كلية الدراسات العليا

Sudan University of Science and Technology

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Using Inverse Power Law Potential and Method of Image to Study Charge Distribution in Conductors

إستخدام جهد القوة العكسي وطريقة الصور لدراسة توزيع الشحنات الكهربيه في الموصلات

A Graduate Project submitted For the Degree complementary of Master (C M D) in physics

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الآيـة

قال تعالى:

بسم الله الرحمن الرحيم

(...يرفع الله الذين ءامنوا منكم والذين أوتوا العلم درجات والله بما تعملون خبير)

سورة المجادلة

صدق الله العظيم

DEDICATATION

I dedicate this work

To Soul of my mother

To my father

My brothers and sisters

To my aunt and her children

To all my friends

A CAKNOWLEDGEMENT

All thanks and gratefulness is for Allah, the Most Gracious, theMost Merciful. I would like to thank the Sudan University Science and Technology to express my sincere gratitude to my supervisors Dr.Ali Sulaiman Mohamed for his helpful discussion in the development of the problem.

ABSTARACT

Various methods to study the electric charge distribution on conductors were presented in details in this thesis. The electrostatic potential is mediated by potential law that varies as inverse power potential instead of coulomb potential.

To simplify this physical problem and its solution, the method of images was used for the different way. Inverse power potential was used to determine the position and magnitude of the image charges on different shapes of conductors. Then the charge density distribution on these conductors was calculated.

The charge density distributions on the surface of these conductors in case of inverse power potential are plotted using Maple program and compared with the charge distribution in the case of using Coulomb law.

المستخلص

في هذه الدراسة تم تقديم بعض الطرق العلمية التي نشرت في دورات علمية متخصصة لدراسة كيفية توزيع الشحنة الكهربية على الموصلات في حالة أن الجهد الناشئ عن الشحنة النقطية يختلف عن الجهد المحسوب بواسطة قانون التربيع العكسي لكولوم ويتغير وفق جهد القوي العكسي .

كما استخدمت طريقة الصور في هذه الرسالة لدراسة توزيع الشحنة الكهربية على الموصلات. وذلك بافتراض أن الجهد الناشئ عن الشحنة النقطية يختلف عن الجهد المحسوب بواسطة قانون التربيع العكسي لكولوم ويتغير وفق جهد القوي العكسي. حيث تم استخدام جهد القوي العكسي بدلا من جهد كولوم بطريقة مختلفه في هذه الدراسة لتحديد موضع صورة الشحنة الكهربية و مقدارها أولا ومن ثم إيجاد كثافة الشحنة السطحية باشتقاق الجهد بالنسبة لوحدة المتجه العمودي.

تم تطبيق طريقة الصور لجهد القوي العكسي على نماذج مختلفة من الموصلات (موصل على شكل لوح متصل بالأرض و موصل كروي متصل بالأرض و موصل كروي مشحون معزول).بعد حساب كثافة الشحنة السطحية لكل هذه النماذج . استخدم برنامج Maple في رسم المنحنيات و مقارنة النتائج المتحصل عليها مع تلك النتائج المحسوبة باستخدام قانون التربيع العكسي لكولوم. من خلال مقارنة النتائج اتضحت اهمية نموذج الصورة في تبسيط المسألة كما أمكن إيجاد كثافة الشحنة السطحية على سطح الموصلات.

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CHAPTER ONE

Introduction

1.1 Introduction

Coulomb's Law is a fundamental principle describing the electric force between isolated charges, and represents the first quantitative law achieved in electromagnetism. The degree of confidence with which the law is experimentally known to hold was investigated after the law was put forth by Coulomb in 1785. The electrodynamics for massive particles suggests that a photon with a finite rest mass will cause a deviation from the inverse square law. So, modern interpretations of the possible deviation from Coulomb's inverse square law are usually associated with the non-zero photon mass. In this article, we first give a historical review of the foundation of Coulomb's inverse square law. Then, the experimental searches for validity of Coulomb's Law, particularly in its inverse square nature, are generally introduced. Based onProca's equations, the unique.

simplest relativistic generalization of Maxwell's equations, the link between the deviation from Coulomb's Law and the upper limit on the photon rest mass based on the concentric-spheres apparatus established in the classical experiment of Cavendish is reviewed. Up to now, all the experiments show no evidence for a positive value, and the experimental result was customarily expressed as an upper limit on the deviation or on the photon rest mass. As a representative method with the double mission of testing of the validity of Coulomb's Law and of the photon rest mass, possible improvements for this kind of experiment are discussed.

The famous inverse square law in electrostatics, first published in 1785 by Charles Augustin de Coulomb, is known as the fundamental law of electrostatics. As the first quantitative law in the history of electricity, Coulomb's inverse square law has played a crucial role and made great contributions to the development of electricity and magnetism, and other related fields. Coulomb's Law, along with the principle.

of superposition, gives Gauss's Law and the conservative nature of the electric field, which may be generalized using the Lorentz transformation to obtain Maxwell's equations. Even then, the validity of Coulomb's Law has been tested.

Continuously over the past centuries. Based on the classical ingenious scheme devised by Henry Cavendish [1,2], modern experiments usually yield not only the result of possible deviation from Coulomb's inverse square law, but that of the upper limit on the photon rest mass [3,7].

The photon, as the fundamental particle of electromagnetic interaction, is generally assumed to be mass less. This hypothesis is based on the fact that a photon cannot stand still for ever. However, a nonzero photon mass could be so small that present-day experiments cannot probe it. Taking into account the uncertainty principle, the photon mass could be estimated using $m_\gamma \approx \hbar/(\Delta t)c^2$ to have a magnitude of about 10^{-66} g while the age of the universe is about 10^{10} years, which gives the ultimate limit for meaningful experimental measurements of the photon mass. Up to now, there is no positive result for the photon rest mass or the deviation from Coulomb's inverse square law. The experimental results just serve to set an upper bound to the photon mass and the deviation from the exponent 2 in the inverse square law. The aim of this paper is to give a review of the main ideas and results of the investigations intended to test Coulomb's Law and pertinently to improve the upper limit on the photon rest mass.

1.2 The Problem

The great triumphs of Maxwell an electromagnetism and quantum electrodynamics were based on the hypothesis that the photon should be a particle with zero rest mass. The photon could carry energy and momentum from place to place and light rays would propagate in vacuum with a constant velocity c being independent of inertial frames, which was the second postulate in Einstein's theory of special relativity. As a result, the velocity of a particle with finite mass would never reach the constant c. The fact that light could not stand still made the assumption reasonable and it was difficult to find any counter-examples in theory. Still, experimental efforts to improve the limits on the rest mass of the photonin other words, to challenge the accepted theories of the timehave continued since the time of Cavendish or earlier, even before the concept of the photon was introduced.So in case of nonzero photon mass the universal constant c will be different and Maxwell equation will reduce to Proca form.

1.3 The Aim of Study

The aim of this thesis is to construct moral idea to study charge distribution in case of potential deviating from inverse square coulombs law and follows inverse power law.

1.4 General Method and Technical Background

From the time of Cavendish or earlier, Coulomb's inverse square law has been tested directly or indirectly. Experiments with higher precision and involving different dimensions have been performed over the years. It is now customary to quote tests of the inverse square law in one of the following two ways [8]:

(a) Assume that the force varies with the distance r between two point charges according to the phenomenological formula $\frac{1}{r^{2+q}}$ and quote a value or limit for q, which represents departure from the Coulomb inverse square law.

(b) Assume that the electrostatic potential has the 'inverse power law as $\frac{1}{r^n}$instead of the Coulomb form 1/r and quote a value or limit forn. Since $\mu_\gamma \approx m_\gamma\, c/\Delta t$ the test of the inverse square law is sometimes expressed in terms of an upper limit on the photon rest mass. Geomagnetic and extraterrestrial experiments give μ_γ or m_γ , while laboratory experiments usually give q and perhaps μ_γ or m_γ .

(c) Using method of images to calculate charge density and maple program to construct graphical representation

1.5 Presentation of the thesis

This thesis contain five chapters, chapter one was an introduction in chapter two photon rest mass and effect of nonzero mass of photon on charge distribution, in chapter threeConductors in the Electrical fieldChapter fourthe method of imageschapter five results and conclusion and recommendations.

CHAPTER TWO

Photon Rest Mass and Effect of Nonzero Mass of Photon

2.1 Introduction

The famous inverse square law in electrostatics, first published in 1785 by Charles Augustine de Coulomb, is known as the fundamental law of electrostatics. As the first quantitative law in the history of electricity, Coulomb's inverse square law has played a crucial role and made great contributions to the development of electricity and magnetism, and other related fields. Coulomb's Law, along with the principle of superposition, gives Gauss's Law and the conservative nature of the electric field, which may be generalized using the Lorentz transformation to obtain Maxwell's equations. Even then, the validity of Coulomb's Law has been tested continuously over the past centuries. Based on the classical ingenious scheme devised by Henry Cavendish [1, 2], modern experiments usually yield not only the result of possible deviation from Coulomb's inverse square law, but that of the upper limit on the photon rest mass [3, 7].

The photon, as the fundamental particle of electromagnetic interaction, is generally assumed to be massless. This hypothesis is based on the fact that a photon cannot stand still for ever. However, a nonzero photon mass could be so small that present-day experiments cannot probe it. Taking into account the uncertainty principle, the photon mass could be estimated using $m_\gamma \approx \hbar/(\Delta t)c^2$ to have a magnitude of about 10^{-66} g while the age of the universe is about 10^{10} years, which gives the ultimate limit for meaningful experimental measurements of the photon mass. Up to now, there is no positive result for the photon rest mass or the deviation from Coulomb's inverse square law. The experimental results just serve to set an upper bound to the photon mass and the deviation from the exponent 2 in the inverse square law.

2.2 The Photon Rest Mass and Related Experiments

2.2.1General Introduction

The great triumphs of Maxwellian electromagnetism and quantum electrodynamics were based on the hypothesis that the photon should be a particle with zero rest mass. The photon could carry energy and momentum from place to place and light rays would propagate in vacuum with a constant velocity c being independent of inertial frames, which was the second postulate in Einstein's theory of special relativity. As a result, the velocity of a particle with finite mass would never reach the constant c. The fact that light could not stand still made the assumption reasonable and it was difficult to find any counter-examples in theory. Still, experimental efforts to improve the limits on the rest mass of the photon—in other words, to challenge the accepted theories of the time—have continued since the time of Cavendish or earlier, even before the concept of the photon was introduced.

A finite photon mass may be accommodated in a unique way by changing the inhomogeneous Maxwell equations to the Proca equations, the theoretical expressions of possible nonzero photon rest mass introduced by Proca [9] and de Broglie [10]. In the presence of sources ρ and J, these
equations may be written as (SI units)

$$\nabla.\mathrm{E} = \frac{\rho}{\varepsilon_0} - \mu_\gamma^2 \phi \qquad (2.1)$$

$$\nabla \times \mathrm{E} = -\frac{\partial B}{\partial t} \qquad (2.2)$$

$$\nabla . B = 0 \qquad (2.3)$$

$$\nabla \times B = \mu_0 J + \mu_0 \varepsilon_0 \frac{\partial \mathrm{E}}{\partial t} - \mu_\gamma^2 A \qquad (2.4)$$

Together with the field strengths $E = -\nabla\varphi - \partial A/\partial t$, $B = \nabla \times A$ and the Lorentz condition

$$\nabla.A+\frac{1}{c^2}\frac{\partial\phi}{\partial t}=0 \qquad (2.5)$$

Where φ and A are the scalar and the vector potentials, which uniquely determine the field, and $\mu^{-1}=\hbar/(m_\gamma c)$ is a characteristic length, with $m_\gamma=0$ as the photon mass. If $m_\gamma=0$, the Proca equations would reduce to Maxwell's equations. The Proca equations, the relativistic ally invariant modification of Maxwell's equations, provide a complete and self-consistent description of electromagnetic Phenomena [7].

In four-dimensional space the Proca equations can be rewritten as

$$(\square^2 - \mu_\gamma)A_\mu = -\mu_0 J_\mu \qquad (2.6)$$

Table 2.1.shows several important limits on the photon rest mass m_γ .

Author (date)	Ref. Experimental scheme	Upper limit on $m\gamma$ /g
Terrestrial results		
Goldhaber et al (1971)	[4] Speed of light	5.6×10^{-42}
Williams et al (1971)	[13] Test of Coulomb's law	1.6×10^{-42}
Chernikov et al (1992)	[14] Test of Ampere's law	8.4×10^{-46}
Lakes (1998)	[15] Static torsion balance	2×10^{-50}
Luo et al (2003)	[16] Dynamic torsion balance	1.2×10^{-50}
Extraterrestrial results		
de Broglie (1940)	[10]Dispersion of starlight	0.8×10^{-39}
Feinberg (1969)	[17]Dispersion of starlight	10^{-44}
Schaefer (1999)	[18] Dispersion of gamma ray bursts	4.2×10^{-44}
Davis et al (1975)	[19]Analysis of Jupiter's magnetic field	8×10^{-49}
Fischbach et al (1994)	[20] Analysis of Earth's magnetic field	1.0×10^{-48}
Ryutov (1997)	[21]Solar wind magnetic f field and plasma	10^{-49}
Gintsburg (1964)	[22] Altitude dependence of geomagnetic field	3×10^{-48}
Patel (1965)	[23] Alfv´en waves in Earth's magnetosphere	4×10^{-47}
Hollweg (1974)	[24] Alfv´en waves in interplanetary medium	1.3×10^{-48}
Barnes et al (1975)	[25] Hydromagnetic waves	3×10^{-50}
DeBernadis et al (1984)	[26]Cosmic ack ground radiation	3×10^{-51}
Williams et al (1971)	[27]Galactic magnetic field	3×10^{-56}
Chibisov (1976)	[5]Stability of the galaxies	3×10^{-60}

Where $A_\mu$and $J_\mu$are the 4-vector of potential (A, iφ/c) and current density (J, icρ), respectively. The d'Alembertiansymbol $\square^2$is equal to $\nabla^2 - \partial^2/\partial(ct)^2$. In free space, the aboveequation reduces to

$$(\square^2 - \mu_\gamma)A_\mu = 0 \qquad (2.7)$$

Which is essentially the Klein–Gordon equation for the photon. The characteristic length scale μ_γ^{-1}, namely the reduced Compton wavelength of the photon, is an effective range in which the electromagnetic interaction would exhibit an exponential damping by exp(μ_γ^{-1}r).

2.3 Effect of Massive Photon on the Static Electric Field

Once the photon is provided with a finite mass, three immediate consequences may be deduced from the Proca equations: the frequency dependence of the velocity of light propagating in free space; the third state of the polarization direction, namely the 'longitudinal photon'; and some modifications in the characteristics of classical static fields. All those effects are useful approaches for laboratory experiments and cosmological observations to determine the upper bound on the photon mass.

What is of interest in this paper is the effect of a massive photon in a static electric field. In the case of a massive photon, the wave equation will be modified for all potentials (including the Coulomb potential) in the form

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \mu^2{}_\gamma\right)\phi = -\frac{\rho}{\varepsilon_0} \tag{2.8}$$

For a point charge and in the static case, this yields a Yukawa type potential,

$$\phi(r) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}\exp(-\mu_\gamma r) \tag{2.9}$$

and the electric field

$$E(r) = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r^2} + \frac{\mu_\gamma}{r}\right)\exp\left(-\mu_\gamma r\right) \tag{2.10}$$

Inspection of equations (2.8),(2.10) shows that if $r \prec\prec \mu_\gamma^{-1} r$ then the inverse square law of forces is a good approximation, but if $r \succ\succ \mu_\gamma^{-1} r$, then the force law departs from the prediction of Maxwell's equations. Up to now, finding the exponential deviation from Coulomb's Law provides the most reliable test for the photon rest mass in terrestrial experiments, in that those laboratory tests have the advantage of free variation of the experimental parameters [7]. As for large scale observations, the limits usually come from the analyses of astronomical data of the cosmological magnetic field. However, those results are essentially order-of-magnitude arguments due to the incomplete knowledge about the structure of the large scale magnetic field [11, 12]. In section 4 we will review those laboratory experiments in detail.

CHAPTER THREE

Conductors in The Electrical Field

3.1 Introduction

Conductors are in all electric devices. They are as common in electrostatics as in other areas of electrical engineering. Nevertheless, it is important to understand how they behave in electrostatics. This behavior explains some useful electromagnetic devices.

In addition, in many non-electrostatic applications conductors behave similarly to the way they do in electrostatics. So this chapter is important beyond its application to electrostatics.

3.2 Behavior of Conductors in the Electrostatic Field

Conductors have a relatively large proportion of freely movable electric charges. The best conductors are metallic (silver, aluminum, copper, gold, etc.). They usually have one free electron per atom, an electron that is not bound to its atom, but moves freely in the space between atoms. Because of their small mass, these free electrons move in response to any electric field, however small, that exists inside a conductor. The same is true for all other conductors, e.g., liquid solutions and semiconductors, except that inside such conductors both positive and negative free charges can exist. The number of free charge carriers is smaller and their mass greater than in metals and electrons, but this has no influence on the behavior of conductors in the electrostatic field.

Let us make an imaginary experiment. Assume that this book is a conductor.

Suppose that it has both free positive and negative charges in equal number. If the book is not situated in the electric field, the number of positive and negative free charge inside any small volume is the same, and there is no surplus electric charge at any point in the book. To be more picturesque, imagine that positive charges are blue and negative yellow. If we mix blue and yellow we get green, so your book will look green both over its surface and at any point inside.

What would happen if we establish an electric field in the book, for example, by means of two electrodes on the two sides of the book, charged with equal charges of opposite sign? Let the positive electrode be on your left. The electric field in the book will then be directed from left to right. You would notice that blue (positive) charges move from left to right (repelled by the positive electrode), and that yellow (negative) charges move from right to left. consequently, the right side of the book will become progressively more blue (positive), and its left side progressively more yellow (negative).

The surplus charges in the body created in this manner are known as electrostatically induced charges. They are, of course, the source of an electric field. Because the positive induced charge is on the right side of the book, and the negative on its left side, this electric field is directed from right to left, i.e., opposite to the initial electric field that produced the charge. As the amount of the induced charge increases, the total field inside the book becomes progressively smaller and the motion of charges inside the book decays. In the end, the electric field of induced charges at all points inside the book cancels out the initial electric field (due to the two charged electrodes). We thus reach electrostatic equilibrium, in which there can be no electric field at any point inside our conductive book.

Form this simple imaginary experiment, we conclude the following:

if we have a conducting body in an electrostatic field, and wait until the drift motion of charges under the influence of field stops (in reality, an extremely rapid process) the electric field of induced charges will exactly cancel out the external field, and the total electric field at all points of a conductor will be zero. Thus the first fundamental conclusion is

Inelectrostatic $E = 0$ inside conductor (3.1)

With this knowledge, let us apply gauss law to an arbitrary closed surface S that is completely inside the conductor. Because vector E is zero at all points on S, the total charge enclosed by S must be zero. This means that all the excess charge (if any) must be distributed over the surfaces of conductor:

In electrostatics, a conductor has charges only on its surface. (3.2)

Because there is no field inside conductors, the tangential component of the electric field strength, E, on the very surface of conductors is also zero (otherwise it would produce organized motion of charge on its surface):

In electrostatics, $E_{\tan gential} = 0$ on conductor surfaces (3.3)

Because the tangential component of E is zero on conductor surfaces, the potential difference between any two points of a conductor is zero. This means that the surface of a conductor in electrostatics is equipotential. Because there is no E inside conductors either, it follows that all points of a conductor have the same potential:

In electrostatics, the surface and volume of a conductor are equipotential (3.4)

Finally, a simple relation exists between the normal component, En of E on a conductor surface, and local surface charge density, σ. To derive this relation, consider a small cylindrical surface, similar to a coin, with a base ΔS and a height Δh → 0. One base is in the conductor and the other in air (Figure. 3.1). Let us apply Gauss' law to the closed surface of the cylinder. There is no flux of E through the base inside the conductor (zero area). The flux of E through the cylinder is thus equal only to

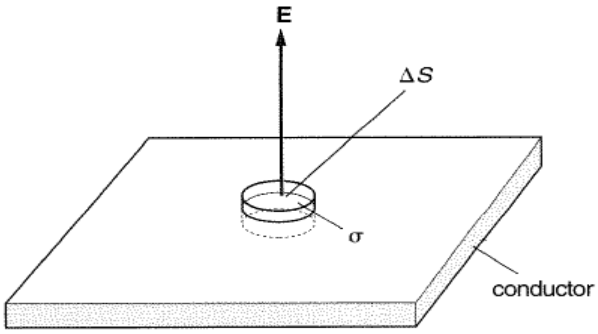

Figure 3.1 A small cylinder of negligible height with one base in the conductor and the other in airEn ΔS. Because the charge enclosed is σ ΔS, using Gauss' law we obtain that on the air side of a conductor surface.

$$E_n = \frac{\sigma}{\varepsilon_0} \quad (3.5)$$

Normal component of electric field strength close to conductor surfacethe simple conclusions in Equation. (3.1) through (3.5) are all we need to know to understand the behavior of conductors in the electrostatic field.

3.2.1 Charged Metal Ball.

Suppose that a metal ball of radius a is situated in a vacuum and has a charge How will the charge be distributed over its surface [We know from Equation (3.2) that Q exists only over the conductor surface.] Because equal charges repel, due to symmetry the charge distribution over the surface of the ball must be uniform. The surface charge density is therefore simple. $\sigma = Q/4\pi a^2$ Let us determine E and V due to this charge.

Due to the uniform charge distribution, vector E is radial and has the same magnitude on any spherical surface concentric to the ball. (Such a surface was said to be an equipotential surface. on can use Gauss' law to find the magnitude of vector E on any of these surfaces:

$$\oint_{sphere} E(r).ds = E(r)4\pi r^2 = \frac{Q}{\varepsilon_0}$$

Note that the sphere encloses no charge if r < a. thus

$$E(r) = \frac{Q}{4\pi\varepsilon_0 r^2} = \frac{\sigma 4\pi a^2}{4\pi\varepsilon_0 r^2} = \frac{\sigma a^2}{\varepsilon_0 r^2}(r \succ a),$$

$$E(r) = 0 \quad r \prec a \tag{3.6}$$

This expression is the same as the one for the field of a point charge Q at the center of the ball. On the surface of the ball (r=a), $E(a) = \sigma/\varepsilon_0$ as predicted by Equation. (3.5). It follows that outside the ball, the potential is the same as that of a point charge Q placed at the center of the ball. Inside the ball the potential is constant, equal to that on the ball surface, that is

$$v(a) = \frac{Q}{4\pi\varepsilon_0 a} = \frac{\sigma a}{\varepsilon_0} \tag{3.7}$$

3.2.2 Charged Metal Wire

.Consider a very long (theoretical, infinitely long) straight metal wire of circular cross section of radius a. let it be charged with Q' per unit length. What are the field and potential everywhere around the wire?

Due to symmetry, the charge will be distributed uniformly over the wire surface. It is not difficult to conclude that, as the result of this symmetrical charge distribution, vector E is radial. Its magnitude depends only on the normal distance r from the wire axis and can be determined by Gauss' law.

For the application of Gauss' law, we adopt the cylindrical surface shown in (Figure3.2). There is not flux through the cylinder bases because vector E is tangential to them. The total flux through the closed surface is therefore equal to the flux through its cylindrical part. Applying Gauss' law gives

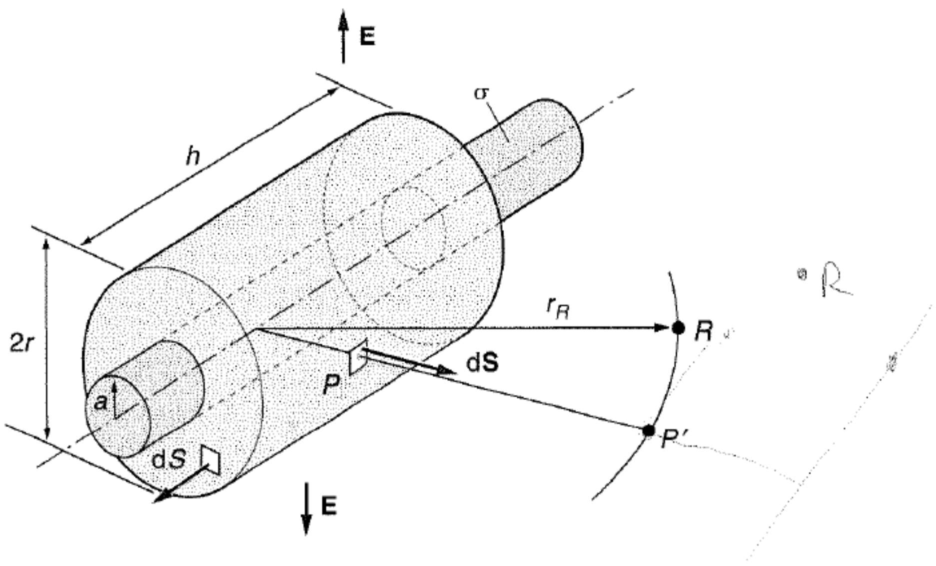

Figure.3.2.Segment of an infinitely long straight wire of circular cross section of radius a.

$$\oint_s E.ds = \int_{belt} Eds = E(r)2\pi rh = \frac{Q_{inclynder}}{\varepsilon_0} = \frac{Q'h}{\varepsilon_0}$$

Note that if $r \prec a$ the surface encloses no charge. Thus,

$$E(r) = \frac{Q'}{2\pi\varepsilon_0 r}\ r \succ a \qquad , \qquad E(r) = 0\ (r \prec a) \qquad (3.8)$$

(Electric field of straight, infinitely long, uniformly charged thin wire)

Because the surface charge density on the cylinder is $\sigma = Q'/(2\pi a), E(r)$ the wire surface can be written in the form. $E(a) = \sigma/\varepsilon_0$ This of course, is the result as obtained by applying Equation (3.5).

The determination of potential is slightly more complicated consider a point P at a containing P and the wire axis. Recall that we can adopt any path from P to R in determining the potential. We choose the simplest: first a radial line from P to the distance rR from the wire axis, and then a line parallel to the axis to R, (Figure.3.2). Along the first path segment, E and the line element are parallel, so $E.dI = E(r)dL = E(r)dL = E(r)dr$, because the line element, dL , becomes the differential increase in r, dr. along the second poth segment $E.dI = 0$. Thus we have

Or

$$V(r)=\int_{P}^{R}E.dI=\int_{\mathrm{r}}^{\mathrm{r_R}}E(r)dr=\frac{Q'}{2\pi\varepsilon_0}\int_{\mathrm{r}}^{\mathrm{r_R}}\frac{dr}{r}$$

$$V(r)=\frac{Q'}{2\pi\varepsilon_0}\ln\frac{r_R}{r} \qquad (3.9)$$

(Potential of straight, infinitely long, uniformly charged thin wire)

We see that in this case we cannot adopt the reference point at infinity, because $\log\infty\rightarrow\ \infty$.

The expressions in Equations (3.8) and (3.9) are also useful for noninfinite wires, as long as we are interested in the field at points close to the wire and away from the ends. Because metallic wires are used often in electrical engineering, these equations are important.

3.3 Charge Distribution on Conductive Bodies of Arbitrary Shapes

Only for symmetrical isolated conductors is the charge distribution on their surface known actually, inferred from symmetry. For conducting bodies of arbitrary shape the determination of charge distribution is one of the most important and the most difficult problems in electrostatics. Except in a few relatively simple cases, it can be determined only numerically. For many applications, it is useful to have a roughidea what the charge distribution is like. In estimating the charge distribution, the following simple reasoning can be of significant help.

We know that on an isolated metal sphere the charge is distributed uniformly. We also know that if the radius of the sphere is a and the surface charge density on it isσ, then the potential of sphere is $V(a)=\sigma a/\varepsilon_0$ (Equation, 3.7). Let us use this expression to estimate the charge distribution on a more complex conducting body.

Consider a charged metal body sketched in (Figure.3.3). It consists of a larger sphere of radius a, onto which are pressed part of two smaller spheres of radii b and c.

Close to points A, B, and C indicated in the figure, the surface charge density is not the same. These three points are, however, at the same potential, V because the body is conductive. because charges that are close to a certain point predominantly contribute to the potential at the point, roughly speaking the surface charge density σ_A is approximately that of a sphere of radius a at the potential V. there for, according to Equation.(3.7) $\sigma_A \approx \varepsilon_0 v/a$ .Similarly, $\sigma_B \approx \varepsilon_0 v/b$, and $\sigma c \approx \varepsilon_0 v/c$ .Thus, for the conducting body shown in (Figure.3.3), $a\sigma_A \approx b\sigma_B \approx c\sigma_c$.

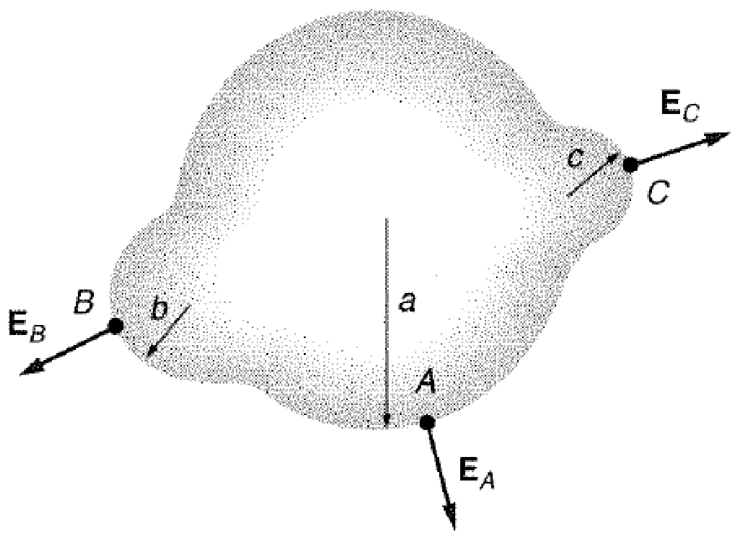

Figure.3.3. A charged metal body

Because the surfaces charge density is proportional to the local electric field strength, $aE_A \approx bE_B \approx cE_c$.

These are simple but important approximate result. They tell us that the surface charge density at different points a metal body is approximately inversely proportional to the curvature of the surface of the body at these points. This means that the largest charge density and electric field strength on charged conductive bodies is around sharp parts of the body.

An application of Equation (3.11), for example, is a simple method for discharging aircraft. During flight, the plane becomes charged due to air friction. this charge could produce large fields during landing that in turn could produce a spark resulting in parts, the charge density and, consequently, the electric field at these points become very high and the air ionizes(i.e., becomes conductive). A large portion of the charge "leaks" through these conducting channels into the atmosphere.

3.4 Charged Conductors

Now suppose we inject charge into the conductor. Since like charges repel, the injected charges will move as far away from each other as possible, without leaving the conductor. This implies that the charge must reside on the surface of the conductor. Moreover, the field within the conductor must be zero. Why? Because the conductor is in equilibrium. By definition, this means that there is no net migration of charge. If there is no net migration of charge, this implies that the free charges experience a net electric force of zero, that is, zero electric field.

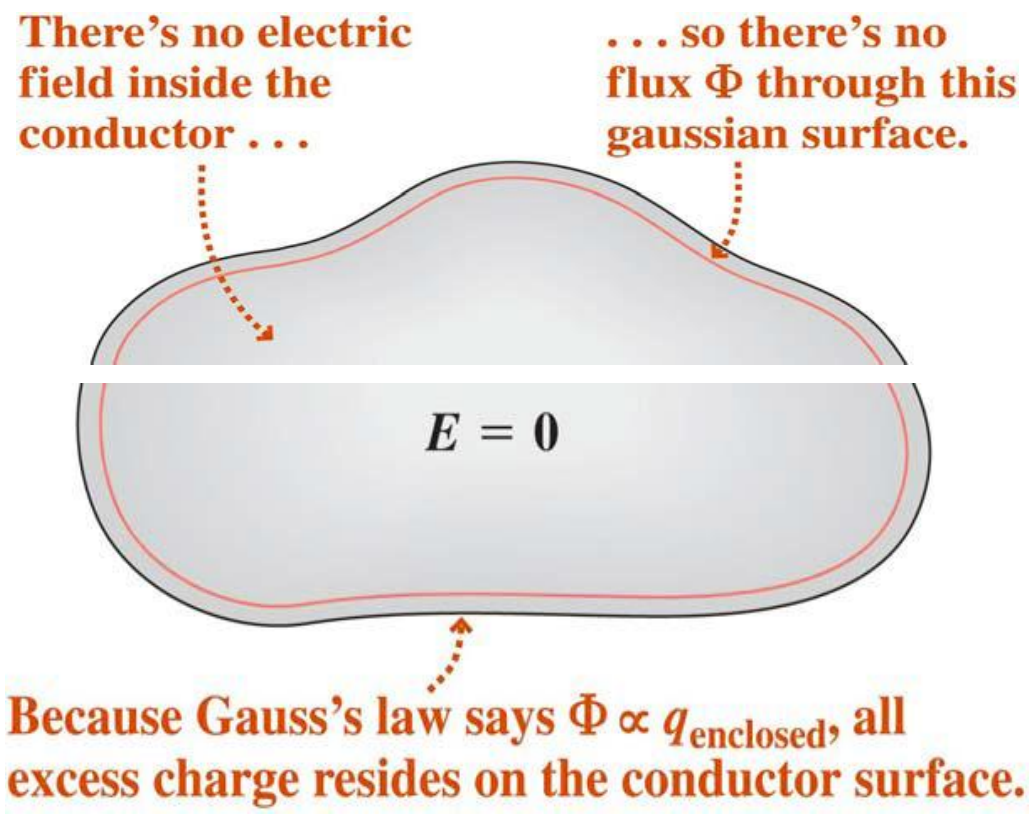

Figure.3.4Injected Charge intotheConductor

3.4.1 A Hollow Conductor

Consider a conductor with a net charge of +1 μC. As noted, the charges will migrate rapidly to the surface of the conductor and distribute themselves in such a way that the electric field within the conductor is zero. Suppose that the cavity within the conductor contains a +2 μC charge. Because the field within the conductor is zero, the field on the Gaussian surface shown is necessarily zero. Therefore, according to Gauss's law, the net charge enclosed by the Gaussian surface must also be zero.

The only way the net charge enclosed by the Gaussian surface can be zero is if, in addition to the +2μC in the cavity, there is also within the Gaussian surface a charge of –2μC. Where does that charge come from? It comes from the free charges in the conductor, which are attracted to the +2μC charge. Where does that charge reside? In the only place it can: on the inner surface of theconductor. Moreover, the inner surface charge is distributed so that its field plus that of the +2μC charge sum to zero, as it must, outside the cavity, that is, within the conductor.

But since the net charge of the conductor is +1μC, and its inner surface has a charge of –2μC, it follows from charge conservation that a charge of +3μC must exist somewhere in or on the conductor. A net charge cannot reside in a conductor in equilibrium. Nor can it reside on the inner surface, because if it did the net charge there would be –2μC + 3μC = 1μC, contradicting Gauss's law. Therefore, the +3μC charge must reside on the conductor's outer surface.

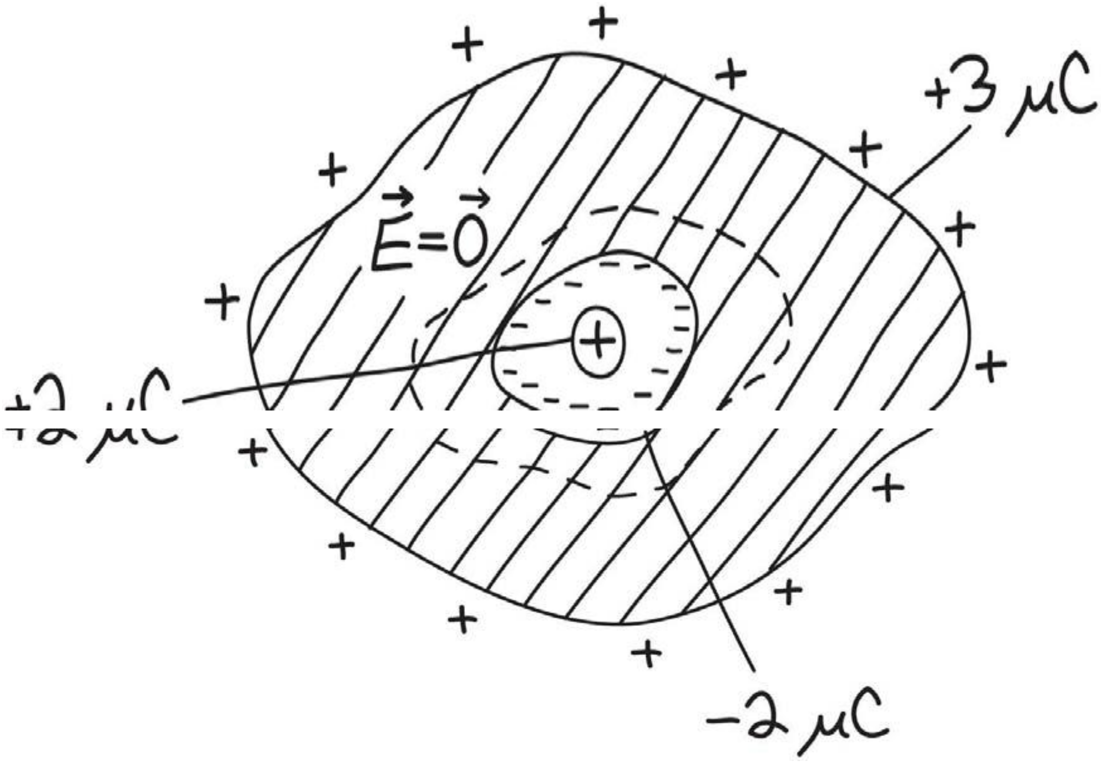

Figure 3.5 a conductor with a net charge

3.4.2 A Hollow Spherical Conductor

Consider a neutral spherical conductor in equilibrium with an internal cavity containing a net charge –q. The fact that the conductor is neutral means that its net charge is zero

1. The charge on the inner surface of the cavity is +q.

2. The charge on the outer surface of the conductor must therefore be –q.

3. Amazingly, in this case, this charge is uniformly distributed. Because the inner surface charge exactly cancels the field in the conductor due to the charge in the cavity and, consequently, the conductor behaves exactly like a neutral spherical conductor without a cavity.

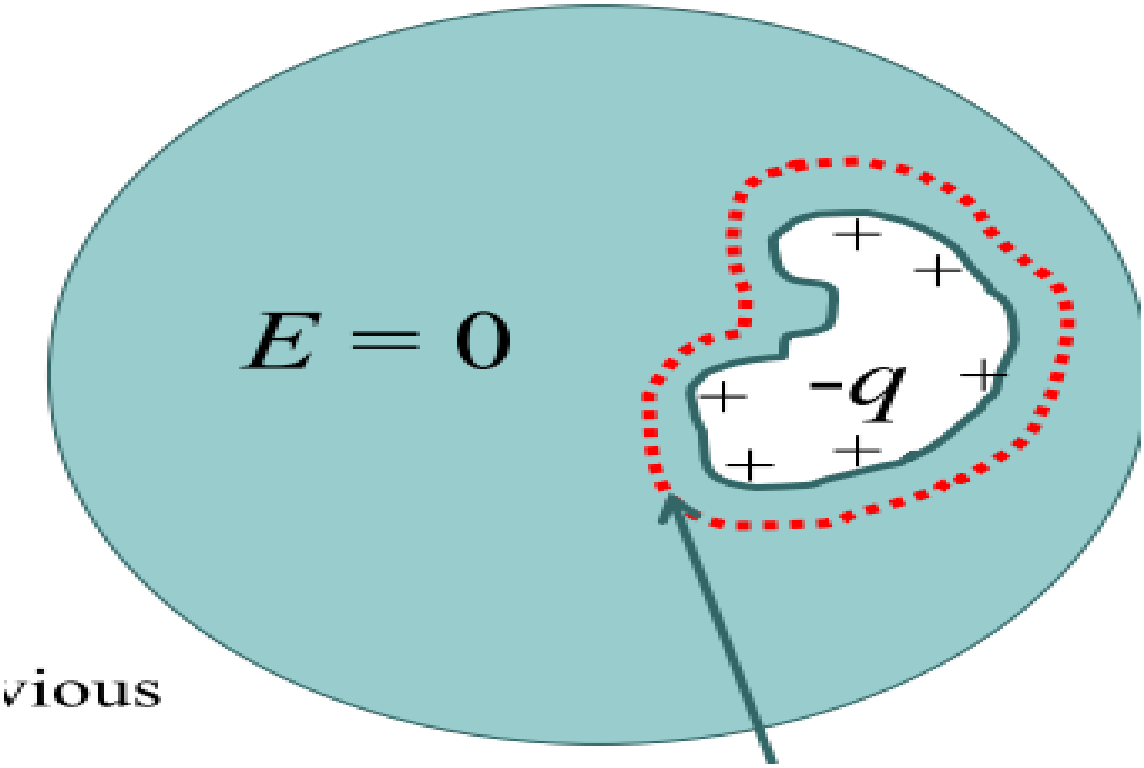

Figure.3.6 Neutral spherical conductor in equilibrium with an internal cavity containing a net charge –q.

CHAPTER FOUR

Method of Images

4.1 Conductors

In a conductor, there are free charges that move in the presence of an electric field. A consequence is that in a purely electrostatic situation, the electric field inside a conductor must be zero. Since the electric field is zero inside the conductor, the electric potential is uniform throughout the conductor.

Any charge on the conductor must reside on the surface of the conductor.

If there are charge free cavities inside the conductor, then these cavities also have uniform potential equal to that of the conductor. This follows from the uniqueness of the solution to the Laplace equation with Dirichlet boundary conditions. Since a cavity has uniform potential the electric field inside the cavity is zero as it is inside the conductor. Since the field is zero in a cavity and conductor there can be no charges on the walls of a cavity. Any charges must lie on the outside of the conductor.

Since the surface of the conductor is an equipotential, the components of electric field in the plane tangent to the conductor are zero. Hence the field is perpendicular to the surface of the conductor. By applying the divergence theorem to a 'Gaussian pillbox', i.e. a small cylinder with axis normal to the surface of the conductor, we find that the surface charge density σ is related to the electric field just outside the conductor by 4ps = n ×E, where n is the outward normal unit vector.

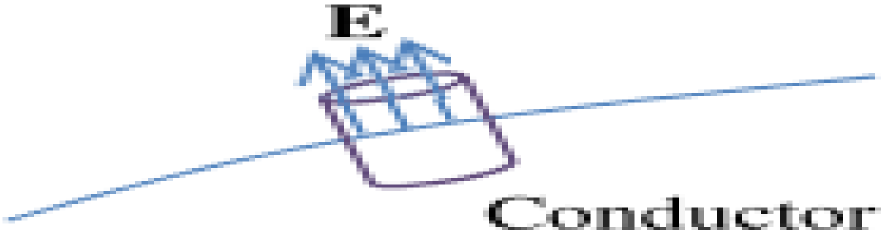

Figure 4.1the field is perpendicular to the surface of the conductor.

4.2 Method of Images

The method of images is useful for finding the potential of a charge distribution in the presence of grounded conductors of simple geometry. The simplest case is when the surface of the conductor is an infinite plane. Since by definition the potential of a grounded conductor is zero (the same as the potential at infinity), the potential on the side of the conductor containingthe charge is the same as for the original charge distribution plus a charge distribution of opposite sign symmetrically placed with respect to the location of the surface of the conductor but with the conductor removed.

Note that the force on the charge q is directed towards the conductor. The work function of a metal is largely the work needed to remove an electron in the presence of its attractive image force.

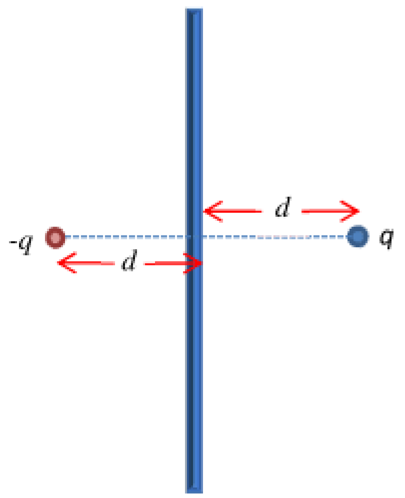

Figure (4.2) grounded conducting plate with charge q located at distanced and its image $q^{/}$ at $d^{/}$.

The method of images can also be used to find the potential due to a charge inside or outside agrounded spherical surface. For example, consider a point charge q, a distance d from the center of a sphere of radius a where d > a. Since there is azimuthal symmetry about the line through the charge and the center of the sphere, we look for a solution involving an image charge $-q'$ placed a distance d'' from thecenter of the sphere on the symmetry axis.

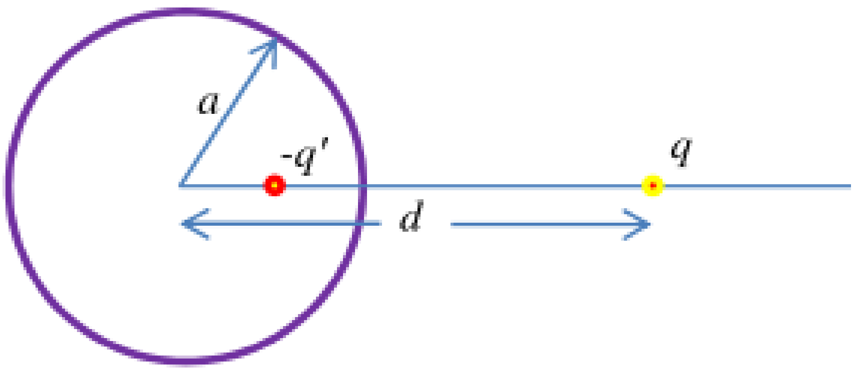

Figure (4.3) grounded conducting sphere with charge q located at distance d and its image $q^{/}$ at $d^{/}$.

The potential due to a charge inside or outside agrounded spherical surface

Let n be a unit vector along the symmetry axis pointingaway from the center of the sphere. The potential at position r is

$$\phi(r) = \frac{q}{|r - dn|} - \frac{q'}{|r - d'n|} \tag{4.1}$$

4.3 The Potential on The Sphere is

$$\phi(a\hat{r}) = \frac{q}{|a\hat{r} - dn|} - \frac{q'}{|a\hat{r} - d'n|} = \frac{q'}{a\left|\hat{r} - \frac{d}{a}n\right|} - \frac{q'}{d'\left|\frac{a}{d'}\hat{r} - n\right|} \tag{4.2}$$

$$\frac{q}{a} = -\frac{q'}{d'}\,\frac{d}{a} = \frac{a}{d'} \tag{4.3}$$

Makes the potential zero on the sphere. Consider two vectors $a - \lambda b$ and $\lambda a - b$. The difference in the squares of the lengths of these two vectors is $(1 - \lambda^2)|a^2 - b^2|$. Hence if a and b have the same length, $a - \lambda b$ and $\lambda a - b$ also have the same length. Since rˆ and n are both unit vectors, we conclude that the above choice for q′ and d′ makes the potential zero on the spherical surface.

The location and magnitude of the image charge are then

$$d' = \frac{a^2}{d}, q' = \frac{a}{d} q \tag{4.4}$$

The potential outside the sphere is

$$\phi(r) = \frac{q}{|r-d|} - \frac{a}{d} \frac{q}{\left| r - \frac{a^2}{d^2} d \right|} \tag{4.5}$$

Where d = dn is the position of the point charge, q.

The force on the charge q from the grounded sphere is the same as that from the image charge. Theonly non-zero component of this force is that along the symmetry axis

$$F = -\frac{qq'}{(d-d')^2} = -q^2 \frac{ad}{(d^2-a^2)^2} \tag{4.6}$$

Note that at a large distance from the sphere, the force varies as the inverse cube of the separation. Thisis because the charge distribution on the sphere is induced by the charge q.

4.4 The Potential of a Point Charge in The Presence of an Insulated, Charge Spherical Conductor

Suppose the total charge on the spherical conductor is *Q*. The charged induced on the conductor bythe point charge is -*q*′. Hence the added charge is *Q*+*q*′. This additional charge distributes itself uniformlyon the surface of the spherical conductor to give an equipotential surface. The potential is

$$\phi(r) = \frac{Q}{|r|} + \frac{q}{|r-d|} + \frac{a}{d} \frac{q}{|r|} - \frac{a}{d} \frac{q}{\left| r - \frac{a^2}{d^2} d \right|} \tag{4.7}$$

The potential outside a grounded conducting sphere placed in a uniform electric field.

The electric field near the midpoint of a line connecting charges Q and $-Q$, separated by a distance $2R$ is nearly uniform over distances small compared to R. The magnitude of the nearly uniform field is

$$E_0 = \frac{2Q}{R^2} \tag{4.8}$$

and is directed towards the charge $-Q$. By letting Q and R go to infinity, keeping $E0$ fixed, we can use the method of images to find the potential outside a grounded conducting sphere placed in a uniform field. Consider the arrangement below:

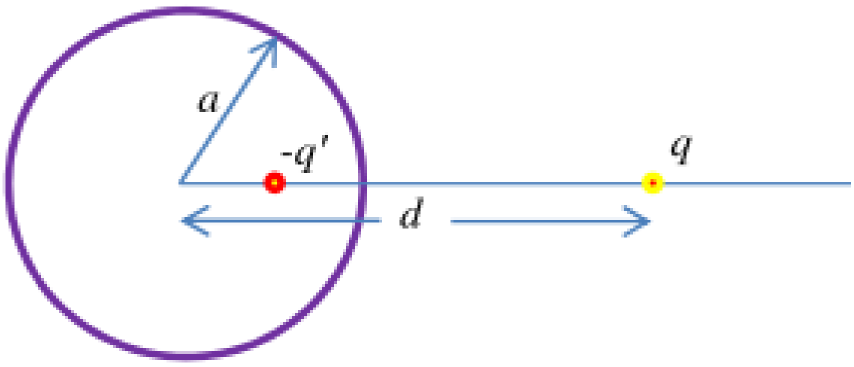

Figure (4.4) grounded charged conducting sphere Q with charge q located at distance d and its image $q^{/}\ at\ d^{/}$.

Using the method of images the potential outside the sphere is

$$\phi(r) = \frac{-Q}{|r-R|} - \frac{a}{R}\frac{-Q}{\left|r-\frac{a^2}{R^2}R\right|} + \frac{Q}{|r+R|} - \frac{a}{R} - \frac{Q}{\left|r+\frac{a^2}{R^2}R\right|}$$

$$= Q\left(\frac{1}{|r-R|} - \frac{1}{|r-R|}\right) + \frac{a}{R}Q\left(\frac{1}{\left|r-\frac{a^2}{R^2}R\right|} - \frac{1}{\left|r-\frac{a^2}{R^2}R\right|}\right) \tag{4.9}$$

With the angle θ as shown, the potential is

$$\phi(r)=\frac{Q}{\left(r^2+2rR\cos\theta+R^2\right)^{\frac{1}{2}}}-\frac{Q}{\left(r^2-2rR\cos\theta+R^2\right)^{\frac{1}{2}}}$$
$$+\frac{a}{R}Q\frac{Q}{\left(r^2-2r\frac{a^2}{R}\cos\theta+\frac{a^4}{R^2}\right)^{1/2}}-\frac{a}{R}Q\frac{Q}{\left(r^2+2r\frac{a^2}{R}\cos\theta+\frac{a^4}{R^2}\right)^{1/2}} \qquad (4.10)$$

$$=\frac{Q}{R\left(1+2\frac{r}{R}\cos\theta+\frac{r^2}{R^2}\right)^{1/2}}-\frac{Q}{R\left(1-2\frac{r}{R}\cos\theta+\frac{r^2}{R^2}\right)^{1/2}}$$
$$+\frac{a}{R}Q\frac{1}{r\left(1+2\frac{r}{R}\cos\theta+\frac{r^2}{R^2}\right)^{1/2}}-\frac{a}{R}Q\frac{1}{r\left(1+2\frac{r}{R}\cos\theta+\frac{r^2}{R^2}\right)^{1/2}} \qquad (4.11)$$

For R >> r, series expansion gives

$$\phi(r)=\frac{Q}{R}\left(1-\frac{r}{R}\cos\theta+\ldots\right)-\frac{Q}{R}\left(1+\frac{r}{R}\cos\theta+\ldots\right)$$
$$+\frac{a}{R}Q\frac{1}{r}\left(1+\frac{a^2}{rR}\cos\theta+\ldots\right)-\frac{a}{R}Q\frac{1}{r}\left(1-\frac{a^2}{rR}\cos\theta+\ldots\right)$$
$$=\frac{2Q}{R^2}\left(-r\cos\theta+\frac{a^3}{r^2}\cos\theta\right) \qquad (4.12)$$

Replacing $2Q/R^2$ by E0, and letting R go to infinity, we finally get

$$\phi(r)=-E_0 r\cos\theta+E_0\frac{a^3}{r^2}\cos\theta \qquad (4.13)$$

Where the first term is the potential due to a uniform field, and the second is that for a dipole aligned ina direction parallel to the field.

The charge density on the sphere is

$$\sigma=\frac{n.E}{4\pi}=-\frac{1}{4\pi}\frac{\partial\phi}{\partial r}(r=a)\frac{3}{4\pi}E_0\cos\theta \qquad (4.14)$$

The net charge on the sphere is zero.

Chapter Five

Results and Conclusion and Recommendations

5.1 Introduction

One of the foundations of electrostatics is Coulomb's law. Major electromagnetic laws are built upon this law.As a direct consequence of this law (or its equivalent, Gauss's law), any excess charge placed on a conductormust lie entirely on its surface. According to Coulomb's law, excess charges given to a conductor will moveaway from each other and distribute themselves about the conductor in such a manner as to reduce the totalamount of repulsive forces within the conductor and that both the charge and the field inside the conductor willvanish [1],[6].

Testing this law has been a subject for many experiments over the past two and a half centuries [1],[2]. Anydeviation from inverse square law would suggest a finite range for electromagnetic force, implying a nonzerophoton rest mass. Rest mass of the photon provides indirect test of the deviations from exactness of Coulomb'law. If the photon mass is zero, Coulomb's inverse-square law is the foundational law in electrostatics. Experimentsmeasure deviations in the exponent of inverse-square law and photon rest mass are increasingly exact.

The most recent ion interferometry experiment measures the value of the exponent to be a few times 10^{-22} anddetect a photon rest mass at the level of9×10^{-50} grams [1]. Detection of any deviation from Coulomb's lawwould have far-reaching implications. Maxwell's equations and much of the standard model would have to bemodified. The notion that absolute electrostatic potential is arbitrary would have to be abandoned, along withmany other tenets of classical electromagnetism [1].

In an interesting papers, Spencer [3] and Griffiths and Uvanovic [4] studied distribution of excess chargewithin a conductor for laws rather than inverse square law such as Yukawa's law or power law. In these twocases they found that some of the charge goes to the surface, and the remainder distributes itself uniformly overthe volume of the conductor.

In this thesis we introduce the method of images to study the distribution of charges in cases where the potentialis depending on the photon rest of mass. And give a theoretical extension work to the experimental resultsthat detect a photon rest mass at the level of 9×10^{-50} grams and as a result a deviating from Coulomb's Law.

This thesis is also important to understand physics of molecules and electron transport through a single moleculewhich offers a highly promising new technology for the production of electronic chip.

5.2 Method and Results

5.2.1 Method of Images for Inverse Power Law Potential and Grounded Spherical Conductor

The reaction field of a point charge due to surrounding medium can be represented by the method of imagecharge. The method of images allows us to solve certain differential form of electric potential problem withoutspecifically solving a differential equation of this problem.

The potential $\Phi(x)$ everywhere outside a conducting sphere can be calculated by using method of images.

As illustrated (Figure 5.1) we consider conducting sphere with radius $R = a$. For convenience, place the sphere atthe origin. We assume a point charge q outside the sphere and defined by position vector y. By symmetry,the image charge lie on the line connecting the charge and the origin of the sphere and will be located inside thesphere at position vector y. If the sphere is grounded then the potential everywhere on the sphere equal zero.

Now we are able to calculate the magnitude and the position vector y' of an image charge q' that is requiredto make the potential equal zero on the surface of the grounded sphere. Total Yukawa potential [4] $\Phi(x)$ dueto the assumed charge q and its image charge q' at any point P is given by Equation (5.1).

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$$\Phi(\vec{r}) = \frac{q}{4\pi\varepsilon_0 \vec{r}^{\,n}} \tag{5.1}$$

$$\Phi(x) = \frac{q}{4\pi\varepsilon_0 |x-y|^n} \tag{5.2}$$

$$\Phi(x) = \frac{q}{4\pi\varepsilon_0 |x-y|^n} + \frac{q'}{4\pi\varepsilon_0 |x-y'|^n} \tag{5.3}$$

$$\Phi(x=a) = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{a^n\left|n-\frac{y}{a}n'\right|^n} + \frac{q'}{y'^n\left|n'-\frac{a}{y'}\right|^n}\right] = 0 \tag{5.4}$$

$$\frac{q}{a^n} = -\frac{q'}{y'^n} \tag{5.5}$$

$$\frac{y}{a} = -\frac{a}{y'}$$

(5.6)

$$q' = \frac{-q}{a^n}\left(\frac{a^{2n}}{y^n}\right) = -q\left(\frac{a}{y}\right)^n \tag{5.7}$$

$$\Phi_1(x) = \frac{q}{4\pi\varepsilon_0 |x-y|^n} \tag{5.8}$$

$$\sigma_1 = -\varepsilon_0 \frac{\partial \Phi(x)}{\partial x}\bigg|_{x=a} \tag{5.9}$$

$$\Phi(x,y,z) = \frac{1}{4\pi}\left(\frac{q}{\left(x^2+y^2+(z-d)^2\right)^{\frac{n}{2}}} + \frac{q'}{\left(x^2+y^2+(z-d)^2\right)^{\frac{n}{2}}}\right) \tag{5.10}$$

$$\sigma = \sigma_1 + \sigma_2 \tag{5.11}$$

$$= -\frac{q}{4\pi}\left[\left(\frac{a}{y}\right)^n \frac{n\left(1-\frac{a}{y}\cos\theta\right)}{a^{n+1}\left(1+\frac{a^2}{y^2}-2\frac{a}{y}\cos\theta\right)^{\frac{n}{2}+1}} - \frac{n\left(\frac{a}{y}\cos\theta\right)}{y^{n+1}\left(1+\frac{a^2}{y^2}-2\frac{a}{y}\cos\theta\right)^{\frac{n}{2}+1}}\right] \tag{5.12}$$

$$\frac{4\pi\sigma}{-q} = \frac{\left(\frac{a}{y}\right)^n n\left(1-\frac{a}{y}\cos\theta\right)}{a^{n+1}\left(1+\frac{a^2}{y^2}-2\frac{a}{y}\cos\theta\right)^{\frac{n}{2}+1}} - \frac{n\left(\frac{a}{y}\cos\theta\right)}{y^{n+1}\left(1+\frac{a^2}{y^2}-2\frac{a}{y}\cos\theta\right)^{\frac{n}{2}+1}} \tag{5.13}$$

$$= \left(\frac{a}{y}\right)^n \frac{ny^{n+1}\left(1-\frac{a}{y}\cos\theta\right) - a^{n+1}n\left(\frac{a}{y}-\cos\theta\right)}{a^{n+1}y^{n+1}\left(1+\frac{a^2}{y^2}-2\frac{a}{y}\cos\theta\right)^{\frac{n}{2}+1}}$$

(5.14)

$$\frac{4\pi a^{n+1}\sigma}{-q} = \left(\frac{a}{y}\right)^n \left[\frac{\left(1-\frac{a}{y}\cos\theta\right) - \frac{a}{y}\left(\frac{a}{y}\cos\theta\right)}{\left(1+\frac{a^2}{y^2}-2\frac{a}{y}\cos\theta\right)^{\frac{n}{2}+1}}\right] \tag{5.15}$$

$$\frac{4\pi a^2\sigma}{-q} = \left(\frac{a}{y}\right)\frac{\left(1-\left(\frac{a}{y}\right)^2\right)}{\left(1+\frac{a^2}{y^2}-2\frac{a}{y}\cos\theta\right)^{\frac{3}{2}}} \tag{5.16}$$

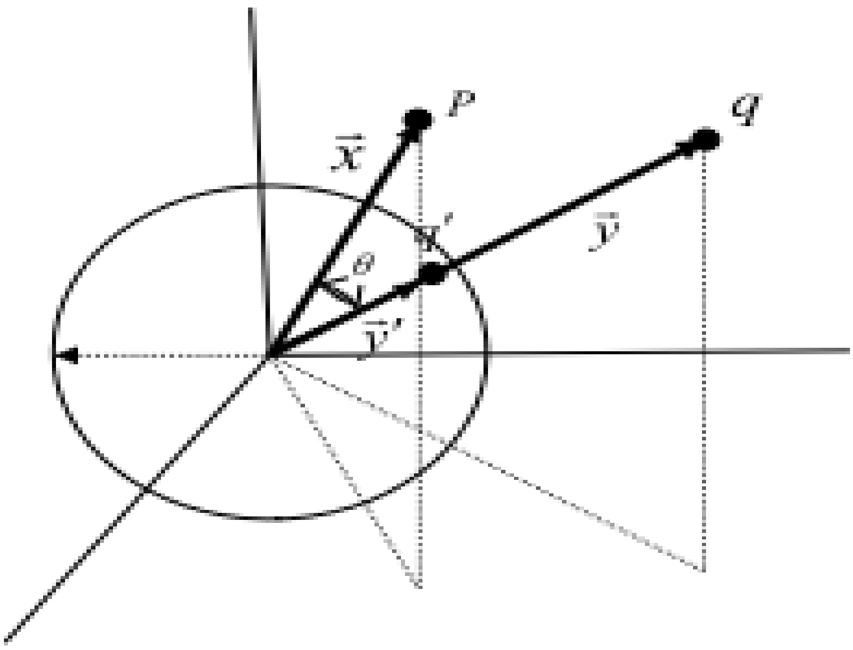

Figure 5.1 Two-dimensional schematic illustration of a conducting sphere of radius *a* with a point charge *q* outside and image charge *q′* inside.

If the sphere is grounded, then the potential at the surface of the sphere vanishes$\Phi(x = a) = 0$, thus:

Where $\hat{n}$ and $\hat{n}'$ are unit vectors in the direction of ***x*** and ***y*** respectively. To satisfy the boundary condition

$\Phi(x = a) = 0$ at $R = a$, we must have:

More generally, the potential in the neighborhood of an uncharged grounded conducting sphere is given by

Equation (5.4):

Substitute Equation (5.5) in Equation (5.4) and then differentiate to get the actual induced charge density on thesurface of the grounded uncharged conducting sphere:

The total charge on the sphere may be found by integrating Equation (5.6) over all angles. The total surface inducedcharge is equal to the magnitude of the image charge for Coulomb potential. But in case of Yukawa potentialthe total surface induced charge is less than the value of the image charge. This result implies that smallportion of the induced charge distributed itself inside the volume of the conducting sphere. The rest of the inducedcharge is distributed itself on the surface of the conducting sphere. Some values of the total induced surfacecharge on grounded conducting sphere are given in Equation (5.15) and (5.16) for both Coulomb when n=1 and inverse power law potentials when n=1.1, 1.2 etc. Which are graphed on (Figures (5.2), (5.6)).

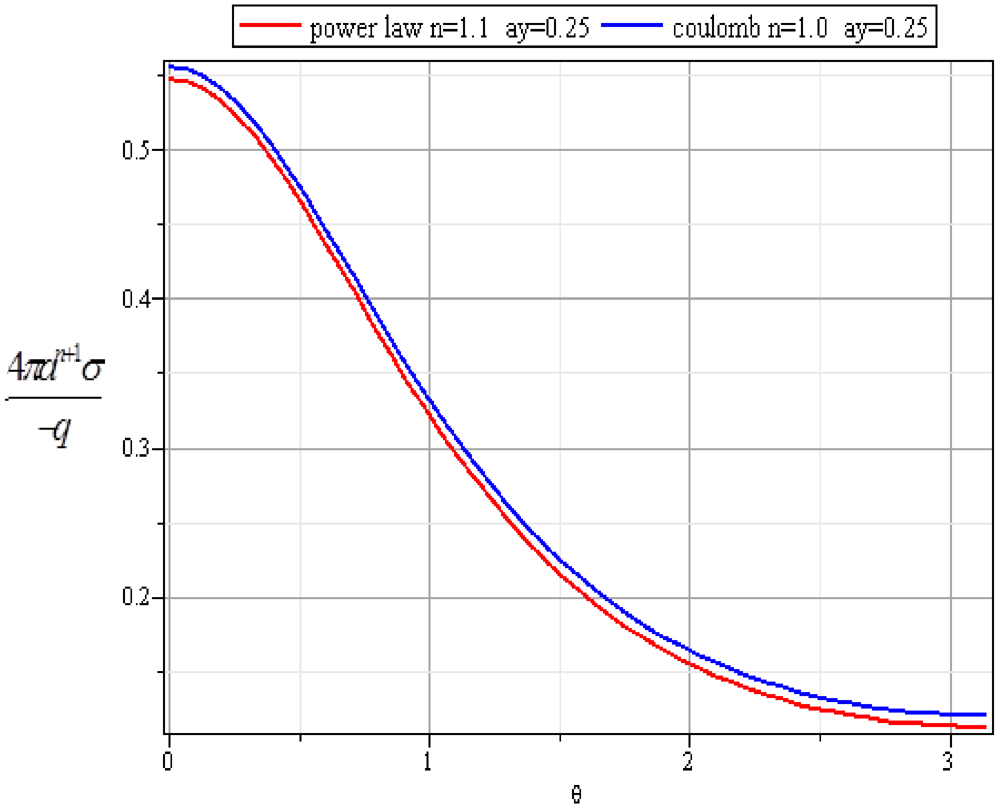

Figure (5.2). Charge distribution on grounded conducting sphere in existence charge q by means method of images in case y inverse power law and coulombs low at $n = 1.1$, $a/y = 0.25$

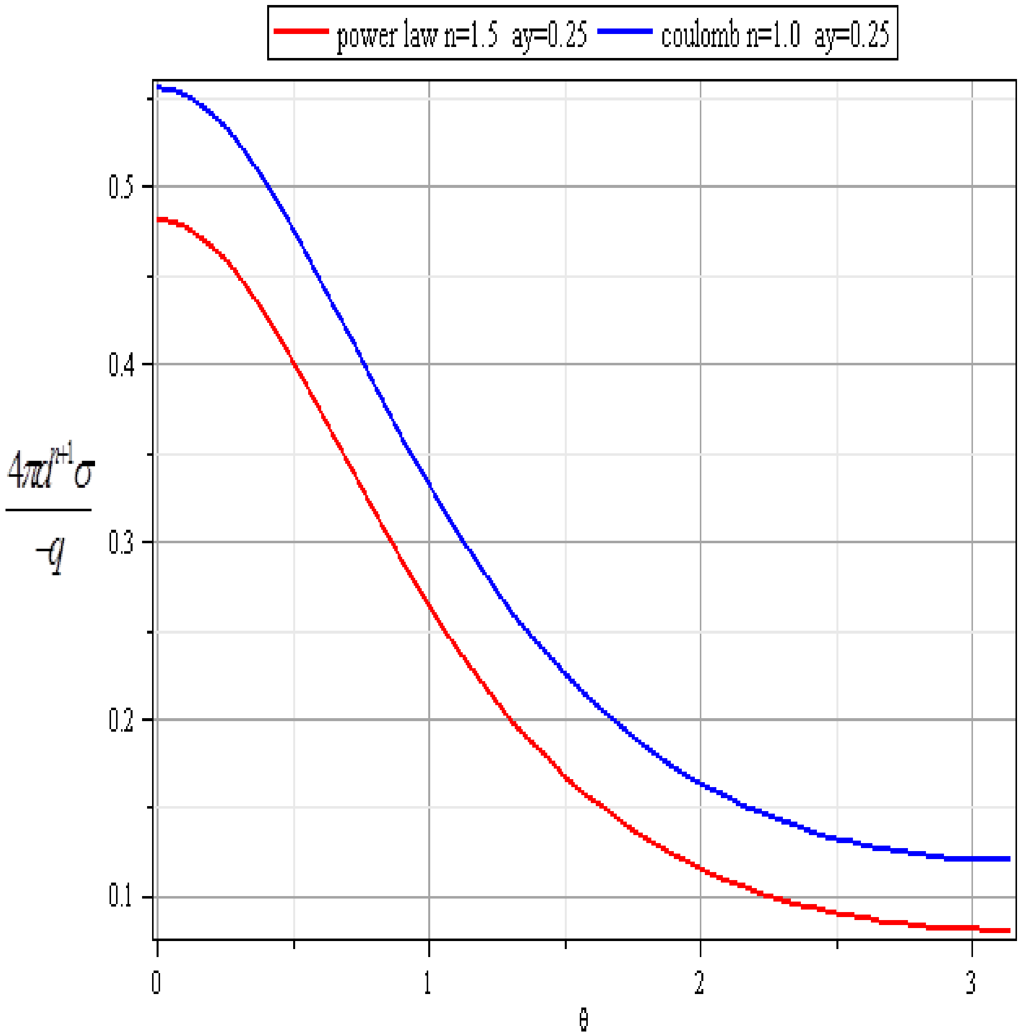

Figure (5.3) Charge distribution on grounded conducting sphere in existence charge q by means method of images in case y inverse power law and coulombs low at $n = 1.5$, $a/y = 0.25$

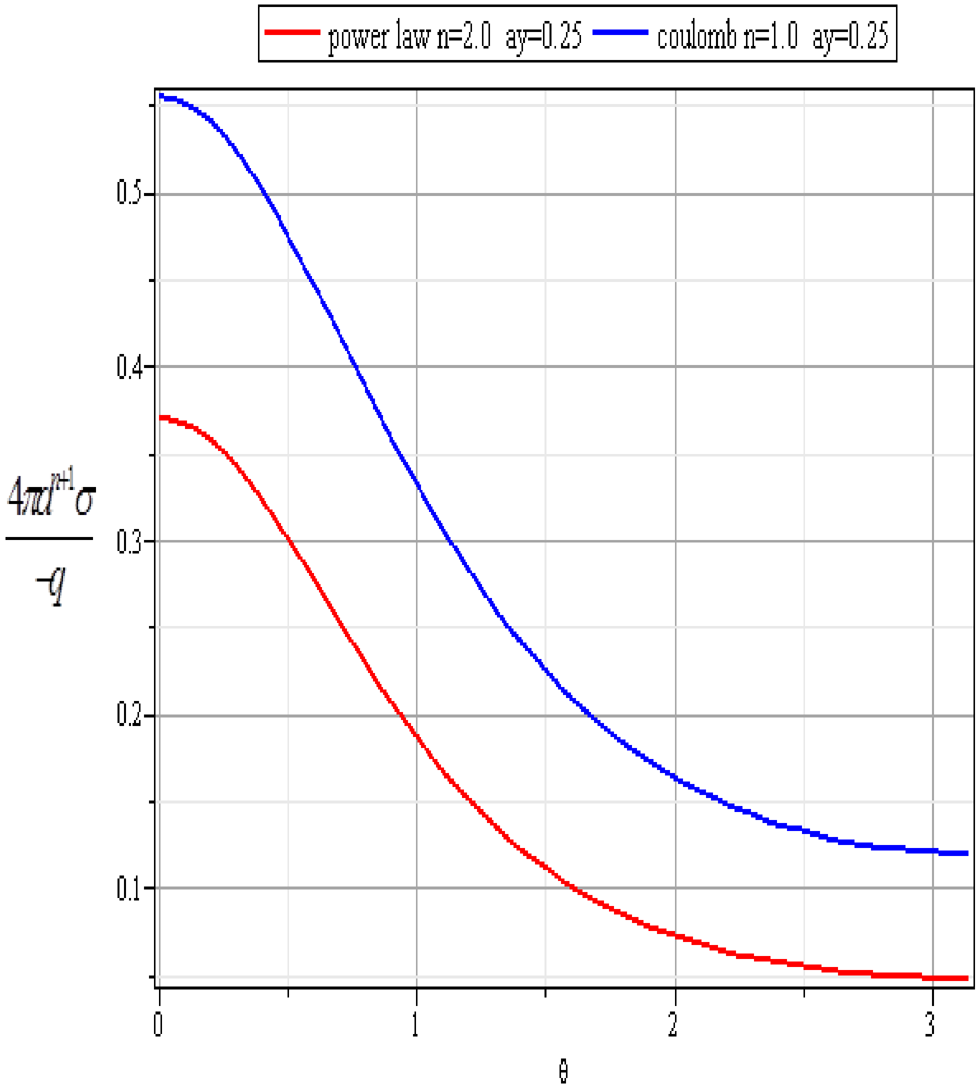

Figure (5.3). Charge distribution on grounded conducting sphere in existence charge q by means method of images in case y inverse power law and coulombs low at $n = 2$, $a/y = 0.25$

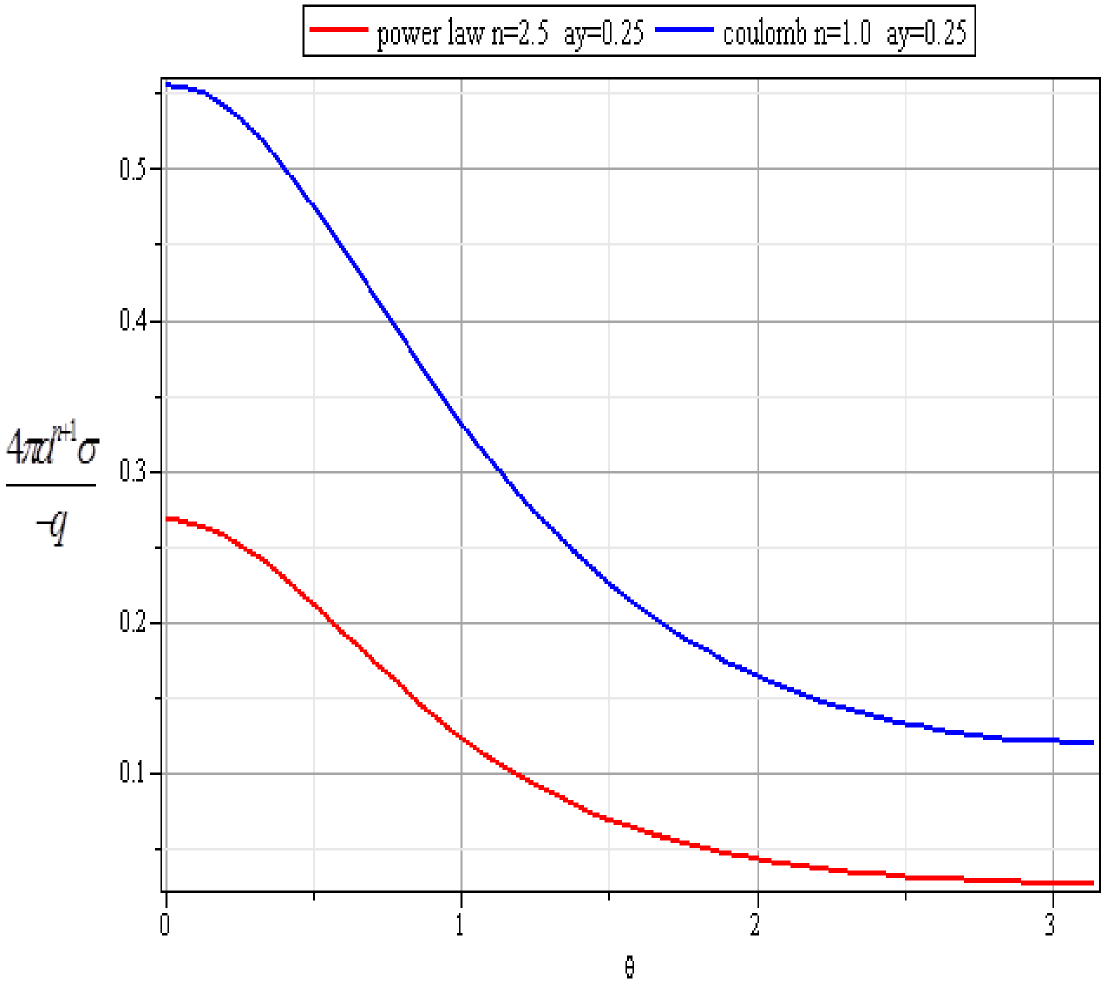

Figure (5.4). Charge distribution on grounded conducting sphere in existence charge q by means method of images in case y inverse power law and coulombs low at $n = 2.5$, $a/y = 0.25$

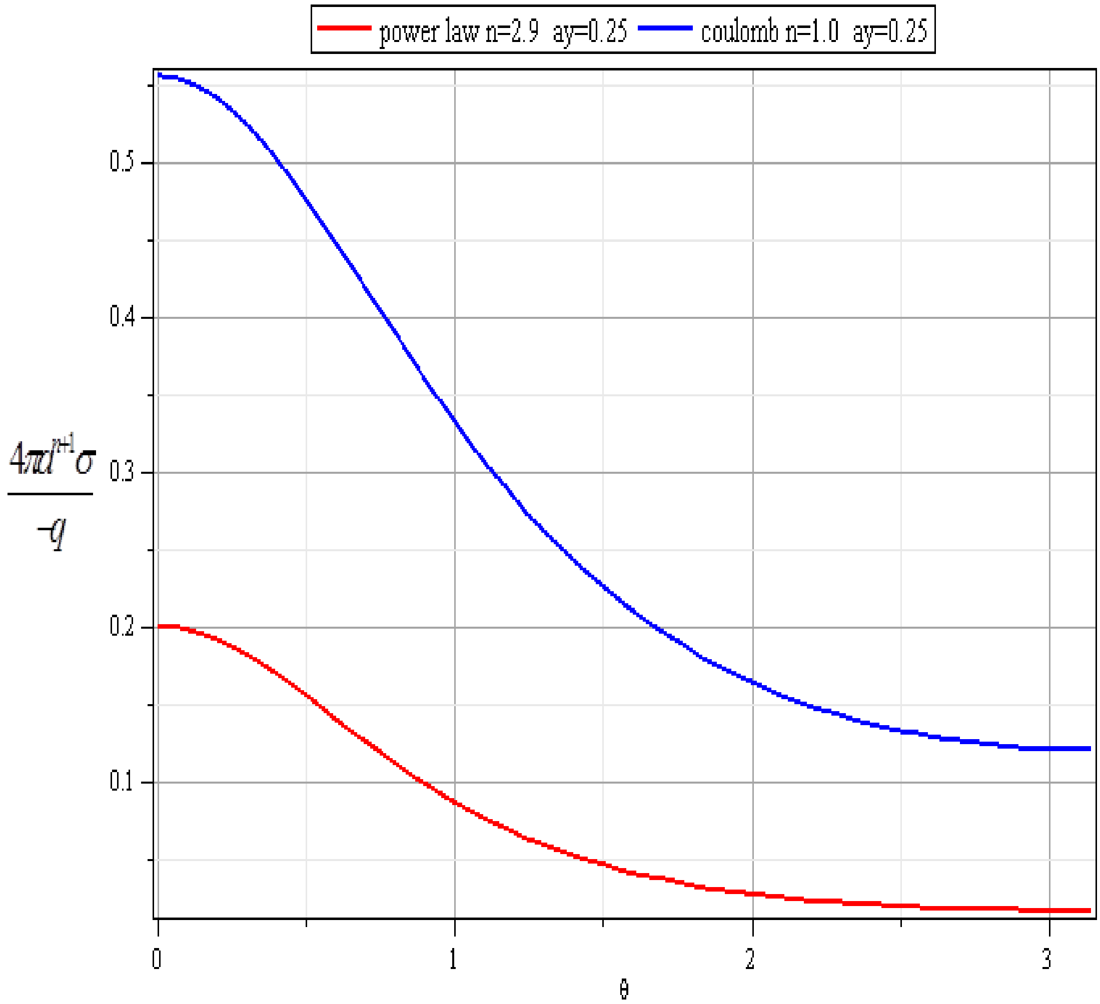

Figure (5.5). Charge distribution on grounded conducting sphere in existence charge q by means method of images in case y inverse power law and coulombs low at $n = 2.9$, $a/y = 0.25$

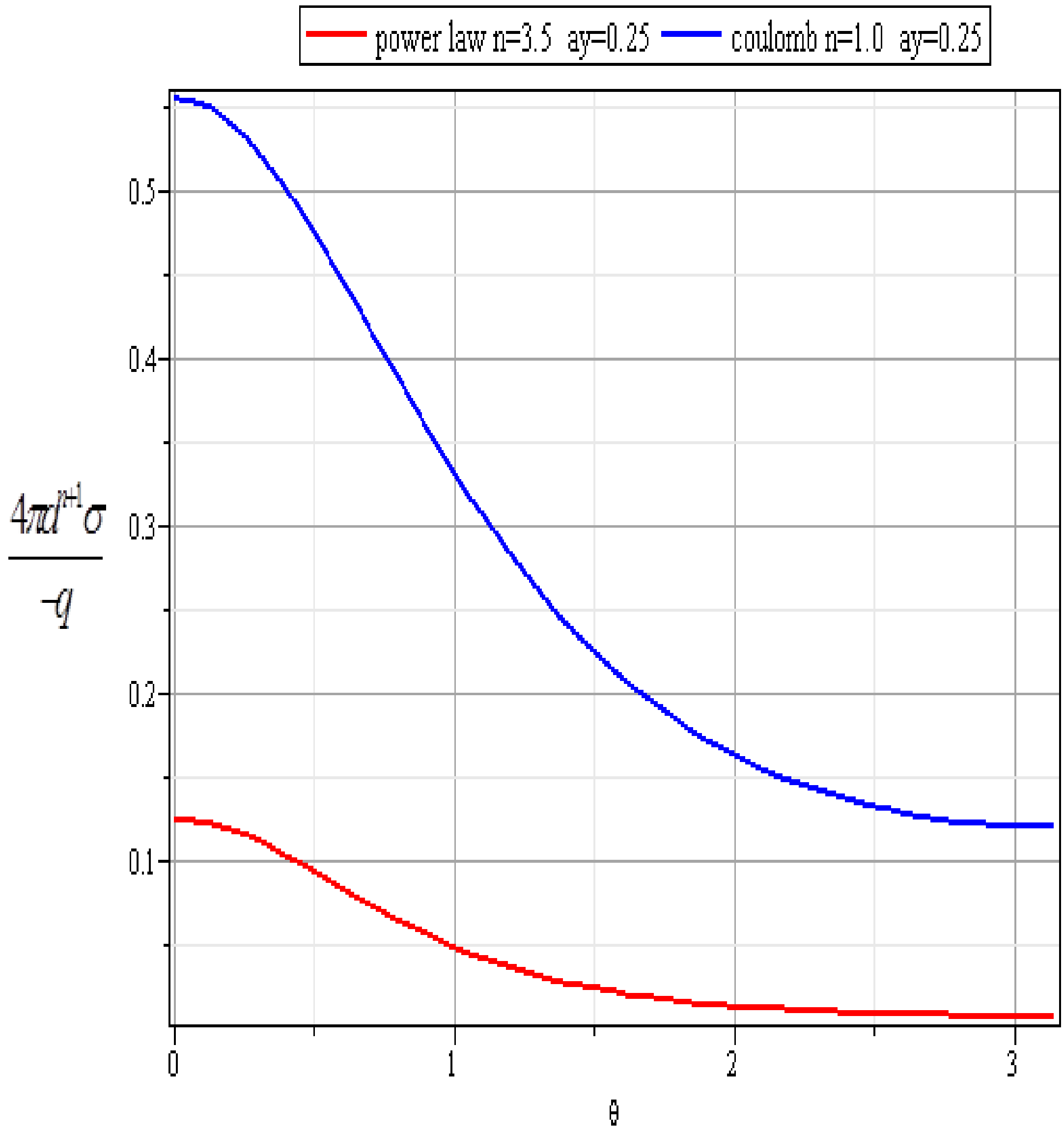

Figure (5.6). Charge distribution on grounded conducting sphere in existence charge q by means method of images in case y inverse power law and coulombs low at $n = 3.5$, $a/y = 0.25$

5.3 Discussion

From (Figure (5.2), (5.6)) it was clear that the area under those curves in case of inverse power law potential compared to coulombs law reveals quite difference (coulombs law state that charge on any conductor reside on the surface of the conductor and inverse power law indicate that some of charge reside on the surface while the remainder of this charge distribute its self in the interior volume of the conductor).

5.4 Conclusion

According to the previous result one concluded that in case of nonzero mass photon even very small mass it will be sensitive to this case and some of this charge will distribute in the interior volume of the conductor when power law used instead of coulomb law.

5.5 Recommendation

1. More experimental work should be held in order to find the upper limit of photon mass.
2. The sensitivity of the equipment which are used to determine the photon mass bust modified to determine the mass of photon accurately.

5.6 Reference

[1] Cavendish H 1773 the Electrical Researches of the Honourable Henry Cavendish ed J C Maxwell (Cambridge:Cambridge University Press, 1879)pp 104–13.

[2] Maxwell J C 1873 A Treatise on Electricity and Magnetism3rd edn (New York: Dover, 1954) pp 80–6.

[3] Kobzarev I Y and Okun L B 1968 Usp. Fiz.Nauk.95 131–7Kobzarev I Y and Okun L B 1968 Sov. Phys. Usp. 11 338–41(Engl. Transl.)

[4] Goldhaber A S and Nieto M M 1971 Rev. Mod. Phys.43 277–96.

[5] Chibisov G V 1976 Usp. Fiz.Nauk.119 551–5Chibisov G V 1976 Sov. Phys. Usp. 19 624–6 (Engl. Transl.)

[6] Goldhaber A S and Nieto M M 1976 Sci. Am. 234 86–96.

[7] Byrne J C 1977 Astrophys. Space Sci. 46 115–32.

[8] Jackson J D 1975 Classical Electrodynamic 2nd edn(New York: Wiley)
pp 5–9.

[9] Proca A 1936 J. Phys. Radium Ser. VII 7 347–53Proca A 1937 J. Phys. Radium Ser. VII 8 23–8.

[10] De Broglie L 1940 La M´echanique Ondulatoire du Photon,Une Nouvelle Th´eorie de Lumi`ere vol 1 (Paris: Hermann)pp 39–40.

[11] Asseo E and Sol H 1987 Phys. Rep. 148 307–46.

[12] Kronberg P P 1994 Rep. Prog. Phys. 57 325–82Kronberg P P 2002 Phys.Today 55 40–6.

[13] Williams E R, Faller J E and Hill H A 1971 Phys. Rev. Lett.26 721–4Williams E R, Faller J E and Hill H A 1970 Bull. Am. Phys.Soc. 15 586–7

Williams E R 1970 PhD Thesis Wesleyan University,Middletown, CT
Soc. 15 586–7Williams E R 1970 PhD Thesis Wesleyan University,Middletown, CT.

[14] Chernikov M A, Gerber C J, Ott H R and Gerber H J 1992Phys. Rev. Lett.68 3383–6.

[15] Lakes R 1998 Phys. Rev. Lett. 80 1826–9.

[16] Luo J, Tu L-C, Hu Z-K and Luan E-J 2003 Phys. Rev. Lett.90 081801.

[17] Feinberg G 1969 Science 166 879–81.

[18] Schaefer B E 1999 Phys. Rev. Lett. 82 4964–6.

[19] Fischbach E, Kloor H, Langel R A, Lui A T Y and Peredo M1994 Phys. Rev. Lett 73 514–17.

[20] Davis L, Goldhaber A S and Nieto M M 1975 Phys. Rev. Lett.35 1402–5.

[21] Ryutov D D 1997 Plasma Phys. Control. Fusion 39 A73–82.

[22] Gintsburg M A 1963 Astron. Zh. 40 703–9Gintsburg M A 1964 Sov. Astron. AJ 7 536–40 (Engl. Transl.)

[23] Patel V L 1965 Phys. Lett. 14 105–6.

[24] Hollweg J V 1974 Phys. Rev. Lett. 32 961–2.

[25] Barnes A and Scargle J D 1975 Phys. Rev. Lett.35 1117–20.

[26] deBernadis P, Masi S, Melchiorri F and Moleti A 1984Astrophys. J. 284 L21–22.

[27] Williams E and Park D 1971 Phys. Rev. Lett. 26 1651–2.

[28] Neyenhuis, B., Christensen, D. and Durfee, D.S. (2007) Testing Nonclassical Theories of Electromagnetism with Ion Interferometry. *Physical Review Letters*, **99**, 200401.http://dx.doi.org/10.1103/PhysRevLett.99.200401.

[29] Tu, L.-C. and Lou, J. (2004) Experimental Tests of Coulomb's Law and the Photon Rest Mass. *Metrologia*, **41**, S136- S146.

[30] Spencer, R.L. (1990) If Coulomb's Law Were Not Inverse Square: The Charge Distribution Inside a Solid Conducting Sphere. *American Journal of Physics*, **58**, 385-390. http://dx.doi.org/10.1119/1.16179.

[31] Griffiths, D.J. and Uvanovic, D.Z. (2001) the Charge Distribution on a Conductor for Non-Coulombic Potentials. *American Journal of Physics*, **69**, 435-440. http://dx.doi.org/10.1119/1.1339279.

[32] Jackson, J.D. (1975) Classical Electrodynamics. 2nd Edition, Wiley, New York.

[33] Griffiths, D.J. (1999) Introduction to Electrodynamics. 3rd Edition, Prentice-Hall, Upper Saddle River.

APPENDEX

البرنامج المستخدم في رسم المنحنيات

<u>اولا</u> في حالة لوح موصل (يوكاوا و كولوم)

```
with(plots):

k:=1.;

                    1.

d:=1.0;

                    1.0

A:=plot((k/(r^2+d^2)+1.0/(r^2+d^2)^1.5)*exp(-k*(r^2+d^2)^0.5),r=0..5,symbolsize=2,thickness=2,color=red,symbol=circle,legend="yukawa k=1.0 d=1"):

B:=plot(1.0/(r^2+d^2)^1.5,r=0..5,color=blue,thickness=1,symbol=diamond,legend="couolomb k=0  d=1"):

display(A,B);
```

<u>ثانيا</u> في حالة كرة موصلة متصل بالأرض(يوكاوا و كولوم)

```
with(plots):

k:=0.00824;

                 k := 0.00824
```

ay:=1.0/2.0;

ay := 0.5000000000

a:=1.0;

a := 1.0

y:=a/2.0;

y := 0.5000000000

z:=(1.25-cos(theta))^0.5;

$$z := (1.25 - \cos(\theta))^{0.5}$$

A:=plot(k*(ay-cos(theta))*(exp(-k*(y-a)*z)-exp(-k*y*z))-k*(1-(ay)*cos(theta))*(exp(-k*(y-a)*z))/(1+(ay)^2-2.0*(ay)*cos(theta))+(1-(ay)*cos(theta))*(exp(-k*(y-a)*z))-ay*(ay-cos(theta))*(exp(-k*y*z))/(1.0+(ay)^2-2.*(ay)*cos(theta))^1.5,theta=0..Pi,color=red):

ay:=1.0/4.0;

ay := 0.2500000000

y:=a/4.0;

y := 0.2500000000

z:=(17.0/16.0-(0.5)*cos(theta))^0.5;

0.5

```
z := (1.062500000 - 0.5 cos(theta))

B:=plot(k*(ay-cos(theta))*(exp(-k*(y-a)*z)-exp(-k*y*z))-k*(1-(ay)*cos(theta))*(exp(-k*(y-a)*z))/(1+(ay)^2-2.0*(ay)*cos(theta))+(1-(ay)*cos(theta))*(exp(-k*(y-a)*z))-ay*(ay-cos(theta))*(exp(-k*y*z))/(1.0+(ay)^2-2.*(ay)*cos(theta))^1.5,theta=0..Pi,color=black):

ay:=1.0/2.0;

ay := 0.5000000000

a:=1.0;

a := 1.0

y:=a/2.0;

y := 0.5000000000

C:=plot(ay*(1.-ay^2)/(1.+ay^2-2.*ay*cos(theta))^1.5,theta=0..Pi,color=blue):

ay:=1.0/4.0;

ay := 0.2500000000

a:=1.0;

a := 1.0

y:=a/4.0;

y := 0.2500000000
```

```
d:=plot(ay*(1.-ay^2)/(1.+ay^2-
2.*ay*cos(theta))^1.5,theta=0..Pi,color=green):

display(A,B,C,d);
```

ثالثا في حالة كرة موصلة مشحونة و معزولة(يوكاوا و كولوم)

```
> with(plots):
> ay:=1.0/2.0;
0.5000000000
> k:=0.00824;
0.00824
> a:=1.0;
1.0
> y:=a/2.0;
0.5000000000
> Qq:=0.0;
0.
> z:=(1.25-cos(theta))^0.5;
(1.25 - cos(θ))^0.5
> A:=plot(ay*(1.0-ay^2)/(1+ay^2-2.*ay*cos(theta))^1.5-
(Qq+ay),theta=0..Pi):
> B:=plot(k*(ay-cos(theta))*(exp(-k*(y-a)*z)-exp(-k*y*z))-k*(1-
(ay)*cos(theta))*(exp(-k*(y-a)*z))/(1+(ay)^2-
2.0*(ay)*cos(theta))+(1-(ay)*cos(theta))*(exp(-k*(y-a)*z))-ay*(ay-
cos(theta))*(exp(-k*y*z))/(1.0+(ay)^2-2.*(ay)*cos(theta))^1.5+(-
```

```
k*(ay-cos(theta))+k*(1-ay*cos(theta)))*exp(-k(y-a)*z)/z-1.0*exp(-
k(y-a)*z),theta=0..Pi):
> display(A,B);
> k:=0.00824;
```

0.00824

```
> ay:=1.0/2.0;
```

0.5000000000

```
> a:=1.0;
```

1.0

```
> y:=a/2.0;
```

0.5000000000

```
> z=(1.25-cos(theta))^0.5;
```

$(1.25-\cos(\theta))^{0.5}=(1.25-\cos(\theta))^{0.5}$

```
> Qq:=-1.0;
```

-1.0

```
> A:=plot(ay*(1.0-ay^2)/(1+ay^2-2.*ay*cos(theta))^1.5-
(Qq+ay),theta=0..Pi):
> B:=plot(k*(ay-cos(theta))*(exp(-k*(y-a)*z)-exp(-k*y*z))-k*(1-
(ay)*cos(theta))*(exp(-k*(y-a)*z))/(1+(ay)^2-
2.0*(ay)*cos(theta))+(1-(ay)*cos(theta))*(exp(-k*(y-a)*z))-ay*(ay-
cos(theta))*(exp(-k*y*z))/(1.0+(ay)^2-2.*(ay)*cos(theta))^1.5+(-
k*(ay-cos(theta))+k*(1-ay*cos(theta)))*exp(-k(y-a)*z)/z-1.0*exp(-
k(y-a)*z),theta=0..Pi):
> display(A,B);
> k:=0.00824;
```

0.00824

> ay:=1.0/2.0;

0.5000000000

> a:=1.0;

1.0

> y:=a/2.0;

0.5000000000

> z=(1.25-cos(theta))^0.5;

$(1.25 - \cos(\theta))^{0.5} = (1.25 - \cos(\theta))^{0.5}$

> Qq:=1.0;

1.0

> A:=plot(ay*(1.0-ay^2)/(1+ay^2-2.*ay*cos(theta))^1.5-(Qq+ay),theta=0..Pi):

> B:=plot(k*(ay-cos(theta))*(exp(-k*(y-a)*z)-exp(-k*y*z))-k*(1-(ay)*cos(theta))*(exp(-k*(y-a)*z))/(1+(ay)^2-2.0*(ay)*cos(theta))+(1-(ay)*cos(theta))*(exp(-k*(y-a)*z))-ay*(ay-cos(theta))*(exp(-k*y*z))/(1.0+(ay)^2-2.*(ay)*cos(theta))^1.5+(-k*(ay-cos(theta))+k*(1-ay*cos(theta)))*exp(-k(y-a)*z)/z-1.0*exp(-k(y-a)*z),theta=0..Pi):

> display(A,B);

k:=0.00824;

0.00824

ay:=1.0/2.0;

0.5000000000

a:=1.0;

1.0

y:=a/2.0;

0.5000000000

z:=(1.25-cos(theta))^0.5;

0.5

(1.25 - cos(theta))

Qq:=3.0;

A:=plot(ay*(1.0-ay^2)/(1+ay^2-2.*ay*cos(theta))^1.5-(Qq+ay),theta=0..Pi):

B:=plot(k*(ay-cos(theta))*(exp(-k*(y-a)*z)-exp(-k*y*z))-k*(1-(ay)*cos(theta))*(exp(-k*(y-a)*z))/(1+(ay)^2-2.0*(ay)*cos(theta))+(1-(ay)*cos(theta))*(exp(-k*(y-a)*z))-ay*(ay-cos(theta))*(exp(-k*y*z))/(1.0+(ay)^2-2.*(ay)*cos(theta))^1.5+(-k*(ay-cos(theta))+k*(1-ay*cos(theta)))*exp(-k(y-a)*z)/z-1.0*exp(-k(y-a)*z),theta=0..Pi):

display(A,B);

رابعا في حالة لوح متصل بالأرض (قانون القوة العكسي و كولوم)

> with(plots):
> n:=1.1;

$n := 1.1$

```
> d:=1.0;
d := 1.0
>
B:=plot(1/(r^2+d^2)^(n/2+1),r=0.0..4.0,color=red,thickness=3,legend="yukawa n=1.1"):
>
A:=plot(1/(r^2+d^2)^(1.5),r=0.0..4.0,color=blue,thickness=3,legend="coulomb n=1"):
> n:=1.5;
n := 1.5
>
C:=plot(1/(r^2+d^2)^(n/2+1),r=0.0..4.0,color=black,thickness=3,legend="yukawa n=1.5"):
> n:=2.0;
n := 2.0
>
l:=plot(1/(r^2+d^2)^(n/2+1),r=0.0..4.0,color=green,thickness=3,legend="yukawa n=2.0"):
>
> n:=2.5;
n := 2.5
> d:=1.0;
d := 1.0
```

```
>
e:=plot(1/(r^2+d^2)^(n/2+1),r=0.0..4.0,color=orange,thickness=3,legend="yukawa n=2.5"):
> n:=2.9;
```

$n := 2.9$

```
>
f:=plot(1/(r^2+d^2)^(n/2+1),r=0.0..4.0,color=brown,thickness=3,legend="yukawa n=2.9"):
> n:=3.0;
```

$n := 3.0$

```
>
g:=plot(1/(r^2+d^2)^(n/2+1),r=0.0..4.0,color=yellow,thickness=4,legend="yukawa n=3.0"):
```

```
> n:=3.5;
```

$n := 3.5$

```
>
h:=plot(1/(r^2+d^2)^(n/2+1),r=0.0..4.0,color=gray,thickness=4,legend="yukawa n=3.5"):
> display(A,B,C,l,e,f,g,h);
```

خامسا كرة موصلة متصلة بالأرض (القوة العكسي و كولوم)

```
> with(plots):
> ay:=1.0/2.0;
```

0.500000000(

> n:=1.1;

1.1

>

> ay:=1.0/2.0;

0.500000000(

>

$A := plot(n*(ay)\verb|^|n*((1-ay*\cos(\theta))-ay*(ay-\cos(\theta)))/(1+ay\verb|^|2-2.0*ay*\cos(\theta))\verb|^|((n+2)/2), \theta = 0.0..\pi, color = red, thickness = 4, legend = \text{"yukawa n=1.1"}) :$

> B:=plot(ay*(1.-ay^2)/(1.+ay^2-2.*ay*cos(theta))^1.5,theta=0..Pi,color=blue,thickness=4,legend="yukawa n=1.1"):

> display(A,B);

> ay:=1.0/2.0;

0.500000000(

> n:=1.5;

1.5

> A:=plot(n*(ay)^n*((1-ay*cos(theta))-ay*(ay-cos(theta)))/(1+ay^2-2.0*ay*cos(theta))^((n+2)/2),theta=0.0..Pi,color=red,thickness=4,legend="yukawa n=1.5"):

> ay:=1.0/2.0;

0.500000000(

> B:=plot(ay*(1.-ay^2)/(1.+ay^2-2.*ay*cos(theta))^1.5,theta=0..Pi,color=blue,thickness=4,legend="yukawa n=1.5"):

> display(A,B);

> ay:=1.0/2.0;

0.5000000000

> n:=2.9;

2.9

> A:=plot(n*(ay)^n*((1-ay*cos(theta))-ay*(ay-cos(theta)))/(1+ay^2-2.0*ay*cos(theta))^((n+2)/2),theta=0.0..Pi,color=red,thickness=4,legend="yukawa n=2.9"):

> ay:=1.0/2.0;

0.5000000000

> B:=plot(ay*(1.-ay^2)/(1.+ay^2-2.*ay*cos(theta))^1.5,theta=0..Pi,color=blue,thickness=4,legend="yukawa n=2.9"):

> display(A,B);

سادسا في حالة كرة معزولة و مشحونة (القوة العكسي و كولوم)

> with(plots):

> ay:=1.0/2.0;

0.5000000000

> n:=1.1;

1.1

> Qq:=-3.0;

−3.0

```
> A:=plot(n*(ay)^n*((1-ay*cos(theta))-ay*(ay-
cos(theta)))/(1+ay^2-
2.0*ay*cos(theta))^((n+2)/2)+n*(Qq+ay^n),theta=0.0..Pi,color=re
d,thickness=2,legend="power law a/y=0.5 n=1.1 Qq=-3"):

> ay:=1.0/2.0;
0.500000000(
> B:=plot(ay*(1.-ay^2)/(1.+ay^2-
2.*ay*cos(theta))^1.5,theta=0..Pi,color=blue,thickness=2,legend="
Coulomb a/y=0.5 n=1.0"):

> display(A,B);
> ay:=1.0/4.0;
0.250000000(
> n:=1.1;
1.1
> Qq:=-3.0;
−3.0
> A:=plot(n*(ay)^n*((1-ay*cos(theta))-ay*(ay-
cos(theta)))/(1+ay^2-
2.0*ay*cos(theta))^((n+2)/2)+n*(Qq+ay^n),theta=0.0..Pi,color=bl
ack,thickness=2,legend="power law a/y=0.25 n=1.1 Qq=-3"):
> ay:=1.0/4.0;
0.250000000(
```

```
> B:=plot(ay*(1.-ay^2)/(1.+ay^2-2.*ay*cos(theta))^1.5,theta=0..Pi,color=green,thickness=2,legend="Coulomb a/y=0.25"):
> display(A,B);
> ay:=1.0/2.0;
0.500000000
> n:=1.1;
1.1
> Qq:=-1.0;
−1.0
> A:=plot(n*(ay)^n*((1-ay*cos(theta))-ay*(ay-cos(theta)))/(1+ay^2-2.0*ay*cos(theta))^((n+2)/2)+n*(Qq+ay^n),theta=0.0..Pi,color=red,thickness=2,legend="power law a/y=0.5 n=1.1 Qq=-1.0"):
> ay:=1.0/2.0;
0.500000000
> B:=plot(ay*(1.-ay^2)/(1.+ay^2-2.*ay*cos(theta))^1.5,theta=0..Pi,color=blue,thickness=2,legend="Coulomb a/y=0.5 n=1.0"):
> display(A,B);
> ay:=1.0/4.0;
0.250000000
> n:=1.1;
1.1
> Qq:=-1.0;
```

−1.0

> A:=plot(n*(ay)^n*((1-ay*cos(theta))-ay*(ay-cos(theta)))/(1+ay^2-2.0*ay*cos(theta))^((n+2)/2)+n*(Qq+ay^n),theta=0.0..Pi,color=black,thickness=2,legend="power law a/y=0.25 n=1.1 Qq=-1.0"):

> ay:=1.0/4.0;

0.2500000000

> B:=plot(ay*(1.-ay^2)/(1.+ay^2-2.*ay*cos(theta))^1.5,theta=0..Pi,color=green,thickness=2,legend="Coulomb a/y=0.25 n=1.0"):

> display(A,B);

> ay:=1.0/2.0;

0.5000000000

> n:=1.5;

1.5

> Qq:=-1.0;

−1.0

> A:=plot(n*(ay)^n*((1-ay*cos(theta))-ay*(ay-cos(theta)))/(1+ay^2-2.0*ay*cos(theta))^((n+2)/2)+n*(Qq+ay^n),theta=0.0..Pi,color=red,thickness=2,legend="power law a/y=0.5 n=1.5 Qq==1.0"):

> ay:=1.0/2.0;

0.5000000000

```
> B:=plot(ay*(1.-ay^2)/(1.+ay^2-
2.*ay*cos(theta))^1.5,theta=0..Pi,color=blue,thickness=2,legend="
Coulomb a/y=0.5 n=1.0"):
> display(A,B);
> ay:=1.0/4.0;
0.250000000(
> n:=1.5;
1.5
> Qq:=-1.0;
−1.0
> A:=plot(n*(ay)^n*((1-ay*cos(theta))-ay*(ay-
cos(theta)))/(1+ay^2-
2.0*ay*cos(theta))^((n+2)/2)+n*(Qq+ay^n),theta=0.0..Pi,color=bl
ack,thickness=2,legend="power law a/y=0.25 n=1.5 Qq=-1.0"):
> ay:=1.0/4.0;
0.250000000(
> B:=plot(ay*(1.-ay^2)/(1.+ay^2-
2.*ay*cos(theta))^1.5,theta=0..Pi,color=green,thickness=2,legend=
"Coulomb a/y=0.25 n=1.0"):
> display(A,B);
> ay:=1.0/2.0;
0.500000000(
> n:=2.9;
2.9
> Qq:=-1.0;
```

```
−1.0
> A:=plot(n*(ay)^n*((1-ay*cos(theta))-ay*(ay-
cos(theta)))/(1+ay^2-
2.0*ay*cos(theta))^((n+2)/2)+n*(Qq+ay^n),theta=0.0..Pi,color=re
d,thickness=2,legend="power law a/y=0.5 n=2.9 Qq=-1.0"):
> ay:=1.0/2.0;
0.500000000(
> B:=plot(ay*(1.-ay^2)/(1.+ay^2-
2.*ay*cos(theta))^1.5,theta=0..Pi,color=blue,thickness=2,legend="
Coulomb a/y=0.5 n=1.0"):
> display(A,B);
> ay:=1.0/4.0;
0.250000000(
> n:=2.9;
2.9
> Qq:=-1.0;
−1.0
> A:=plot(n*(ay)^n*((1-ay*cos(theta))-ay*(ay-
cos(theta)))/(1+ay^2-
2.0*ay*cos(theta))^((n+2)/2)+n*(Qq+ay^n),theta=0.0..Pi,color=bl
ack,thickness=2,legend="power law a/y=0.25 n=2.9 Qq=-1.0"):
> ay:=1.0/4.0;
0.250000000(
```

```
> B:=plot(ay*(1.-ay^2)/(1.+ay^2-
2.*ay*cos(theta))^1.5,theta=0..Pi,color=green,thickness=2,legend=
"Coulomb a/y=0.25 n=1.0"):
> display(A,B);
> ay:=1.0/2.0;
```

0.500000000

```
> n := 1.1;
```

1.1

```
> Qq := -2.0;
```

−2.0

```
> A := plot(n*(ay)^n*((1-ay*cos(θ))-ay*(ay-cos(θ)))/(1
      + ay^2-2.0*ay*cos(θ))^((n + 2)/2) + n*(Qq + ay^n), θ
      = 0.0..π, color = red, thickness = 2, legend
      = "power law a/y=0.5 n=1.1 Qq=-2.0" :
```

```
> ay := 0.5;
```

0.5

```
> B := plot(ay*(1.-ay^2)/(1. + ay^2-2.*ay*cos(θ))^1.5, θ = 0
      ..π, color = blue, thickness = 2, legend
      = "Coulomb a/y=0.5 n=1.0" :
```

```
> DISPLAY(A,B);
```