Sudan University of Science and Technology Post Graduate Studies

The Relationship between Visual Illusion and Landscape Drawing

العلاقة بين الإيهام البصري والمنظور في رسم أو تصوير المشهد الطبيعي

Thesis Summitted for (PHD) in Fine Art (Painting)

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Honorable Grants

To our old masters those who show us the way to differentiate between Knowledge and science. Those old masters. Descending from profit Mohamed downed to his grandsons such as Gaffer bin Mohamed Albggir Elhazn and Al Masoodi. . To all those brilliants I offer this humble contribution to be between their hands

I'm hounarable to offer the findings of this study to my sincere master Sheikh Noureldeen Elsheikh Ahmed who had helped and supported me from the early stages of this study and continued his support up to the fulfillment of my research goals.

Compliments

First of all thanks to Allah Subhanaho wa taala for giving me the strength and knowledge to fulfill my ambition for doing this research.

I would like to send my sincere thanks and gratitude's to everyone who helped me to achieve this research, my thanks and gratitude to professor Chris Johnson the head of the Institute of the Ophthalmology, University of Iowa, and Professor Buggies from the Department of Physics, for their help and support when I started this research at the University of Aiwa USA. thanks also extends to professor Malik Hussain, Dr. Abu bakr Alhadi , Eng.Salah El-khair, and my cousins Hafiz Ali, Gasmallah Ali & Wigdan Ali.for their support

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Abstract

The geometry of linear perspective of landscape drawing and rendering remains one of the long standing scientific problem that has not been solved because its experimental behavior lacks theoretical explanation as a visual phenomenon. This study suggests the use of Ophthalmology and Geometrical Optics to explain the real geometry of linear perspective. It utilizes eye geometry to answer the question: why seen objects vanish in/around the horizon line? Or the question; what do eyes do to maintain the vanishing attitude. For answering these questions, the geometry of optics should be put to a comparative analytical research under a hypothesis that says: there is a different geometry of visual optics than both the geometric performance of linear perspective and the nowadays believes about visual optics.

The new visual geometry is based on eye contribution of a shape of a pyramid or a cone of visual rays which are responsible for the vanishing attitude on the horizon.

The eye contributes four cones or pyramids of visions that stand with the seen figure down to the retina of the eye up to the horizon.

مستخلص الدراسة

يظل الرسم الهندسي للمنظور الخطي والذي يختص برسم المناظير العامة أو مقاربات الرسم المعماري يشكل معضلة علمية ظلت باقية دون أي حلول ومعالجات وذلك نتيجة لسلوكها التجريبي الذي يفتقر لمفاهيم نظرية بوصفه ظاهرة طبيعية تتصل بالخبرة البصرية.

تقترح هذه الدراسة استخدام علميّ معارف علم طب العيون وفيزياء البصريات وذلك لتوضح الهندسة الحقيقية للمنظور الخطي.

وهي تستخدم العين للاجابة عن السؤال : لماذا تتلاشى الاجسام المبصره في / أو حوالي خط الأفق أو بصورة أخرى السؤال عم تقوم به العين ممايؤدي الى ظاهرة التلاشي البصري.

وللاجابة عن هذه الاسئلة ، ينبغي وضع ماهو معلوم حالياً من فيزياء البصريات أمام مقاربة تحليلية لبحوث حيث يفترض أن تكون موضوعة تحت فرضيات وهي تقول أن هنالك اختلاف في هندسة البصريات في كلاهما.

اضافةً أن هنالك جزء آخر نستخدم فيه المنهجية التاريخية التحليلية وذلك بالنظر فيما يخص بتجربة الفنون والرسم المعماري وتجربة تاريخ الفنون كي يتم اكمال نقص الدراسة العملي خاصة بعد أن جمعنا الجزء النظري في المنهجيتين المعمليتين أعلاهما.

وتتوصل الدراسة في نهاية أمرها الى رسم هندسة بصريات جديدة لاول مرة في تاريخ علوم البصريات في مختلف اجزاءه العلمية المعملية أو الفنية وذلك بصياغة شبكة بترتيب خاص تبعث به العين وبها وحدها تتمكن من إلتقاط الصور.

وتستند الهندسة البصرية الجديدة على مساهمة العين في شكل هرم أو مخروط من الأشعة البصرية المسؤولة عن موقف التلاشي في الأفق.

تساهم العين بأربعة مخاريط أو أهرامات رؤى تقف مع الشكل المشاهد وصولا إلى شبكية العين **حتى الأفق**.

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1. Terminologies and Keywords

Keywords:

Geometrical optics

Orthogonal lines

Vanishing point

Opaque surface

Visual perception

Pyramid of vision

Chapter One

Introduction

1/1 Introduction , the proposal & literature review

Why do we see objects vanish in vanishing points on the horizon? If we know that the vanishing and the horizon line phenomena are just illusions that do not really exist, what do eyes do to formulate these optical illusions?

These are some of the why-of-vision questions which were asked within the study of perspective and persisted for more than five hundred years, since perspective knowledge was established in the fifteenth century, without answers.

Visual perception which is a modern complicated term in visual art studies, is considered to cover many branches of art and design by focusing on aspects of knowledge that deal with visual images. Visual perception studies include Painting, Drawing, The History of Art, Architecture, Sculpture, Cinema, Theatre and many other minor fields that apply art and design such as graphics, fashion and industrial design.etc. The term 'visual perception' did not exist out of a sudden invention of the last decades of the 20th century. It is a result of a long-term process through the human practice of vision. In its conceptual form, it was well known for hundreds of centuries before the actual term appeared. The dialogue between the practice of vision and visual knowledge is as old as the time when humans were not distinguished from other mammals. They looked, gazed, learned and benefited from the services of their efficient vision tools. Since then these humans were too satisfied with their vision tool's services to pose any sort of questions about its efficiency. But, unlike contemporary scientists who feel free to say that whenever prehistoric animal hunters needed to cooperate to hunt, the hunters needed to invent a language to support their activity, they did that when they presented their hunting activity in paintings on their cave

walls. Thus, academic scholars can say that when the cave paintings appeared, it was

the beginning of a long-term process, leading to the question about the logic and the grammar of vision.

Therefore, it was visual art that became the activity through which humanity had the ability to pose questions about visual perception, especially when simulating drawn and painted shapes and pictures to natural ones. Looking decisively at this early beginning of visual art can lead us to see how cave painting and the design of stone tools were the earliest human work that deserves to be called human creativity. This creativity has to be classified as other than the natural work of hunting and food gathering. Related to this concept it is so easy to say that visual art's long process of practicing painting or sculpture or tool design, from cave painting up to modern art, was the main historical factor that established the triumph of the experimental mind.

Linear perspective is an artistic knowledge that had aroused and spread within the environment of Drawing, Rendering and Architectural Presentation. It is a method used in depicting what was usually seen in landscapes and other objects that fall within the visual scope.

Exploring or inventing Linear Perspective was not more than an incident that had taken place in the year 1425 AC, at Florence of Italy. That incident was considered as a huge transition that has lead Italy, and then the rest of the European world, towards historic achievement of their Renaissance. That incident was the discovery, or the creation of linear perspective. The direct cause of the consideration of that incident had depended conceptually on a fact that the discovery or creation of linear perspective could not be considered only as an innovation of art, whatever might be the consideration of Art effect in the European civilization, but for Art, then, was not distinguished from other members

of the world of science and knowledge. Artists of that time were Mathematicians, Architects, Astronomers and Artists in the same time.

So, it could not be said, but perspective was an innovation of the scientific mind of that time.

Historically there were three epochs of art styles in Drawing and Painting had taken place. Two of them were before the European Renaissance, the third came after. Those three epochs which tell the story of the development of art styles in their different ages of drawing styles came as follows:

1/ the old Egyptian and the Babylonian Art

2/ the Greek up to Medieval Art

3/ the European Renaissance up to Modern Art

These three ages of Art split the history of art into three main streams of development of art styles on its way to obtain the most life-like performance of drawing styles. The two earlier seemed to lack this symptom due to the shortage of experience that hadn't been achieved then. Their ways of drawing were very primitive specially when expressing depth or the third dimension.

The Egyptian and the Babylonian drawings had presented flatness without any effect of depth or forms. Specials selected features of frontals and profile poses were mixed so as to cook the nearest appearance to natural shapes and forms of figures.

Thereafter, some expressions of forms had appeared in the Greek and the Christian art, but there hadn't been any knowledge achieved about the third dimension in picture making.

Subsequently, the big change between those two ages and the one which came after, The Renaissance was the achievement of the knowledge of how to express depth in pictures, or in other words; how to create the third dimension which expresses forms, voids and volumes of objects.

This achievement had happened in the Italian Renaissance within the experimental traditions in the 15th and the 16th centuries between Florence where perspective was invented and Rome where it was accepted and experienced.

Then it is quite natural for perspective to grow up affected by the environment within which it had been born and raised, the reason that it had been split into two characters. The first was specified as projection, the second was its consideration as a member of the family of Mathematics. Actually the splitting case suits the European environment of that time when and where the experimental mind had begun to grow, at the same time when and where the European Renaissance had been born.

Within such environment of the growth of the experimental mind, there were some knowledge which was accepted before, can hardly be considered under the name “science" such as “Euclidian Optics” which had decided that all visuals were done as a result of projection of rays moving from the eye. Then the first character was projection taken from the environment of Euclidian Geometry.

Queen Nefertiti

A sample of the law of frontalaty style which is how the old Egyptian draw the human figure. (Figure 1)

Resourrce:https://images.fineartamerica.comthe-ancient-egyptian-goddess-isis-leading-queen-nefertari-ben-morales-correa

This style is based on two poses profile from the head and the legs frontal for the chest and the hand.

1/2 The Proposal:

1/2/1 The Research Problem

It is clear that we face contradiction between the traditional geometric solutions of linear perspective that refers to the experimental background, and the natural optics of vision, which is to some extent theoretically and practically stable.

1/2/2 The Problem Statement

a) Actually, perspective was not just an initiation of knowledge that helped architects and painters to practice depth and forms in drawings and present vanishing attitude of figures, but a revolutionary experimental movement of the scientific mind which was lead by the explorers of the third dimension, to explore the practice of descriptive geometry and projective geometry. Despite all of these achievements, perspective did not cross the borders of knowledge and entered the world of science, because of some scientific complexities and mysteries that surround its inventors experimental attitude. This situation may stand as a reason of its lack of theoretical explanation and its inability to face inquiries like what we have asked in the above introduction.

b) Anyway, whatever might it acts theoretically or practically, it was clear that the two fields, perspective geometric solutions and visual observation of landscapes, were not working cooperatively, but performing contradictory bases that leads to a scientific conflict. The reason of this scientific conflict was because of the geometric formula of linear perspective bases that had been built on the concepts of the Euclidian geometry and the Euclidian theory of light transformation, which were out of date scientifically.

c) Therefore, the geometric solutions of the Renaissance Masters might be accepted as an achievement done due to the scientific mentality

of their time. But it would not be the same to nowadays mentality, because it will not be accepted since the time that Alhazen had corrected the theory of light transformation, and changed the older theory about the function of the eye that became a light absorbent tool not a producer of visual rays as it was believed in the Euclidian theory of light transformation.

d) Consequently teaching linear perspective in its geometric form has become a complicated job, not easily explained and educated. This fact in particular caused a big turnover that contradicts the teaching process of geometric linear perspective which was supposed to be done and practiced manually.

The substitute to that was a new trend that uses computer based process as a new tool. In fact it is easier for designer to use the computer based tool, because it gives a readymade landscape that saves time and effort of the designer. But for students it differs so much, because they are still in need of a tool that emphasizes their designing abilities. As a result, students are actually in need of the manual process which they miss because they were taught only the new computer based techniques.

e) Thereafter, missing the tool that helps in formulating designing skills was not the sole problem we should face. We are faced nowadays by the problem of losing skilful teachers, lecturers and supervisors who can teach or lead research on solving the scientific problems of this significant designing tool.

1 / 2 /3 Research Goals and Objectives:

This research aims at emphasizing the usage of the manual tools of drawing linear perspective, as an added value that improves design ability in Arts education.

It is well known that the environment of fine arts and design is quite separate from the environment of scientific experimental work.

The reach objectives are as follow:

- To cross the border of improving the scientific skills for perspective study
- To train the students to follow their unanswered questions about the visual phenomenon of visual perception
- To give the students the ability and knowledge of using both the theoretical and practical parts of linear perspective
- To show the contribution of artist in solving problems in Ophthalmology and Optics.

1 / 2 /4 Research Methodology:

We recognize that there are two streams that divide the field of this study between them. the first one is the field of art which is usually the study of linear perspective finds itself within its environment , further more the methodology of research which we have to keep working with it for this part should be the historical analytical method of research .

We know however, that the word of perspective can't be said only from the world of Art . another fields should have be considered such as Ophthalmology & Optics . we know that the tow fields added later are fields within the empirical science environment that uses lab rotary & it ,s experimental engine. For this reason there will be another part of the mythology of research for this study which we can call empirical exploring analytical method of research.

To conclude the case of the methodology of research of this study we can say it consists of two parts:

1/ empirical analytical that deal with Ophthalmology & Optics part of the research.

2 / historical analytical research methodology that deals with the part of art & design.

1 /2/5 Main Research Question:

Is it true that a drawn on normal paper size landscapes following the rules of the living geometric linear perspective can neither represent the real scenery of this landscape, nor provide the real grammar of how its figures vanish?

If the answer is no, other questions need to be answered within this study. These questions as follows:

Why and how do objects vanish visually?

What do eyes do to obtain this vanishing attitude?

1 /2 / 6 Hypothesis of the Research

For the research questions were asked above, there are assumed facts that represent their answers. These assumptions will be considered as hypothesis of this research study. They are as follows:

The geometric linear perspective living practice are/is quite different from the actual geometry of vision.

1 /2/7 Research Obstacle

Two main obstacles had faced this research study, one of them is objective, the other one is subjective.

as for the first one the researcher faced a case of total blindness . This had happened in the first months of 1016 after the researcher had complete collecting his data & began writing his thesis. After that the researcher had struggle very hard to write his first paper & to make his three seminars which was very successful after that he tried to seek help to

complete the writing of the thesis which had taken a long time with the achievement which is demonstrated on this research.

The second one deals with the research of linear perspective. In itself four or five art historians is the only ones whom they were encourage enough to write about the linear perspective. Even those five art

historians they didn't explain the theory of vanishing attitude but they have just shown what the old masters have done.

We assure that all what was explained by those art historians are problematic experiences put the linear perspective outside the world of science up to our recent time. so, we can say there is no text which can participate in any discussion especially when maintaining the problematic linear perspective as we proposed here down this research

1/ 3 Literature Review

1/3 /1 the Literature Review

a. It appears that the scientific conflict faces those who want to pose any theoretical face to linear perspective if they tried to do that while they depend on its Renaissance geometric formula. The following texts were supposed to reflect the living scientific references that explain perspective, but unfortunately they were instead replicating the conflict. These texts are:

1: The first text is taken from a short review by Dr. Kim H. Veltman, the Scientific Director of the Maastricht McLuhan Institute. He wrote a very short but effective critique of Dr. S. Edgerton, who contributed a book on perspective titled (The Renaissance Rediscovery of Linear Perspective.) S. Edgerton Jr, (July 1977), the Renaissance Rediscovery of Linear Perspective, The Art Bulletin, New York, Vol. 59, No. 2, (pp.281-282). The importance of Dr Veltman's short quotation is that he put his finger on the spot of the confusion.

He stated: "The reason for Edgerton's ongoing confusion is quite obvious. He does not distinguish clearly between the objective relationship that linear perspective establishes with the measured world and the subjective interpretations of visual perception-let alone make more subtle distinctions, as has Gombrich, between perspective relating to the 'what' but not the 'how' of vision.".

2: The second text is taken from Professor Martin Kemp's book (The Science of Art; Optical Themes in Western Art from Brunelleschi to Seurat.) Martin Kemp, (1990) " The Science of Art" Yale University Press, New Haven and London, , P334

It appears that Professor Kemp was aware enough of the conflict about the how or what of vision. His awareness however, did not lead him to maintain its disorder, but he just wrote: "The first and most historically orthodox of these questions concern the explanations as to why there should be so much shared ground between visual art and optical science in this particular period. In other words, what explanatory causes can be assembled? The second question concerns the status of the optical 'truth' with which our predominantly naturalist art has been concerned. On the surface, this question is not historical, but cannot be disentangled from interpretation of the history, since our view of the visual status of the techniques will radically affect where we look for our historical explanation. If, for instance, we believe that orthodox perspective is not more or less than an artificial convention based on a manner of 'seeing' peculiar to a particular period, we will formulate a different kind of explanatory model than if we believe that it stands in some privileged relationship how the world is 'really seen 'and that it was, like the law of gravitation, waiting to be discovered."

3: The third text refers to Professor Joseph W. Dauben of Columbia University, USA, who wrote a book and added an educational film

explaining how mathematics was the scientific roots of linear perspective/ Joseph W. Dauben, The Art of Renaissance Science

(www.crs4.it/Ars/arshtml/arch1.html He passed on the same confusion that existed in the minds of Renaissance masters instead of explaining the mathematical roots of perspective. He said: "We don't know how far Brunelleschi intellectualized his system in mathematical terms, but it was not long before someone did. In 1435, Leon Battista Alberti, another architect, published a treatise on perspective (Della Pitture) in 1435. Once Alberti's treatise was published, knowledge of perspective no longer had to be passed on by word of mouth." But when we study Alberti's treatise, we can actually discover a scientific simplicity leading us to assume that Prof. Dauben did not actually study or even understand Alberti's treatise when he said: "Once Alberti's treatise was published, knowledge of perspective no longer had to be passed on by word of mouth." Or when he said: "For some it became a matter of consuming artistic, even philosophical interest" because Alberti did not write in his treatise any explanations theoretically or practically concerning perspective, except a little quotation about "the far point" which had been referred to after as "the vanishing point." Actually when studying Professor Dauban's explanations, we recognize that he did not realize that Brunelleschi or Alberti were not conscious of the concept of visual vanishing so as to state the rules of perspective. If not so, Professor Dauban should have been aware of a historical understanding of the new visual vanishing phenomenon or even the knowledge of drawing three dimensional or performing depth in pictures.

4:The fourth text was taken from a mathematics book titled (Introduction to Projective Geometry) by Professor C. R. Waylie, the head of the Mathematics Department at the University of Utah, USA.) C. R. WaylieJr Introduction to Projective Geometry McGraw hill book company , New

York He was not only trying to criticize the geometric form of perspective, but he was actually trying to give reasons why mathematicians do not think the same way about the geometric suggestions of perspective. Indirectly he stated that the geometric suggestions made for perspective were not real; or in other words, they are not natural. He stated, "Presumably, the scene in which an artist is interested always lies on the opposite side of the picture plane from the viewing point. Hence, the picture itself always lies in the half of the picture plane which is on the same side of the object plane as the viewing point. However, in the mathematics discussion of perspective such restrictions are unnecessary and unnatural, and shall assume that our transformations extend over the entire object and image planes."

5. The fifth text seems to be touching other sides of linear perspective. The Professor of architecture of the University of Dundee Dr. Lornes Holms discussed the dialogue between the subjectivity of the psychoanalysis and the objectivity of architecture. He researched the relation between space and psyche. He suggested that Brunelleschi's invention is a compared conception to psychoanalysis done by Lacan. He also touched the living scientific problem when he posted a question asking: 'seeing through what.

1) Lorens Holm, "Brunelleschi, Lacan, Le Corbusier" Ruotledge Taylor &Francis Group, London and New York, page 37. 2010, page 37.

so as to investigate Albrecht Durer's quotation: "Perspective is a Latin word which means 'seeing through.' Ibid, page 37 then he continued in exploring different answers of different way of seeing and thinking when he said: "Theoreticians and practitioners of perspective like Alberti or Leonardo said like seeing through window. Alberti first theorized the perspective image as a planar cross section through the pyramid of vision. A physiologist might reply: seeing through the eye. S/he is attentive to the

way optics and biology intersect at the cornea, and probably is not worried about the fact that the retina images is an image to everyone except the person upon whose retina it is projected. And those of us who are attentive to the conundrum of subjectivity might say: seeing through the subject, an answer that is both more or less obvious, more or less plagued by problems of self-reference. Ibid page 37

a. The painter David Hockney and the professor of architecture of University College of London Professor Philip Steadman addressed neither the geometric linear perspective nor the objectivity of its missed visual perception, but both of them being fully aware of styles and skills of drawing and painting, they put what was thought to be mathematical experiments of the Renaissance masters to careful studies and investigation. These investigations led them to state an amazing result, that the Renaissance masters were drafting their drawings by using a camera obscura. This statement unconstrained the fact that art historians' contribution assumed to be misleading the point, caused by their judgment to the case from outside its technical field. More of that these two scholars had opened the door wide to huge size assumptions that the linear perspective:

b. as some mathematician had stated, Ibid, page 37.has nothing to do with the field of mathematics' studies.

c. is not an invention but a randomly gained knowledge.

d. is not a projection which was thought to be a concern of visual rays while it is not, but a practitioner mentality's terminology that refer to the camera obscura.

1/3/2-Analysis of the Literature Review

Perspective, as an invented knowledge, helped painters and architects to draw their landscapes and to be the leaders of one of the main cultural trends of the twentieth century named 'visual perception.' Despite that,

perspective did not earn the capability of crossing the borders of the field of knowledge to earn a seat in the field of science, and be able to answer questions, such as what were asked above, about its identity. One of the main reasons for this shortage is its lack of roots. Art historians assume that perspective was born within the family of mathematics, but mathematic professors neglect this assumption saying that perspective has nothing to do with mathematics, because it has no law or theory or even an equation. They support their argument by saying that if perspective is a branch of mathematics, it should have been taught by the professors of mathematics, but it is usually taught by the professors of painting, drawing and architecture. Omer Elamin Ahmed, (200) " Perspective between Theory & Practice", MA Thesis, College of Fine & Applied Art, U of Sudan,. Such a disorder of roots can be taken as a reason for the limitation of perspective within the borders of knowledge, but that is not the case, it is only the tip of the iceberg, for just mentioning the lack of theory may point at a deeper scientific conflict more than a shortage.

All art historians recognized when dealing with the lack of naturalism in painting before the Italian Renaissance, that the simplicity of the older mind controlled the situation. It was easier for artists before the Italian Renaissance to achieve natural simulation of shapes in sculpture where sculptors could be supported by their direct sensation of touching and measuring forms, but it was too complicated to practice that in drawing or painting. The impact of visual perception such as perspective and fore \shortening had not yet been absorbed. In other words, there was a lack of practicing depth and of feeling the third dimension in picture making; only flatness or a combination of easily drawn poses of some selected frontals or profiles of the human body or portraits in painting and drawing were the solutions to overcome the problem of comparing the

really seen visually and the drawn shapes. The lack of perspective was controlling the situation since the Egyptian and Babylonian down to the medieval art. It was a time span of 5,400 years of experimenting with flatness and formality in painting and drawing which enabled artists to cross the borders of absorbing the ABC is of foreshortening and depth instead of flatness, and then be able to practice the linear perspective in Florence in 1425 AC with the newly born experimental mind. So it was again visual art that opened the door for the second time for the human experimental mind.

e. Followed by the School of Alexandria in 200 BC, the Athens School put visual knowledge in the form of scientific study for the first time in human history. Two of the schools' masters, Aristotle and then Euclid, made two assumptions about the transformation of visual rays from the eye. These assumptions, as they were stated by most art historians, initiated the knowledge which was developed in the European Renaissance by Brunelleschi, Alberti, Albert Durer and Leonardo Da Vinci, to what was known as the mathematical form of linear perspective. From that time up to the twentieth century, following historians' statements, perspective was presented as mathematical knowledge depending on Euclidean geometry.

f. A historical void should have to take its time to formulate and prepare the upcoming experimental mind to be ready for great changes in human history. Twentieth century historians and scientists noted these changes and considered them as a declaration of the triumph of the scientific mind in its struggle to exist. Here we can come to an agreement about the fact that linear perspective was not only a landmark on the long track of natural simulation in picture making, but aside from that it can be considered as a step of the scientific mind to maintain the experimental attitude at first, and after that the door will open wide towards

understanding the logic and grammar of living phenomena such as perspective. This agreement will not lessen consideration of the geometric suggestions as a well organized, brilliant, suitable and efficient method to do the simulation process by the mentality of that time. Subsequent to that, the next five centuries of the practicing of the scientific mind may not be totally free of time voids essential for more experimenting. For the scientific mind should have to look back and retrace its footprints to see if it had or had not corrected some misleading steps which were taken during the 350 hundred centuries of practice towards its triumph. Moreover, the contemporary situation of the non rooted perspective may strengthen the debaters with the opinion of the relative mentality, because of the said scientific conflict that faces those who want to justify these geometric suggestions in a scientific order. The seriousness of this conflict is its automatic reaction against posing theory out of the geometric roots of perspective due to the cause of an unanswered methodological question: Was perspective an invention, as Professor Martin Kamp has stated directly, or was it just a discovery leading to answer the why question of vision, as some other historians noted debating the invention opinion? Marin Kemp, (1990)"The Science of Art, Optical Themes in Western Art…" Yale University Press, p 09. Without giving a direct answer to this question, it will be impossible to accept the logic of either side. It seems unreasonable to take one side's answer to the question and neglect the other's opinion, because when speaking about the suggested geometric roots, perspective was an invention, but when we realize that seen pictures exist as a result of visual observation, we will be asked to accept the word "invention" cautiously, for perspective should hence be treated as a discovery of living visual phenomena more than an invention.

g. Thereafter another question will appear automatically, emphasizing the importance of the scientific mind retracing footprints: Can we then define those Renaissance masters' efforts as research about visual phenomena? The direct answer will be no; they actually posed a geometric method to draw simulated shapes to visual illusion.

Chapter Two:

Linear Perspective:

Is a method of drawing used by artist and Architectural designers to draw their landscape drawing or to depict their drawn figures or to rendered their suggested designs for their suggested building.

It consist of a way to trace what eyes do to form the illusion on which pictures depend to formulated their seen shape, the illusion done by the eyes consist of a process of decreasing value of shapes, whenever shapes has gone far from the eyeuntil the shape vanishes at the horizon.

This Illusion is been taken as a visual phenomenon called the vanishing attitude of shapes. So we can say as a conclusion for this case that linear perspective is an outline drawing that depicts the phenomenon of vanishing landscapes and Architectural presentation. (See figure ??)

As we have stated before that linear perspective had been born in the environment of Euclidean geometry in The Renascence time in Italy, where the geometric solution has taken place and became the way to solve the problem of linear perspective instead of visual perception.

In this case projection of a simulation of rays not like the visual rays is been set to calculate the way they can pose a simulated drawing to the need to draw shapes.

This geometric method was shaped on suggested projectile orthogonal lines which represent the visual rays of vision. When drawing a picture, these orthogonal lines move from the eyes of the beholder parallel to the ground level and then change their direction from the ground level upright, rising vertically towards the picture plane, passing through its horizontal lower line which is sited over the ground level. Then using the vanishing point which is positioned in the conjunction of the central vertical line and the horizon line, and after using an elevation and a plan to maintain the measures of e.g. a building, its landscape can be drawn,

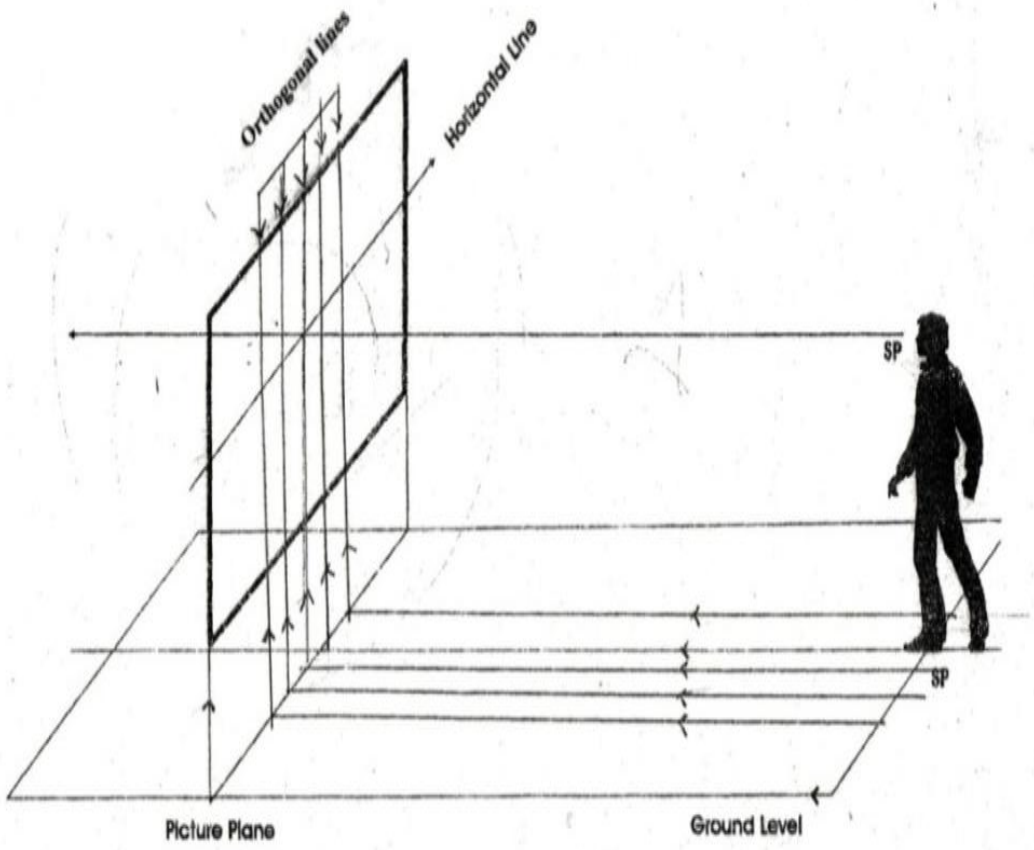

Orthogonal lines
Horizontal Line
SP
SP
Picture Plane
Ground Level

Figure (2-1)

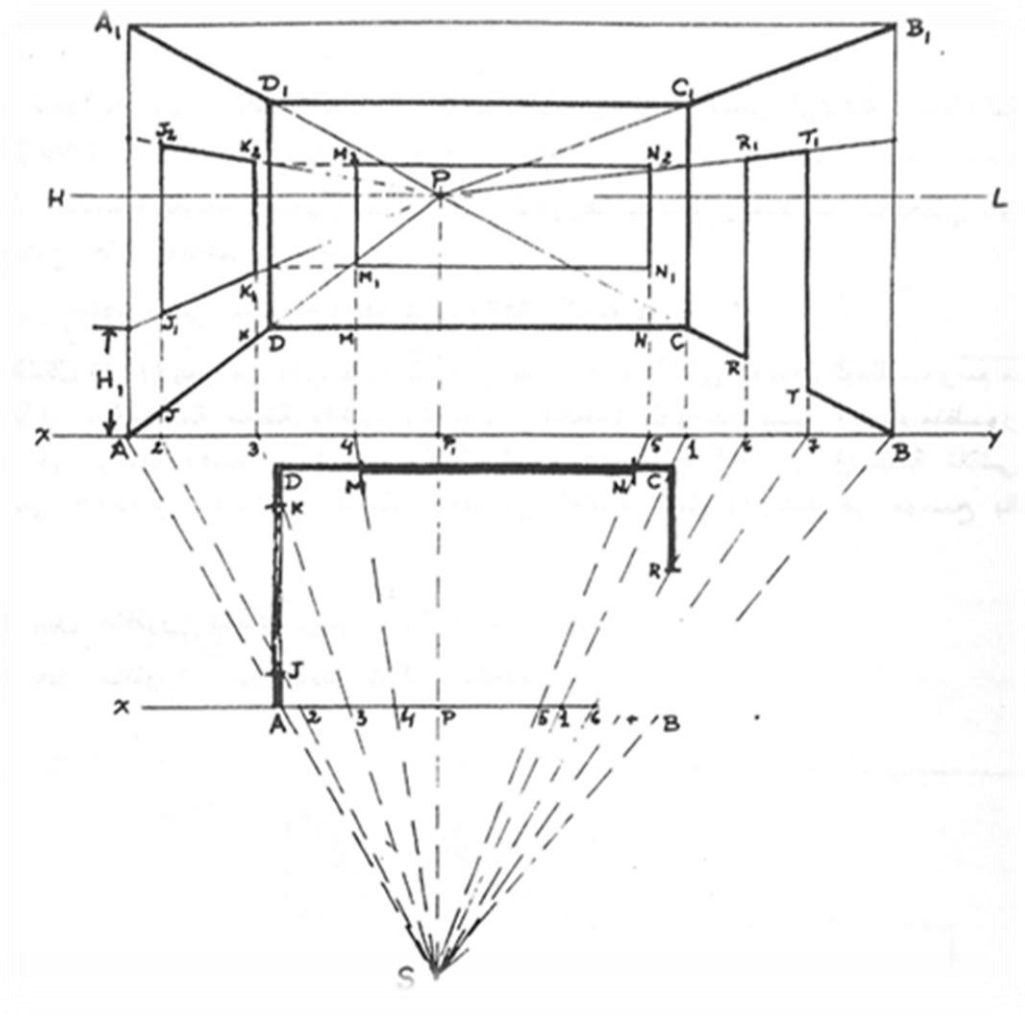


Figure (2-2)

The orthogonal lines in figure 2-1 move parallel to the picture horizontal lines right angle over the ground level to formulate a process of projected rays as in figure 2-2 that changes two dimensional shapes into 3 dimensional shapes.

Drawing by the researcher

Fig. 1.2 Woodcut from Dürer's "A Treatise on Measurement."

Figure (3)

An experiment of the renaissance optics shown on wood cut, mostly showing Albert Durer and Leonardo Davinci. Resource C.R. Wayliejr (Introduction to Projective Geometry, McGraw hill book company, New York.)

Chapter Three:

The Eye Anatomy and Function :

3/1/1 The Eye Anatomy and Function:

We recognize that the study cases divided into two main fields of study:

The first one is the field of geometric linear perspective that does not act as representative of neither visual phenomenon nor yields to analytical enquiry to find and pose its theoretical performance.

The second field refers to the eyes contribution that can be indicated by the term: visual perception, another feature may appear when putting this field to investigation. We recognize here that there are two factors facing each other as active in research. The first one is the physiology of the eye; the other one is the vanishing attitude. The first one is the tool of production, while the other one is the product. What we need to explain is the process that takes place between the producer and the product.

3/1/2 The Anatomy and Physiology of the Eye

A step back is needed for ensuring the validity of existing ideas and beliefs about the actual geometry of visual perception. The importance of the availability of those ideas and beliefs is that: they are needed to be put under analytical procedure.

Consequently, the dialogue between the producer and the product forwards the coming question: what are the eyes and what do they do to maintain their activity?

As for the anatomical side of the human eye, we all, happily, agree with what was given scientifically as anatomy of the eye. In addition, we agree with the eye's dimensions presented by the American Academy of Ophthalmology in their book Clinical Optics 2010-2012 page 104 shown below as figures (4).

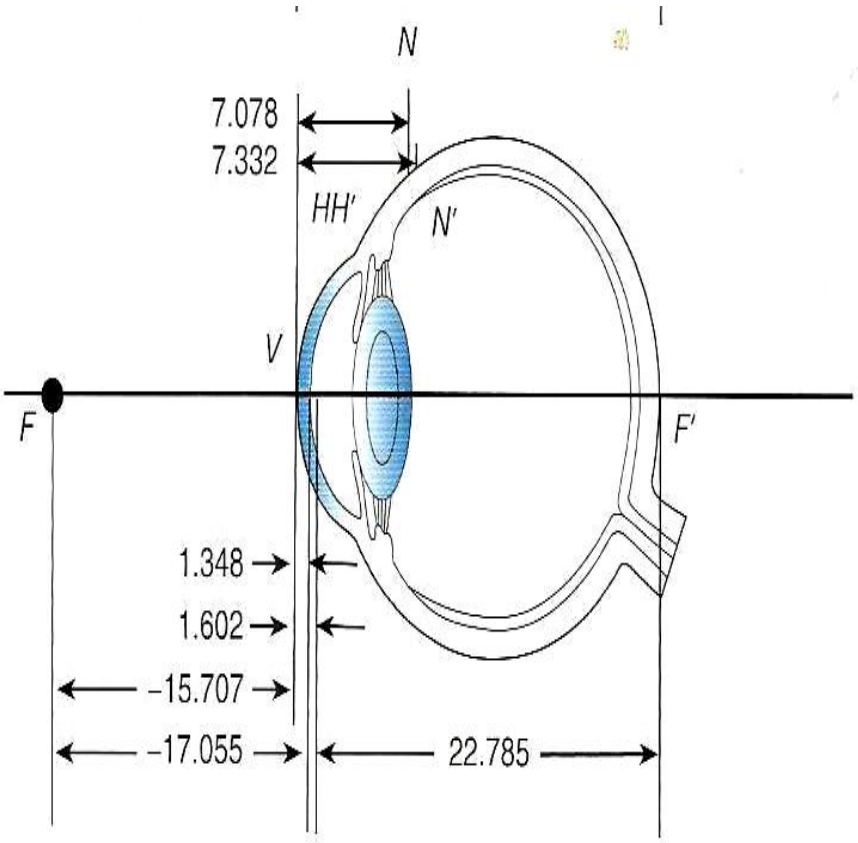


Figure (4)

Eye anatomy and dimensions. Resource the American Academy of Ophthalmology

Reference mentioned above

As image projectile transformation is referred to as study case, the parts of the eye that would be considered as the main ones are (from out to in):

1/the cornea

2/the iris

3/ the pupil

4/ the lens

5/ the retina

The intention of this study is to analyze what was thought of or believed as correct optical Ophthalmology. Opticians and ophthalmologists agree

up to the moment that there are two pyramids or cones of vision that represent the visual rays between seen objects and the retina. (see the American Academy of Ophthalmology; Reference mention above)

They specify that those two pyramids or cones are responsible for constructing rays of images between the cornea and the retina passing through the lens.

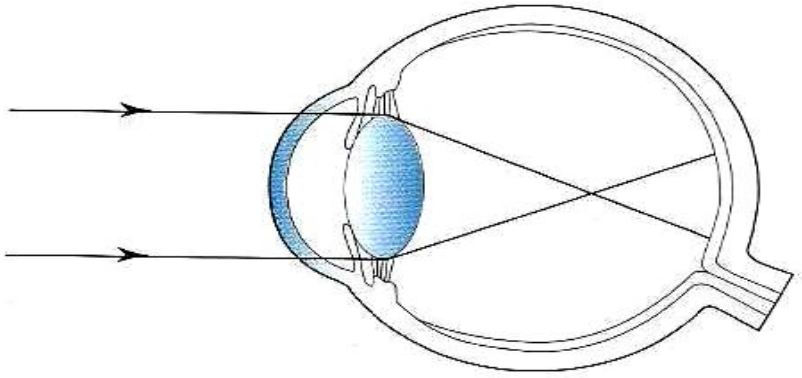


Figure (5)

What Ophalmologist and Opticians present the picture transformation to the retina.

Resource the American Academy of Ophthalmology, Reference mention above.

These two visual pyramids, which lie between the lens and the retina, are divided by the focal point of the eye lens. Therefore, the image is projected usually upside down on the retina. See figure (5).(see the American Academy of Ophthalmology; Reference mention above)

The interpretation of this idea is the following: since the cornea opens wide for image rays, nothing prevents these rays to project in full size through both the cornea and the lens, and to formulate an upside down projection of the seen object on the retina.

To accomplish critics that oppose these conclusions we can say the following:

The existing believes about the size of light projection of image rays that covers the whole eye lens or perhaps half of it, is not valid particularly if it depends on the size coverage of light rays on the cornea. Actually, those who believe in that do not take into their justification the forbiddance of light transformation to the lens done by the opaque surface of the iris.

See Figure (6).(see the American Academy of Ophthalmology; Reference mention above)

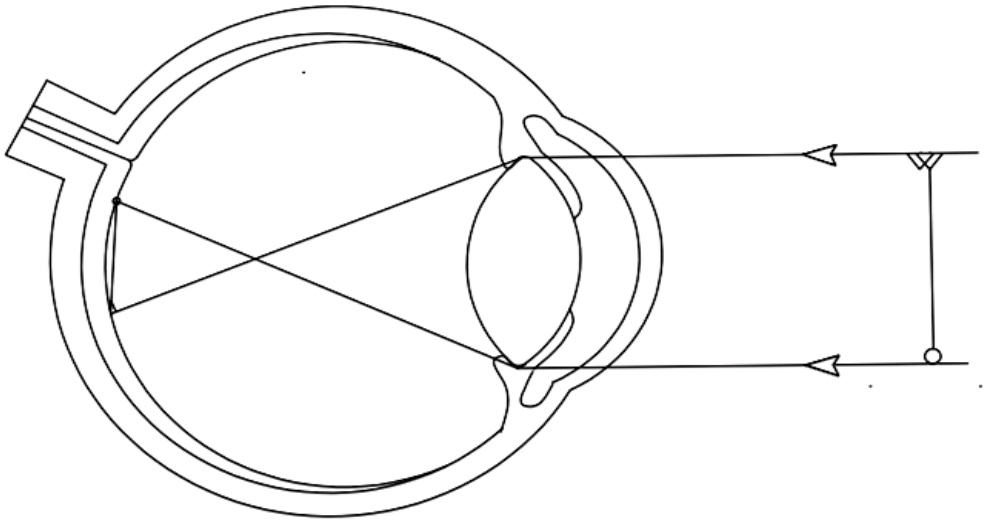


Figure (6)

The old idea of the full picture projection through the lense

Resource the American Academy of Ophthalmology, reference mention above.

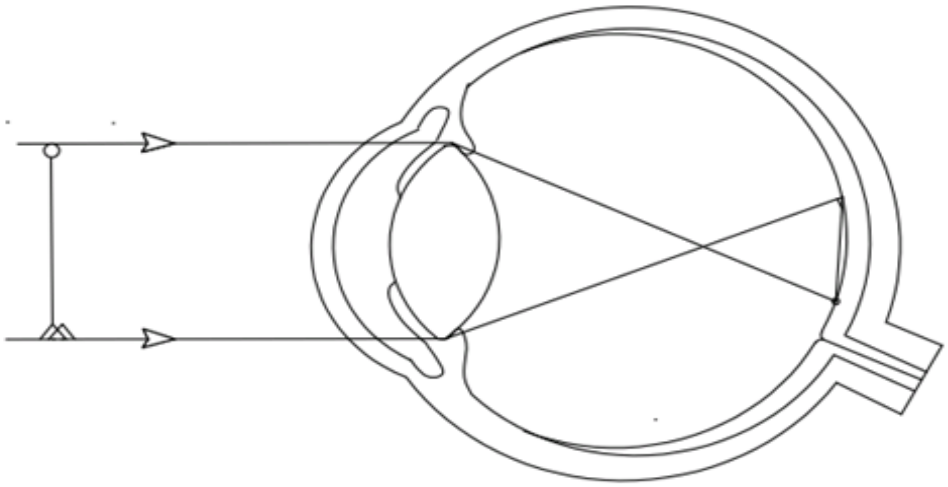


Figure (7)

Visual rays doing the impossible job of crossing the opaque surface of the iris.

The visual rays crossing the opaque surface of the iris.

Resource the American Academy of Ophthalmology.

Reference mention above.

If we take into consideration the opaque cover of the iris, the pupil, then should be the sole entrance for the visual rays; then there would be no way for the above mentioned idea of the full size transformation of image rays.

Usually, the pupil, which is 2-3 millimeters wide, is considered as responsible for controlling the density of light, which is a negotiable fact. If it controls the density of light, it should have to use a shading screen to prevent the unwanted light, because, without using a shading screen, the size of the pupil would not affect the quality or the quantity of light.

To sum up the idea of the sole entrance of image rays, we can say that it provides a new approach to visual perception that needs in depth investigations, which we will do in the following part below.

3/2/1**Explaining the Research Laboratory Experiments**

As a research tool we should have set and design a systematic eye typical to the eye provided in the chapter above so as to run the research of projecting images on the retina.

This eye should have to comprise a pinhole that stands at the pupil of the eye which is 2-3 mm wide. A lens after the pinhole, which is sited one mm from it, is taking place in the inner part directly after the pinhole. Then a suitable space of a transparent object takes its position up to the retina. This suggested systematic eye is closely similar to the pinhole camera and in the same time in and out movable lens makes it similar to Alhazan black pinhole box experiment see pictures (1,2,3)& drawing (1,2,3) below.

When trying to follow the image projection through the systematic eye. we put the pinhole to a selected image then place the lens and then receive the image on the transparent systematic eye retina. Then we try to do that again after removing the lens to see how the image could be received in this case.

When receiving the image through the lens it is clear that we are using the systematic eye as a human eye or a pinhole camera. The second case when removing the lens we use the systematic eye as Alhazan pinhole black box.

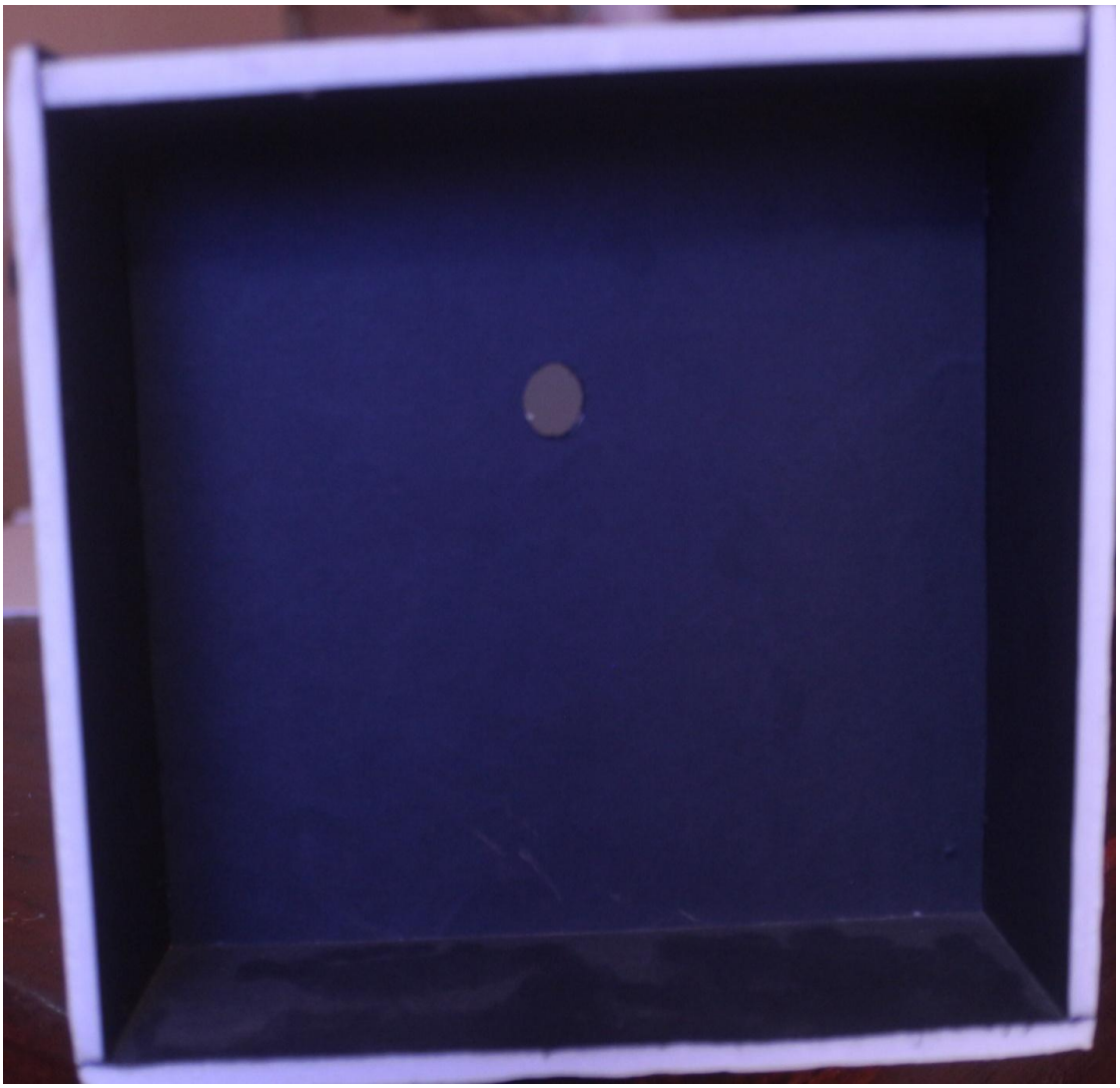
Picture (1)

The pinhole camera aril view.

Designed and presented by the researcher with the help of the designer Salah Alkhair.

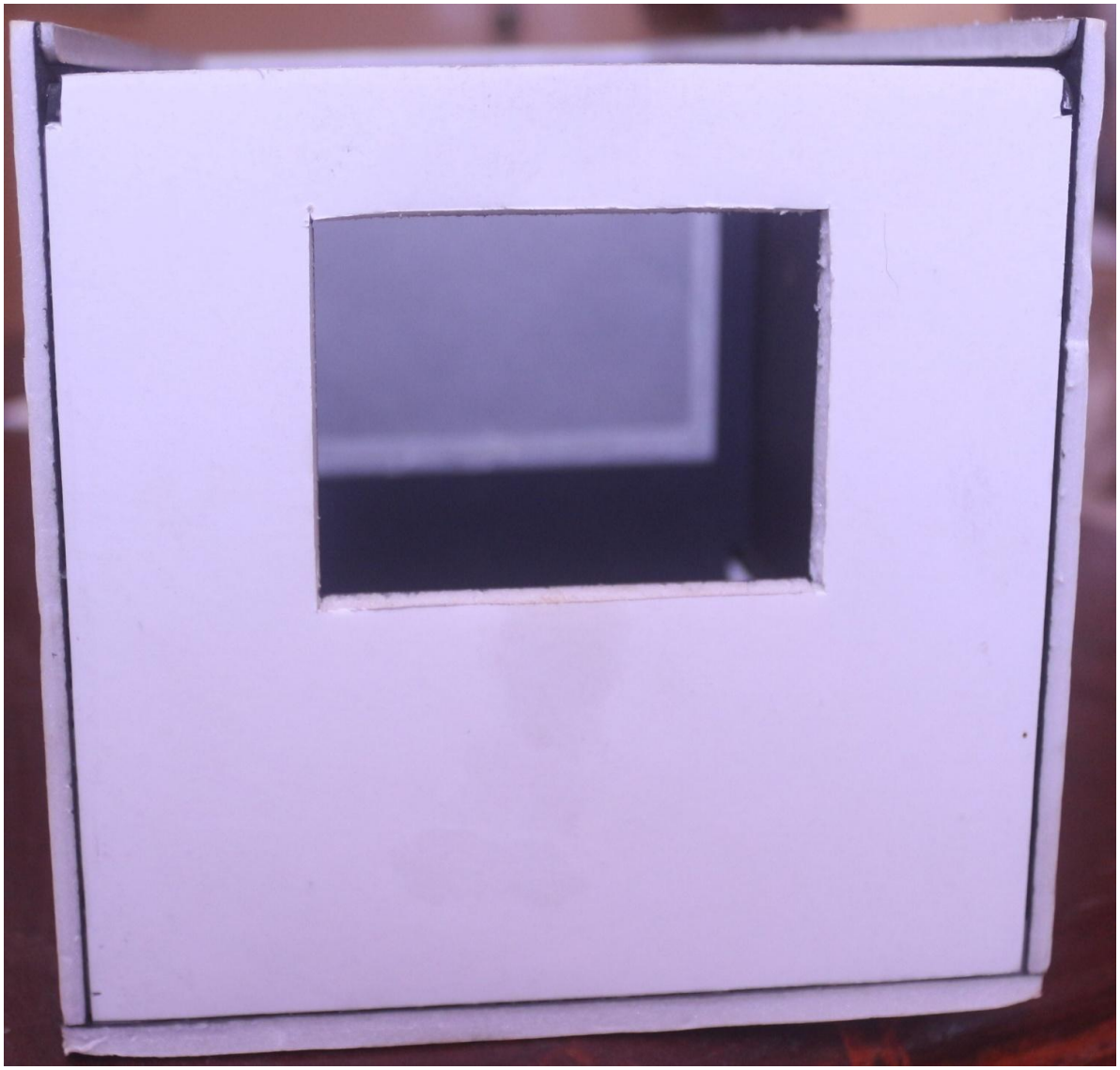
Picture (2)

Side view for the pinhole camera, designed and presented by the researcher with help of designer Salah Alkhair.



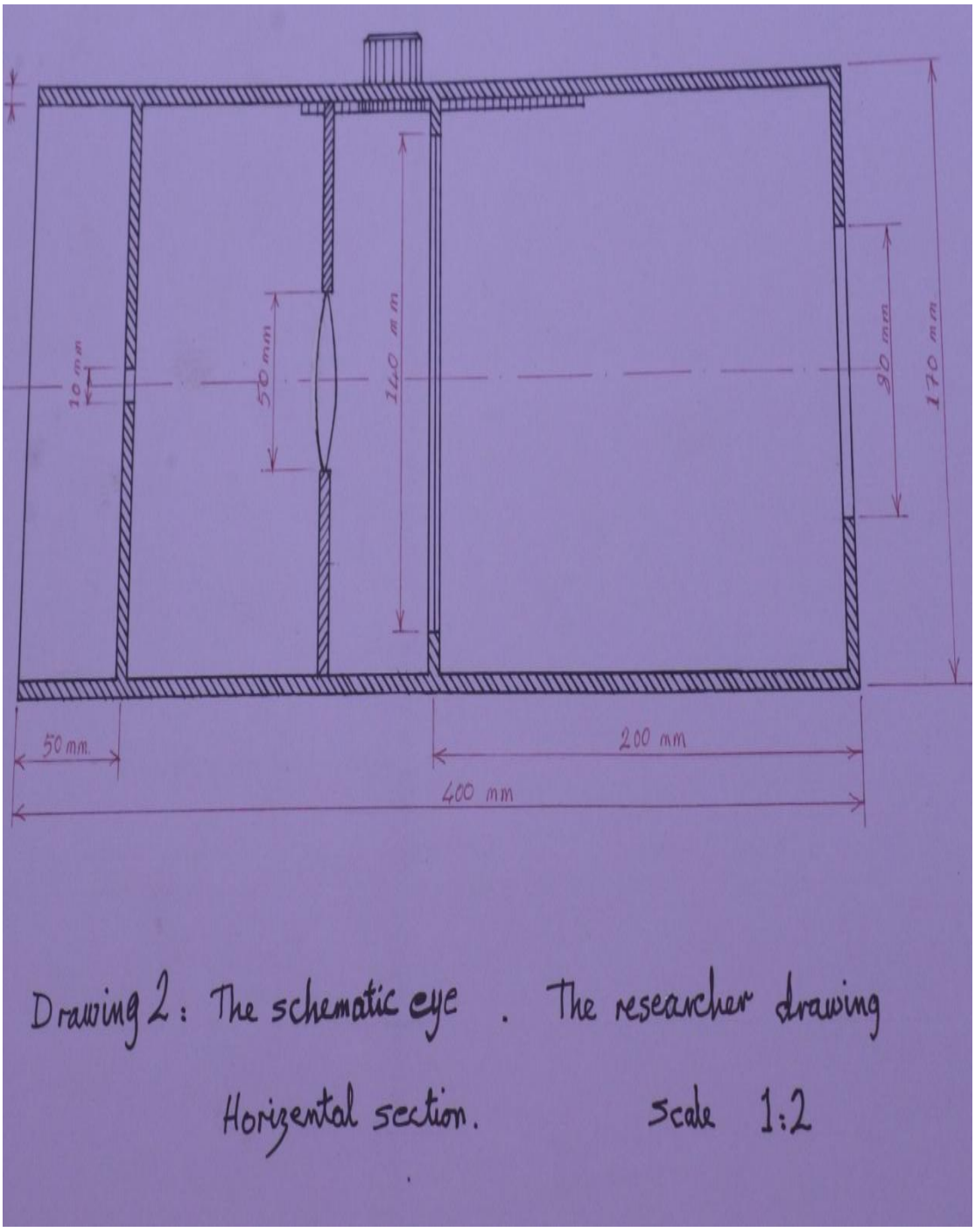
Picture (3)

Frontal view of the pinhole camera designed and presented by the researcher with the help of the designer Salah Alkhair.



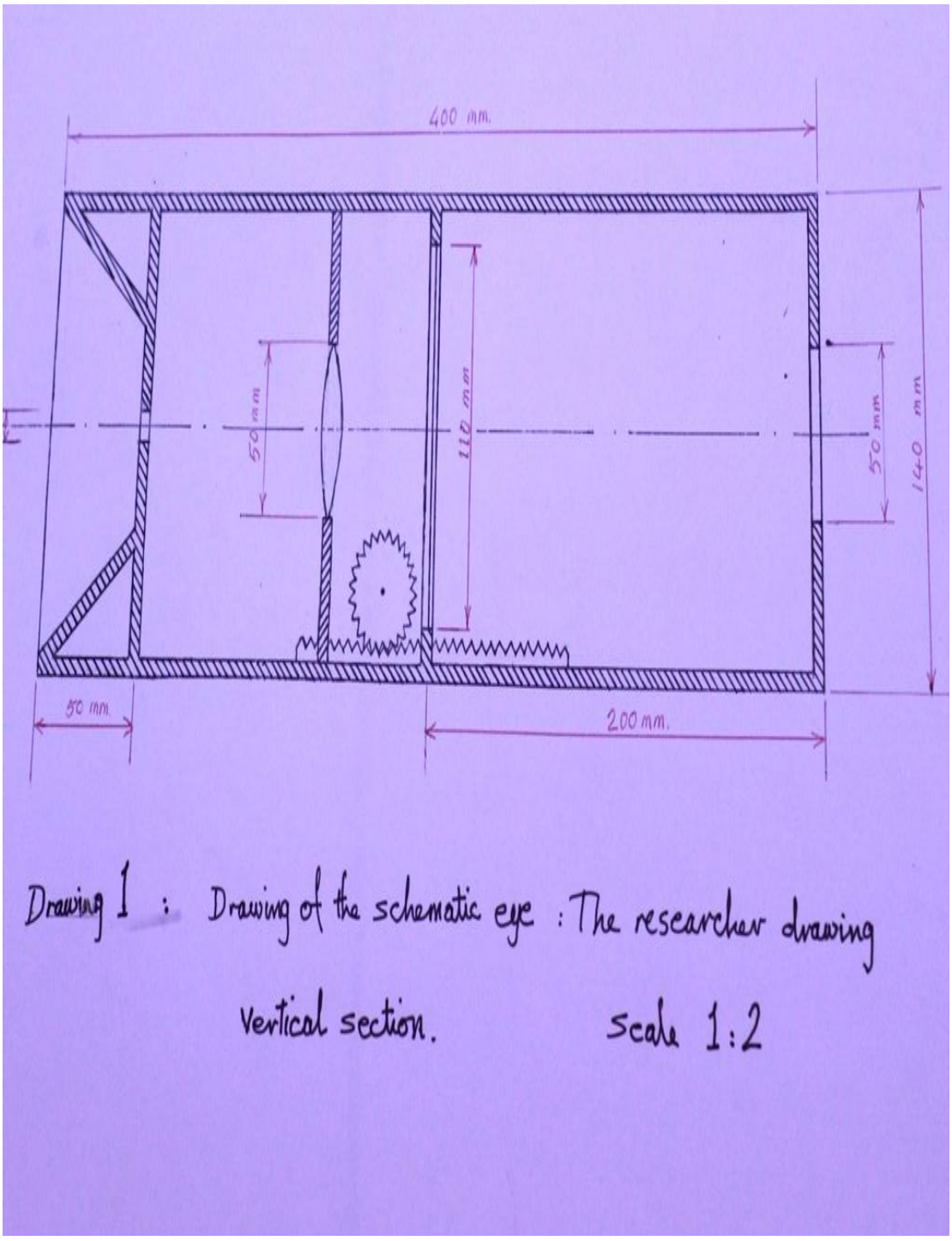
Picture (4)

The back view of the pinhole camera designed by the researcher with the help of the designer Salah Alkhair.



Drawing (1)

Horizontal of the pinhole camera. Drawn by the researcher with help of the designer Salah Alkhair.



Drawing (2)

Vertical section of the pinhole camera. Drawn by the researcher with help of the designer Salah Alkhair.

3/2/2The experiment results:

When using the lens we usually receive a well focused upside-down small projected image.

When removing the lens we receive an upside down unfocused image but in a bigger scale than the first one.

Therefore, it is very clear that the upside-down turn of the image is not done by the lens but it is done by the pinhole or the pupil of the eye.

Consequently, we can assure that this is a very strong first confirmed fact that have identified from our experiment, which will have big consequences in changing the ideas of how image projected on the retina.

Also this confirmed fact will help this research enough to navigate on correcting the rest of all criteria of the process of image projecting on the retina.

3/2/3 About the research findings:

As a result of the idea of the pupil's entrance, the optical properties of the final image we see in front of us are closely related to the visual rays pathway-through the pupil. In addition, the properties of the lens and the properties of the retina's image are constrained by the same said pupil's pathway-through.

From the beginning we have to state the fact whether the pupil has the ability of controlling the density of light or not, it should have the ability of earning its main responsibility which is allowing the visual rays to pass through. Then, naturally, the pupil will be the gatekeeper of image rays. It keeps rays carefully controlled by banding them into one pathway through beam or band of rays. As a result a new geometry should have to take place, and many new facts could be observed here, for example:

- There is a new job for the pupil that it can help the visual rays pass through it to the inner part of the eye. In this case the pupil will; change its function to a gate keeper and a band for banding the beams of visual rays to the minimum size.
- Changing the function of the pinhole camera and collecting the beans of the entering visual rays affects the geometry of visual rays to put a big change by making two pyramids of vision one of them outside the eye made of the entering visual rays and the second inside the eye meeting the other pyramids with its head and stands opposite to it on the lens of the eye.
- This form of new geometry should have to extend to cover all the way being taken by the visual rays, starting from the vanishing point outside the eye to the position of this image projection on the retina.
- Due to that, there is a different geometry of landscape scenery that takes place between the spectator point and the vanishing point on the horizon.

We recognize that there is specific feature fits the eye when related to Alhazen sole entrance light box which can be identified the same as the eye with the pupil, then, we say the fact that the human eye is the same as the pinhole box of Alhazen or the pinhole camera which we had set as an experiment of this research see figure (8).

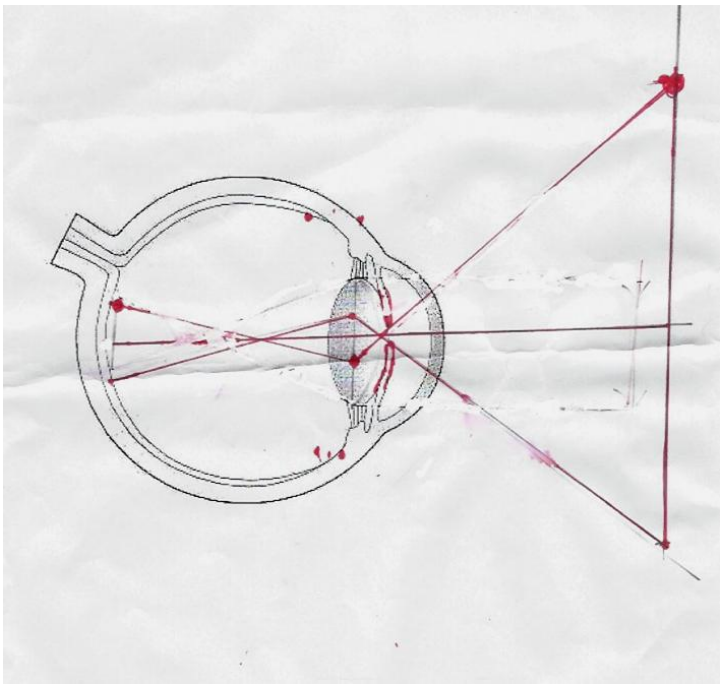


Figure (8)

The sole entrance of the visual rays throw the pupil.

Researcher drawing. Scale: 1:10

The entering visual rays formulate a pyramid outside the eye

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The main research of the experiment of the pinhole camera constrained huge facts that fulfill the assumptions such as:

- The image projection on the retina of the pinhole camera is projected in small figure upside down.
- The lens of the pinhole camera which measures not more than 4.5 mm, affects the image only on its size which can be part of the retina of the pinhole camera.

There will be a different discipline which stands between the vanishing point and the retina of the eye consist of four pyramids or cones of visual rays. See figure (9).

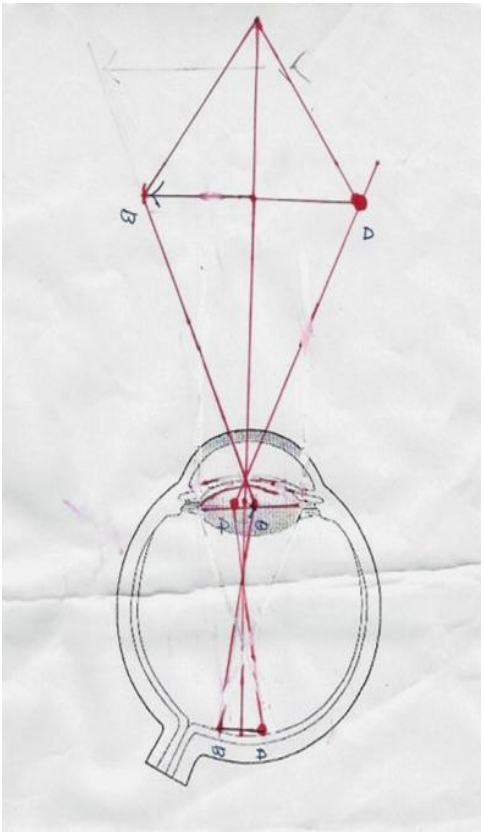

Figure (9)

The four pyramids system of vision: the researcher drawing. Scale 1: 50 0. Point: No decrease no increase of picture. Between the 0. Point and the pupil, columns are increased in size. Between the 0. Point the vanishing point, columns are decreased in size.

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These four pyramids of vision are divided into two groups. Group (1) consists of two pyramids or cones of visual rays sited between the pupil and the vanishing point. Group (2) is composed of two pyramids or cons of visual rays sited between the pupil and the retina. These pyramids or cones of vision are situated as follows:

a. Pyramid (A): the head angle of this pyramid is banded in the pupil band and its body extends from the pupil up to its base outside the eye touching the unseen frame that surrounds the seen image.
b. Pyramid (B) starts its head angle from the vanishing point on the horizon, and its body extends up to its base that duplicates the base of pyramid (B) on the unseen frame of the seen image.
c. Pyramid (C) it is sited inside the eye; its head angle is banded in the inner side of the pupil's band and its base on the vertical central line of the lens.
d. Pyramid (D) starts from the vertical central lines of the lens down to the retina.

3/2/4Function of the four pyramids of vision:

These pyramids work in cooperative discipline and individually in the same time. That means any of the four pyramids has its own individual function done along with their collective ones in the same time. These individual functions go like that:

1/ pyramid (A) is responsible of picking up the image and let it be projected through the pupil to inner part of the eye.

2/ pyramid (C) is responsible for projecting the image upside down on the central line of the lens.

3/ pyramid (D) it is affected by the lens function. It is produces between the central line of the lens to the focal retina.

4/pyramid (B) carries and presents the landscape's vanishing attitude.

More about the four Pyramids

While we recognize that all of the four pyramids or cones of visual rays work cooperatively with all the eye parts, still there are special functions that may concern one or two part in particular. Within that, we take in consideration what happens between pyramid (A) and (C). Especially when their two head angle meet by their heads. What happens is the increase of the angles whenever the seen object got nearer, decrease when the object got more and more far, due to that the further the objects, the more its image decrease its projected space in the retina, and this is the reason of the perspective effect on images. Besides, the furthest place may provide a semi zero cross-headed angles of pyramids (A) and (C).in this zero case object appears to vanish and on what we call the vanishing point.

Accordingly we can say that, the eye contribution is a perfectly and finely made discipline of four pyramids of vision, which there would be no image transformation out of or without it. However the eye is light rays receptor, but it doesn't receive the light rays idly, it is usually received with great managing effort that it can be considered as a contributor of a system of visual rays. See figure (8) and (9).

Chapter 4

Data Analyses and Discussion

4 /1 Analysis of the Research Data &Result :

The logic of shapes of image transformation:-

- As we have seen about the wrong way presentation of image transformation which shows how the light rays
- transform through and opaque surface which is impossible.
- Because it is unreasonable to state that an image was transform through an opaque surface.
- We agree that the theory of the sole entrance for picture transformation which is the pupil of the eye.
- Following this theory of the sole picture transformation as we had explained above requires the system of the four pyramids of vision. See figure (10). furthermore, the logic of drawing works the same as the picture transformation of the sole image entrance.
- To describe the system theory which the logic that rules this system we should say that:-
- always the presentation was usually drawn out of scale , to say that the proportion of the figure of the image is unreasonable to the actual scale of the eye.
- the dimension of the figure and how it was laid to the eye in the retina is usually not calculated in the presentation system

- It is crucial to know that any figure which is 4 meters away from the eye with the length of 4 meters enters the pupil through 3 mm entrance which is the pupil of the eye.

More analysis for the experiment data and results

The actual dimensions of the transform picture through the eye:

- To conclude the case of the picture transformation an image usually cross its way through the supposed picture frame, the vanishing point and the retina of the eye.
- These three elements, the eye, the supposed picture frame , the vanishing point , are usually centered on one line, within this line we will find the system of the four pyramids of vision which were the controllers of the system transformation. We can describe the system as follows :-

1-The first pyramid starts from the vanishing point 4 to 5 thousands meters away up to the infinite picture frame which is sited 4 meters in front of the eye. See fig (10)

2-The second pyramid is opposite to the first one .it starts from the picture frame, four meters away from the eye down to the pupil of the eye to pass the pupil which bands rays of the pyramid to the pupil size. See fig (10)

3-The third is opposite to the second, starts from the pupil down to the central line of the eye lens, where the image on the entrance line of the lens project upside down.

4-The fourth pyramid starts from of the central vertical line of the lens so as to project a small upside down image on the retina.

Therefore; there are stable and proved facts and statements that support the theory of perspective and visual perception. These facts apply new discipline and theory that suggest revising old optics disciplines as it is known now a day's within the academic world.

- We are very near to state the fact that Aristotle and Euclid had stated facts which could be in a way or another recognized, they decided that the eyes are sender

of visual rays that touch forms and see it, but we have the confirmed proof that the eye is a contributor of a system of four pyramids rather than a sender of rays as Alhazen has stated, in this case they are very near to what is right but Alhazen has stated a scientific fact changing their approach to the visual theory.(see drawing 1,2&3)

- Then this research can maintain an easy-pass way that purifies the two ideas of the old masters to new findings of the eyes visual system of the four pyramids.

4/2 More theoretical preparation to the visual vanishing attitudes:

The four pyramid system is usually divided into two parts, two pyramids outside the eye and two pyramids inside the eye. The well known dimension that rules the forms of these pyramids specially the two outside the eye is like that:

4 meters away starting from the pupil up to the end of the first pyramid which formulate an unseen 4*6 meters (see fig 11) rectangle ,which is actually stands as this base of the pyramid, where 6 is the horizontal length and 4 is the vertical width of the rectangle base of the pyramid. The 4*6 meters shape outside the eye is usually called the nasal photo or the infinity invisible frame that keeps its full dimensions of the seen (see fig 11) picture actually as it is.eg 4 meters is taken actually as 4 meters on this frame.

Another pyramid of vision with a base more far than the 4*6 pyramid, 4 meters will be less than its actual dimensions, and that is because this new pyramid has a less angles on the pupil than the first pyramid see figure (11).

While the opposite case of the pyramid with more near place to the pupil the four meters will be longer than its actual dimensions and that is because of the increasing factor of the angles of

That means any form nearer to the eye than this infinity place should be seen bigger than it's actual size.

The second part of this two pyramids outside the eye is formed starting from the infinity shape rectangle and then extends 4 thousands to 5 thousands meters up to the end of the pyramids on the horizon where every 4 meters seen on this pyramids is acting less than its actual form gradually from its starting point and finishing into the vanishing point with the zero form. Due to that, it is very clear that perspective theory is based on the vanishing attitude of the second pyramid mentioned in the above paragraph please see fig (11).

We recognize that these two pyramids outside the eye constitute the grammar of visual perception that depends on these two pyramids geometry. Therefore the geometry of visual perception which is in the same time the geometry of the linear perspective, should face a replacement case of this new geometry for the Ophthalmology Optics and the theory of perspective in Architecture Presentation, fine art, Landscape Drawings and Painting.

More after, the old scientific form of linear perspective had driven perspective out of the world of science, as we had agreed above in this study. Linear perspective should be rehearsed and maintained by adding this geometry of visual perception

perception to complete the scientific form by adding it as the theoretical part of this important study.

4 / 3 Inside the two pyramids outside the eyes:

Then analyzing the pyramids that start from the infinity rectangle up to the vanishing point will be analyzing the image that we see actually.

This image is composed of certain elements that form the items of the seen picture. The first element is the 3 levels that shape the inside horizontal shape of the pyramids.

The first and the second elements are the main sides of the pyramid, the first one is the upper one which is usually the image of the sky and the lower one is usually the image of the ground, both of them are levels that extend horizontally (see fig 12). Starts by the 6 meter length of the infinity shape in front of the eye and extends 4-5 miles deep to the vanishing point on the horizon. So these two levels shape the horizontal attitude of the image because they are actually found as an image of the sky and the ground.

The third level is invisible because it represents no picture but a function. It is actually divides the two picture of the upper side of the pyramid and the lower side of the ground level (see fig 12).

rdingly it cannot be seen as a level but as line that separates the two spaces of the visual pyramids to upper and lower. There are other three vertical levels which are also invisible and represent no actual existing figures. Two of them stand at the left and the right sides levels of the pyramids. The third vertical one stands at the middle of the pyramid that divides the image into two left and right side. Knowing that the image was divided into two upper and lower part, now we recognize is divided into four parts which are: upper left, upper right, lower left and lower right (see fig 13).

To explain more about perspective we should analyze the picture given by the first pyramid that starts from the vanishing pointon the area. we recognize that there are two vertical and horizontal lines. the effect of the two central lines, the horizontal and the vertical we can add that, the horizontal line is hypothetical and represents in the same time the horizon as its last effect. As we had agreed it divides the picture into two parts upper and lower which we can say upper the eye level and lower the eye level.

These two parts of the two zones upper& lower the eye zones which are the sky level and the ground level of vision have their effect on visuals such as e.g. a box. Whenever it is seen on one of them, their effect are very well known when a solid box is seen up or down the eye level. When it is up the eye level we usually see its bottom when its down the eye level we see its upper side. See fig (13) .

We can also add that more effects can happen to the seen box if we add the central visual line, there will be four zones that affect the picture of the box, first upper left where we can see three faces of the box bottom, frontal, and right. Upper right where we can see other sides of the box frontal, bottom and left, lower left where we can see upper, frontal, and right sides of the box. Lower right where we can see, frontal, upper, and left sides of the box. See fig (13).

Figure (10)

the main theory of vanishing attitude of objects in land scape shows that 4000 meters is the depth of the vanishing point as it appears in the postal columns drawn and presented by the researcher.

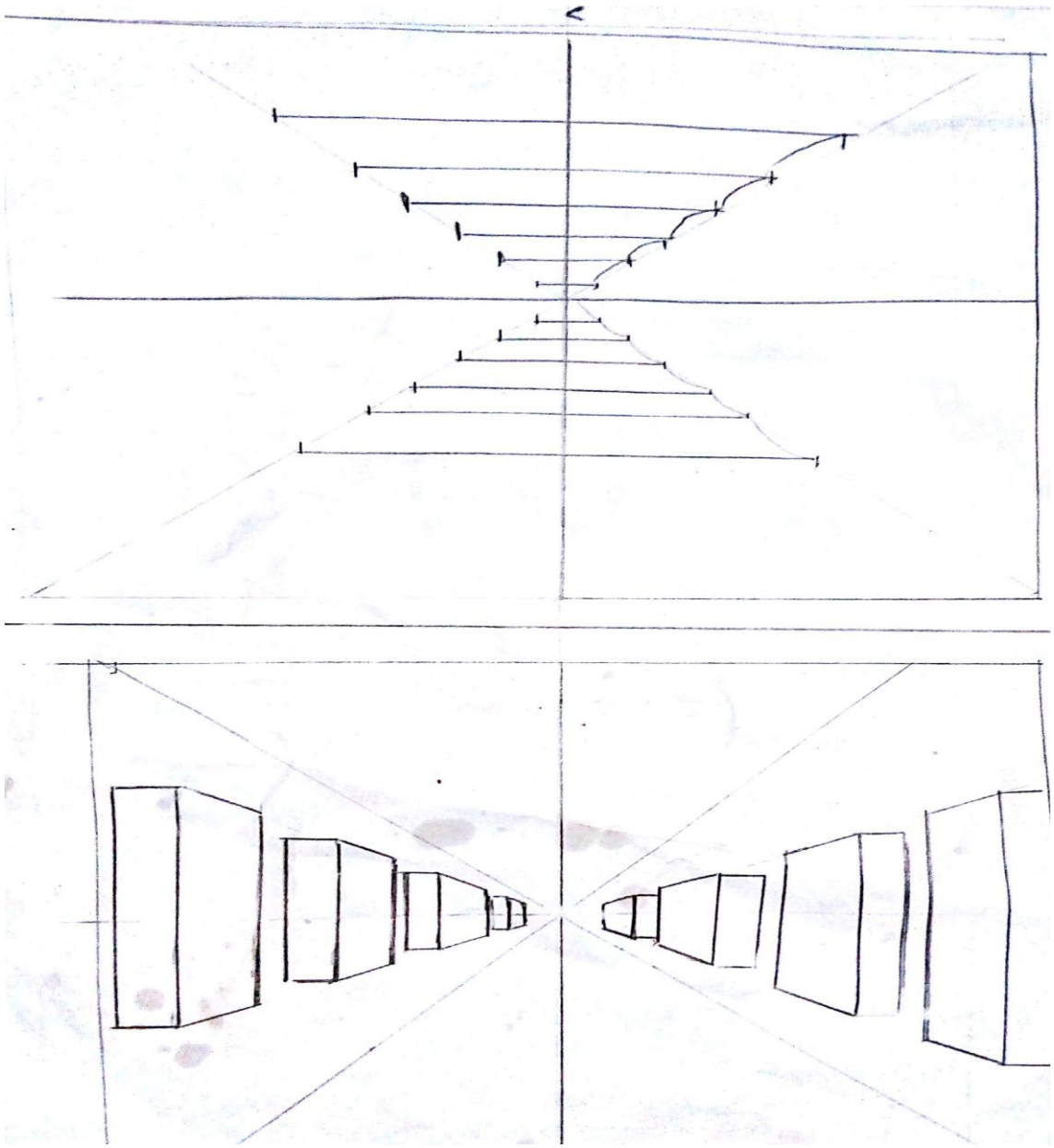

Figure (11)

Four visual zones takes their place when the eye level and central visual lines are centered horizontal and vertically on the seen picture, and they divide the picture to four deferent visual zones.

Chapter 5

Result, Conclusion and Recommendation

5/1 the results of the research:-

- The eye is the tool that put in discipline four pyramids of vision starting from the vanishing point on the horizon going up to the eye and then down to the retina,
- There is very important fact that could be taken as the main research result. That the eye is not an idle receptor of the image. Despite the modern research of Alhazen which had stated that the eye is not a sender of the visual rays but a receptor of light. As we discovered that the eye doesn't receive the light or the visual rays idly, it is a contributor of a system that arranges these light rays in a certain discipline.
- It is true as it had been prove above that the eye is a producer of the system of the four pyramids of the visual rays that can put the image to perspective theory as it was shown above and the visual field theory of the retina as it was described above.
- This system produces the discipline of linear perspective which is engaged with the second pyramid of vision between the infinity frame four meters in front of the eye and the sole entrance which is the pupil. As we had decided above that the sole entrance of the visual rays to the eye which is the pupil makes the eye the same as Alhazen pinhole light box or the pinhole camera that projects the image of any objects in front of the pinhole bigger if it's going near to the pinhole or smaller if it's going far from the pinhole.

5/2 Conclusion

To conclude the case; the two methodological' tracks of research had been followed by putting the geometric formula of linear perspective as an investigative case study. The two tracks are the empirical analytical methodology and the descriptive analytical historical methodology. The first one had covered the Ophthalmology and optical experiments of research, while the second one had covered the Art and Design part of the research.

Then we approached the conclusion that filled the case of experimenting a more scientific method to substitute the old geometric formula, and then we had observed new suggestion to trace the visual function of the eye that completes the theoretical part of the linear perspective and complete its scientific situation theoretically and practically, which can be easily done after posing the four pyramids as anew visual geometry in state of Orthogonal lines, and that will be very helpful in maintaining the teaching process of linear perspective .

5/3 Recommendations:

1. As an early result that emphasis the comments that captivate the chance of administrating the research of the missed side of the geometric linear perspective so as to avoid missing the right time of doing that. If it is not, we would surely miss the well-equipped student who should have to provide the well-equipped designer, more than that, we are going to miss the expert teachers and lecturers and research supervisors.
2. It is assured that the study of linear perspective is a contemporary task that had been solved already within this research, so; that needs a new curriculum of a new harmonized between the theoretical part which had been decided on this research and the practical part after mending its main facts due to the new founding s of this research.

3. Continuing the joint research between the three main fields" art, optics and ophthalmology" of this research to excavate down to the real fact to the retina projection; is it through four pyramids of vision and then agree to the idea that say the image projection on the retina is upside down..
4. To carry on the field of Art work and the architecture drawing so as to arrange the scientific environment of the visual phenomena instead of the traditional experimental linear perspective in its right place and then complete its theoretical form.

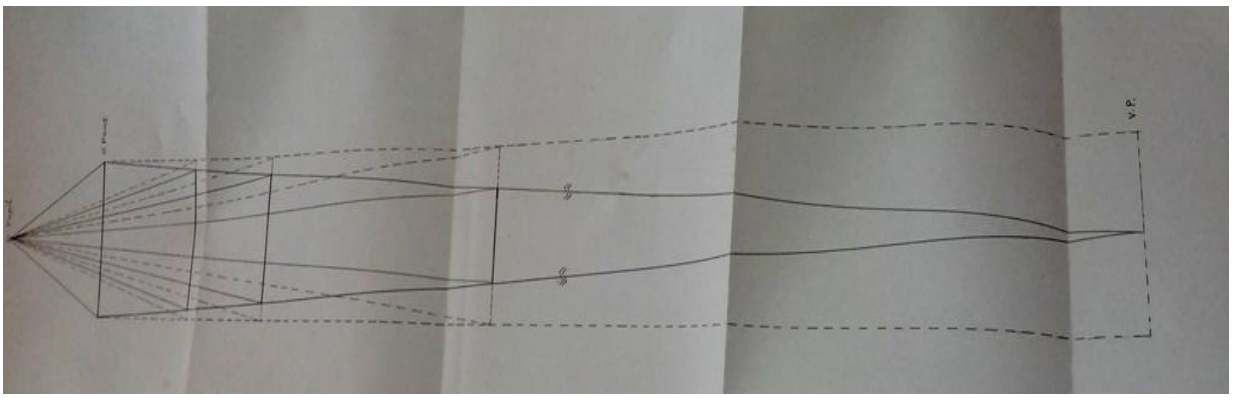

Figure (12)

Figure (12) the actual size of five posts in the seen landscape compare to the illusional size done by the eyes for the same posts.

Drawing by the researcher

Figure (12) should be enlarged to cover from page 88 up to page 103

Chapter6

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