بسم الله الرحمن الرحيم

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جامعة السودان للعلوم والتكنولوجيا
كلية الدراسات العليا

تعميم مبدأ تكافؤ الكتلة القصورية مع الكتلة الجاذبة وصياغة نظرية جديدة للجاذبية من التعميم تتوافق مع مبادئ ميكانيكا الكم

Generalization of Equivalence Principle of Inertia Mass with Gravity Mass and Formation of New Gravity Theory from the Generalization in Harmony with Quantum Mechanics Principles

A thesis Submitted for fulfillment for requirements of the degree of Doctor Philosophy in Physics

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الآيـــة

بِسۡــــمِ ٱللَّهِ ٱلرَّحۡمَٰنِ ٱلرَّحِيــمِ

وَيَسْأَلُونَكَ عَنِ الرُّوحِ ۖ قُلِ الرُّوحُ مِنْ أَمْرِ رَبِّي وَمَا أُوتِيتُمْ مِنَ الْعِلْمِ إِلَّا قَلِيلًا ﴿٨٥﴾

صـدق الـلــه الـعـظـيـم

سورة الإسراء

الاهـداء

الى تلك الشمعة التي مازالت تضئ لي دربي

من اسكب همومي عليها دون رحمة فتلقفها لهفة لتحيلها لي فرحة

أمي

الى ابنائي أعز ما امتلكت في هذه الدنيا

أيهم وهيفار

الى من تشرفت ان أكون اخا لهم وخالا وعما لأبنائهم

سندي وعزوتي

اخواتي واخواني

الى من الجأ لنصحه حين تغلق كل الدروب فتفاجئني حكمته العميقة

أبي

الي من حملت ابنائي واهدتني جوهرتان

زوجتي

الي أصدقاء غابوا عن الذاكرة وتشتت بهم الطرق

شــكر وعــرفــان

الشكر أولا واخير لله سبحانه وتعالى ولدعوات امي التي لم تتوقف منذ ان تنشقت اول انفاس الحياة في هذه الدنيا. كما أخص بالشكر الجزيل الى مشرفي بروف مبارك درار عبد الله الذي لا أجد من الكلمات ما توفيه حقه فيما قدمه الى من دعم بكل اشكاله وبصبر وتواضع العلماء جعلني اتلمس طريقي واخذ بيدي في كثير من الأمور التي تتجاوز حتى حدود هذا البحث، اسأل الله ان يجازيه خيرا وان يزيدنا علما من علمه.

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المستخلص

تمت دراسة تكافؤ الكتلة القصورية والكتلة الجاذبة في مجال الجاذبية الضعيف والقوي، حيث تبين تكافؤهما. كما تمت مناقشة تحويل لورنتز للجسيمات المتسارعة وكذلك الفوتون (جسيمات عديمة الكتلة). كما تم إيجاد مترية الفضاء لنظرية GSR في المجال الضعيف ثم في المجال القوي. ومن ثم تم تطبيق المترية على معادلة شرودنكر وإيجاد الحلول لها. اتضح انه في حالة تطبيق مترية الفضاء للمجال الضعيف فان الحلول تعطي للجسيمات ذات الكتلة ومضاد الجسيمات ذات الكتلة قيم طاقة متساوية لكن متعاكسة وكذلك بالنسبة للجسيمات عديمة الكتلة ومضاداتها. وعند تطبيق مترية المجال القوي ل GSR في معادلة شرودنكر للجسيمات عديمة الكتلة ومضاداتها وجد انها تتصرف بطريقة مغايرة للجسيمات ذات الكتلة، حيث اتضح ان مضاد الجسيمات عديمة الكتلة تتعامل مع الجاذبية باعتبارها قوة تنافر، وقاد هذا الامر لاعتبار ان مضاد الجسيمات عديمة الكتلة هي المسؤولة عن الطاقة المظلمة التي تسبب توسع الكون.

Abstract

The equivalence principle between inertial and gravity mass had been studied in weak and strong gravity field. Also the Lorentz transformation for acceleration particle and photon (massless particle) had been discussion. Also the metric tensor for GSR in weak and strong field was found, and they applied in Schrödinger equation and found its solutions. By applying the metric tensor of weak field, the solutions gave the particles with mass and anti-mass particles an equal value of energy but opposites and so for massless and anti-massless. By applying the metric tensor of strong gravity field of GSR in Schrödinger equation for the massless particles and anti-massless particles it found that they acting different comparing to particles with mass، where the anti-massless particles acting with the gravity as repulsion force، and this was led to concerned the anti-massless particles are responsibly about the dark energy which expanding the universe.

CHAPTER ONE

INTRODUCTION

1.1. Gravity and Quantum Mechanics

Gravity force is one of the forces that affect any matter. Gravitational forces described by Isaac Newton as inverse square of distance **[1]**. He used the fact that all bodies fall at rate independent of their masses which proved first by Galileo Galilean **[1,2]**. He was well aware that conclusions might be only approximately true, and that the inertial mass entering in his second law might not be precisely the same as a gravitation mass appearing in the law of gravitation **[2]**. Fiendish Wilhelm Bessel and Rolan von Eotvos were succeeded by different method to show that the ratio of gravity mass and inertia mass does not differ from one substance to another. Einstein took this conclusion of the equivalence between inertial and gravitational mass to led him to his principle of equivalence **[3]**. Actually first suggestion of an inverse square law was by Ismaed Bullia Idus in 1640, anyway, it was certainty Newton who in 1665 first deduced the invers square law from his observations. In the following centuries Newton's law of gravity met with a brilliant series of successes in explaining the motion of plants and moon **[4]**. But LeVerrie had calculate that the observed precession of the perihelia of Mercury faster than that would be expected according to Newton's theory. This calculation was confirmed by Simon Newcomb in 1882**[4]**. However Newtonian mechanics defined a family of references frame the so-called inertial frame with in which the laws of physics take the same form. The invariance of law of motion under Galileo group transformation is called Galilean invariance or principle of Galilean relativity **[5]**. This relativity depends in concept of absolute space and time. In 1880's Ernst Mach produce the first constructive against Newtonian absolute space. He discussed that the mass of the earth and other celestial bodies determines the inertial frame, this called Mach's principle **[4,5]**. In 1887 Michelson and E.W. Morley showed that the velocity of light is constant in any direction in any frame however the way you used to measure it still constant **[4]**. This problem solved by Albert Einstein in 1905 by replaced Galilean relativity by other relativity depends on the universal speed of light as the maximum limit of velocity which known as special relativity (SR) **[2]**. But under this new transformation mechanic of Newton is not invariant, only the electromagnetic theory satisfied the special relativity, therefor Einstein was led to modify the law of motion to be Lorentz-invariant, also this motivated Einstein to find invariant formula of gravity law, so in 1907 he introduced the principle of equivalence (EP) of gravitation and inertia to calculate the red shift of light in gravity field **[2]**. Einstein and other tries many times to find the formula of invariant gravity law until the year 1913 when the mathematician Marcal Grossman inspired Einstein to view that gravitational field can be identified with ten components of matric tenser of Riemannian space-time geometry **[1,6]**. The principle of equivalence is incorporated into this formula through the requirement that the physical equation be invariant under general coordinate transformation not just Lorentz transformation. Thus in series of papers presented by Einstein to Prussian Academy he succeeded to form the field equations for the metric tensor and calculate the deflection of light and the precession of perihelia of Mercury **[1,7]**. In 1916 he presented the final formula **[8]**. Other way of improving and developing the physics was beginning in 1900 by Max Plank who suggested the concept of the quantum mechanics by assumed that the emission and absorption of radiation always takes place in discrete levels of energy on energy quanta **[9,10]**. This branch of physics developed first by classical mechanics. Other revolutionary ideas in this branch was by Bohr, Einstein, De Brule, Schrodinger, Dirac, and others. These ideas succeeded to describe the atomic structure and the behavior of elementary particle **[9,10,11]**. First use of special relativity in quantum mechanics was by Klein and Gordon and later by Dirac. The mixing of special relativity and quantum mechanics led to discover and predict of electron's spin and discovering of positron and other **[9]**. But there is problem to introduce a quantum theory of relativity where while the special relativity is in both of quantum mechanics and general relativity but the theory of general relativity does not match well with quantum mechanics **[4,12,13,14]**. In 2003 M.D. Abdallah introduce a general formula of special relativity to include the effect of fields **[15]**. This theory succeeded to explain and confirm and predict many phenomena. These offers were led by M.D. Abdallah and others to explain the behavior of elementary particle in strong and weak field **[16,17,18,19,20]**. The theory derived from modified the formula of Lorentz transformation and by other way **[15]**.

1.2. Research Problem

The concept of mass (inertial and gravitation) is ambiguous in SR. this is due to the fact that the concept of field is not present in SR expressing of energy.

1.3. Literature Review

Different attempts were made to form quantum theory of gravitation, but all these offers are uncompleted, even the new theory like string theory does not give a complete answer about some phenomenon like the expansion of the universe. Even the new version which described by eleven space-time axis also is unsuccessful **[21,22,23]**. All these theories need more than four component of space time like Kaloza theory. The problem of the rotating galaxies does not have answer in general relativity except we assume of huge missing of matter that called the dark matter which clearly cannot detect by any way. General relativity because it is nonlinear theory cannot combine with quantum mechanics which it is a linear and static theory. The version of relative quantum mechanics contains theory of special relativity without take the effect of field on the special relativity and the mass is taken as the function of position only instead of both position and gravity field.

1.4. Aim of the Work

The aim of this work is to relate inertial to gravitational mass in the presence of field. This requires using generalized special relativity theory, where the mass is function of both position and field.

1.5. Presentation of the Thesis

This thesis contains five chapters, where chapter one gives a summary of the evolution of the two branch of physics, quantum mechanics and general relativity and the link between them. Chapter two present the main concept of formula of equivalence principle and general relativity and quantum mechanics specific Schrödinger, and Kelin-Gordien formula. Then it introduces the generalize special relativity and their relative equation of mass, energy, length, and time with aim of Savickas ideas **[22-24]**. Chapter three introduce new derived of generalize of special relativity in the language of metric tensor and other way to derived it. Then derived generalized formula of Kelin-Gordien equation by using the generalize special relativity. Chapter four is present mathematical method to calculate the general formula of $g_{00}$for strong gravity field and doped the formula in quantum and then we try to find all possible solutions. chapter five left to discussion the results and conclusion.

Chapter two

Theoretical Background

2.1. Introduction

The concept of mass in physics is one of the main basic concepts in physics. This needs digging deeper to give full view of this concept. This need reviewing SR, general relativity, and quantum mechanics.

2.2. Newton's Gravitational law

Newton's gravitational law is concerned with the gravity force F exerts by a large a body of mass M on a small mass having gravity mass m_g, at distance r. This force is given by

$$F = G\frac{m_g M}{r^2} \qquad (\mathbf{2.2.1})$$

Where G is called gravitational constant. This force causes the particle to move with acceleration a. Thus

$$F = m_i a \qquad (\mathbf{2.2.2})$$

Where m_i is called inertial mass.

2.3. Equivalence principle

The principle of equivalence principle rests on the equality of gravitational and inertial mass, that no external static homogeneous gravitational field could be detected in a freely falling elevator, for the observers, their test bodies, and the elevator itself would respond to the field with the same acceleration **[4,5,6,21]**. This can easily prove for a system of particle N, moving with nonrelativistic velocities under the influence of forces $f(x_N - x_M)$ (gravitational forces) and an external gravitational field g. The equations of motion are

$$m_N \frac{d^2}{dt^2} x_N = m_N \nabla\phi + \sum f(x_N - x_M) \quad (\mathbf{2.3.1})$$

Where ϕ denotes to field, so $\nabla\phi = g$, and g is acceleration. Suppose that we perform a non-Galilean space-time coordinate transformation

$$\bar{x} = x - \frac{gt^2}{2}, \;\; \bar{t} = t \quad (\mathbf{2.3.2})$$

Thus

$$m_N \frac{d^2}{dt^2}\left(\bar{x}_{N+}\frac{gt^2}{2}\right) = m_N \mathrm{g} + \sum f\left(\bar{x}_{N+}\frac{gt^2}{2} - (\bar{x}_{M+}\frac{gt^2}{2})\right) \quad (\mathbf{2.3.3})$$

$$\Longrightarrow \; m_N \frac{d^2}{dt^2}\bar{x}_N + \frac{m_N g}{2}\frac{d^2}{dt^2}t^2 = m_N g + \sum f(\bar{x}_N - \bar{x}_M) \quad (\mathbf{2.3.4})$$

$$\Longrightarrow m_N \frac{d^2}{dt^2}\bar{x}_N = \sum f(\bar{x}_N - \bar{x}_M) \quad (\mathbf{2.3.5})$$

It's clearly that transformation made to equation $(\mathbf{2.3.1})$ cancel the g by an inertial force in equation $(\mathbf{2.3.5})$. Hence the original observer O who uses coordinate xt, and the free falling observer $\bar{O}$ who uses $\bar{x}\bar{t}$,will detect no difference in the laws of mechanics, except that O will say that he feels gravitational field and $\bar{O}$ will say that he does not. Equivalence principle says that this cancellation of gravitational by inertial force and hence their equivalence, will obtain for all free falling system, whether or not they can describe by simple equation such as equation $(\mathbf{2.3.1})$. One can rewrite the first term in equation $(\mathbf{2.3.1})$ in the right hand as **[15,25]**

$$mg = ma = m\nabla\phi \quad (\mathbf{2.3.6})$$

So that means $ma = m\nabla\phi$, and it is to find that

$$a = \nabla\phi \quad (\mathbf{2.3.7})$$

$$\therefore a \int dx = \int \nabla\phi \qquad (\mathbf{2.3.8})$$

$$\Rightarrow ax = \phi \qquad (\mathbf{2.3.9})$$

One can note that from equation $(\mathbf{2.3.1})$ to equation $(\mathbf{2.3.5})$ that the second term in the right hand is invariant under the transformation while the term in the left hand is variant under transformation and it produce the term that cancels the gravitational force which its invariant and $\nabla\phi = \nabla\bar{\phi}$.

2.4.Galilean Transformation

Galilean transformation is used to transform between two reference frames coordinate, which they differ only by constant relative motion within the constructs of Newtonian physics. Let's we have two coordinate (x, y, z, t) and $(\grave{x}, \grave{y}, \grave{z}, \grave{t})$, the transformation of these two frames in constant velocity v is

$$\grave{x} = x - vt \qquad (\mathbf{2.4.1})$$

$$\grave{y} = y \qquad (\mathbf{2.4.2})$$

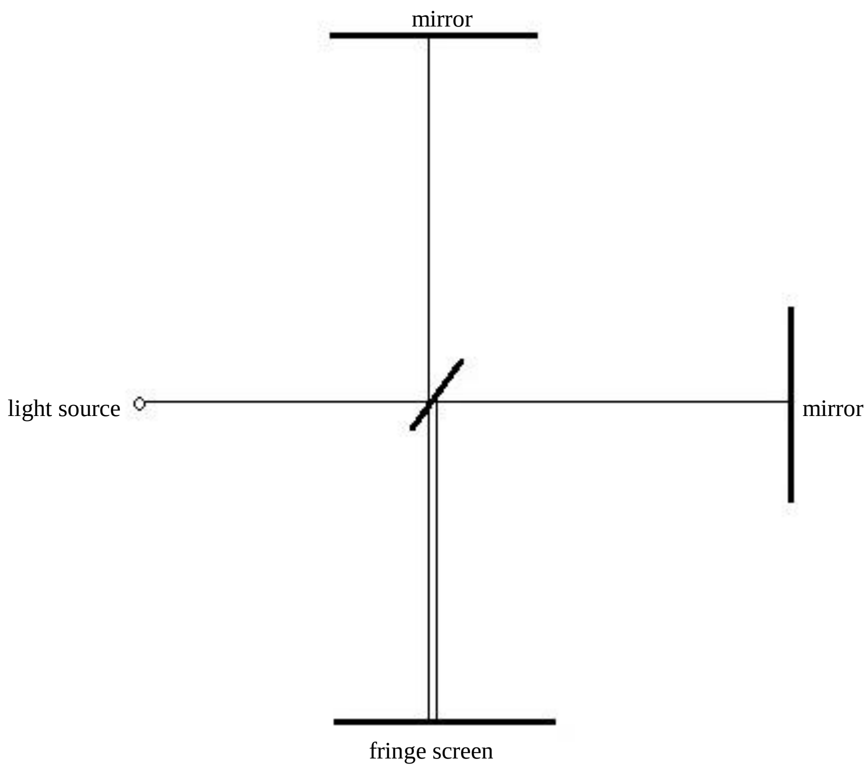
$$\grave{z} = z \qquad (\mathbf{2.4.3})$$

$$\grave{t} = t \qquad (\mathbf{2.4.4})$$

Theses equations are valid only at speeds much less than the speed of light.

2.5.Michelson Morley Experiment

The experiment was designed to compare the speed of light in perpendicular direction, in an attempt to detect the relative motion of through the Aether wind. The result was negative. They found no significant difference between the speed of light in the direction of movement through the presumed Aether, and the speed at right angles. The idea behind the experiment that since the earth orbits around the sun at speed about $30\ Km/s$, so two possibilities were considered in the view of existence of Aether: first, the Aether is stationary and only partially dragged by earth, second, the Aether is completely dragged by earth and thus shares its motion at earth's surface. According to this hypothesis, earth and Aether are in relative motion. Although it would be possible, in theory, for the Erath's motion to match that of the Aether at one moment in time, it was not possible for the earth to remain at rest with respect to the Aether at all time, because of the variation in both the direction and the speed of the motion. The device used in this experiment known as Michelson interferometer. The arrangement of experiment shown below.



Michelson-Morley experiment

The interference between two light beams produce a shape of fringe, according to above hypothesizes this shape should be change if we rotate the device by 180^0or by time when the position of earth change relative to sun. But the fringe shape still has no significant different, and thus the result of experiment is negative, which means the speed of light is independent from any frame or any motion.

2.6.Special relativity

In the view of the results of Michelson-Morley experiment Einstein assumed that the speed of light is constant and it is the limit of velocities, no particle can reach this speed or move faster than the speed of light, so he conclude that the Galilean transformation must change to anther formula that concerned the limit of speed. Other hypothesize was that the laws of physics must be invariant under any transformation between two reference frame. According to that he introduces new transformation known as Lorentz transformation

2.7.Lorentz transformation

Lorentz transformation is a linear transformation between two references coordinate $\grave{S}\&S$ that move at constant velocity relative to each other. The simple form of the transformation along x-axies is

$$\grave{x} = \gamma(x - vt) \quad \mathbf{(2.7.1)}$$

$$\grave{y} = y \quad \mathbf{(2.7.2)}$$

$$\grave{z} = z \quad \mathbf{(2.7.3)}$$

$$\grave{t} = \gamma\left(t - \frac{vx}{C^2}\right) \quad \mathbf{(2.7.4)}$$

Where $(x, y, z, t)\&(\grave{x}, \grave{y}, \grave{z}, \grave{t})$ represent an event's coordinate in two references $\grave{S}\&S$ with relative velocity v, C is speed of light, and $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{C^2}}}$ is Lorentz factor. The inverse Lorentz transformation along x-axies take the form

$$x = \gamma(\grave{x} + v\grave{t}) \quad \mathbf{(2.7.5)}$$

$$y = \grave{y} \quad \mathbf{(2.7.6)}$$

$$z = \grave{z} \quad \mathbf{(2.7.7)}$$

$$t = \gamma\left(\grave{t} + \frac{v\grave{x}}{C^2}\right) \quad (\mathbf{2.7.8})$$

This transformation has some consequences in time, length, and the energy when we take the effect of relativity due to relative velocity of the two frames $\grave{S}\&S$.

2.8. Time dilation

In the view of relativity and Lorentz transformation, let there be two events at which the moving clock indicates $t_1\&t_2$, thus

$$\grave{t}_1 = \frac{t_1 - \frac{vx_1}{C^2}}{\sqrt{1 - \frac{v^2}{C^2}}} \quad (\mathbf{2.8.1})$$

$$\grave{t}_2 = \frac{t_2 - \frac{vx_2}{C^2}}{\sqrt{1 - \frac{v^2}{C^2}}} \quad (\mathbf{2.8.2})$$

Since the clock remains at rest in the inertial frame, it follows $x_1 = x_2$, thus the interval

$$\grave{t}_2 - \grave{t}_1 = \Delta\grave{t} = \gamma(t_2 - t_1) = \gamma\Delta t = \frac{\Delta t}{\sqrt{1 - \frac{v^2}{C^2}}} \quad (\mathbf{2.8.3})$$

Where Δt is the time interval between two co-local events for an observer in some inertial frame, $\Delta\grave{t}$ is the time interval between those some events as measured by anther observer inertial moving with velocity v with respect to former observer. The physical meaning of equation $(\mathbf{2.8.3})$ is that the duration of the clock cycle of a moving clock is found to be increased. It is measured to be running slow. Actually this dilation become important only in high speed near speed of light.

2.9. Length contraction

In an inertial reference frame S, $x_1 \& x_2$ denote to the length of an object in motion in this frame. The proper length of this object in reference $\grave{S}$ could be calculated using Lorentz transformation, thus

$$\grave{x}_1 = \gamma(x_1 - vt_1) \quad \mathbf{(2.9.1)}$$

$$\grave{x}_2 = \gamma(x_2 - vt_2) \quad \mathbf{(2.9.2)}$$

Since $t_1 = t_2$ and by setting $L = x_2 - x_1$, and $\grave{L}_0 = \grave{x}_2 - \grave{x}_1$ the proper length in $\grave{S}$ is given by $\grave{L}_0 = \gamma L$ with respect to which the measured length S is contracted by $L = {}^{\grave{L}_0}/_{\gamma}$. According to the relativity principle objects that at rest in S have to be contraction in $\grave{S}$ as well. By exchanging the above signs and primes symmetrically, it follows

$$L_0 = \gamma \grave{L} \quad \mathbf{(2.9.3)}$$

Thus the contract length as measured in $\grave{S}$ is given by $\grave{L} = {}^{L_0}/_{\gamma}$.

2.10. Relativistic energy

The formula of relativistic energy takes the form

$$E = mC^2 = \gamma m_0 C^2 \quad \mathbf{(2.10.1)}$$

Where m_0 is the rest mass connected to $m = \gamma m_0$ **[33],** thus

$$E = \frac{m_0 C^2}{\sqrt{1 - \frac{v^2}{C^2}}} \quad \mathbf{(2.10.2)}$$

In low speed (Newtonian limit) by using Taylor series

$$E = m_0 C^2 \left(1 + \frac{1}{2}\left(\frac{v}{C}\right)^2 + \frac{3}{8}\left(\frac{v}{C}\right)^4 + \cdots\right) \quad \mathbf{(2.10.3)}$$

$$E \approx m_0 C^2 + \frac{1}{2} m_0 v^2 \quad \mathbf{(2.10.4)}$$

The total energy is a sum of the rest energy and the Newtonian kinetic energy, but it seems there is something missing, where the potential energy does not appear in the classical limit.

2.11. Theory of General Relativity

Einstein form his gravity law in non-Euclidean form unless the space is empty from any mas. To introduce the final formula of Einstein's field equation we need first to give a view of some important tensors and equation that used by him to structure the general relativity, since he assumed that the gravity field which produced by static mass is curved the space-time, so that he used the Riemannian geometry to calculate the curvature of space-time due to mass. First formula is the matric tensor which describe the distance between two point

$$ds^2 = g^{ij} dx_i dx_j \qquad (\mathbf{2.11.1})$$

Where $g_{ij} = \frac{1}{g^{ij}}$ and $g_{ij}$is metric tensor and x_i, x_jdenote to the four coordinate$(-ict, x_2, x_3, x_4)$ and $dx_{11} = C^2 dt^2, dx_{22} = dx^2, dx_{33} = dy^2$, $dx_{44} = dz^2$ in the Cartesian coordinate. The geodesic is the path that point is following in the gravity field so $[\mathbf{6, 26, 27}]$

$$\frac{d^2 x^i}{ds^2} + \Gamma^i_{jk} \frac{dx^j}{ds} \frac{dx^k}{ds} = 0 \qquad (\mathbf{2.11.2})$$

Where Γ^i_{jk} is Chrestofel notation and it is not a tensor, it take the form

$$\Gamma^i_{jk} = \frac{\partial^2 x_\sigma}{\partial x_j \partial x_i} \frac{\partial x_i}{\partial x_\sigma} \qquad (\mathbf{2.11.3})$$

Equation $(\mathbf{2.11.2})$ produce four equations that determine the geodesic path. Second Riemannian- Christopher tensor which calculate the change of any vector in Euclidean or non when this vector transform covariance one round around curvature, so it's coordinate change and this tensor determining that change if it's not equal to zero, it take the formula

$$R^{i}_{jkl} = \Gamma^{i}_{rk}\Gamma^{r}_{jl} - \Gamma^{i}_{rl}\Gamma^{r}_{jk} + \frac{\partial}{\partial x_k}\Gamma^{i}_{jl} - \frac{\partial}{\partial x_l}\Gamma^{i}_{jk} \qquad (\mathbf{2.11.4})$$

Third is Ricc tensor, it's case of Riemannian- Christopher tensor when the upper index equal to one of lower indexes, so that index called dummy and it eliminate each other around the summation, it takes the form

$$R^{i}_{jkl} \Longrightarrow i = l \Longrightarrow R^{i}_{jkl} = R_{jk} \qquad (\mathbf{2.11.5})$$

And $R = g^{jk}R_{jk}$ it's symmetric tensor and measuring the space curve. Finally, Einstein tensor and the formula of field equation as he introduced in 1916 paper **[4]**

$$G_{ij} = R_{ij} - \frac{1}{2}g_{ij}R = -kT_{ij} \qquad (\mathbf{2.11.6})$$

Where $T_{ij}$is momentum-energy tensor and $k$is constant obtained from Poisson equation. As we show the field equation is really complex, hard to find a solution, and it's not have a quantum terms.

2.12. Quantum Mechanics

The beginning of quantum mechanics was by Plank, when he assumed that the electromagnetic emission and absorption in discrete quanta, each of them equal to **[11]**

$$E = h\nu \qquad (\mathbf{2.12.1})$$

Where E is energy, h is plank constant, and ν is frequency. Then De Broglie suggestion that matter has a duel character, particle like and wave like in the atomic level. He found that the relation between momentum p of the particle and the wave length λ of the corresponding wave is

$$\lambda = \frac{h}{p} \qquad \mathbf{(2.12.2)}$$

These two equations were the mean structure of which called the old quantum mechanics. Then Heisenberg presented the principle of uncertainly, it stats that it's impossible to specify precisely and simultaneously the value of both members of particular pairs of physics variables that describe the behavior of an atomic system, and magnitude of the product of the uncertainties in the knowledge of the two variables must be at least Plank's constant h divided by 2π, so for that

$$\Delta x . \Delta p_x \geq \frac{h}{2\pi} \qquad \mathbf{(2.12.3)}$$

$$\Delta \vartheta . \Delta J_z \geq \frac{h}{2\pi} \qquad \mathbf{(2.12.4)}$$

$$\Delta t . \Delta E \geq \frac{h}{2\pi} \qquad \mathbf{(2.12.5)}$$

Where x is position that pairs with p_x the component of the momentum, and here ϑ is angular position that pairs with J_z the component of angular momentum perpendicular to the plane of the orbital, and t is time where E is energy. In order to understand the implications of the uncertainly principle in more physical terms Bohr introduce the complementarity principle which stats that atomic phenomena cannot be described with the completeness demanded by classical dynamical; some of the elements that complement each other to make up a complete classical description are actually mutually exclusive, and these complementary elements are all necessary for the description of various aspects of the phenomena **[9,11,28]**.

2.13. The Non-Relativistic Schrodinger Equation

Schrodinger form his wave equation by generalized the properties of the wave amplitude. Where for a continues traveling harmonic wave the wave length and the momentum are related by equation $(\mathbf{2.12.2})$ and the energy frequency by equation $(\mathbf{2.12.1})$, so these two equation can be rewrite in terms of the universal constant $\hbar = \frac{h}{2\pi}$ so

$$p = \hbar k \qquad (\mathbf{2.13.1})$$

$$E = \hbar \omega \qquad (\mathbf{2.13.2})$$

Where $k = \frac{2\pi}{\lambda}$ and $\omega = 2\pi\nu$. A wave function $\Psi(x,t)$ that represents a particle of completely undetermined position traveling in the position x direction with precisely known momentum p and Kinetic energy E, would then be expected to one of the form

$$\cos(kx - \omega t)\,, \sin(kx - \omega t)\,, e^{i(kx-\omega t)}, e^{-i(kx-\omega t)} \qquad (\mathbf{2.13.3})$$

Or some linear combination of them. The wave equation that describes the motion of transverse waves on a string or plane sound waves can be describe by

$$\frac{\partial^2 \Psi}{\partial t^2} = \gamma \frac{\partial^2 \Psi}{\partial x^2} \qquad (\mathbf{2.13.4})$$

Where γ is the square of the wave velocity. Substitution of the form $(\mathbf{2.13.3})$ into equation $(\mathbf{2.13.4})$ shows that each of the four harmonic solutions, and hence any linear combination of them, satisfies this differential equation, if only we put

$$\gamma = \frac{\omega^2}{k^2} = \frac{E^2}{p^2} = \frac{p^2}{2m} \qquad (\mathbf{2.13.5})$$

Where m is the mass of the particle that is to be described by equation $(\mathbf{2.13.4})$. Because of the structure of equation $(\mathbf{2.13.5})$ it's apparent that the coefficient γ in equation $(\mathbf{2.13.4})$ involves the parameters of the motion (E or p). in looking further for a suitable equation, it's helpful to note that the differentiation with respect to x of wave function $(\mathbf{2.13.3})$ has the general effect of multiplication of the function by k with differentiation with respect to t has the general effect of multiplication by ω. then the relation $E = \frac{p^2}{2m}$ which is equivalent to the relation $\omega = \frac{\hbar k^2}{2m}$ suggests that the differential equation for which we are looking contains a first derivative with respect to t and a second derivative with respect to x so equation $(\mathbf{2.13.4})$ therefor not useful and hence

$$\frac{\partial \Psi}{\partial t} = \gamma \frac{\partial^2 \Psi}{\partial x^2} \qquad (\mathbf{2.13.6})$$

Substitution shows that the first two of the wave function introduced in equation $(\mathbf{2.13.3})$ are not solution of equation $(\mathbf{2.13.6})$, but that either of the last two maybe if the constant γ is suitably chosen. In particular, if we chose

$$\gamma = \frac{i\omega}{k^2} = \frac{i\hbar}{p^2} = \frac{i\hbar}{2m} \qquad (\mathbf{2.13.7})$$

Then the third of the wave function in $(\mathbf{2.13.3})$ satisfies equation $(\mathbf{2.13.7})$. More ever the value of γ given by equation $(\mathbf{2.13.7})$ involves only the constant $\hbar$ and m. Thus the Schrodinger wave equation for a free particle of mass m, which from equations $(\mathbf{2.13.6}), (\mathbf{2.13.7})$ may be written

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} \quad (\mathbf{2.13.8})$$

To includes the effect of external forces that can act on the particle, we shall assume for the present that these forces are of such a nature (electrostatic, gravitational, nuclear, etc…) that they can be combined into a single force f that is derivable from a potential energy U

$$f(r,t) = -\nabla U((r.t) \quad (\mathbf{2.13.9})$$

Just as the classical relation between energy and momentum is used to infer the structure of equation $(\mathbf{2.12.3})$, so it's desirable now to start from corresponding classical relation that includes external forces. This is simply expressed in terms of the potential energy

$$E = \frac{p^2}{2m} + U(r,t) \quad (\mathbf{2.13.10})$$

Here E is total energy and the first and second terms on the right side are the Kinetic and potential energy of the particle, so equation $(\mathbf{2.13.8})$ be generalized into

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + U(x,t)\Psi \quad (\mathbf{2.13.11})$$

It can be written in three dimensions in terms of

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\Psi + U(x,t)\Psi \quad (\mathbf{2.13.12})$$

This is the non-relative Schrodinger wave equation that describes the motion of a particle of mass m in a force field **[10,11,28, 29]**.

2.14. The Relative Formula of Schrödinger Equation

To derive a relative formula of Schrodinger wave equation we must replace the classical form of energy by its relative form, where from the special relativity the relative energy is given as

$$E = \sqrt{p^2C^2 + m^2C^4} \quad (\mathbf{2.14.1})$$

The operator of p is

$$\hat{p} = -i\hbar\nabla \quad (\mathbf{2.14.2})$$

Also the operator of E is

$$\hat{E} = i\frac{\partial}{\partial t} \quad (\mathbf{2.14.3})$$

So, one can write that

$$\sqrt{(-i\hbar\nabla)^2 + m^2C^4}\Psi = i\frac{\partial}{\partial t}\Psi \quad (\mathbf{2.14.4})$$

Acutely the square root can be easily defined by Fourier transformation, but due to asymmetry of space and time that make this formula impossible to include external electromagnetic field in a relativistic invariant way. So Klein and Gordon instead began with the square of equation $(\mathbf{2.14.1})$, so

$$E^2 = m^2C^4 + p^2C^2 \quad (\mathbf{2.14.5})$$

And equation $(\mathbf{2.14.4})$ goes to be

$$-\hbar^2C^2\nabla^2\Psi + m^2C^4\Psi = -\hbar^2\frac{\partial^2}{\partial t^2}\Psi \quad (\mathbf{2.14.6})$$

Rearranging terms yields

$$\frac{1}{C^2}\frac{\partial^2}{\partial t^2}\Psi - \nabla^2\Psi + \frac{m^2C^2}{\hbar^2}\Psi = 0 \quad (\mathbf{2.14.7})$$

Equation $(\mathbf{2.14.7})$ can be written in a covariant notation as

$$(\square + \mu^2)\Psi = 0 \quad (\mathbf{2.14.8})$$

Where the operator $\square$ is called D'Alemert operator, it takes the form

$$\square = \frac{1}{C^2}\frac{\partial^2}{\partial t^2} - \nabla^2 \quad (\mathbf{2.14.9})$$

And

$$\mu = \frac{mC}{\hbar} \quad (\mathbf{2.14.10})$$

Equation $(\mathbf{2.14.7})$ is a relative form of Schrodinger equation for free particle. It correctly describes the spin less relativistic composite particle like pion such as higgs boson since it's a spin-zero particle, and it became the first observed ostensibly elementary particle to be described by the Klein-Gordon equation. The Klein-Gordon equation admits plane wave solutions corresponding to Eigen states of four momentum $p_\mu = (\frac{E}{C}, p)$ in the form

$$\Psi_p(r,t) = Ne^{\frac{i}{\hbar}(pr - Et)} = Ne^{i(Kr - \omega t)} \quad (\mathbf{2.14.11})$$

Where N is normalization constant, $E = \hbar\omega$, and $p = \hbar K$. The Klein-Gordon equation is invariant under Lorentz transformation **[9,11,28,29,30]**.

2.15. Previews Studies

2.15.1. Generalized of Special Relativity in a Curved Space Time GSR

In special relativity of Einstein, time t, length L, mass m, and the energy E are velocity dependent. They take the form

$$t = \gamma t_0 \quad (\mathbf{2.15.1.1})$$

$$L = \frac{L_0}{\gamma} \quad (\mathbf{2.15.1.2})$$

$$m = \gamma m_0 \quad (\mathbf{2.15.1.3})$$

$$E = mC^2 = \gamma m_0 C^2 \quad (\mathbf{2.15.1.4})$$

Where

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{C^2}}} \quad (\mathbf{2.15.1.5})$$

The frame work is based on two postulates, first; the speed of light C is constant in vacuum and it's independent from direction of source motion. Second; that the laws of physics should be invariant due to any transform from any inertial frame to other. Beside that the space –time is continuous and it take the general form of Monkoviski space as

$$ds^2 = g_{00}C^2dt^2 + g_{ij}dx_{ij} \ \&i, j = 1,2,3 \quad (\mathbf{2.15.1.6})$$

And for Cartesian Euclidian space where $dx_{ij} = dx^2 + dy^2 + dz^2$,and $g_{00} = 1, g_{11} = g_{22} = g_{33} = -1$, that yields

$$ds^2 = C^2dt^2 - dx^2 - dy^2 - dz^2 \quad (\mathbf{2.15.1.7})$$

And the factor γ can be derivate as

$$\frac{1}{\gamma^2}=\frac{ds^2}{C^2dt^2}=1-\frac{v^2}{C^2} \qquad (\mathbf{2.15.1.8})$$

So $\gamma=\frac{1}{\sqrt{1-\frac{v^2}{C^2}}}$, the frame of special relativity is Euclidian geometry, it does not take the effect of fields. Many offers tried to include the effect of field, like the model of Savakas, and later on the model of generalized special relativity GSR theory. Both theories modified special relativity to take the effect of field ϕ, where in GSR the equation of relativistic of t, L, m, and E take the form **[17]**

$$t=\frac{t_0}{\sqrt{g_{00}-\frac{v^2}{C^2}}} \qquad (\mathbf{2.15.1.9})$$

$$L=L_0\sqrt{g_{00}-\frac{v^2}{C^2}} \qquad (\mathbf{2.15.1.10})$$

$$m=\frac{g_{00}m_0}{\sqrt{g_{00}-\frac{v^2}{C^2}}} \qquad (\mathbf{2.15.1.11})$$

$$E=mC^2=\frac{g_{00}m_0C^2}{\sqrt{g_{00}-\frac{v^2}{C^2}}} \qquad (\mathbf{2.15.1.12})$$

Where $g_{00}=1+\frac{2\phi}{C^2}$,derived from Monkoviski space with the weak field approximation. The mean different between Savakis model and GSR is the expression of relative mass, while in GSR take the form of equation $(\mathbf{2.15.1.11})$ but in Savakis model the g_{00} in the numerator is absent, thus

$$m = \frac{m_0}{\sqrt{g_{00} - \frac{v^2}{C^2}}} \quad \mathbf{(2.15.1.13)}$$

$$\therefore E = mC^2 = \frac{m_0 C^2}{\sqrt{g_{00} - \frac{v^2}{C^2}}} \quad \mathbf{(2.15.1.14)}$$

In GSR the expression of mass can drive by obtained the volume V, where the effect of motion and gravity can be take the formula

$$V = \frac{V_0}{\gamma} = V_0\sqrt{g_{00} - \frac{v^2}{C^2}} \quad \mathbf{(2.15.1.15)}$$

By using the expression for Hamiltonian in general relativity from Energy-momentum tensor, one can write **[1,2,8,17,27]**

$$H = \rho C^2 = g_{00}T^{00} = g_{00}\rho_0\left(\frac{dx_0}{ds}\right)^2 \quad \mathbf{(2.15.1.16)}$$

$$\therefore \rho C^2 = g_{00}\rho_0 C^2\gamma^2 = g_{00}\frac{m_0}{V_0}C^2\gamma^2 = g_{00}\frac{m_0}{V}C^2\gamma \quad \mathbf{(2.15.1.17)}$$

$$but\ \rho C^2 = \frac{mC^2}{V} \quad \mathbf{(2.15.1.18)}$$

So

$$\frac{mC^2}{V} = g_{00}\frac{m_0}{V}C^2\gamma \quad \mathbf{(2.15.1.19)}$$

That yields

$$m = g_{00}m_0\gamma = \frac{g_{00}m_0}{\sqrt{g_{00} - \frac{v^2}{C^2}}} \quad \mathbf{(2.15.1.20)}$$

GSR succeeded to explain many phenomena, such as conversion of electron neutrinos, gravitational red shift, and other **[17,18,19,20]**. But the most interesting result is that in the frame of GSR and Savakis model the accelerator particle can in some case exceeded the speed of light without falling in the problem of imaginary mass **[16]**.

2.15.2. EGSR Model for Dirar and others

In this model they drive a general Lorentz transformation for motion in the presence of motion and field, and it holds for any fields including gravity, electromagnetic, weak and strong nuclear field. The derivation is based on homogeneity of space, constancy of light space. The mean equation of this model based on the factor

$$\gamma = \frac{1}{\sqrt{1 - \frac{2v_x^2 - 4\emptyset + 2v_x\sqrt{v_x^2 - 4\emptyset}}{4C^2}}} \qquad (\mathbf{2.15.2.1})$$

Thus

$$x = \frac{\grave{x} + \left(\frac{v_x^2 - 2\emptyset + v_x\sqrt{v_x^2 - 4\emptyset}}{2}\right)\grave{t}}{\sqrt{1 - \frac{2v_x^2 - 4\emptyset + 2v_x\sqrt{v_x^2 - 4\emptyset}}{4C^2}}} \qquad (\mathbf{2.15.2.2})$$

$$\grave{t} = \frac{t - \left(\frac{v_x^2 - 2\emptyset + v_x\sqrt{v_x^2 - 4\emptyset}}{2}\right)x}{\sqrt{1 - \frac{2v_x^2 - 4\emptyset + 2v_x\sqrt{v_x^2 - 4\emptyset}}{4C^2}}} \qquad (\mathbf{2.15.2.3})$$

Where

$$v_x^2 = v_{0x}^2 + 4\emptyset \qquad (\mathbf{2.15.2.4})$$

And

$$x = \grave{x} + vt - at^2 = \grave{x} + \frac{1}{2}(v - at)t + \frac{1}{2}vt$$

$$= \grave{x} + \frac{v_0 + v}{2}t \quad (\mathbf{2.15.2.4})$$

Thus

$$x = \grave{x} + v_m t \quad (\mathbf{2.15.2.5})$$

This transformation reduces to SR when there is no field **[34]**

2.15.3. Savakas Model

In this model Savakas try to found a formula that related between classical mechanics and the idea of Einstein's relativity by formed the Newtonian mechanics in curved space, also he suggested this model to simplify the complex calculations of GR. The mean equations depends on the factor γ, thus

$$\gamma = \frac{1}{\sqrt{\left(\frac{C_{eff}}{C}\right)^2 - \left(\frac{v}{C}\right)^2}} \quad (\mathbf{2.15.3.1})$$

Where

$$C_{eff} = \sqrt{g_{44}}C \quad (\mathbf{2.15.3.2})$$

And

$$\sqrt{g_{44}} = 1 + \frac{2\phi}{C^2} \quad (\mathbf{2.15.3.3})$$

The relative mass takes the formula

$$m = \frac{m_0}{\sqrt{\left(\frac{C_{eff}}{C}\right)^2 - \left(\frac{v}{C}\right)^2}} = \frac{m_0}{\sqrt{1 + \frac{2\phi}{C^2} - \left(\frac{v}{C}\right)^2}} \quad (\mathbf{2.15.3.4})$$

In this model Savakas succeeded to point the relation between classical mechanics, GR, and quantum mechanics **[35, 36, 37]**

2.15.4. Dirar and Husham model

In this model they generalized nonlinear Lorentz transformation for particles moving in a potential field. The transformation is based on the usual Newtonian relation displacement in terms of initial velocity for constant acceleration. The major equation is the factor

$$\gamma = \frac{1}{\sqrt{\left(1+\frac{v_0}{C}+\frac{\emptyset}{2C^2}\right)\left(1-\frac{v_0}{C}-\frac{\emptyset}{2C^2}\right)}} \qquad (\mathbf{2.15.4.1})$$

Thus

$$\dot{x} = \frac{x+v_0t+\frac{\emptyset}{2x}t^2}{\sqrt{\left(1+\frac{v_0}{C}+\frac{\emptyset}{2C^2}\right)\left(1-\frac{v_0}{C}-\frac{\emptyset}{2C^2}\right)}} \qquad (\mathbf{2.15.4.2})$$

This model reduces to SR when $\emptyset = 0$. Also $\emptyset$ is taken to be as

$$\emptyset = ax \qquad (\mathbf{2.15.4.3})$$

Where a denotes to constant acceleration[**38].**

2.15.5. Dirar and Nuha Model

In this model they using GSR to drive special relativistic and generalized special relativistic Lorentz transformation by using the concept of curved space and the expression of the electric and magnetic force on electrons beside the expression of displacement current. The factor γ was found as

$$\gamma = \left[1-\frac{\left(v-\frac{\emptyset}{C}\right)^2}{C^2}\right]^{-\frac{1}{2}} \qquad (\mathbf{2.15.5.1})$$

Where

$$v_m = v-\frac{\emptyset}{C} \qquad (\mathbf{2.15.5.2})$$

Also the factor γ for pulse of light in field that cause a constant acceleration was found to be

$$\gamma = \left[1 - \frac{\left(v + \frac{\emptyset}{2C}\right)^2}{C^2}\right]^{-\frac{1}{2}} \qquad (\mathbf{2.15.5.3})$$

This prove that GSR stands for all kinds of field such gravity, electromagnetic, weak nuclear field, and strong nuclear field. And the model is reducing for ordinary SR in absence of field. **[39, 40]**

2.15.6. Other studies

GSR used in many studies for explaining and predicting many phenomena such as Using generalized special relativity together with Newton's laws of gravitation for treating particles as quantum strings **[41]**, also for proving the gravitation red shift **[42]**, also for estimating the proton (nucleon) mass to explain the mass defect, and estimating the neutrino masses to explain the conversion of electron neutrinos **[43, 44].**

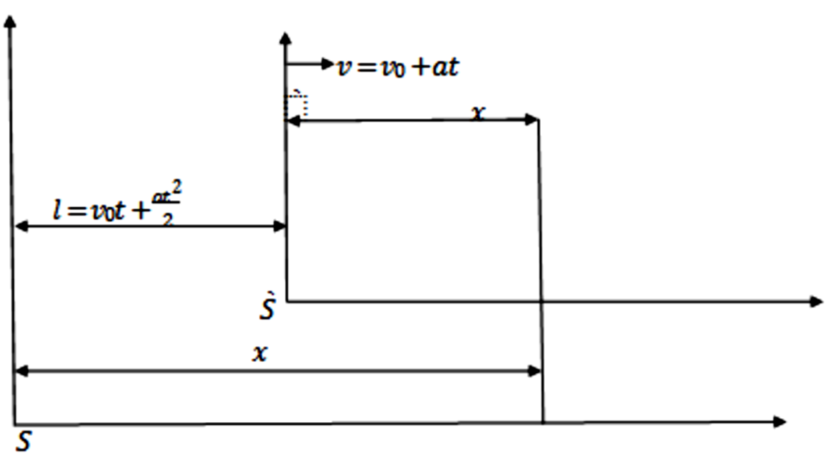
Chapter three
Inertial Mass in Curved Space

3.1 Introduction

Equivalence principle is related to inertia. Equivalence principle emerged when Galileo found that the acceleration of a mass due to gravitation is independent of the amount of mass being accelerated. Galileo obtained his results with balls rolling down nearly frictionless inclined planes to slow the motion and increase the timing accuracy. Increasingly precise experiments have been performed by Loránd Eötvös using the torsion balance pendulum **[3]**. From Newton second law one can estimate the inertial mass due to force which accelerate the body, and the gravitation mass can be estimated from Newton law of gravity. Albert Einstein developed his general theory of relativity starting from the assumption that this correspondence between inertial and gravitational mass is not accidental. However, in his theory, gravitation is not a force and thus not subject to Newton's third law, so the equality of inertial and gravitational mass remains as puzzling **[5]**. Einstein also referred to two reference frames, S and $\grave{S}$. S is a uniform gravitational field, whereas $\grave{S}$ is uniformly accelerated such that objects in the two frames experience identical forces. Einstein combined the equivalence principle with special relativity to predict that time is affected by a gravitational potential, and light rays bend in a gravitational field, before he developed the concept of curved space-time. So the original equivalence principle, as described by Einstein, predicted that freefall and inertial motion were physically equivalent. Inertial mass is the mass of an object measured by its resistance to acceleration. This definition has been championed by Ernst Mach and has been developed by Percy W. Bridgman **[5,4]**. In special relativity, there are two kinds of mass: rest mass and relativistic mass, which increases with velocity. Rest mass is the Newtonian mass as measured by an observer at rest. Relativistic mass is proportional to the total quantity of energy in a body or system. But there is no term refers to the effect of gravity field or any fields on mass. The next work is concerned with the effect of gravity field and acceleration on the mass. Then one can verify equivalence principle.

3.2 The Equivalence Principle on the Basis of Average Velocity Lorentz Transformation

The equivalence principle states the laws of nature takes the same form in accelerated frame and the frame permeated by gravitational field. This can be studied here with in frame work of lorentz transformation based on average velocity assumption as proposed by M.Dirar and other**[15]**. In this version the relativistic mass is given by



$$m = \frac{m_0}{\sqrt{1 - \frac{v_m^2}{c^2}}} \qquad (3.2.1)$$

Where the average velocity which was assumed to be invariant

$$v_m = \frac{v_0 + v}{2} \qquad (3.2.2)$$

For particle moving with initial velocity v_0 against gravity, the final velocity is given according to the ordinary rectilinear motion with constant acceleration to be

$$v^2 = v_0^2 - 2ax = v_0^2 - 2\varphi \quad (\mathbf{3.2.3.})$$

Where φ stands for the potential per unit mass. Thus according to equation $(\mathbf{3.2.3})$, $(\mathbf{3.2.2})$ and $(\mathbf{3.2.1})$

$$v_m^2 = \frac{v_0^2 + 2vv_0 + v^2}{4} = \frac{2v^2 + 2\varphi + 2v\sqrt{v^2 + 2\varphi}}{4}$$

$$= \frac{v^2 + \varphi + v\sqrt{v^2 + 2\varphi}}{2} \quad (\mathbf{3.2.4.})$$

Thus equation $(\mathbf{3.2.1})$ become

$$m = \frac{m_0}{\sqrt{1 - \frac{\varphi}{2c^2} - \left(\frac{v^2 + v\sqrt{v^2 + 2\varphi}}{2c^2}\right)}} \quad (\mathbf{3.2.5.})$$

Consider now a particle falling in a gravitational field such that its speed at a certain point x is v and its potential energy is φ. According to equation $(\mathbf{3.2.5})$ its mass is given by

$$m_g = m = \frac{m_0}{\sqrt{1 - \frac{\varphi}{2c^2} - \left(\frac{v^2 + v\sqrt{v^2 + 2\varphi}}{2c^2}\right)}} \quad (\mathbf{3.2.6.})$$

Which stands for gravitational mass. Now consider a particle moving upward with speed v_0. For an elevator moving with acceleration a up ward its velocity after time t when its displacement is x becomes

$$v^2 = v_0^2 - 2ax \quad (\mathbf{3.2.7})$$

Thus according to equations $(\mathbf{3.2.1})$, $(\mathbf{3.2.2})$ and $(\mathbf{3.2.7})$ one gets

$$v_m^2 = \frac{v_0^2 + 2vv_0 + v^2}{4}$$

$$v_m^2 = \frac{v^2 + ax + v\sqrt{v^2 + 2ax}}{2}$$

Thus equation $(\mathbf{3.2.1})$ gives the inertial mass $m_i$is gives

$$m_i = m = \frac{m_0}{\sqrt{1 - \frac{ax}{2c^2} - \left(\frac{v^2 + v\sqrt{v^2 + 2ax}}{2c^2}\right)}} \quad (\mathbf{3.2.8.})$$

But numerically

$$ax = \varphi \quad (\mathbf{3.2.9.})$$

Thus

$$m_i = m = \frac{m_0}{\sqrt{1 - \frac{\varphi}{2c^2} - \left(\frac{v^2 + v\sqrt{v^2 + 2\varphi}}{2c^2}\right)}} \quad (\mathbf{3.2.10.})$$

A direct comparison of equations $(\mathbf{3.2.6})$ and $(\mathbf{3.2.10})$ gives

$$m_g = m_i \quad (\mathbf{3.2.11.})$$

This means that the inertial and gravitational masses are equivalent. Another version of equivalence principle states that the laws of nature for elevator in free fall in a gravitational fields takes the same form as that in free space. Now consider an elevator which is in free fall with a particle of rest mass m_0 inside it. According to equation $(\mathbf{3.2.2})$

$$v_m = \frac{v_0 + v}{2}$$

Since the particle is at rest all times, it follows that

$$v_0 = 0 \;\&\; v = 0 \quad (\mathbf{3.2.12.})$$

Thus

$$v_m = 0 \quad (\mathbf{3.2.13.})$$

Thus in view of equation $(\mathbf{3.2.1})$ the gravitational mass is

$$m_g = \frac{m_0}{\sqrt{1-0}} = m_0 \quad (\mathbf{3.2.14.})$$

For a particle of rest mass inside an elevator in free space that have rest mass m_0:

$$v_0 = 0 \;\&\; v = 0 \;\&\; v_m = 0 \quad (\mathbf{3.2.15.})$$

Thus according to equation $(\mathbf{3.2.1})$ the inertial mass is given by

$$m_i = \frac{m_0}{\sqrt{1-0}} = m_0 \quad (\mathbf{3.2.16.})$$

3.3 Inertia and gravitational mass in a curved space Lorentz transformation

The equation of motion of an accelerated particle due the action of gravity field or due to the elevator acceleration a with respect to a particle in free space is given by

$$\frac{d^2x^\lambda}{dt^2} + \Gamma^\lambda_{\mu\nu}\frac{dx^\mu}{dt}\frac{dx^\nu}{dt} = 0 \quad (\mathbf{3.3.1})$$

For slow moving particle in x-direction in a weak gravitational field, equation $(\mathbf{3.3.1})$ is reduced to

$$\frac{d^2x}{dt^2} + c^2\Gamma_{00}^1 = 0 \qquad (\mathbf{3.3.2})$$

The connection in a weak field limit is given by

$$\Gamma_{00}^1 = -\frac{1}{2}\nabla h_{00} \qquad (\mathbf{3.3.3})$$

Thus the acceleration a can curve the space according to equation $(\mathbf{3.3.2})$, and $(\mathbf{3.3.3})$ to get

$$a = \frac{d^2x}{dt^2} = -c^2\Gamma_{00}^1 = \frac{c^2}{2}\nabla h_{00} = \frac{c^2}{2}\frac{\partial h_{00}}{\partial x} \qquad (\mathbf{3.3.4})$$

For constant acceleration

$$\int adx = \frac{c^2}{2}\int dh_{00}$$

$$2ax = c^2 h_{00} \qquad (\mathbf{3.3.5})$$

By using the definition of force therefor,

$$F = ma = -\nabla V = -m\nabla\varphi = -\frac{mc^2}{2}\nabla h_{00} \qquad (\mathbf{3.3.6})$$

Thus

$$\varphi = -\frac{c^2}{2}h_{00} \qquad \& \qquad h_{00} = -\frac{2\varphi}{c^2} \qquad (\mathbf{3.3.7})$$

Therefore, the time metric takes the form

$$g_{00} = -1 + h_{00} = -\left(1 + \frac{2\varphi}{c^2}\right) \qquad (\mathbf{3.3.8})$$

In view of equation $(\mathbf{3.3.5})$ and $(\mathbf{3.3.8})$ it also takes the form

$$g_{00} = -1 + h_{00} = -1 + \frac{2ax}{c^2} = -\left(1 - \frac{2ax}{c^2}\right) \quad (\mathbf{3.3.9})$$

Taking into account equation $(\mathbf{3.3.5})$ and $(\mathbf{3.3.7})$ one gets

$$\frac{2\varphi}{c^2} = -\frac{2ax}{c^2} \quad (\mathbf{3.3.10})$$

The mass in a curved space takes the form **[15]**

$$m = \frac{m_0}{\sqrt{-g_{00} - \frac{v^2}{c^2}}} \quad (\mathbf{3.3.11})$$

Using equations $(\mathbf{3.3.11})$ and $(\mathbf{3.3.8})$ the gravitational mass is given by

$$m_g = \frac{m_0}{\sqrt{-g_{00} - \frac{v^2}{c^2}}} = \frac{m_0}{\sqrt{\left(1 + \frac{2\varphi}{c^2}\right) - \frac{v^2}{c^2}}} \quad (\mathbf{3.3.12})$$

With the aid of equation $(\mathbf{3.3.11})$ and $(\mathbf{3.3.9})$ the inertial mass is given by

$$m_i = \frac{m_0}{\sqrt{-g_{00} - \frac{v^2}{c^2}}} = \frac{m_0}{\sqrt{1 - \frac{2ax}{c^2} - \frac{v^2}{c^2}}} \quad (\mathbf{3.3.13})$$

Taking into account equations $(\mathbf{3.3.9})$, and $(\mathbf{3.3.8})$; it follows that

$$g_{00} = -\left(1 + \frac{2\varphi}{c^2}\right) = -\left(1 - \frac{2ax}{c^2}\right) \quad (\mathbf{3.3.14})$$

Therefore

$$m_g = m_i \quad (\mathbf{3.3.15})$$

i.e. the inertial and gravitational mass are equal. For an elevator freely falling with a particle in a gravitational field, the particle is at rest with respect to him. Thus no acceleration is observed, i.e.

$$v = 0 \quad \& \quad a = 0 \qquad (\mathbf{3.3.16})$$

According to equation $(\mathbf{3.3.14})$ and $(\mathbf{3.3.12})$

$$m_g = m_0 \qquad (\mathbf{3.3.17})$$

For the particle in free space, which is at rest with respect to an elevator

$$v = 0 \quad \& \quad a = 0 \qquad (\mathbf{3.3.18})$$

Again equation $(\mathbf{3.3.14})$ and $(\mathbf{3.3.13})$ gives

$$m_i = m_0 \qquad (\mathbf{3.3.19})$$

Thus

$$m_g = m_i \qquad (\mathbf{3.3.20})$$

The equality of gravitational and inertial mass is studied with in the frame work of velocity invariance and curved space Lorentz transformation. Two scenarios are proposed. In the first approach the particle mass falling freely in a gravitational field is compared with the one in free space observed by an observer in an elevator moving with respect to him with acceleration equal to the gravity acceleration. According to this version the velocity invariant Lorentz transformation shows the equality of gravitational mass and inertial mass as shown by equation $(\mathbf{3.2.11})$. This equality is related to the equality of potential per unit mass and work done due to acceleration [see equation $(\mathbf{3.2.9})$]. The curved space Lorentz transformation shows also equality of gravity and inertial mass [see equation $(\mathbf{3.3.15})$]. This is due to the fact that both field and acceleration deform the space as shown by equation $(\mathbf{3.3.14})$. In the second version the elevator falling with a particle in a gravitational field is compared with that in free space at rest with respect to the particle. In velocity invariant Lorentz transformation gravitational mass is equal to the rest mass [see equation $(\mathbf{3.2.14})$ and $(\mathbf{3.2.16})$]. In the curved space Lorentz transformation, the fact that the particle is at rest in both make acceleration vanishes. Thus the space is Minkowskian and the inertial and gravitational mass are equal to rest mass in both frames [see equations $(\mathbf{3.3.16-3.3.20})$].The velocity invariant model and curved space Lorentz transformation shows the equality of gravitational and inertial mass. This equality is related to the equality of potential and acceleration work done per unit mass.

3.4 GSR in language of metric tensor

Generalized special relativity theory represented the case of free falling where the velocity depends on the position and the field, and the relation between acceleration and field is given as in equation **(3.3.10)**, so

$$ax = -\varphi \quad (\mathbf{3.4.1})$$

We shall start from Euclidean space of four coordinate, so

$$C^2 d\tau^2 = ds^2 = g^{\mu\nu} dx_\mu dx_\nu = C^2 dt^2 - dx^2 - dy^2 - dz^2 \quad (\mathbf{3.4.2})$$

The factor γ in special relativity can be directly derived from above equation as

$$\gamma = \frac{dt}{d\tau} = \frac{Cdt}{ds} = \frac{1}{\sqrt{1-\frac{v^2}{C^2}}} \quad (\mathbf{3.4.3})$$

Since in the case of free falling the velocity is changing due to field it's useful to recall equation $(\mathbf{3.2.3})$

$$v^2 = v_0^2 - 2\varphi \quad (\mathbf{3.4.4})$$

where $\varphi = -\frac{Gm}{r}$. Now let's find formula from equation $(\mathbf{3.4.2})$ to replace v^2 in $(\mathbf{3.4.4})$, so

$$\frac{ds^2}{dt^2} = C^2 - v^2 \qquad (\mathbf{3.4.5})$$

This lead to

$$v^2 = C^2 - \frac{ds^2}{dt^2} = C^2\left(1 - \frac{ds^2}{C^2 dt^2}\right) = v_0^2 - 2\varphi \qquad (\mathbf{3.4.6})$$

Rearrangement, yields

$$C^2 - \frac{ds^2}{dt^2} = v_0^2 - 2\varphi \qquad (\mathbf{3.4.7})$$

$$\therefore C^2dt^2 - ds^2 = v_0^2dt^2 - 2\varphi dt^2 \qquad (\mathbf{3.4.8})$$

$$ds^2 = C^2dt^2 + 2\varphi dt^2 - v_0^2dt^2 \qquad (\mathbf{3.4.9})$$

$$ds^2 = \left(1 + \frac{2\varphi}{C^2}\right)C^2dt^2 - \frac{dx^2}{dt^2}dt^2 \qquad (\mathbf{3.4.10})$$

$$\therefore ds^2 = \left(1 + \frac{2\varphi}{C^2}\right)C^2dt^2 - dx^2 \qquad (\mathbf{3.4.11})$$

Equation $(\mathbf{3.4.11})$ represent the metric tensor of GSR, its solution the case of free falling in the Cartesian coordinate, also it represents the case of acceleration in one direction according to equivalence principle. This metric can be used for free accelerated particle, it should be also used for photons in field with respect to direction of photon and field. It's easy to drive the factor γ by using equation **(3.4.3)**, so

$$\gamma = \frac{1}{\sqrt{1 + \frac{2\varphi}{C^2} - \frac{v^2}{C^2}}} \qquad (\mathbf{3.4.12})$$

This is the same factor of γ in the GSR, so the metric in equation $(\mathbf{3.4.11})$ represent the space components of GSR derived easily from newton's law of motion under acceleration in street line. The classical approximate for $\frac{2\varphi}{C^2} - \frac{v^2}{C^2} \ll 1$ in term of total energy can be write from the relative energy as

$$E = \gamma m_0 C^2 = \frac{m_0 C^2}{\sqrt{1 + \frac{2\varphi}{C^2} - \frac{v^2}{C^2}}} = m_0 C^2 \left(1 - \frac{\varphi}{C^2} + \frac{v^2}{2C^2}\right) \qquad (\mathbf{3.4.13})$$

$$E = m_0 C^2 - m_0 \varphi + \frac{m_0 v^2}{2} = m_0 C^2 - V + T \qquad (\mathbf{3.4.14})$$

Note that we derive this equation by assuming that the particle is accelerated is positive and its potential energy decreasing, if one starting with negative accelerating which that means its potential energy increasing the factor metric tensor in equation $(\mathbf{3.4.11})$ become

$$\therefore ds^2 = \left(1 - \frac{2\varphi}{C^2}\right) C^2 dt^2 - dx^2 \qquad (\mathbf{3.4.15})$$

Thus, γ become

$$\gamma = \frac{1}{\sqrt{1 - \frac{2\varphi}{C^2} - \frac{v^2}{C^2}}} \qquad (\mathbf{3.4.16})$$

So the total energy in classical limit become

$$E=\gamma m_0 C^2=\frac{m_0 C^2}{\sqrt{1-\frac{2\varphi}{C^2}-\frac{v^2}{C^2}}}=m_0 C^2\left(1+\frac{\varphi}{C^2}+\frac{v^2}{2C^2}\right) \quad (\mathbf{3.4.17})$$

$$E=m_0 C^2+m_0\varphi+\frac{m_0 v^2}{2}=m_0 C^2+V+T \quad (\mathbf{3.4.18})$$

In view of equations $(\mathbf{3.4.11-3.1.18})$ one can define here the term $\mp m_0\varphi$ as the inertia resistance or the increasing and decreasing of inertia mass due to accelerated particle in gravity field, where in special relativity the mass have a limit velocities under light speed and the mass become infinity if it's velocity reach the speed of light due to inertia resistance, but in this model the particle that have mass, the inertia resistance appears as increasing or decreasing in their potential energy, this make the particles if we lie say it reach the speed of light its mass does not become infinity, but its potential energy increasing, now let's compare if the acceleration due to field or due to constant force applied to particle from equation $(\mathbf{3.4.1})$ where

$$ax=-\varphi=\frac{GM}{x^2}x=gx \quad (\mathbf{3.4.19})$$

The left side of equation $(\mathbf{3.4.19})$ assume that the accelerate a is constant due to constant force, where $a=\frac{F}{m}$ so there is no limit for the value ax and F is not function of distance x , while the right side represent of the acceleration due to field gravity generated by M as function of $\frac{1}{x}$, where $g=\frac{GM}{x^2}=\frac{F}{m}$, it's clearly here F is function of distance $\frac{1}{x^2}$ this is the mean different between acceleration due gravity field verses acceleration due constant force. But actually we assumed that the gravity field in specific limited region can be treatment as inertia force. This lead us to equivalence principle, one can say that the equivalence principle should be generalize to include the effect of strong field which produce not constant acceleration in any region, so for that the equivalence may not be accurate, however the equivalence principle can be used as well as studding the motion of plants around their star because here the angular acceleration can be nearly constant, but in the case of free falling it's better to replace $ax = -\varphi$.

3.5 The Relative Energy in GSR and Savickas model

As we shown in chapter two in equation $(\mathbf{2.15.1.20})$, and $(\mathbf{2.15.1.13})$ there is different equations between GSR and Savickas model to describe the relative mass and the relative energy, the question here is there a relation between these two expressions? Let's start with Savickas equation **[22]**,

$$m = \frac{m_0}{\sqrt{g_{00} - \frac{v^2{}_1}{C^2}}} \qquad (\mathbf{3.5.1})$$

$$\therefore E = mC^2 = \frac{m_0 C^2}{\sqrt{g_{00} - \frac{v^2{}_1}{C^2}}} \qquad (\mathbf{3.5.2})$$

In GSR **[17]**

$$m = g_{00} m_0 \gamma = \frac{g_{00} m_0}{\sqrt{g_{00} - \frac{v^2{}_2}{C^2}}} \qquad (\mathbf{3.5.3})$$

$$E = mC^2 = g_{00} m_0 \gamma C^2 = \frac{g_{00} m_0 C^2}{\sqrt{g_{00} - \frac{v^2{}_2}{C^2}}} \qquad (\mathbf{3.5.4})$$

Now let's find the relation between $v^2{}_1$ and $v^2{}_2$ by making equation $(\mathbf{3.5.1})$ equaled to $(\mathbf{3.58})$, thus

$$\frac{m_0}{\sqrt{g_{00}-\frac{v^2{}_1}{C^2}}}=\frac{g_{00}m_0}{\sqrt{g_{00}-\frac{v^2{}_2}{C^2}}} \quad (\mathbf{3.5.5})$$

Where $g_{00} = 1 + \frac{2\varphi}{C^2}$ that yields

$$\frac{1}{\sqrt{1+\frac{2\varphi}{C^2}-\frac{v^2{}_1}{C^2}}}=\frac{1+\frac{2\varphi}{C^2}}{\sqrt{1+\frac{2\varphi}{C^2}-\frac{v^2{}_2}{C^2}}} \quad (\mathbf{3.5.6})$$

So

$$\left(1+\frac{2\varphi}{C^2}\right)\sqrt{1+\frac{2\varphi}{C^2}-\frac{v^2{}_1}{C^2}}=\sqrt{1+\frac{2\varphi}{C^2}-\frac{v^2{}_2}{C^2}} \quad (\mathbf{3.5.7})$$

In classical limit where $\frac{2\varphi}{C^2}-\frac{v^2{}_1}{C^2} \ll 1$ and $\frac{2\varphi}{C^2}-\frac{v^2{}_2}{C^2} \ll 1$ one can write that

$$\left(1+\frac{2\varphi}{C^2}\right)(1+\frac{\varphi}{C^2}-\frac{v^2{}_1}{2C^2}) = (1+\frac{\varphi}{C^2}-\frac{v^2{}_2}{2C^2}) \quad (\mathbf{3.5.8})$$

$$1+\frac{\varphi}{C^2}-\frac{v^2{}_1}{2C^2}+\frac{2\varphi}{C^2}+\frac{2\varphi^2}{C^4}-\frac{\varphi v^2{}_1}{C^4}=1+\frac{\varphi}{C^2}-\frac{v^2{}_2}{2C^2} \quad (\mathbf{3.5.9})$$

By neglected the terms of C^{-4}, one can get

$$-\frac{v^2{}_1}{2C^2}+\frac{2\varphi}{C^2}=-\frac{v^2{}_2}{2C^2} \quad (\mathbf{3.5.10})$$

This equation gives

$$\frac{v^2{}_1}{2}=\frac{v^2{}_2}{2}+2\varphi \quad (\mathbf{3.5.11})$$

$$\therefore v^2{}_1 = v^2{}_2 + 4\varphi \quad (\mathbf{3.5.12})$$

This gives two results, first in the classical limit the both theory is equivalences, but it seems that GSR is more general than Savickas model, so it represents also the strong field, second result that the velocity is taken as the effective value of root mean square where $v_{rms} = \frac{v}{\sqrt{2}}$

Actually it seems that the factor γ which derived in chapter two in equations $(\mathbf{2.15.1.16-2.15.1.20})$ is more general and it valued for strong field and weak field, so by taken $\gamma=\frac{g_{00}}{\sqrt{g_{00}-\frac{v^2{}_2}{C^2}}}$ this lead us to form more general metric tensor instead of metric tensors in equations $(\mathbf{3.4.11})$, and $(\mathbf{3.4.15})$. let's starting from

$$\gamma=\frac{g_{00}}{\sqrt{g_{00}-\frac{v^2{}_2}{C^2}}}\qquad(\mathbf{3.5.13})$$

By using the same way used in derived the GSR metric tensor in weak field where

$$\frac{1}{\gamma^2}=\frac{ds^2}{C^2dt^2}=\frac{1+\frac{2\varphi}{C^2}-\frac{v^2{}_2}{C^2}}{\left(1+\frac{2\varphi}{C^2}\right)^2}\qquad(\mathbf{3.5.14})$$

$$\therefore ds^2=\frac{\left(1+\frac{2\varphi}{C^2}-\frac{v^2{}_2}{C^2}\right)}{\left(1+\frac{2\varphi}{C^2}\right)^2}C^2dt^2\qquad(\mathbf{3.5.15})$$

Rearrangement

$$ds^2=\frac{C^2dt^2}{\left(1+\frac{2\varphi}{C^2}\right)}-\frac{dx^2}{\left(1+\frac{2\varphi}{C^2}\right)^2}\qquad(\mathbf{3.5.16})$$

Equation $(\mathbf{3.5.16})$ can be written as

$$ds^2=\left(1+\frac{2\varphi}{C^2}\right)^{-1}C^2dt^2-\left(1+\frac{2\varphi}{C^2}\right)^{-2}dx^2\qquad(\mathbf{3.5.17})$$

Equation $(\mathbf{3.5.17})$ is matric tensor of GSR in strong field for free falling in the direction of the x-axies parallel to the force line of field, and for accelerated particle in very high accelerated. Its easily to prove that it goes to equation $(\mathbf{3.4.11})$ for weak field where $\frac{2\varphi}{C^2} \ll 1$.we use simple mathematic method to calculate the matric tensor of GSR in weak field and some approximations, so starting from $(\mathbf{3.5.17})$ one can write that

$$ds^2 = \left(1 - \frac{\varphi}{C^2}\right)^2 C^2dt^2 - \left(1 - \frac{\varphi}{C^2}\right)^4 dx^2 \quad (\mathbf{3.5.18})$$

This can be reached by using that

$$\frac{1}{\left(1 + \frac{2\varphi}{C^2}\right)} = \frac{1}{\sqrt{\left(1 + \frac{2\varphi}{C^2}\right)}\sqrt{\left(1 + \frac{2\varphi}{C^2}\right)}} = \left(1 - \frac{\varphi}{C^2}\right)^2 \quad (\mathbf{3.5.19})$$

In general, for $\frac{2\varphi}{C^2} \ll 1$ one can find that

$$\frac{1}{\left(1 + \frac{2\varphi}{C^2}\right)^n} = \left(1 - \frac{\varphi}{C^2}\right)^{2n} \quad (\mathbf{3.5.20})$$

So equation $(\mathbf{3.5.18})$ valued for $\frac{2\varphi}{C^2} \ll 1$. Now let's calculate the brackets and we shall have neglected any terms of power more than C^{-2} , that

$$ds^2 = \left(1 - \frac{2\varphi}{C^2} + \frac{\varphi^2}{C^4}\right) C^2dt^2 - \left(1 - \frac{2\varphi}{C^2} + \frac{\varphi^2}{C^4} - \frac{2\varphi}{C^2} + \frac{4\varphi^2}{C^4} - \frac{2\varphi^3}{C^6} + \frac{\varphi^2}{C^4} - \frac{2\varphi^3}{C^6} + \frac{2\varphi^4}{C^8}\right) dx^2 (\mathbf{3.5.21})$$

By neglected any terms of power more than C^{-2}, it yields

$$ds^2 = \left(1 - \frac{2\varphi}{C^2}\right) C^2dt^2 - \left(1 - \frac{4\varphi}{C^2}\right) dx^2 \quad (\mathbf{3.5.22})$$

Equation $(\mathbf{3.5.22})$ is equivalence exactly to equation $(\mathbf{3.4.11})$, and we can prove that by calculate the factor γ for equation $(\mathbf{3.5.22})$, so

$$\frac{1}{\gamma^2} = \frac{ds^2}{C^2 dt^2} = \left(1 - \frac{2\varphi}{C^2}\right) - \left(1 - \frac{4\varphi}{C^2}\right)\frac{v^2{}_2}{C^2} \qquad (\mathbf{3.5.23})$$

$$\therefore \gamma = \frac{1}{\sqrt{\left(1 - \frac{2\varphi}{C^2}\right) - \left(\frac{v^2{}_2}{C^2} - \frac{v^2{}_2 4\varphi}{C^4}\right)}} \qquad (\mathbf{3.5.24})$$

Neglected the term of C^{-4} that gives

$$\gamma = \frac{1}{\sqrt{1 - \frac{2\varphi}{C^2} - \frac{v^2{}_2}{C^2}}} \qquad (\mathbf{3.5.25})$$

Substitute $v^2{}_2 = v^2{}_1 - 4\varphi$, yields

$$\gamma = \frac{1}{\sqrt{1 - \frac{2\varphi}{C^2} - \frac{v^2{}_1 - 4\varphi}{C^2}}} \qquad (\mathbf{3.5.26})$$

Rearrangement,

$$\gamma = \frac{1}{\sqrt{1 + \frac{2\varphi}{C^2} - \frac{v^2{}_1}{C^2}}} \qquad (\mathbf{3.5.27})$$

Thus, equation $(\mathbf{3.5.27})$ is the same of equation $(\mathbf{3.4.12})$, which accentual that the metric tensor of space in equation $(\mathbf{3.5.17})$ is correct formula for strong field describe the free falling and more general than metric tensor of GSR presented in equation $(\mathbf{3.4.11})$. these two metric should be a solution of general relativity in the case of free falling in weak, and strong gravity field. It also represents of constant accelerated due to constant force.

3.6 New Lorentz Field Dependent Lorentz Transformation Due to Photon Direction Change

New Lorentz transformation accounting for the effect of the field on space time and mass was derived. This transformation is based on the effect of the field on the photon trajectory by preserving the photon speed but changing its direction. According to this version the photon is accelerated by the field due to the change of its direction. This transformation shows that length, space and mass are effected by the field. It also shows the beauty of Einstein curved space concept and its advantage to describe fields more correctly than the Newton laws. One of the important theories in physics is Einstein's special relativity, it is radical theory that made modifications to concept of space, time and energy **[1,25]**. It is formed from two hypothesizes, first, the homogeneity of the space and other is invariability of light speed in vacuum as maximum velocity of transforming any kind of signal. In Newtonian mechanics the concept of space, time and mass are absolute and they have the same value at any references frames, while they are not absolute in special relativity, they are relative quantities according to the velocities of the observers in inertial frames of references. The hypothesis of invariability of light speed is came from Einstein's conclusion for the results of Michelson-Morley experiments. Einstein used the above hypothesizes and Lorentz transformation to obtain the expressions of time, length, mass and energy **[2]**. Special relativity theory passed many tests that confirmed its accurate result and it prove its self as extended theory for Newtonian mechanics in high speed near light speed **[1,2,4,12]**. It succeeded to explaining many physical phenomena like pair production, photoelectric effect and meson decay **[4]**. But special relativity suffers from noticeable setbacks, some of these setbacks is related to the fact that classical limit of energy expression in special relativity does not coincide with Newtonian energy expression, where the term of potential energy been missing. That means that the energy expression by special relativity does not satisfy the correspondence principle. Also the special relativity can't explain the gravitation red shift for photon due to gravitation field **[15]**. The photon mass is proportional to its frequency, that conflict with fact that the mass in special relativity is not a function of the field potential. The same holds for the time, length and mass expressions in special relativity, which does not recognize the effect of gravitational field in the weak limit, which is not in conformity with that of general relativity where time and length are effected by gravitational field. However, Einstein's general relativity is generally covariant theory of gravity. Many offers were made to correct and modify special relativity to include the effect of gravity and other fields. These offers concentrated to the mass notion and energy without considering the effect of both motion beside fields on time, length and mass. Some offers were made to involve the curvature effect of space time on energy and momentum **[7,15]**, but their energy expression is incomplete because they deal with equation of motion instead of Hamiltonian. These drawbacks motivate searching for new model accounting for the effect of fields. The next work concerns on these problems**.**

3.7 Lorentz Transformation for Accelerated Photon

Consider the Lorentz transformation

$$x = \gamma\left(\grave{x} + v\grave{t} - \frac{a\grave{t}^2}{2}\right) \quad \mathbf{(3.7.1)}$$

$$\grave{x} = \gamma\left(x - vt + \frac{at^2}{2}\right) \quad \mathbf{(3.7.2)}$$

Consider the two frames $(x.t)$ and $(\grave{x}.\grave{t})$ have their origin coincide at $= \grave{t} = 0$. If a pulse of light is received from a source S then its position in the two frames becomes at t and $\grave{t}$ respectivity

$$x = ct \quad (\mathbf{3.7.3 \cdot \boldsymbol{a}})$$

$$\grave{x} = c\grave{t} \quad (\mathbf{3.7.3 \cdot \boldsymbol{b}})$$

Substitute $(\mathbf{3.7.3.\boldsymbol{a}})$ and $(\mathbf{3.7.3.\boldsymbol{b}})$ in $(\mathbf{3.7.1})$ yields

$$ct = \gamma\left(c\grave{t} + v\grave{t} - \frac{a\grave{t}^2}{2}\right) = \gamma\left((c+v)\grave{t} - \frac{a\grave{t}^2}{2}\right)$$

$$t = \gamma\left(\left(1 + \frac{v}{c}\right)\grave{t} - \frac{a\grave{t}^2}{2c}\right)$$

$$t = C_1\grave{t} + C_2\grave{t}^2 \quad (\mathbf{3.7.4})$$

Where

$$C_1 = \gamma\left(1 + \frac{v}{c}\right) \quad (\mathbf{3.7.5 \cdot \boldsymbol{a}})$$

$$C_2 = -\frac{\gamma a}{2c} \quad (\mathbf{3.7.5 \cdot \boldsymbol{b}})$$

Substituting $(\mathbf{3.7.3.\boldsymbol{a}})$ and $(\mathbf{3.7.3.\boldsymbol{b}})$ in $(\mathbf{3.7.2})$ gives

$$c\grave{t} = \gamma\left(ct - vt + \frac{at^2}{2}\right)$$

$$\grave{t} = \gamma\left(\left(1 - \frac{v}{c}\right)t + \frac{at^2}{2c}\right)$$

$$\grave{t} = C_3 t + C_4 t^2 \quad (\mathbf{3.7.6})$$

Where

$$C_3 = \gamma\left(1 - \frac{v}{c}\right) \quad (\mathbf{3.7.7 \cdot \boldsymbol{a}})$$

$$C_4 = \frac{\gamma a}{2c} \quad (\mathbf{3.7.7 \cdot \boldsymbol{b}})$$

Substitute ($\mathbf{3.7.6}$) in ($\mathbf{3.7.4}$) to get

$$t = C_1(C_3 t + C_4 t^2) + C_2(C_3 t + C_4 t^2)^2 \quad (\mathbf{3.7.8})$$

$$t = C_1 C_2 t + C_1 C_4 t^2 + C_2 C_3^2 t^2 + 2C_2 C_3 C_4 t^3 + C_2 C_4^2 t^4 \quad (\mathbf{3.7.9})$$

Comparing the coefficients of $t.t^2.t^3$and t^4 on both sides gives

$$C_1 C_3 = 1 \quad (\mathbf{3.7.10})$$

$$C_1 C_4 = -C_2 C_3^2 \quad (\mathbf{3.7.11})$$

$$2C_2 C_3 C_4 = 0 \quad (\mathbf{3.7.12})$$

$$C_2 C_4^2 = 0 \quad (\mathbf{3.7.13})$$

From ($\mathbf{3.7.5.\boldsymbol{a}}$) and ($\mathbf{3.7.7.\boldsymbol{a}}$) equations, ($\mathbf{3.7.10}$) becomes

$$\gamma^2\left(1 + \frac{v}{c}\right)\left(1 - \frac{v}{c}\right) = 1$$

$$\therefore \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (\mathbf{3.7.14})$$

From ($\mathbf{3.5.7.\boldsymbol{a}}$) and ($\mathbf{3.7.7.\boldsymbol{b}}$) equation, ($\mathbf{3.7.11}$) becomes

$$\gamma^2\left(1 + \frac{v}{c}\right)\frac{a}{2c} = \frac{\gamma^3 a}{2c}\left(1 - \frac{v}{c}\right) \quad (\mathbf{3.7.15})$$

From ($\mathbf{3.7.5.\boldsymbol{b}}$), ($\mathbf{3.7.7.\boldsymbol{a}}$) and ($\mathbf{3.7.7.\boldsymbol{b}}$) equation ($\mathbf{3.7.12}$) becomes

$$-\frac{2\gamma^3 a^2}{4c^2}\left(1-\frac{v}{c}\right)=0 \quad (\mathbf{3.7.16})$$

From $(\mathbf{3.5.7.}\boldsymbol{b})$ and $(\mathbf{3.7.7.}\boldsymbol{b})$ equation, $(\mathbf{3.7.13})$ becomes

$$-\frac{\gamma^3 a^3}{8c^3}=0 \quad (\mathbf{3.7.17})$$

In view of equation $(\mathbf{3.7.14})$ γ take the same special relativity form. However, equations $(\mathbf{3.7.15})$, and $(\mathbf{3.7.16})$ shows that the Lorentz transformation $(\mathbf{3.7.1})$ and $(\mathbf{3.7.2})$ gives consistent results only when $(a = 0)$. This requires trying another transformation to take care of effect of fields. One can assume that the light is accelerated due to the effect of field on photon trajectory. It is well known in mechanics that any particle can be accelerated if its magnitude of velocity v is constant when it change its direction. This happens for particles having constant speed v and moving in a circular orbit, thus changing its direction regularly and possessing an acceleration

$$a=\frac{v^2}{r} \quad (\mathbf{3.7.18})$$

towards the Centre of a circular orbit. According to general relativity (GR) the photon move in a curved trajectory in a gravitational field, although the magnitude of photon speed c is constant, but it is accelerated due to the change of photon direction, since the change of photon direction decreases its speed in the original direction. For example if the photon change its direction by $\Delta\emptyset$ during time interval Δt, its acceleration becomes

$$a=\frac{\Delta c}{\Delta t}=\frac{c-c\sin\Delta\emptyset}{\Delta t}\approx\frac{c(1-\Delta\emptyset)}{\Delta t} \quad (\mathbf{3.7.19})$$

This means that SR and GR are not in conflict with each other, this shows how beauty is Einstein relativity compared to Newton's laws. The photon acceleration can be found by using the relation between work done and energy change according to gravity red shift. The change in photon energy is given by

$$\Delta E = h\grave{f} - hf = V \quad \mathbf{(3.7.20)}$$

Where V is the field potintial. Here one assume that V is potintial of any field; not gravity field only. The change of energy is equal to the work done, again assuming constant mass and constant acceleration, one gets

$$F \cdot x = max = V \quad \mathbf{(3.7.21)}$$

The photon displacement can be found by using the expression for photon interval in a curved space, to get

$$0 = c^2 d\tau^2 = g_{00} c^2 dt^2 - g_{xx} dx^2 \quad \mathbf{(3.7.22)}$$

Assuming that the photon obeys static isotropic constraints $g_{00} = g_{xx}$, one gets

$$dx^2 = g_{00}^2 c^2 dt^2 = \left(1 + \frac{2\varphi}{c^2}\right)^2 c^2 dt^2 \quad \mathbf{(3.7.23)}$$

$$dx = \left(1 + \frac{2\varphi}{c^2}\right) cdt$$

Thus integrating both sides yields

$$x = \left(1 + \frac{2\varphi}{c^2}\right) ct \quad \mathbf{(3.7.24)}$$

Similar relation can be obtained by finding the photon acceleration by assuming $x = ct$ to get

$$a = \frac{V}{mx} = \frac{\varphi}{x} = \frac{\varphi}{ct} \qquad (\mathbf{3.7.25})$$

In view of equation $(\mathbf{3.7.3.}\boldsymbol{a})$ and $(\mathbf{3.7.25})$ the position is given by

$$x = ct - \frac{at^2}{2} = ct - \frac{\varphi t^2}{2ct} = ct - \frac{\varphi}{2c}t \qquad (\mathbf{3.7.26})$$

Similarly, equation $(\mathbf{3.7.3.}\boldsymbol{b})$ and $(\mathbf{3.7.25})$ gives

$$\grave{x} = c\grave{t} + \frac{a\grave{t}^2}{2} = c\grave{t} + \frac{\varphi\grave{t}^2}{2\grave{x}} = c\grave{t} + \frac{\varphi}{2c}\grave{t} \qquad (\mathbf{3.7.27})$$

Consider the Lorentz transformation

$$x = \gamma\left(\grave{x} + v\grave{t} - \frac{a\grave{t}^2}{2}\right) \qquad (\mathbf{3.7.28})$$

Where the average velocity v_m is given by

$$v_m = \frac{v + v_0}{2} = \frac{v + v - a\grave{t}}{2} = v - \frac{a\grave{t}}{2} \qquad (\mathbf{3.7.29})$$

Thus

$$\grave{l} = v\grave{t}^2 - \frac{a\grave{t}^2}{2} = \left(v - \frac{a\grave{t}}{2}\right)\grave{t} = v_m\grave{t} \qquad (\mathbf{3.7.30})$$

Thus

$$x = \gamma(\grave{x} + v_m\grave{t}) \qquad (\mathbf{3.7.31})$$

Similarly

$$\grave{x} = \gamma(x - v_m t) \qquad (\mathbf{3.7.32})$$

For static source in a frame S the photon is not accelerated , thus

$$x = ct \qquad (\mathbf{3.7.33})$$

But the observer in $\grave{S}$ sees the source S is accelerated and the photon moves in curved space , thus

$$\grave{x} = c\grave{t} + \frac{\varphi}{2c}\grave{t} \quad (\mathbf{3.7.34})$$

By substituting $(\mathbf{3.7.33})$ and $(\mathbf{3.7.34})$ in $(\mathbf{3.7.31})$ yields

$$ct = \gamma\left(c + \frac{\varphi}{2c} + v_m\right)\grave{t} \quad (\mathbf{3.7.35})$$

Similarly if the source is at rest in frame $\grave{S}$ the photon position is given by

$$\grave{x} = c\grave{t} \quad (\mathbf{3.7.36})$$

Since $\grave{S}$ is accelerated with respect to S due to the field effect , therefore the photon move in a curved space , thus it is accelerated, hence

$$x = ct - \frac{\varphi}{2c}t \quad (\mathbf{3.7.37})$$

Substitute $(\mathbf{3.7.36})$, $(\mathbf{3.7.37})$ in $(\mathbf{3.7.32})$ to get

$$c\grave{t} = \gamma\left(c - \frac{\varphi}{2c} - v_m\right)t \quad (\mathbf{3.7.38})$$

From $(\mathbf{3.7.34})$ and $(\mathbf{3.7.37})$

$$\grave{t} = \frac{\grave{t}}{c}\gamma^2\left(c + \frac{\varphi}{2c} + v_m\right)\left(c - \frac{\varphi}{2c} - v_m\right) \quad (\mathbf{3.7.39})$$

So

$$\gamma = \frac{1}{\sqrt{1 - \left(\frac{\varphi}{2c^2} + \frac{v_m}{c}\right)^2}} \quad (\mathbf{3.7.40})$$

Thus the generalized special relativistic energy is given by

$$E = \gamma m_0 c^2 = \frac{m_0 c^2}{\sqrt{1 - \left(\frac{\varphi}{2c^2} + \frac{v_m}{c}\right)^2}} \qquad (\mathbf{3.7.41})$$

Neglect the term consisting of c^2, yields

$$\gamma = \frac{1}{\sqrt{1 - \frac{v_m^2}{c^2}}} \qquad (\mathbf{3.7.42})$$

Where

$$v_m = \frac{v + v_0}{2} \qquad (\mathbf{3.7.43})$$

But when the particle moves against the field

$$v^2 = v_0^2 - 2ax = v_0^2 - 2\varphi$$

$$\therefore v_0^2 = v^2 + 2\varphi \qquad (\mathbf{3.7.44})$$

by Assuming that $v$and v_0 represent the average values that related to maximum valuesv_{max} and v_{0max} according to relations $v = \frac{v_{max}}{\sqrt{2}}$ and $v_0 = \frac{v_{0max}}{\sqrt{2}}$

$$\therefore v_{0max}^2 = v_{max}^2 + 4ax \qquad (\mathbf{3.7.45})$$

then

$$v_m^2 = \left(\frac{v_{max} + v_{0max}}{2}\right)^2 = \frac{v_{max}^2 + 2v_{max}v_{0max} + v_{0max}^2}{4}$$

$$= \frac{v_{max}^2 + \left(2v_{max}\sqrt{v_{max}^2 + 4ax}\right) + v_{max}^2 + 4ax}{4}$$

$$v_m^2 = \frac{2v_{max}^2 + \left(2v_{max}^2\sqrt{1 + \frac{4ax}{v_{max}^2}}\right) + 4ax}{4}$$

$$\approx \frac{2v_{max}^2 + 2v_{max}^2\left(1 + \frac{2ax}{v_{max}^2}\right) + 4ax}{4} \quad (\mathbf{3.7.46})$$

$$\therefore v_m^2 = \frac{4v_{max}^2 + 8ax}{4} = v_{max}^2 + 2ax = v_{max}^2 + 2\varphi \quad (\mathbf{3.7.47})$$

But from equation ($\mathbf{3.7.38}$) for $\frac{v_m^2}{c^2} < 1$ then

$$\gamma = 1 + \frac{v_m^2}{2c^2} = 1 + \frac{v_{max}^2 + 2\varphi}{2c^2} \quad (\mathbf{3.7.48})$$

$$\therefore \gamma = 1 + \frac{1}{C^2}\left(\frac{v_{max}^2}{2} + \varphi\right) \quad (\mathbf{3.7.49})$$

Thus equation ($\mathbf{3.7.41}$) and ($\mathbf{3.7.38}$) gives

$$E = \gamma m_0 c^2 = m_0 c^2 + \frac{1}{2} m_0 v_{max}^2 + m_0 \varphi = m_0 c^2 + T + V \quad (\mathbf{3.7.50})$$

Thus the generalized special relativity energy relation satisfies the Newtonian limit. This is since the energy include kinetic beside potential energy term. The gravitational red shift of photons can also be explained by using GSR. Assuming photon in free space so its potential energy $V = 0$, by using ($\mathbf{3.7.50}$) and plank hypothesis , one can get

$$hf = m_0 c^2 + T \quad (\mathbf{3.7.51})$$

if the photon enters gravitational field its frequency ($\mathbf{3.7.52}$) changes also to $\grave{f}$. Thus equation ($\mathbf{3.7.50}$) gives

$$h\grave{f} = m_0 c^2 + T + V = hf + V \quad (\mathbf{3.7.52})$$

thus fortunately equation ($\mathbf{3.7.48}$) explains the gravitational red shift. Equations ($\mathbf{3.7.15}$), ($\mathbf{3.7.16}$) and ($\mathbf{3.7.17}$) can only be satisfied if the acceleration a vanishes. This means that Lorentz transformation ($\mathbf{3.7.1}$) and ($\mathbf{3.7.2}$) are not suitable for describing the behavior of particles in fields by using the acceleration concept and assuming a to be invariant. However if one uses the concept of potential φ and assume it to be invariant, one can find useful expression for γ similar to that obtained by other researchers **[17,18,19,20,22,23,24]**. This expression explains the gravitational red shift and satisfies the Newtonian limit. The fact that the potential invariance concept is suitable for Lorentz transformation of fields may be related to the fact that

$$g_{00} = -\left(1 + \frac{2\varphi}{c^2}\right) \rightarrow \varphi = -\frac{c^2}{2}(g_{00} + 1) \qquad (\mathbf{3.7.53})$$

Which means that φ reflects space deformation. Inserting this relation in equation ($\mathbf{3.7.23}$) gives

$$x = ct - \frac{\varphi}{2c}t = ct + \frac{c}{4}(g_{00} + 1)t = \frac{5}{4}ct + \frac{c}{4}g_{00}t \qquad (\mathbf{3.7.54})$$

Which means that the field deform the space. This makes the relation

$$l = vt - \frac{at^2}{2} \qquad (\mathbf{3.7.55})$$

which describes the motion in Euclidean space invalid. The fact that the field deform space in generalized special relativity conforms completely with GR, which shows also that the gravity field deform the space. The space deformation shows also that the photon acceleration is not due to the change of light speed but due to the change of photon(light) direction. This result shows that GSR and GR are not in conflict with each other but they integrate each other. This shows how beauty is the Einstein space deformation concept. The Lorentz transformation based on photon motion in a curved space relation, resembles that obtained by others. It also satisfies Newtonian limit and predict gravitational red shift. This model is in agreement with previous GSR models.

3.8 Derivation of v_{0max}^2 & v_{max}^2 by using SR and GSR metric tensor

To reach the equation $(\mathbf{3.7.45})$ that described the effective value of velocity we should starting from the equation

$$v^2 = v_0^2 + 2\varphi \quad (\mathbf{3.8.1})$$

As we know v_0 is a constant velocity, its space-time follows the special relativity metric tensor as

$$ds^2 = C^2 dt^2 - dx^2 \quad (\mathbf{3.8.2})$$

$$\therefore \frac{ds^2}{dt^2} = C^2 - v_0^2 \rightrightarrows\ v_0^2 = C^2\left(1 - \frac{ds^2}{C^2 dt^2}\right) = C^2\left(1 - \frac{1}{\gamma_s^2}\right) \quad (\mathbf{3.8.3})$$

Where $\gamma_s = \frac{1}{\sqrt{1-\frac{v_0^2}{C^2}}}$. but v in equation $(\mathbf{3.8.1})$ is not constant value, it accelerated along; suppose, x-axis due gravity field, so its space-time follows the GSR metric tensor as we find in equation $(\mathbf{3.4.11})$ for weak field

$$ds^2 = \left(1 + \frac{2\varphi}{C^2}\right) C^2 dt^2 - dx^2 \quad (\mathbf{3.8.4})$$

That means

$$v^2 = C^2\left(1 - \frac{1}{\gamma_G^2}\right) \quad (\mathbf{3.8.5})$$

Where $\gamma_{G=\frac{1}{\sqrt{1+\frac{2\varphi}{C^2}-\frac{v^2}{C^2}}}}$. Now let's substitute equation $(\mathbf{3.8.3})$, and $(\mathbf{3.8.5})$ in equation $(\mathbf{3.8.1})$ to get the effective velocity, we get

$$C^2\left(1-\frac{1}{\gamma_G^2}\right)=C^2\left(1-\frac{1}{\gamma_s^2}\right)+2\varphi \quad (\mathbf{3.8.6})$$

$$C^2\left(1-(1+\frac{2\varphi}{C^2}-\frac{v^2}{C^2})\right)=C^2\left(1-(1-\frac{v_0^2}{C^2})\right)+2\varphi \quad (\mathbf{3.8.7})$$

This yields

$$-2\varphi+v^2=v_0^2+2\varphi \quad (\mathbf{3.8.8})$$

$$\therefore v_{eff}^2=v_{0eff}^2+4\varphi \quad (\mathbf{3.8.9})$$

This can be done also if the particle accelerated positive or negative by using equation $(\mathbf{3.4.16})$. we conclude that by comparing $(\mathbf{3.8.9})$ and $(\mathbf{3.8.1})$

$$v^2=\frac{v_{eff}^2}{2} \;\&\; v_0^2=\frac{v_{0eff}^2}{2} \quad (\mathbf{3.8.10})$$

3.9 Schrödinger Equation in GSR

Schrodinger equation describe the behaver of particles according to classical mechanic. It does not take the relative effect for particles. The relative version of it was derived by Klein and Gordon. Then Deric present his equation to describe the particle that have spin. In this section we will introduce a new version of Schrödinger equation to take the effect of accelerated particle depends on GSR theory and Savakis model in weak field. The relative energy for particle can be taken as

$$E=mC^2=\gamma m_0C^2 \quad (\mathbf{3.9.1})$$

By substitute the factor γ, yileds

$$E = \frac{m_0C^2}{\sqrt{\frac{m^2C^2}{m^2C^2}\left(1+\frac{2\varphi}{C^2}-\frac{v^2}{C^2}\right)}} = \frac{m_0C^2}{\sqrt{1+\frac{2m^2C^2\varphi}{E^2}-\frac{p^2C^2}{E^2}}} \quad (\mathbf{3.9.2})$$

$$\rightrightarrows \frac{m_0C^2}{\sqrt{\frac{E^2+2m^2C^2\varphi-p^2C^2}{E^2}}}$$

Rearrangement

$$m_0C^2 = \sqrt{E^2+2m^2C^2\varphi-p^2C^2} \quad (\mathbf{3.9.3})$$

$$\therefore\ m_0^2C^4 = E^2+2m^2C^2\varphi-p^2C^2 \quad (\mathbf{3.9.4})$$

This can write as

$$E^2+2m^2C^2\varphi = E^2+\frac{2E^2\varphi}{C^2} = m_0^2C^4+p^2C^2 \quad (\mathbf{3.9.5})$$

So

$$\left(1+\frac{2\varphi}{C^2}\right)E^2 = m_0^2C^4+p^2C^2 \quad (\mathbf{3.9.6})$$

Now, let's substitute the operation of p^2, and E^2, since $\hat{p}^2 = -\hbar^2\nabla^2$, $\hat{E}^{2=-}\hbar^2\frac{\partial^2}{\partial t^2}$

$$-\left(1+\frac{2\varphi}{C^2}\right)\hbar^2\frac{\partial^2}{\partial t^2}\Psi(\mathrm{t},\bar{\mathrm{r}}) = m_0^2C^4\Psi(\mathrm{t},\bar{\mathrm{r}}) - C^2\hbar^2\nabla^2\Psi(\mathrm{t},\bar{\mathrm{r}}) \quad (\mathbf{3.9.7})$$

Rearrangement

$$\left(1+\frac{2\varphi}{C^2}\right)\frac{1}{C^2}\frac{\partial^2}{\partial t^2}\Psi(\mathrm{t},\bar{\mathrm{r}}) - \nabla^2\Psi(\mathrm{t},\bar{\mathrm{r}}) + \frac{m_0^2C^2}{\hbar^2}\Psi(\mathrm{t},\bar{\mathrm{r}}) = 0 \quad (\mathbf{3.9.8})$$

Comparing with equation $(\mathbf{2.14.7})$, it can find that it modified D'Alemert operator by factor $\left(1+\frac{2\varphi}{C^2}\right)$, and it is general form of Klien-Gorden equation for accelerated particle due to gravity felid. It can be written in a covariant notation as

$$\left(\left(1+\frac{2\varphi}{C^2}\right)\square+\mu^2\right)\Psi=0 \quad \mathbf{(3.9.9)}$$

Equation $(\mathbf{3.9.8})$ can be in Deric notation as

$$\left(1+\frac{2\varphi}{C^2}\right)\frac{1}{C^2}\frac{\partial^2}{\partial t^2}|\Psi(\mathrm{t},\bar{\mathrm{r}})\rangle-\nabla^2|\Psi(\mathrm{t},\bar{\mathrm{r}})\rangle+\frac{m_0^2C^2}{\hbar^2}|\Psi(\mathrm{t},\bar{\mathrm{r}})\rangle=0 \quad \mathbf{(3.9.10)}$$

Tet's take the effect of conjugate $\dot{\Psi}(\mathrm{t},\bar{\mathrm{r}})$ by multiply equation $(\mathbf{3.9.10})$ by $\langle\Psi(\mathrm{t},\bar{\mathrm{r}})|$, so

$$\left(1+\frac{2\varphi}{C^2}\right)\frac{1}{C^2}\langle\Psi(\mathrm{t},\bar{\mathrm{r}})|\frac{\partial^2}{\partial t^2}|\Psi(\mathrm{t},\bar{\mathrm{r}})\rangle-\langle\Psi(\mathrm{t},\bar{\mathrm{r}})|\nabla^2|\Psi(\mathrm{t},\bar{\mathrm{r}})\rangle$$
$$+\frac{m_0^2C^2}{\hbar^2}\langle\Psi(\mathrm{t},\bar{\mathrm{r}})|\Psi(\mathrm{t},\bar{\mathrm{r}})\rangle \quad \mathbf{(3.9.11)}$$

This gives

$$\left(1+\frac{2\varphi}{C^2}\right)\frac{1}{C^2}\left(-\frac{E^2}{\hbar^2}\right)+\frac{p^2}{\hbar^2}+\frac{m_0^2C^2}{\hbar^2}=0 \quad \mathbf{(3.9.12)}$$

Multiply by $\hbar^2$ and then arrangements, yields

$$\left(1+\frac{2\varphi}{C^2}\right)E^2=m_0^2C^4+p^2C^2 \quad \mathbf{(3.9.13)}$$

$$\Longrightarrow E^2=\frac{m_0^2C^4+p^2C^2}{\left(1+\frac{2\varphi}{C^2}\right)} \quad \mathbf{(3.9.14)}$$

Thus, equation $(\mathbf{3.9.14})$ as we shown in equations $(\mathbf{3.9.2-3.9.6})$ derived from equation $(\mathbf{3.9.1})$ the relative energy from GSR, thus

$$E=mC^2=\gamma m_0C^2 \quad \mathbf{(3.9.15)}$$

This prove that equations $(\mathbf{3.9.8, 3.9.9})$ is valued for accelerated particle due weak gravity field, it is invariant, and it goes to Klein-Gordon equation when $\varphi=0$.

Chapter four

Schrodinger equation in weak and strong field

4.1. Introduction

from chapter three a new version of relative Schrodinger equation that described accelerated spineless particle due weak gravity field, and its invariant under Lorentz transformation as we shown that the energy is conserved in equations $(\mathbf{3.9.11-3.9.15})$. Now let's do that for photon that is in weak gravity field. The metric tensor of GSR as we shown in chapter three equation $(\mathbf{3.4.11})$

$$ds^2 = \left(1+\frac{2\varphi}{C^2}\right)C^2dt^2 - dx^2 \quad (\mathbf{4.1.1})$$

For photon $ds^2 = 0$, so

$$\left(1+\frac{2\varphi}{C^2}\right)C^2dt^2 - dx^2 = 0 \quad (\mathbf{4.1.2})$$

$$\therefore \left(1+\frac{2\varphi}{C^2}\right) - \frac{dx^2}{C^2dt^2} = \left(1+\frac{2\varphi}{C^2}\right) - \frac{C_{eff}^2}{C^2} = 0 \quad (\mathbf{4.1.3})$$

It yields

$$C_{eff}^2 = C^2\left(1+\frac{2\varphi}{C^2}\right) \quad (\mathbf{4.1.4})$$

This equation can be used to drive the gravitation red shift in weak gravity field, that

$$E = h\nu = \frac{h}{\lambda}C \quad (\mathbf{4.1.5})$$

The difference in light energy presented as

$$\Delta E = h\nu - h\nu_0 = \frac{h}{\lambda}\left(C_{eff} - C\right) \quad (\mathbf{4.1.6})$$

Then

$$\frac{\Delta E}{E} = \frac{C - C_{eff}}{C} = \frac{C\sqrt{1 + \frac{2\varphi}{C^2}} - C}{C} \qquad (\mathbf{4.1.7})$$

And for $\frac{2\varphi}{C^2} \ll 1$, one can write

$$\frac{\Delta E}{E} = \frac{\Delta \nu}{\nu} = \frac{C\left(\left(1 + \frac{\varphi}{C^2}\right) - 1\right)}{C} = \frac{\varphi}{C^2} = -\frac{GM}{rC^2} \qquad (\mathbf{4.1.8})$$

Equation $(\mathbf{4.1.8})$ is the same formula of the gravitation red shift of spectral line. Equation $(\mathbf{4.1.4})$ does not means that the photon in gravity field exceeds the speed of light in vacuum, it means that its direction changes due to curve of space time, but since there is a gravity spectral line shift that means its frequency change due to equation $(\mathbf{4.1.8})$, but we now that in the electromagnetic spectrum all waves have a constant velocity equal to speed of light and their frequencies vary depends on their wave lengths, if the frequency high that means its wave length short to stand the speed constant where $C = \nu\lambda$, but equation $(\mathbf{4.1.8})$ that described well the gravity red shift, not in this model but in any success model that describe the effect of gravity for the line spectral like general relativity and others, there is no change in the wave length, the effect only in the frequency, the frequency increased but the wave length does not change, in the view of equation $(\mathbf{4.1.4})$, and $(\mathbf{4.1.8})$ we conclude that if we take the space-time in not Euclidean that means the addition energy is stored temporary in the increasing of frequency, but if we assumed that the gravity field curve the space-time that means the beam light passes throw curved line to pass this addition energy due gravity field, or one can say the beam light have potential energy.

4.2. Estimate g_{00}In Strong Gravity Field For Photon In GSR Model

As we shown in chapter three in equations $(\mathbf{3.3.9})$, and $(\mathbf{3.5.17})$ that g_{00} in strong field goes to that in weak field easily, we search here for general form of g_{00} by using simple mathematics that describe the effect of gravity field on the time coordinate for photon. Let's recall equation $(\mathbf{4.1.4})$ that

$$C_{eff}^2 = C^2\left(1 + \frac{2\varphi}{C^2}\right) \qquad (\mathbf{4.2.1})$$

The term $\left(1 + \frac{2\varphi}{C^2}\right)$ equivalence to g_{00}, which derived from weak field approximation. If we assumed that its approximated from a general nonlinear form, so one can start with this formula of Taylor series

$$C_{eff}^2 = C^2\, e^{\frac{2\varphi}{C^2}} \qquad (\mathbf{4.2.2})$$

Now let's do simple operation by multiply by $\frac{i}{i}$ where $i = \sqrt{-1}$, yields

$$C_{eff}^2 = C^2\, e^{\frac{2\varphi i}{iC^2}} \qquad (\mathbf{4.2.3})$$

Now let

$$\frac{2\varphi}{iC^2} = -\frac{2\varphi i}{C^2} \equiv \theta \qquad (\mathbf{4.2.4})$$

$$C_{eff}^2 = C^2 e^{i\theta} \qquad (\mathbf{4.2.5})$$

By using Euler identity, that

$$C_{eff}^2 = C^2 e^{i\theta} = C^2(\cos\theta + i\sin\theta) \qquad (\mathbf{4.2.6})$$

Substitute $(\mathbf{4.2.4})$ in $(\mathbf{4.2.6})$ to get

$$C_{eff}^2 = C^2\left(\cos\theta(-\frac{2\varphi i}{C^2}) + i\sin(-\frac{2\varphi i}{C^2})\right) \qquad (\mathbf{4.2.7})$$

Since $\cos(-\theta) = \cos\theta$, and $\sin(-\theta) = -\sin\theta$ that yields

$$C_{eff}^2 = C^2\left(\cos(\frac{2\varphi i}{C^2}) - i\sin(\frac{2\varphi i}{C^2})\right) \qquad (\mathbf{4.2.8})$$

We have the identical

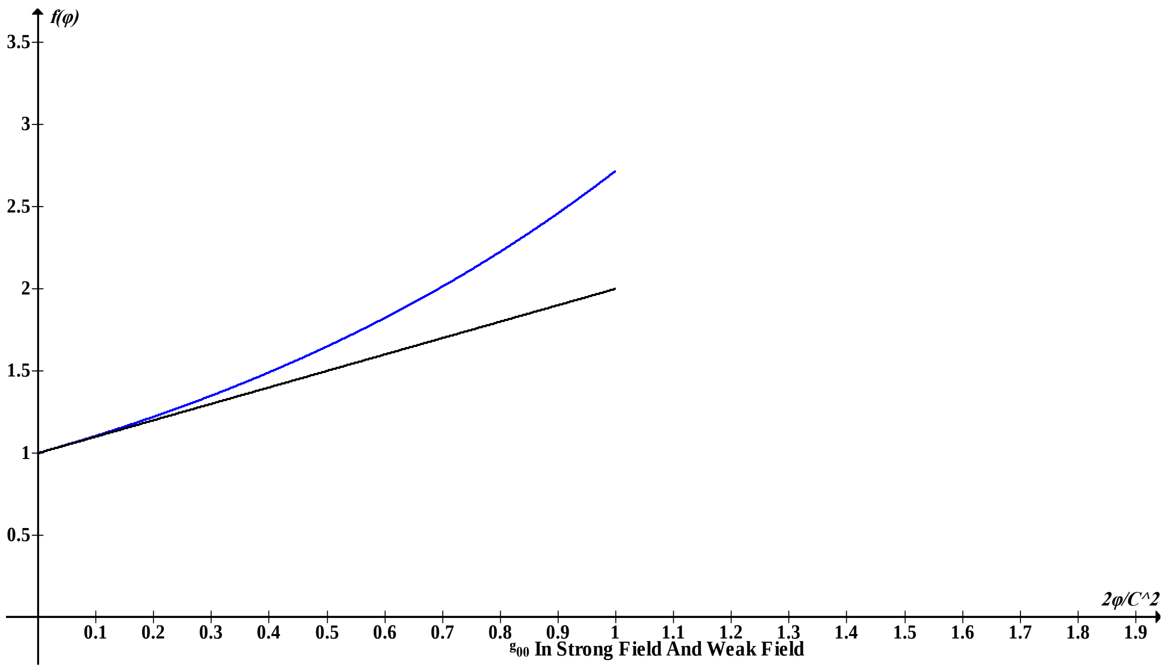
$$\sin i\theta = i\sinh\theta \ \ \& \ \cos i\theta = \cosh\theta \qquad (\mathbf{4.2.9})$$

$$\therefore C_{eff}^2 = C^2 \left(\cosh(\frac{2\varphi}{C^2}) + \sinh(\frac{2\varphi}{C^2}) \right) \quad (\mathbf{4.2.10})$$

In view of equation $(\mathbf{4.2.1} - \mathbf{4.2.10})$, it seems that g_{00} in strong field gravity takes the formula

$$g_{00} = \cosh(\frac{2\varphi}{C^2}) + \sinh(\frac{2\varphi}{C^2}) \quad (\mathbf{4.2.11})$$

It is true for any value of $\frac{2\varphi}{C^2}$, even near horizon of black hole. For $\frac{2\varphi}{C^2} \ll 1$ it becomes $\left(1 + \frac{2\varphi}{C^2}\right)$.



g_{00} In Strong Field And Weak Field

This curve show that g_{00} how change in weak field approximation in equation $(\mathbf{3.4.12})$ vs strong field derived in equation $(\mathbf{4.2.11})$. thus, g_{00}in weak field nearly linear function while in strong field is nonlinear. We prove that in equations $(\mathbf{3.5.17} - \mathbf{3.5.27})$ strong field metric goes to weak field metric in GSR for weak field according to equation $(\mathbf{3.5.12})$. one can generalize the metric tensor of GSR that describe free fall according to $(\mathbf{4.2.11})$, and $(\mathbf{3.5.17})$ as

$$ds^2 = \left(\cosh(\frac{2\varphi}{C^2}) + \sinh(\frac{2\varphi}{C^2})\right)^{-1} C^2dt^2 - \left(\cosh(\frac{2\varphi}{C^2}) + \sinh(\frac{2\varphi}{C^2})\right)^{-2} dx^2 \quad (\mathbf{4.2.12})$$

And therefor

$$\gamma = \frac{\cosh(\frac{2\varphi}{C^2}) + \sinh(\frac{2\varphi}{C^2})}{\sqrt{\cosh(\frac{2\varphi}{C^2}) + \sinh(\frac{2\varphi}{C^2}) - \frac{v^2}{C^2}}} \quad (\mathbf{4.2.13})$$

Equation $(\mathbf{4.2.13})$ can go to equation $(\mathbf{3.5.13})$ by using Taylor series representation, that are

$$\sinh x = \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!} = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots \text{ for all } x \quad (\mathbf{4.2.14})$$

$$\cosh x = \sum_{n=0}^{\infty} \frac{x^{2n}}{2n!} = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots \text{ for all } x \quad (\mathbf{4.2.15})$$

So equation $(\mathbf{4.2.13})$ goes to be

$$\gamma = \frac{1 + \frac{(\frac{2\varphi}{C^2})^2}{2!} + \frac{(\frac{2\varphi}{C^2})^4}{4!} + \cdots + (\frac{2\varphi}{C^2}) + \frac{(\frac{2\varphi}{C^2})^3}{3!} + \frac{(\frac{2\varphi}{C^2})^5}{5!} + \cdots}{\sqrt{1 + \frac{(\frac{2\varphi}{C^2})^2}{2!} + \frac{(\frac{2\varphi}{C^2})^4}{4!} + \cdots + (\frac{2\varphi}{C^2}) + \frac{(\frac{2\varphi}{C^2})^3}{3!} + \frac{(\frac{2\varphi}{C^2})^5}{5!} + \cdots - \frac{v^2}{C^2}}} \quad (\mathbf{4.2.16})$$

For weak field all terms that bigger than C^{-2} can be neglected, so equation $(\mathbf{4.2.16})$ become

$$\gamma = \frac{1 + (\frac{2\varphi}{C^2})}{\sqrt{1 + (\frac{2\varphi}{C^2}) - \frac{v^2}{C^2}}} = \frac{g_{00}}{\sqrt{g_{00} - \frac{v^2}{C^2}}} \quad (\mathbf{4.2.17})$$

And this is the same equation $(\mathbf{3.5.13})$ that related to equation $(\mathbf{3.4.12})$ throw equation $(\mathbf{3.5.12})$ as we shown in chapter three. So this led us to say that equation $(\mathbf{4.2.12})$ is general form of space-time metric that describe GSR for free falling.

4.3. Using of GSR in Strong Gravity Field to Explain the Dark Energy & Expanding of Universe)

As we shown in chapter three relative quantum mechanics equation by using the factor the relative equation $(\mathbf{3.9.1})$ that gives us the quantum formula in equation $(\mathbf{3.9.10})$. now let's start with equation $(\mathbf{4.2.13})$ and then the relative energy become

$$E = m_0 C^2 \gamma = \frac{m_0 C^2 \left(\cosh(\frac{2\varphi}{C^2}) + \sinh(\frac{2\varphi}{C^2})\right)}{\sqrt{\cosh(\frac{2\varphi}{C^2}) + \sinh(\frac{2\varphi}{C^2}) - \frac{v^2}{C^2}}} \qquad (\mathbf{4.3.1})$$

Let's for simplify denotes to $(\frac{2\varphi}{C^2}) \equiv \eta$, so

$$E = m_0 C^2 \gamma = \frac{m_0 C^2 (\cosh\eta + \sinh\eta)}{\sqrt{\cosh\eta + \sinh\eta - \frac{v^2}{C^2}}} \qquad (\mathbf{4.3.2})$$

With aid of some calculation equation $(\mathbf{4.3.2})$ becomes

$$1 = \frac{m_0 C^2 (\cosh\eta + \sinh\eta)}{\sqrt{E^2(\cosh\eta + \sinh\eta) - p^2 C^2}} \qquad (\mathbf{4.3.3})$$

Then

$$E^2(\cosh\eta + \sinh\eta) - p^2 C^2 = m_0^2 C^4 (\cosh\eta + \sinh\eta)^2 \qquad (\mathbf{4.3.4})$$

$$\therefore E^2 = m_0^2 C^4 (\cosh\eta + \sinh\eta) + \frac{p^2 C^2}{(\cosh\eta + \sinh\eta)} \qquad (\mathbf{4.3.5})$$

Now, let's substitute the operation of p^2, and E^2, since $\hat{p}^2 = -\hbar^2 \nabla^2$, $\hat{E}^{2=} -\hbar^2 \frac{\partial^2}{\partial t^2}$ yields

$$-\hbar^2 \frac{\partial^2}{\partial t^2} = m_0^2 C^4 (\cosh\eta + \sinh\eta) - \frac{C^2 \hbar^2 \nabla^2}{(\cosh\eta + \sinh\eta)} \qquad (\mathbf{4.3.6})$$

Now let's assume that $\cosh\eta \equiv \begin{vmatrix} 1 & 0 \\ 0 & \cosh\eta \end{vmatrix} = I_c$ & $\sinh\eta \equiv \begin{vmatrix} 0 & -1 \\ \sinh\eta & 0 \end{vmatrix} = I_s$ or any combined of matrixes verify that. However, equation ($\mathbf{4.3.6}$) can be written as

$$-\hbar^2 \frac{\partial^2}{\partial t^2} = m_0^2 C^4 (I_c + I_s) - \frac{C^2 \hbar^2 \nabla^2}{(I_c + I_s)} \qquad (\mathbf{4.3.7})$$

That yields

$$-\frac{(I_c + I_s)}{C^2} \frac{\partial^2}{\partial t^2} = \frac{m_0^2 C^2}{\hbar^2} (I_c + I_s)^2 - \nabla^2 \qquad (\mathbf{4.3.8})$$

In Dirac notation

$$-\frac{(I_c + I_s)}{C^2} \frac{\partial^2}{\partial t^2} |\Psi(\mathrm{t}, \bar{\mathrm{r}})\rangle + \nabla^2 |\Psi(\mathrm{t}, \bar{\mathrm{r}})\rangle - \frac{m_0^2 C^2}{\hbar^2} (I_c + I_s)^2 |\Psi(\mathrm{t}, \bar{\mathrm{r}})\rangle = 0 \qquad (\mathbf{4.3.9})$$

Now let's calculate equation ($\mathbf{4.3.9}$) for particle with mass only, that

$$\left(\frac{(I_c + I_s)}{C^2} \frac{\partial^2}{\partial t^2} |\Psi(\mathrm{t}, \bar{\mathrm{r}})\rangle \right) + \left(\frac{m_0^2 C^2}{\hbar^2} (I_c + I_s)^2 |\Psi(\mathrm{t}, \bar{\mathrm{r}})\rangle \right) = 0 \qquad (\mathbf{4.3.10})$$

This can be simplifying as

$$(I_c + I_s) \left(\frac{1}{C^2} \frac{\partial^2}{\partial t^2} |\Psi(\mathrm{t}, \bar{\mathrm{r}})\rangle + \frac{m_0^2 C^2}{\hbar^2} (I_c + I_s) |\Psi(\mathrm{t}, \bar{\mathrm{r}})\rangle \right) = 0 \qquad (\mathbf{4.3.11})$$

So since $(I_c + I_s) \neq 0$, that means

$$\left(\frac{1}{C^2} \frac{\partial^2}{\partial t^2} |\Psi(\mathrm{t}, \bar{\mathrm{r}})\rangle + \frac{m_0^2 C^2}{\hbar^2} (I_c + I_s) |\Psi(\mathrm{t}, \bar{\mathrm{r}})\rangle \right) = 0 \qquad (\mathbf{4.3.12})$$

Equation ($\mathbf{4.3.12}$) can be written as

$$\left(\frac{1}{C} \frac{\partial}{\partial t} + i \frac{m_0 C}{\hbar} \sqrt{(I_c + I_s)} \right) \left(\frac{1}{C} \frac{\partial}{\partial t} - i \frac{m_0 C}{\hbar} \sqrt{(I_c + I_s)} \right) |\Psi(\mathrm{t}, \bar{\mathrm{r}})\rangle = 0 \qquad (\mathbf{4.3.13})$$

We have here two solutions where

$$|\Psi(\mathrm{t}, \bar{\mathrm{r}})\rangle = e^{-\frac{i}{\hbar}(p.\mathrm{r} - \mathrm{Et})} \qquad (\mathbf{4.3.14})$$

- First if

$$\left(\frac{1}{C}\frac{\partial}{\partial t}+i\frac{m_0C}{\hbar}\sqrt{(I_c+I_s)}\right)|\Psi(\mathrm{t},\bar{\mathrm{r}})\rangle=0 \qquad (\mathbf{4.3.15})$$

That yields

$$\frac{i}{\hbar C}E+i\frac{m_0C}{\hbar}\sqrt{(I_c+I_s)}=0 \qquad (\mathbf{4.3.16})$$

So

$$E=-\sqrt{(I_c+I_s)}\,m_0C^2 \qquad (\mathbf{4.3.17})$$

- Second

$$\left(\frac{1}{C}\frac{\partial}{\partial t}-i\frac{m_0C}{\hbar}\sqrt{(I_c+I_s)}\right)|\Psi(\mathrm{t},\bar{\mathrm{r}})\rangle=0 \qquad (\mathbf{4.3.18})$$

Then

$$E=\sqrt{(I_c+I_s)}\,m_0C^2 \qquad (\mathbf{4.3.19})$$

These two solution denote to matter and anti-matter in gravity field. We now that from equation $(\mathbf{4.2.14},\mathbf{4.2.15})$

$$\sqrt{(I_c+I_s)}\cong\sqrt{1+(\frac{2\varphi}{C^2})} \qquad (\mathbf{4.3.20})$$

This gives the energy of matter equal to

$$E=\sqrt{1+(\frac{2\varphi}{C^2})}\,m_0C^2\cong\left(1+(\frac{\varphi}{C^2})\right)m_0C^2=m_0C^2+m_0\varphi \qquad (\mathbf{4.3.21})$$

And the energy of anti-matter equal to

$$E = -\sqrt{1 + (\frac{2\varphi}{C^2})}\, m_0 C^2 \cong -\left(1 + (\frac{\varphi}{C^2})\right) m_0 C^2$$

$$= -m_0 C^2 - m_0 \varphi \quad (\mathbf{4.3.22})$$

And the different of energy equal to

$$\Delta E = 2m_0 C^2 + 2m_0 \varphi \quad (\mathbf{4.3.23})$$

And if the matter bumped into anti-matter they should eliminate each other, this verified by the sum of equations $(\mathbf{4.3.21})$ to $(\mathbf{4.3.22})$, yields

$$E_+ + E_- = m_0 C^2 + m_0 \varphi - m_0 C^2 - m_0 \varphi = 0 \quad (\mathbf{4.3.24})$$

Equation $(\mathbf{4.3.24})$ gives big mystery problem in physics, since the theories of evolutionary the universe say that there is equal amount in the beginning of the Bing bang from matter and anti-matter, and they should have eliminated each other, so how are there a matter in the universe, unless the matter were very much than anti-matter in the beginning of Bing bang, this big mystery remains unsolved, no such theory can explain this yet. But we will give some good ideas to solve this mystery after solving the mystery of dark energy, so now let's back to equation $(\mathbf{4.3.9})$ and we will have to solved for massless particle like photons. Equation $(\mathbf{4.3.9})$ for massless particle becomes

$$\frac{(I_c + I_s)}{C^2} \frac{\partial^2}{\partial t^2} |\Psi(\mathrm{t}, \bar{\mathrm{r}})\rangle - \nabla^2 |\Psi(\mathrm{t}, \bar{\mathrm{r}})\rangle = 0 \quad (\mathbf{4.3.25})$$

So that lead us to

$$\left(\frac{\sqrt{(I_c + I_s)}}{C} \frac{\partial}{\partial t} - \nabla\right)\left(\frac{\sqrt{(I_c + I_s)}}{C} \frac{\partial}{\partial t} + \nabla\right) |\Psi(\mathrm{t}, \bar{\mathrm{r}})\rangle = 0 \quad (\mathbf{4.3.26})$$

Equation $(\mathbf{4.3.26})$ have two solutions where here

$$|\Psi(\mathrm{t},\bar{\mathrm{r}})\rangle = e^{-i(K.\mathrm{r}-\omega\mathrm{t})} \qquad (\mathbf{4.3.27})$$

- First solution, if

$$\left(\frac{\sqrt{(I_c + I_s)}}{C}\frac{\partial}{\partial t} - \nabla\right)|\Psi(\mathrm{t},\bar{\mathrm{r}})\rangle = 0 \qquad (\mathbf{4.3.28})$$

Then

$$\left(\frac{\sqrt{(I_c + I_s)}}{C} i\omega + i\mathrm{K}\right) = 0 \qquad (\mathbf{4.3.29})$$

Then

$$\omega = -\frac{KC}{\sqrt{(I_c + I_s)}} \qquad (\mathbf{4.3.30})$$

- Second solution if

$$\left(\frac{\sqrt{(I_c + I_s)}}{C}\frac{\partial}{\partial t} + \nabla\right)|\Psi(\mathrm{t},\bar{\mathrm{r}})\rangle = 0 \qquad (\mathbf{4.3.31})$$

Then

$$\left(\frac{\sqrt{(I_c + I_s)}}{C} i\omega - iK\right) = 0 \qquad (\mathbf{4.3.32})$$

That gives

$$\omega = \frac{KC}{\sqrt{(I_c + I_s)}} \qquad (\mathbf{4.3.33})$$

These two solution denote to massless particle and anti-massless particles in gravity field. now from equation $(\mathbf{4.2.14, 4.2.15})$, equation $(\mathbf{4.3.30})$ becomes

$$\omega = -\frac{KC}{\sqrt{1+(\frac{2\varphi}{C^2})}} \cong -KC\left(1-\frac{\varphi}{C^2}\right) = -KC + \frac{K\varphi}{C} \quad (\mathbf{4.3.34})$$

Also equation $(\mathbf{4.3.33})$ becomes

$$\omega = \frac{KC}{\sqrt{1+(\frac{2\varphi}{C^2})}} = KC\left(1-\frac{\varphi}{C^2}\right) = KC - \frac{K\varphi}{C} \quad (\mathbf{4.3.35})$$

And the different of energy equal to

$$\Delta\omega = 2KC - 2\frac{K\varphi}{C} \quad (\mathbf{4.3.36})$$

And since

$$\Delta E = \hbar\Delta\omega = 2pC - 2\frac{p\varphi}{C} \quad (\mathbf{4.3.37})$$

And if the massless particle bumped into anti-massless particle they should eliminate each other, this verified by the sum of equations $(\mathbf{4.3.34})$ to $(\mathbf{4.3.35})$, yields

$$\omega_+ + \omega_- \equiv E_+ + E_- = KC - \frac{K\varphi}{C} - KC + \frac{K\varphi}{C} = 0 \quad (\mathbf{4.3.38})$$

Since we assumed that

$$\varphi = -ax \quad (\mathbf{4.3.39})$$

So for particle have mass

$$\Delta E = 2m_0C^2 - 2m_0ax \quad (\mathbf{4.3.40})$$

And for massless particle

$$\Delta E = 2pC + 2\frac{pax}{C} \quad (\mathbf{4.3.41})$$

Actually this is interested result, which means that the accelerated particle for constant acceleration the energy gap is decreasing, so if we accelerate particle the gap between particle and anti-particle decreasing and this related to the increasing of probability to eliminate each other, while for accelerate massless particle the energy gap increasing and this related to decreasing of probability to eliminate each other. This maybe the key to explanation the mystery of dark energy and the addition amount of matter that does not disappear. So let's drop that to the Bing bang theory, which said that when the singularity expanding, that contains all the energy in our universe, it expanding with high acceleration and it is just pure energy, none of any familiar particles were made, just a huge amount of energy expanding, so in view of equation $(\mathbf{4.3.41})$ the energy gap increasing very much and the probability of interaction between massless and anti-massless very low, after a few time when the universe become cold to create the particle with mass and anti-mass the gap energy is very high between mass and anti-mass, so every one of these evaluated alone and create two kind of matter that energy gap between them very high and there is small probability to interacting with each other. So this equation predicts that a huge amount of anti-matter but in high level of energy, and to reach that gap we need to accelerate the particle according to equation $(\mathbf{4.3.40})$ for very long distance to decrease the gab energy. Also in other view equation $(\mathbf{4.3.40})$ deals with the pair production, that accelerate matter and anti-matter when they interacting they produce energy less than double relative energy zero by $(-2m_0ax)$. While there is no evidence for an anti-massless particle but one can relate its negative energy to the dark energy which causes the expanding of universe, thus we can assume the dark matter as the anti-massless particle. One can calculate its field from equation $(\mathbf{4.3.34})$

$$\omega_{-} = -KC + \frac{K\varphi}{C} \qquad (\mathbf{4.3.42})$$

And

$$\varphi = \frac{\omega_{-}C + KC^2}{K} \qquad (\mathbf{4.3.43})$$

It is clearly that from equations $(\mathbf{4.3.34}, \mathbf{4.3.35})$ the massless particle and anti-massless particle interacting with different way in gravity field, while massless like photon in gravity field its direction change to the source of gravity the anti-mass less does the same against the gravity source, so the curvature for photon (massless) is positive but for anti-massless is negative, this also another explanation for the expanding the universe due to existence of anti-mass particle. We treat here the case just for gravity field, it is one of four fundamental forces, so if we consider the other fields that decreasing the gap energy between matter and anti-matter, but massless and anti-massless they interacting fundamentally with gravity field force.

Chapter five

Discussion and Conclusion

5.1 Discussion

In the weak gravity field, the equivalence principle between inertia and gravity mass already verified, in view of equations $(\mathbf{3.2.5})$ to equation $(\mathbf{3.2.16})$ we found that the principle of equivalence in strong gravity field still standing according to equation $(\mathbf{2.15.1.20})$. this means GSR is follow the equivalence principle in weak and strong gravity fields. For photon (massless particle) the effect of gravity field is due to curvature of space-time according to equation $(\mathbf{3.7.19})$, which means the field deform the space-time in GSR frame work, that make Lorentz transformation take the form of equation $(\mathbf{3.7.54})$. GSR and Savickas model are related to each other in terms of equation $(\mathbf{3.5.12})$. this was done in chapter three section six, but it seems GSR more general than Savickas model. Also equation $(\mathbf{3.8.9})$ deals with the relation between velocity of accelerated particle that its initial velocity was constant and it gives two times increasing of the factor 2φ. This can be in general ideas to calculate the perihelion advance of Mercury to exact value that was calculated from GR from Schortizshield solution. Where historical, first calculate of perihelion and photon geodesic by Einstein was two time less than the correct answer that given by Einstein himself after he formed the last formula of GR, so this can be calculate by equation $(\mathbf{3.8.9})$ also. The metric tensor of GSR in weak gravity field in equation $(\mathbf{3.4.11})$ is the same matric used by Einstein to give the first value for perihelion of Mercury, which it did not give the exact value, so one can use the general form of metric tensor for GSR in equation $(\mathbf{3.5.17})$ and $(\mathbf{4.2.12})$ to calculate more accurate value for perihelion of Mercury.

The results of inserting weak GSR tensor to Schrodinger equation gives two solutions for mass and anti-mass as expected, and two solutions for massless and anti-massless particle as usually expected. The interesting results the we obtained when we apply GSR in strong gravity field equation $(\mathbf{4.3.9})$ to Schrodinger equation, in the first two solutions for mass particle and anti-mass particle the energy level was shifted both for them in same direction, but the most amazing results that for the two solutions for massless and anti-massless particle, we found that massless (like photons, gluons, and gravitons) particle interacting with gravity by changing their geodesic around the source of gravity, while anti-massless interacting with changing their geodesic opposite to the source of gravity, which means it have negative energy that not attracted but repulsion with the gravity field, this can be seen in equation $(\mathbf{4.3.34})$ and $(\mathbf{4.3.35})$. this behavior of anti-mass particle in strong gravity field maybe explain for negative energy (dark energy) that work against gravitation and causes the expanding of the universe. In the GR equation the universe is expanding, but Einstein refuse that and he but correct term called Λ to make the universe stable, later after the observations of Edwin Halley that universe actually expanding, Einstein say that he had a big mistake by adding this term, but according to all visible matter the universe should not expanding unless there is a negative energy did that, so again Λ returns to give meaning of the negative dark energy, so in view of equation $(\mathbf{4.3.34})$ shows that GR describe the behavior of mass and anti-mass particle and massless particle but it does not describe the behavior of anti-mass particle in gravity field. So by combing of GSR in quantum mechanics we found that from solutions that anti-massless particle maybe one reasons of expanding the universe. Also the equations give way to produce spontaneously the mass and anti-mass just by accelerate the mass to decreasing the gab energy between mass and anti-mass particle, that actually one of the mains ways used in the accelerator like CERN to produce the anti-mass particle by accelerate the particle, so to produce the anti-massless particle we need a field gravity accelerator to accelerate the massless into high amount of gravitation field. This can be happening near black hole when the massless (photon) trapped in orbital around the black hole, so should produce anti-massless with negative energy repulsing from the field black hole, this can be related to Hawking' radiation **[31,32]**.

5.2 Conclusion

Quantum mechanics with frame work of GSR in weak and strong field can be the first step to form quantum gravity theory. It gives an excellent explanations and expectations for many phenomena, which means that GR and GSR is not with conflict in principle with the quantum theory.

5.3 Recommendation

1. The use of GSR with quantum mechanics needs more study in the field of Dirac quantum formula to include the particle with spin $\frac{1}{2}$.
2. The metric tensor in strong field derived from GSR needs more investigations. It can be used to calculate the perihelion of plants as Schortizshield solutions.
3. That anti-massless particles that accelerated with strong gravity field near black holes needs more concentrating, it may be the key to explain the dark energy and dark matter and expanding the universe.

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