



بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

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**Calculation of Cumulative Water Influx Using Van-
Everdingen Model with superposition concept by MATLAB
program**

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استهلال

قال تعالى:

(أَمَّنْ هُوَ قَانِتٌ آنَاءَ اللَّيْلِ سَاجِدًا وَقَائِمًا يَحْذَرُ الْآخِرَةَ
وَيَرْجُو رَحْمَةَ رَبِّهِ ۗ قُلْ هَلْ يَسْتَوِي الَّذِينَ يَعْلَمُونَ وَالَّذِينَ
لَا يَعْلَمُونَ ۗ إِنَّمَا يَتَذَكَّرُ أُولُو الْأَلْبَابِ (٩))

صدق الله العظيم

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ABSTRACT

Natural influx of water in oil reservoir surrounded by water aquifers play a very important role in increasing oil recovery. Such reservoir caused by production of hydrocarbon promotes the influx of water from surrounding aquifer to offset it. this can drive more hydrocarbons towards the surface thus enhancing the oil recovery.

In this research a computer program has been designed based on dimensionless solution of diffusivity equation (Everdingen), The program has been validated with good results obtained.

In addition to that a cumulative water influx for x field has been calculated to reflect the validation of the program.

التجريد

الجريان الطبيعي للماء في مكامن الزيت المحاطة بمياه جوفية تلعب دور مهم في زيادة انتعاش الزيت. هذه المكامن سببت انتاج الهيدروكربونات و ذلك يعزز جريان الماء حول المياه الجوفية و تعويضها .

ذلك يساعد على دفع اكبر كمية من الهيدروكربونات الى السطح ثم تحسين انعاش الزيت . في هذا البحث صمم برنامج يعتمد على الحل اللابعدى لمعادلة الانتشارية هذا البرنامج تم التأكد من صحته و الحصول على نتائج جيدة بالاضافة الى انه تم حساب جريان الماء التراكم.

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CHAPTER ONE

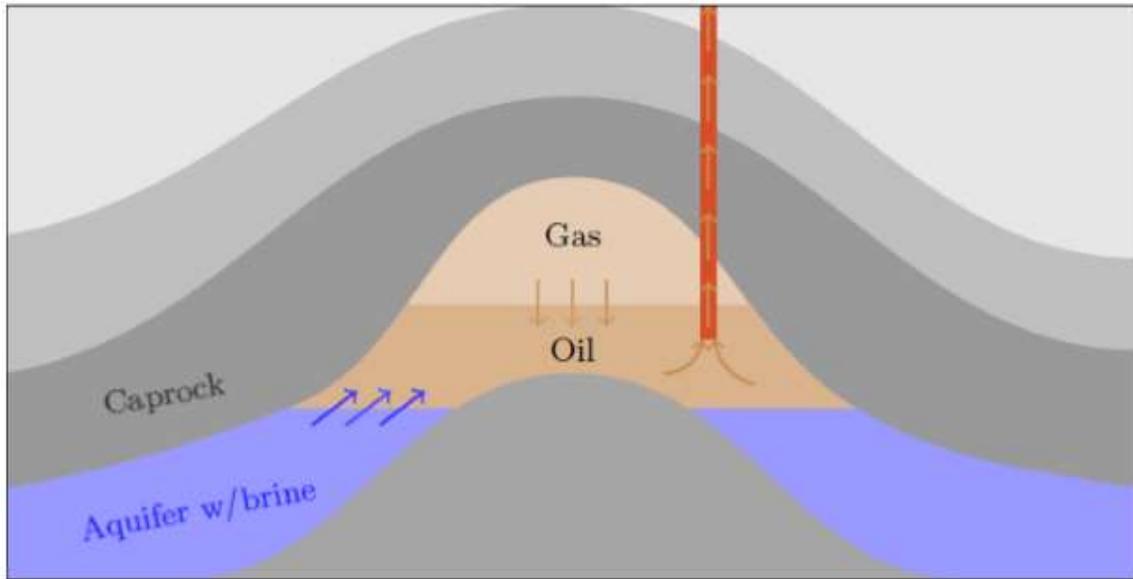
INTRODUCTION

1.1 Introduction:

Aquifer is one of the sources of water influx into the reservoir. Other sources of water influx into the reservoir include recharge of the reservoir by surface water from outcrops and water injection from the surface to supplement a weak aquifer. Water influx contributes to the total driving energy used for production of oil and gas from the reservoir to the surface. Other driving energy for production of hydrocarbon includes fluid expansion due to change in condition such as pressure and temperature, gravity-drainage drive due to fluid density differences, gas cap drive due to expansion of gas in the gas cap or expansion of liberated solution gas, and formation, and connate water compressibility (Ahmed, 2006; Fekete, 2014). Suppose that an aquifer underlies the reservoir B and they are hydraulically connected to each other, once the reservoir pressure starts to decline due to production, the aquifer will react by encroaches water into the reservoir to offset the reservoir pressure from declining thus increasing hydrocarbon recovery. This tendency of water to encroach into the reservoir is what referred in this research as water influx. The conceptual influx of water into the petroleum reservoir is illustrated in Fig 1.1.

Estimation of water influx volumes into the reservoir is significant in number of applications such as material balance for estimation of reserves, reservoir simulation studies for model calibration, production scheduling and setting up development strategies to optimize hydrocarbon recovery(Jassim A. Al-Ghanim February 2012). An accurate estimation of water influx into the reservoir is required with the aid of an efficiency aquifer model that can capture the real dynamics of petroleum subsurface system. Further, it is important to characterize the aquifer behavior before start aquifer modeling or inclusion of aquifer into the reservoir simulation model. This is because during aquifer characterization, the understanding of aquifer properties and strength is increased. Aquifer characterization is the challenging task in aquifer modeling. This is because most of aquifer properties such as aquifer size, aquifer permeability, aquifer porosity and water encroachment angle are uncertain. One of the main reason is the cost of drilling wells into the aquifer to gain necessary information is often not justified(Craft 1991). This is reasonable; however, the uncertainties associated with aquifer properties should be reduced to have an efficient aquifer model. For example, uncertainties of aquifer model parameters can be reduced by using history matching method or material balance method(Petrowiki 2015).

The inclusion of aquifer into reservoir simulation model cannot be isolated from aquifer characterization. It should begin with aquifer characterization to increase the understanding of its properties and strength. In addition, the inclusion of aquifer into the reservoir simulation model may help to capture uncertainties in reservoir simulation model and thus increasing its predictive capability in terms of hydrocarbon recovery factor for better management of the reservoir.



**Fig 1.1 Conceptual influx of water into the petroleum reservoir
(Ahmed, 2006).**

1.2 Problem Statement:

The accurate estimation of water influx into a petroleum reservoir is very important in many reservoir engineering applications, such as material balance calculation, design of pressure maintenance programs, and advanced reservoir simulation studies. These applications have heavily relied on the classical work of van Everdingen and Hurst for finite and infinite edge-water drive reservoirs. However, for both types of water drive reservoirs, the calculation of water influx is not a straight forward task. Table lookup and interpolation between time entries are needed, and furthermore for finite aquifers, interpolation between tables may be also required. To that a program will be developed to reduce the time of calculations and errors.

1.3 Objective:

Design a computer program to compute calculations of van Everdingen and Hurst for infinite aquifer.

1.4 Outline:

The chapter outlines of this research are as follows. Chapter 1 has explained an introduction of research topic, problem statement, objectives, and methodology. Chapter 2 will contain review of water influx models covering an introduction of reservoirs and reservoir fluids classification, reservoir drive, reservoir-aquifer system, their classification, factors causing water influx into the reservoir, the significance of estimating water influx into the reservoir. Chapter 3 will review of water influx model and their equations. Chapter 4 will contain the methodology of this program covering data collection, estimation of water influx manually and by the program, and comparison of this two estimation. will contain results and discussion of results. Chapter 5 conclusion and recommendations.

CHAPTER TWO

LITERATURE REVIEW

2.1 Literature review:

2.1.1 Water influx model:

Water influx models are mathematical models that simulate and predict aquifer performance. They predict the cumulative water influx history when successfully integrated with a reservoir simulator, the net result is a model that effectively simulates the performance of a water drive reservoir.

The mathematical water influx models that are commonly used in the petroleum industry include;

- Pot aquifer
- Schilthuis's steady-state
- Hurst's modified steady-state
- The van Everdingen-Hurst unsteady-state

These models apply to different flow regimes, including steady-state, modified steady-state (Schilthuis 1936) pseudo-steady-state (Hurst 1943); (Hurst 1958); Leung, 1986), and unsteady-state (Fetkovich, 1971; Leung, 1988).

van Everdingen and Hurst (1949) presented the most commonly used water-influx model. This model is basically a solution of the radial diffusivity equation; hence, it yields an accurate estimate of water encroachment for practically all flow regimes, provided that the flow

geometry is actually radial. van Everdingen and Hurst solutions are for both the constant-terminal-rate case and the constant-terminal-pressure case of finite and infinite edge-water aquifers.

Coats (1962) developed a model that takes into consideration the vertical flow of water into the reservoir. However, his model has two major drawbacks:

(1) the presented solution is for the constant-terminal-rate case only, which permits the calculation of the pressure from a known water influx rather than the reserve, and

(2) the model is only applicable to infinite aquifers and does not provide a solution for the finite aquifers.

Allard and Chen (1988) expanded Coats (1962) model for bottom-water drive reservoirs by presenting a constant-terminal-pressure solution that covers finite and infinite aquifers.

Eventhough the van Everdingen-Hurst (1949), Coats (1962), and Allard-Chen (1988) models offer accurate solutions for edge and bottom- water drive reservoirs, they suffer a major limitation because the results of these models are presented in table forms, which greatly limits their application in computer analysis and reservoir simulation studies.

2.2 Theoretical background:

2.2.1 Classification of reservoirs and reservoir fluids:

2.2.1.1 Type of reservoir: -

Accordingly, reservoir can be classified into basically two types:

I. Oil reservoirs:

If the reservoir temperature T is less than the critical temperature T_c of the reservoir fluid, the reservoir is classified as an oil reservoir.

Depending upon initial reservoir pressure p_i , oil reservoirs can be sub classified into the following categories: -

1. Under saturated oil reservoir: If the initial reservoir pressure p_i is greater than bubble point pressure P_b of the reservoir fluid, the reservoir is labeled under saturated oil reservoir.
2. Saturated oil reservoir: When the initial reservoir pressure is equal to the bubble point pressure P_b of the reservoir fluid, is called a saturated oil reservoir.
3. Gas cap reservoir: If the initial reservoir pressure is below the bubble point pressure of the reservoir fluid the reservoir is termed gas cap or two phase reservoir, in which the gas or vapor phase is underlain by an oil phase.

II. Gas reservoirs: -

If the reservoir temperature is greater than the critical temperature of the hydrocarbon fluid, the reservoir is considered a gas reservoir.

2.2.1.2 Classification of crude oil:

In general, crude oils are commonly classified in to the following types:

- A. Ordinary black oil.
- B. Low-shrinkage crude oil.
- C. High-shrinkage crude oil.
- D. Near-critical crude oil.

The above classifications are including physical properties, composition, gas- oil ratio, appearance, and pressure-temperature phase diagrams.

A. Ordinary black oil:

If should be noted that quality lines which are approximates equally spaced characterize black oil phase diagram.

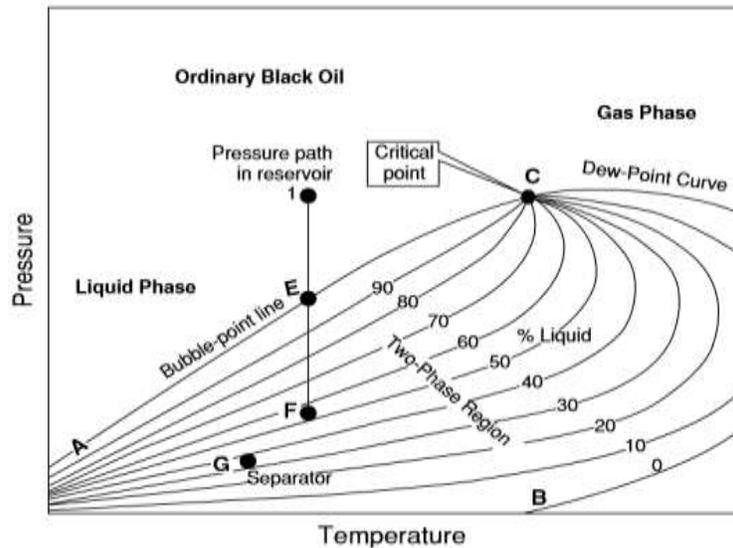


Fig 2.1 A typical phase diagram for Ordinary black oil (Ahmed, 2006).

B. Low-shrinkage crude oil:

The diagram is characterized by quality lines that are closely spaced near the dew-point curve.

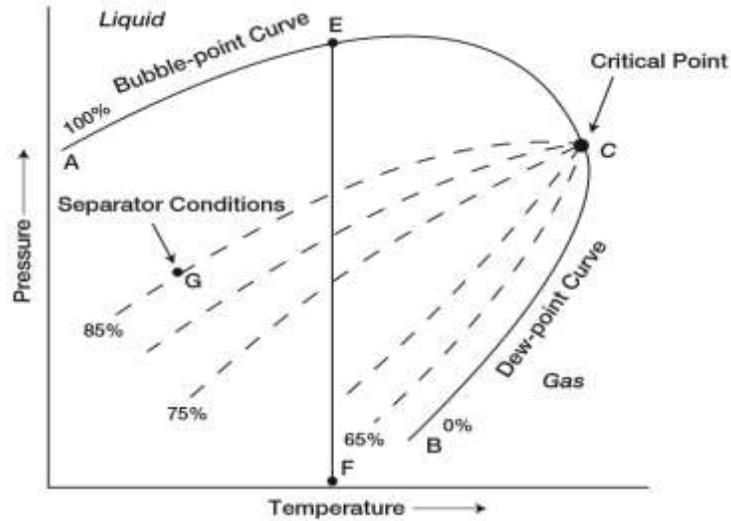


Fig 2.2 A typical phase diagram for low-shrinkage oil (Ahmed, 2006).

C. Volatile crude oil:

If quality lines are close together near the bubble-point and are more widely spaced at lower pressures.

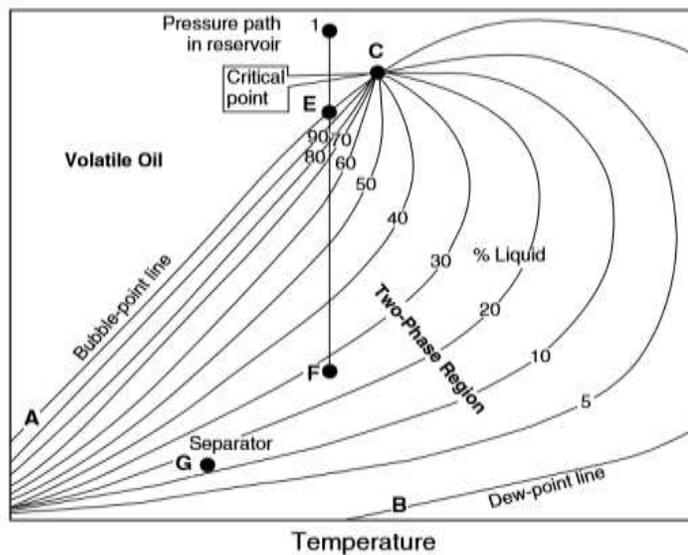


Fig 2.3 A typical phase diagram for Volatile crude oil.

D. Near-critical crude oil:

If the reservoir temperature T is near the critical temperature T_c of the hydrocarbon system, the hydrocarbon mixture is identified as a near-critical crude oil.

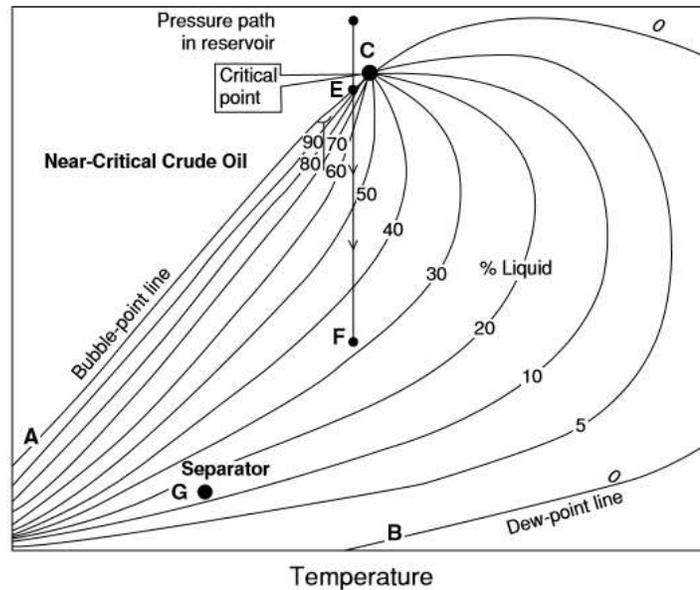


Fig2.4 A typical phase diagram for Near-critical crude oil (Ahmed, 2006).

2.2.1.3 Type of natural gas:

On the basis of their phase diagrams and prevailing reservoir conditions, natural gases can be classified into the four categories:

1. Retrograde gas –condensate reservoir:

If the reservoir temperature T lies between the critical temperature T_c and ricondentherm T_{ct} of the reservoir fluid, the reservoir is classified as a retrograde gas-condensate reservoir.

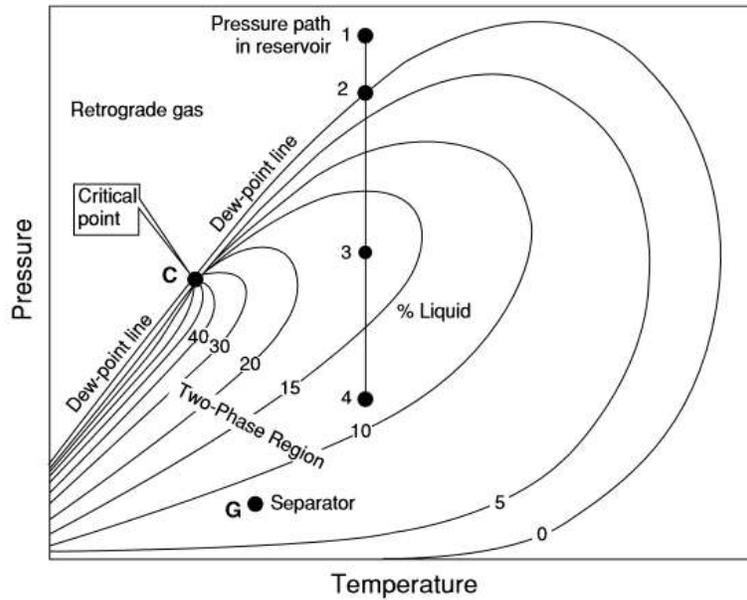


Fig 2.5 A typical phase diagram of a retrograde system (Ahmed, 2006).

2. Near-critical gas condensate reservoir:

If the reservoir temperature is near the critical temperature, the hydrocarbon mixture is classified as a near-critical gas-condensate.

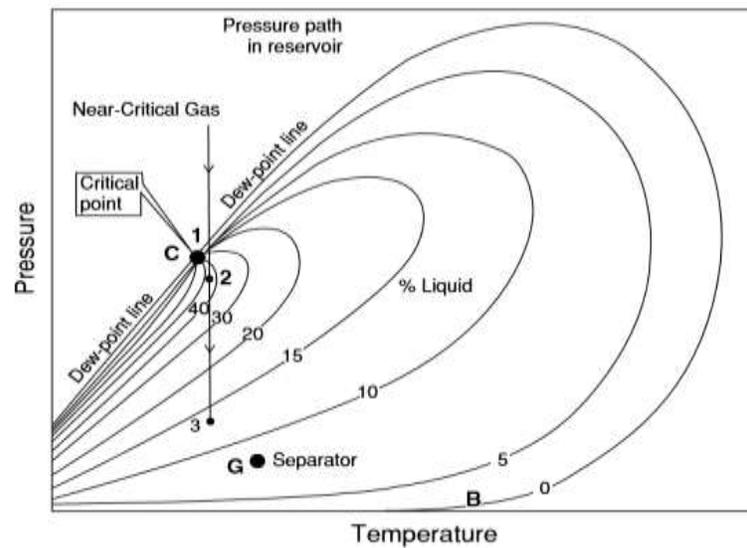


Fig 2.6 A typical phase diagram for a near-critical gas condensate reservoir (Ahmed, 2006).

3. Wet-gas reservoir:

If the reservoir temperature is above the cricondentherm of the hydrocarbon mixture, because the reservoir temperature exceeds the cricondentherm of the hydrocarbon system.

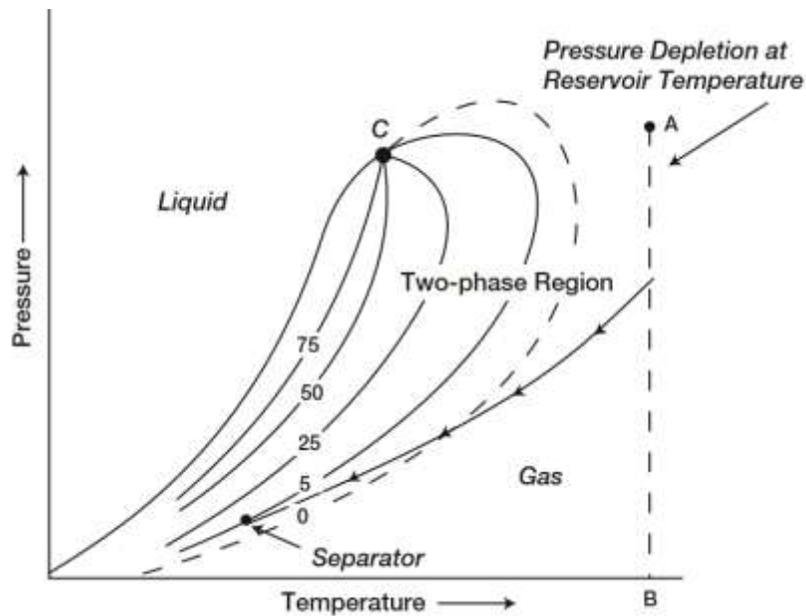


Fig 2.7 Phase diagram for a wet gas (Ahmed, 2006).

4. Dry-gas reservoir:

The hydrocarbon mixture exists as a gas both in the reservoir and in surface facilities. The only liquid associated with the gas from a dry-gas reservoir is water (Ahmed 2006).

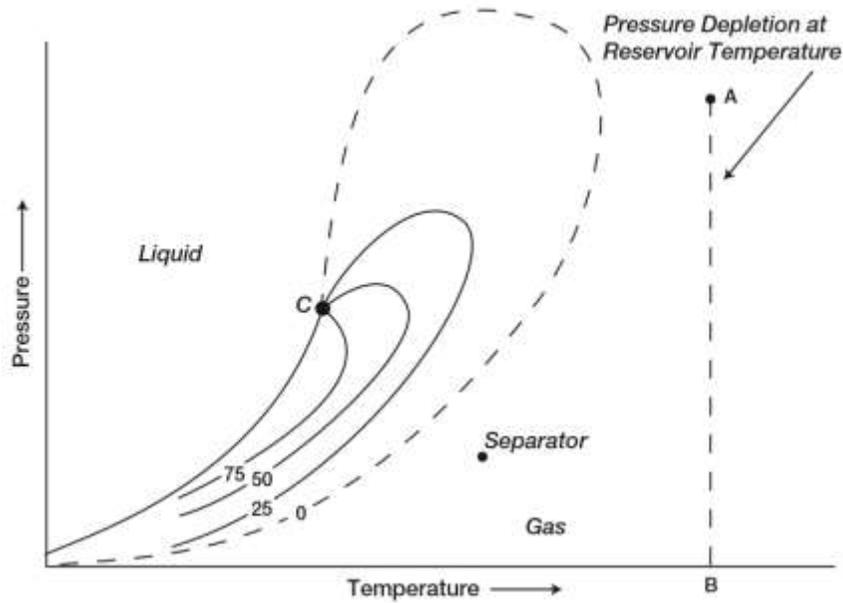


Fig 2.8 Phase diagram for a dry gas (Ahmed, 2006).

2.2.2 Reservoir Drives:

Recovery of hydrocarbons from an oil reservoir is commonly recognized to occur in several recovery stages. These are:

1. Primary recovery:

This is the recovery of hydrocarbons from the reservoir using the natural energy of the reservoir as a drive.

2. Secondary recovery:

This is recovery aided or driven by the injection of water or gas from the surface.

3. Tertiary recovery (Enhanced Oil Recovery, EOR):

There are a range of techniques broadly labeled 'Enhanced Oil Recovery' that are applied to reservoirs in order to improve flagging production.

4. Infill recovery:

Is carried out when recovery from the previous three phases have been completed. It involves drilling cheap production holes between existing boreholes to ensure that the whole reservoir has been fully depleted of its oil.

➤ Primary Recovery Drive Mechanisms:

During primary recovery the natural energy of the reservoir is used to transport hydrocarbons towards and out of the production wells. There are several different energy sources, and each gives rise to a drive mechanism. Early in the history of a reservoir the drive mechanism will not be known. It is determined by analysis of production data (reservoir pressure and fluid production ratios). The earliest possible determination of the drive mechanism is a primary goal in the early life of the reservoir, as its knowledge can greatly improve the management and recovery of reserves from the reservoir in its middle and later life.

There are basically six driving mechanism that provides the natural energy necessary for oil recovery:

- Solution Gas Drive
- Gas cap drive
- Water drive
- Gravity drainage drive
- Combination drive

**Table 2.1 Recovery factor ranges for each drive mechanism
(GLOVER)**

Drive mechanism	Energy source	R.F, % OOIP
Solution gas drive	Evolved solution gas and expansion	20 – 30
Evolved gas		18 - 25
Gas expansion		2 – 5
Gas cap drive	Gas cap expansion	20 – 40
Water drive	Aquifer expansion	20 – 60
Bottom		20 - 40
Edge		35 – 60
Gravity derange	Gravity	50 – 70

A combination or mixed drive occurs when any of the first three drives operate together, or when any of the first three drives operate with the aid of gravity drainage.

The reservoir pressure and GOR trends for each of the main (first) three drive mechanisms is shown as Figures 2.9 and 2.10. Note particularly that water drive maintains the reservoir pressure much higher than the gas drives, and has a uniformly low GOR.

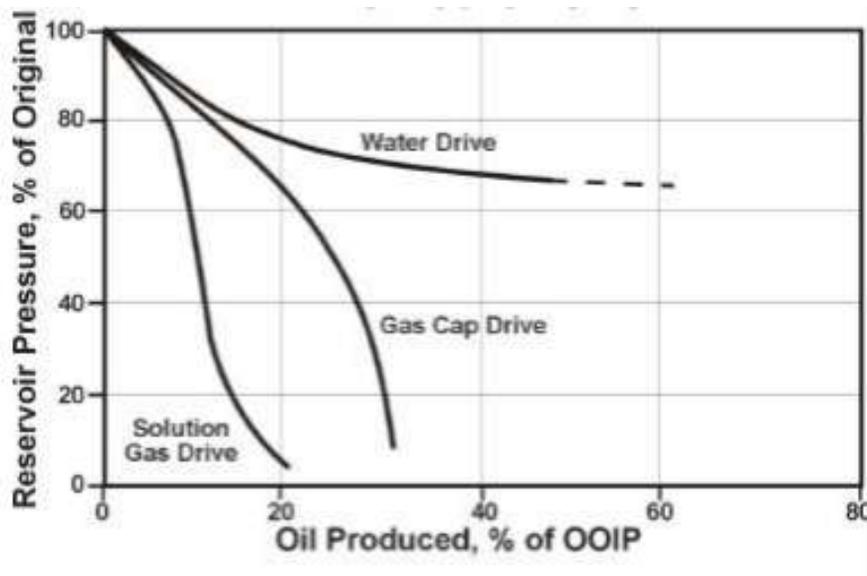


Fig 2.9 Reservoir pressure trends for drive mechanism (Glover).

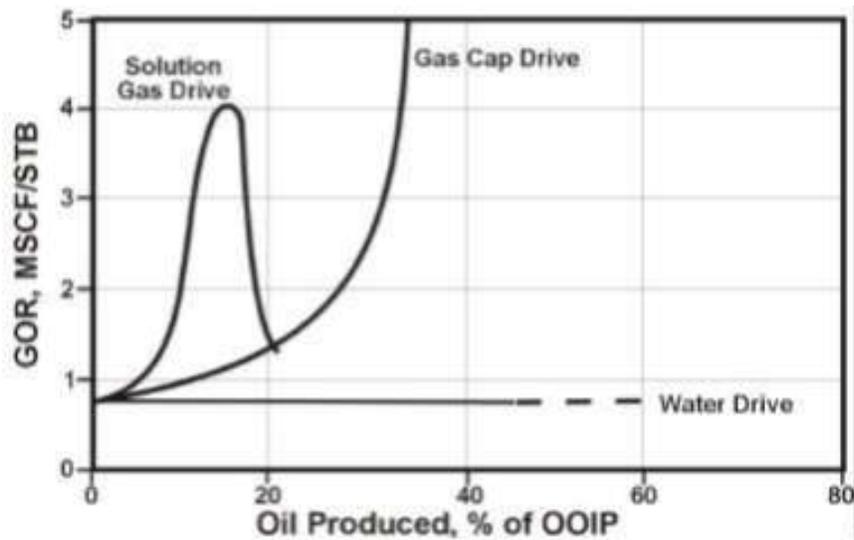


Fig 2.10 GOR trends for drive mechanism (Glover).

a) Solution Gas Drive:

A solution gas drive reservoir is initially either considered to be under saturated or saturated depending on its pressure:

- Under saturated: Reservoir pressure $>$ bubble point of oil.
- Saturated: Reservoir pressure \leq bubble point of oil.

For an under saturated reservoir no free gas exists until the reservoir pressure falls below the bubble point. In this regime reservoir drive energy is provided only by the bulk expansion of the reservoir rock and liquids (water and oil).

b) Gas cap drive:

Gas cap reservoirs produce very little or no water.

The recovery of gas cap reservoirs is better than for solution drive reservoirs (20% to 40% OOIP). The recovery efficiency depends on the size of the gas cap, which is a measure of how much latent energy there is available to drive production, and how the reservoir is managed, i.e. how the

energy resource is used bearing in mind the geometric characteristics of the reservoir, economics and equity considerations

c) Water Drive:

The drive energy is provided by an aquifer that interfaces with the oil in the reservoir at the oil-water contact (OWC). As production continues, and oil is extracted from the reservoir, the aquifer expands into the reservoir displacing the oil. Clearly, for most reservoirs, solution gas drive will also be taking place, and there may also be a gas cap contributing to the primary recovery. Two types of water drive are commonly recognized:

- Bottom water drive (Figure 2.11)
- Edge water drive (Figure 2.11)

The pressure history of a water driven reservoir depends critically upon:

- (i) The size of the aquifer.
- (ii) The permeability of the aquifer.
- (iii) The reservoir production rate.

If the production rate is low, and the size and permeability of the aquifer is high, then the reservoir pressure will remain high because all produced oil is replaced efficiently with water. If the production rate is too high then the extracted oil may not be able to be replaced by water in the same timescale, especially if the aquifer is small or low permeability. In this case the reservoir pressure will fall (Glover).

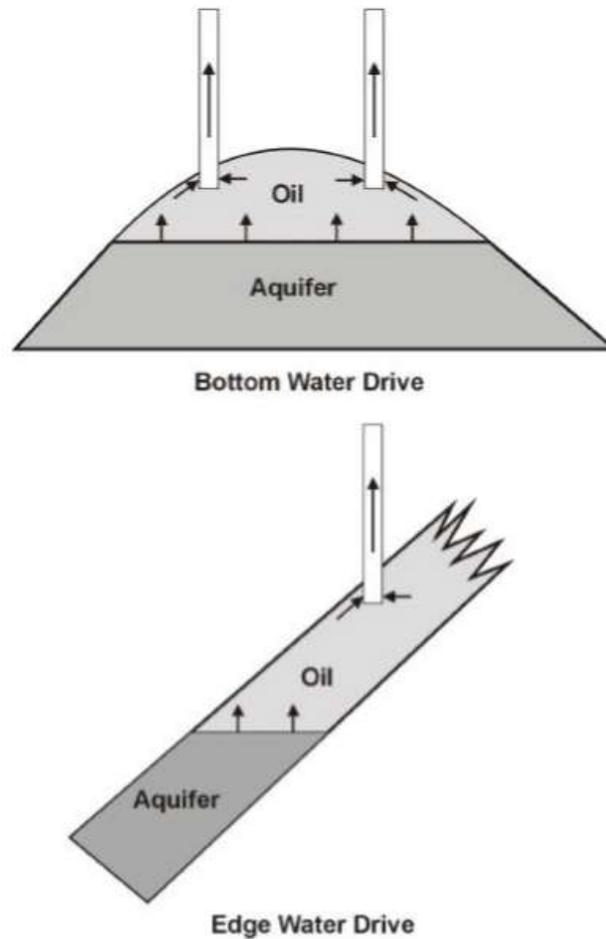


Fig 2.11 Water drive (Glover).

d) Gravity Drainage

The density differences between oil and gas and water result in their natural segregation in the reservoir. This process can be used as a drive mechanism, but is relatively weak, and in practice is only used in combination with other drive mechanisms. Figure 2.12 shows production by gravity drainage.

The best conditions for gravity drainage are:

- Thick oil zones.
- High vertical permeability.

The rate of production engendered by gravity drainage is very low compared with the other drive mechanisms examined so far. However, it is extremely efficient over long periods and can give rise to extremely high recoveries (50-70% OOIP, Table 2.1).

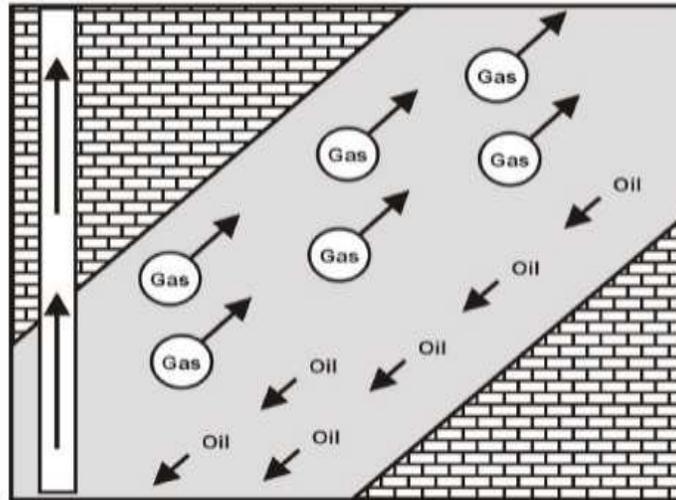


Fig 2.12 Gravity drainage (Glover).

e) Combination or Mixed Drive

In practice a reservoir usually incorporates at least two main drive mechanisms(Glover).

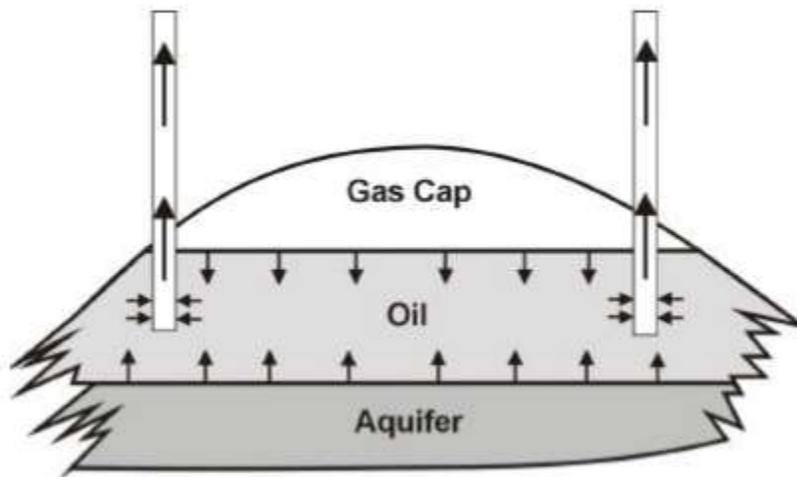


Fig 2.13 Mixed drive reservoir (Glover).

2.1.3 Classification of Reservoir-Aquifer System:

A reservoir-aquifer system refers to the reservoir which is bounded with an aquifer. A reservoir is a subsurface body of rock having sufficient porosity and permeability to store and transmit fluids. While, a water bearing rock is called an aquifer. The fluid referred in this contest are oil, gas, and water. Reservoir-aquifer systems are commonly classified based on the following four categories.

- Flow regimes
- Flow geometry
- Outer boundary conditions and
- Degree of pressure maintenance

2.1.3.1 Flow regimes:

Three flow regimes are basically considered in describing fluid flow in the reservoir. These are steady state, semi-steady state, and unsteady state. Steady state flow regime occurs when the rate of change of pressure at every location in the reservoir is zero. This is when the reservoir is supported by pressure maintenance operations or when there is a recharge from strong aquifer. Semi-steady state occurs when the rate of change of pressure is constant and unsteady state flow regime occurs when the rate of change of pressure is not zero or constant (Ahmed, 2006).

2.1.3.2 Flow geometry:

The reservoir-aquifer system can be classified based on flow geometry as edge water drive, bottom water drive and linear water drive based on the direction of water encroachment into the reservoir. In edge water drive, water encroaches through the flanks of the reservoir when the reservoir

pressure declines due to production. In bottom water drive, water encroach the reservoir in vertical direction from the bottom especially when the aquifer completely underlies the reservoir. In linear water drive, water encroach from one side of the reservoir in linear direction. Figure 5 illustrate in detail (Ahmed, 2006).

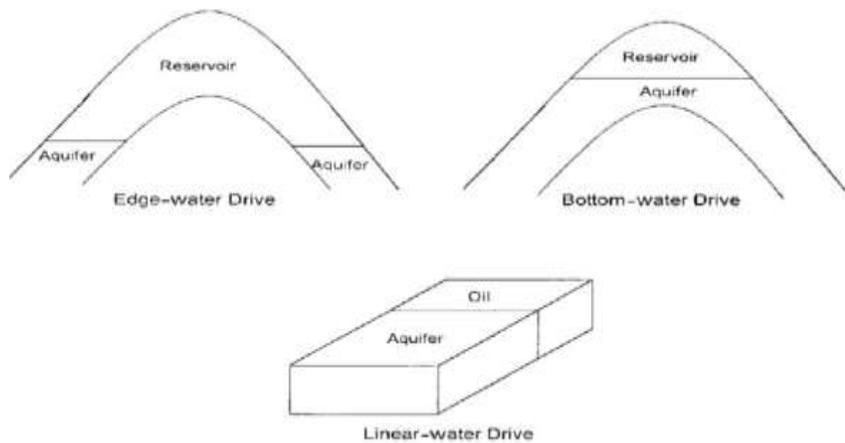


Fig 2.14 flow geometries describing water influx into the reservoir (Ahmed, 2006).

2.1.3.4 Outer boundary conditions:

The reservoir-aquifer classification based on outer boundary condition can be infinite or finite. Infinite system is when the pressure changes at the reservoir aquifer boundary is not felt at the aquifer boundary. While the finite system is when the change of pressure at the reservoir-aquifer boundary is felt at the aquifer boundary (Ahmed, 2006).

2.1.3.5 Degree of pressure maintenance:

The reservoir-aquifer classification based on the degree of pressure maintenance can be grouped into three categories. Active water drive, partial water drive and limited water drive. In active water drive system, there is

100 percent voidage replacement, meaning that the rate of water influx is equals to the total production rate. The plot of oil rate versus time on the semi logarithm scale tends to be flat as shown in fig 2.15 (AAPG, 2016).

Furthermore, it is possible to have a reservoir-aquifer system with combination of the categories. For example, an aquifer in which water encroaches into the reservoir from the bottom can be of significant larger size to be regarded as an infinite.

2.1.4 Factors Causing Water Influx into the Reservoir:

In response to the pressure drop due to production, and when the reservoir is hydraulically connected with an aquifer and the size of the aquifer is larger enough with high permeability, water in the aquifer begins to expand and moves into the reservoir to offset the pressure decline. Apart from expansion of water in the aquifer, there are other factors causing water influx into the reservoir. These include expansion of other known or unknown accumulation of hydrocarbon in the aquifer rock, compressibility of the aquifer rock and artesian flow especially when the water bearing formation is located structurally high than the pay zone (Craft, Applied Petroleum Reservoir Engineering (2nd ed.), 1991).

2.1.5 Significance of Estimating Water Influx into the Reservoir:

The accurate estimation of water influx into the petroleum reservoir is very important in various applications such as material balance calculations, reservoir simulation studies, production scheduling and setting up development strategies to optimize hydrocarbon recovery (Jassim A. Al-Ghanim, Prediction of Water Influx of EdgeWater Drive Reservoirs Using Nonparametric Optimal Transformations, February 2012). For example, in water drive reservoir, estimation of initial oil in place or amount of oil

produced at specific interval of time requires amount of water influx into the reservoir to be known. Likewise, in reservoir simulation studies, inclusion of aquifer into reservoir simulation model can help to reduce model uncertainties in case when water influx into the reservoir is significant.

CHAPTER THREE

METHODOLOGY

Water influx models are mathematical models that simulate and predict cumulative water influx into the reservoir. Various researchers have proposed models that estimates cumulative water influx into the reservoir. The following are some of models applied in estimating water influx into the reservoir listed based on authors.

3.1 Pot aquifer:

In this model, the aquifer pressure is assumed to be at equilibrium with the boundary pressure. The model is valid when the fluid transmissibility between the aquifer and the reservoir is very large. The simplest model that can be to estimate the water influx into a gas or oil reservoir is based on the basic definition of compressibility. A drop in the reservoir pressure, due to the production of fluid, causes the aquifer water to expand and flow into the reservoir.as shown in equation.(Ahmed 2006)

The compressibility is defined mathematically as:

$$\Delta V = c V \Delta p \quad (3.1)$$

Applying the above basic compressibility definition to the aquifer gives:

$$W_i = \frac{\pi(ra^2 - re^2)h\theta}{5.615} \quad (3.2)$$

$$W_e = (c_w + c_f) W_i (p_i - p) \quad (3.3)$$

Where:

W_e = cumulative water influx, bbl.

c_w = aquifer water compressibility, psi^{-1}

c_f = aquifer rock compressibility, psi^{-1}

Wi = initial volume of water in the aquifer, bbl.

pi = initial reservoir pressure, psi

p = current reservoir pressure

ra = radius of the aquifer, ft.

re = radius of the reservoir, ft.

h = thickness of the aquifer, ft.

φ = porosity of the aquifer.

3.2 Schilthuis's steady- state:

The model holds the steady state condition, that is the rate of change of pressure is equal to zero, and assumes the aquifer volume is very large than the reservoir volume such that the pressure at the external boundary of aquifer remains constant at initial pressure throughout the entire field life (Craft, Hawkins, & Terry, 1991; Fekete, 2014; Petrowiki, 2015). The rate of water influx into the reservoir is proportional to the pressure drawdown,

$(p_i - p)$ and can be determined by using Darcy equation as shown in Equation 3.5 (Craft, Hawkins, & Terry, 1991; Ahmed, 2006).

$$\frac{d_{we}}{dt} = ew = \left\{ \frac{0.000708kh}{\mu_w \ln\left(\frac{r_a}{r_e}\right)} \right\} (p_i - p) \quad (3.4)$$

$$\frac{dwe}{dt} = ew = C(p_i - p) \quad (3.5)$$

Where:

ew = rate of water influx, bbl/day.

k = permeability of the aquifer, md.

h = thickness of the aquifer, ft.

r_a = radius of the aquifer, ft.

r_e = radius of the reservoir.

t = time, days.

The parameter C is called the water influx constant and is expressed in bbl/day/psi.

3.3 Hurst's modified steady –state:

Hurst (1943) proposed that the “apparent” aquifer radius r_a would increase with time and, therefore the dimensionless radius r_a/r_e may be replaced with a time dependent function, as:(Ahmed 2006)

$$r_a/r_e = a_t \quad (3.6)$$

$$e_w = \frac{dw_e}{dt} = \left[\frac{0.00708 kh}{\mu_w \ln\left(\frac{r_a}{r_e}\right)} \right] (p_i - p) \quad (3.7)$$

The Hurst modified steady-state equation can be written in a more simplified form as:

$$ew = \frac{W_e}{dt} = \frac{C(p_i - p)}{\ln(at)} \quad (3.8)$$

$$W_e = c \int_0^t \left[\frac{0.00708 kh (p_i - p)}{\mu_e \ln(a_t)} \right] dt \quad (3.9)$$

3.4 The van Everdingen steady state

The van Evergreen and Hurst model represents a mathematical model that estimate the cumulative water influx into the reservoir by using superposition principle. The authors solved the radial diffusivity equation for water influx into the reservoir by applying Laplace transformation. The

detail of radial diffusivity equation and its derivation of the solution can be found through the following paper (van Everdingen & Hurst, 1949). The model is applicable for determining water influx of the following systems: edge water-drive system, bottom water drive system and linear water-drive system. The authors proposed solutions to the dimensionless diffusivity equation shown in Equation 3.10 for constant terminal rate and constant terminal pressure boundary conditions (Klins, Bouchard, & Cable, 1988; Ahmed, 2006).

In constant terminal rate boundary condition, the rate of water influx at the reservoir aquifer boundary is assumed to be constant and pressure drop at the interface of the reservoir aquifer system is calculated as a function of time.

While, for constant terminal pressure boundary condition, the constant pressure drop is assumed over finite period and water influx is calculated. In addition, various researchers recommend calculation of water influx into the reservoir-aquifer boundary rather than pressure (Dake, 1978; Klins, Bouchard, & Cable, 1988; Craft, Hawkins, & Terry, 1991; Ahmed, 2006). This is because water influx into the reservoir is a function of time and pressure drop at the inner boundary condition of reservoir-aquifer system.

$$\frac{\partial P_D}{\partial r_D} + \frac{1}{r_D} \frac{\partial P_D}{\partial r_D} = \frac{\partial P_D}{\partial t_D} \quad (3.10)$$

$$\text{Dimensionless radius:} \quad r_D = \frac{r_a}{r_e} \quad (3.11)$$

$$\text{Dimensionless pressure} \quad :$$

$$P_D = \frac{(P_i - p)}{p_i - p_{wf}} \quad (3.12)$$

Total compressibility: $c = c_w + c_f$ (3.13)

Dimensionless time: $t_d = 0.0002637 \frac{kt}{\phi \mu c_t r_d^2}$ (3.14)

Dimensionless water influx for infinite aquifer is:

$$w_e = B \Delta p w_{ed} \quad (3.15)$$

With:

$$B = 1.119 \phi c_t r_e^2 h$$

assuming that the water is encroaching in a radial form then we introduce the encroachment angle to the water influx constant B as:

$$f = \frac{\phi}{360} \quad (3.17)$$

$$B = 1.119 \phi c_t r_e^2 h f \quad (3.18)$$

Where:-

We = cumulative water influx, bbl.

B = water influx constant, bbl/psi.

Δp = pressure drop at the boundary, psi.

WeD = dimensionless water influx.

CHAPTER FOUR

4.1 Introduction:

In this chapter a validity check will be done to verify the designed program results and then the designed program should be used to calculate cumulative water influx for given case study.

4.2 Error Calculation:

To calculate an error, the calculation will be done manually and using program as shown below:

4.2.1 Manual Calculation:

For the following data it is required to calculate the cumulative water influx after 24 months. Using the data given in table to calculate the cumulative water influx at the end of 6, 12, 18, and 24 months.

Table (4.1) the reservoir aquifer system characterized by following properties.

Aquifer properties	value
radius, ft	2000
h, ft	25
k, md	100
ϕ , %	20
μ_w , cp	0.8
c_w , psi ⁻¹	0.7×10^{-6}
c_f , psi ⁻¹	0.3×10^{-6}

The predicted boundary pressure at the end of each specified time period is given below:

Table (4.2): Boundary pressure vs time

Time, month	Boundary pressure, psi
0	2500
6	2490
12	24720
18	2444
24	2408

calculate the cumulative water influx at the end of 6, 12, 18, and 24 months using the following steps.

Water influx after 6 months:

$$B = (1.119) (0.2) (1 \cdot 10^{-6}) (2000) * (2000) * (25) (360/360)$$

$$B = 22.4 \text{ bbl/psi}$$

$$t_D = 6.328 * 10^{-3} \frac{100t}{(0.8)(0.2)(1 * 10^{-6})(2000)^2}$$

$$t_D = 0.9888t = 0.9888 (182.5) = 180.5$$

WeD at $t_D = 180.5$ to give:

$$\text{WeD} = 69.46$$

$$\Delta p_1 = \frac{p_i - p_1}{2} = \frac{2500 - 2490}{2} = 5 \text{ psi}$$

Calculate the cumulative water influx at the end of 182.5 days due to the first pressure drop of 5 psi:

$$\text{We} = B * \text{WeD} * \Delta p$$

$$\text{We} = (22.4) (5) (69.46) = 7779.52 \text{ bbl}$$

Cumulative water influx after 12 months:

$$\Delta p_2 = \frac{p_i - p_2}{2} = \frac{2500 - 2472}{2} = 14 \text{psi}$$

$$t_D = 0.9888 (365) = 361, \text{ WeD} = 123.5$$

$$(\text{We}) \Delta p_1 = (22.4)(5)(123.5) = 13832 \text{bbl}$$

$$(\text{We}) \Delta p_2 = (22.4)(14)(69.46) = 21782.65 \text{bbl}$$

$$\text{We} = 13832 + 21782.65 = 35614.65 \text{bbl}$$

Water influx after 18 months

$$\Delta p_3 = \frac{p_1 - p_3}{2} = \frac{2490 - 2444}{2} = 23 \text{psi}$$

$$t_D = 0.9888 (547.5) = 541.5, \text{ WeD} = 173.7$$

$$(\text{We}) \Delta p_1 = (22.4) (5) (173.7) = 19454.4 \text{bbl}$$

$$(\text{We}) \Delta p_2 = (22.4) (14) (123.5) = 38729.6 \text{bbl}$$

$$(\text{We}) \Delta p_3 = (22.4) (23) (69.4) = 35754.88 \text{bbl}$$

$$\text{We} = 19454.4 + 38729.6 + 35754.88 = 93938.88 \text{bbl}$$

Water influx after 24 months:

$$\Delta p_4 = \frac{p_2 - p_4}{2} = \frac{2472 - 2408}{2} = 32 \text{psi}$$

$$t_D = 0.9888 (730) = 722, \text{ WeD} = 221.8$$

$$(\text{We}) \Delta p_1 = (22.4) (5) (221.8) = 24841.6 \text{bbl}$$

$$(\text{We}) \Delta p_2 = (22.4) (14) (173.7) = 54472.32 \text{bbl}$$

$$(\text{We}) \Delta p_3 = (22.4) (23) (123.5) = 63627.2 \text{bbl}$$

$$(\text{We}) \Delta p_4 = (22.4) (32) (69.4) = 49745.92 \text{bbl}$$

$$\text{We} = 24841.6 + 54472.32 + 63627.2 + 49745.92 = 192687.04 \text{bbl}$$

4.2.2 Program Calculation:

The same above mentioned data are entered to the designed program, when click calculate bottom a group of mathematical operation is performed including reading the data of van Everdingen table and the interpolation of dimensionless water influx and also superposition concept all these operations are performed in a second that is means reducing calculation time and increase calculation accuracy.

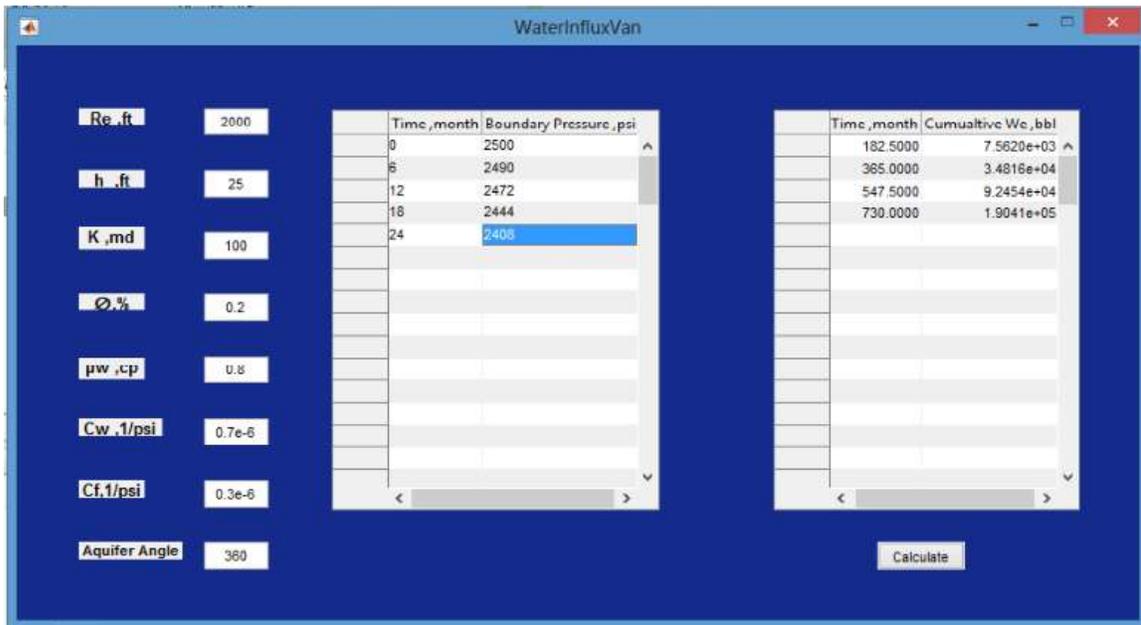


Figure (4.1): Program Calculations

4.3 Comparisons:

The table below show the difference between calculated value of cumulative water influx estimated manually and using the designed program value:

Table (4.3) The difference between calculated value and designed program value.

We at end of specified time	Manual	Program	Error %
We at end of 6 month	7779.52	7562	2.8
We at end of 12 month	35614.65	34816	2.24
We at end of 18 month	93969.8	92454	1.6
We at end of 24 month	192687.04	190410	1.18

A.A.E is 1.9 %

4.4 Case study

The following data are available for X field it is required to calculate the cumulative water influx after 120 months.

Table (4.4) the following data are available for X field.

Aquifer properties	Value
radius, ft	2600
h, ft	25
k, md	100
ϕ , %	12
μ_w , cp	0.18
cw, psi ⁻¹	5×10^{-6}
cf, psi ⁻¹	3×10^{-6}

Table (4.5): X field boundary pressure history

Time, month	Boundary pressure, psi
0	3600
6	3360
12	3125
18	2996
24	2966
30	2940
36	2860
42	2850
48	2842
54	2830
60	2800
66	2796
72	2790
78	2784
84	2774
90	2766
96	2754
102	2740
108	2733
114	2726
120	2720

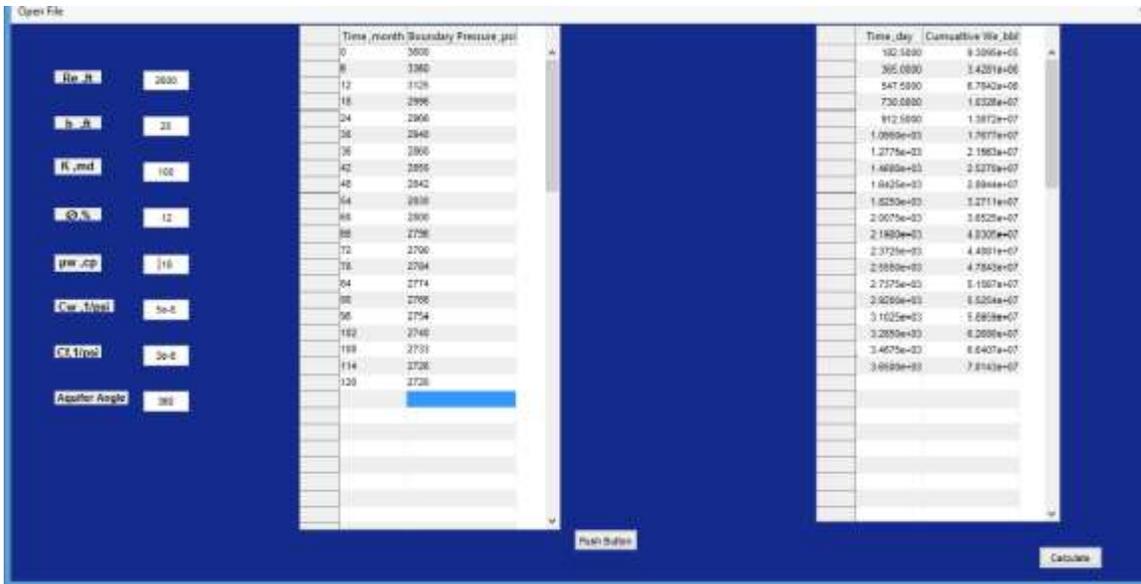


Figure (4.2): Cumulative water influx after 120 months

CHAPTER FIVE

Conclusion and Recommendations

5.1 Conclusion:

- 1) The program is designed and absolute average error has been calculated as 1.9 %.
- 2) The cumulative water influx for X field is calculated after 120 months using the designed program and it is equal to (7.0143e+7bbl)

5.2 Recommendations:

- 1) It is recommended to develop this program to including the calculation of cumulative oil production by using MBE prediction.
- 2) Production data to be used for material balance analysis should be carefully obtained.
- 3) Microsoft excel can be used for calculations to be able to avoid human error and inaccurate result.

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