

Dedication

To my family and to my wife.

To my beloved child Ahmed

ACKNOWLEDGEMENTS

All praise is due to Allah the Almighty for enabling me to achieve this research.

Countless thanks are due to my Supervisor Dr.Moh.Ali.Bashir and co-supervisor Dr.Emad.Abd Allah. for their guidance and beneficial advice. Likewise, I am grateful to Sudan University of Science & Technology especially the College of Graduate studies which offered me this opportunity to complete this study and also to my uncle's family and my wife's family.

Abstract

The aim of this dissertation was to study global existence of solutions of nonlinear integral equations with applications to Volterra and Fredholm integral equations using some analytical and numerical methods.

Chapter one includes introduction of the integral equation(IE) and construction methods of the IE .

Chapter two includes existence of solutions of linear integral equations using Adomain decomposition method and Adomain modified decomposition method and noise terms phenomenon and fuzzy integral equation and homotopy perturbation method.

Chapter three includes existence of solutions of nonlinear integral equation using successive approximation method and Adomain decomposition method and Laplace transform method and direct computational method.

Chapter four includes existence and uniqueness of continuous solutions of linear and nonlinear Volterra-Fredholm integral equation in cone metric space.

Chapter five includes numerical methods (variation iteration method and homotopy perturbation method) for linear and nonlinear Fredholm integral equation.

المستخلص

هدفت هذه الأطروحة لدراسة الوجود الشمولي لحل المعادلات التكاملية غير الخطية مع تطبيقات معادلات فولتيرا وفريدهولم التكاملية باستخدام بعض الطرق التحليلية والعديدية.

الفصل الأول يتضمن مقدمة للمعادلة التكاملية وطرق بناء المعادلة التكاملية.

الفصل الثاني يتضمن وجود حلول المعادلات التكاملية الخطية باستخدام طريقة التفكيك لأدوميان وطريقة التفكيك المعدلة لأدوميان وظاهرة حدود الضوضاء ومعادلة التكامل الغائمة وطريقة ارتجاج (اضطراب) الهوموتوبي.

الفصل الثالث يحتوي على وجود حلول المعادلات التكاملية غير الخطية باستخدام طريقة التقريب المتتابع وطريقة التفكيك لأدوميان وطريقة تحويل لابلاس وطريقة الحوسبة المباشرة.

الفصل الرابع يتضمن وجود ووحداية الحلول المستمرة لمعادلات فولتيرا - فريدهولم التكاملية الخطية وغير الخطية في الفضاء المترى المخروطي.

الفصل الخامس يتضمن الطرق العددية (طريقة تكرار التغيرات وطريقة ارتجاج الهوموتوبي) لمعادلة تكامل فريدهولم الخطية وغير الخطية.

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