

## **Dedication**

***This research is dedicated with love and  
affection  
To my parents,  
my brothers and my sister***

## **Acknowledgements**

Weeping may endure for a night but joy comes in the morning. I thank the Almighty God for His unprecedented love and grace upon me. I would like to express my sincere gratitude to the Sudan university of science and technology (SUST) for granting me this wonderful opportunity to do a high graduate studies (MSC) in mathematical sciences. A million thanks to Academy of Engineering Science for making Academy of Engineering Science a global reality. My loving kindness goes to my parents, brothers and my sister. I would like to give a special appreciation to my supervisor, **Prof. Mohammed Ali Basheir**, for his steadfast love, guidance and support in making this research a success. I love you all and I pray that you guys will always be there for me. One love!!

## **Abstract**

In this research we discussed the application of differential form to Maxwell's equations.

Maxwell's equations, which depict classical electromagnetic theory, are pulled apart and brought together into a modern language of differential geometry. A background of vector fields and differential forms on a manifold is introduced, as well as the Hodge star operator, which eventually lead to the success of rewriting Maxwell's equations in terms of differential forms. In order to appreciate the beauty of differential forms, we first review these equations in covariant form which are shown afterwards to be consistent with the differential forms when expressed explicitly in terms of components. I declaration , the undersigned, hereby declare that the work contained in this research is my original work, and that any work done by others or by myself previously has been acknowledged and referenced accordingly.

## الخلاصة

فى هذا البحث ناقشنا تطبيق الصيغ التفاضلية لمعادلات ماكسويل.

معادلات ماكسويل وهى المعادلات التى تصور النظرية الكهرومغناطيسية الكلاسيكية التى جمعت فى لغة حديثة وهى الهندسة التفاضلية، عرضنا خلفية عن فضاءات المتجه والصيغ التفاضلية وكذلك مؤثر هودج والتى تؤدى فى النهاية الى نجاح إعادة صياغة معادلات ماكسويل فى صورة صيغ تفاضلية. اولاً استعرضنا هذه المعادلات فى شكل متغير مشارك والتى تظهر بعد ذلك لتكون متنسقة مع الصيغ التفاضلية من حيث المركبات. انا الموقع ادناه اعلن ان العمل الوارد فى هذا البحث هو عملى وان اى عمل قمت به او قام به اخرون معترف به ومشار إليه.

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