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كلية الدراسات العليا

Evolution of the Inverse Fine Structure beyond the Standard Model

نشأة معكوس البنية الدقيقة في ما بعد النموذج القياسي

A Dissertation submitted to the college of graduate studies, in partial fulfillment of the requirements for the degree of M. Sc. in Physics.

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الآيـة

قال تعالى:

(قُلْ سِيرُوا فِي الْأَرْضِ فَانْظُرُوا كَيْفَ بَدَأَ الْخَلْقَ ۚ ثُمَّ اللَّهُ يُنْشِئُ النَّشْأَةَ الْآخِرَةَ ۚ إِنَّ اللَّهَ عَلَىٰ كُلِّ شَيْءٍ قَدِيرٌ)

صدق الله العظيم

سورة العنكبوت، الآية (20)

Dedication

To:

My parents,

Colleagues

And friend

With deep love

Author

Acknowledgment

I wish to express my sincere gratitude and appreciation to Dr: Ammar Ibrahim Abdalgabar, for his advice, assistance and encouragement throughout the study.

I am very grateful to my mother for her support and help, during all my study period.

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Abstract

In this dissertation, the one-loop renormalization group equations for the gauge couplings in five and six dimensional models as well as 5D Minimal Super-symmetric Standard Model were derived. We considered different localization of matter fields that is, all fields propagate in the full space-time (bulk case), or some of the fields restricted in the brane (brane case). We studied the evolution of the inverse fine structure constantsα^{-1}, which is linked to the gauge couplings constant. We found that the three inverse fine structure constants approximately unified at some high-energy scale at $E = 10^{4.31} GeV$ and $E = 10^{3.60} GeV$ in five and six dimensional models respectively. We also compare the 5D SM result with 6D SM results, and we found that there is only one different which is, the running in 6D SM is faster than 5D SM. Furthermore, a successful unification of the three gauge couplings was achieved in 5D MSSM plus additional fields in both scenarios. We found a precise unification is achieved at energy scale$E = 10^{4.48} GeV$.

ملخص البحث

في هذه الأطروحة، تم اشتقاق معادلات مجموعة المعايرة لحلقة واحدة لثوابت البنية الدقيقة في 5Dو 6D أبعاد وكذلك في نموذج التناظر الفائق في 5D. حيث تم اعتماد حالات مختلفة لمجالات المادة التي تمثل جميع الحقول المنتشرة في الوقت والفراغ الكامل (brane case) أو بعض الحقول المقيدة . وتم دراسة نشاة الثوابت التركيبية الدقيقة والتي ترتبط بثابت القياس للقوي. وجد أن مقلوب ثوابت البنية الدقيقة الثلاثة توحدت تقريبًا في بعض نطاق الطاقة العالية عند $E = 10^{4.31} GeV$ و $E = 10^{3.60} GeV$ في 5Dو 6D أبعاد على التوالي .قورنت نتيجة 5D SMمع نتائج6D SM ، ووجد أن هناك اختلاف في نشاة الثوابت   6D SM وهي أسرع من 5D SM.علاوة على ذلك، تم تحقيق توحيد ناجح لثوابت المقياس الثلاثة في 5D MSSM مضاف اليها مجالات إضافية في كلا الحالتين .وتم ايجاد توحيد دقيق عند نطاق الطاقة $E = 10^{4.48} GeV$.

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Chapter One

Introduction

1.1 Introduction:

The standard model (SM) of particle physics is currently accepted theory and its predictions have been examined experimentally, however the standard model doesn't answer many problems, such as the origin of fermion masses and their associated mixing angles, gauge couplings unification, hierarchy problem that is why there is a huge different between the weak scale and Planck scale, in other word why the weak scale is much smaller than Planck scale,. These issues require the existence of new physics beyond the standard model, among other model super-symmetry (SUSY) is assumed the best candidate for theory beyond the standard model. Super-symmetry it is minimal version called Minimal Super-symmetric Standard Model (MSSM) naturally solve some issues with the SM. It solved the gauge couplings unification free and solves the hierarchy problem; furthermore, the lightest super-symmetry particle is a good candidate for dark matter.

One of the advantage feature of extra-dimensional models is the effect of the huge number of KK modes on the renormalization group (RG) running of physical parameters. The RG running in extra-dimensional models has been investigated and studied see for example (N.~Maru, 2010) and reference therein. It has been proven that the RG evolution changes from the usual logarithmic running in four-dimensional standard models to an effective power-law running at high energies in extra dimension. This means that a sizable running could take place at relatively low energy scales. As such extra space-time dimensions naturally lead to unification of gauge coupling constants at intermediate mass scales.

1.2 Problem of the study:

Many unified theories have been proposed, but we need data to choose which, if any, of these theories describes nature. In many grand unified theory the gauge couplings constants (which define the electromagnetic, weak and strong interactions or forces, are combined into one single force.) are predicted to meet at some high-energy unification scale. In the standard model the gauge; couplings do not meet at single point. However, the unification works very well in Super-symmetry theory but at high scale approximately$10^{16}\mathrm{GeV}$, such a high-energy scale is beyond the reach of any present or future experiments. Extra dimensions offer power law running, that brings down the unification scale to low energy range (N.~Maru, 2010).

1.3 Objectives of the Study:

There are many different ways to build a realistic models with an extra dimensional space-time, the easiest one is the universal extra dimension (UED) model in which case all particles (bulk case) or some (brane case) do propagate in higher dimension space time. Therefore, we will study the unification of gauge coupling constants in various scenarios from the standard model all the way to six dimensional models.

1.4 Outline of the Study:

The outline of the dissertation is as follow:

We introduce in chapter one a general introduction of particle physics. Chapter two will discuss the theory of the standard model, super-symmetry and extra dimension models in five and six dimensional models. In chapter three, we will apply the technique of renormalization group equations to the evolution of gauge couplings for different field localization. Chapter four devoted to our numerical results, discussions and conclusions.

Chapter Two

The Standard Model

2.1 Introduction:

In this chapter we shall introduce the standard model of elementary particle physics, and discuss of its related issues, we also present beyond the standard model, such as super-symmetric and extra dimension models.

2.2 What is Standard Model?

The SM is the theory that describes the interaction between elementary particles and based on gauge group theory. As we mentioned earlier that its predictions have been tested experimentally to a high level of accuracy, such as the structures of the neutral and charged current, which agree with experiments? The SM asserts that the material is made up of elementary fermions interacting through fields; the particles associated with interacting fields are called bosons mediator (Guigg, 1983).

The different kinds of elementary particles in the SM are given as follows:

$$\text{Quarks} = \binom{u}{d}\ ,\binom{c}{s}\binom{t}{b}, \tag{2.1}$$

$$\text{Leptons} = \binom{\nu_e}{e},\binom{\nu_\mu}{\mu},\binom{\nu_\tau}{\tau} \tag{2.2}$$

$$Gauge\ bosons \left\{ \begin{pmatrix} photon\ \gamma \\ Weak\ gauge \\ gluon\ g \end{pmatrix} W^{\pm}, Z^0, Higges\ boson\ H \right\}$$

Where H responsible of mass

The material in our universe is assumed to be built from small number of fundamental constituents, the quarks and the leptons, all of them are fermions, they have electrical charges and spin ($\frac{1}{2}$). For each of these particles there is an antiparticle with the opposite value of electric charge and magnetic moment ,but with identical mass and lifetime.

Quarks interact through strong interaction (with colors), weak interaction and electromagnetic (with charge) (L.F.Li, 1991).

Leptons interact through weak interaction and electromagnetic interaction (with charge).

Higgs boson have spin (0), is introduced by Higgs mechanism to give mass to elementary particles. The spontaneous symmetry breaking predicted it and must be scalar and neutral (Quigg, 2007).

Gauge boson have spin (0 or 1), the two or more of them can exist in the same place at the same time unlike fermions also interact with the Higgs field. Some gauge bosons like $W^{\pm}$ and Z^0 have mass while others such as photons do not have mass as we will see later (Falcone, 2002).

Table 2.1: Building Blocks of elementary particles and some of their quantum number:

Name	Spin	Baryon Number B	Lepton Number L	Charge Q
Quarks				
U (up)	$\frac{1}{2}$	$\frac{1}{3}$	0	$+\frac{2}{3}$
D (down)	$\frac{1}{2}$	$\frac{1}{3}$	0	$-\frac{1}{3}$
Leptons e (electron) ν (neutrino)	$\frac{1}{2}$ $\frac{1}{2}$	0 0	1 1	-1 0
Gauge bosons	1	0	0	0

γ (photon) $W^{\pm}$, Z (weak bosons)	1	0	0	±1,0
g, (i=1,000,8 gluons)	1	0	0	0

2.3 The Lagrangian of the Standard Model:

The complete SM lagrangian, which is based on gauge group, $SU(3)_C \times SU(2)_L \times U(1)_Y$ is given by:

$$L_{SM} = L_{Gauge} + L_{Fermion} + L_{Higgs} + L_{Yukawa} + L_{Gauge.fixing} + L_{Ghost} \qquad (2.3)$$

We shall now briefly present in details the contents of each sector of this lagrangian.

2.3.1 Fermion Sector:

The SM contains three copies of chiral fermions (called generations) with different gauge transformation. The fermion lagrangian follows the usual covariant Dirac form as:

$$L_{Fermion} = \sum_{\psi} i\bar{\psi}\gamma_{\mu}D^{\mu}\psi \qquad (2.4)$$

With the covariant derivatives read as:

$$D_{\mu}\binom{u}{d}_L = [\partial\mu - ig_s \frac{\lambda^a}{2} G_{\mu}^a - \mathrm{i}g\frac{\sigma^a}{2} W_{\mu}^a - \mathrm{i}g^{`} \frac{1}{6} B_{\mu}]\binom{u}{d}_L \qquad (2.5)$$

$$D_{\mu}u_R = [\partial\mu - ig_s \frac{\lambda^a}{2} G_{\mu}^a - \mathrm{i}g^{`} \frac{2}{3} B_{\mu}]u_R, \qquad (2.6)$$

$$D_{\mu}d_R = [\partial\mu - ig_s \frac{\lambda^a}{2} G_{\mu}^a - \mathrm{i}g^{`} \frac{1}{3} B_{\mu}]d_R, \qquad (2.7)$$

$$D_\mu \binom{\nu}{e}_L = [\partial\mu - \text{ig}\frac{\sigma^a}{2} W_\mu^a + \text{i}g^` \frac{1}{2} B_\mu] \binom{\nu}{e}_L, \tag{2.8}$$

And

$$D_\mu e_R = [\partial\mu + ig^` B_\mu] e_R \tag{2.9}$$

Where γ_μ are the usual Dirac matrices, $g^`$ is the coupling strength of the hypercharge interaction , Y is the hypercharge , σ^a are the generators of SU$(2)_L$ (simply the three Pauli matrices), and λ^a are generators of SU$(3)_C$ (the eight Gell Mann matrices) (Guigg, 1983).

2.3.2 Gauge Boson Sector:

The gauge boson sector contain 12 gauge fields which mediate the interactions among the fermion field the photon (γ, mediates the electromagnetic interactions), the three weak gauge bosons ($W^\pm$ and Z) mediate the weak interactions and eight gluons ($g_\propto$,$\propto=$ 1,2, … . ,8)mediate the strong interactions (A.J.G.hey, 1993). The gauge field dynamics are embedded in the lagrangian in terms of field strength tensors as:

$$L_{Gauge} = -\frac{1}{4} G_{\mu\nu}^A G^{A\mu\nu} - \frac{1}{4} W_{\mu\nu}^a W^{a\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} \tag{2.10}$$

Where the first term is given by:

$$G_{\mu\nu}^A = \partial_\mu G_\nu^A - \partial_\nu G_\mu^A - ig_s f^{ABC} G_\mu^B G_\nu^C \tag{2.11}$$

Where g_s is the coupling strength of the strong interaction, A, B, C run from 1to 8 and f^{ABC} are the (ant symmetric) structure constants ofSU (3). In addition, the second term represents

$$W_{\mu\nu}^a = \partial_\mu W_\nu^a - \partial_\nu W_\nu^a - \text{ig} \in^{abc} W_\mu^b W_\nu^c \tag{2.12}$$

a, b, c Run from 1 to 3and $\in^{abc}$ is the very ant-symmetric three – index tensor with $\in^{abc}= 1, g$ is the coupling strength of the weak interaction.

While for the field strength of the $U(1)_Y$ gauge boson has the same form as electromagnetism and given by:

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \tag{2.13}$$

2.3.3 The Higgs Mechanism:

As was mentioned before, fermion mass term will violate the gauge symmetries of our standard model lagrangian. As such, we need some how to gives mass to the SM particles and keep the lagrangian invariant under gage symmetries. This can be achieved through the mechanism of spontaneous gauge symmetry breaking also known as the Higgs mechanism, this mechanism assumed the existence of a new complex scalar field Φ which is a doublet under the $SU(2)_L$ group , and singlet with respect to SU$(3)_c$ with hyper charge $y_\Phi = 1$ (P.~W.~Higgs, 1964)

$$\Phi = \begin{pmatrix} \Phi^\dagger \\ \Phi^0 \end{pmatrix} = \begin{pmatrix} \Phi_1 + i\Phi_2 \\ \Phi_3 + i\Phi_2 \end{pmatrix} \tag{2.14}$$

Where$\Phi_1 \Phi_2 \Phi_3$andΦ_4 are real scalar. This new scalar Φ add extra terms to the SM lagrangain:

$$L_{Higgs} = \left(D_\mu \Phi\right)^\dagger (D^\mu \Phi) - V(\Phi) \tag{2.15}$$

Where the covariant derivative $D_\mu$is defined as:

$$D_\mu = \partial_\mu - i\frac{g^`}{2} B_\mu - ig\frac{\sigma^a}{2} W_\mu^a \tag{2.16}$$

The most general gauge invariant renormalizable potential for the new scalar field Φ is given by:

$$V(\,\Phi) = -\frac{1}{2}\mu^2\,\Phi^\dagger\,\,\Phi + \frac{\lambda}{4} \tag{2.17}$$

We choose the vacuum expectation value (VEV) in the neutral direction as the photon is neutral, so Φ becomes:

$$\langle\Phi\rangle = \frac{1}{\sqrt{2}}\binom{0}{\nu} \qquad (2.18)$$

With this particular choice of the ground state, the electroweak gauge group $\mathrm{SU}(2)_L \times U(1)_Y$ is broken to electromagnetism U $(1)_{em}$

$$SU(2)_l \times U(1)_Y \xrightarrow{<\Phi>} U(1)_{em} \qquad (2.19)$$

2.3.4 Yukawa Sector:

Yukawa interactions represent the couplings between fermion doublets and the scalar fieldΦ. Fermion masses originate from Yukawa interactions after the spontaneous symmetry breaking take place (Quigg, 2007) (P.~W.~Higgs, 1964).

Yukawa coupling is uniquely fixed by gauge invariance and the lagrangian, as given by:

$$L_{Yukawa} = Y_{ij}^{d} q_l^{-i} \Phi d_R^i + Y_{ij}^{u} \Phi^{\sim} U_R^j + Y_{ij}^{e} l_L^{-i} \Phi e_R^j + h.c \qquad (2.20)$$

Where Y is 3× 3 complex matrices, the so-called Yukawa coupling constants, h.c. means the Hermitian conjugate and $\Phi^{\sim}$ is defined by:

$$\widetilde{\Phi} = \begin{pmatrix} \overline{\Phi_2^*} \\ \Phi_1^* \end{pmatrix} \qquad (2.21)$$

2.4 Problems with the Standard Model:

Although not the most successfully theory of particles physics, the SM does not give enough explanation for number of theoretical and experimental observations (Majee, March, 2008). Some of them are mention below.

2.4.1 Gravity is not included:

Despite the fact that the unification of electromagnetic and weak interaction was a achieved in the SM and the strong interaction appears to be part of the unification, the SM does not include the effects of gravity, the gravity effect might be important at energies of the order of the Planck scale, $M_{PL} = 10^{18}$ GeV (J.donoghue, 1994).

2.4.2 The Hierarchy Problem:

The hierarchy problem is the equation of why there is such a huge difference between the electroweak scale $M_{Ew} = 0\ (100)$ GeV and the Planck scale $mpl = 0$ (10^{18}) GeV. This is also known as the naturalness problem (Weinberg S. , 1996).

2.4.3 Dark Matter:

The SM explains about 5% of the energy present in the universe. About 26% should be dark matter, which would behave just like other matter, but only interacts via weak interaction with the SM. The SM does not supply any fundamental particles that are good dark matter candidates (Collaboration], 2011).

2.4.4 Neutrino Masses:

According to the SM, neutrinos are massless particles. However, neutrino oscillation experiments have shown that neutrinos do have mass (A.D.Martin, 1984).

2.4.5 Matter Antimatter Asymmetry:

The SM predicts that matter and antimatter should have been created in equal amounts if the initial conditions of the universe in evolve disproportionate matter relative to antimatter. No mechanism sufficient to explain this problem (Veltman, 2003).

2.5 Beyond the Standard Model

Physics beyond the SM (BSM) refers to the theoretical developments needed to explain some unanswered issues in SM raised in previous section, such as super-symmetric, extra dimensions or a combination between them 5D super-symmetric (H.~-U.~Yee, 2003). Here in this dissertation we will try to study all of them to search for unification.

2.5.1 Super-symmetry:

Super-symmetry is space-time symmetry, which relates the bosonic degrees of freedom to the fermion degrees of freedom, (Ammar Ibrahim Abdalgabar, May 21,2014). The good idea of super-symmetric helps to solve the gauge hierarchy problem (J. Louis, 1998).

In super-symmetric transformation a boson transform to a fermion and vice versa thus if Q is the generator transformation then:

$$\mathrm{Q}|\mathrm{fermion}> \equiv |boson> \ and \ Q|boson> \equiv |fermion> \qquad (2.22)$$

Table 2.2: Super-symmetric partners with the SM members:

Names		Spin 0	Spin $\frac{1}{2}$	SU (3),SU(2) ,U(1)Y
Squares	Q	($\tilde{u}_l$ d_l)	(u_l d_l)	(3,2,$\frac{1}{6}$)
Quarks	Ũ	$u_R^\dagger$	$u_R^\dagger$	($\overline{3}$,1,$\frac{-2}{3}$)
	$\tilde{d}$	$d_R^\dagger$	$d_R^\dagger$	($\overline{3}$,1,$\frac{1}{3}$)
S leptons	L	($\tilde{\nu}$ $e_L^{\sim}$)	(ν e_L)	(1,2,$\frac{-1}{2}$)
Leptons	$\overline{e}$	$e_R^{\sim *}$	$e_R^\dagger$	(1,1,1)
Higgs	H_1	($H_1^\dagger$ H_1^0)	($H_1^{\sim +}$ $H_1^{\sim 0}$)	(1,2,$\frac{+1}{2}$)
Higgins	H_2	(H_2^0 H_2^-)	($H_2^{\sim 0}$ $H_2^{\sim -}$)	(1,2,$\frac{-1}{2}$)
		Spin $\frac{1}{2}$	Spin 1	
Gluion,gluon		$\tilde{g}$		(8,1,0)
Winos , w-boson		$W^{\pm}$,W^0	$W^{\pm}$,W^0	
Bion, b-boson		$B^{\sim 0}$	B^0	(1,1,0)

2.5.2 Extra Dimensions:

In the SM, the hierarch problem is arising due to the hug ratio of the Planck scale, M_{PL} or GUT scale, M_G to the electroweak scale. In order to solve the hierarchy problem, if we incorporate any new physics it should appear around that scale to address the hug ratio. More recently, new kind of physics, Extra Dimension (ED), was introduced in particle physics. Historically, Kaluza and Klein dated this idea back in 1920, to unify the electromagnetic interaction with the gravitational one by generating the photon from the extra components of the five dimensional metric (T.~Kaluza, 1921).

2.5.3 Universal Extra Dimension (UED):

A model in which all the SM particles are allowed to access the extra dimensions is known as the Universal Extra Dimension (UED) model also known as the ACD model after its proposers Appelquist, Cheng and Dobrescu (T.~Appelquist, 2001). The UED is model which place particles of the standard model in the bulk of one or more compactified extra dimension, also makes for an interesting TeV scale physic scenario, as it features a tower of Kaluza Klein (KK) state for each of the SM fields all of which have full access to the extended space time manifold (H.~-U.~Yee, 2003). In next subsection, we will each particle lagrangian in this model.

2.5.3.1 Scalar Particle in ED:

In addition to the four space-time co-ordinates $X(x,t)$, let us denote the extra space-type co-ordinate by y, compactified on circle of radius R. Thus, the lagragian of a free complex scalar $\Phi(x,y)$ with mass m will be a function of both x and y co-ordinates with a condition that at y=2πR, will match with that at y=0, i.e. it has a periodicity of 2πR along the y direction. So one can us Fourier series expansion

$$\Phi(x,t) = \frac{1}{\sqrt{2\pi R}}\Phi^0(x) + \sum_{n=1}^{\infty}\frac{1}{\sqrt{\pi R}}\left[\Phi_n^+(x)\cos\left(\frac{ny}{R}\right) + \Phi_n^-(x)\sin\left(\frac{ny}{R}\right)\right] \qquad (2.25)$$

The five dimensional actions is given by

$$S_\Phi^5 = \frac{1}{2}\int d^4x\, d_y\{\, (\partial^A\Phi)^\dagger(\partial_A\Phi) - m^2\phi^\dagger\,\phi \,\} \qquad (2.24)$$

With $A = 0,1,2,3,5$

With the use of equation (2.34) if we replace the scalar field Φ and integrate out the extra dimension y, then the action will correspond to large number Kaluza–Klein (KK) modes as

$$S_{\Phi}^{5} = \frac{1}{2}\int d^4x \left\{(\partial^A\Phi^0)^\dagger(\partial_A\Phi^0) - m^2\Phi^{0\dagger}\,\Phi^0\right\}$$

$$+ \int d_x^4 \left\{(\partial^\mu\Phi_n^-)^\dagger(\partial_\mu\Phi_n^-) - m^2\Phi_n^{-\dagger}\,\Phi_n^-\right\}, \quad (2.25)$$

Where the n-th KK state mass is given as

$$M_n^2 = \frac{m^2+n^2}{R^2} \quad (2.26)$$

In four-dimensional effective theory, thus, in addition to the zero mode field, we are getting two different sets –one is even and another odd under the transformation $y \to -$y of field when the extra space dimension is compactified on the circle S^1 (H.~-U.~Yee, 2003).

2.5.3.2 Fermion Particle in ED:

In some models, only the scalar bosons are allowed to access the extra dimensions while fermions are kept in a fixed point of the extra dimension, called "brane". In such cases, the above compactification is quite natural but what happens if we intend to allow the fermions as well to have access the extra dimension. Do we have the same set of Kaluza-Klein states for the fermion fields or something else?

Let us consider a fermion in the five-dimensional field, where the extra space dimension is compactified in the way as we discussed in previous section. The five-dimensional spinor can be written as a two component four-dimensional spinor:

$$\Psi = \begin{pmatrix}\Psi_R \\ \Psi_L\end{pmatrix} \quad (2.27)$$

Note that in the five-dimensional field theory, one can construct the five Γ^A matrices with

A=0, 1,2,3,5, from the usual four-dimensional ones as follows:

$$\Gamma_\mu = \Upsilon_\mu \quad and\, \Gamma^5 = i\Upsilon^5 \tag{2.28}$$

In 5D, the fifth component of the Γ_A is constructed from the γ^5 matrix, which is used, in four dimensions, to define the chiral operator $P_{R/L} = (1 \pm \gamma^5)$. So, in five-dimensions, and it is true for any odd number of dimensions, there is no chiral operator. To be clear, in $P_{R/L}$ the Subscripts L and R are just two component notations only.

Let us consider the action for a massless fermion as:

$$S = \int d^4x\, dy\, i\bar{\Psi}\Gamma^A\partial_A\Psi = \int d^4x\, dy\left(i\bar{\Psi}\Upsilon_\mu\partial_\mu\Psi + \bar{\Psi}\Upsilon^5\partial y\Psi\right) \tag{2.29}$$

Due to the symmetry of the fermion field at the point y $Y = 0$ and $Y = 2\pi R$, we can have the Fourier series expansion of the fermion field as:

$$\Psi_{L/R}(x, y) =$$

$$\frac{1}{\sqrt{2\pi R}}\Psi^0_{L/R}(x) + \sum_{n=1}^{\infty}\frac{1}{\sqrt{\pi R}}\left\{\Psi^{+n}_{L/R}(x)\, cos\left(\frac{ny}{R}\right) + \Psi^{-n}_{L/R}(x,y)\, sin\left(\frac{ny}{R}\right)\right\} \qquad (2\cdot 30)$$

Once we put these fermions into eqn. (2.29), we end up with a few phenomenological problems. For instant, let us use the zero mode term in eqn. (2.29), then we have

$$\sim \int d^4X\, \left\{\Psi^{0-}_L i\Upsilon_\mu\partial_\mu\Psi^0_L + \Psi^{0-}_R i\Upsilon_\mu\partial_\mu\Psi^0_R\right\} \tag{2.31}$$

Thus, for each massless field in five-dimension we are having two massless zero modes in the four-dimensional effective theory. The four-dimensional fermion is thus vector like in nature.

It is well known that fermions in the SM are chiral in nature, the left chiral part transforms as a doublet under $SU(2)$ gauge transformations and the right chiral part transforms trivially. If the dimensional reduction doubles the state, can we regain our chiral nature of the fermion in its zero modes.

To regain the chiral nature we have to compactify on an $\frac{S_1}{Z_2}$ orbifold instead of a circle. The expansions of different kind of field for the $\frac{S_1}{Z_2}$ orbifold will be discussed. In that case, although the higher KK modes of the chiral fermion behave as vector but the zero modes remains a chiral one.

2.5.3.3. Gauge fields and Gauge fixing in ED:

The lagrangian for an Abelian gauge field and gauge fixing given by:

$$\mathcal{L}_{Gauge+GF} = \int dx_5(-\frac{1}{4}F^{MN}F_{MN} - \frac{1}{2\xi}(\partial_\mu A^\mu - \xi(\partial_5 A_5)^2 \qquad (2.32)$$

Where ξ is the gauge fixing parameter and $F_{MN} = \partial_M A_N - \partial_N A_M$. The gauge fixing term eliminates the mixing between $A_\mu$and the extra polarization A_5.

In the Feynman-'t Hooft gauge $\xi = 1$, the equations of motion for A_5can be obtained by subject the action under variation principle**:**

$$(\partial_5^2 - \partial_\mu^2)A_5 = 0 \qquad (2.33)$$

$$A_\mu^a(x,y) = \frac{1}{\sqrt{2\pi R}}A_\mu^{(0)a}(x) + \sum_{n=1}^{\infty}\frac{1}{\sqrt{\pi R}}\left[A_\mu^{(n)a}(x)\cos(\frac{ny}{R})\right] \qquad (2.34)$$

Where

$$A_\mu^a(x,y) = (G_\mu^a(x,y), W_\mu^a(x,y), B_\mu(x,y)) \qquad (2.35)$$

The effective 4D lagrangian after integrating the fifth coordinate yield

$$
\begin{aligned}
\mathcal{L}_{4D} \quad = \quad & -\frac{1}{4}\Big(G_{\mu\nu}^{(0)a}G^{(0)a\mu\nu} + G_{\mu\nu}^{(n)a}G^{(n)a\mu\nu} \ + 2G_{\mu 5}^{(n)a}G^{(n)a\mu 5}\Big) \\
& -\frac{1}{4}\Big(W_{\mu\nu}^{(0)a}W^{(0)a\mu\nu} + W_{\mu\nu}^{(n)a}W^{(n)a\mu\nu} + 2W_{\mu 5}^{(n)a}W^{(n)a\mu 5}\Big) \\
& -\frac{1}{4}\Big(B_{\mu\nu}^{(0)}B^{(0)\mu\nu} + B_{\mu\nu}^{(n)}B^{(n)\mu\nu} \\
& + 2B_{\mu 5}^{(n)}B^{(n)\mu 5}\Big) \qquad (2\cdot 36)
\end{aligned}
$$

Chapter Three

Renormalization Group Equations (RGEs) For Gauge Coupling Constants

3.1 Introduction:

This chapter will discuss the Renormalization Group Equations (RGES) that will be used in our calculations. We will supply all our REGs for various models that are considered in this dissertation.

3.2 Renormalization and Renormalization Group Equations:

Before we calculate our desired RGEs, we want define the RGEs. The renormalization group in quantum field theory (GFT), tell us how different couplings change with energy. What is the renormalization? In QFT, the green function is a most important quantity to be calculated. In perturbative QFT, these quantities are divergent. The systematic to isolate these divergences is known as the renormalization. There are different methods to cancel the infinities. In order to renormalize the theory we need reference point to start with, which is also arbitrary. Different choices of this reference point will lead to different sets of parameters for the theory, but physics should not depend on the arbitrary choice of the reference point and must be invariant. This invariance leads to the renormalization group (Collins, 1984).

In QFT, it is a useful to examine the behavior of physics parameter at different scale knowing the same at it other scale. Thus, measuring the observable will allow energy experiment, one can compare with the values predicted from a theory at higher scale, e.g. at the GUT scale and certify about the correctness of the theory (Weinberg S. , 1996).

3.3 RGEs for 4D SM Case:

The renormalization group equations are:

$$16\pi^2 E \frac{dg_i}{dE} = b_i g_i^3 = \beta_{SM}(g_i) \tag{3.1}$$

Where i stand for $\mathrm{U}(1)_Y$, $\mathrm{SU}(2)_L$ and $\mathrm{SU}(3)_C$ and the right-hand-side is known as the β function of corresponding coupling.

$$b_{G=}^{i}\left[\frac{11}{3}C_2(G)-\frac{4}{3}n_g C(R)-\frac{1}{3}n_h C_2(R)\right] \tag{3.2}$$

Where n_g the number of Fermions and n_h the number of Higgs scalar, the$C_2(G)$, C(R), C_2(R) refer to the gauge boson fermions ,and Higgs scalar contribution respectively. We will show below how to calculate equation (3.2) by using evaluating Feynman diagrams in figure below, the calculation of all these diagrams are similar, so we will do the detail calculation of figure (a) and we give the result for other diagrams as can be found in many text book and articles (L.~-X.~Liu A. a., 2011) and reference therein.

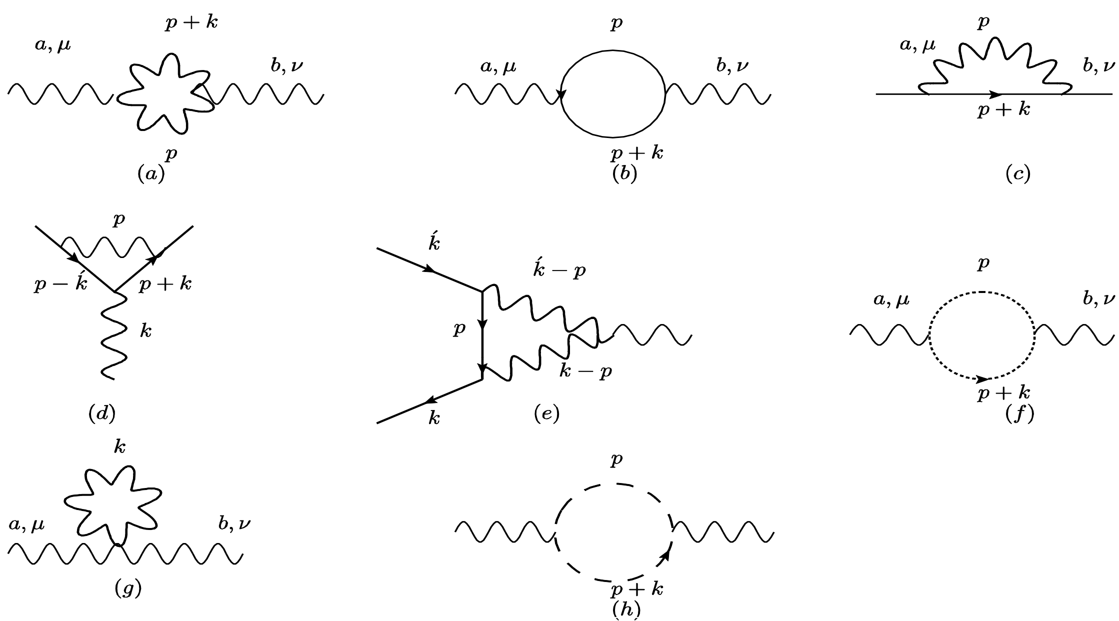

Figure (3.1): show all diagrams contributing to self-gauge boson

We will calculate the contribution from figure (3.1 a) in details: Using Feynman rules (Guigg, 1983) and converts it to an integral we get:

$$F_a = \int_{-\infty}^{+\infty} \frac{d^4p}{(2\pi)^2} Tr[-ig\gamma^{\mu}\lambda^{a} \times \frac{ig^{\mu\nu}}{(p+k)^2 - m^2} \times -ig\gamma^{\nu}\lambda^{b} \times \frac{ig^{\mu\nu}}{p^2 - m^2} \qquad (3.3)$$

$$F_a = g^2 Tr(\lambda^a \lambda^b) \int_{-\infty}^{+\infty} \frac{d^4p}{(2\pi)^2} Tr\left[\frac{\gamma^{\mu} g^{\mu\nu}}{(p+k)^2 - m^2} \times \frac{\gamma^{\nu} g^{\mu\nu}}{p^2 - m^2}\right] \qquad (3.4)$$

Put:

$$Tr(\lambda^a \lambda^b) = C_2(R), \qquad and\ Tr(\gamma^{\mu}\gamma^{\nu}) = 4g^{\mu\nu} \qquad (3.5)$$

Then equation (3.4) becomes:

$$F_a = \frac{4g^2 (g^{\mu\nu})^3 C_2(R)^2}{(2\pi)^4} \int_{-\infty}^{+\infty} d^4p\ Tr\left[\frac{1}{((p+k)^2 - m^2)(p^2 - m^2)}\right] \qquad (3.6)$$

To calculate the above integral we using Feynman integral parameterization:

$$\frac{1}{ab} = \int_{0}^{1} dz \frac{1}{[b + (a-b)z]^2} \qquad (3.7)$$

Now let us call:

$$a = (p+k)^2 - m^2 = p^2 - 2pk + k^2 - m^2, b = p^2 - m^2 \qquad (3.8)$$

From equations (3.7) and equation (3.8) equation (3.6) becomes:

$$F_a = \frac{4g^2 (g^{\mu\nu})^3 C_2(R)^2}{(2\pi)^4} \int_{-\infty}^{+\infty} d^4p\ \left[\frac{1}{[(p^2 - m^2) + (2pk + k^2)z]^2}\right] \qquad (3.9)$$

Now let:

$$D = [(p^2 - m^2) + (2pk + k^2)z]^2 \qquad (3.10)$$

Introduce new variable q:

$$q = p + kz \rightarrow p = q - kz \qquad (3.11)$$

Then equation (3.10) can be written as:

$$D = [(q - kz)^2 - m^2 + (2(qk - k^2z) + k^2)z]^2 \tag{3.12}$$

After a little algebra, the above equation simplified to:

$$D = [q^2 + k^2z(1-z) - m^2]^2 = [q^2 + a]^2 \tag{3.13}$$

Where:

$$a = k^2z(1-z) - m^2 \tag{3.14}$$

Insert equation (3.13) in equation (3.9) we obtain:

$$F_a = \frac{4g^2(g^{\mu\nu})^3 C_2(R)^2}{(2\pi)^4} \int_{-\infty}^{+\infty} d^4p \left[\frac{1}{q^2 + a}\right] \tag{3.15}$$

Using the standard integral of beta function:

$$\int_{-\infty}^{+\infty} d^4p \left[\frac{1}{q^2 + a}\right]^n = \frac{i(\pi)^{\frac{d}{2}}\Gamma\left(n - \frac{d}{2}\right)}{a^{2-\frac{d}{2}}} \tag{3.16}$$

but we have $\mathrm{n} = 2, \mathrm{d} = 4 - \varepsilon$ (3.17)

Substituting equation (3.17) in equation(3.16), we get:

$$\int_{-\infty}^{+\infty} d^4p \left[\frac{1}{q^2 + a}\right]^n = \frac{i\pi^2\pi^{-\frac{\varepsilon}{2}}\Gamma\frac{\varepsilon}{2}}{a^{\frac{\varepsilon}{2}}} = i\pi^2\pi^{-\frac{\varepsilon}{2}}\Gamma\frac{\varepsilon}{2}\int_0^1 a^{-\frac{\varepsilon}{2}}\,dz \tag{3.18}$$

We have also:

$$\Gamma(\frac{\varepsilon}{2}) = \frac{2}{\varepsilon} - \gamma, a^{-\frac{\varepsilon}{2}} = 1 - \frac{\varepsilon}{2}\log a \tag{3.19}$$

Then equation (3.18) becomes:

$$\int_{-\infty}^{+\infty} d^4p \left[\frac{1}{q^2+a}\right] = i\pi^2\pi^{-\frac{\varepsilon}{2}}(\frac{2}{\varepsilon} - \gamma)\int_0^1(1 - \frac{\varepsilon}{2}\log a)dz = i\pi^2\pi^{-\frac{\varepsilon}{2}}\left(\frac{2}{\varepsilon} - \gamma\right) \tag{3.20}$$

Therefore, our figure (3.1a) gives us:

$$F_a = \frac{4g^2(g^{\mu\nu})^3 C_2(R)}{16\pi^4}\left[i\pi^2\pi^{-\frac{\varepsilon}{2}}\left(\frac{2}{\varepsilon} - \gamma\right)\right] = \frac{4ig^2(g^{\mu\nu})^3 C_2(R)\left(\frac{2}{\varepsilon} - \gamma\right)}{16\pi^4\pi^{\frac{\varepsilon}{2}}} \tag{3.21}$$

Other figures give us:

$$F_b = i(k^2 g^{\mu\nu} - k^\mu k^\nu)\delta^{ab}\left[\frac{-g^2}{(4\pi)^2}\frac{4}{3}n_f C(r)\right]\left(\Gamma\left(2-\frac{d}{2}\right)\right) \tag{3.22}$$

$$F_c = i\frac{g^2}{(4\pi)^2}[C_2(R)\not{k}]\Gamma\left(2-\frac{d}{2}\right) \tag{3.23}$$

$$F_d = \frac{ig^3}{(4\pi)^2}t^a\gamma^\mu\left[C_2(r) - \frac{1}{2}C_2(G)\right]\Gamma\left(2-\frac{d}{2}\right) \tag{3.24}$$

$$F_e = \frac{3}{2}\frac{ig^3}{(4\pi)^2}[C_2(G)t^a\gamma^\mu]\Gamma\left(2-\frac{d}{2}\right) \tag{3.25}$$

$$F_f = \frac{ig^2 C_2(G)\delta^{ab}}{6(4\pi)^2}\left[\frac{g^{\mu\nu}k^2}{2} + k^\mu k^\nu\right]\Gamma\left(2-\frac{d}{2}\right) \tag{3.26}$$

$$F_g = 0 \tag{3.27}$$

$$F_h = \frac{-ig^2 C_2(R)\delta^{ab}}{6(4\pi)^2}[g^{\mu\nu}k^2 - k^\mu k^\nu]\Gamma\left(2-\frac{d}{2}\right) \tag{3.28}$$

Therefore summing all these contributions yield:

$$b_G^i = \left[\frac{11}{3}C_2(G) - \frac{4}{3}n_g C(R) - \frac{1}{3}n_h C_2(R)\right] \tag{3.29}$$

For the representation R;$C_2(G)$, $C(R)$ and $C_2(R)$ refer to the gauge bosons, Fermions and Higgs scalar contributions, respectively. Normally gauge bosons are in the adjoint representation of the group (for SU (N),$C_2(G) = N$). The fermions and the Higgs fields are in the fundamental representation; we choose for the fundamental representation in which $C(R) = C_2(R) = \frac{1}{2}$.

3.3.1. Numerical Coefficients b_G^i in the SM ($SU(3) \times SU(2) \times U(1)$):

Consider first the strong interaction $SU(3)$, here which $C_2(G) = 3$, $C(R) = \frac{1}{2}$, $n_g = 3$ and $C_2(R) = 0$ because the Higgs fields does not carry color. Substitute these group factors in to equation (3.29), we get:

$$b_3 = \left[\frac{11}{3}\times 3 - \frac{4}{3}\times 3\times 1\right] = 11 - 4 = 7 \tag{3.30}$$

Now for the weak interaction $SU(2)$; we have which $C_2(G) = 2$, $C(R) = C_2(R) = \frac{1}{2}$ and $n_g = 3$, $n_h = 1$, we obtain:

$$b_2 = \left[\frac{11}{3} \times 2 - \frac{4}{3} \times 3 \times 1 - \frac{1}{3} \times 1 \times \frac{1}{2}\right] = \frac{44 - 24 - 1}{6} = \frac{19}{6} \tag{3.31}$$

Finally we consider the $U(1)$ hypercharge, here we set $C_2(G) = 0$, $C(R) = C_2(R) = (\frac{Y}{2})^2$ and $n_g = 3, n_h = 1$ yields:

$$b_1 = -\frac{4}{3} \times 3 \left[2 \times \left(\frac{1}{6}\right)^2 \times 3 + \left(\frac{2}{3}\right)^2 \times 3 + \left(-\frac{1}{3}\right)^2 \times 3 + 2 \times \left(-\frac{1}{2}\right)^2 + (-1)\right] \times \frac{1}{2}$$
$$-\frac{1}{3} \times \left(\frac{1}{2}\right)^2 \times 2 = -\frac{41}{6} \tag{3.32}$$

b_1 should be normailized according to SU(5) unified model by the factor $\left(\frac{3}{5}\right)$.

Therefore:

$$b_1 = -\frac{41}{6} \times \frac{3}{5} = -\frac{41}{10} \tag{3.33}$$

$$\therefore b_G^{SM} = \left[-\frac{41}{10}, \frac{19}{6}, 7\right] \tag{3.34}$$

3.3.2. RGEs for Gauge Couplings in Five Dimensional (Bulk case)

We shall consider places all SM particles in the bulk of one, or more compactified extra dimensions as result all the fields have KK expansions. The zero-mode will be associated as the standard model fields and the n^{th} will be the new excited KK modes and will contribute at energy$\geq R^{-1}$.

The gauge coupling constants RGEs in ED generally will run with energy scale as

$$16\pi^2 \frac{dg_i}{dt} = b_i^{SM} g_i^3 + (S(t) - 1) b_i^{5D} g_i^3 \tag{3.35}$$

The beta-function coefficients b_i^{SM} are those of the usual SM given in (3.34), which are the contribution of the zero-mode, while the coefficients b_i^{5D} originates from the n^{th} new excited KK modes and are given by $b_i^{5D} = \left(\frac{81}{10}, -\frac{7}{6}, \frac{5}{2}\right)$, and,

$S(t) = m_Z R e^t = \mu R$, represent the sum of all new excited KK modes in the energy interval $m_Z < \mu < \Lambda$ (Λ is the cut-off scale in which the couplings constant unified). The contributions of new excited KK modes come from A_5^n fields belonging to the adjoint representation as pictured in figure (3.2). Therefore, these new fields will modify our RGEs and change equation (3.29) to the following equation:

$$b_G^{5D} = \left[\frac{11}{3}C_2(G) - \frac{8}{3}n_g C(R) - \frac{1}{3}n_h C_2(R) - \frac{1}{6}C_2(G)\right] \tag{3.36}$$

Where the first term is the contribution of A_μ^n and the second term is the contribution of $\Psi_{\square/R}^n$ (it is different from the zero mode by factor 2 this is because the fermions appear twice in Fourier expansion). The third term comes from the Φ^n Higgs bosons; the last term is originated from our new fieldsA_5^n.

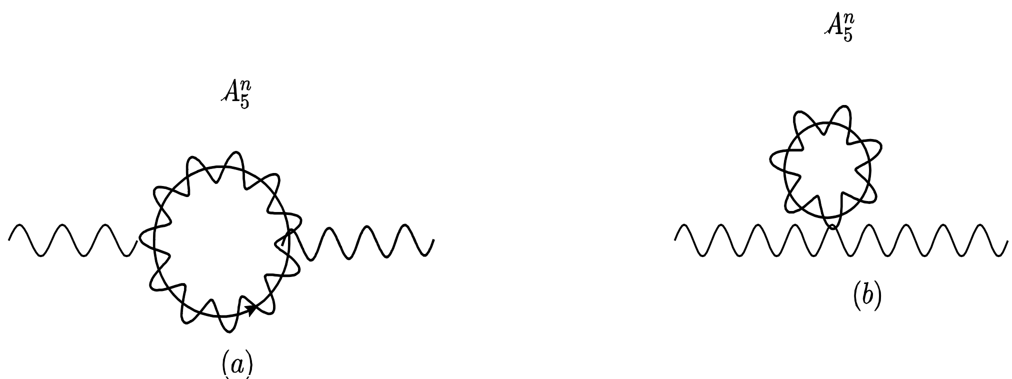

Figure (3.2): Contribution of KK states to the self-gauge bosons

Similarly, we obtain the numerical coefficients in ED as:

$$\mathrm{b_i^{5D}} = \left(\frac{81}{10}, -\frac{7}{6}, \frac{5}{2}\right) \tag{3.37}$$

3.3.3. RGEs for Gauge Couplings in Five Dimensional (Brane case):

Now for this case, we assumed some gauge and the Higgs live in the bulk and matter fields (fermions) live in the brane. The zero-mode will be associated as the standard model fields (as usual) and the n^{th} will be the new excited KK modes if the particle live in the bulk. However, if the particles live in the brane the do not have KK modes and will not contributes to the ED. As such, our numerical coefficients generally in ED will be modified to

$$b_G^{5D} = \left[\frac{11}{3}C_2(G) - \frac{1}{6}C_2(G) - \frac{1}{3}n_h C_2(R)\right] - \frac{8}{3}\eta \tag{3.38}$$

Where η is the number of fermions generations among interaction and takes the values ($\eta = 0,1,2,3$): Thus in ED the numerical coefficients is summarized into

$$b_G^{5D} = \left(-\frac{1}{10}, \frac{41}{6}, \frac{21}{2}\right) - \frac{8}{3}\eta \tag{3.39}$$

Note that if we assume all the matter fields live in the bulk as well in which case $\eta = 3$ leads to equation (3.37). So for the brane case $\eta = 0$, we get $b_G^{5D} = \left(-\frac{1}{10}, \frac{41}{6}, \frac{21}{2}\right)$.

3.3.4. RGEs for Gauge Couplings in Six Dimensional Model (Bulk and Brane cases):

In six dimensional models the theory is not that different from the five dimensional the only different that we can mention is that instead of A_5^n in five dimensional case will have two adjoint fields A_5^n and A_6^n and the way we compactify the six dimensional to yield the effective four dimensional SM. Therefore, the RGEs running in six dimensional model is similar to five dimensional models and will be in general:

$$16\pi^2 \frac{\mathrm{dg_i}}{\mathrm{dt}} = \mathrm{b_i^{SM} g_i^3} + (S(t)^2 - 1)\mathrm{b_i^{6D} g_i^3} \tag{3.40}$$

Where

$$b_G^{6D} = \left[\frac{11}{3}C_2(G) - \frac{2}{6}C_2(G) - \frac{1}{3}n_h C_2(R)\right] - \frac{8}{3}\eta \tag{3.41}$$

$$b_G^{6D} = \left[-\frac{1}{10}, \frac{13}{2}, 2\right] - \frac{8}{3}\eta \tag{3.42}$$

The numerical coefficients b_G^{6D} for the two scenarios bulk $\eta = 3$, and for the brane case $\eta = 0$.

3.3.5. Numerical Coefficients b_G^i in the MSSM ($SU(3) \times SU(2) \times U(1)$):

Now the calculation of the coefficients b_G in four dimensions MSSM is given by:

$$b_G^{MSSM} = \left[\frac{11}{3}C_2(G) - \frac{2}{3}C_2(G) - \frac{4}{3}n_g C(R) - \frac{2}{3}n_g C(R) - \frac{1}{3}n_h C_2(R) - \frac{2}{3}n_h C_2(R)\right] \tag{3.43}$$

Where first two terms present the SM gauge bosons and gauginos contributions respectively, the third and fourth terms is the contribution of the SM fermions and sfermions respectively, and the last two terms correspond to the Higgs and Higgsinos contributions respectively. Plugging the values of $C_2(G), C(R)$ and $C_2(R)$for each group into the above equation, we obtain

$$b_G^{MSSM} = \left[-\frac{33}{5}, -1, 3\right] \tag{3.44}$$

3.3.6. Numerical Coefficients b_G^i in 5D MSSM ($SU(3) \times SU(2) \times U(1)$):

The RGES for five dimensional MSSM (Brane and Bulk cases) follow:

$$16\pi^2 \frac{dg_i}{dt} = b_i^{MSSM} g_i^3 + (S(t) - 1) b_i^{5DMSSM} g_i^3 \tag{3.45}$$

Where the numerical coefficients $b_G^{5D\ MSSM}$ is given by:

$$b_G^{5D\ MSSM} = \left[-\frac{11}{3}C_2(G) + \frac{2}{3}C_2(G) + C_2(G) + 4\eta C(R) + 2n_h C_2(R)\right] \tag{3.46}$$

Here the first two terms is the gauge bosons and gauginos contributions respectively, the third is the contribution from the super-field χ scalar and fermion, the fourth terms correspond to η generations of matter contribution and the last term is the Higgs super-field (mirror to 4D MSSM) and Higgs super-field Φ^c contribution. Substituting the values of $C_2(G), C(R)$ and $C_2(R)$for each group into the above equation, we get:

$$b_G^{5DMSSM} = \left[\frac{6}{5}, -2, -6\right] + 4\eta \tag{3.47}$$

3.3.7. RGEs for 4D MSSM Case+$F^{\pm}$ $\frac{Y}{2} = \pm 1$:

We assumed singlet particle named by $F^{\pm}$it has hypercharge= ± 1 in 4D MSSM. Calculate the coefficientb_1:

$$b_1 = [\frac{11}{3} C_2(G) - \frac{2}{3} C_2(G) - \frac{4}{3} n_g (\frac{Y}{2})^2 - \frac{2}{3} n_g (\frac{Y}{2})^2 - \frac{1}{3} n_h (\frac{Y}{2})^2 - \frac{2}{3} n_h (\frac{Y}{2})^2]$$

$$= -\frac{40}{6} - \frac{20}{6} - 1 - 2 = -\frac{39}{5} \tag{3.48}$$

b_1 should be normailized according to SU(4) unified model by the factor $\left(\frac{3}{5}\right)$

$$b_1 = -\frac{39}{5} \tag{3.49}$$

b_2 and b_3 do not change.

Chapter Four

Numerical Results, Discussions and Conclusions

4.1 Numerical Results

We employed the technique of the RGEs at on-loop level for inverse fine structure constants beyond the SM; we performed numerically the RGEs by using dedicated numerical packages Mathematical version 9. For our numerical evaluation, we have chosen the compactification scale to be $R^{-1} = 1$ TeV with the initial values adopted at the M_Z scale as shown in table 4.1. In five dimensional and six dimensional models, we considered different possibilities of localization for the matter fields, such as the case of bulk or brane localized fields. We will discuss in both scenarios the evolution of the inverse fine structure constants, which is related to the gauge couplings by the relation $\alpha^{-1} = 4\pi/g^2$. Note that in brane case the SM chiral fermions are located on a boundary and do not have Kaluza-Klein (KK) states that contribute to our RGEs only zero mode will contributes. The SM Higgs live in the bulk and the gauge fields live in the bulk. We will also compare 5D and 6D models. Furthermore, we also present the evolution of the inverse fine structure constants in 5D MSSM and its extensions with additional fields.

Table 4.1 shows the initial values at $\mathbf{M_z}$ scale (where$\mathbf{M_z = 91.1876GeV}$) used in our numerical results. Data is taken from Ref (Z.~-z.~Xing H. a., 2008)**.**

Parameter	**Value (90% CL)**
$g_1(M_z)$	0.461655
$g_2(M_z)$	0.651434
$g_3(M_z)$	1.21978

4.2 Discussions

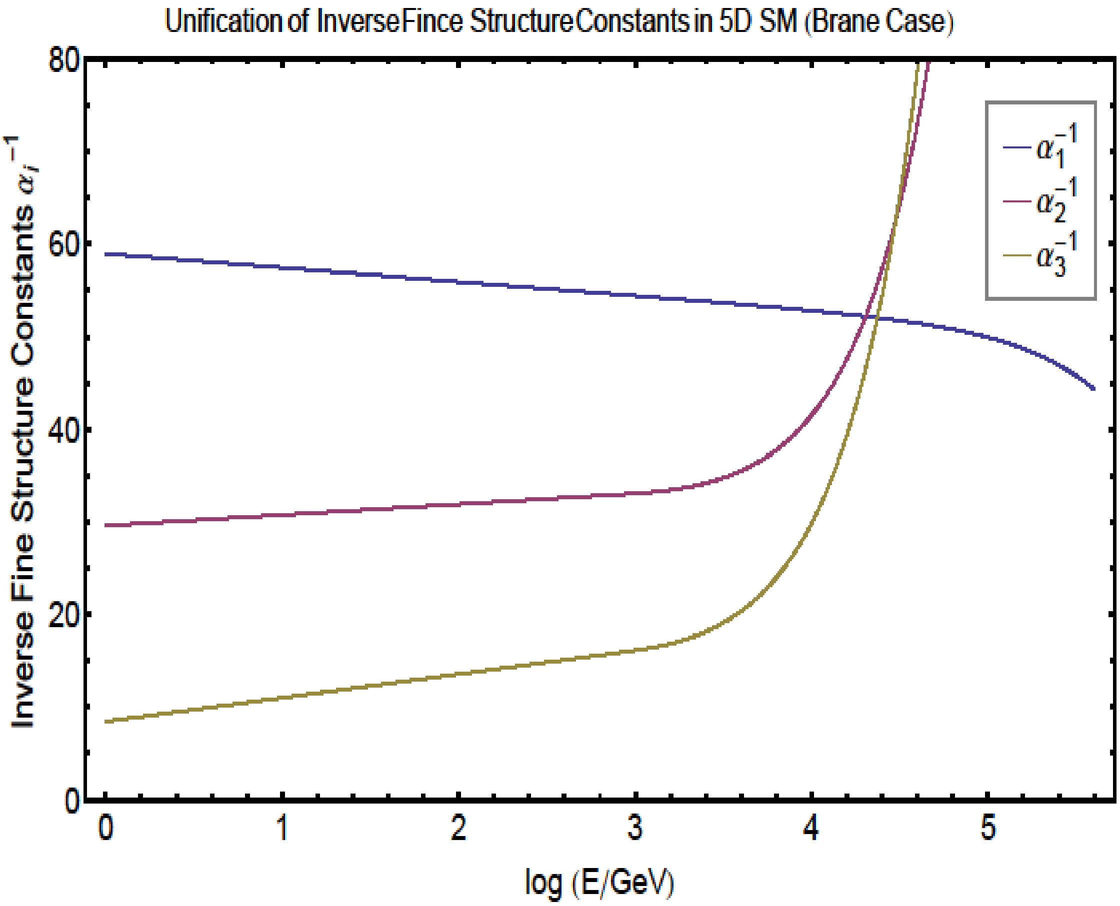

Figure (4.1): present the unification of inverse fine structure constants in 5D standard model (brane case) as function of energy scale for compactification radius$R^{-1} = 1TeV$.

Extra dimensions suggested the existence of new particle (KK particles) which contributes at energy$E \geq R^{-1}$, as one cross the threshold energy$E = R^{-1}$ the KK particle will contribute to our RGEs for the gauge couplings constant and their effect are shown in figure 4.1 and figure 4.2 Note that, the three inverse fine structure constants approximately unify at some high-energy scale. Nevertheless, α_i^{-1} unify approximately at $E = 10^{4.31} GeV$. The only different between brane and bulk case in five dimensions (see figure 4.1 and 4.2) is that the running of α_2^{-1} is increased in brane localized matter field and decreases in the bulk scenario.

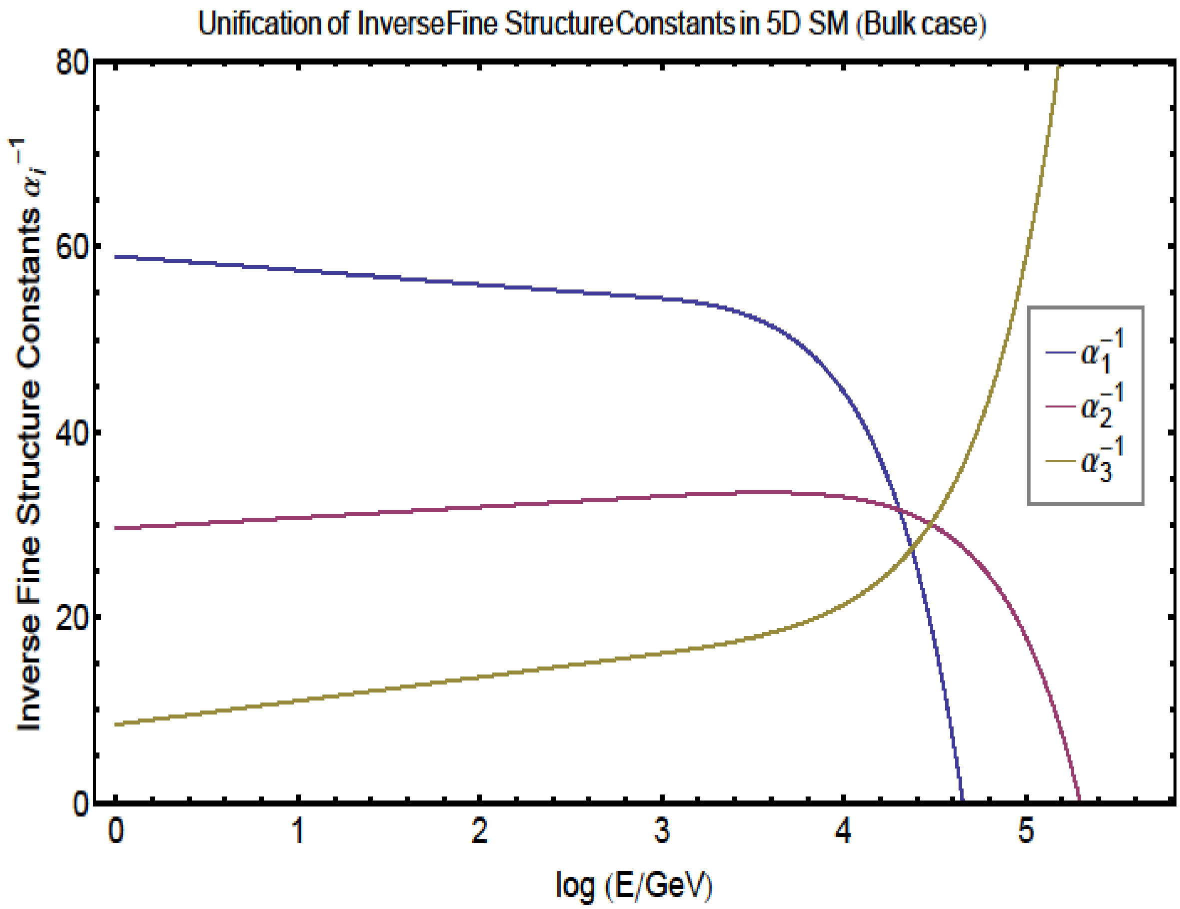

Figure (4.2) present the evolution of inverse fine structure constants in 5D standard model (bulk case) as function of energy scale for compactification radius$R^{-1} = 1TeV$.

We show in figure 4.3 and figure 4.4 the evolution of the inverse fine structure constants in the brane-localized fields and bulk fields scenarios for compactification scale $1\,TeV$ for the six extra-dimensional models. We can see that the three coupling constants, as expected in extra dimensional theories, unify at some value. As depicted in these figures, for $1\,TeV$ we see approximate unification at$E = 10^{3.60}GeV$.

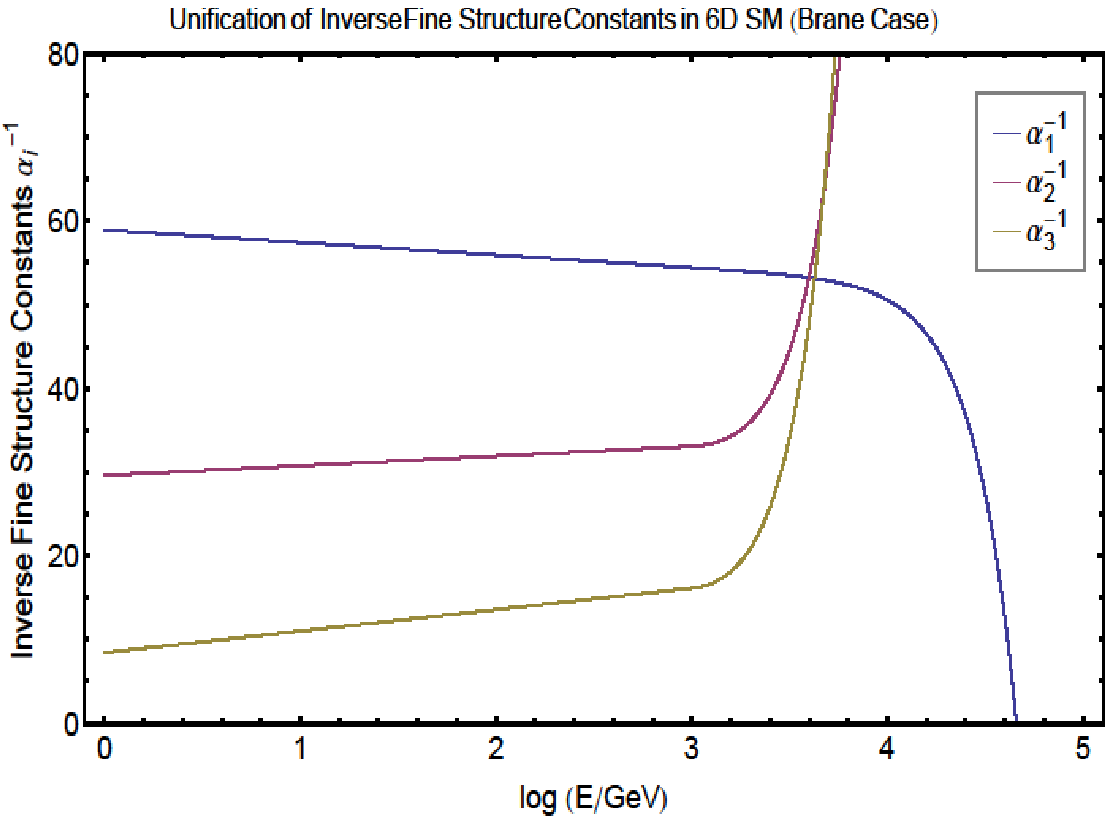

Figure (4.3): the evolution of inverse fine structure constants in 6D standard model (brane case) as function of energy scale for compactification radius$R^{-1} = 1TeV$.

In comparison between the evolution of coupling constants in five and six dimensional models for a compactification scale of 1 TeV, we see that in both cases the coupling constants have similar behavior; however, in the six dimensional case we get asymptotes at lower $E = 10^{3.60} GeV$ values, that is, a lower energy scale compare to five dimensional case where $E = 10^{4.31} GeV$. As such, the range of validity for the six dimensional models is less than the five dimensional models; this because of the $S^2(t)$ factor appeared in equation (3.25), there is only a linear dependence on five dimensional models.

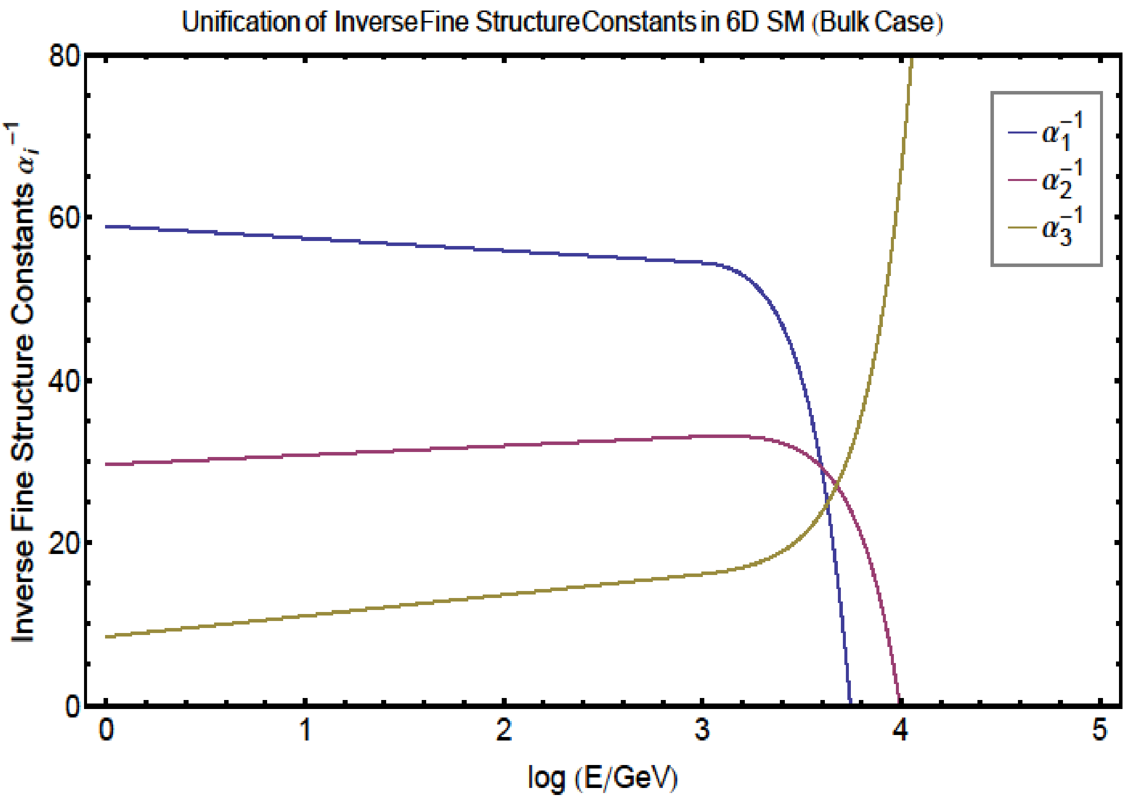

Figure (4.4): the evolution of inverse fine structure constants in 6D standard model (bulk case) as function of energy scale for compactification radius $R^{-1} = 1TeV$.

We have studied also the super-symmetry in extra dimension plus additional field in order to have a precise unification that is all the couplings constants meet at single point-called the unification scale. We plot in figure 4.5 the evolution of inverse fine structure constants in $5D\ MSSM + F^{\pm}$. As can be seen in this figure, the precise unification is achieved at energy scale $E = 10^{4.48} GeV$.

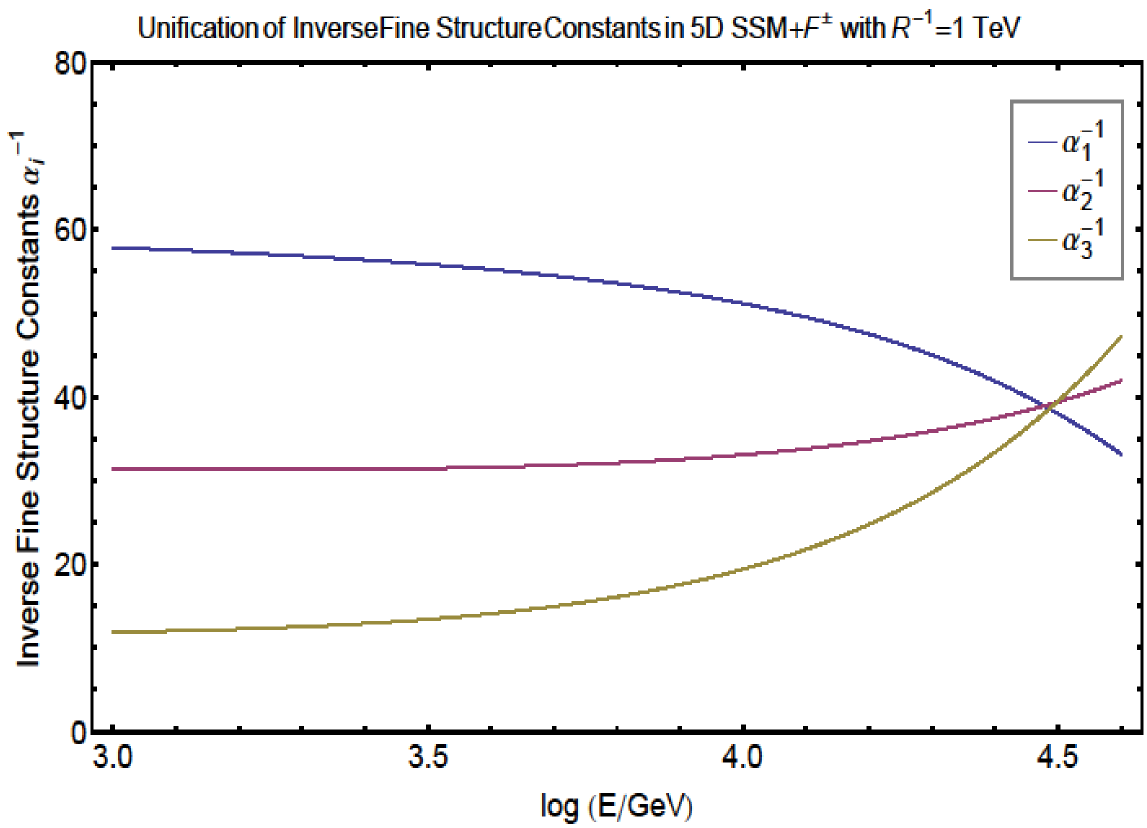

Figure (4.5): the evolution of inverse fine structure constants in 5D SSM+$F^{\pm}$ $R^{-1} = 10TeV$

4.3 Conclusions:

In conclusion, we studied the evolution of gauge coupling constants beyond the standard model. We derived the renormalization group equation at one loop for the gauge coupling constants in five and six dimensional model (bulk and brane cases) as well as 5D MSSM plus additional fields. We showed that the three inverse fine structure constants approximately unified at some high-energy scale at $E = 10^{4.31}GeV$ and $E = 10^{3.60}GeV$ in five and six dimensional models respectively. We also have studied the unification in 5DMSSM plus additional fields; we found a precise unification is achieved at energy scale$E = 10^{4.48}GeV$.

4.4 Recommendation:

This work can be extended in a number of ways. One might suspect that higher-loop corrections to the unification might be sizable. Therefore it is important to confirm these results and conclusions made at one loop that are sensitive to this scale and are still consistent and under control at two (and higher) loops.

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