{m_c c} = \left(\frac{G\hbar}{c^3}\right)^{\frac{1}{2}} \sim 1.6 \times 10^{-33} cm \qquad (3.6.36)$$

(Equivalent Planck's length) One can calculate the critical density σc of the material when the particles are considered as a hollow sphere surrounded by thin layer or membrane. In this case the surface density is given by

$$\sigma = \frac{m_c}{A} \;,\; m_c = \frac{\hbar}{r_c c} \;,\; A = 4\pi r_c^2 \qquad (3.6.37)$$

$$\sigma = \left(\frac{\hbar}{r_c c}\right)\left(\frac{1}{4\pi r_c^2}\right) = \frac{\hbar}{4\pi r_c^3 c} \qquad (3.6.38)$$

Where

$$m_c = \frac{\hbar}{r_c c} \qquad (3.6.39)$$

$$\sigma = \frac{m_c}{4\pi r_c^2} \sim 6.7 \times 10^{59} g.cm^{-2} \qquad (3.6.40)$$

Thus the critical density satisfies

$$\sigma_c = \frac{m_c}{r_c^2} = \left(\frac{c^7}{G^3\hbar}\right)^{\frac{1}{2}}$$

Where

$$\sigma_c = 4\pi\sigma \sim 8.4 \times 10^{60} g.cm^{-2} \qquad (3.6.41)$$

According to this model the universe began at a time and specific place, at the critical point (rc , tc), where all fundamental forces are unified into a single force. The Planck time is thus given by

$$t_c = \frac{r_c}{c} = \left(\frac{G\hbar}{c^3}\right)^{\frac{1}{2}}\left(\frac{1}{c}\right) = \left(\frac{G\hbar}{c^5}\right)^{\frac{1}{2}} \sim 5.4 \times 10^{-44} S \qquad (3.6.42)$$

The value speed of light c at the critical point (rc , tc). Is given by

$$c = \frac{r_c}{t_c} \sim 3 \times 10^{10} cm.S^{-1} \qquad (3.6.43)$$

It is also very interesting to note that according to equations (3-6-11) - (3-6-21) that the stars having short and long range gravity force have finite self-energy that is formed when the radius is very small, provided that the mass should be less than a critical value. This means that only elementary particles having very small radius and very small mass can have self-energy due to the transformation of potential field energy to rest mass energy, where equations (3-6-17), (3-6-19) and (3-6-20) gives:

$$V = m\varphi = \frac{mc^2}{2} = \frac{GMm}{r_c}$$

It is very interesting to note that the radius for self-energy is that of black holes. It is also very interesting note that, using quantum oscillator and relativistic energy expressions (3-6-25) and (3-6-26) beside Newtonian potential relation a useful expressions for Planck mass, length and time are obtained in equations (3-6-34), (3-6-36) and (3-6-42). The numerical values of these parameters agree with standard values.

3.7 Energy-Energy-Momentum Relation and Eigen Equations In a Curved Space Time

With the aid of the expression of time and distance in a curved space time a useful expression of energy and momentum Eigen equation similar to that is a curved space is found. These expressions are used to derive the corresponding relations in the Euclidean space. The corresponding expressions of energy-momentum relations for both curved and Euclidian space gives a relation between energy and momentum similar to the energy and momentum Eigen equations. The expression of mass in a curved space is similar to that of the generalized relativity. [55]

3.8 Dirac Generalized Relativistic Quantum Wave Function Which Gives Right Electrons Number in Each Energy Level by Controlling Quantized Atomic Radius

A linear energy-momentum Dirac relation was found using generalized special relativistic energy expression equation was used to find Dirac generalized relativistic quantum equation. This equation enables obtaining the number of particles for each atomic energy level. The wave function give an expression for the number of electrons in each energy levels if the atomic radius is quantized and satisfy certain constraints. It shows also that the stable atom corresponds to the minimum relativistic Einstein energy. [56]

3.9 United Nations Educational Scientific and Cultural Organization And International Atomic Energy Agency.

To describe our cosmos a generalized gravitational model depending on a quadratic lagrangian has been constructed. Solutions of this model are non-singular and do not possess the horizon, entropy, flatness or the age problem.

Unlike general relativity, where the flat space is characterized by a critical density, the flat space here does not contain matter at all. Moreover, the expansion of the universe is caused by matter only while in the case of empty space the expansion ceases. According to this approach vacuum energy, , decays where its initial value is very large and at present is negligibly small in conformity with both particle physics and present observations. The solutions are non-singular and do not possess the horizon, flatness and age problem [57].

3.10 Gravitomagnetism:

A novel explanation of the precession of planets and binary pulsars.

Owing to gravitomagnetism, the precession of planetary and pulsars orbits is due to the gravitomagnetic field.

The model unifies gravitational laws with electromag-netism ones through analogy. According to this unification, a phenomenon occurring in one discipline is typical to that of the other discipline The gravitomagnetic field resulting from the orbital and spin motion of celestial objects will induce a Larmor-like precession of the axis of rotation of these objects, analogous to the precession predicted by GTR, which is attributed to the curvature of space-time. The calculated values of the present gravitational phenomena according to our generalized Newton law of gravitation are in close agreement with those calculated from GTR. [58]

3.11 Massive photons propagation in gravitational field

The photons which are massive, moving inse a gravity field behave like the radiation emitted by a black hole. A black hole emitting such a radiation develops an entropy that is found to increase linearly with black hole mass, and inversely with the photon mass. The created photons could be seen as resulting from quantum fluctuation during an uncertainty time. The gravitational force on the photon is that of an entropic nature, and varies inversely with the square of the entropy. The power of the massive photon radiation is found to be analogous to Larmor power of an accelerating charge.

The photon placed under gravity behaves like a particle with mass that depends on the gravitational acceleration. Unlike the standard entropy for a black hole that is directly proportional to *M*2, the entropy of a black hole emitting such a radiation is found to be directly proportional to its mass, *M*. Thus, its entropy is additive like other thermodynamic systems. The created particle near the black hole horizon takes too long time to be emitted. A single photon needs about one year to be created near the Earth surface. The gravitational force on the massive photon is found to be of a quantum character. The radiated power by massive photon is analogous to that of the classical Larmor power. Using the Heisenberg uncertainty relation, the power radiated by a black hole as massive photon is greater than that of a massless photon, due to the black body radiation,

CHAPTER FOUR

Gravitation Waves and Origin of Mass

4.1 Introduction

The behavior of exotic objects like black holes and pulsars beside binary stars need a new version to explain their behavior. This model tries to explain some of the new astronomical observations associated with these exotic objects.

4.2 Emission of Gravitational waves by Black holes

Gravitational waves

The ideal model treats black holes as perfect spherical bodies. This requires finding the radial parts of the generalized general relativity equations. Thus, the covarient derivative of the scalar curvature R along the radius r according to equation takes the form

$$R_{;i;j} = \frac{\partial R_{;j}}{\partial x^j} - \Gamma^{\lambda}_{ij} R_{;\lambda} \tag{4.1.1}$$

$$R_{;0} = R_{;t} = \frac{\partial R}{\partial t} = R'$$

$$R_{;1} = R_{;r} = \frac{\partial R}{\partial r} = R^{\cdot}$$

$$R_{;t;i} = \frac{\partial R_{;t}}{\partial x^i} - \Gamma^{\lambda}_{ti} R_{;\lambda} \tag{4.1.2}$$

$$R_{;t;t} = \frac{\partial R'}{\partial t} - \Gamma^{\lambda}_{tt} R_{;\lambda} = R'' - \Gamma^{\lambda}_{tt} R_{;\lambda}$$

$$= R'' - \Gamma^{t}_{tt} R' - \Gamma^{r}_{tt} R^{\cdot}$$

$$R_{;}t,r = \frac{\partial R'}{\partial r} - \Gamma^{\lambda}_{rt} R_{;}\lambda = R^{\cdot\prime} - \Gamma^{t}_{tr} R' - \Gamma^{t}_{tr} R^{\cdot}$$

$$R_{;}r,t = \frac{\partial R^{\cdot}}{\partial t} - \Gamma^{\lambda}_{rt} R_{;}\lambda = R^{\cdot\prime} - \Gamma^{t}_{rt} R' - \Gamma^{r}_{rt} R^{\cdot}$$

$$R_{;}r;r = \frac{\partial R^{\cdot}}{\partial r} - \Gamma^{\lambda}_{rr} R_{;}\lambda = R^{\cdot\cdot} - \Gamma^{t}_{rr} R' - \Gamma^{r}_{rr} R^{\cdot} \tag{4.1.3}$$

$$\Box^{2} R = g^{\rho\sigma} R_{;}\rho;\sigma = g^{tt} R_{;}t;t + g^{tr} R_{;}t;r + g^{rt} R_{;}r;t + g^{rr} R_{;}r;r$$

$$= \frac{\beta R + 2\gamma}{6\alpha} \tag{4.1.4}$$

But

$$g^{tr} = 0 \quad g^{rt} = 0 \tag{4.1.5}$$

Thus

$$\Box^{2} R = g^{tt} R_{;}t;t + g^{rr} R_{;}r;r$$

$$= g^{tt}[R'' - \Gamma^{t}_{tt} R' - \Gamma^{r}_{tt} R^{\cdot}] + g^{rr}[R^{\cdot\cdot} - \Gamma^{t}_{rr} R' - \Gamma^{r}_{rr} R^{\cdot}] \tag{4.1.6}$$

From tensor relations, one gets

$$\Gamma^{\gamma}_{\lambda\mu} = \frac{1}{2} g^{v\gamma}\left[\partial_{\lambda} g_{\mu v} + \partial_{\mu} g_{\lambda v} - \partial_{v} g_{\lambda\mu}\right]$$

$$\Gamma^{t}_{tt} = \frac{1}{2} g^{vt}[\partial_{t} g_{tv} + \partial_{t} g_{tv} - \partial_{v} g_{tt}]$$

$$= \frac{1}{2} g^{tt}[2\partial_{t} g_{tt} - \partial_{t} g_{tt}] = \frac{1}{2} g^{tt} \frac{\partial g_{tt}}{\partial_{t}} = \frac{1}{2} g^{tt} g'_{tt}$$

$$\Gamma^{r}_{tt} = \frac{1}{2} g^{vt}[\partial_{t} g_{tv} + \partial_{t} g_{tv} - \partial_{v} g_{tt}] = \frac{1}{2} g^{rr}[\partial_{t} g_{tr} + \partial_{t} g_{tr} - \partial_{r} g_{tt}] = -\frac{1}{2} g^{rr} g^{\cdot}_{tt}$$

$$\Gamma^{t}_{rr} = \frac{1}{2}g^{vt}[\partial_r g_{rv} + \partial_r g_{rv} - \partial_v g_{rr}] = \frac{1}{2}g^{tt}[\partial_r g_{rt} + \partial_r g_{rt} - \partial_t g_{rr}]$$

$$= -\frac{1}{2}g^{tt}\frac{\partial g_{rr}}{\partial_t} = -\frac{1}{2}g^{tt}g'_{rr}$$

$$\Gamma^{r}_{rr} = \frac{1}{2}g^{rr}[\partial_r g_{rr} + \partial_r g_{rr} - \partial_r g_{rr}] = \frac{1}{2}g^{rr}\frac{\partial g^{rr}}{\partial r} \qquad (4.1.8)$$

$$\frac{1}{2}g^{rr}g^{\cdot rr}$$

The equation of motion in the field is given by

$$\frac{dx^{\lambda}}{dt^2} + c^2\Gamma^{\lambda}_{00} = 0 \qquad (4.1.9)$$

The motion in one dimension towards the center in given by

$$\frac{d^2x^1}{dt^2} + c^2\Gamma^{1}_{00} = 0 \qquad (4.1.10)$$

where

$$x^1 = r \qquad x^0 = ict \qquad (4.1.11)$$

In view of equation (4.1.8)

$$\Gamma^{r}_{tt} = \Gamma^{1}_{00} = \Gamma^{r}_{00} = -\frac{1}{2}g^{rr}\frac{\partial g_{00}}{\partial r} \qquad (4.1.12)$$

Thus from eqn (4.1.10), (4.1.11) and (4.1.12)

$$\ddot{r} = \frac{d^2r}{dt^2} = -\frac{c^2}{2}g^{rr}\nabla_r g_{00} = -\frac{c^2}{2}g^{rr}\frac{\partial g_{00}}{\partial r} = -\frac{c^2}{2}g^{rr}\nabla g_{00}$$

Thus from eqn (4.1.8)

$$\Gamma^{r}_{tt} = -\frac{1}{2}g^{rr}g^{\cdot}_{tt} = -\frac{1}{2}g^{rr}\frac{\partial g_{00}}{\partial r} = -\frac{1}{2}g^{rr}\nabla g_{00} \qquad (4.1.13)$$

$$\Gamma_{tt}^{r} = \frac{\ddot{r}}{c} = \frac{g_{rr}^{2}\ddot{r}}{c^{2}} \qquad (4.1.14)$$

Using (4.1.8), one also gets

$$\Gamma_{rr}^{t} = -\frac{1}{2}g^{tt}g_{rr}^{'} = -\frac{1}{2}g^{00}\frac{\partial g_{rr}}{\partial t} \qquad (4.1.15)$$

The proper interval is given by

$$ds^{2} = g_{\mu v}dx^{\mu}dxv \qquad (4.1.16)$$

For spherically symmetric direction independent, the proper interval is given by

$$ds^{2} = g_{00}d^{2}x^{0} + g_{rr}dr^{2}$$

$$= c^{2}g_{00}dt^{2} + g_{rr}dr^{2} \qquad (4.1.17)$$

For astronomical object in the form of sphere

$$g_{rr} = 1 \qquad g_{rr} = 1 \qquad (4.1.18)$$

Thus from egn (4.1.13)

$$\ddot{r} = -\frac{c^{2}}{2}\frac{\partial g_{00}}{\partial r} \qquad (4.1.19)$$

$$g_{00} = -\frac{2}{c^{2}}\int \ddot{r}\, dr + c_{0} \qquad (4.1.20)$$

To satisfy Minkoskian limit in vacuum or free space

$$\ddot{r} = 0 \qquad c_{0} = 1 \qquad (4.1.21)$$

Where

$$g_{00} = \zeta_{00} = 1 \qquad (4.1.22)$$

Thus

$$g_{00} = -\frac{2}{c^2}\int \ddot{r}\, dr + c_0 \tag{4.1.23}$$

$$= -\frac{2}{c^2}\int \ddot{r}\, dr + 1$$

But

$$F = m\ddot{r} = -\nabla V = -m\nabla\emptyset \tag{4.1.24}$$

There fore

$$g_{tt} = g_{00} = \frac{2}{c^2}\int \nabla\emptyset dr + 1 = \frac{2}{c^2}\int \frac{\partial\emptyset}{\partial r} dr + 1 = \frac{2\emptyset}{c^2} + 1 \tag{4.1.25}$$

Thus this relation (4.1.25) is valid even for strong field. Using the formal definitions (4.1.6)

$$\Box^2 R = g^{tt}\left[R'' - \frac{g^{tt}}{2}\frac{\partial g_{tt}}{\partial t}R' - \frac{1}{2}g^{rr}\frac{\partial g_{tt}}{\partial r}R^{\cdot}\right] + g^{rr}\left[\ddot{R} + \frac{1}{2}g^{tt}\frac{\partial g_{rr}}{\partial t}R' - \frac{1}{2}g^{rr}\frac{\partial g_{rr}}{\partial r}R^{\cdot}\right] \tag{4.1.26}$$

Since

$$g^{rr} = g_{rr} = 1 \tag{4.1.27}$$

$$g^{tt} = g^{00} = g_{00}^{-1} = \left(1 + \frac{2\emptyset}{c^2}\right) = g^{00}(r)$$

It follows that

$$\frac{\partial g_{tt}}{\partial t} = 0 \qquad \frac{\partial g_{rr}}{\partial t} = 0 \qquad \frac{\partial g^{rr}}{\partial t} = 0 \tag{4.1.28}$$

$$\Box^2 R = g^{00}\left[R'' - \frac{1}{2}(\nabla g_{00})R^{\cdot}\right] + R^{\cdot\cdot} \tag{4.1.29}$$

Since for linear Lagrangian GFE reduced to GR, and since the non-linear term gives successful cosmological model

Thus, one select L to be

$$L = -\alpha R^2 + \beta R + \gamma \quad (4.1.30)$$

Thus the

$$\Box^2 R = \frac{\beta R + 2\gamma}{6\alpha} \quad (4.1.31)$$

Hence

$$g^{00}\left[R'' - \frac{1}{2}(\nabla g_{00})R'\right] + R^{\cdot\cdot} = \frac{\beta R + 2\gamma}{6\alpha} \quad (4.1.32)$$

But

$$g^{00} = g_{00}^{-1} \quad (4.1.33)$$

$$R'' - \frac{1}{2}(\nabla g_{00})R' + R^{\cdot\cdot} = \left(\frac{\beta R + 2\gamma}{6\alpha}\right) g_{00} \quad (4.1.34)$$

To simplify this equ, one can split R to time and r dependent parts to get

$$R(r,t) = h(t)f(r) \quad (4.1.35)$$

Thus

$$R = hf, \quad R' = hf' \qquad R^{\cdot\cdot} = hf^{\cdot\cdot} \quad (4.1.36)$$

Farther simplification can be made by considering the field out-side the source. Hence

$$(\gamma = 0) \quad (4.1.37)$$

$$h'' f + \frac{1}{2}(\nabla g_{00})hf' + g_{00}hf^{\cdot\cdot} = \frac{g_{00}\beta R}{6\alpha} \quad (4.1.38)$$

dividing both sides by (hf) yields

$$\frac{h''}{h} + \frac{1}{2}(\nabla g_{00})\frac{f'}{f} + \frac{g_{00}f^{\cdot\cdot}}{f} = g_{00}b3 \quad (4.1.39)$$

Where

$$b_3 = \frac{\beta}{6\alpha} \tag{4.1.40}$$

Considering the particles as strings, the time metric is given by

$$g_{00} = 1 + \frac{2\emptyset}{c^2} = 1 + \frac{2}{c^2}\left(\frac{1}{2}\frac{k}{m}r^2\right)$$

$$= 1 + \frac{k}{m}r^2 = 1 + a_1 r^2 \tag{4.1.41}$$

Where

$$a_1 = \frac{k}{c^2 m} \tag{4.1.42}$$

Therefore

$$\nabla g_{00} = 2a_1 r \tag{4.1.43}$$

One can solve eqn (4.1.39) by suggesting

$$h = A_0 e^{-iwt} \tag{4.1.44}$$

thus

$$h' = \frac{\partial h}{\partial x} = \frac{\partial h}{ic\partial t} = -\frac{iwh}{ic} = \frac{-wh}{c} \tag{4.1.45}$$

$$h'' = \frac{-wh'}{c} = \frac{w^2}{c^2}h \tag{4.1.46}$$

$$h'' = -b4h \tag{4.1.47}$$

Thus

$$b4 = \frac{-w^2}{c^2} \tag{4.1.48}$$

Thus from eqn (4.1.39) and (4.1.46)

$$g_{00}f^{\cdot\cdot} + \frac{1}{2}(\nabla g_{00})f^{\cdot} = (g_{00}b3 + b4)f \qquad (4.1.49)$$

To solve eqn (4.1.49) consider the solution

$$f = (r + b_1)e^{b_2 r}$$

$$f^{\cdot} = (b_1)e^{b_2 r} + b_2(r + b_1)e^{b_2 r}$$

$$= (b_1 + b_1 b_2 + b_2 r)e^{b_2 r}$$

$$f^{\cdot\cdot} = (b_2)e^{b_2 r} + (b_1 + b_1 b_2 + b_2 r)b_2 e^{b_2 r}$$

$$f^{\cdot\cdot} = (b_1 b_2 + b_1 b_2^2 + b_2 + b_2^2 r)e^{b_2 r} \qquad (4.1.50)$$

A direct insertion of eqn (4.1.50) in (4.1.49) gives

$$(1 + a_1 r^2)(b_1 b_2 + b_1 b_2^2 + b_2 + b_2^2 r) + a_1 r(b_1 + b_1 b_2 + b_2 r)$$
$$= (b_3 + a_1 b_3 r^2)(r + b_1) + b_4(r + b_1) \qquad (4.1.51)$$

$$b_1 b_2 + b_1 b_2^2 + b_2 + b_2^2 r + a_1 b_1 b_2 r^2 + a_1 b_1 b_2^2 r^2 + a_1 b_2 r^2 + a_1 b_2^2 r^3$$
$$+ (a_1 b_1 + a_1 b_1 b_2)r + a_1 b_2 r^2$$

$$= b_3 r + b_3 b_1 + a_1 b_3 r^3 + a_1 b_1 b_3 r^2 + b_1 b_4 + b_4 r \qquad (4.1.52)$$

$$(b_1 b_2 + b_1 b_2^2 + b_2) + (b_2^2 + a_1 b_1 + a_1 b_1 b_2)r$$
$$+ (a_1 b_1 b_2 + a_1 b_1 b_2^2 + a_1 b_2 + a_1 b_2)r^2 + a_1 b_2^2 r^3$$

$$= (b_1 b_3 + b_1 b_4) + (b_3 + b_4)r + a_1 b_1 b_3 r^2 + a_1 b_3 r^3 \qquad (4.1.53)$$

Taking the coefficients of r^n

$$r^0: b_1 b_2 + b_1 b_2^2 + b_2 = b_1 b_3 + b_1 b_4 \qquad (4.1.54)$$

$$r: b_2^2 + a_1 b_1 + a_1 b_1 b_2 = b_3 + b_4 \qquad (4.1.55)$$

$$r^2: a_1 b_1 b_2 + a_1 b_1 b_2^2 + 2a_1 b_2 = a_1 b_1 b_3 \qquad (4.1.56)$$

$$a_1 b_2^2 = a_1 b_3 \quad (4.1.57)$$

From (4.1.57):

$$b_3 = b_2^2 \quad (4.1.58)$$

From (4.1.56):

$$a_1 b_1 b_2 + a_1 b_1 b_2^2 + 2a_1 b_2 = a_1 b_1 b_2^2$$

$$a_1 b_1 b_2 + 2a_1 b_2 = 0$$

$$b_1 = -2 \quad (4.1.59)$$

From (4.1.55), (4.1.58), (4.1.59)

$$b_2^2 - 2a_1 - 2a_1 b_2 = b_2^2 + b_4$$

$$b_4 = -2a_1 - 2a_1 b_2 \quad (4.1.60)$$

From (4.1.54), (4.1.58), (4.1.59):

$$-2b_2 - 2b_2^2 + b_2 = -2b_2^2 - 2b_4$$

$$-2b_4 = -b_2$$

$$b_4 = \frac{1}{2} b_2 \quad (4.1.61)$$

Sub (4.1.61) in (4.1.60)

$$\frac{1}{2} b_2 = -2a_1 - 2a_1 b_2$$

$$b_2 = -4a_1 - 4a_1 b_2$$

$$(1 + 4a_1) b_2 = -4a_1$$

$$b_2 = \frac{-4a_1}{(1 + 4a_1)} \quad (4.1.62)$$

Thus from eqn (4.1.42)

$$b_1 = \frac{-4\left(\frac{k}{mc^2}\right)}{1+\frac{4k}{mc^2}} = \frac{-4k}{mc^2+4k} = -\gamma_0 \qquad (4.1.63)$$

Thus inserting eqn (4.1.63) and (4.1.59)

$$f = (r-2)e^{-\gamma_0 r} \qquad (4.1.64)$$

In view of eqns (4.1.37), (4.1.45) and (4.1.64)

$$R = A_0 e^{-iwt}(r-2)e^{-\gamma_0 r}$$

$$R = A_0 (r-2)e^{-\gamma_0 r}e^{-iwt} \qquad (4.1.65)$$

Which describes stationary, standing, spatial decaying, time oscillating gravitational wave.

Equation (4.1.26) can be written using curved space coordinate

$$dt_c = \sqrt{g_{tt}}\,.dt$$

$$dr_c = \sqrt{g_{rr}}\,.dr \qquad (4.1.66)$$

To get

$$\Box^2 R = \frac{\partial^2 R^{\cdot}}{\partial t_c^{\,2}} - \frac{1}{2}g^{tt}\frac{\partial g_{tt}}{\partial t_c}\frac{\partial R}{\partial t_c} - \frac{1}{2}g^{tt}\frac{\partial g_{tt}}{\partial r_c}\frac{\partial R}{\partial r_c}$$

$$+\frac{\partial^2 R^{\cdot}}{\partial r_c^{\,2}} + \frac{1}{2}g^{rr}\frac{\partial g_{rr}}{\partial t_c}\frac{\partial R}{\partial t_c} - \frac{1}{2}g^{rr}\frac{\partial g_{tt}}{\partial r_c}\frac{\partial R}{\partial r_c}$$

$$\Box^2 R = R'' - \frac{1}{2}g^{tt}g'_{tt}R' - \frac{1}{2}g^{tt}g^{\cdot}_{tt}R^{\cdot} + R^{\cdot\cdot} + \frac{1}{2}g^{rr}g'_{rr}R' - \frac{1}{2}g^{rr}g^{\cdot}_{rr}R^{\cdot} \qquad (4.1.67)$$

Following Schwarzschild solution

$$g_{rr} = g_{tt}^{-1} = \left(1 + \frac{2\emptyset}{c^2}\right)^{-1} = \left(1 + \frac{2V}{mc^2}\right)^{-1} \qquad (4.1.68)$$

Where one assume the black hole as a harmonic oscillater, with

$$V = \frac{1}{2}kr^2 \qquad (4.1.69)$$

Since the potential V is part of the total energy mc^2 thus

$$V < mc^2$$

$$mc^2 > \frac{1}{2}kr^2$$

$$m > \frac{kr^2}{2c^2} \qquad (4.1.70)$$

In this case

$$g_{rr} \approx 1 \qquad g^{rr} \approx 1 \qquad g_{tt} \approx 1 \qquad g^{tt} \approx 1 \qquad (4.1.71)$$

Therefore equation (4.1.67) becomes

$$\Box^2 R = R'' + \ddot{R} \qquad (4.1.72)$$

In view of eqn (4.1.31), with ($\gamma = 0$)

$$R'' + \ddot{R} = \frac{\beta}{6\alpha}R \qquad (4.1.73)$$

Suggesting again

$$R = f(r)h(t) = fh \qquad (4.1.74)$$

In view of eqns (4.1.41) and (4.1.74)

$$fh'' + h\ddot{f} = b_3 fh$$

$$\frac{h''}{h} + \frac{f^{\cdot\cdot}}{f} = b_3 \qquad (4.1.75)$$

Using equations (4.1.45) and (4.1.47) beside (4.1.48)

$$\frac{f^{\cdot\cdot}}{f} = b_3 + b_4 = b_3 - \frac{w^2}{c^2} \qquad (4.1.76)$$

One can solve this equation by suggesting

$$f = e^{ikr}$$

$$f^{\cdot} = ikf \qquad f^{\cdot\cdot} = -k^2 f \qquad (4.1.77)$$

A direct substitution of (4.1.77) in (4.1.76) gives

$$-k^2 = b_3 - \frac{w^2}{c^2}$$

$$k^2 = \frac{w^2}{c^2} - b_3 = \frac{w^2}{c^2} - \frac{\beta}{6\alpha} \qquad (4.1.78)$$

In view of equations (4.1.45), (4.1.74) and (4.1.78)

$$R = A_0 e^{i(kr - wt)} \qquad (4.1.79)$$

Thus gravitational waves can be generated provided that

$$k^2 > 0 \qquad (4.1.80)$$

According to eqn (4.1.78) this require

$$\frac{w^2}{c^2} - \frac{\beta}{6\alpha} > 0$$

$$w^2 > \frac{\beta}{6\alpha} c^2 \qquad (4.1.81)$$

Since

$$w = \frac{2\pi c}{\lambda} = ck \qquad (4.1.82)$$

Let the critical wave number defined by

$$k_c^2 = \frac{\beta}{6\alpha} \qquad (4.1.83)$$

thus condition (4.1.81) can be rewritten as

$$\frac{w^2}{c^2} > k_c^2$$

$$k^2 > k_c^2$$

$$k > k_c \qquad (4.1.84)$$

$$\frac{1}{\lambda} > \frac{1}{\lambda_c}$$

$$\lambda < \lambda_c \qquad (4.1.85)$$

4.2. Black hole and Elementary particle self-energy and radius

When the field is weak, the time metric is given by

$$\mathrm{g}_{00} = \left(1 + \frac{2\emptyset}{\mathrm{c}^2}\right)^{\frac{1}{2}} \qquad (4.2.1)$$

The total energy can be given by

$$\mathrm{E} = \frac{\mathrm{g}_{00}\mathrm{m}_0\mathrm{c}^2}{\sqrt{\mathrm{g}_{00} - \frac{\mathrm{v}^2}{\mathrm{c}^2}}} \qquad (4.2.2)$$

With mo, c, v standing for the rest mass, speed of light in vacuum, and the particle velocity respectively.

Consider the particle to be at rest, as far as we need the minimum energy. Thus according to equations (4.2.1) and (4.2.2) and since (v=0). it follows that

$$E = \left(1 + \frac{2\emptyset}{c^2}\right)^{+\frac{1}{2}} m_0 c^2 \qquad (4.2.3)$$

When the elementary particle is affected by short range nuclear field $\emptyset_n$ and gravitational field $\emptyset_g$, in this case

$$\emptyset = \frac{c_0}{r} e^{c_1 r} - \frac{GM}{r} \qquad (4.2.4)$$

Where the nuclear and gravitational potentials per unit mass are given by

$$\emptyset_n = \frac{c_0}{r} e^{c_1 r} \qquad (4.2.5)$$

$$\emptyset_g = -\frac{GM}{r} \qquad (4.2.6)$$

Then by defining

$$m_0 c^2 = c_2 \qquad (4.2.7)$$

Equation (4.2.3) and (4.2.4) beside (4.2.7) gives

$$E = \left(1 + \frac{2c_0}{c^2 r} e^{c_1 r} - 2\frac{GM}{c^2 r}\right)^{+\frac{1}{2}} c_2 \qquad (4.2.8)$$

The radius for which the energy is minimum can be found by minimizing E

$$\frac{dE}{dr} = \frac{c_2}{2}\left(1 + \frac{2c_0}{c^2 r} e^{c_1 r} - 2\frac{GM}{c^2 r}\right)^{\frac{-1}{2}} \left(\frac{-2c_0}{c^2 r} e^{c_1 r} + \frac{2c_0 c_1}{c^2 r} e^{c_1 r} + 2\frac{GM}{c^2 r^2}\right) \qquad (4.2.9)$$

The radius which make the energy minimum is thus given by setting E is minimum when

$$\frac{dE}{dr} = 0 \quad (4.2.10)$$

This is satisfied when

$$\frac{-2c_0}{c^2 r} e^{c_1 r} + \frac{2c_0 c_1}{c^2 r} e^{c_1 r} + 2\frac{GM}{c^2 r^2} = 0 \quad (4.2.11)$$

Since the radii of elementary particles are so small, or if one consider small radius black hole, one has

$$r \to 0 \quad (4.2.12)$$

Thus one can use the Tayler expansion for the exponential function to get

$$e^{c_1 r} \cong 1 + c_1 r \quad (4.2.13)$$

A direct substitution of this in (4.2.11) gives

$$-c_0(1 - c_1 r)(1 + c_1 r) + GM = 0 \quad (4.2.14)$$

$$1 - c_1^2 r^2 = \frac{GM}{c_0} \quad (4.2.15)$$

$$r^2 = \frac{c_0 - GM}{c_0 c_1^2}$$

Thus, the radius at which the energy is minimum is given by

$$r_0 = r = \frac{1}{c_1}\sqrt{1 - \frac{GM}{c_0}} \quad (4.2.16)$$

It is very interesting to note that for massive body additional the radius become smaller due to strong gravity attraction. Means that the radius of massive black hole is very small. To this find the additional self-mass one can consider the energy in equation (4.2.3) to having vacuum term, which is given by

$$\emptyset_v = \frac{V_v}{M} \tag{4.2.17}$$

Thus according to equation (4.2.4) and (4.2.17) the total potential is given by

$$\emptyset = \emptyset_n + \emptyset_n + \emptyset_v = \frac{c_0}{r} e^{c_1 r} - \frac{GM}{r} + \frac{V_v}{M} \tag{4.2.18}$$

Inserting (4.2.18) in (4.2.3) gives

$$E = m_0 c^2 \left(1 + \frac{2\emptyset}{c^2}\right)^{\frac{1}{2}} = m_0 c^2 \left(1 + \frac{2c_0 c_1}{c^2 r} e^{c_1 r} + 2\frac{GM}{c^2 r^2} + \frac{2V_v}{c^2 M}\right)^{\frac{1}{2}} \tag{4.2.19}$$

Thus the mass which makes the energy minimum, requires

$$\frac{dE}{dM} = 0 \tag{4.2.20}$$

$$\frac{dE}{dM} = \frac{1}{2} m_0 c^2 \left(1 + \frac{2}{c^2}\left(\frac{c_0}{r} e^{c_1 r} + \frac{GM}{r} + \frac{V_v}{M}\right)\right)^{\frac{1}{2}} \left(-\frac{G}{r} - \frac{V_v}{M^2}\right)\left(\frac{2}{c^2}\right) \tag{4.2.21}$$

$$\frac{G}{r} + \frac{V_v}{M^2} = 0$$

$$\frac{V_v}{M^2} = -\frac{G}{r}$$

$$\frac{M^2}{V_v} = -\frac{r}{G}$$

$$M^2 = -\frac{rV_v}{G}$$

$$M = \sqrt{-\frac{rV_v}{G}} \tag{4.2.22}$$

Since the mass is real and not imaginary. This requires vacuum energy to be attractive, i.e

$$V_v = -V_0 \quad (4.2.23)$$

$$M = \sqrt{\frac{rV_0}{G}} \quad (4.2.24)$$

It is clear that the mass increases when vacuum energy increases which means that mass is generated by vacuum. Thus, more vacuum energy generates more massive body

Where r is the radius for minimum energy i.e (see equation 4.2.16)

$$r = r_0 = \frac{1}{C_0}\sqrt{1 - \frac{GM}{C_0}} \quad (4.2.25)$$

$$M = \sqrt{\frac{r_0 V_0}{G}} \quad (4.2.26)$$

$$M^2 = \frac{r_0 V_0}{G} = \frac{V_0}{GC_1}\sqrt{1 - \frac{GM}{C_0}} \quad (4.2.27)$$

Since short range nuclear force in much stronger than the gravity force, it follows that

$$\frac{GM}{C_0} \ll 1 \quad (4.2.28)$$

$$M = \sqrt{\frac{V_0}{GC_1}} \quad (4.2.29)$$

This equation again shows that mass is generated by vacuum. The more vacuum energy generates more massive body, but more generally the mass can be found by squaring equation (4.2.27) to get

$$M^4 = \frac{V_0^{\ 2}}{(GC_1)^2}\left(1 - \frac{GM}{C_1}\right)$$

$$\frac{(GC_1)^2}{V_0^{\ 2}} M^4 + \frac{G}{C_0} M - 1 = 0 \qquad (4.2.30)$$

Consider now elementary particles.

Since the mass of elementary particles is so small. Thus

$$M^4 \ll M \qquad (4.2.31)$$

Therefore

$$M = \frac{C_0}{G} \qquad (4.2.32)$$

However for massive black holes

$$M^4 > M \qquad (4.2.33)$$

Thus equation (4.2.30) gives

$$M^4 = \frac{V_0^{\ 2}}{(GC_1)^2}$$

$$M = \pm \sqrt{\frac{V_0}{GC_1}} \qquad (4.2.34)$$

Which again gives an expression similar to eqn (4.2.29) but with additional negative mass solution. This gives possibility of generating anti particles which canforms with elementary particles theories which propose that particles are created by photons in pairs. However black holes have positive mass. But one can also propose existence of anti-particles black holes with negative masses.

One can also try to find the radius of elementary particles and black holes in the presence of vacuum and strong nuclear force. This is done when the generalized potential special relativity (gpsr) energy is given by

$$E = m_0 c^2 \left(\emptyset_0 - \frac{c_2}{r} e^{c_1 r} \right)^{\frac{1}{2}} \qquad (34.2.5)$$

Where

$$c_2 = \frac{2c_0}{c^2} \qquad \emptyset_0 = 1 + \frac{2}{c_2} \emptyset_v \qquad (4.2.36)$$

Here outside the body ($c_0 = 0$)

It is important to note that, the shortrange force in eqn (4.2.33) is the attractive gravity force, to find the minimum radius under the effect of a shortrange attractive gravity force, one applies the condition of minimum energy, where

$$\frac{dE}{dr} = \frac{1}{2} m_0 c^2 \left(\emptyset_0 - \frac{c_2}{r} e^{c_1 r} \right)^{\frac{-1}{2}} \left(\frac{c_2}{r^2} e^{c_1 r} - \frac{c_2 c_1}{r} e^{c_1 r} \right) \qquad (4.2.37)$$

For minimum energy

$$\frac{dE}{dr} = 0$$

$$\frac{1}{2} m_0 c^2 \left(\emptyset_0 - \frac{c_2}{r} e^{c_1 r} \right)^{\frac{-1}{2}} \left(\frac{c_2}{r^2} e^{c_1 r} + \frac{c_2 c_1}{r} e^{c_1 r} \right) = 0$$

$$\frac{c_2}{r^2} [e^{c_1 r} - c_1 r e^{c_1 r}] = 0 \qquad (4.2.38)$$

For the black hole r may by sometimes large, thus

$$e^{c_1 r}(1 - c_1 r) = 0$$

$$1 - c_1 r = 0$$

$$r = \frac{1}{c_1} \quad (4.2.39)$$

Thus, the minimum radius is given by

$$r_0 = \frac{1}{c_1} \quad (4.2.40)$$

For (out-side the body $c_0 = c$) for small distances

$$e^{c_1 r} = (1 + c_1 r) \quad (4.2.41)$$

$$(1 + c_1 r)(1 - c_1 r) = 0$$

$$(1 - c_1^2 r^2) = 0$$

$$c_1^2 r^2 = 1$$

$$r = \pm \frac{1}{c_1} \quad (4.2.42)$$

Again the minimum radius is given by

$$r_0 = \frac{1}{c_1} \quad (4.2.43)$$

In view of eqn (4.2.35) it is clear that stronger attractive force requires c_1 to be large. This makes r_0 very small. This is quite reasonable, since stronger attractive force causes the body to be shrinked more thus causing its radius to be very small. Equation (4.2.42) gives additional possibility by all awing a possibility of having short attractive force which diminish itself away from the source. Thus requires according to equation (4.2.35)

$$c_1 = -c_3 \tag{4.2.44}$$

Thus according to eqn (42) the minimum radius is given by

$$r_0 = \frac{1}{c_3} \tag{4.2.45}$$

4.3 Discussion

Using Reimann geometry the generalized field equation of the gravitational field has been exhibited in a very general form in equations (4.1.4). Restricting our-selves to spherically symmetric bodies the GFE is reduced to eqn (4.1.33). The time metric g_{00} is shown by equations (4.1.10-4.1.25) to represent all fields including strong fields. The GFE has been solved by using the method of speration of variables, by splitting R to time and radial parts as shown by eqn (4.1.36). The time part solution (4.1.45) suggests time oscillating field. However the radial solution (4.1.64) indicates radially decaying wave. Equation (4.1.65) shows generation of gravitational waves by black holes in the form of non-travelling standing radial decaying wave, but when one uses curved coordinates (see eqn (4.1.66)), and bearing in mind that the potential energy is less that the total energy, the GFE (4.1.67) reduces to (4.1.72). The solution (4.1.79) predicts generation of travelling oscillating gravitational field provided that the black hole mass exceeds certain critical mass $m_c = \frac{Kr^2}{2c^2}$ as eqn (4.1.70) indicates. The gravitational waves generated should also have shorter wave length less than a certain critical value determined by eqn (4.1.85).

4.4 Conclusion:

The theoretical model based on potential dependent special relativity can successfully construct anon singular model to describe the behavior and find the self-energy and the radius of generated elementary particles and black holes. According to this model the self-mass is dependent on vacuum energy and gravitational and short-range force coupling constants. The radius depends on the short-range coupling constant, as well as the mass in some cases also.

The theoretical model based on the GFE shows that a gravitational wave in the form of standing time oscillating and radially decaying wave can be generated by any black hole. However if the black hole a quire a mass larger than acritical value, a travelling gravitational wave can be generated with wave lengths, shorter than a certain critical value.

4.5 Future Work

The constructed model can be used as well to study the behavior of the binary stars and pulsars as well as neutron stars. The Plank era and a quantum cosmological model at the early universe specially at Planck time where unification takes place and the elementary particles are generated can also be constructed. The behavior of vacuum beside its history can also be envistigated

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