Sudan University of Science and Technology
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Performance Analysis of Massive MIMO for Maximal Spectral Efficiency

A Research Submitted In Partial fulfillment for the Requirements of the Degree of BEng (Honors) in Electronics Engineering

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الإستهلال

قال تعالى
والله أخرجكم من بطون أمهاتكم لا تعلمون شيئا وجعل لكم السمع والأبصار والأفئدة ۙ لعلكم تشكرون ألم يروا إلى الطير مسخرات في جو السماء ما يمسكهن إلا الله ۗ إن في ذلك لآيات لقوم يؤمنون

سورة النحل

الإهداء

لك انت سر النجاح سبب الافراح بلسم الجراح عبق الجنان وينبوع الحنان، مهد الامان عطر يفوح على مر الزمان .. تقف الكلمات اجلالا لقدرك العظيم.. امي العزيزة...

ولك انت من احمل اسمه وملامحه بكل فخر.. من اسيقتني الشهامة والرجولة والكرم والعطاء.. من كان لي عمودا فقريا.. ابي العزيز...

الي من شاركونا نفس الرحم والحنان وكانوا لنا الحياة والدعم.. اخوتي الاعزاء...

الي رفقاء الدرب الطويل..سند الصعاب واعز الاحباب.. رفقاء الدراسة ...

الى كل من ساندنا وساعدنا ودعمنا.. اهدي هذا البحث...

شكر وعرفان

الى اساتذتنا الكرام الذين قدموا لنا الكثير باذلين بذلك جهودا كبيرة في بناء جيل الغد...
في الحياة .. الى الذين مهدوا لنا طريق العلم والمعرفة ...
الى جميع اساتذتنا الافاضل ...
ْكن عالما..فان لم تستطع فأحب العلماء ،فان لم تستطع فلا تبغضهمْ
ان قلت شكرا فشكري لن يوفيكم   حقا سعيتم فكان السعي مشكورا
ان جف حبري عن التعبير يكتبكم   قلب به صفاء الحب تعبيرا

وأخص بالتقدير والشكر الدكتورة: ابتهال حيدر..
التي لن نوفيها حقها في الشكر والعرفان...

Abstract

Massive MIMO is a promising technique to increase the spectral efficiency (SE) of cellular network, by deploying hundreds or thousands of antennas at the base station (BS) to perform coherence transceiver processing. This research focusing into study, analyze the performance of massive MIMO for maximal spectral efficiency. All this accomplished by using MATLAB software program which simulate the performance of massive MIMO. We used complex signal processing technique to obtain optimal performance, compared between this technique in simulation according to the propagation environment, which divided into three cases. First, the best case in which all UEs in are at the cell edge further from the (BS). Second, the average case, in which averaging over uniform UE locations in all cells. Finally, the worst case in which all UEs in other cells are at the cell edge closest to BS. From result it will be observed that the spectral efficiency dramatically increases while increasing the number of antennas in the (BS). This increased SE give a high throughput.

المستخلص

تقنية متعدد المدخلات متعدد المخرجات الهائلة من التقنيات الواعدة لزيادة الكفاءة الطيفية في مجال الاتصالات الخلوية وذلك بتزويد المحطة الرئيسية بمئات او ألاف الهوائيات وذلك لعملية ارسال واستقبال متماسكة. الغاية من هذا البحث دراسة وتحليل ادائية هذه التغنية لزيادة الكفاءة الطيفية وقد تم استخدام الماتلاب في محاكاة اداء هذه التقنية. استخدمت تقنيات مقعدة من معالجات الاشارة للحصول علي أحسن أداة تمت المقارنة هذه في المحاكي استنادا علي وسط الانتشار المقسم الي ثلاثة حالات. اولا افضل حالة و هي عندما يكون كل المستخدمين بكونون علي اطراف الخلية وبعين عن محطة الخلية الرئسية. ثانيا الحالة المتوسطة و هي جالة توزيع المستخدمين داخل الخلية يكون بشكل متناسق. اخيرا سوا حالة و هي عندما يكون كل المستخدمين في الخلايا المجاورة موزعين علي اطراف الخلية قريبة من المحطة الرئيسية. من النتائج تمت ملاحظة ان كفاءة الطيفية تزيد لصورة درامية عند زيادة عدد الهوائيات في المحطة الرئيسية وزيادة الكفاءة الطيفية تعني زيادة سرعة ارسال واستقبال البيانات.

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List of Abbreviations

BS	Base Station
CSI	Channel State Information
DL	Downlink
DPC	Dirty Paper Coding
FD	Full Duplex
FDD	Frequency Division Duplexing
HD	Half Duplex
i.i.d.	Independent and Identically Distributed
LTE	Long Term Evolution
LoS	Line-of-Sight
LS	Least-Squares
MIMO	Multiple-Input Multiple-Output
MISO	Multiple-Input Single-Output
ML	Maximum Likelihood
MRC	Maximum Ratio Combining
MU-MIMO	Multiuser MIMO
OFDM	Orthogonal Frequency Division Multiplexing
SINR	Signal-to-Interference-plus-Noise Ratio
SISO	Single-Input Single-Output
SNR	Signal-to-Noise Ratio
TDD	Time Division Duplexing
UL	Uplink
ZF	Zero-Forcing
3GPP-LTE	Third Generation Partnership Project long term evaluation

WIMAX	Wide World Interoperability of Microwave access
HF	High Frequency
UHF	Ultra High Frequency
SHF	Super High Frequency
ITU-R	International Telegraph Union Radio-communication
C-MIMO	Centralized Multiple Input Multiple Output
D-MIMO	Distributed Multiple Input Multiple Output
ADC	Analog to Digital Converter
3D	Three Dimension

List of Symbols

j	imaginary unit $j = \sqrt{-1}$
$(.)^T$	transpose
T_c	Coherence time
W_c	Coherence bandwidth
S	Coherence block length
B	pilot books of size
β	Pilot reuse factor
K	number of users per cell
M	number of antennas
P	Signal power constraint
σ^2	Noise variance
ρ/σ^2	Cellular system: reference SNR
ζ_{UL}	the fractions of UL transmission
ζ_{dl}	the fractions of DL transmission
$\mathbb{C}$	Set of complex numbers
R	Set of real numbers
Y_j	The received UL signal
$x_{\iota k}$	the symbol transmitted by UE k in cell ι
$h_{j\iota k}$	the channel response between BS j and UE k in cell ι
$p_{\iota k}$	the transmit power in UL
n_j	the additive noise
z_{jk}	The received DL signal
$S_{\iota m}$	the symbol intended for
$W_{\iota m}$	precoding vector
I_M	the $M \times M$ identity matrix
$\mathbf{0}$	vector of zeroes

I_j	the interference
κ	the pathloss exponent
$\mu_{j\iota}^{(\omega)}$	the Propagation parameters
$dj(z\iota m)$	the variance of channel attenuation from BS j
$dl(z\iota m)$	the variance of channel attenuation from BS ι

Chapter One
Introduction

1.1 Overview

Cellular communication networks are continuously evolving to keep up with the rapidly increasing demand for wireless data services. Higher area throughput (in bit/s per km^2) has traditionally been achieved by a combination of three multiplicative factors [1], more frequency spectrum (Hz), higher cell density (more cells per km^2), and higher spectral efficiency (bit/s/Hz/cell). The massive MIMO concept is based on equipping base stations (BSs) with hundreds or thousands of antenna elements which, unlike conventional cellular technology, are operated in a coherent fashion. This can provide unprecedented array gains and a spatial resolution that allows for multi-user MIMO communication to tens or hundreds of user equipments (UEs) per cell, while maintaining robustness to inter-user interference. The research on massive MIMO has so far focused on establishing the fundamental physical (PHY) layer properties; in particular, that the acquisition of channel state information (CSI) is limited by the channel coherence block (i.e., the fact that channel responses are only static in limited time/frequency blocks) and how this impacts the SEs and the ability to mitigate inter-cell interference [2–4]. In addition, the aggressive multiplexing in massive MIMO has been shown to provide major improvements in the overall energy efficiency [5]- [6], while [7, 8] have shown that the hardware impairments of practical transceivers have smaller impact on massive MIMO than contemporary systems. The importance of resource allocation for massive MIMO was described in [9], where initial guidelines were given. A main insight is that the limited number of orthogonal pilot sequences needs to be allocated intelligently among the UEs to reduce interference, which can be done by capitalizing on path loss differences [10, 11], and spatial correlation [9, 10, 12]. It is shown that how the coherence block length, number of antennas, pilot allocation, hardware impairments, and other system parameters determine the answer. To this end, we derive new SE expressions which are valid for both uplink (UL) and downlink (DL) transmission, with random user locations and power control that yields uniform UE performance.

1.2 Problem statement

Communication system, in past years, was suffering from noticed increase in users who use wireless traffic which impact decrease the speed of transceiver, marked interference

1.3 Proposed solution

By applying the massive MIMO could be able to maximize the spectral efficiency which cause decries interference and improve the transceiver speed.

1.4 Objectives

The objectives of this study are as follows.

- Using two linear processing schemes: (1) maximum ratio(MR); and (2) zero forcing (ZF), the impact of the following parameters will be studied
  - signal to noise ratio
  - path loss (κ)
  - coherence block length S
- To compare between the spectral efficiency and number of antennas (M)in the BS with the linear processing scheme according to the propagation environment

The simulation by default will compare the relationship between SISO, SIMO, MISO, MIMO and massive MIMO number of antennas in BS and spectral efficiency.

1.5 Methodology

First a literature review will be conducted. The implementation of massive MIMO system model will be performed using the MATLAB software program.

The next step is to analyze the performance of massive MIMO per cell according to propagation environment to the UEs in the cell. We consider three propagation environments with different severity of inter-cell interference

1. Average case: Averaging over uniform UE locations in all cells.
2. Best case: All UEs in other cells are at the cell edge furthest from BS j (for each j).
3. Worst case: All UEs in other cells are at the cell edge closest to BS j (for each j).

1.6 Thesis organization

This thesis is organized as follows. Chapter one provides an introduction. Chapter two presents an background and literature review. Chapter three discusses the considered system model. Chapter four provides simulation results and discussion. Finally, chapter five concludes the thesis.

Chapter Two
Literature Review

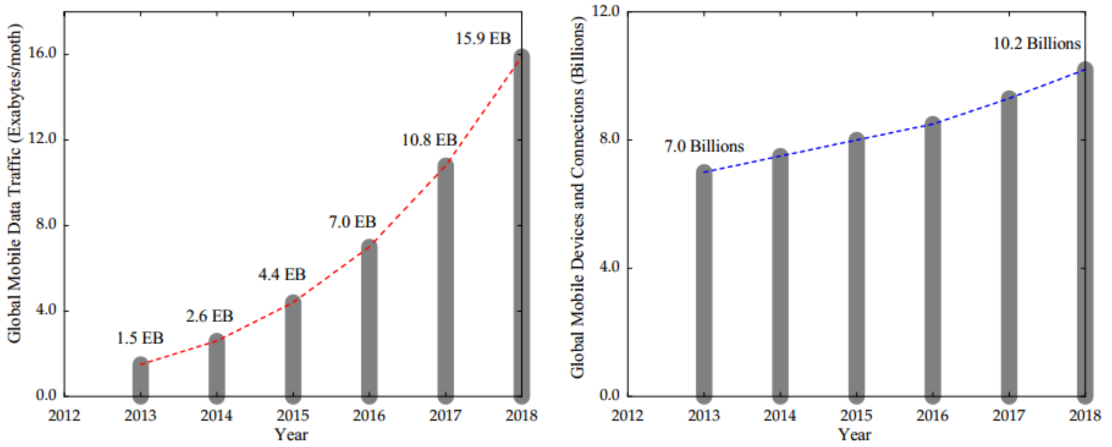
2.1 Background

During the last years, data traffic (both mobile and fixed) has grown exponentially due to the dramatic growth of smart phones, tablets, laptops, and many other wireless data consuming devices. The demand for wireless data traffic will be even more in future. Figures 2.1 shows the demand for mobile data traffic and the number of connected devices. Global mobile data traffic is expected to increase to 15.9 Exabytes per month by 2018, which is about an 6-fold increase over 2014. In addition, the number of mobile devices and connections are expected to grow to 10.2 billion by 2018. New technologies are required to meet this demand. Related to wireless data traffic, the key parameter to consider is wireless throughput (bits/s) which is defined as:

$$\text{Throughput} = \text{(Hz) Bandwidth} \times \text{(bits/s/Hz) efficiency Spectral} \quad (2.1)$$

Clearly, to improve the throughput, some new technologies which can increase the bandwidth or the spectral efficiency or both should be exploited. In this thesis, we focus on techniques which improve the spectral efficiency. A well-known way to increase the spectral efficiency is using multiple antennas at the transceivers. In wireless communication, the transmitted signals are being attenuated by fading due to multipath propagation and by shadowing due to large obstacles between the transmitter and the receiver, yielding a fundamental challenge for reliable communication. Transmission with multiple-input multiple-output (MIMO) antennas is a well-known diversity technique to enhance the reliability of the communication.

Furthermore, with multiple antennas, multiple streams can be sent out and hence, we can obtain a multiplexing gain which significantly improves the communication capacity. MIMO systems have gained significant attention for the past decades, and are now being incorporated into several new



(a) Global mobile data traffic.

(b) Global mobile devices and connections growth.

Figure 2.1: Demand for mobile data traffic and number of connected devices. (Source: Cisco [3])

generation wireless standards (e.g., LTE-Advanced, 802.16m). The effort to exploit the spatial multiplexing gain has been shifted from MIMO to multiuser MIMO (MU-MIMO), where several users are simultaneously served by a multiple-antenna base station (BS). Multi-user MIMO (MU-MIMO) is a set of multiple-input and multiple-output technologies for wireless communication, in which a set of users or wireless terminals, each with one or more antennas, communicate with each other. In contrast, single-user MIMO considers a single multi-antenna transmitter communicating with a single multi-antenna receiver. MU-MIMO does not only reap all benefits of MIMO systems, but also overcomes most of propagation limitations in MIMO such as ill-behaved channels. Specifically, by using scheduling schemes, we can reduce the limitations of ill-behaved channels. Line-of-sight propagation, which causes significant reduction of the performance of MIMO systems, is no longer a problem in MU-MIMO systems. Thus, MU-MIMO has attracted substantial interest.
MU-MIMO systems, where a (BS) with a hundred or more antennas simultaneously serves tens (or more) of users in the same time frequency resource, are known as Massive MIMO systems (also called very large MU-MIMO, hyper-MIMO, or full-dimension MIMO systems).

The main benefits of Massive MIMO systems are explained in the following subsections.

2.1.1 Huge spectral efficiency and high communication reliability

Massive MIMO inherits all gains from conventional MU-MIMO, i.e., with M-antenna BS and K single-antenna users, we can achieve a diversity of order M and a multiplexing gain of min (M, K). By increasing both M and K, we can obtain a huge spectral efficiency and very high communication reliability.

2.1.2 High energy efficiency

In the uplink Massive MIMO, coherent combining can achieve a very high array gain which allows for substantial reduction in the transmit power of each user. In the downlink, the BS can focus the energy into the spatial directions where the terminals are located. As a result, with massive antenna arrays, the radiated power can be reduced by an order of magnitude, or more, and hence, we can obtain high energy efficiency. For axed number of users, by doubling the number of BS antennas, while reducing the transmit power by two, we can maintain the original the spectral efficiency, and hence, the radiated energy efficiency is doubled.

2.1.3 Simple signal processing

For most propagation environments, the use of an excessive number of BS antennas over the number of users yields favorable propagation where the channel vectors between the users and the BS are pair wisely (nearly) orthogonal. Under favorable propagation, the effect of interfuse interference and noise can be eliminated with simple linear signal processing (liner precoding in the downlink and linear decoding in the uplink). As a result, simple linear processing schemes are nearly optimal. Another key property of Massive MIMO is channel hardening. Under some conditions, when the number of BS antennas is large, the channel becomes (nearly) deterministic, and hence, the effect of small-scale fading is averaged out. The system scheduling, power control, etc., can be done over the large-scale fading time scale instead of over the small-scale fading time scale. This simplifies the signal processing significantly. In this research we focus on the point of huge spectral efficiency

2.2 SISO, SIMO, MISO, MIMO terminology

2.2.1 SISO Systems

The modest form is known as SISO - Single Input Single Output . This is commendably a standard radio channel. The transmitter and receiver both operates with one antenna, and no diversity and additional processing is required

Figure 2.2: SISO : Single input singel output.

The advantages and disadvantages are as follows. The advantage of a SISO system, it does not requires processing in various forms of diversity, also the SISO system is very simple. Whereas SISO system has limited Performance, fading and interference affect the system, also its Bandwidth is limited by Shannon's law.

2.2.2 SIMO Systems

SIMO is also a form of MIMO, having single antenna at transmitter and multiple antennas at receiver, which is recognized as receive diversity. SIMO systems are helpful up to some extent to conquer the fading effects

Figure 2.3: SIMO : Single input Multi output.

There are two forms of SIMO that can be used, which can be explained as follows.

- *Switched diversity SIMO*: This form of SIMO appearances for the resilient signal and switches to that antenna.

- *Maximum ratio combining SIMO*: This form of SIMO receives both signals and adds them to give a combined result.

The advantages and disadvantages can be summarized as well. The advantage of SIMO is that it is comparatively easy to implement. SIMO also have some disadvantages, such as needs processing at the receiver. SIMO can be used in many applications but in case of mobile phones where receiver is positioned, processing is limited because of size and also drains the battery.

2.2.3 MISO Systems

MISO(Multiple input single output) also known as transmit diversity. In MISO systems, same data are transmitted excessively from both transmit antennas, and the receiver receives the optimal signal to obtain the desired information.

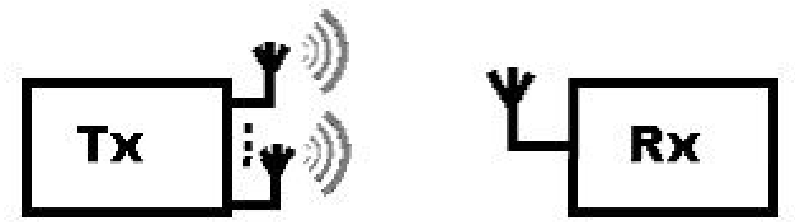


Figure 2.4: MISO : Multi input singel output.

The advantage of using MISO is that the multiple antennas and the redundancy coding / processing is moved from the receiver to the transmitter. In instances such as cellphone UEs, this can be a significant advantage in terms of space for the antennas and reducing the level of processing required in the receiver for the redundancy coding. This has a positive impact on size, cost and battery life as the lower level of processing requires less battery consumption.

2.2.4 MIMO Systems

When talk about more than one antenna at both transmitter and receiver, that system is known as MIMO system. MIMO stances for Multiple-Input, Multiple-Output can be used to provide improvements in both channel robustness as well as channel throughput . MIMO is an important factor of wireless communication standards such as IEEE 802.11n (Wi-Fi), IEEE 802.11ac (Wi-Fi), 4G, 3GPP Long Term Evolution, WiMAX and HSPA+.

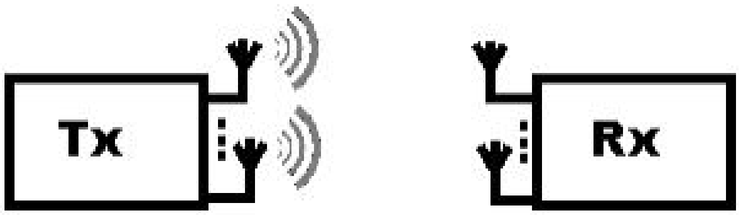


Figure 2.5: MIMO : Multi input Multi output.

2.3 Channel Impairments

2.3.1 Fading

The performance of wireless communication systems is mainly governed by the wireless channel environment. As opposed to the typically static and predictable characteristics of a wired channel, the wireless channel is rather dynamic and unpredictable, which makes an exact analysis of the wireless communication system often difficult. In recent years, optimization of the wireless communication system has become critical with the rapid growth of mobile communication services and emerging broadband mobile Internet access services. In fact, the understanding of wireless channels will lay the foundation for the development of high performance and bandwidth-efficient wireless transmission technology. In wireless communication, radio propagation refers to the behavior of radio waves when they are propagated from transmitter to receiver. In the course of propagation, radio waves are mainly affected by three different modes of physical phenomena: reflection, diffraction, and scattering [[13], [14]]. Reflection is the physical phenomenon that occurs when a propagating electromagnetic wave impinges upon an object with very large dimensions compared to the wavelength, for example, surface of the earth and building. It forces the transmit signal power to be reflected back to its origin rather than being passed all the way along the path to the receiver. Diffractionrefers to various phenomena that occur when the radio path between the transmitter and receiver is obstructed by a surface with sharp irregularities or small openings. It appears as a bending of waves around the small obstacles and spreading out of waves past small openings. The secondary waves generated by diffraction are useful for establishing a path between the transmitter and receiver, even when a line-of-sight path is not present. Scattering is the physical phenomenon that forces the radiation of an electromagnetic wave to deviate from a straight path by one or more local obstacles, with small dimensions compared to the wavelength. Those obstacles that induce scattering, such as foliage, street signs, and lamp posts, are referred to as the scatters. In other words, the propagation of a radio wave is a complicated and less predictable process that is governed by reflection, diffraction, and scattering, whose intensity varies with different environments at different instances. A unique characteristic in a wireless channel is a phenomenon called ‘fading,’ the variation of the signal amplitude over time and frequency. In contrast with the additive noise as the most common source of signal degradation, fading is another source of signal degradation that is characterized as a non-additive signal disturbance in the wireless channel. Fading may either be due to multipath propagation, referred to as multi-path (induced) fading, or to shadowing from obstacles that affect the propagation of a radio wave, referred to as shadow fading. The fading phenomenon in the wireless communication channel was initially modeled for HF (High Frequency, 330 MHz), UHF (Ultra HF, 3003000 GHz), and SHF (Super HF, 330 GHz) bands in the 1950s and 1960s. Currently, the most popular wireless channel models have been established for 800MHz to 2.5 GHz by extensive channel measurements in the field. These include the ITU-R standard channel models specialized for a single-antenna communication system, typically referred to as a SISO (Single Input Single Output) communication, over some frequency bands. Meanwhile, spatial channel models for a multi-antenna communication system, referred to as the MIMO (Multiple Input Multiple Output) system, have been recently developed by the various research and standardization activities such as IEEE 802, METRA Project, 3GPP/3GPP2, and WINNER Projects, aiming at high-speed wireless transmission and diversity gain. The fading phenomenon can be broadly classified into two different types: large-scale fading and small-scale fading. Large-scale fading occurs as the mobile moves through a large distance, for example, a distance of the order of cell size [1]. It is caused by path loss of signal as a function of distance and shadowing by large objects such as buildings, intervening terrains, and vegetation.

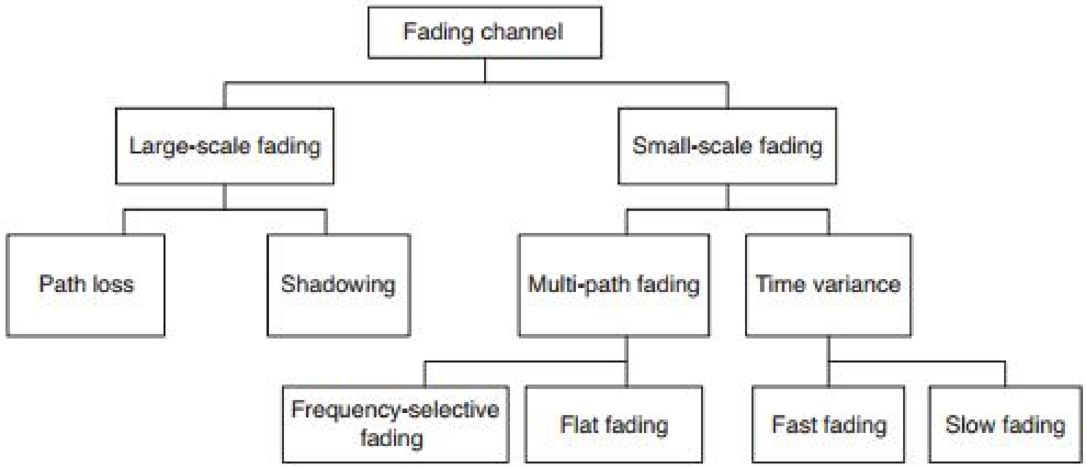


Figure 2.6: Classification of fading Channel

Shadowing is a slow fading process characterized by variation of median path loss between the transmitter and receiver in fixed locations. In other words, large-scale fading is characterized by average path loss and shadowing. On the other hand, small-scale fading refers to rapid variation of signal levels due to the constructive and destructive interference of multiple signal paths (multi-paths) when the mobile station moves short distances. Depending on the relative extent of a multipath, frequency selectivity of a channel is characterized (e.g., by frequency-selective or frequency flat) for small-scaling fading. Meanwhile, depending on the time variation in a channel due to mobile speed (characterized by the Doppler spread), shortterm fading can be classified as either fast fading or slow fading. Figure 2.6 classifies the types of fading channels.

Large-scale fading is manifested by the mean path loss that decreases with distanceand shadowing that varies along the mean path loss. The received signal strength may be different even at the same distance from a transmitter, due to the shadowing caused by obstacles on the path. Furthermore, the scattering components incur small-scale fading, which finally yields a short-term variation of the signal that has already experienced shadowing.

2.3.2 Noise

Noise: It's unwanted electrical or electromagnetic energy that degrades the quality of signals and data. Noise occurs in digital and analog systems, and can affect files and communications of all types, including text, programs, images, audio, and telemetry. Noise may be but not limited into following two categories:

1. external noise: noise whose source is external
    - Atmospheric noises.
    - Extraterrestrial noises.
    - Man-made noises or industrial noises.
2. Internal noise: noises which get generated within the receiver or communication system.
    - Thermal noises or white noise
    - Shot noise

2.4 Related Work

Jingxian Wu et al., [14] reviewed exact closed-form expressions for the short-term Rayleigh fading-averaged spectral efficiency of cellular systems with channel-aware schedulers that operate with non-identical co-channel interferers and noise Kamga et al., [15] reviewed the spectral efficiency of massive MIMO systems in both centralized (C-MIMO) and distributed (D-MIMO) settings, was analytically investigated, based on a novel comprehensive analytical channel model where major natural environmental and antenna physical parameters were accounted for, including path loss, shadowing, multi-path fading and antenna correlation
Zhang et al ., [16] reviewed the uplink spectral efficiency (SE) of massive MIMO systems with low-resolution ADCs over Rician fading channels, where both perfect and imperfect channel state information are considered
Bjornson et al., [17] reviewed analyze how the optimal number of scheduled users, K?, depends on M and other system parameters and new SE expressions are derived to enable efficient system-level analysis with power control, arbitrary pilot reuse, and random user locations. The value of K? in the large-M regime is derived in closed form, while simulations are used to show what happens at finite M, in different interference scenarios, with different pilot reuse factors, and for different processing schemes beside concentrates on frames that carry user-specific signals, in particular, payload data and pilots

Ngo et al ., [18] reviewed that, when the number of BS antennas M grows without bound, we can reduce the transmitted power of each user proportionally to 1/M if the BS has perfect channel state information (CSI), and proportionally to $1/\sqrt{M}$ if CSI is estimated from uplink pilots. This holds true even when using simple, linear receivers. We also derive closed-form lower bounds on the uplink achievable rates for finite M, for the cases of perfect and imperfect CSI, assuming MRC, ZF, and minimum mean-squared error (MMSE) receivers, respectively, beside the tradeoff between spectral efficiency and energy efficiency. For imperfect CSI, in the low transmit power regime, we can simultaneously increase the spectral-efficiency and energy-efficiency. We further show that in large-scale MIMO, very high spectral efficiency can be obtained even with simple MRC processing at the same time as the transmit power can be cut back by orders of magnitude and that this holds true even when taking into account the losses associated with acquiring CSI from uplink pilots. MRC also has the advantage that it can be implemented in a distributed manner, i.e., each antenna performs multiplication of the received signals with the conjugate of the channel, without sending the entire baseband signal to the BS for processing. Quantitatively, our energy-spectral efficiency tradeoff analysis incorporates the effects of small-scale fading but neglects those of large-scale fading, leaving an analysis of the effect of large-scale fading for future work in

kammoun et al., [19] show twofold. First is to provide an information-theoretic channel model for 3D massive MIMO systems and second is to predict and analyze the performance of these systems by characterizing the distribution of the MI

Hoydis et al ., [20] show assess to which extent the above conclusions hold true for large, but finite N provide a definition of massive MIMO as an operating condition of cellular systems where multiuser interference and noise are small compared to pilot contamination.

Tai Do et al ., [21] viewed proposes anti-jamming strategies based on pilot retransmission for a single user uplink massive MIMO under jamming attack. A jammer is assumed to attack the system both in the training and data transmission phases. We first derive an achievable rate which enables us to analyze the effect of jamming attacks on the system performance

2.5 Contributions

The main contribution is increasing the throughput by regardless of bandwidth and focus on spectral efficiency and to achieve the propose we use Massive MIMO instead of point to point MIMO because Massive MIMO has shown over 10 times spectral efficiency increase over a point-to-point MIMO under realistic propagation environment with simpler signal processing algorithms so by overabundance the throughput we scale up the covered area and increase the user deserve service so as to mitigate the effects of noise, fading, and multi-user interference.

Chapter Three
System Model

3.1 Introduction

We consider a cellular network where payload data is transmitted with universal time and frequency reuse. Each cell is assigned an index in the set L , where the cardinality $|L|$ is the number of cells. The BS in each cell is equipped with an array of M antennas and communicates with K single-antenna UEs at the time, out of a set of $K(max)$ UEs. We are interested in massive MIMO topologies where M and K max are large and fixed, while K is a design parameter and all UEs have unlimited demand for data. The subset of active UEs changes over time, thus the name UE $k \in 1, ..., K$ in cell $l \in L$is given to different UEs at different times. The geographical position $z_{lk} \in \mathbb{R}^2$ of UE k in cell l is therefore an ergodic random variable with a cell-specific distribution. This model is used to study the average performance for a random rather than fixed set of interfering UEs. The time- frequency resources are divided into frames consisting of Tc seconds and Wc Hz, This leaves room for $S = T_c W_c$ transmission symbols per frame. We assume that the frame dimensions are such that T c is smaller or equal to the coherence time of all UEs, while W c is smaller or equal to the coherence bandwidth of all UEs.

These channel responses are drawn as realizations from zeromean circularly symmetric complex Gaussian distributions:

$$h_{jlk} \sim \mathcal{CN}(0, d_j(z_{lk}), \mathbf{I}_M) \tag{3.1}$$

where I_M is the $M \times M$ identity matrix. This is a theoretical model for non-line-of-sight propagation that is known to give representative results with both few and many BS antennas. The deterministic function $d_j(z)$ gives the variance of the channel attenuation from BS j to any UE position z. The value of $d_j(z_l k)$ varies slowly over time and frequency, thus we assume that the value is known at BS j for all l and k and that each UE knows its value to its serving BS. The exact UE positions $z_l k$ are unknown

We consider the time-division duplex (TDD) protocol , where $B \geq 1$ out of the S symbols in each frame are reserved for UL pilot signaling. There is no DL pilot signaling and no feedback of CSI, because the BSs can process both UL and DL signals using the UL channel measurements due to the channel reciprocity in TDD systems.

The remaining S – B symbols are allocated for payload data and are split between UL and DL transmission. We let ζ (ul) and ζ (dl) denote the fixed fractions allocated for UL and DL, respectively. These fractions can be selected arbitrarily, subject to the constraint $\zeta^{(ul)} + \zeta(dl) = 1$ and that $\zeta^{(ul)}(S-B)$ and $\zeta^{(ul)}(S-B)$ are positive integers.

3.2 Uplink Model

The received UL signal $y_j \in \mathbb{C}^M$ at BS j in a frame is modeled as

$$Y_j = \sum_{\iota \in \varsigma} \sum_{k=1}^{K} \sqrt{P\iota_k} h_{j\iota k} x_{\iota k} + n_j \tag{3.2}$$

where $h_{jlk} \in \mathbb{C}^N$ denotes the channel response between BS j and UE k in cell l , $x_{lk} \in \mathbb{C}$is the symbol transmitted by UE$k$in cell l. This signal is normalized as $\mathbb{E}|x_{lk}|^2 = 1$, while the corresponding UL transmit power is defined by $p_{lk} \geq 0$. The additive noise $n_j \in \mathbb{C}^M$ is modeled as $n_j \sim CN(0, \sigma^2 I_M)$, where σ^2 is the noise variance.

Contrary to most previous works on massive MIMO, which assume fixed UL power, we consider statistics-aware power control the symbols from UE k in cell l have the transmit power$p_{lk} = \frac{\rho}{d_l(z_{lk})}$, where $\rho > 0$ is a design parameter . This power-control policy inverts the average channel attenuation $d_l(z_{lk})$ and has the merit of making the average effective channel gain the same for all UEs: $\mathbb{E} p_l k ||h_{llk}||^2 = M\rho$ Hence, this policy guarantees a uniform user experience, saves valuable energy at UEs, and avoids near-far blockage where weak signals drown in stronger signals due to the finite dynamic range of analog-to-digital converters (ADCs).

3.3 Downlink Model

the received DL signal $z_{jk} \in \mathbb{C}$ at UE k in cell j in a frame is modeled as

$$z_{jk} = \sum_{\iota \in \varsigma} \sum_{m=1}^{K} h_{\iota jk}^{T} \mathbf{w}_{\iota m} s_{\iota m} + \eta_{jk} \tag{3.3}$$

where $(.)^T$ denotes transpose, $s_l m$ is the symbol intended for UE m in cell l, $\mathbf{w}_{lm} \in \mathbb{C}^M$ is the corresponding precoding vector, and $||\mathbf{w}_{lm}||^2$ is the allocated DL transmit power, The additive noise at UE$k$in cell j is modeled as $\eta_{jk} \sim CN(0, \sigma^2)$.
The UL/DL system models in 3.2and 3.3 assume perfect synchronization across all cells, as commonly done in the massive MIMO literature. Local synchronization is achievable, for example, using the cyclic prefix in OFDM-based systems, but network-wide synchronization is probably infeasible over large coverage areas. The processing techniques analyzed in this project can thus be used to suppress the strong interference between the closest tiers of neighboring cells, while the interference from distant cells is asynchronously received and practically insuppressible. We expect that the simplified synchronization modeling used here and elsewhere has negligible impact on the system performance, since the insuppressible distant interferers are weak as compared to(partially suppressed) interference from neighboring cells.

3.4 Linear Processing

To obtain optimal performance, complex signal processing techniques must be implemented. For example, in the uplink, the maximum likelihood (ML) multiuser detection can be used. With ML multiuser detection, the BS has to search all possible transmitted signal , and choose the best one. The BS can use linear processing schemes (linear receivers in the uplink and linear precoders in the downlink) to reduce the signal processing complexity. These schemes are not optimal. However, when the number of BS antennas is large, that linear processing is nearly-optimal. We consider both conventional linear processing schemes such as maximum ratio (MR) combining/transmission and zero forcing (ZF).

3.4.1 Maximum Ratio Combining

In this technique, the received signals are adjusted both in magnitude and phase by the weights in the combining filter to maximise the Signal-to-Noise-Ratio (SNR) at the output of the combiner [22, 23]. The weighting applied to each diversity branch is adjusted independently from other branches according to the SNR at that branch. The received signal at k^{th} branch, y_k , and the output of the MRC combiner, d, are given by

$$d = \sum_{k=1}^{M} w_k^H y_k \tag{3.4}$$

$$y_k = h_k u + n \tag{3.5}$$

$$w_k^H = h_k^H \tag{3.6}$$

Where $[.]^H$ represents the Hermitian or complex conjugate. The transmitted signal, u is corrupted by the channel effects characterized by h_k , while w_k is the associated weight of the k^{th} antenna element.

3.4.2 Zero Forcing

In a zero-forcing combiner, the combiner coefficients W are chosen to remove undesired interference leaving only the desired signal. This technique assumes the channel characteristic is known or estimated from the pilot bits.The output of the zero-forcing combiner is given by [24]

$$\mathbf{y} = \mathbf{Hu} + \mathbf{n} \tag{3.7}$$

$$d = \mathbf{W^H y} \tag{3.8}$$

$$\mathbf{W^H} = (\mathbf{H^H H})^{\mathbf{-1}} \mathbf{a^H} \tag{3.9}$$

Where d is an estimate of the users' signal vector, $\mathbf{y}$ is a received signal vector corrupted by the channel effects characterized by matrix $\mathbf{H}$ of size $M \times N$ as given in (3.5). $\mathbf{W}$ is a corresponding weight matrix of size $N \times M$ to the antenna elements and $(.)^{-1}$ is the inverse matrix.

3.5 Pilot Contamination

Ideally every terminal in a Massive MIMO system is assigned an orthogonal uplink pilot sequence. However, the maximum number of orthogonal pilot sequences that can exist is upper bounded by the duration of the coherence interval divided by the channel delay-spread. In [2], for a typical operating scenario, the maximum number of orthogonal pilot sequences in a one millisecond coherence interval is estimated to be about 200. It is easy to exhaust the available supply of orthogonal pilot sequences in a multi-cellular system.

The effect of re-using pilots from one cell to another, and the associated negative consequences, is termed "pilot contamination". More specifically, when the service-array correlates its received pilot signal with the pilot sequence associated with a particular terminal it actually obtains a channel estimate that is contaminated by a linear combination of channels to the other terminals that share the same pilot sequence. Downlink beamforming based on the contaminated channel estimate results in interference that is directed to those terminals that share the same pilot sequence. Similar interference is associated with uplink transmissions of data.

This directed interference grows with the number of service-antennas at the same rate as the desired signal [2]. Even partially correlated pilot sequences result in directed interference. Pilot contamination as a basic phenomenon is not really specific to massive MIMO, but its effect on massive MIMO appears to be much more profound than in classical MIMO [2,4]. In [2] it was argued that pilot contamination constitutes an ultimate limit on performance, when the number of antennas is increased without bound, at least with receivers that rely on pilot-based channel estimation. While this argument has been contested recently [25], at least under some specific assumptions on the power control used, it appears likely that pilot contamination must be dealt with in some way. This can be done in several ways

- The allocation of pilot waveforms can be optimized. One possibility is to use a less aggressive frequency re-use factor for the pilots (but not necessarily for the payload data)—say 3 or 7. This pushes mutually-contaminating cells farther apart. It is also possible to coordinate the use of pilots or adaptively allocate pilot sequences to the different terminals in the network [12]. Currently, the optimal strategy is unknown.

- Clever channel estimation algorithms [25], or even blind techniques that circumvent the use of pilots altogether [26], may mitigate or eliminate the effects of pilot contamination. The most promising direction seems to be blind techniques that jointly estimate the channels and the payload data.

- New precoding techniques that take into account the network structure, such as pilot contamination precoding [27], can utilize cooperative transmission over a multiplicity of cells—outside of the beamforming operation—to nullify, at least partially, the directed interference that results from pilot contamination. Unlike coordinated beamforming over multiple cells which requires estimates of the actual channels between the terminals and the service-arrays of the contaminating cells, pilot-contamination precoding requires only the corresponding slow-fading coefficients. Practical pilot-contamination precoding remains to be developed.

3.6 Computing Spectral Efficiency

Let $j(\beta) \subset L$ be the subset of cells that uses the same pilots as cell j. In the UL, an achievable SE in cell j is

$$SE_j^{(ul)} = K\zeta^{(ul)}(1 - \frac{B}{S}) \log_2 (1 + \frac{1}{I_j^{scheme}}) \quad [bit/s/HZ/cell] \tag{3.10}$$

Let $L_l(\beta) \subset L$ be the subset of cells that uses the same pilots as cell j. in the DL, , an achievable SE in cell j is

$$SE_j^{(dl)} = K\zeta^{(dl)}(1 - \frac{B}{S}) \log_2 (1 + \frac{1}{I_j^{scheme}}) \quad [bit/s/HZ/cell] \tag{3.11}$$

where the interference term

$$I_j^{scheme} = \sum_{l \in \iota_j(\beta)\setminus\{j\}} \left(\mu_{jl}^{(2)} + \frac{\mu_{jl}^{(2)} - \left(\mu_{jl}^{(1)}\right)^2}{G^{scheme}} \right) + \frac{\left(\sum_{l\in\iota} \mu_{jl}^{(1)} Z_{jl}^{scheme} + \frac{\sigma^2}{\rho}\right)\left(\sum_{l\in\iota_j(\beta)} \mu_{jl}^{(1)} + \frac{\sigma^2}{\rho}\right)}{G^{scheme}} \tag{3.12}$$

The SE expression manifests the importance of pilot allocation, since the interference term in equation (3.12) contains summations that only consider the cells that use the same pilots as cell j The first term describes the pilot contamination, while the second term mention the inter-user interference, where the interference term I_j^{scheme} is defined in (3.12) and depends on G^{scheme} and Z_{jl}^{scheme}. The parameter values with MR and ZF,

- In the term MR

$$G^{MR} = M \tag{3.13}$$

$$Z_{jl}^{MR} = K \tag{3.14}$$

- In the term ZF

$$G^{ZF} = M - K \tag{3.15}$$

$$Z_{jl}^{ZF} = \begin{cases} K\left(1 - \frac{\mu_{jl}^{(1)}}{\sum_{l\in\iota(\beta)}\mu_{jl(1)} + \frac{\sigma^2}{B_\rho}}\right) & \text{if} l \in \iota_j(\beta), \\ K & \text{if} l \notin \iota_j(\beta), \end{cases} \tag{3.16}$$

3.7 Propagation parameters

The hexagonal grid is infinitely large, to avoid edge effects and to give all cells the same properties. The cell radius is denoted by $r > 0$ and is the distance from the cell center to the corners. Each cell can be uniquely indexed by a pair of integers $\alpha_j(1), _j(2) \in Z$, where Z is the set of integers. This integer pair specifies the location of BS j:

$$b_j = \sqrt{3}\begin{bmatrix}\sqrt{3}r/2 \\ r/2\end{bmatrix}\alpha_j^{(1)} + \begin{bmatrix}0 \\ \sqrt{3}r\end{bmatrix}\alpha_2^{(2)} \in \mathbb{R}^2 \tag{3.17}$$

Every cell on the hexagonal grid has 6 interfering cells in the first surrounding tier, 12 in the second tier, etc. this limits which pilot reuse factors that give symmetric reuse patterns: $\beta \in \{1, 3, 4, 7, 9, 12, 13, ...\}$ [28]

$$\mu_{j\iota}^{(\omega)} = \mathbb{E}_{\mathbf{z}_{\iota m}}\left\{\left(\frac{d_j(z_{\iota m})}{d_\iota(z_{\iota m})}\right)^{\omega}\right\} = \mathbb{E}_{\mathbf{z}_{lm}}\left\{\left(\frac{||\mathbf{z}_{lm} - \mathbf{b}_l||}{||\mathbf{z}_{lm} - \mathbf{b}_j||}\right)^{\kappa\omega}\right\} \tag{3.18}$$

consider a classic pathloss model where the variance of the channel attenuation in 3.1 is $d_j(z) = \frac{C}{||z-b_j||^\kappa}$, where $||.||$is the Euclidean norm, C > 0 is a reference value, and $\kappa \geq 2$ is the pathloss exponent

The latter two are the average ratio between the channel variance to BS j and the channel variance to BS ι, for an arbitrary UE in cellι , and the second-order moment of this ratio, respectively. These parameters are equal to 1 for j =ι and otherwise go to zero as the distance between BS j and cell ι increases Based on equations of the spectral efficiency is $SE^{(ul)}$ and $SE^{(dl)}$, the sum of the per-cell achievable SEs in the UL and DL are given by the following [28]

$$SE_j = SE_j^{(ul)} + SE_j^{(dl)} \tag{3.19}$$

$$K(1 - \frac{B}{S})\log_2(1 + \frac{1}{I_j^{scheme}}) \quad [bit/s/HZ/cell] \tag{3.20}$$

where $M \to \infty$. This SE is maximized jointly for all cells when the number of scheduled UEs is either $K = [S/2\beta]$, the asymptotically optimal SE is [28]

$$SE_j^{\infty} = \frac{S}{4\beta} \log_2 \left(1 + \frac{1}{\sum_{\iota \in \varsigma_j(\beta)/j} \mu_{j\iota}^{(2)}} \right) \tag{3.21}$$

where the interference term I_j^{scheme} for UE k is given in (3.11) and (3.10) for MR and ZF . This SE can be divided between the UL and DL arbitrarily using any positive fractions $\zeta^{(ul)}$ and $\zeta^{(dl)}$, with $\zeta^{(ul)} + \zeta^{(ul)} = 1$

The SE increases linearly with the frame length S, the asymptotically optimal scheduling gives $B = [S/2]$ for any β, which means that half the frame is allocated to pilot transmission. The rationale is that the SE gain from adding an extra UE outweighs the pre-log loss at the existing UEs if at least half the frame is used for data (a criterion independent of β). The asymptotically optimal β cannot be computed in closed-form, but we notice that a larger β leads to fewer interferers in $L_j(\beta)$ and also reduces the pre-log factor; hence, a larger β brings SINR improvements until a certain point where the pre-log loss starts to dominate

Chapter Four
Simulation Results

4.1 Simulation Assumptions

We simulate the SE in an arbitrary cell on the hexagonal grid and take all non-negligible interference into account. The UEs can be anywhere in the cells, but at least 0.14r from the serving BS (this makes the analysis independent of r). Since the SE expressions in system model are the same for the UL and DL, except for the fractions $\zeta(ul)$ and $\zeta(dl)$

We simulate the sum of these SEs and note that it can be divided arbitrarily between the UL and DL. The same linear processing schemes are used in both directions. The simulations consider MR and ZF combining, and all results are obtained by computing the closed-form expressions from system model for different parameter combinations. For each number of antennas, M, we optimize the SE with respect to the number of UEs K and the pilot reuse factor β (which determine $B = \beta K$) by searching the range of all reasonable integer values. We set the coherence block length to $S = 400$ (e.g., 2 ms coherence time and 200 kHz coherence bandwidth) , set the SNR to $\rho/\sigma^2 = 5dB$, and pick $\kappa = 3.7$ as pathloss exponent. Note that there are various values for the path loss based on the propagation environment

- for free space, $\kappa = 2$,
- Urban Area 2.7 to 3.5
- Suburban Area 3 to 5
- Indoor (line-of-sight) 1.6 to 1.8

We consider three propagation environments with different severity of inter-cell interference:

- Average case: Averaging over uniform UE locations in all cells.

- Best case: All UEs in other cells are at the cell edge furthest from BS j (for each j).
- Worst case: All UEs in other cells are at the cell edge closest to BS j (for each j).

The corresponding values of the parameters $\mu_{jl}^{(1)}$ and $\mu_{jl}^{(2)}$ were computed by Monte-Carlo simulations with 10^3 UE locations in each cell. The best case is overly optimistic since the desirable UE positions in the interfering cells are different with respect to different cells. However, it gives an upper bound on what is achievable by coordinated scheduling across cells. The worst case is overly pessimistic since the UEs cannot all be at the worst locations, with respect to all other cells, at the same time. The average case is probably the most applicable in practice, where the averaging comes from UE mobility, scheduling, and random switching of pilot sequences between the UEs.

4.2 Simulation Flow

This flow chart figure [4.2] takes points in the complex plane and check if they are inside a hexagon of specified size (and a rotation with two sides being parallel to the horizontal axis). The check can be done by input the point = point in the complex plane radius = Radius (length to corners) of the hexagon in the complex plane and then we calculate the angle and the distance of the point then we save the different locations of UEs in matrix telling if the points are inside the hexagon other wise if the point out of the hexagon we ignoring it and add new point

After we input the location of the point in complex plane and doing the hexagon test then flow chart figure [4.1] check that if the point is not in forbidden area and achieve the pathloss exponent then we determine the exactly position of the point if the point is in uniform location in the cell we save this point in average case if the point is in the cell edge furthest from BS then save the point in best case otherwise if the point is in the cell edge closest from BS save the point in worst case After all points saved in their location we finally calculate propagation parameter for the different cases.

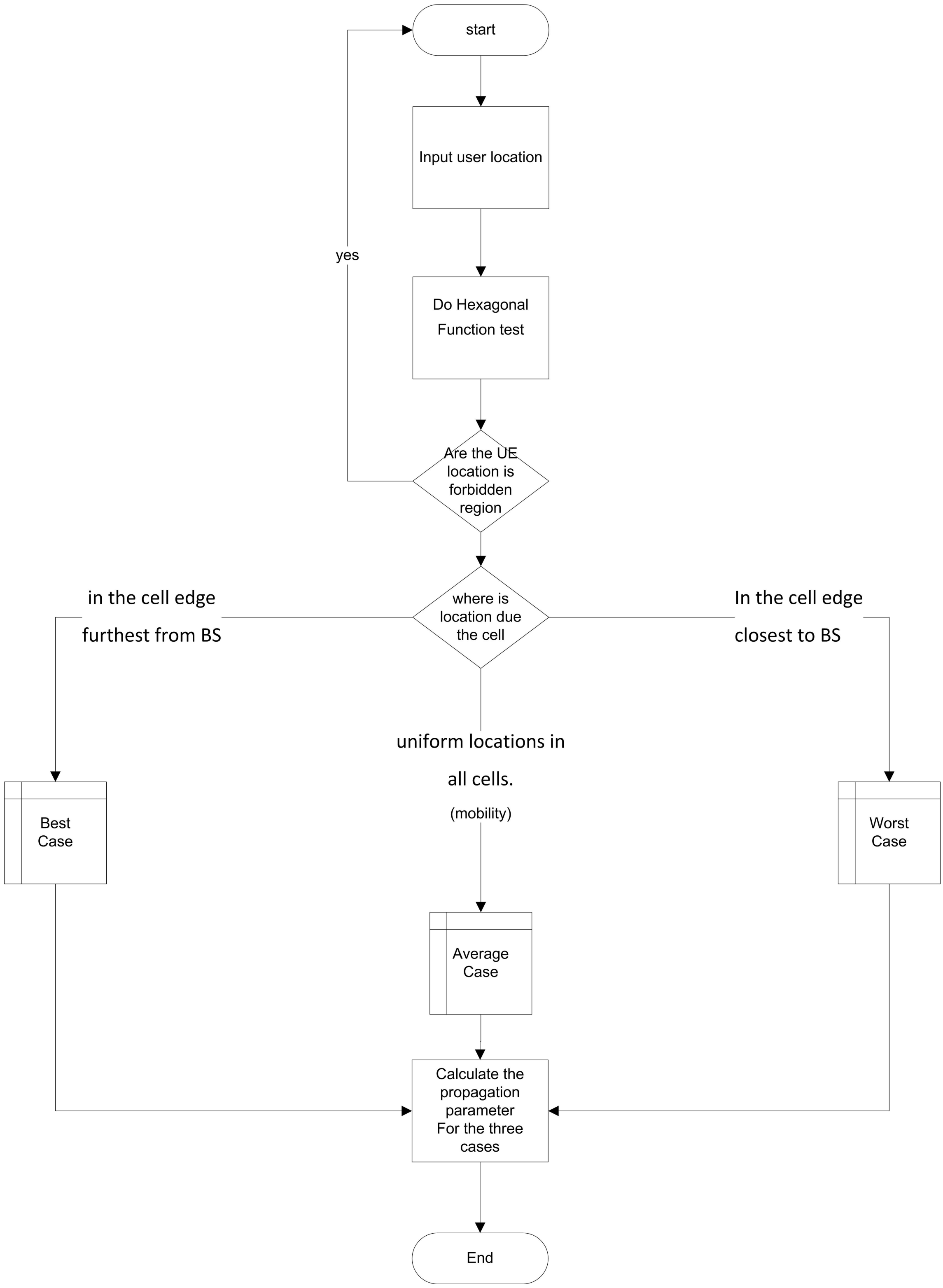


Figure 4.1: Flowchart show how the compute environment and the htree cases production

4.3 SE and Number of Antennas

In the fallowing cases there are a clear increase in SE ,while we increasing the number of antennas(M).The Mean different came in which way we use the Linear processing to reduce the different type of interference and as we mentioned before we well take ZF and MR to compare

Results for the average case are shown in Fig 4.1, the best case in Fig 4.2, and the worst case in Fig 4.3.

The enhance SE and the corresponding K^* are shown the figures respectively.The achievable SEs (per cell) are very different between the best case interference and the two other cases this confirms the fact that results from single-cell analysis of massive MIMO is often not applicable to multi-cell cases (and vice versa). ZF brings much higher SEs than MR under the best case inter- cell interference, since then the potential gain from mitigating intra-cell interference is very high. In the realistic average case, the optimized SEs are rather similar for MR and ZF; particularly in the practical range of $10 \leq M \leq 200$ antennas. In all cases, the largest differences appear when the number of antennas is very large (notice the logarithmic M-scales). At least $M = 10^5$ is needed to come close to comparing between this cases .

4.4 Impact of Other Parameters

We would like to change some of the parameter and figure out the new effect of it in the simulation result.

4.4.1 Coherence block length

The first parameter we change is *Coherence block length S* from 400 to 800 while the other parameters is constant (Pathloss = 3.7 and SNR= 5 dB) and then we noticed that the are increasing in spectral efficiency appreciably in the average case and ZF is sort of similar to MR other wise in the best case also we found the same result of increasing the spectral efficiency and ZF achieve high result comparing to the MR but in the worst case there are no noticed change in the value of the spectral efficiency as the fallowing figures present

4.4.2 Pathloss exponent

the second parameter we change is *Pathloss exponent* from 3.7 to 5 while the other parameters is constant (S = 400 and SNR= 5 dB) and then we noticed that the are increasing in spectral efficiency in the average case and ZF is sort of similar to MR other wise in the best case and the worst case there are no noticed change in the value of the spectral efficiency as the fallowing figures present

4.4.3 Signal-to-Noise Ratio

the third parameter we change is *Signal-to-Noise* from 5 dB to -10 dB while the other parameters is constant (S = 400 and Pathloss= 3.7) and then we noticed that the are increasing in spectral efficiency in the average case and ZF is sort of similar to MR other wise in the best case and the worst case there are no noticed change in the value of the spectral efficiency as the fallowing figures present

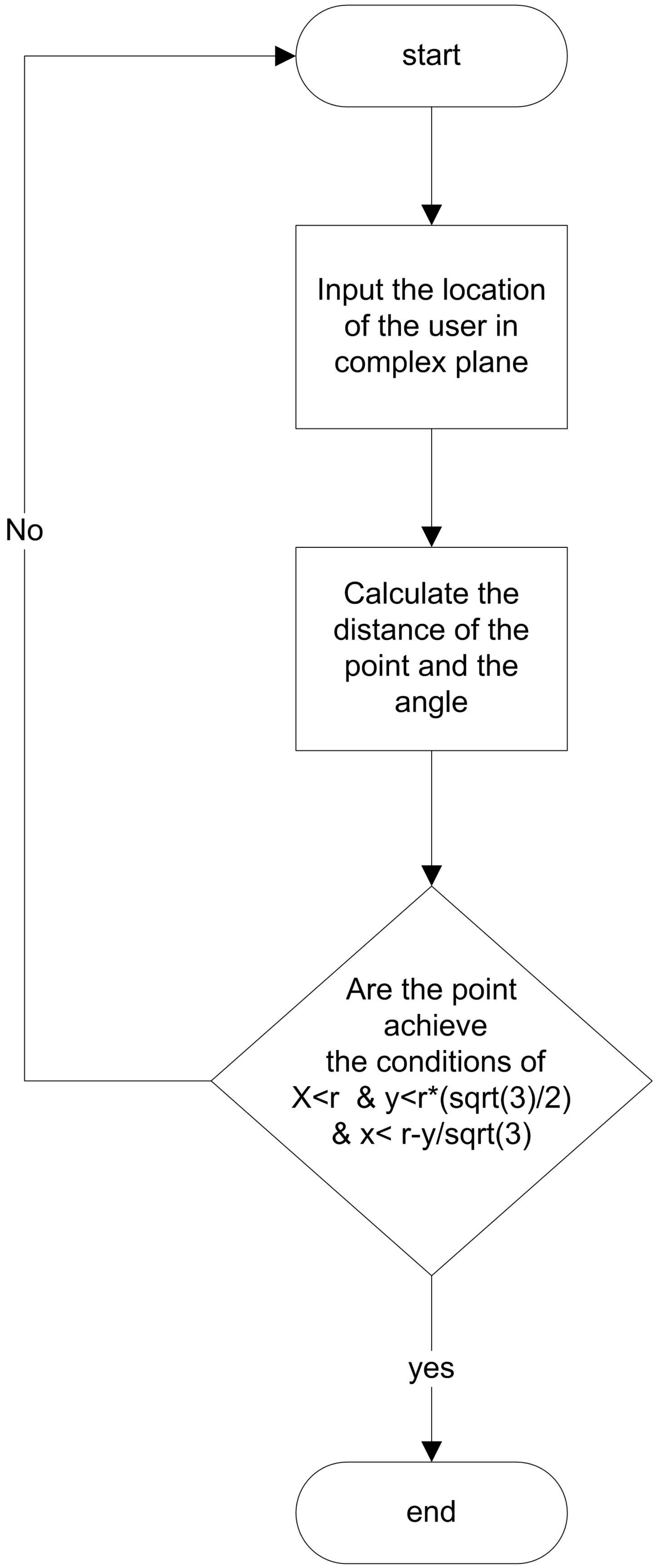


Figure 4.2: Flowchart show the process of calculate the location of the point

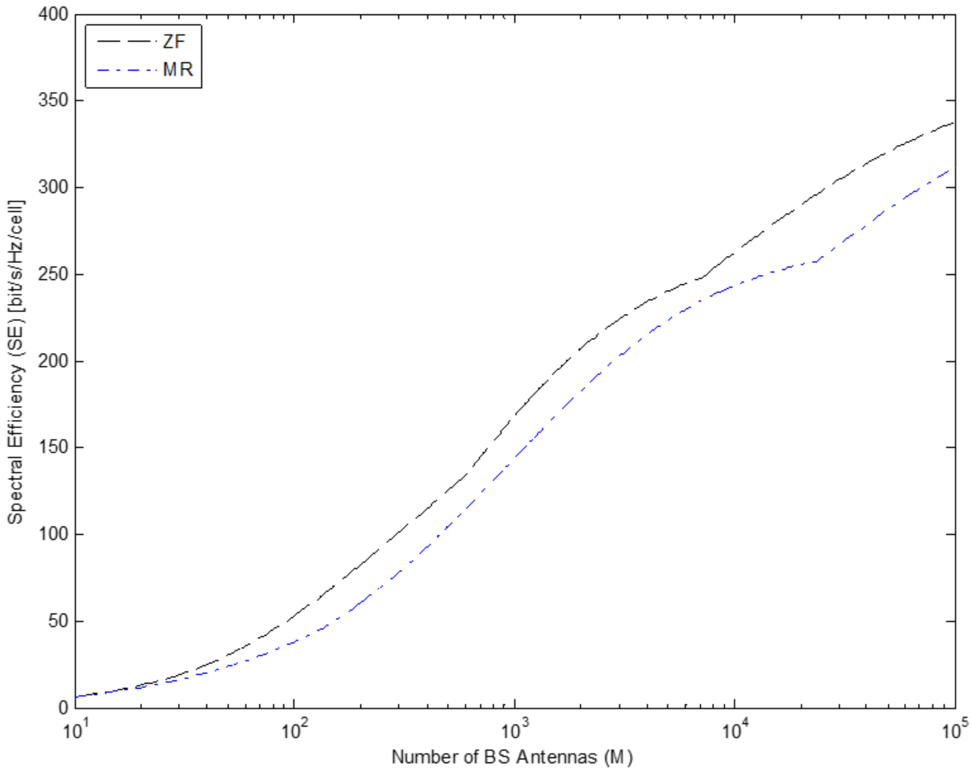


Figure 4.3: Simulation of enhanced SE, as a function of M, with average inter-cell interference.

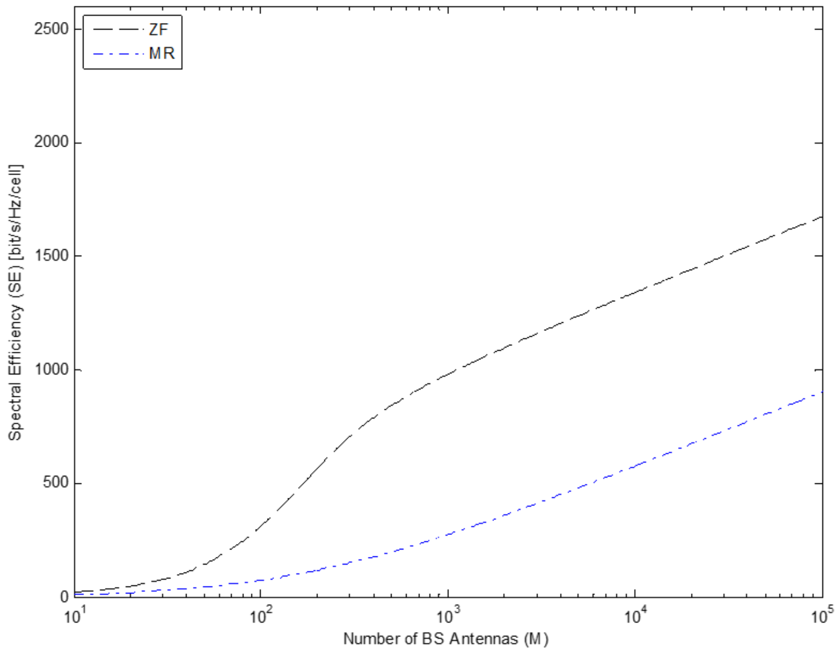


Figure 4.4: Simulation of enhanced SE, as a function of M, with best-case inter-cell interference.

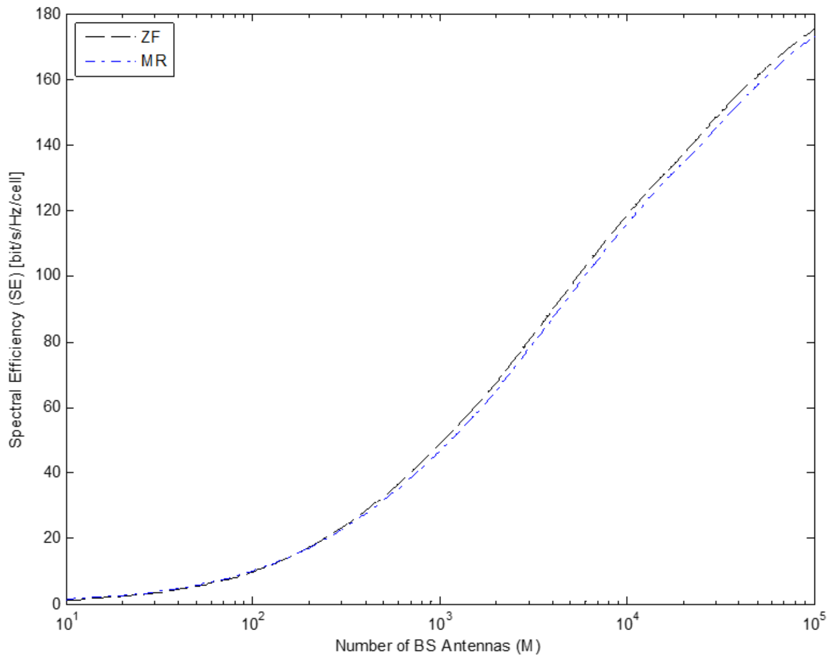


Figure 4.5: Simulation of enhanced SE, as a function of M, with worst-case inter-cell interference

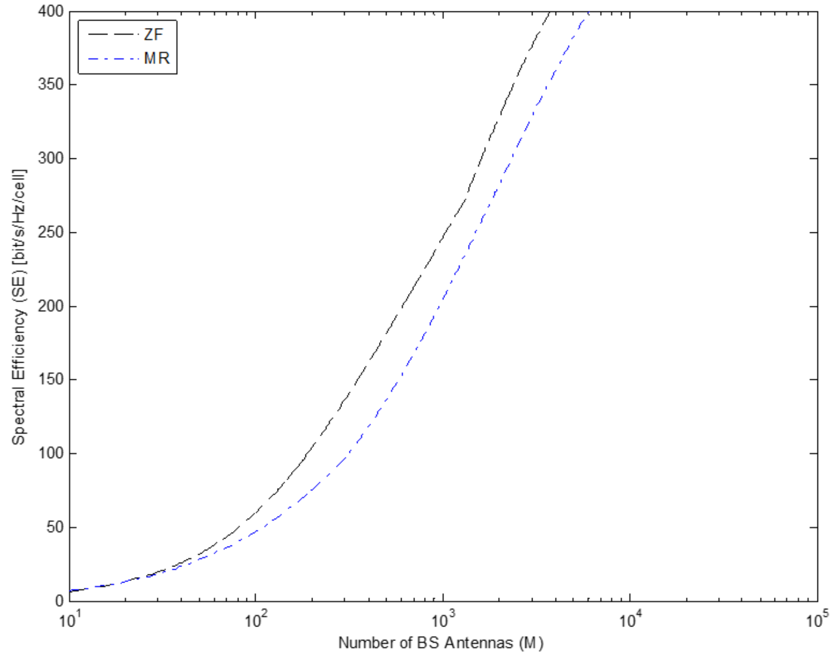


Figure 4.6: Average case with change in Coherence block length $S = 800$

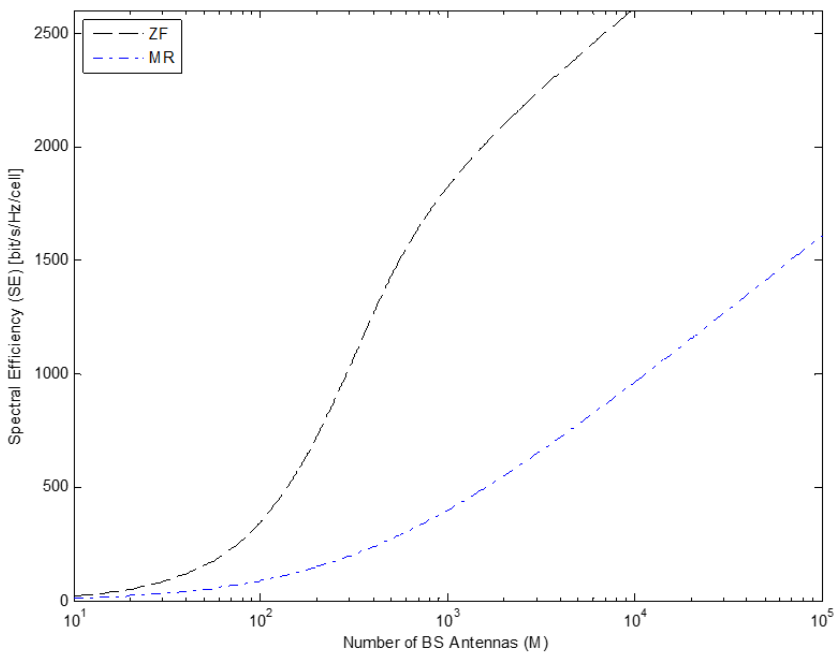


Figure 4.7: Best case with change in Coherence block length $S = 800$

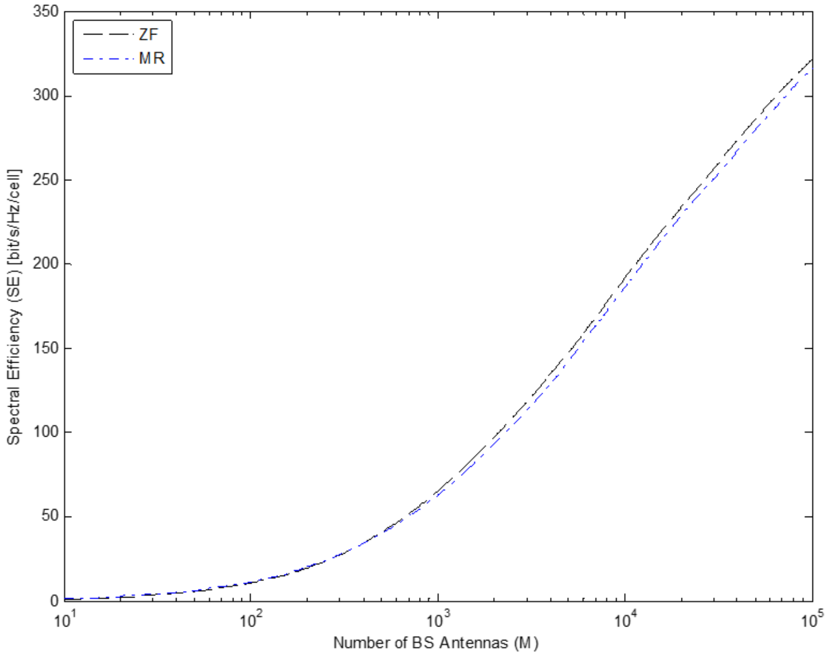


Figure 4.8: Worst case with change in Coherence block length $S = 800$

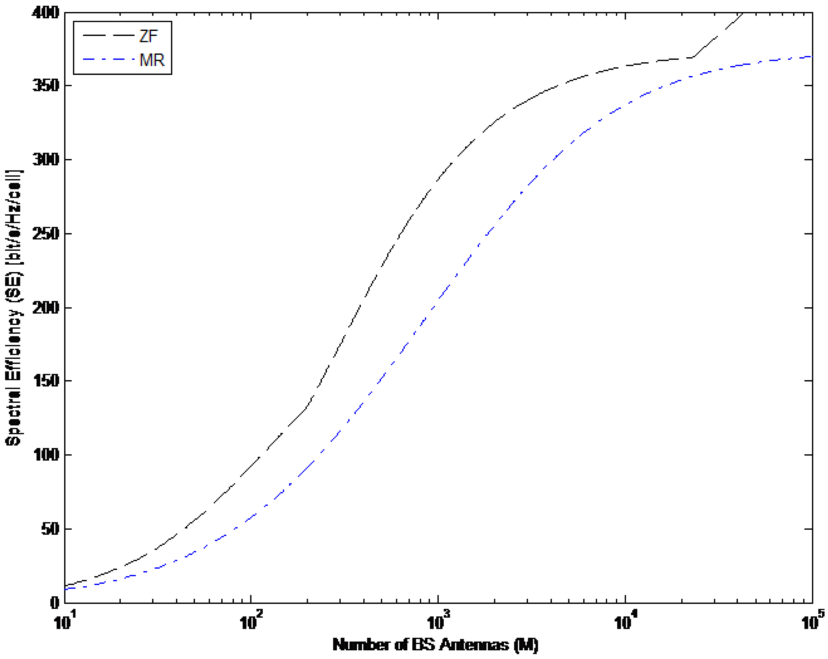


Figure 4.9: Average case with change $\kappa = 5$

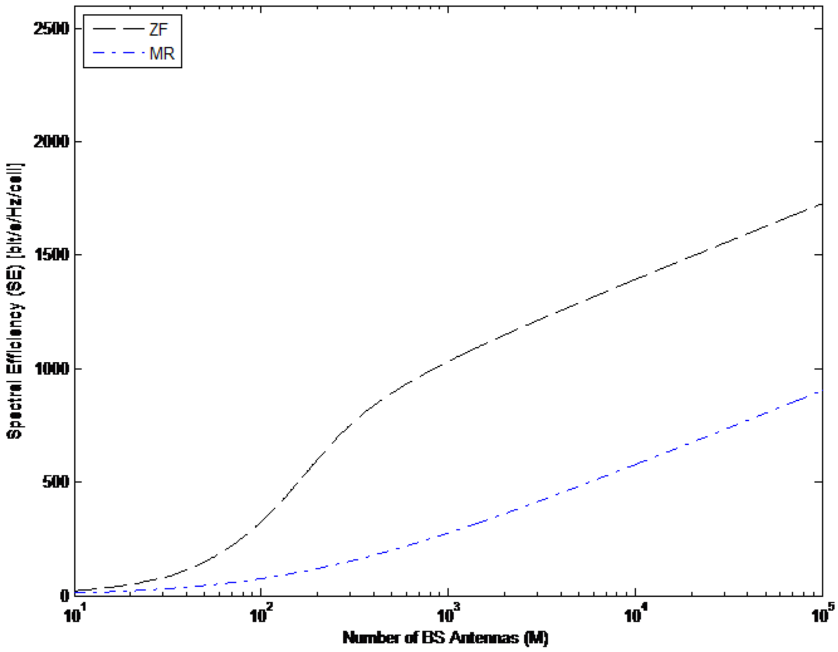


Figure 4.10: Best case with change $\kappa = 5$

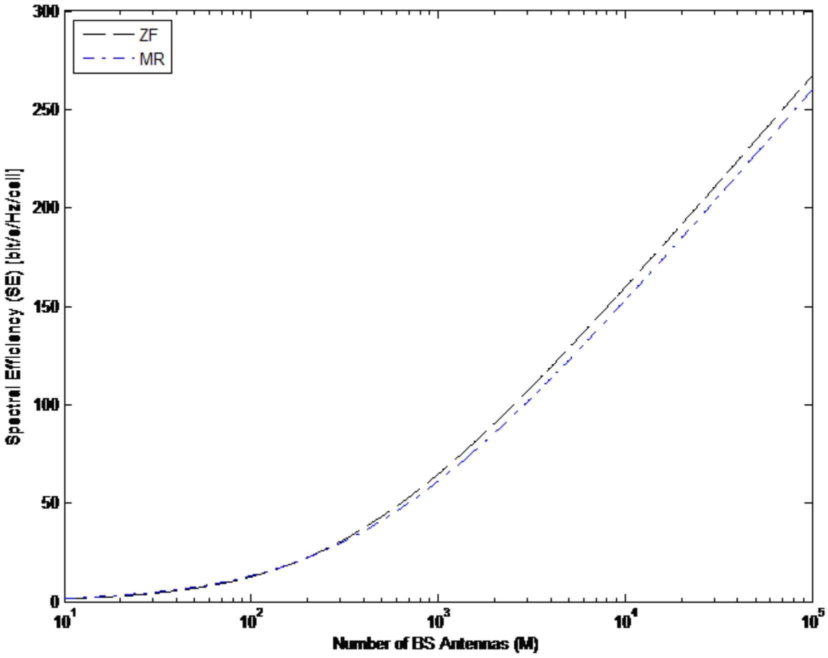


Figure 4.11: Worst case with change $\kappa = 5$

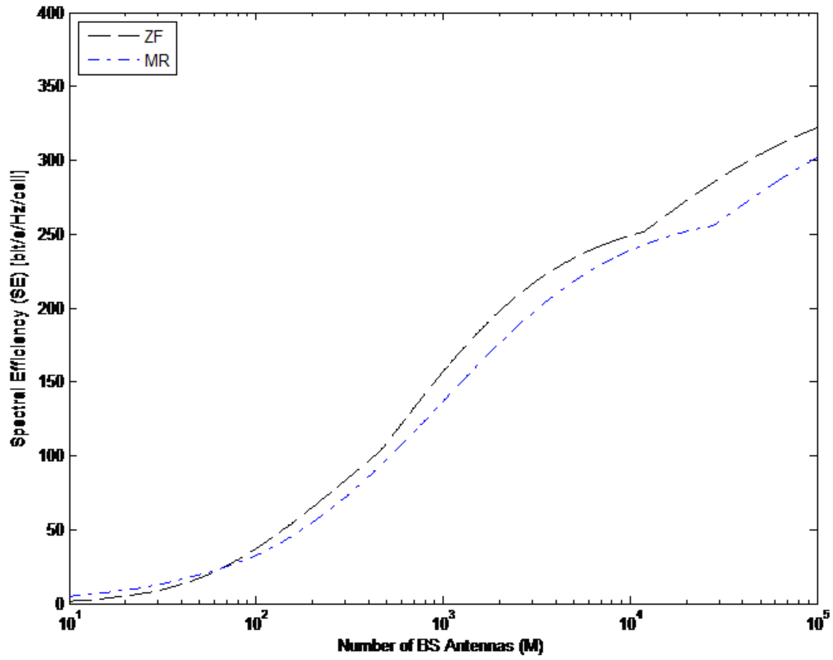


Figure 4.12: Average case with change $SNR = -10dB$

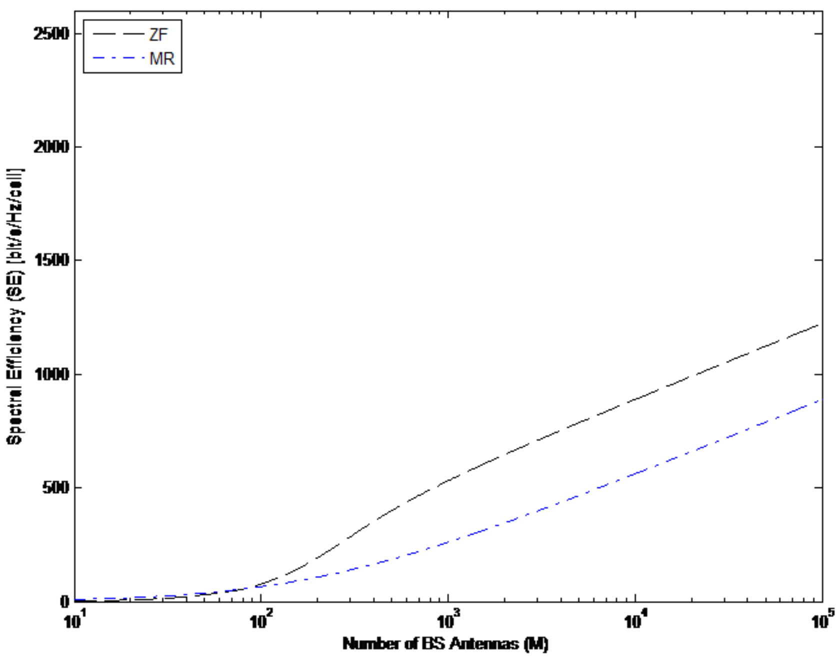


Figure 4.13: Best case with change $SNR = -10dB$

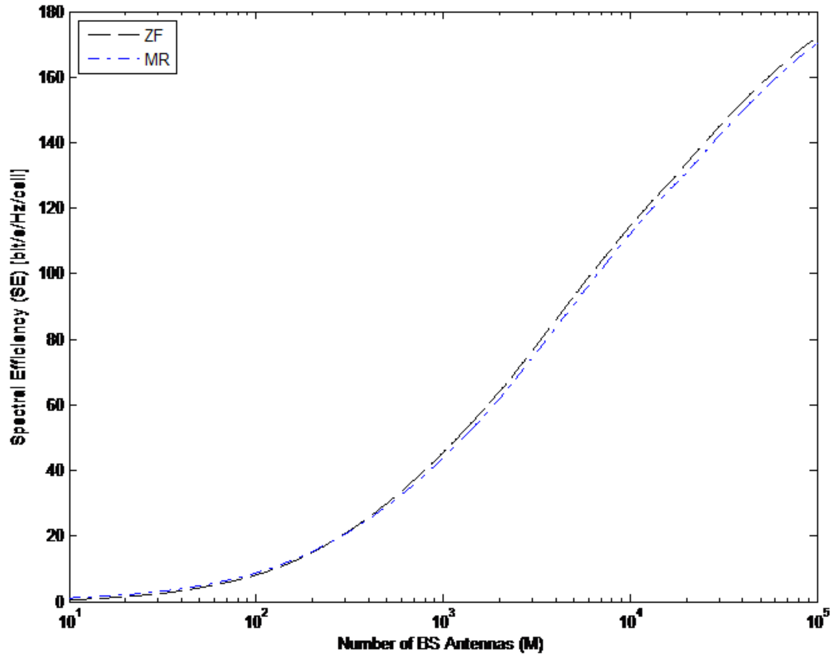


Figure 4.14: Worst case with change $SNR = -10dB$

Chapter Five
Conclusions and Recommendations

5.1 Conclusions

This project concerning with how to maximal the spectral efficiency by applying spatial multiplexing (massive MIMO) . ZF give high spectral efficiency per cell when it compared with MR, whats mean the reduce of the interference is better in ZF than MR

The study, analyze , plan of the software program to simulate the performance of massive MIMO has being done by using MATLAB software program. The linear schemes which has taken under consideration are zero forcing and the maximum ratio methods. The implementation of massive MIMO by using different linear processing schemes under different interference situations. The spectral efficiency is directly proportional with number of antennas in the (BS).

5.2 Recommendations

From the results we suggest the following recommendation for future work:

- Analyze and implement the massive MIMO using other linear processing technique, or non linear processing technique such as (DP) and (SIC)

- Optimize the number of UEs antennas (K) per-cell.
- Pilot contamination imposes much more severe limitations on massive MIMO than on traditional MIMO systems. The effect of massive MIMO in power consumption

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Appendix A

A.1 Program 1

```matlab
function okay = checkHexagonal(points,radius)
%This function takes points in the complex plane and check ...
    if they are
%inside a hexagon of specified size (and a rotation with ...
    two sides being
%parallel to the horizontal axis.
%
%INPUT
%points = Matrix with points in the complex plane
%radius = Radius (length to corners) of the hexagon in the ...
    complex plane
%
%OUTPUT
%okay   = Matrix with booleans telling if the points are ...
    inside the hexagon

%Extract distances and angle
angles = angle(points);
distances = abs(points);

%Symmetry allows us to rotate all angles to lie in the area ...
    0, pi/3
angles_modulus = mod(angles,pi/3);

%Extract the Cartesian coordinates for the rotated points
x = distances .* cos(angles_modulus);
y = distances .* sin(angles_modulus);

%Check if the points are in the hexagon, in an area limited ...
    by three lines
okay = (x<radius) & ( y<radius*(sqrt(3)/2)) & (x < radius - ...
    y/sqrt(3));
```

A.2 Program 2

```
function [muValues1Mean,muValues2Mean,....
reuseMu1Mean,reuseMu1Mean2,reuseMu1MeanNext ...
reuseMu1Mean2Next,reuseMu2Mean, ...
reuseMuMeanVariance,muValues1Worst, ...
muValues2Wors
reuseMu1Worst,reuseMu1Worst2,reuseMu1WorstNext...
 ,reuseMu1Worst2Next,reuseMu2Worst...
reuseMuWorstVariance,muValues1Best,muValues2Best...
 ,reuseMu1Best,reuseMu1Best2,..
reuseMu1BestNext,reuseMu1Best2Next,reuseMu2Best...
 ,reuseMuBestVariance,reuseFactor]
  = computeEnvironment(kappa,forbiddenRegion,monteCarloUEs)
%This function performs Monte Carlo simulations to compute ...
    various sums of
%the mu-parameters



%Set number of UE locations in the Monte Carlo simulations, ...
    if this number
%is not set as an input parameter
if nargin<3
    monteCarloUEs = 1000;
end


%Define matrix for storing UE locations
UElocations = zeros(1,monteCarloUEs);


%Define cell dimensions (the unit or exact size doesn't ...
    matter since
%everything is the mu-parameters are scale invariant)
intersiteDistance = 0.5; %Distance between neighboring BSs

dmax = intersiteDistance/2; %Cell radius
dmin = dmax * forbiddenRegion; %Shortest distance from a BS
```

```
%Generate UE locations randomly with uniform distribution ...
    inside the cells
nbrToGenerate = monteCarloUEs;
notFinished = true(monteCarloUEs,1);


%Iterate the generation of UE locations until all of them ...
    are inside a
%hexagonal cell
while nbrToGenerate>0

    %Generate new UE locations uniformly at random in a ...
        circle of radius dmax
    UElocations(1,notFinished) =
    sqrt( rand(1,nbrToGenerate)*(dmax^2-dmin^2)+ dmin^2 ) ...
    .* exp(1i*2*pi*rand(1,nbrToGenerate));

    %Check which UEs that are inside a hexagonal and ...
        declare as finished
    finished = checkHexagonal(UElocations(1,:)',dmax);

    %Update which UEs that are left to generate
    notFinished = (finished==false);

    %Update how many UEs that are left to generate
    nbrToGenerate = sum(notFinished);

end


%Angle between each edge point (360/6 = 60)
baseAngle = 60;

%Select how many tiers of BSs should be considered around ...
    the cell of
%interest
howFar = 5;


%Placeholders for storing results for the mean interference ...
    case
muValues1Mean = zeros(6,6);
muValues2Mean = zeros(6,6);
```

```
muValues1Mean(1,1) = 1;
muValues2Mean(1,1) = 1;

reuseMu1Mean = zeros(6,6);
reuseMu1Mean2 = zeros(6,6);
reuseMu1MeanNext = zeros(6,6);
reuseMu1Mean2Next = zeros(6,6);

reuseMu2Mean = zeros(6,6);
reuseMuMeanVariance = zeros(6,6);


%Placeholders for storing results for the worst ...
    interference case
muValues1Worst = zeros(6,6);
muValues2Worst = zeros(6,6);
muValues1Worst(1,1) = 1;
muValues2Worst(1,1) = 1;

reuseMu1Worst = zeros(6,6);
reuseMu1Worst2 = zeros(6,6);
reuseMu1WorstNext = zeros(6,6);
reuseMu1Worst2Next = zeros(6,6);

reuseMu2Worst = zeros(6,6);
reuseMuWorstVariance = zeros(6,6);


%Placeholders for storing results for the best interference ...
    case
muValues1Best = zeros(6,6);
muValues2Best = zeros(6,6);
muValues1Best(1,1) = 1;
muValues2Best(1,1) = 1;

reuseMu1Best = zeros(6,6);
reuseMu1Best2 = zeros(6,6);
reuseMu1BestNext = zeros(6,6);
reuseMu1Best2Next = zeros(6,6);

reuseMu2Best = zeros(6,6);
reuseMuBestVariance = zeros(6,6);

```

```

%Placeholder for storing reuse factors
reuseFactor = zeros(6,6);

%Define the position of one of the neighboring cells, as ... seen from the
%origin.
nextNeighbor = sqrt(3)*dmax*exp(1i*pi*(30/180));


%Go through neighboring cells at different distances using the
%parameterization in Eq. (31). Only one search direction is ... considered, but
%there are six neighboring cells at the same distance.
for alpha1 = 1:1:howFar
    for alpha2 = 0:1:howFar

        %Put out another BS using coordinates u and v
        BSlocations =
        sqrt(3)*alpha1*dmax*exp(1i*pi*(30/180)) + ... sqrt(3)*alpha2*dmax*1i; ...
        %Coordinates based on Eq. (31)


        %Compute the reuse factor for the hexagonal ... topology when the
        %current neighboring cell is the first one that ... reuses the same
        %pilot sequences
        reuseFactor(alpha1+1,alpha2+1) = ... alpha1^2+alpha2^2+alpha1*alpha2;


        %Mean interference

        %Compute the mu^(1) and mu^(2) values for mean ... interference
        %seen from the base station in the origin
        muValues1Mean(alpha1+1,alpha2+1) =
        mean((abs(UElocations(:))./abs(UElocations(:) ...
        +BSlocations)).^kappa);
        muValues2Mean(alpha1+1,alpha2+1) =
        mean((abs(UElocations(:))./abs(UElocations(:) ...
```

```
            +BSlocations)).^(2*kappa));

            %Store the sum of interference for the cells that ...
                have the same
            %pilot sequences as the cell in the origin, when ...
                the pilot reuse
            %factor is alpha1^2+alpha2^2+alpha1*alpha2
            reuseMu1Mean(alpha1+1,alpha2+1) =
            reuseMu1Mean(alpha1+1,alpha2+1) + ...
            muValues1Mean(alpha1+1,alpha2+1); %Sum of mu^(1)
            reuseMu1Mean2(alpha1+1,alpha2+1) =
            reuseMu1Mean2(alpha1+1,alpha2+1) + ...
            muValues1Mean(alpha1+1,alpha2+1)^2; %Sum of (mu^(1))^2
            reuseMu2Mean(alpha1+1,alpha2+1) =
            reuseMu2Mean(alpha1+1,alpha2+1) + ...
            muValues2Mean(alpha1+1,alpha2+1); %Sum of mu^(2)
            reuseMuMeanVariance(alpha1+1,alpha2+1) =
            reuseMuMeanVariance(alpha1+1,alpha2+1) ...
            + muValues2Mean(alpha1+1,alpha2+1) - ...
                muValues1Mean(alpha1+1,alpha2+1)^2;..

            %Store the sum of interference for the cells that ...
                have the same
            %pilot sequences as one of the neighbors of the ...
                cell in the origin,
            %when the pilot reuse factor is ...
                alpha1^2+alpha2^2+alpha1*alpha2
            newMu1ReuseNext =
            mean((abs(UElocations(:))./abs(UElocations(:)+nextNeighbor)).^ka
            newMu1ReuseNextOneStep =
            mean((abs(UElocations(:))./abs(UElocations(:)+BSlocations+nextNe
            .^kappa);
            reuseMu1MeanNext(alpha1+1,alpha2+1) =
            reuseMu1MeanNext(alpha1+1,alpha2+1) + ...
                newMu1ReuseNext ...
            + newMu1ReuseNextOneStep;
            reuseMu1Mean2Next(alpha1+1,alpha2+1) =
            reuseMu1Mean2Next(alpha1+1,alpha2+1) + ...
                newMu1ReuseNext.^2 ...
            +newMu1ReuseNextOneStep.^2;


            %Worst-case interference
```

```
        %Compute the mu^(1) and mu^(2) values for ...
            worst-case interference,
        %seen from the base station in the origin
        muValues1Worst(alpha1+1,alpha2+1) =
        max((abs(UElocations(:))./abs(UElocations(:)+BSlocations))...
        .^kappa);
        muValues2Worst(alpha1+1,alpha2+1) =
        max((abs(UElocations(:))./abs(UElocations(:)+BSlocations))...
        .^(2*kappa));

        %Store the sum of interference for the cells that ...
            have the same
        %pilot sequences as the cell in the origin, when ...
            the pilot reuse
        %factor is alpha1^2+alpha2^2+alpha1*alpha2
        reuseMu1Worst(alpha1+1,alpha2+1) =
        reuseMu1Worst(alpha1+1,alpha2+1) +...
        muValues1Worst(alpha1+1,alpha2+1); %Sum of mu^(1)
        reuseMu1Worst2(alpha1+1,alpha2+1) =
        reuseMu1Worst2(alpha1+1,alpha2+1) ...
        + muValues1Worst(alpha1+1,alpha2+1).^2;  %Sum of ...
            (mu^(1))^2
        reuseMu2Worst(alpha1+1,alpha2+1) =
        reuseMu2Worst(alpha1+1,alpha2+1) ...
        + muValues2Worst(alpha1+1,alpha2+1); %Sum of mu^(2)
        reuseMuWorstVariance(alpha1+1,alpha2+1) =
        reuseMuWorstVariance(alpha1+1,alpha2+1)...
        + muValues2Worst(alpha1+1,alpha2+1) -...
        muValues1Worst(alpha1+1,alpha2+1)^2;

        %Store the sum of interference for the cells that ...
            have the same
        %pilot sequences as one of the neighbors of the ...
            cell in the origin,
        %when the pilot reuse factor is ...
            alpha1^2+alpha2^2+alpha1*alpha2
        newMu1ReuseNext =
        max((abs(UElocations(:))./abs(UElocations(:)+nextNeighbor)).^kap
        newMu1ReuseNextOneStep =
        max((abs(UElocations(:))./abs(UElocations(:) ...
        +BSlocations+nextNeighbor)).^kappa);
        reuseMu1WorstNext(alpha1+1,alpha2+1) =
```

```
        reuseMu1WorstNext(alpha1+1,alpha2+1) ...
        + newMu1ReuseNext + newMu1ReuseNextOneStep;
        reuseMu1Worst2Next(alpha1+1,alpha2+1) =
        reuseMu1Worst2Next(alpha1+1,alpha2+1)...
        + newMu1ReuseNext.^2 + newMu1ReuseNextOneStep.^2;



        %Best-case interference

        %Compute the mu^(1) and mu^(2) values for best-case ... interference
        %seen from the base station in the origin
        muValues1Best(alpha1+1,alpha2+1) =
        min((abs(UElocations(:))./abs(UElocations(:) ...
        +BSlocations)).^kappa);
        muValues2Best(alpha1+1,alpha2+1) =
        min((abs(UElocations(:))./abs(UElocations(:) ...
        +BSlocations)).^(2*kappa));

        %Store the sum of interference for the cells that ... have the same
        %pilot sequences as the cell in the origin, when ... the pilot reuse
        %factor is alpha1^2+alpha2^2+alpha1*alpha2
        reuseMu1Best(alpha1+1,alpha2+1) =
        reuseMu1Best(alpha1+1,alpha2+1) + ...
        muValues1Best(alpha1+1,alpha2+1); %Sum of mu^(1)
        reuseMu1Best2(alpha1+1,alpha2+1) =
        reuseMu1Best2(alpha1+1,alpha2+1) +...
        muValues1Best(alpha1+1,alpha2+1).^2; %Sum of (mu^(1))^2
        reuseMu2Best(alpha1+1,alpha2+1) =
        reuseMu2Best(alpha1+1,alpha2+1) +...
        muValues2Best(alpha1+1,alpha2+1); %Sum of mu^(2)
        reuseMuBestVariance(alpha1+1,alpha2+1) =
        reuseMuBestVariance(alpha1+1,alpha2+1)...
        + muValues2Best(alpha1+1,alpha2+1) -...
        muValues1Best(alpha1+1,alpha2+1)^2;

        %Store the sum of interference for the cells that ... have the same
        %pilot sequences as one of the neighbors of the ... cell in the origin,
```

```
        %when the pilot reuse factor is ...
            alpha1^2+alpha2^2+alpha1*alpha2
        newMu1ReuseNext =
        min((abs(UElocations(:))./abs(UElocations(:)+nextNeighbor)).^kap
        newMu1ReuseNextOneStep =
        min((abs(UElocations(:))./abs(UElocations(:)+...
        BSlocations+nextNeighbor)).^kappa);
        reuseMu1BestNext(alpha1+1,alpha2+1) =
        reuseMu1BestNext(alpha1+1,alpha2+1) +...
        newMu1ReuseNext + newMu1ReuseNextOneStep;
        reuseMu1Best2Next(alpha1+1,alpha2+1) =
        reuseMu1Best2Next(alpha1+1,alpha2+1) + ...
        newMu1ReuseNext.^2 + newMu1ReuseNextOneStep.^2;


        %Consider the next two cells with the same reuse ...
            factor (there are
        %two neigbors instead of one in the second ...
            interfering tier)
        for index = 0:1

            %Compute location of the next BS that use the ...
                same reuse factor
            BSlocation2 =
            BSlocations + ...
                BSlocations*exp(1i*pi*((index*baseAngle)/180));


            %Mean interference

            %Compute the mu^(1) and mu^(2) values for mean ...
                interference
            %seen from the base station in the origin
            newMu1Reuse = mean((abs(UElocations(:))./...
            abs(UElocations(:)+BSlocation2)).^kappa);
            newMu2Reuse = mean((abs(UElocations(:))./...
            abs(UElocations(:)+BSlocation2)).^(2*kappa));
            newMu1ReuseNext = mean((abs(UElocations(:))./...
            abs(UElocations(:) ...
            +BSlocation2+nextNeighbor)).^kappa);

            reuseMu1Mean(alpha1+1,alpha2+1) =
            reuseMu1Mean(alpha1+1,alpha2+1) + ...
```

```
                newMu1Reuse; %Add to sum of mu^(1)
                reuseMu1Mean2(alpha1+1,alpha2+1) =
                reuseMu1Mean2(alpha1+1,alpha2+1) +...
                newMu1Reuse.^2; %Add to sum of (mu^(1))^2
                reuseMu2Mean(alpha1+1,alpha2+1) =
                reuseMu2Mean(alpha1+1,alpha2+1) + ....
                newMu2Reuse; %Add to sum of mu^(2)
                reuseMuMeanVariance(alpha1+1,alpha2+1) =
                reuseMuMeanVariance(alpha1+1,alpha2+1) ...
                + newMu2Reuse - newMu1Reuse^2;
                reuseMu1MeanNext(alpha1+1,alpha2+1) =
                reuseMu1MeanNext(alpha1+1,alpha2+1) + ...
                newMu1ReuseNext;
                reuseMu1Mean2Next(alpha1+1,alpha2+1) =
                reuseMu1Mean2Next(alpha1+1,alpha2+1) + ...
                newMu1ReuseNext.^2;


                %Worst-case interference

                %Compute the mu^(1) and mu^(2) for the next BS, ...
                    for worst-case
                %interference seen from the base station in the ...
                    origin
                newMu1Reuse = max((abs(UElocations(:))./...
                abs(UElocations(:)+BSlocation2)).^kappa);
                newMu2Reuse = max((abs(UElocations(:))./...
                abs(UElocations(:)+BSlocation2)).^(2*kappa));
                newMu1ReuseNext = max((abs(UElocations(:))./...
                abs(UElocations(:) ...
                +BSlocation2+nextNeighbor)).^kappa);

                %Store the results
                reuseMu1Worst(alpha1+1,alpha2+1) =
                reuseMu1Worst(alpha1+1,alpha2+1)...
                + newMu1Reuse; %Add to sum of mu^(1)
                reuseMu1Worst2(alpha1+1,alpha2+1) =
                reuseMu1Worst2(alpha1+1,alpha2+1) ...
                + newMu1Reuse.^2; %Add to sum of (mu^(1))^2
                reuseMu2Worst(alpha1+1,alpha2+1) =
                reuseMu2Worst(alpha1+1,alpha2+1) ...
                + newMu2Reuse; %Add to sum of mu^(2)
                reuseMuWorstVariance(alpha1+1,alpha2+1) =
```

```
            reuseMuWorstVariance(alpha1+1,alpha2+1)...
            + newMu2Reuse - newMu1Reuse^2;
            reuseMu1WorstNext(alpha1+1,alpha2+1) =
            reuseMu1WorstNext(alpha1+1,alpha2+1)...
            + newMu1ReuseNext;
            reuseMu1Worst2Next(alpha1+1,alpha2+1) =
            reuseMu1Worst2Next(alpha1+1,alpha2+1)...
            + newMu1ReuseNext.^2;


            %Best-case interference

            %Compute the mu^(1) and mu^(2) for the next BS, ...
                for best-case
            %interference seen from the base station in the ...
                origin
            newMu1Reuse = min((abs(UElocations(:))./...
            abs(UElocations(:)+BSlocation2)).^kappa);
            newMu2Reuse = min((abs(UElocations(:))./
            abs(UElocations(:)+BSlocation2)).^(2*kappa));
            newMu1ReuseNext = min((abs(UElocations(:))./...
            abs(UElocations(:)...
            +BSlocation2+nextNeighbor)).^kappa);

            %Store the results
            reuseMu1Best(alpha1+1,alpha2+1) =
            reuseMu1Best(alpha1+1,alpha2+1) + ...
            newMu1Reuse; %Add to sum of mu^(1)
            reuseMu1Best2(alpha1+1,alpha2+1) =
            reuseMu1Best2(alpha1+1,alpha2+1) +...
            newMu1Reuse.^2; %Add to sum of (mu^(1))^2
            reuseMu2Best(alpha1+1,alpha2+1) =
            reuseMu2Best(alpha1+1,alpha2+1) +...
            newMu2Reuse; %Add to sum of mu^(2)
            reuseMuBestVariance(alpha1+1,alpha2+1) =
            reuseMuBestVariance(alpha1+1,alpha2+1) ...
            + newMu2Reuse - newMu1Reuse^2;
            reuseMu1BestNext(alpha1+1,alpha2+1) =
            reuseMu1BestNext(alpha1+1,alpha2+1) +...
            newMu1ReuseNext;
            reuseMu1Best2Next(alpha1+1,alpha2+1) =
            reuseMu1Best2Next(alpha1+1,alpha2+1) +...
            newMu1ReuseNext.^2;
```

```
            %Consider the next three cells with the same ...
                reuse factor
            %(there are three neigbors instead of two in ...
                the third interfering tier)
            for index2 = index:1

                %Compute location of the next BS that use ...
                    the same reuse factor
                BSlocation3 = BSlocation2 + BSlocations*...
                exp(1i*pi*((index2*baseAngle)/180));


                %Mean interference

                %Compute the mu^(1) and mu^(2) for the next ...
                    BS, for mean
                %interference seen from the base station in ...
                    the origin
                newMu1Reuse = mean((abs(UElocations(:))./....
                abs(UElocations(:) ...
                +BSlocation3)).^kappa);
                newMu2Reuse = mean((abs(UElocations(:))./...
                abs(UElocations(:) ...
                +BSlocation3)).^(2*kappa));
                newMu1ReuseNext = ...
                    mean((abs(UElocations(:))./...
                abs(UElocations(:) ...
                +BSlocation3+nextNeighbor)).^kappa);

                reuseMu1Mean(alpha1+1,alpha2+1) =
                reuseMu1Mean(alpha1+1,alpha2+1) ...
                + newMu1Reuse; %Add to sum of mu^(1)
                reuseMu1Mean2(alpha1+1,alpha2+1) =
                reuseMu1Mean2(alpha1+1,alpha2+1)...
                + newMu1Reuse.^2; %Add to sum of (mu^(1))^2
                reuseMu2Mean(alpha1+1,alpha2+1) =
                reuseMu2Mean(alpha1+1,alpha2+1)...
                + newMu2Reuse; %Add to sum of mu^(2)
                reuseMuMeanVariance(alpha1+1,alpha2+1) =
                reuseMuMeanVariance(alpha1+1,alpha2+1) ...
```

```
        + newMu2Reuse - newMu1Reuse^2;
        reuseMu1MeanNext(alpha1+1,alpha2+1) =
        reuseMu1MeanNext(alpha1+1,alpha2+1) + ...
        newMu1ReuseNext;
        reuseMu1Mean2Next(alpha1+1,alpha2+1) =
        reuseMu1Mean2Next(alpha1+1,alpha2+1) +...
        newMu1ReuseNext.^2;


        %Worst-case interference

        %Compute the mu^(1) and mu^(2) for the next ...
            BS, for
        %worst-case interference seen from the base ...
            station in the origin
        newMu1Reuse = max((abs(UElocations(:))./...
        abs(UElocations(:) ...
        +BSlocation3)).^kappa);
        newMu2Reuse = max((abs(UElocations(:))./...
        abs(UElocations(:) ...
        +BSlocation3)).^(2*kappa));
        newMu1ReuseNext = max((abs(UElocations(:))./...
        abs(UElocations(:) ...
        +BSlocation3+nextNeighbor)).^kappa);

        %Store the results
        reuseMu1Worst(alpha1+1,alpha2+1) =
        reuseMu1Worst(alpha1+1,alpha2+1)...
        + newMu1Reuse; %Add to sum of mu^(1)
        reuseMu1Worst2(alpha1+1,alpha2+1) =
        reuseMu1Worst2(alpha1+1,alpha2+1) ...
        + newMu1Reuse.^2; %Add to sum of (mu^(1))^2
        reuseMu2Worst(alpha1+1,alpha2+1) =
        reuseMu2Worst(alpha1+1,alpha2+1) ...
        + newMu2Reuse; %Add to sum of mu^(2)
        reuseMuWorstVariance(alpha1+1,alpha2+1) =
        reuseMuWorstVariance(alpha1+1,alpha2+1) + ...
        newMu2Reuse - newMu1Reuse^2;
        reuseMu1WorstNext(alpha1+1,alpha2+1) =
        reuseMu1WorstNext(alpha1+1,alpha2+1)...
        + newMu1ReuseNext;
        reuseMu1Worst2Next(alpha1+1,alpha2+1) =
        euseMu1Worst2Next(alpha1+1,alpha2+1)...
```

```
            + newMu1ReuseNext.^2;


            %Best-case interference

            %Compute the mu^(1) and mu^(2) for the next ...
                BS, for
            %best-case interference seen from the base ...
                station in the origin
            newMu1Reuse = ...
                min((abs(UElocations(:))./abs(UElocations(:)...
            +BSlocation3)).^kappa);
            newMu2Reuse = ...
                min((abs(UElocations(:))./abs(UElocations(:)...
            +BSlocation3)).^(2*kappa));
            newMu1ReuseNext = ...
                min((abs(UElocations(:))./abs(UElocations(:)...
            +BSlocation3+nextNeighbor)).^kappa);

            %Store the results
            reuseMu1Best(alpha1+1,alpha2+1) =
            reuseMu1Best(alpha1+1,alpha2+1)...
            + newMu1Reuse; %Add to sum of mu^(1)
            reuseMu1Best2(alpha1+1,alpha2+1) =
            reuseMu1Best2(alpha1+1,alpha2+1) ...
            + newMu1Reuse.^2; %Add to sum of (mu^(1))^2
            reuseMu2Best(alpha1+1,alpha2+1) =
            reuseMu2Best(alpha1+1,alpha2+1) ...
            + newMu2Reuse; %Add to sum of mu^(2)
            reuseMuBestVariance(alpha1+1,alpha2+1) =
            reuseMuBestVariance(alpha1+1,alpha2+1)...
            + newMu2Reuse - newMu1Reuse^2;
            reuseMu1BestNext(alpha1+1,alpha2+1) =
            reuseMu1BestNext(alpha1+1,alpha2+1) ...
            + newMu1ReuseNext;
            reuseMu1Best2Next(alpha1+1,alpha2+1) =
            reuseMu1Best2Next(alpha1+1,alpha2+1) ...
            + newMu1ReuseNext.^2;

        end

    end

```

```
    end

end
```

A.3 Program 3

```
%Initialization
close all;
clear all;


%%Simulation parameters

%Initiate the random number generators
% with a random seed
randn('state',sum(100*clock));

%Pathloss exponent
kappa = 3.7;

%Number of directions to look for interfering cells
%(for hexagonal cells)
directions = 6;

%Percentage of the radius inside the cell where no UEs are ...
    allowed
forbiddenRegion = .14;

%Parameters for the Monte Carlo simulations
monteCarloUEs = 1000; %Number of random UE locations per cell

%Compute various combinations of the
%mu-parameters Propagation parameter Eq, using
%Monte Carlo simulations
[muValues1Mean,muValues2Mean,reuseMu1Mean,
reuseMu1Mean2,reuseMu1MeanNext,reuseMu1Mean2Next,...
reuseMu2Mean,reuseMuMeanVariance
,muValues1Worst,muValues2Worst,
reuseMu1Worst,reuseMu1Worst2,...
reuseMu1WorstNext,reuseMu1Worst2Next,reuseMu2Worst,
```

```
reuseMuWorstVariance,muValues1NorthWest, ...
muValues2NorthWest,reuseMu1NorthWest,reuseMu1NorthWest2, ...
reuseMu1NorthWestNext,reuseMu1NorthWest2Next,
reuseMu2NorthWest, ...
reuseMuNorthWestVariance,reuseFactor] =
computeEnvironment(kappa,forbiddenRegion,monteCarloUEs);




    %Select range of BS antennas
    %Number of different cases
    nbrOfMvalues = 1000;
     %Spread out antenna numbers equally in log-scale
    Mvalues = round(logspace(1,5,nbrOfMvalues));

    %Coherence block length
    S = 400 * ones(1,2);

    %Inverse SNR value
    sigma2rho = 1/10^(5/10) * ones(1,2); %5 dB

    %EVM value
    epsilon2 = [0 0.1^2];




%Define the range of UEs to consider
Kvalues = 1:max(S);


%Compute the sum of all mu values in Propagation parameter Eq
mu1all_mean = 1+directions*(sum(muValues1Mean(:))-1);
mu1all_worst = 1+directions*(sum(muValues1Worst(:))-1);
mu1all_NorthWest = 1+directions*(sum(muValues1NorthWest(:))-1);


%Extract only reuse factors smaller or equal to 7
reuseIndices = find(reuseFactor>0 & reuseFactor<=directions+1);
for j = 1:length(reuseIndices);
    if sum(reuseFactor(reuseIndices(j))==
    reuseFactor(reuseIndices(1:j-1)))>0
```

```
            reuseIndices(j)=1;
        end
end
reuseIndices = reuseIndices(reuseIndices>1);



%%Compute spectral efficiencies according to Equations .

%Placeholders for storing spectral efficiencies
SE_MR_mean =
zeros(length(Mvalues),max(S),length(reuseIndices),length(S));
SE_ZF_mean =
zeros(length(Mvalues),max(S),length(reuseIndices),length(S));

SE_MR_worst =
zeros(length(Mvalues),max(S),length(reuseIndices),length(S));
SE_ZF_worst =
zeros(length(Mvalues),max(S),length(reuseIndices),length(S));

SE_MR_NorthWest =
zeros(length(Mvalues),max(S),length(reuseIndices),length(S));
SE_ZF_NorthWest =
 zeros(length(Mvalues),max(S),length(reuseIndices),length(S));



%Go through the different reuse factors
for j = 1:length(reuseIndices);

     %Extract the reuse factor
     currentReuseFactor =
     reuseFactor(reuseIndices(j));

     %Extract sum of mu-values for current reuse factor
%    for mean interference
     mu1reuse_mean =
     directions*reuseMu1Mean(reuseIndices(j));
     mu2reuse_mean =
     directions*reuseMu2Mean(reuseIndices(j));
     variance_mean =
     directions*reuseMuMeanVariance(reuseIndices(j));

```

```
    %Extract sum of mu-values for current reuse factor
%   for worst interference
    mu1reuse_worst =
    directions*reuseMu1Worst(reuseIndices(j));
    mu2reuse_worst =
    directions*reuseMu2Worst(reuseIndices(j));
    variance_worst =
    directions*reuseMuWorstVariance(reuseIndices(j));

    %Extract sum of mu-values for current reuse factor
    %for NorthWest interference
    mu1reuse_NorthWest =
    directions*reuseMu1NorthWest(reuseIndices(j));
    mu2reuse_NorthWest =
    directions*reuseMu2NorthWest(reuseIndices(j));
    variance_NorthWest =
    directions*reuseMuNorthWestVariance(reuseIndices(j));

    %Number of neighbors that use each of the other sets of ...
        pilots
    neighborsPerOtherPilot =
    directions/(currentReuseFactor-1);



    for n = 1:length(Mvalues)


        for m = 1:length(S)


            for K = 1:S(m)


                B = currentReuseFactor*K;

                if B < S(m)





                    %Maximum ratio (MR) combining/precoding
```

```
%
%Achievable spectral efficiency using ...
    the formula in
%Theorem 1, for mean, worst, and ...
    NorthWest case interference
SINR_MR_mean =
B*(1-epsilon2(m))/(epsilon2(m)*B + ...
    (mu1all_mean*K...
+ ...
    sigma2rho(m))*(B*(mu1reuse_mean+1)+sigma2rho(m))/
((1-epsilon2(m))...
*Mvalues(n)) + mu2reuse_mean*B +....
B*variance_mean*(1/((1-epsilon2(m))...
*Mvalues(n))));
SE_MR_mean(n,K,j,m) =
K*(1-B/S(m))*log2(1+SINR_MR_mean);

SINR_MR_worst =
B*(1-epsilon2(m))/(epsilon2(m)*B + ...
    (mu1all_worst*K ...
+ sigma2rho(m))*(B*(mu1reuse_worst+1)+...
sigma2rho(m))/((1-epsilon2(m))...
*Mvalues(n)) + mu2reuse_worst*B + ...
B*variance_worst*(1/((1-epsilon2(m))...
*Mvalues(n))));
SE_MR_worst(n,K,j,m) =
K*(1-B/S(m))*log2(1+SINR_MR_worst);

SINR_MR_NorthWest =
B*(1-epsilon2(m))/(epsilon2(m)*B + ...
(mu1all_NorthWest*K ...
+ ...
    sigma2rho(m))*(B*(mu1reuse_NorthWest+1)+...
sigma2rho(m))/((1-epsilon2(m))...
*Mvalues(n)) + mu2reuse_NorthWest*B +...
B*variance_NorthWest*...
(1/((1-epsilon2(m))*Mvalues(n))));
SE_MR_NorthWest(n,K,j,m) = K*(1-B/S(m))...
*log2(1+SINR_MR_NorthWest);


%Zero-forcing (ZF) combining/precoding
%
```

```
            %Achievable spectral efficiency using ...
                the formula in
            %Theorem 1, for mean, worst, and ...
                NorthWest case interference
            if Mvalues(n)-K>0

                %Compute one of the terms in ...
                    Theorem 1
                term2_ZF_mean =
                (directions*reuseMu1Mean2(reuseIndices(j))+1^2).
                /(B*(mu1reuse_mean+1)+sigma2rho(m));
                term2_ZF_worst =
                (directions*reuseMu1Worst2(reuseIndices(j))+1^2)
                /(B*(mu1reuse_worst+1)+sigma2rho(m));
                term2_ZF_NorthWest =
                (directions*reuseMu1NorthWest2(reuseIndices(j)).
                +1^2)/(B*(mu1reuse_NorthWest+1)+sigma2rho(m));

                SINR_ZF_mean =
                B*(1-epsilon2(m))/(epsilon2(m)*B + ...
                    mu2reuse_mean*B ...
                + B*variance_mean/(Mvalues(n)-K)/...
                (1-epsilon2(m)) +  (K*(mu1all_mean ...
                    -....
                (1-epsilon2(m))*B*term2_ZF_mean) +...
                sigma2rho(m)  )*(B*(mu1reuse_mean+1)...
                +sigma2rho(m))/(Mvalues(n)-K)/(1-epsilon2(m))  ..
                    );
                SE_ZF_mean(n,K,j,m) =
                K*(1-B/S(m))*log2(1+SINR_ZF_mean);

                SINR_ZF_worst =
                B*(1-epsilon2(m))/(epsilon2(m)*B + ...
                    mu2reuse_worst*B...
                + B*variance_worst/(Mvalues(n)-K)/....
                (1-epsilon2(m)) +  (K*(mu1all_worst...
                - (1-epsilon2(m))*B*term2_ZF_worst) ...
                    ....
                + sigma2rho(m)  )*...
                (B*(mu1reuse_worst+1)+sigma2rho(m))/...
                (Mvalues(n)-K)/(1-epsilon2(m))  );
                SE_ZF_worst(n,K,j,m) =
                K*(1-B/S(m))*log2(1+SINR_ZF_worst);
```

```
                                SINR_ZF_NorthWest =
                                B*(1-epsilon2(m))/(epsilon2(m)*B + ...
                                mu2reuse_NorthWest*B + ...
                                    B*variance_NorthWest/....
                                (Mvalues(n)-K) ...
                                /(1-epsilon2(m))
                                +  (K*(mu1all_NorthWest - ...
                                (1-epsilon2(m))*B*term2_ZF_NorthWest) ...
                                    ...
                                + sigma2rho(m)  ...
                                    )*(B*(mu1reuse_NorthWest+1)...
                                +sigma2rho(m))
                                /(Mvalues(n)-K)/(1-epsilon2(m))  );
                                SE_ZF_NorthWest(n,K,j,m) =
                                K*(1-B/S(m))...
                                *log2(1+SINR_ZF_NorthWest);

                            end

                        end

                    end

                end

            end

end



%%Compute optimal number of UEs, K, for different system ...
    parameters
%Placeholders for storing simulation results for optimal ...
    number of UEs
optimalK_MR_mean = zeros(length(Mvalues),3,length(S));
optimalK_ZF_mean = zeros(length(Mvalues),3,length(S));

optimalK_MR_worst = zeros(length(Mvalues),3,length(S));
optimalK_ZF_worst = zeros(length(Mvalues),3,length(S));

optimalK_MR_NorthWest = zeros(length(Mvalues),3,length(S));
```

```
optimalK_ZF_NorthWest = zeros(length(Mvalues),3,length(S));


%Go through different number of antennas
for n = 1:length(Mvalues)

  %Go through different reuse factors
  for j = 1:length(reuseIndices)

     currentReuseFactor = reuseFactor(reuseIndices(j));

    for m = 1:length(S)
                [maxValue,maxIndex] = max(SE_MR_mean(n,:,j,m));
     if maxValue > optimalK_MR_mean(n,2,m)
       optimalK_MR_mean(n,:,m) =
       [maxIndex maxValue currentReuseFactor];
        end

       [maxValue,maxIndex] = max(SE_ZF_mean(n,:,j,m));
        if maxValue > optimalK_ZF_mean(n,2,m)
          %Store optimal number of UEs along with the ...
              optimized SE
          %and the corresponding reuse factor
        optimalK_ZF_mean(n,:,m) =
         [maxIndex maxValue currentReuseFactor];
         end




   %Optimize for worst interference case
        [maxValue,maxIndex] = max(SE_MR_worst(n,:,j,m));
         if maxValue > optimalK_MR_worst(n,2,m)

          %and the corresponding reuse factor
                  optimalK_MR_worst(n,:,m) =
                  [maxIndex maxValue currentReuseFactor];
              end

              [maxValue,maxIndex] = max(SE_ZF_worst(n,:,j,m));
              if maxValue > optimalK_ZF_worst(n,2,m)
                 %Store optimal number of UEs along with the ...
                     optimized SE
```

```
                    %and the corresponding reuse factor
                    optimalK_ZF_worst(n,:,m) =
                    [maxIndex maxValue currentReuseFactor];
                end




                %Optimize for NorthWest interference case
                [maxValue,maxIndex] = ...
                    max(SE_MR_NorthWest(n,:,j,m));
                if maxValue > optimalK_MR_NorthWest(n,2,m)
                    %Store optimal number of UEs along with the ...
                        optimized SE
                    %and the corresponding reuse factor
                    optimalK_MR_NorthWest(n,:,m) =
                    [maxIndex maxValue currentReuseFactor];
                end

                [maxValue,maxIndex] = ...
                    max(SE_ZF_NorthWest(n,:,j,m));
                if maxValue > optimalK_ZF_NorthWest(n,2,m)
                    %Store optimal number of UEs along with the ...
                        optimized SE
                    %and the corresponding reuse factor
                    optimalK_ZF_NorthWest(n,:,m) =
                    [maxIndex maxValue currentReuseFactor];
                end



            end

        end

end





%%Plot simulation results
 %Simulations from Section IV.A
```

```
%Plot Figure 4(a)
figure(4);


hold on; box on;


plot(Mvalues,optimalK_ZF_mean(:,2,1),'k--','LineWidth',1);
plot(Mvalues,optimalK_MR_mean(:,2,1),'b-.','LineWidth',1);

xlabel('Number of BS Antennas (M)');
ylabel('Spectral Efficiency (SE) [bit/s/Hz/cell]');
legend('ZF','MR','Location','NorthWest');
set(gca,'Xscale','log');
axis([10 1e5 0 400]);





%Plot Figure 5(a)
figure(5);


hold on; box on;


plot(Mvalues,optimalK_ZF_NorthWest(:,2,1),'k--','LineWidth',1);
plot(Mvalues,optimalK_MR_NorthWest(:,2,1),'b-.','LineWidth',1);

xlabel('Number of BS Antennas (M)');
ylabel('Spectral Efficiency (SE) [bit/s/Hz/cell]');
legend('ZF','MR','Location','NorthWest');
set(gca,'Xscale','log');
axis([10 1e5 0 2600]);
```

```
%Plot Figure 6(a)
figure(6);
hold on; box on;
plot(Mvalues,optimalK_ZF_worst(:,2,1),'k--','LineWidth',1);
plot(Mvalues,optimalK_MR_worst(:,2,1),'b-.','LineWidth',1);
xlabel('Number of BS Antennas (M)');
ylabel('Spectral Efficiency (SE) [bit/s/Hz/cell]');
legend('ZF','MR','Location','NorthWest');
set(gca,'Xscale','log');
```