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# Sudan University of Sciences \&Technology Collage of Graduate Studies 

Department of Electrical Engineering

# Damping of Low-Frequency Oscillations in 

Inter Connected Power Systems
تخميد تردد التذبذبات المنخفضة في أنظمة الققرة المتر ابطة داخلياً

A Thesis Submitted in Partial Fulfillment for the Requirements of the Degree of M.Sc. In Electrical Engineering (Power \& Machines)

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## الآيه

\}السَّ نُورُ السنَّمَاوَاتِ وَالْأْزْضِ مَثَلُ نُورِهِ كَمِشْكَاةٍ فِيهَا مِصْبَاحِ

 تَمْسَسْنُ نَارُ نُّورٌ عَلَى نُورٍ يَهْْرِي السَّ لِنُورِهِ مَن يَشَتَاء وَيَضْرِبُ السَّهُ الْأَمْنَّلَ لِلْنَّاسِ وَالسَّهُ كِكُلِ شَيْءٍ عَليمٌ صدق الله العظيم

سوره النور الآية 35

## DEDICATION

For the woman who support and encourage me in all of my life.... my mother

For that who are great and most significant

Person in my eyes......
my father

For my sister .... For my brother.

For that great man who help and support me in this thesis
my supervisor

Dr. Mohammed Osman Hassan

For the partners whose participate me in the way of science....
My sincere friends

For the people who pays all of their efforts and their times to give me the knowledge.

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## ABSTRACT

Stability study is the most important studies in power system; which keep the system secure and continuous operation. Stability of power system means the ability of the synchronous generators to be in synchronism when small or large disturbances occur in load or network of power system. If one of the generators losses the synchronism the other generators will be overload and make the network separate which cause blackout of the system; so that the instability problem must be solved quickly.

This thesis concerned to study and analyzes the small signal, transient stability problem, and damping of low frequency oscillations by using power system stabilizer for multi machines system. The nonlinear equations which represent the system have been linearized and then placed in state space form in order to study and analysis the dynamic performance of the system. IEEE system (10 machines-39 bus bars) has been taken as case study. The AVR system causes negative damping of oscillation; so that the PSS was designed and added in optimal location by using participation factor technique to increase the damping ratio. The eigenvalue and time domain simulation methods were used to analyze the system stability; finally, the responses and results have been represented for several cases by using time domain simulation. Result obtained showed that the system without PSS is oscillatory and after add PSS to the optimal $\mathrm{M} / \mathrm{Cs}$ the system return back to stability conditions after few cycles.


## (المستخلص

إستقرارية النظام من أهم الدراسات في منظومة التدرة الكهربائية, حيث تحافظ على أمن النظام وإستمرارية الخدمة, والمقصود بإستقرارية منظومة القدرة الكهربائية مقدرة الآلات المتزامنة بها على الإحتفاظ بتزامنها بعد أي تغيير يحدث تريجياً او فجائيأ لشبكة او أحمال منظومة القدرة . عند فقد أحد المولدات للتزامن يحصل تحميل زائد لبقية المولات مما يؤدي الي إنقطاع كامل للكهرباء وخروج الشبكة من الخدمة لذا يجب حل ومعالجة مشكلة عدم الإستقرارية بسرعة.

هذا البحث يهتم بدراسة وتحليل الإستقرارية لنظام متعدد الماكينات تحت تأثير التيغيرات الطفيفة والتنيرات العابرة و اخماد تذبذبات الترددات المنخفضه باستخدام مثبت نظام القدره. حيث تم تمثل النظام في صورة معادلات تفاضلية غير خطية ث تحويلها إلي معادلات خطية رومن ثم وضعها في صورة فراغ الحالة من أجل تحليل الاداء الديناميكي للنظام. ولغرض الدراسة أخذنا نظام يتكون من عشرة ماكينات وتسعة وثلاثون قضبان تجميع. و بما أن نظام متحكم الجهـ الأتوماتيكي يسبب اخماداً سالباً للتذبذبات التي تحصل نتيجة للتينيرات الطفيفة في المنظومة مثل تغيير الحمل، أو التغيرات العابرة مثل الأعطال تم تصميم مثبت نظام التدرة الكهربائية وإضافته في الموقع المناسب بإستخدام تتنية معامل الإشتراك أو السساههة وذلك لزيادة نسبة التخميد. ثم حلت إستقراية النظام عن طريق نظريتي أقطاب معادلة الخصائص، و نظام المحاكاة في حيز الزمن. واخيراً عرضت الإستجابات والنتائج لعدد من التغيرات والإضطرابات التي تحدث للنظام وتتثيلها في نظام المحاكاة الزمني.من النتائج وجد ان النظام بلا مثبت نظام القدره يكون متذبذب وبعد توصيل مثبت القدرة في الماكينات المناسبه النظام يرجع الى شروط الاستقرارية بعد عدد قليل من الدورات.


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## LIST OF SYMPOLS

| $\mathrm{E}_{\mathrm{q}}{ }^{\prime}$ | q -axis component of the transient internal emf <br> proportional to the field winding flux linkages. |
| :---: | :--- |
| $\mathrm{E}_{\mathrm{fd}}$ | Generator field voltage |
| $\mathrm{E}_{\mathrm{d}}{ }^{\prime}$ | d-axis component of the transient internal emf <br> proportional to flux linkages in the q-axis solid steel rotor <br> body |
| $\omega_{\mathrm{i}}$ | Rotor angle |
| H | Inertia constant |
| D | Damping coefficient |
| $\mathrm{T}_{\mathrm{do}}{ }^{\prime}$ | open-circuit d-axis transient time constants |
| $\mathrm{T}_{\mathrm{qo}}{ }^{\prime}$ | open-circuit q-axis transient time constants |
| $\lambda$ | flux linkage |
| $J_{\mathrm{D}}$ | moment of inertia |
| $\mathrm{T}_{\mathrm{m}}$ | The mechanical torque applied to the shaft |
| $\mathrm{T}_{\mathrm{e}}$ | The Electrical torque |
| $\mathrm{T}_{\mathrm{fw}}$ | The damping torque coefficient |
| $\theta$ | Rotor angular displacement |
| $v_{\mathrm{fdi}}$ | the field voltage |
| $i_{\mathrm{fdi}}$ | the field current |
| $\mathrm{r}_{\mathrm{fdi}}$ | the resistance current |
| $\psi_{\mathrm{fdi}}$ | flux components in the dq0 reference system |
| $v_{\mathrm{Di}}, v_{\mathrm{Qi}} \& v_{\mathrm{Oi}}$ | the stator voltage in the dq0 reference system |
| $i_{\mathrm{Qi}} \& i_{\mathrm{Oi}}$ | the stator current in the dq0 reference system |
|  | the stator flux linkage in the dq0 reference system |


| $x_{\mathrm{Di}}, x_{\mathrm{Qi}}$ | $\mathrm{d} \& \mathrm{q}$-axis reactance |
| :---: | :--- |
| $x_{\mathrm{di}}^{\prime}, x_{\mathrm{qi}}{ }^{\prime}$ | $\mathrm{d} \& \mathrm{q}$-axis transient reactance |
| $\mathrm{V}_{\mathrm{t}}$ | The terminal voltage |
| $\mathrm{V}_{\infty}$ | Infinite bus voltage |
| $\Delta \mathrm{P}_{\mathrm{e}}$ | Electrical power deviation |
| $\mathrm{G}_{\mathrm{W}}$ | Washout stage transfer function |
| $\mathrm{G}(\mathrm{s})$ | Lead-lag transfer function |
| $\mathrm{K}_{\mathrm{pss}}$ | Power system stabilizer gain |
| $\mathrm{T}_{\mathrm{w}}$ | Washout time constant |
| Z | Damping ratio |
| $\omega_{\mathrm{n}}$ | Un-damped natural frequency |
| $\mathrm{K}_{\mathrm{A}}$ | Automatic voltage regulator gain |
| $\mathrm{T}_{\mathrm{A}}$ | Automatic voltage regulator time constant |
| $\mathrm{T}_{1}, \mathrm{~T}_{2}, \mathrm{~T}_{3}, \mathrm{~T}_{4}$ | Power system stabilizer time constants |

## LIST OF ABBREVIATIONS

| AC | Alternating Current |
| :--- | :--- |
| DC | Direct Current |
| LFO | low frequency oscillations |
| AVR | Automatic Voltage Regulator |
| PSS | Power System Stabilizer |
| HVDC | High Voltage Direct Current |
| SMIB | Single Machine Infinite Bus |
| GEPS | Generator, Exciter system, and power system |
| PF | Particle Swarm Optimization |
| OPLI | Multi-Objective Honey Bee Mating Optimization Factor |
| PSO | Genetic Algorithm |
| MOHBMO | Flexible Alternating Current Transmission System |
| GA |  |
| FACTS |  |

## CHAPTER ONE

## INTRODUCTION

### 1.1 Background:

In an electrical power system many equipment and controllers have been used to maintain the balance between load and generation in a reliable manner with high degree of quality [1].

Stability of power systems has been and continues to be of major concern in system operation. This arises from the fact that in steady state (under normal conditions) the average electrical speed of all the generators must be the same. This is termed as the synchronous operation of a system. Any disturbance small or large can affect the synchronous operation.

The stability of a system determines whether the system can settle down to a new or original steady state after the transients disappear [2].

Modern power systems, apart from a large number of generators and associated controllers, there are many types of load, ranging from a simple resistive load to more complicated loads with electronic controllers. The influence of more and more controllers and loads, increase the complexity and nonlinearity of power systems. As a result, power systems are viewed as complex nonlinear dynamical system that shows a number of instability problems. stability problems can be broadly classified into three main categories, namely voltage, angular and frequency stability problems. Though instability eventually blackout or collapse never happened in a pure form of voltage or angular or frequency problems, the initial part of the incident can be clearly related to one of the categories.

Operation engineers faced with transient stability problem and researchers struggled to find counter measures to overcome it. Transient stability problem, considered as part of the angular related problem, is defined as the ability of power system to maintain synchronism when subjected to large disturbances. When the system faces large disturbances such as large load
increase, loss of tie lines, loss of generating units, maintaining constant electrical speed among all the generators were challenging as some machines speed up while some other slow down to adjust to post disturbance situation. If there is no control mechanism to keep the speeding up or slowing down generators within the allowable speed limits, there is a good chance that these generators would fall out of the grid by losing synchronism. Hence, fast exciter or Automatic Voltage Regulators (AVR) was introduced in the system as one of the remedial measures to solve the problem. However, the fast AVR could not do the "fine adjustment" to control oscillation in the speed. Then, Power System Stabilizer (PSS) was introduced in generator to give that fine adjustment to damp out power oscillations that are referred to as electromechanical or low frequency oscillations (LFO).

The ability of synchronous machines of an interconnected power system to remain in synchronism after being subjected to a small disturbance is known as small signal stability that is subclass of angular related stability problem. Lack of sufficient synchronizing torque results in "aperiodic" or nonoscillatory instability, whereas lack of damping torque results in low frequency oscillations. Low frequency oscillations are generator rotor angle oscillations having a frequency between $0.1-2.0 \mathrm{~Hz}$ and are classified based on the source of the oscillation. The root cause of electrical power oscillations are the unbalance between power demand and available power at a period of time. In the earliest era power system development, the power oscillations are almost non-observable because generators are closely connected to loads, but now days, large demand of power to the farthest end of the system that forces to transmit huge power through a long transmission line, which results an increasing power oscillations [1].

### 1.2 Statement of Problem:

The electric power system must be stable to ensure continuity of supply. stability problems may lead to partial or full one of causes Low frequency oscillations, these frequencies may sustain and grow to cause system separation
or black out if adequate damping is not available. It has always been traditional to use the speed input to the AVR to improve system damping, in this thesis the damping of the frequency oscillations by using power system stabilizer (PSS) device is proposed.

### 1.3 Objectives:

> To determine how accurately a small disturbance (low frequency oscillations) behavior in a power system be modeled and predicted using transient and dynamic stability analysis.
> Study the response of generators to external network disturbances (faults).
> Designing of power system stabilizer (PSS).

### 1.4 Methodology / Approach:

To achieve the thesis objectives, first effect of low frequency oscillations in power system stability will be investigated, followed by eigenvalue analysis and time domain simulation of the system oscillatory behavior in low frequency range, and how it can be damp out by using power system stabilizer (PSS).

Finally, modeling of (PSS) that used for damping low frequency oscillations and investigate the methods to control the PSS output and using (NPLAN) to simulate the output.

### 1.5 Thesis Lay-out:

> Chapter One: presents a general introduction to the power system stability, low frequency oscillations (LOF) and power system stabilizer (PSS), research objectives, statement of problem and Methodology.
> Chapter Two: includes the definition, classification of stability, low frequency oscillations (LFO) classification and modeling of power system.
> Chapter Three: linearization of multi-machine system, power system stabilizer (PSS) design.
> Chapter Four: represent the results of the eigenvalue analysis and time domain simulation of small signal and transient stability the case study ( 10 machine, 39 bus bar), and PSS optimized parameters.

Chapter Five: represents thesis conclusion and recommendations.

## CHAPTER TWO

## DEFINITION OF STABILITY AND MODELING OF POWER SYSTEM

The problem of interest is one where a power system operating under a steady load condition is perturbed, causing the re- adjustment of the voltage angles of the synchronous machines. If such an occurrence creates an unbalance between the system generation and load, it results in the establishment of a new steady-state operating condition, with the subsequent adjustment of the voltage angles. The perturbation could be a major disturbance such as the loss of a generator, a fault or the loss of a line, or a combination of such events. It could also be a small load or random load changes occurring under normal operating conditions.

Adjustment to the new operating condition is called the transient period. The system behavior during this time is called the dynamic system performance, which is of concern in defining system stability. The main criterion for stability is that the synchronous machines maintain synchronism at the end of the transient period.

### 2.1 Definition of stability:

This primitive definition of stability requires that the system oscillations be damped. This condition is sometimes called asymptotic stability and means that the system contains inherent forces that tend to reduce oscillations. This is a desirable feature in many systems and is considered necessary for power systems.

The definition also excludes continuous oscillation from the family of stable systems, although oscillators are stable in a mathematical sense. The reason is practical since a continually oscillating system would be undesirable for both the supplier and the user of electric power. Hence the definition describes a practical specification for an acceptable operating condition [4]. Figure (2.1) represent classification of power system stability.


Figure 2.1 Classification of power system stability.
The disturbance can be divided into two categories (a) small and (b) large. A small disturbance is one for which the system dynamics can be analyzed from linearized equations (small signal analysis). The small (random) changes in the load or generation can be termed as small disturbances. The tripping of a line may be considered as a small disturbance if the initial (pre-disturbance) power flow on that line is not significant. However, faults which result in a sudden dip in the bus voltages are large disturbances and require remedial action in the form of clearing of the fault. The duration of the fault has a critical influence on system stability.

Although stability of a system is an integral property of the system, for purposes of the system analysis, it is divided into two broad classes [2].

### 2.2 Classification of Power System Stability:

## a. Steady-State or Small Signal Stability:

Small-signal (or small disturbance) stability is the ability of the power system to maintain synchronism under small disturbances such as small variations in loads and generations. Physically power system stability can be broadly classified into two main categories - angle stability or rotor angle stability and voltage stability [3].

## b. Transient Stability

A power system is transiently stable for a particular steady-state operating condition and for a particular (large) disturbance or sequence of disturbances if, following that (or sequence of) disturbance(s) it reaches an acceptable steady-state operating condition.

It is important to note that, while steady-state stability is a function only of the operating condition, transient stability is a function of both the operating condition and the disturbance(s). This complicates the analysis of transient stability considerably. Not only system linearization cannot be used, repeated analysis is required for different disturbances that are to be considered.

Another important point to be noted is that while the system can be operated even if it is transiently unstable, small signal stability is necessary at all times. In general, the stability depends upon the system loading. An increase in the load can bring about onset of instability. This shows the importance of maintaining system stability even under high loading conditions [2].

### 2.3 Power System Low-frequency Oscillations:

Power system low-frequency oscillations are the oscillations of active power delivered along particular transmission corridors in a power system with the oscillation frequency from 0.1 Hz up to a couple of Hz . Once started, the oscillations can continue for a while and then disappear, or grow continuously to cause power system collapse.

Manifestation of a power oscillation is the oscillation of relative movement of angular positions of generators in the power system. The oscillation can be triggered by severe faults, such as a three-phase to-earth short circuit along or tripping of a transmission line. It can also occur under normal operating conditions when the power system is only subject to small disturbances. Hence, if the power system collapse is caused by the power oscillation, it could belong to the problem of power system large-signal rotor angle (angular) stability or small-signal angular stability [5].

## Types of Oscillation

The LFO can be classified as local and inter-area mode:
1- Local modes are associated with the swinging of units at a generating station with respect to the rest of the power system. Oscillations occurred only to the small part of the power system. Typically, the frequency range is $1-3 \mathrm{~Hz}$.

2- Inter area modes are associated with swinging of many machines in one part of the system against machines in other parts. It generally occurs in weak interconnected power systems through long tie lines. Typically, frequency range is $0.2-0.8 \mathrm{~Hz}$.

Besides these modes, there can be other modes associated with controllers which happen due to poor design of controllers. Torsional oscillation is another type of oscillation that happened in series capacitor compensated system and the frequency of oscillation is typically in sub synchronous frequency range.

### 2.4 Power System Modeling:

In this section all of the power system component which uses in this thesis were modeled. Figure (2.2) below represents the power system components.


Figure 2.2 Power System Components

### 2.4.1 Synchronous Machine Modeling:



Figure 2.3: The windings in the synchronous generator and their axes.
Referring to figure (2.3) shows the coil orientation assumed polarities, and rotor position reference. The rotor windings have axes 120 electrical degrees apart and assumed to have an equivalent sinusoidal distribution, the following windings are depicted:

- The three stator windings denote $\mathrm{a}, \mathrm{b}$, and c .
- Field winding denoted F . This winding carries the field current, which gives rise to the field flux. This rotating flux induces the voltages in the stator windings.
- Short circuited damper winding in the d-axis denoted by D.
- Short circuited damper winding in the q -axis denoted by Q .

The basic voltage Equations which descript the machine in a-b-c reference system are:

## Stator and rotor Equations

$$
\begin{array}{rlrl}
v_{a i} & =i_{a i} r_{a i}+\frac{d}{d t} \Psi_{a i} & \mathrm{i}=1, \ldots . \mathrm{n} \\
v_{b i} & =i_{b i} r_{b i}+\frac{d}{d t} \Psi_{b i} & \mathrm{i}=1, \ldots . \mathrm{n} \\
v_{c i} & =i_{c i} r_{c i}+\frac{d}{d t} \Psi_{c i} & & \mathrm{i}=1, \ldots . \mathrm{n}
\end{array}
$$

$$
\begin{equation*}
v_{f d i}=i_{f d i} r_{f d i}+\frac{d}{d t} \Psi_{f d i} \quad i=1, \ldots . \mathrm{n} \tag{2.4}
\end{equation*}
$$

Where $\Psi$ is flux linkage, $v, i$, and $r$ is winding voltage, current and resistance in the a-b-c reference system respectively.

## Park's transformation

It is convenient to transform all synchronous machine stator and network variable into a reference frame that converts balance three-phase sinusoidal variations into constant. Such a transformation is
$v_{d q o} \triangleq T_{d q o} v_{a b c}, i_{d q o} \triangleq T_{d q o} i_{a b c}, \Psi_{d q o} \triangleq T_{d q o} \Psi_{a b c}$
where

$$
\begin{align*}
& v_{a b c} \triangleq\left[v_{a} v_{b} v_{c}\right]^{t}, i_{a b c} \triangleq\left[i_{a} i_{b} i_{c}\right]^{t}, \Psi_{a b c} \triangleq\left[\Psi_{a} \Psi_{b} \Psi_{c}\right]^{t}  \tag{2.6}\\
& v_{d q o} \triangleq\left[v_{d} v_{q} v_{o}\right]^{t}, i_{d q o} \triangleq\left[i_{d} i_{q} i_{o}\right]^{t}, \Psi_{d q o} \triangleq\left[\Psi_{d} \Psi_{q} \Psi_{o}\right]^{t} \tag{2.7}
\end{align*}
$$

$T_{d q o}=\frac{2}{3}\left[\begin{array}{ccc}\cos \omega_{s} t & \cos \left(\omega_{s} t-\frac{2 \pi}{3}\right) & \cos \left(\omega_{s} t+\frac{2 \pi}{3}\right) \\ -\sin \omega_{s} t & -\sin \left(\omega_{s} t-\frac{2 \pi}{3}\right) & -\sin \left(\omega_{s} t+\frac{2 \pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2}\end{array}\right]$
$T_{d q o}^{-1}=\left[\begin{array}{ccc}\cos \omega_{s} t & -\sin \omega_{s} t & 1 \\ \cos \left(\omega_{s} t-\frac{2 \pi}{3}\right) & -\sin \left(\omega_{s} t-\frac{2 \pi}{3}\right) & 1 \\ \cos \left(\omega_{s} t+\frac{2 \pi}{3}\right) & -\sin \left(\omega_{s} t+\frac{2 \pi}{3}\right) & 1\end{array}\right]$
From (2.1) - (2.4), kirchhoff's and faraday's laws are
$v_{\mathrm{abc}}=\mathrm{r}_{\mathrm{s}} \mathrm{i}_{\mathrm{abc}}+\frac{d}{a t} \Psi_{\mathrm{abc}}$
which, when transformed by using (2.8) and (2.9) are
$v_{\mathrm{dqo}}=\mathrm{r}_{\mathrm{s}} \mathrm{i}_{\mathrm{dqo}}+T_{d q o} \frac{d}{a t}\left(T_{d q o}^{-1} \Psi_{\mathrm{dqo}}\right)$
$\left[\begin{array}{c}V_{D i} \\ V_{Q i} \\ V_{O i}\end{array}\right]=T_{d q o} T_{d q o}^{-1}\left[\begin{array}{c}V_{d i} \\ V_{q i} \\ V_{o i}\end{array}\right]=\frac{1}{\sqrt{2}} T_{d q o}\left[\begin{array}{c}V_{a i} \\ V_{b i} \\ V_{c i}\end{array}\right] \quad \mathrm{i}=1, \ldots . \mathrm{n}$
$\left[\begin{array}{l}I_{D i} \\ I_{Q i} \\ I_{O i}\end{array}\right]=T_{d q o} T_{d q o}^{-1}\left[\begin{array}{c}I_{d i} \\ I_{q i} \\ I_{o i}\end{array}\right]=\frac{1}{\sqrt{2}} T_{d q o}\left[\begin{array}{c}I_{a i} \\ I_{b i} \\ I_{c i}\end{array}\right] \quad \mathrm{i}=1, \ldots . \mathrm{n}$

$$
\left[\begin{array}{l}
\Psi_{D i}  \tag{2.14}\\
\Psi_{Q i} \\
\Psi_{O i}
\end{array}\right]=T_{d q o} T_{d q o}^{-1}\left[\begin{array}{l}
\Psi_{d i} \\
\Psi_{q i} \\
\Psi_{o i}
\end{array}\right]=\frac{1}{\sqrt{2}} T_{d q o}\left[\begin{array}{l}
\Psi_{a i} \\
\Psi_{b i} \\
\Psi_{c i}
\end{array}\right] \quad \mathrm{i}=1, \ldots \mathrm{n}
$$

Where $T_{d q o}$ is the machine transformation[6] .
After evaluation, the system in dqo coordinates has the forms

## Stator and rotor equations

$\frac{d}{d t} \Psi_{D i}=i_{D i} r_{s i}+\omega_{i} \Psi_{Q i}+v_{D i} \quad \mathrm{i}=1, \ldots \mathrm{n}$
$\frac{d}{d t} \Psi_{Q i}=i_{Q i} r_{s i}-\omega_{i} \Psi_{D i}+v_{Q i} \quad \mathrm{i}=1, \ldots . \mathrm{n}$
$\frac{d}{d t} \Psi_{O i}=i_{O i} r_{S i}+v_{O i}$

$$
\begin{equation*}
\mathrm{i}=1, \ldots \mathrm{n} \tag{2.16}
\end{equation*}
$$

Where $v_{D i}, v_{Q i}, v_{O i}, i_{D i}, i_{Q i}, i_{O i}, \Psi_{D i}, \Psi_{Q i}, \Psi_{O i}$ are the stator voltage, current, and flux linkage in the dq0 reference system, $\mathrm{r}_{\mathrm{si}}$ is the stator resistance, where $\omega_{i}$ is the rotor speed.
$v_{f d i}=i_{f d i} r_{f d i}+\frac{d}{d t} \Psi_{f d i}$

$$
\begin{equation*}
\mathrm{i}=1, \ldots \mathrm{n} \tag{2.18}
\end{equation*}
$$



Where $v_{f d i}, i_{f d i}, r_{f d i}$ and $\Psi_{f d i}$ is field voltage, current, resistance and flux component in the dqo reference system respectively

Figure 2.4: model of synchronous machine in dqo axis

## Motion equations

$$
\begin{array}{ll}
\frac{d \delta}{d t}=\frac{d \theta_{i}}{d t}-\omega_{s} & i=1, \ldots, n \\
\dot{\delta}=\omega_{i}-\omega_{s} & i=1, \ldots, n
\end{array}
$$

$$
\begin{array}{ll}
\frac{2 H_{i}}{\omega_{s}} \frac{d^{2} \delta}{d t^{2}}=\frac{2 H_{i}}{\omega_{s}} \frac{d \omega_{i}}{d t}=\left(T_{m i}-T_{e i}-T_{f w i}\right) & i=1, \ldots, n \\
\dot{\omega}_{l}=\frac{\omega_{s}}{2 H_{i}}\left(T_{m i}-T_{e i}-T_{f w i}\right) & i=1, \ldots, n \tag{2.22}
\end{array}
$$

Where
$\mathrm{H}=$ The inertia time constant in seconds.
$T_{m i}=$ The mechanical torque.
$T_{e i}=$ The electrical torque.
$T_{f w i}=$ The damping torque coefficient given in per unit
$\delta=$ The angular position of the rotor.
$\omega_{s}=$ The synchronous speed.
$\omega_{i}=$ The rotor speed.

### 2.4.2 Transmission Line Modeling:

Electrical power is transferred from generating stations to consumers through overhead lines and cables.

Overhead lines are used for long distance in open country and rural areas, where as cables are used for underground transmission in urban areas and for underwater crossings [7].

Transmission lines are modeled as nominal $\pi$ circuit as shown in figure (2.5)


Figure 2.5: Transmission Lines as a nominal $\pi$ circuit.
Z : represents the series impedance of the line.
$\frac{y}{2}$ : represents half of the total line charging $y$, at each node.

### 2.2.3 Transformer Modeling:

Transformers enable utilization of different voltage levels across the system. Transformers are generally used as inter connecting (IC) transformers and generator transformers.

In digital computer analysis of power flow, it is not convenient to represent an ideal transformer. Transformers are usually with off-nominal-turns- ratio and are modeled as equivalent $\pi$ circuit as shown in figure (2.6) [7].


Figure 2.6: The transformer as equivalent $\pi$ circuit.
Where,

$$
\begin{equation*}
\mathrm{y}_{\mathrm{t}}=\frac{1}{z_{t}} \tag{2.23}
\end{equation*}
$$

$z_{t}$ : represents the series impedance at nominal- turns - ratio.
a: represents per unit off-nominal tap position.
The transmission network represented by and algebraic equation given by
$\bar{I}=Y_{B U S} \bar{V}$
Where,
$\bar{I}$ : Vector of injected bus currents.
$Y_{B U S}$ : Bus admittance matrix.
$\bar{V}:$ Vector of bus voltages.
The above equation is obtained by writing the network equations in the nodeframe of reference taking ground as the reference [7].

### 2.4.4 Excitation System Modeling:

The synchronous generator is provided with two automatic (feedback) controllers for the regulation of the terminal voltage and frequency. These controllers indirectly influence the reactive power and active power outputs of the generator respectively. The regulation of the voltage is the faster of the two controllers and has bearing on the system stability much more than the regulation of speed.

The main objective of the excitation system is to control the field current of the synchronous machine. The field current is controlled so as to regulate the terminal voltage of the machine. As the field circuit time constant is high (of the order of a few seconds), fast control of the field current requires field forcing. Thus exciter should have a high ceiling voltage which enables it to operate transiently with voltage levels that are 3 to 4 times the normal. The rate of change of voltage should also be fast. Because of the high reliability required, unit exciter scheme is prevalent where each generating unit has its individual exciter [2]. Figure (2.7) represent the element of excitation system.


Figure 2.7 functional block diagram of a synchronous generator excitation control system (Element of Excitation System).

1. Exciter provides dc power to the synchronous machine field winding consisting the power stage of the excitation system.
2. Regulator processes and amplifies input control signals a level and form appropriate for control of the exciter. This includes both regulating and excitation system stabilizing functions (rate feedback or lead-lag compensation).
3. Terminal voltage transducer and load compensator since generator terminal voltage, rectifies and filters it to dc quantity, and compares it with a reference which represents the desired terminal voltage. In addition, load (or line-drop, or reactive) compensation may be provided, if it is desired to hold constant voltage at some point electrically remote from the generator terminal (for example, partway through the step-up transformer).
4. Power system stabilizer provides an additional input signal to the regulator to damp power system oscillations. Sime commonly used input signals are rotor speed deviation, accelerating power, and frequency deviation.
5. Limiters and protective circuits these include a wide array of control and protective functions which ensure that the capability limits of the exciter and synchronous generator are not exceeded. some of the commonly used functions are the field-current limiter, maximum excitation limiter, terminal voltage limiter, volt-per-hertz regulator and protection, and under excitation limiter. These are normally distinct circuits and their output signals may be applied to the excitation system at various locations as summing input or a gated input [7].

There are three distinct types of excitation systems based on the power source for exciter:

1. DC Excitation Systems (DC) which utilize a DC generator with commutator. source for exciter.
2. AC Excitation Systems (AC) which use alternators and either stationary or rotating rectifiers to produce the direct current needed.
3. Static Excitation Systems (ST) in which the power is supplied through transformers and rectifiers. The first two types of exciters are also called rotating exciters which are mounted on the same shaft as the generator and driven by the prime mover.

Modern Automatic Voltage Regulators (AVR) are continuously acting electronic regulators with high gain and small time constants.

The exciters can be one of the following types:

1. Field controlled dc generator - commutator
2. a) Field controlled alternator with non-controlled rectifier (using diodes)
i) with slip rings and brushes (stationary rectifier)
ii) brushless, without slip rings (rotating rectifier)
b) Alternator with controlled rectifier
3. Static exciter with
a) potential source controlled rectifier in which the excitation power is supplied through a potential transformer connected to generator terminals
b) Compound source (using both current and voltage transformers at the generator terminals) with
(i) non-controlled rectifier (control using magnetic elements such as saturable reactors)
(ii) controlled rectifier (for controlling the voltage).

The advantages of the static excitation system are

1) Expanded rang for excitation voltage and current with high amplification.
2) Very fast response.
3) Simplicity of design.


Figure 2.8: Block diagram of static excitation system.

Figure (2.8) shows simple type of exciter (static exciter).
High performance excitation systems are essential for maintaining steady state and transient stability of modern synchronous generators, apart from providing fast control of the terminal voltage. Bus fed static exciters with thyristor controllers are increasingly used for both hydraulic and thermal units. They are characterized by high initial response and increased reliability due to advances in thyristor controllers. Block diagram of exciter simple with limits system is shown in Figure (2.9) The time constant $T A$ of the regulator in the range of 0.01 to 0.02 sec . The other time constant $T R$, is necessary for filtering of the rectified terminal voltage waveform. The voltage regulator and the exciter can be modelled as a gain in series with an optional block of Transient Gain Reduction (TGR). The role of TGR is primarily to provide satisfactory operation on open circuit. The Automatic Voltage Regulator (AVR) gain is typically around $200 \mathrm{pu} / \mathrm{pu}$. The exciter ceiling is typically 8.0 pu. These parameters permit the exciter to reach $90 \%$ of the ceiling voltage (from the rated-load field voltage) within 25 ms for a sustained drop in the terminal voltage not exceeding 5\%.


Figure 2.9: Block diagram of exciter simple with limits system.
It is well established that fast acting exciters with high gain AVR can contribute to oscillatory instability in power systems. This type or-instability is characterized by low frequency ( 0.2 to 2.0 Hz ) oscillations which can persist (or even grow in magnitude) for no apparent reason. This type of instability can endanger system security and limit power transfer. The major factors that contribute to the instability are:
(a) loading of the generator or tie line
(b) power transfer capability of transmission lines
(c) power factor of the generator (leading power factor operation is more problematic than lagging power factor operation).
(d) AVR gain [2].

### 2.4.5 Power System Stabilizer (PSS) Modeling:

The stabilization provided by PSS is not to be confused with that by Excitation System Stabi1izer(ESS). While ESS is designed to provide effective voltage regulation under open or short circuit conditions, the objective of PSS is to provide damping of the rotor oscillations whenever there is a transient disturbance. The damping of these oscillations (whose frequency varies from 0.2 to 2.0 Hz ) can be impaired by the provision of high gain AVR, particularly at high loading conditions when a generator is connected through a high external impedance (due to weak transmission network).

It consists of a washout circuit, dynamic compensator, torsional filter and limiter. The function of each of the components of PSS with guidelines for the selection of parameters (tuning) are given next.

It is to be noted that the major objective of providing PSS is to increase the power transfer in the network, which would otherwise be limited by oscillatory instability. The PSS must also function properly when the system is subjected to large disturbances. The block diagram in figure (2.10) shows the element of two stage of PSS.


Figure 2.10: Block diagram of two stage PSS.
a) Gain (K ${ }_{P s S}$ ):

The gain of PSS is to be chosen to provide adequate damping of all critical mode under various operating conditions. Although PSS may be tuned to give optimum damping under such condition, the performance will not be
optimal under other conditions. The critical modes include not only local and inter area modes, but other modes.
b) Washout Circuit:

The washout circuit is provided to eliminate steady-state bias in the output of PSS which will modify the generator terminal voltage. The PSS is expected to respond only to transient variations in the input signal (say rotor speed) and The gain of PSS is to be chosen to provide adequate damping of not to the dc offsets in the signal. This is achieved by subtracting from it the low frequency components of the signal obtained by passing the signal through a low pass filter. The washout circuit acts essentially as a high pass filter and it must pass all frequencies that are of interest.

## c) Dynamic compensator:

The compensator is made up to double lead-lag stage where Ks is the gain of PSS and the time constants, T1 was chosen to provide a phase lead for the input signal in the range of frequencies that are of interest $(0.1$ to 3.0 Hz$)$. With static exciters, only one lead-lag stage may be adequate.
d) Torsional Filter:

The torsional filter in the PSS is essentially a band reject or a low pass filter to attenuate the first torsional mode frequency.

For stabilizers derived from accelerating power, torsional filter can have a simple configuration of a low pass filter independent of the frequency of the torsional mode to be filtered out.

Torsional filter is necessitated by the adverse interaction of PSS with the torsional oscillations. This can lead to shaft damage, particularly at light generator loads when the inherent mechanical damping is small. Even if shaft damage does not occur; stabilizer output can go into saturation (due to torsional frequency components) making it ineffective. The criteria for designing of the torsional filter are:

1. The maximum possible change in damping of any torsional mode is less than some fraction of the inherent torsional damping.
2. The phase lag of the filter in the frequency range of 1 to 3 Hz is minimized.

## e) Limiter:

The output of the PSS must be limited to prevent the PSS acting to counter the action of AVR. For example, when load rejection takes place, the AVR acts to reduce the terminal voltage when PSS action calls for higher value of the terminal voltage (due to the increase in speed or frequency). It may even be desirable to trip the PSS in case of load rejection.

The negative limit of PSS output is of importance during the back swing of the rotor (after initial acceleration is over). The AVR action is required to maintain the voltage (and thus prevent loss of synchronism) after the angular separation has increased. PSS action in the negative direction must be curtailed more than in the positive direction. Hydro uses a -0.05 pu . as the lower limit and 0.1 to 0.2 as the higher limit. Recent studies have shown that higher negative limit can impair first swing stability.

The operation is permitted only if the following conditions are satisfied simultaneously
(a) a drop in the terminal voltage in excess of the preset value
(b) field voltage is at positive ceiling
(c) rise in speed above a preset value.

## f) Input signal:

The common signal which used as input to the PSS are: rotor speed deviation $\Delta \omega$, bus frequency $\Delta f$, electrical power $\Delta p$.The signal must be obtained from local measurements. Since the basic function of power system stabilizer (PSS) is to add damping to the rotor oscillations [2]. In this work a speed signal is used as input signal.

The control equation is given by:
Output $(\mathrm{PSS})=\frac{S \text { Kpss Tw }(1+\mathrm{ST} 1)(1+\mathrm{ST} 3)}{(1+S T w)(1+\mathrm{ST} 2)(1+\mathrm{ST} 4)}$.Input
Where;

Kpss : Power System Stabilizer gain.
$\mathrm{T}_{\mathrm{w}}$ : washout time constant.
$\mathrm{T}_{1}, \mathrm{~T}_{2}, \mathrm{~T}_{3}, \mathrm{~T}_{4}$ :Time constant selected to provide a phase lead for the input signal in the range of frequencies of interest.

## CHAPTER THREE

## LINEARIZED MODEL OF MULTI- MACHINES SYSTEM and DESIGN of PSS

### 3.1 Introduction:

When the system is subjected to a small load change, it tends to acquire a new operating state. During the transition between the initial state and the new state the system behavior is oscillatory. If the two states are such that all the state variables change only slightly (i.e., the variable $x_{i}$ changes from $x_{i 0}$ to $\mathrm{x}_{\mathrm{io}}+\Delta \mathrm{x}_{i}$ where $\Delta \mathrm{x}_{i}$ is a small change in $\mathrm{x}_{\mathrm{i}}$ ), the system is operating near the initial state. The initial state may be considered as a quiescent operating condition for the system. To examine the behavior of the system when it is perturbed such that the new and old equilibrium states are nearly equal, the system equations are linearized about the quiescent operating condition. By this we mean that first-order approximations are made for the system Equations. The new linear Equations thus derived are assumed to be valid in a region near the quiescent condition.

As an example of product nonlinearities, consider the product $\mathrm{x}_{\mathrm{i}} \mathrm{x}_{\mathrm{j}}$. Let the state variables $x_{i}$ and $x_{j}$ have the initial values $x_{i o}$ and $x_{j o}$. Let the changes in these variables be $\Delta \mathrm{x}_{i}$ and $\Delta \mathrm{x}_{j}$. Initially their product is given by $\mathrm{X}_{\mathrm{io}} \mathrm{X}_{\mathrm{jo}}$. The new value becomes

$$
\left(\mathrm{x}_{i o}+\Delta \mathrm{x}_{i}\right)\left(\mathrm{x}_{j o}+\Delta \mathrm{x}_{j}\right)=\mathrm{x}_{i o} \mathrm{x}_{j o}+\mathrm{x}_{i o} \Delta \mathrm{x}_{j}+\mathrm{x}_{j o} \Delta \mathrm{x}_{i}+\Delta \mathrm{x}_{i} \Delta \mathrm{x}_{j}
$$

The last term is a second-order term, which is assumed to be negligibly small. Thus for a first-order approximation, the change in the product $\mathrm{x}_{\mathrm{i}} \mathrm{x}_{\mathrm{j}}$ is given by

$$
\left(\mathrm{x}_{i o}+\Delta \mathrm{x}_{i}\right)\left(\mathrm{x}_{j o}+\Delta \mathrm{x}_{j}\right)-\mathrm{x}_{i o} \mathrm{x}_{j o}=\mathrm{x}_{j o} \Delta \mathrm{x}_{i}+\mathrm{x}_{i 0} \Delta \mathrm{x}_{j}
$$

We note that $\mathrm{x}_{\mathrm{jo}}$ and $\mathrm{x}_{\mathrm{io}}$ are known quantities and are treated here as coefficients, while $\Delta \mathrm{x}_{i}$ and $\Delta \mathrm{x}_{j}$ are "incremental" variables.

The trigonometric nonlinearities are treated in a similar manner as

$$
\cos \left(\delta_{0}+\Delta \delta\right)=\cos \delta_{0} \cos \Delta \delta-\sin \delta_{0} \sin \Delta \delta
$$

with $\cos \Delta \delta \cong 1$ and $\sin \Delta \delta \cong \Delta \delta$.therefore,

$$
\cos \left(\delta_{0}+\Delta \delta\right)-\cos \delta_{0} \cong\left(-\sin \delta_{0}\right) \Delta \delta
$$

The incremental change in $\cos \delta$ is then $\left(-\sin \delta_{0}\right) \Delta \delta$; the incremental variable is $\Delta \delta$ and its coefficient is $-\sin \delta_{0}$. Similarly, we can show that the incremental change in the term $\sin \delta$ is given by

$$
\sin \left(\delta_{0}+\Delta \delta\right)-\sin \delta_{0} \cong\left(\cos \delta_{0}\right) \Delta \delta
$$

The equivalent circuit of MMS can be shown below:


Figure 3.1: Interconnection of the synchronous machine dynamic circuit and the rest of the network.

- third Order Model Differential Equations:

$$
\begin{align*}
& \dot{E}_{q \mathrm{i}}^{\prime}=-\frac{1}{T_{d o i}^{\prime}}\left[-E_{q i}^{\prime}-\left(x_{d i}-x_{d i}^{\prime}\right) I_{d i}+E_{f d i}\right]  \tag{3.1}\\
& \dot{\delta}_{\mathrm{i}}=\omega_{\mathrm{i}}-\omega_{\mathrm{s}}  \tag{3.2}\\
& \dot{\omega}_{\mathrm{i}}=\frac{\omega_{s}}{2 H_{i}}\left[T_{m i}-E_{q i}^{\prime} I_{q i}-\left(x_{q i}^{\prime}-x_{d i}^{\prime}\right) I_{d i} I_{q i}-D_{i}\left(\omega_{i}-\omega_{s}\right)\right] \tag{3.3}
\end{align*}
$$

## * Algebraic Equations:

- Stator Algebraic Equations:

$$
\begin{align*}
0 & =v_{i} e^{j \theta_{i}}+j x_{d i}^{\prime}\left(I_{d i}+j I_{q i}\right) e^{j\left(\delta_{i}-\frac{\pi}{2}\right)}-  \tag{3.4}\\
& {\left[\mathrm{E}_{d i}^{\prime}+\left(\mathrm{x}_{q i}^{\prime}-\mathrm{x}_{d i}^{\prime}\right) \mathrm{I}_{q i}+j E_{q i}^{\prime}\right] e^{j\left(\delta_{i}-\frac{\pi}{2}\right)} }
\end{align*}
$$

$$
\text { For } \mathrm{i}=1, \ldots, \mathrm{n}
$$

In polar form:

$$
\begin{align*}
& E_{q i}^{\prime}-V_{i} \cos \left(\delta_{i}-\theta_{i}\right)-x_{d i}^{\prime} I_{d i}=0  \tag{3.5}\\
& E_{d i}^{\prime}-V_{i} \sin \left(\delta_{i}-\theta_{i}\right)-x_{q i}^{\prime} I_{q i}=0 \tag{3.6}
\end{align*}
$$

- Network Equations:


Figure 3.2: Synchronous machine two-axis model dynamic circuit

$$
(\mathrm{i}=1, \ldots, \mathrm{n})
$$

- Generator Buses:

$$
\begin{align*}
& I_{d i} V_{i} \sin \left(\delta_{i}-\theta_{i}\right)+I_{q i} V_{i} \cos \left(\delta_{i}-\theta_{i}\right) \\
& +P_{l i} V_{i}-\sum_{k=1}^{n} V_{i} V_{k} Y_{i k} \cos \left(\theta_{i}-\theta_{k}-\alpha_{i k}\right)=0  \tag{3.7}\\
& I_{d i} V_{i} \cos \left(\delta_{i}-\theta_{i}\right)-I_{q i} V_{i} \sin \left(\delta_{i}-\theta_{i}\right) \\
& +Q_{l i} V_{i}-\sum_{k=1}^{n} V_{i} V_{k} Y_{i k} \sin \left(\theta_{i}-\theta_{k}-\alpha_{i k}\right)=0 \tag{3.8}
\end{align*}
$$

$$
\text { For } \mathrm{i}=1, \ldots, \mathrm{n}
$$

## - Load Buses:

$$
\begin{align*}
P_{l i} V_{i}+Q_{l i} V_{i} & =\sum_{k=1}^{n} V_{i} V_{k} Y_{i k} \cos \left(\theta_{i}-\theta_{k}-\alpha_{i k}\right)  \tag{3.9}\\
P_{l i} V_{i}+Q_{l i} V_{i} & =\sum_{k=1}^{n} V_{i} V_{k} Y_{i k} \sin \left(\theta_{i}-\theta_{k}-\alpha_{i k}\right) \tag{3.10}
\end{align*}
$$

$$
\text { For } \mathrm{i}=1, \ldots, \mathrm{n}
$$

### 3.2 Linearization:

Let the state-space vector $\mathbf{x}$ have an initial state $\mathrm{x}_{0}$ at time $\mathrm{t}=\mathrm{t}_{0}$, at occurrence of a small disturbance, i.e., after $\mathrm{t}=\mathrm{t}_{0}+$, the states will change slightly from their previous positions or values. Thus

$$
x=x+\Delta x
$$

The state-space model is in the form
$\dot{x}=f(\mathrm{x}, \mathrm{t})$
Which
$\dot{x}_{0}+\Delta \dot{x}=\mathrm{f}\left[\left(\mathrm{x}_{0}+\Delta \mathrm{x}\right), \mathrm{t}\right]$
From which we obtain the linearized state-space equation

$$
\Delta \dot{x}=\mathrm{A}\left(\mathrm{x}_{0}\right) \Delta \mathrm{x}+\mathrm{B}\left(\mathrm{x}_{0}\right) \mathrm{u}
$$

Where A and B are state matrixes. Therefore, the system variable be:

$$
\begin{array}{lcl}
\mathrm{V}_{\mathrm{i}}=\mathrm{V}_{\mathrm{i}}+\Delta \mathrm{V}_{\mathrm{i}} & \delta_{i}=\delta_{i 0}+\Delta \delta_{i} & \omega_{i}=\omega_{i 0}+\Delta \omega_{i} \\
\theta_{i}=\theta_{i 0}+\Delta \theta_{i} & E_{q i}^{\prime}=E_{q i 0}^{\prime}+\Delta E_{q i}^{\prime} & E_{f d i}=E_{f d i}+\Delta E_{f d i}
\end{array}
$$

### 3.2.1 The Linearization of the Differential Equations:

The linearization of the third order differential equations (3.1) (3.3) yields

$$
\begin{align*}
& \frac{d \Delta E_{q i}^{\prime}}{d t}=-\frac{1}{T_{d o i}^{\prime}}\left[\Delta E_{q i}^{\prime}-\left(\mathrm{x}_{d i}-\mathrm{x}_{d i}^{\prime}\right) \Delta \mathrm{I}_{d i}+\Delta E_{f d i}\right]  \tag{3.11}\\
& \frac{d \Delta \delta_{i}}{d t}=\Delta \omega_{i} \tag{3.12}
\end{align*}
$$

$$
\begin{align*}
& \frac{d \Delta \omega_{i}}{d t}=\frac{1}{M_{i}}\left[\Delta T_{M i}-E_{q i 0}^{\prime} \Delta I_{q i}+\left(\mathrm{x}_{d i}^{\prime}-\mathrm{x}_{q i}^{\prime}\right) \mathrm{I}_{q i} \Delta \mathrm{I}_{d i}\right.  \tag{3.13}\\
& \left.+\left(\mathrm{x}_{d i}^{\prime}-\mathrm{x}_{q i}^{\prime}\right) I_{d i} \Delta I_{q i}-D_{i} \Delta \omega_{i}-\Delta E_{q i 0}^{\prime} I_{q i}-\Delta E_{d i 0}^{\prime} I_{d i}-\Delta E_{d i 0}^{\prime} \Delta I_{d i}\right]
\end{align*}
$$

Writing (3.12) through (3.15) in matrix notation, we obtain

$$
\begin{align*}
& {\left[\begin{array}{c}
\Delta \dot{\delta}_{i} \\
\Delta \dot{\omega}_{i} \\
\Delta \dot{E}_{q i}^{\prime}
\end{array}\right] }=\left[\begin{array}{ccc}
0 & 1 & 0 \\
0 & -\frac{D_{i}}{M_{i}} & -\frac{I_{q i}}{M_{i}} \\
0 & 0 & -\frac{1}{T_{q o i}^{\prime}}
\end{array}\right]\left[\begin{array}{c}
\Delta \delta_{i} \\
\Delta \omega_{i} \\
\Delta E_{q i}^{\prime}
\end{array}\right]+ \\
& {\left[\begin{array}{cc}
\frac{-I_{q i}\left(x_{d i}^{\prime}-x_{q i}^{\prime}\right)-E_{d i}^{\prime}}{M_{i}} & \frac{-I_{d i}\left(x_{d i}^{\prime}-x_{q i}^{\prime}\right)-E_{q i}^{\prime}}{M_{i}}
\end{array}\right]\left(\begin{array}{l}
\frac{-\left(x_{d i}-x_{d i}^{\prime}\right)}{T_{d o i}^{\prime}} \\
\Delta I_{d i} \\
\Delta I_{q i}
\end{array}\right]+} \\
& {\left[\begin{array}{cc}
0 & 0 \\
\frac{1}{M_{i}} & 0 \\
0 & 0 \\
\binom{\Delta T_{m i}}{\Delta V_{r e f}}
\end{array}\right.} \tag{3.14}
\end{align*}
$$

### 3.2.2 The System with AVR:

we add a fast exciter whose state space equation is:
$\stackrel{\bullet}{V}_{f}=\frac{1}{T_{R i}}\left(V_{c}-V_{f}\right)$
$\dot{E}_{f d i}=-\frac{1}{T_{A i}} E_{f d i}+\frac{K_{A i}}{T_{A i}}\left(V_{r e f}-V_{i}\right)$
The linearized form of (3.17) is:

$$
\begin{align*}
& \dot{V}_{f}=\frac{1}{T_{R i}}\left(V_{c}-\Delta V_{f}\right)  \tag{3.17}\\
& \Delta \dot{E}_{f d i}=-\frac{1}{T_{A i}} \Delta E_{f d i}+\frac{K_{A i}}{T_{A i}}\left(\Delta V_{r e f}-\Delta V_{i}\right) \tag{3.18}
\end{align*}
$$

Writing (3.11) through (3.17), (3.18) in matrix notation, we obtain

$$
\begin{align*}
& {\left[\begin{array}{c}
\Delta \dot{\delta}_{i} \\
\Delta \dot{\omega}_{i} \\
\Delta \dot{E}_{q i}^{\prime} \\
\Delta \dot{V}_{f}^{\prime} \\
\Delta \dot{E}_{f d i}^{\prime}
\end{array}\right]=\left[\begin{array}{ccccc}
0 & 1 & 0 & 0 & 0 \\
0 & -\frac{D_{i}}{M_{i}} & -\frac{I_{q i}}{M_{i}} & 0 & 0 \\
0 & 0 & -\frac{1}{T_{d o i}^{\prime}} & \frac{1}{T_{d o i}^{\prime}} & 0 \\
0 & 0 & 0 & -\frac{1}{T_{R i}} & 0 \\
0 & 0 & 0 & 0 & -\frac{1}{T_{A i}}
\end{array}\right]\left[\begin{array}{c}
\Delta \delta_{i} \\
\Delta \omega_{i} \\
\Delta E_{q i}^{\prime} \\
\Delta V_{f} \\
\Delta E_{f d i}
\end{array}\right]+} \\
& {\left[\begin{array}{cc}
0 & 0 \\
\frac{-I_{q i}\left(x_{d i}^{\prime}-x^{\prime}{ }_{q i}\right)-E_{d i}^{\prime}}{M_{i}} & \frac{-I_{d i}\left(x_{d i}^{\prime}-x^{\prime}{ }_{q i}\right)-E^{\prime}{ }_{q i}}{M_{i}} \\
\frac{-\left(x_{d i}-x^{\prime}{ }_{d i}\right)}{T^{\prime}{ }_{d o i}} & 0 \\
0 & 0 \\
0 & 0
\end{array}\right]\binom{\Delta I_{d i}}{\Delta I_{q i}}+} \\
& {\left[\begin{array}{cc}
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & -\frac{K_{A i}}{T_{A i}}
\end{array}\right]\binom{\Delta \theta_{i}}{\Delta V_{i}}+\left[\begin{array}{cc}
0 & 0 \\
\frac{1}{M_{i}} & 0 \\
0 & 0 \\
0 & 0 \\
0 & -\frac{K_{A i}}{T_{A i}}
\end{array}\right]\binom{\Delta T_{M i}}{\Delta V_{r e f}}} \tag{3.19}
\end{align*}
$$

Put

$$
\Delta I_{g i}=\binom{\Delta I_{d i}}{\Delta I_{q i}}, \quad \Delta V_{g i}=\binom{\Delta \theta_{i}}{\Delta V_{i}}, \quad \Delta V_{i}=\binom{\Delta T_{M i}}{\Delta V_{r f}}
$$

Then

$$
\begin{equation*}
\Delta \dot{x}_{i}=A_{1 i} \Delta x_{i}+B_{1 i} \Delta I_{g i}+B_{2 i} \Delta V_{g i}+E_{i} \Delta V_{i} \quad \mathrm{i}=1, \ldots, \mathrm{n} \tag{3.20}
\end{equation*}
$$

### 3.2.3 Linearization of Algebraic Equations:

The linearization of algebraic equations (3.4) and (3.6) in matrix form yields

$$
\begin{align*}
& {\left[\begin{array}{ccccc}
-V_{i 0} \cos \left(\delta_{i 0}-\theta_{i 0}\right) & 0 & 0 & 0 & 0 \\
V_{i 0} \sin \left(\delta_{i 0}-\theta_{i 0}\right) & 0 & 0 & 0 & 0
\end{array}\right]\left[\begin{array}{c}
\Delta \delta_{i} \\
\Delta \omega_{i} \\
\Delta E_{q i}^{\prime} \\
\Delta V_{f} \\
\Delta E_{f d i}
\end{array}\right]+\left[\begin{array}{cc}
0 & x_{q i}^{\prime} \\
-x_{d i}^{\prime} & 0
\end{array}\right]\binom{\Delta I_{d i}}{\Delta I_{q i}}}  \tag{3.21}\\
& +\left[\begin{array}{cc}
V_{i 0} \cos \left(\delta_{i 0}-\theta_{i 0}\right) & -\sin \left(\delta_{i 0}-\theta_{i 0}\right) \\
-V_{i 0} \sin \left(\delta_{i 0}-\theta_{i 0}\right) & -\cos \left(\delta_{i 0}-\theta_{i 0}\right)
\end{array}\right]\binom{\Delta \theta_{i}}{\Delta V_{i}}=0
\end{align*}
$$

Rewriting (3.21) we obtain

$$
\begin{equation*}
0=C_{1 i} \Delta x_{i}+D_{1 i} \Delta I_{g i}+D_{2 i} \Delta V_{g i} \tag{3.22}
\end{equation*}
$$

In matrix notation, (3.23) can be written as

$$
\begin{equation*}
0=C_{1} \Delta x+D_{1} \Delta I_{g}+D_{2} \Delta V_{g} \tag{3.23}
\end{equation*}
$$

Where $\mathrm{C}_{1}, \mathrm{D}_{1}$, and $\mathrm{D}_{2}$ are block diagonal.

### 3.2.4 Linearization the Network Equations:

For generator buses (3.7) and (3.8)

$$
\begin{align*}
& {\left[\begin{array}{l}
0 \\
0 \\
0 \\
0 \\
0
\end{array}\right]=\left[\begin{array}{ccccc}
{\left[I_{d i} V_{i 0} \cos \left(\delta_{i 0}-\theta_{i 0}\right)-I_{q i} V_{i 0} \sin \left(\delta_{i 0}-\theta_{i 0}\right)\right]} & 0 & 0 & 0 & 0 \\
{\left[-I_{d i} V_{i 0} \sin \left(\delta_{i 0}-\theta_{i 0}\right)-I_{q i} V_{i 0} \cos \left(\delta_{i 0}-\theta_{i 0}\right)\right]} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{array}\right]\left[\begin{array}{c}
\Delta \delta_{i} \\
\Delta \omega_{i} \\
\Delta E_{q i}^{\prime} \\
\Delta V_{f} \\
\Delta E_{f i i}
\end{array}\right]+} \\
& {\left[\begin{array}{cc}
V_{i 0} \sin \left(\delta_{i 0}-\theta_{i 0}\right) & V_{i 0} \cos \left(\delta_{i 0}-\theta_{i 0}\right) \\
V_{i 0} \cos \left(\delta_{i 0}-\theta_{i 0}\right) & -V_{i 0} \sin \left(\delta_{i 0}-\theta_{i 0}\right) \\
0 & 0 \\
0 & 0 \\
0 & 0
\end{array}\right]\left(\begin{array}{l}
\Delta I_{d i} \\
\Delta I_{q i}
\end{array}\right]+} \\
& {\left[\begin{array}{ll}
-I_{d i} V_{i 0} \cos \left(\delta_{i 0}-\theta_{i 0}\right)+I_{q i} V_{i 0} \sin \left(\delta_{i 0}-\theta_{i 0}\right) & I_{d i} V_{i 0} \sin \left(\delta_{i 0}-\theta_{i 0}\right)+I_{q i} V_{i 0} \cos \left(\delta_{i 0}-\theta_{i 0}\right) \\
+V_{i 0} \sum_{\substack{k=1 \\
k \neq i}}^{n} V_{k i} Y_{i k} \sin \left(\theta_{i 0}-\theta_{k o}-\alpha_{i k}\right) & +V_{i 0} \sum_{k=10}^{n} V_{k 0} Y_{i k} \cos \left(\theta_{i 0}-\theta_{k 0}-\alpha_{i k}\right)+\frac{\partial P_{i 0}\left(V_{i}\right)}{\partial V_{i}}
\end{array}\right]} \\
& I_{d i} V_{i 0} \sin \left(\delta_{i 0}-\theta_{i 0}\right)+I_{q i} V_{i 0} \cos \left(\delta_{i 0}-\theta_{i 0}\right) \quad I_{d i} V_{i 0} \cos \left(\delta_{i 0}-\theta_{i 0}\right)-I_{q i} V_{i 0} \sin \left(\delta_{i 0}-\theta_{i 0}\right) \\
& -\mathrm{V}_{i o} \sum_{\substack{k=1 \\
k \neq i}}^{n} V_{k 0} Y_{i k} \sin \left(\theta_{i o}-\theta_{k o}-\alpha_{i k}\right) \quad-V_{i o} \sum_{k=1}^{n} V_{k o} Y_{i k} \sin \left(\theta_{i o}-\theta_{k o}-\alpha_{i k}\right)+\frac{\partial Q_{i j}\left(V_{V}\right)}{\partial V_{i}}  \tag{i}\\
& \begin{array}{ll}
0 & 0 \\
0 & 0 \\
0 & 0
\end{array} \\
& +\left[\begin{array}{cc}
-V_{i o} \sum_{\substack{k=10 \\
k \neq i}}^{n} V_{k o} Y_{i k} \sin \left(\theta_{i o}-\theta_{k o}-\alpha_{i k}\right) & -V_{i o} \sum_{k=1}^{n} V_{k o} Y_{i k} \cos \left(\theta_{i o}-\theta_{k o}-\alpha_{i k}\right) \\
-V_{i 0_{i o}} \sum_{k=10}^{n} V_{k o} Y_{i k} \cos \left(\theta_{i o}-\theta_{k o}-\alpha_{i k}\right) & -V_{i o} \sum_{k=1}^{n} V_{k o} Y_{i k} \sin \left(\theta_{i o}-\theta_{k o}-\alpha_{i k}\right) \\
0 & 0 \\
0 & 0 \\
0 & 0
\end{array}\right]\left(\begin{array}{c}
\Delta \theta_{K} \\
\left.\Delta V_{K}\right) \\
\\
0
\end{array}\right. \\
& \mathrm{i}=1, \ldots, \mathrm{n} \tag{3.24}
\end{align*}
$$

Rewriting (3.24) we obtain

$$
\begin{align*}
0 & =\left[\begin{array}{ccc}
C_{21} & & \\
& \ddots & \\
& & C_{2 n}
\end{array}\right]\left(\begin{array}{l}
\Delta x_{1} \\
\vdots \\
\Delta x_{n}
\end{array}\right)+\left(\begin{array}{lll}
D_{31} & & \\
& \ddots & \\
& & D_{3 n}
\end{array}\right)\left(\begin{array}{l}
\Delta I_{g 1} \\
\vdots \\
\Delta I_{g m}
\end{array}\right)+ \\
& {\left[\begin{array}{ccc}
D_{41,1} & \cdots & D_{41, \mathrm{n}} \\
\vdots & \vdots & \vdots \\
D_{4 n, 1} & \cdots & D_{4 n, n}
\end{array}\right]\left(\begin{array}{l}
\Delta V_{g 1} \\
\vdots \\
\Delta V_{g n}
\end{array}\right)+\left[\begin{array}{ccc}
D_{51, n+1} & \cdots & D_{51, m} \\
\vdots & \vdots & \vdots \\
D_{5 n, n+1} & \cdots & D_{5 n, m}
\end{array}\right]\left[\begin{array}{l}
\Delta V_{\ell \mathrm{n}+1} \\
\vdots \\
\Delta V_{\ell \mathrm{n}}
\end{array}\right] } \tag{3.25}
\end{align*}
$$

Where the various sub matrices of (3.25) can be easily identified. In matrix notation, (3.26) is

$$
\begin{equation*}
0=C_{2} \Delta x+D_{3} \Delta I_{g}+D_{4} \Delta V_{g}+D_{5} \Delta V_{\ell} \tag{3.26}
\end{equation*}
$$

Where

$$
\Delta V_{\ell}=\binom{\Delta \theta_{K}}{\Delta V_{K}}
$$

For the non-generator buses $\mathrm{i}=1, \ldots, \mathrm{n}$
Note that $\mathrm{C}_{2}, \mathrm{D}_{3}$ are block diagonal, whereas $\mathrm{D}_{4}, \mathrm{D}_{5}$ are full matrices.
For load buses (3.9) and (3.10) can be given as:

$$
\begin{align*}
& {\left[\begin{array}{cc}
\sum_{\substack{k=1 \\
k \neq i}}^{n} V_{i o} V_{k o} Y_{i k} \sin \left(\theta_{i o}-\theta_{k o}-\alpha_{i k}\right) & -\sum_{k=1}^{n} V_{k o} Y_{i k} \cos \left(\theta_{i o}-\theta_{k o}-\alpha_{i k}\right)+\frac{\partial P_{l i}\left(V_{i}\right)}{\partial V_{i}} \\
-\sum_{\substack{k=1 \\
k \neq i}}^{n} V_{i o} V_{k o} Y_{i k} \cos \left(\theta_{i o}-\theta_{k o}-\alpha_{i k}\right) & -\sum_{k=1}^{n} V_{k o} Y_{i k} \sin \left(\theta_{i o}-\theta_{k o}-\alpha_{i k}\right)+\frac{\partial Q_{i i}\left(V_{i}\right)}{\partial V_{i}} \\
0 & 0 \\
0 & 0 \\
0 & 0
\end{array}\right]\binom{\Delta \theta_{i}}{\Delta V_{i}}} \\
& +\left[\begin{array}{cc}
-\mathrm{V}_{i o} \sum_{\substack{k=1 \\
k \neq i}}^{n} V_{k o} Y_{i k} \sin \left(\theta_{i o}-\theta_{k o}-\alpha_{i k}\right) & -\mathrm{V}_{i o} \sum_{k=1}^{n} Y_{i k} \cos \left(\theta_{i o}-\theta_{k o}-\alpha_{i k}\right) \\
-\mathrm{V}_{i o} \sum_{\substack{k=1 \\
k \neq i}}^{n} V_{k o} Y_{i k} \sin \left(\theta_{i o}-\theta_{k o}-\alpha_{i k}\right) & -\mathrm{V}_{i o} \sum_{k=1}^{n} Y_{i k} \sin \left(\theta_{i o}-\theta_{k o}-\alpha_{i k}\right) \\
0 & 0 \\
0 & 0 \\
0 & 0
\end{array}\right]\binom{\Delta \theta_{K}}{\Delta V_{K}} \tag{3.27}
\end{align*}
$$

Rewriting (3.27) we obtain
$0=\left[\begin{array}{ccc}D_{6 \mathrm{~m}+1,1} & \cdots & D_{6 m+1, \mathrm{~m}} \\ \vdots & \vdots & \vdots \\ D_{6 n, 1} & \cdots & D_{6 n, \mathrm{~m}}\end{array}\right]\left(\begin{array}{l}\Delta V_{g 1} \\ \vdots \\ \Delta V_{g m}\end{array}\right)+\left[\begin{array}{ccc}D_{7 \mathrm{~m}+1, \mathrm{~m}+1} & \cdots & D_{7 m+1, n} \\ \vdots & \vdots & \vdots \\ D_{7 \mathrm{n}, \mathrm{m}+1} & \cdots & D_{7 n, \mathrm{n}}\end{array}\right]\left[\begin{array}{l}\Delta V_{\ell m+1} \\ \vdots \\ \Delta V_{\ell \mathrm{n}}\end{array}\right]$
In matrix notation, (3.28) can be written as

$$
\begin{equation*}
0=D_{6} \Delta V_{g}+D_{7} \Delta V_{\ell} \tag{3.29}
\end{equation*}
$$

Where $\mathrm{D}_{6}$ and $\mathrm{D}_{7}$ are full matrices.
Rewritten equation (3.20),(3.23),(3.26) and (3.29) together as follow :

$$
\begin{align*}
& \Delta \dot{x}_{i}=A_{1 i} \Delta x_{i}+B_{1 i} \Delta I_{g i}+B_{2 i} \Delta V_{g i}+E_{i} \Delta V_{i}  \tag{3.30}\\
& 0=C_{1} \Delta x+D_{1} \Delta I_{g}+D_{2} \Delta V_{g}  \tag{3.31}\\
& 0=C_{2} \Delta x+D_{3} \Delta I_{g}+D_{4} \Delta V_{g}+D_{5} \Delta V_{\ell}  \tag{3.32}\\
& 0=D_{6} \Delta V_{g}+D_{7} \Delta V_{\ell} \tag{3.33}
\end{align*}
$$

From (3.31) we obtain

$$
\begin{equation*}
\Delta I_{g}=-D_{1}^{-1} C_{1} \Delta X-D_{1}^{-1} D_{2} \Delta V_{g} \tag{3.34}
\end{equation*}
$$

Substitute (3.34) in (3.30) as shown

$$
\begin{equation*}
\mathrm{C}_{2} \Delta X+\mathrm{D}_{3}\left(-D_{1}^{-1} C_{1} \Delta X-D_{1}^{-1} D_{2} \Delta V_{g}\right)+D_{4} \Delta V_{g}+D_{5} \Delta V_{\ell}=0 \tag{3.35}
\end{equation*}
$$

Let

$$
\left[D_{4}-D_{3} D_{1}^{-1} D_{2}\right]=K_{1}
$$

And

$$
\left[C_{2}-D_{3} D_{1}^{-1} C_{1}\right]=K_{2}
$$

Then

$$
\begin{equation*}
K_{2} \Delta X+K_{1} \Delta V_{g}+D_{5} \Delta V_{\ell}=0 \tag{3.36}
\end{equation*}
$$

Substitute (3.36) in (3.30) we find

$$
\begin{align*}
& \Delta \dot{X}=\left(A_{1}-B_{1} D_{1}^{-1} C_{1}\right) \Delta \mathrm{X}+\left(-B_{1} D_{1}^{-1} D_{2}+B_{2}\right) \Delta V_{g}+E_{1} \Delta u  \tag{3.37}\\
& K_{2} \Delta X+K_{1} \Delta V_{g}+D_{5} \Delta V_{\ell}=0  \tag{3.38}\\
& \quad 0=D_{6} \Delta V_{g}+D_{7} \Delta V_{\ell} \tag{3.39}
\end{align*}
$$

Equation (3.37)-(3.39) can be put in a form of matrix shown below

$$
\left(\begin{array}{c}
\Delta \dot{X}  \tag{3.40}\\
0 \\
0
\end{array}\right)=\left(\begin{array}{ccc}
A_{1}-B_{1} D_{1}^{-1} C_{1} & -B_{1} D_{1}^{-1} D_{2}+B_{2} & 0 \\
K_{2} & K_{1} & D_{5} \\
0 & D_{6} & D_{7}
\end{array}\right)\left(\begin{array}{c}
\Delta X \\
\Delta V_{g} \\
\Delta V_{\ell}
\end{array}\right)+\left(\begin{array}{c}
E_{1} \\
0 \\
0
\end{array}\right) \Delta u
$$

Let
$A^{\prime}=\left[A_{1}-B_{1} D_{1}^{-1} C_{1}\right]$
$B^{\prime}=-B_{1} D_{1}^{-1} D_{2}+B_{2}$
So,

$$
\begin{align*}
& \Delta \dot{X}=A^{\prime} \Delta \mathrm{X}+B^{\prime} \Delta V_{g}+E_{1} \Delta u  \tag{3.41}\\
& K_{2} \Delta X+K_{1} \Delta V_{g}+D_{5} \Delta V_{l}=0  \tag{3.42}\\
& 0=D_{6} \Delta V_{g}+D_{7} \Delta V_{\ell} \tag{3.43}
\end{align*}
$$

From (3.43)

$$
\begin{equation*}
\Delta V_{\ell}=-D_{7}^{-1} D_{6} \Delta V_{g} \tag{3.44}
\end{equation*}
$$

Substitute (3.43) in (3.39) we find
$K_{2} \Delta X+F \Delta V_{g}=0$
(3.45)

Where

$$
\begin{equation*}
F=K_{1}-D_{5} D_{7}^{-1} D_{6} \tag{3.46}
\end{equation*}
$$

From (3.45)

$$
\begin{equation*}
\Delta V_{g}=-F^{-1} K_{2} \Delta X \tag{3.47}
\end{equation*}
$$

Substitute (3.47) in (3.41) we find [9]

$$
\begin{equation*}
\Delta \dot{X}=A_{S Y S} \Delta \mathrm{X}+E_{1} \Delta u \tag{3.48}
\end{equation*}
$$

Where

$$
\begin{equation*}
A_{S Y S}=A^{\prime}-B^{\prime} F^{-1} K_{2} \tag{3.49}
\end{equation*}
$$



### 3.3 Methods of Analysis of Small Signal Stability:

1. Eigenvalue analysis.
2. Synchronizing and damping torque analysis.
3. Frequency response- and residue-based analysis.
4. Time-domain solution analysis.

### 3.3.1 Eigenvalue Analysis for Small Signal Stability:

Eigenvalue analysis is used to study oscillatory behavior of power systems and hence has been described in detail. The system is linearized about an operating point and typically involves computation of eigenvalues. After linearization of system equations, the system can be described as
$\Delta \dot{x}=\mathrm{A} \Delta \mathrm{x}(\mathrm{t})+\mathrm{B} \Delta \mathrm{u}(\mathrm{t})$
$\lambda=\sigma \pm \mathrm{j} \omega$
Where:
A is the state matrix and
$\lambda$ is the eigenvalues of the matrix A .
The small signal stability can be assessed by the eigenvalues $\lambda_{i}$ of the state matrix A. For any eigenvalue $\lambda_{i}$, there exists at least one nonzero column vector $r_{i}$ that satisfies
A. $r_{i}=\lambda_{i} . r_{i}$

The vector $r_{i}$ is called a right eigenvector of its eigenvalue $\lambda_{i}$. A vector Ii that satisfies

AT . $\mathrm{I}_{\mathrm{i}}=\lambda_{\mathrm{i}} \cdot \mathrm{I}_{\mathrm{i}}$
is called a left eigenvector of its eigenvalue $\lambda_{i}$.
The stability of a power system is determined by the eigenvalues as follows:
(i) A real eigenvalue corresponds to a non-oscillatory mode. A negative real eigenvalue represents a decaying mode. The larger its magnitude, the faster is the decay. A positive real eigenvalue represents aperiodic instability.
(ii) Complex eigenvalues occur in conjugate pairs (since the state matrix is real), and each pair correspond to an oscillatory mode. The real component of the eigenvalues gives the damping, and the imaginary component gives the frequency of oscillation. A negative real part represents a damped oscillation whereas a positive real part represents oscillation of increasing amplitude.

For a complex pair of eigenvalues $\lambda=\sigma \pm \mathrm{j} \omega$ the frequency of oscillation in Hz is given by

$$
\begin{equation*}
\mathrm{f}=\frac{\omega}{2 \pi} \tag{3.52}
\end{equation*}
$$

The damping ratio is given by

$$
\begin{equation*}
\zeta=\frac{-\sigma}{\sqrt{\left(\sigma^{2}+\omega^{2}\right)}} \tag{3.53}
\end{equation*}
$$

The damping ratio $\zeta$ determines the rate of decay of the amplitude of the oscillation. The time constant of amplitude decay is $[1 /|\sigma|][9]$.

## Advantages of Eigenvalue or Modal Analysis:

With eigenvalue techniques, oscillations can be characterized easily, quickly and accurately. Different modes, which are mixed with each other in curves of time-domain simulation, are identified separately. Root loci plotted with variations in system parameters or operating conditions provide valuable insight into the dynamic characteristics of the system.

Using eigenvectors coherent groups of generators which participate in a given swing mode can be identified. In addition, linear models can be used to design controllers that damp oscillations. Further, information regarding the most effective site of controller, tuning of existing one, installation of new controller can be decided.

Eigenvalue or modal analysis describes the small-signal behavior of the system about an operating point, and does not take into account the nonlinear behavior of components such as controller's limits at large system perturbations. Further, design and analysis carried out using various indices such as participation factors, residues, etc. may lead to many alternate options. These options need to be verified for their effectiveness using system responses for small/large disturbances. In such cases, time-domain simulations are very essential. In this thesis time-domain simulation, and modal analysis in the frequency domain should be used in a complement manner in analyzing smallsignal stability of power systems [10].

### 3.3.2 Time Domain Simulation:

The time domain simulations were performed to validate the results of modal analysis. The effectiveness of the PSSs is assessed by their ability to damp low frequency oscillations under various operating conditions. Furthermore, the PSS must be able to stabilize the system under transient conditions. Therefore, two types of time domain simulations are performed; small signal and transient simulations.

### 3.4 Development of a Location Selection Indicator of PSS:

During the application of PSS to a multi machine power system to achieve the largest improvement in damping, first, the best location of PSS must be identified among a number of interconnected machines. Study reveals that the PSS displaces the swing mode from its critical position to a more desirable position, changing the response of the excitation system. Based on the change of the exciter transfer function, a simple and easy indicator called OPLI to identify the best location of the PSS in a multi machine system [8].

### 3.4.1 Participation Factor:

Participation factor is a tool for identifying the state variables that have significant participation in a selected mode among many modes in a multi--generator power system. It is natural to say that the significant state variables for an eigenvalue $\lambda_{\mathrm{p}}$ are those that correspond to large entries in the corresponding eigenvector $\phi_{\mathrm{p}}$. However, the problem of using right and left eigenvector entries individually for identifying the relationship between the states and the modes is that the elements of the eigenvectors are dependent on dimension and scaling associated with the state variables. As a solution of this problem, a matrix called the participation matrix $(\mathrm{P})$ is suggested in which the right and left eigenvectors entries are combined, and it is used as a measure of the association between the state variables and the modes: $p=\left[\begin{array}{llll}p_{1} & p_{2} & \ldots & p_{r}\end{array}\right]$

$$
p=\left[\begin{array}{c}
p_{1}  \tag{3.54}\\
p_{2} \\
\vdots \\
p_{r}
\end{array}\right]=\left[\begin{array}{c}
\phi_{1 p} \psi_{p 1} \\
\phi_{2 p} \psi_{p 2} \\
\vdots \\
\phi_{r p} \psi_{p r}
\end{array}\right]
$$

Where:
$\phi_{\mathrm{kp}}$ is the element on the kth row and pth column of the modal matrix.
$\Phi$ is the kth entry of the right eigenvector $\phi_{\mathrm{p}}$.
$\psi_{\mathrm{pk}}$ is the element on the pth row and kth column of the modal matrix.
$\psi$ is the kth entry of the left eigenvector $\psi$ p.
The element $\mathrm{P}_{\mathrm{kp}}=\phi_{\mathrm{kp}} \psi_{\mathrm{pk}}$ is termed the participation factor. It is a measure of the relative participation of the kth state variable in the pth mode, and vice versa.

Since $\phi_{\mathrm{kp}}$ measures the activity of the variable $\mathrm{X}_{\mathrm{k}}$ in the pth mode, and $\psi_{\mathrm{pk}}$ weighs the contribution of this activity to the mode, the product $\mathrm{P}_{\mathrm{kp}}$ measures the net participation. The effect of multiplying the elements of the left and right eigenvectors makes the $\mathrm{P}_{\mathrm{kp}}$ dimensionless. In view of the eigenvector normalization, the sum of the participation factors associated with any mode ( $\sum_{p=1}^{r} p_{k p}$ ) or with any state variable ( $\sum_{k=1}^{r} p_{k p}$ ) is equal to 1 . For a given autonomous linear system
$\Delta \dot{x}=\mathrm{A}_{\mathrm{sys}} \Delta x$
Participation factor is actually a measurement of sensitivity of the eigenvalue $\lambda p$ to the diagonal element $\mathrm{a}_{\mathrm{kk}}$ of the state matrix A , defined as:
$\mathrm{p}_{\mathrm{kp}}=\frac{\partial \lambda p}{\partial a_{k k}}, \mathrm{k}=1,2, \ldots \mathrm{r}$
$p_{\mathrm{kp}}=\frac{\psi_{k p} \phi_{k p}}{\psi_{p}^{T} \phi_{p}}, \mathrm{k}=1,2, \ldots \mathrm{r}$
where $\psi_{\mathrm{kp}}$ and $\phi_{\mathrm{kp}}$ are the kth entries in the left and right eigenvector associated with the pth eigenvalue [8].

In power system, participation factors give good indications for power system stabilizers placement. As previously mentioned, PSS is a device use to provide supplementary signal to add damping at the generator shaft. Hence, the
participation factor is use to identify the states that correspond have the highest participation in the mode (in general rotor speed and rotor angle). If the corresponding rotor angle and/or rotor speed participation factor of a generator in a mode is zero, then that particular generator state does not contribute to the damping of the mode. However, if the participation factor is real positive, adding damping at the generator will increase the damping of the mode whereas if negative, it will have adverse effects [9].

In this thesis participation factor (PF) method is considered.

### 3.4.2 Sensitivity of PSS Effect:

The PSS installed on a machine in a power system is a closed-loop controller. If a machine is selected for installation of PSS, for best effect, first, the amplitude of PSS input that is measured by the right eigenvector corresponding to speed change $\Delta \omega$ should be relatively large, and second, the control effect of PSS measured by the coefficient $\mathrm{S}_{\mathrm{ji}}$ should be strong.

The control effect of PSS on the system (by the PSS output state $\Delta \mathrm{V}$ si and the system mode $\lambda_{\mathrm{j}}$ ) can be calculated by $\mathrm{S}_{\mathrm{ji}}=\psi \mathrm{j}, \Delta \mathrm{E}_{f d i}$, where $\psi \mathrm{j}, \Delta \mathrm{E}_{f d i}$ is the left-eigenvector entry of the $\mathrm{j}^{\text {th }}$ mode $(\lambda \mathrm{j})$ corresponding to the state variable $\Delta \mathrm{E}_{f d i}$ of the ith machine. In order to take into consideration, the effect of both the PSS input and the PSS control in selecting the PSS location, SPE for the $\mathrm{i}^{\text {th }}$ machine has been considered as
$\mathrm{SPE}_{\mathrm{i}}=\phi_{j, \Delta \omega \mathrm{i}} \psi_{j, \Delta \mathrm{Efdi}}$
for $\mathrm{i}=1,2, \ldots, \mathrm{~m}$ (the number of machines) where $\phi_{j, \Delta \omega \mathrm{i}}$ is the right-eigenvector entry and $\psi_{j, \Delta \mathrm{Efdi}}$ is the left-eigenvector entry of the jth mode corresponding to the state $\Delta \omega_{\mathrm{i}}$ and $\Delta \mathrm{E}_{\text {fdi }}$ of the ith machine. SPE measures both the activity of PSS input ( $\Delta \omega_{\mathrm{i}}$ ) participating in a certain oscillatory mode and the control effect of PSS, on this mode. The larger the magnitude of the SPE, the better is the overall performance of the PSS. In a multi machine power system, there may be several swing modes that are of interest, and for each mode, a set (SPEi,
$\mathrm{i}=1,2, \ldots$.,m) can be calculated by Equation (3.56). The SPE with the largest magnitude of any ith machine identifies the best location of PSS.

### 3.4.3 Optimum PSS location index:

The newly proposed concept of OPLI is based on the change of exciter transfer function with respect to the PSS transfer function in a certain swing mode. The PSS on a machine is a closed-loop controller and that considers usually the machine speed or power as its input and introduces a damping so that the system moves from a less stable region to a more stable region. As the PSS acts through the excitation system, the effect of displacement of swing modes due to the installation of PSS will change the response of the excitation system. The response of the excitation system at a swing mode 10 can be obtained by replacing $\lambda^{\prime}$ for " s " in its transfer function $\mathrm{G}_{\mathrm{ex}}(\mathrm{s})$. The change of response of the excitation system with respect to the PSS response for a swing mode $\lambda^{\prime}$ is determined by the proposed index OPLI and is defined by
$\left|\operatorname{OPLI}_{\mathrm{i}}\right|=\frac{\left|G_{e x i}\left(\lambda^{\prime}\right)-G_{e x i}\left(\lambda^{0}\right)\right|}{\left|G_{P S S}\left(\lambda^{\prime}\right)\right|}$
for $\mathrm{i}=1,2, \ldots, \mathrm{~m}$ (the number of machines).
Here, $\lambda^{\prime}$ and $\lambda^{0}$ are the critical swing modes before and after the installation of PSS, respectively. The magnitude of OPLI measures the effect of PSS on the exciter response in a swing mode $\lambda^{\prime}$ of interest. The larger value of the OPLI, the larger is the control effect of PSS on the exciter and the better is the overall performance of PSS in the power system [9]. Figure (3.3) represent the location of PSS.


Figure 3.3: Location of PSS in a power system.

### 3.5 Design of Power System Stabilizer:

### 3.5.1 Operating Principle:

The basic function of power system stabilizer (PSS) is to add damping to the generator rotor oscillations by controlling its excitation by using auxiliary stabilizing signal(s). Based on the automatic voltage regulator (AVR) and using speed deviation, power deviation or frequency deviation as additional control signals, PSS is designed to introduce an additional torque coaxial with the rotational speed deviation, so that it can increase low-frequency oscillation damping and enhance the dynamic stability of power system.


Figure (3.4): Torque analysis between AVR and PSS.
As shown in Figure (3.4) the torque analysis between AVR and PSS.
under some conditions, such as much impedance, heavy load need etc., the additional torque $\Delta \mathrm{M}_{\mathrm{e} 2}$ provided by the AVR lags the negative feedback voltage $\left(-\Delta \mathrm{V}_{\mathrm{t}}\right)$ by one angle $\Delta \varphi_{x}$ which can generate the positive synchronizing torque and the negative damping torque component to reduce the low frequency oscillations damping. On the other hand, the power system stabilizer, using the speed signal $(\Delta \omega)$ as input signal, will have a positive damping torque component $\Delta M_{p 2}$. So, the synthesis torque with positive synchronous torque and the damping torque can enhance the capacity of the damping oscillation [7].

### 3.5.2 Phase compensation method:

The steps involved in designing a PSS are as follows:
a. Computation of GEPS(s).
b. Design of compensator using phase compensation technique.
c. Determination of compensator gain.

## a. Computation of GEPS(s):

A PSS acts through generator, exciter system, and power system (GEPS). Therefore, a PSS must compensate the phase lag through the GEPS. To obtain the phase information of GEPS, the frequency response of the transfer function between the exciter reference input (i.e., PSS output) and the generator electrical torque should be observed. In computing this response, the generator speed and rotor angle should remain constant, otherwise, when the excitation of a generator is modulated, the resulting change in electrical torque causes variations in rotor speed and angle and that in turn affect the electrical torque. As we are interested only in the phase characteristics between exciter reference input and electrical torque, the feedback effect through rotor angle variation should be eliminated by holding the speed constant. This is achieved by removing the columns and rows corresponding to rotor speed and angle from the state matrix.
$\operatorname{GEPS}(\mathrm{s})=\frac{\Delta T_{e}(s)}{\Delta v_{s}(s)}$

Where
$\Delta T_{e}(s)$ is the electrical torque and $\Delta v_{s}(s)$ is the terminal voltage .

## b. Design of Compensator Gc (s):

If a PSS is to provide pure damping torque at all frequencies, ideally, the phase characteristics of PSS must balance the phase characteristics of GEPS at all frequencies.

The following criteria are chosen to design the phase compensation for PSS:

- Compute the phase angle from the GEPs transfer function.
- Substitute the phase angle in the compensator transfer function

$$
\begin{equation*}
\mathrm{G}_{\mathrm{c}}(\mathrm{~s})=K_{p s s i} \frac{s T_{w i}\left(1+s T_{1 i}\right)\left(1+s T_{3 i}\right)}{\left(1+s T_{w i}\right)\left(1+s T_{2 i}\right)\left(1+s T_{4 i}\right)} \tag{3.61}
\end{equation*}
$$

An improvement in the damping torque component is reflected in an increase in the damping factor of the mode [10].

The proposed optimized PSS parameters set should minimize the objective function given below.
$J=\max \operatorname{Re}(\lambda i)$
Where, $\lambda i$ is the closed loop eigenvalue of the system. Damping ratio about 0.05 is considered to be sufficient. The parameters constraints are the PSS parameters settings. Thus, the tuning problem can be formulated as an optimization problem as follows

Min J
Subject to

$$
\begin{aligned}
K_{p s s i}^{\min } & \leq K_{p s s i} \leq K_{p s s i}^{\max } \\
T_{1 i}^{\min } & \leq T_{1 i} \leq T_{1 i}^{\max } \\
T_{2 i}^{\min } & \leq T_{2 i} \leq T_{2 i}^{\max }
\end{aligned}
$$

The objective of optimization is to maximize to damping ratio up to 0.05 . The parameters to be optimized ( $K_{p s s i}, T_{1 i}$, and $T_{2 i}$ ). Assuming $T_{1 i}=T_{3 i}, T_{2 i}=T_{4 i}$, in this study $T_{w}$ is set as 10 sec , and for convinces $T_{2}$ is set to 0.05 , the OPSS parameters settings ( $K_{p s s i}, T_{1 i}$ ) are to be computed. Where $\mathrm{i}=1,2 \ldots n$. Where n is number of machines [11].

To determine $T_{1}$ and $T_{2}$ the phase angle lead $\emptyset_{\mathrm{m}}$ to be provided by the compensator is related to $\mathrm{T}_{1}$ and $\mathrm{T}_{2}$ as
$\sin \left(\emptyset_{m}\right)=\frac{1-\alpha}{1+\alpha}$
Where $\alpha$ is the ratio between $\mathrm{T}_{2}$ and $\mathrm{T}_{1}$
$\alpha=\frac{T_{2}}{T_{1}}$
The center frequency at which it offers a phase lead $\emptyset_{\mathrm{m}}$ is given by
$\omega_{\mathrm{m}}=\frac{1}{\sqrt{\alpha} T_{1}}$
Typically $\frac{T_{1}}{T_{2}}$ must be less than 10 .

## c. Determination of compensator gain:

To set the gain of the PSS, the following criterion are generally employed

1. Based on the gain for instability

$$
K_{P S S}=\frac{K_{P S S}^{*}}{2}
$$

Where,
$K_{P S S}^{*}$ is the instability gain, which determine by trial and error.
2. Damping factor of the critical mode: Here, the gain is selected such that damping factor for the mode is above some typical value say 0.05 .
3. High frequency gain: The high frequency gain of PSS is given by $K_{\text {PSS }} \frac{T 1}{T 2}$ this should not be too high, as it would lead to noise amplification decreasing the effectiveness of a PSS [10].

### 3.5.3 Residues method:

Another way to obtain the best place to install PSS controllers is to use the residues of the open-loop transfer function of this device. The residues $\left(\mathrm{R}_{\mathrm{ijk}}\right)$ provide information about the observability and controllability of the PSS input-output set $\left(\mathrm{C}_{\mathrm{j}}-\mathrm{B}_{\mathrm{k}}\right)$ in a predefined eigenvalue ( $\lambda_{\mathrm{i}}$ ) according to the right $\left(\varphi_{\mathrm{i}}\right)$ and left ( $\psi_{\mathrm{i}}$ ) eigenvectors as described in equation (3.67).

$$
\begin{equation*}
R_{i j k}=C_{i} \varphi_{i} \psi_{i} B_{k} \tag{3.67}
\end{equation*}
$$

Therefore, the bigger the residues obtained the bigger the damping insert on the system. It is emphasized that the residues obtained from each possible installation of the PSS provide the best localization of these devices on the system.

Once the residues are obtained, this index gives the procedure to design and obtain the PSS parameters To design the controller, it is necessary to calculate the time constants $\mathrm{T}_{1}=\mathrm{T}_{3}, \mathrm{~T}_{2}=\mathrm{T}_{4}$ and the gain $\mathrm{K}_{\mathrm{pss}}$ in order to introduce the necessary phase compensation to the displacement of the eigenvalue of interests.

An eigenvalue ( $\lambda_{\mathrm{i}}$ ) can be displaced to the left semi-plane including PSS, in such way that the real part becomes more negative, increasing the damping of the oscillatory mode. Equation (3.68) shows the relation between the displacement of the eigenvalue and the correspondent residue.
$\Delta \lambda_{i}=R_{i j k} P S S\left(\lambda_{i}\right)=R_{i j k} K\left(p s s H\left(\lambda_{i}\right)\right)$
Thus, consider that the angle to be compensated by the controller is $\beta$, and $\omega_{\mathrm{i}}$ is the frequency in rad/s of the electromechanical mode of interests, and $\lambda_{\text {ides }}$ the desired position of the eigenvalue, then the equations below provides the procedure to obtain the parameters of the controller [12].
$\beta=180^{\circ}-\arg \left(R_{i j k}\right)$
$\alpha=\frac{1-\sin \left(\frac{\beta}{2}\right)}{1+\sin \left(\frac{\beta}{2}\right)}$
Where $\alpha$ is the ratio between $\mathrm{T}_{2}$ and $\mathrm{T}_{1}$
$\alpha=\frac{T_{1}}{T_{2}}$

$$
\begin{equation*}
K_{P S S}=\left|\frac{\lambda_{\text {ides }}-\lambda_{i}}{R_{i j k} H\left(\lambda_{i}\right)}\right| \tag{3.70}
\end{equation*}
$$

the center frequency at which it offers a phase lead is given by $\omega_{\mathrm{i}}=\frac{1}{\sqrt{\alpha} T_{2}}$

### 3.6 Interfacing PSS to the System Matrix:

To see the performance of the system with PSS, the system matrix needs to be modified to account for PSS.

The state space equations for two stages PSS can be written as

$$
\begin{align*}
& v_{1}^{\cdot}=\frac{1}{T_{w}}\left[\mathrm{~K}_{\mathrm{PSS}} \Delta \omega-v_{1}\right]  \tag{3.72}\\
& v_{2}^{\cdot}=\frac{1}{T_{2}}\left[\mathrm{~K}_{\mathrm{PSS}} \frac{\left(T_{2}-T_{1}\right)}{T_{2}} \Delta \omega-\frac{\left(T_{2}-T_{1}\right)}{T_{2}} v_{l}-v_{2}\right]  \tag{3.73}\\
& v_{s}^{\cdot}=\frac{1}{T_{4}}\left[\mathrm{~K}_{\mathrm{PSS}} \frac{T_{1}}{T_{2}} \frac{\left(T_{4}-T_{3}\right)}{T_{4}} \Delta \omega-\frac{T_{1}}{T_{2}} \frac{\left(T_{4}-T_{3}\right)}{T_{4}} v_{l}+\frac{\left(T_{4}-T_{3}\right)}{T_{4}} v_{2}-v_{s}\right]  \tag{3.74}\\
& E_{f d}^{\cdot}=-\frac{1}{T_{r}} E_{f d}+\frac{K_{r}}{T_{r}} v_{r e f}-\frac{K_{r}}{T_{r}} v_{k}+\frac{K_{r}}{T_{r}} v_{s} \tag{3.75}
\end{align*}
$$

$T w$ is usually chosen in range of $(10-20) \mathrm{sec}, T 1=T 3$, and $T 2=T 4$.
The linearized equations:

$$
\begin{align*}
& \Delta v_{1}^{\cdot}=\frac{1}{T_{w}}\left[\mathrm{~K}_{\mathrm{PSS}} \Delta \omega-\Delta v_{1}\right]  \tag{3.76}\\
& \Delta v_{2}^{\cdot}=\frac{1}{T_{2}}\left[\mathrm{~K}_{\mathrm{PSS}} \frac{\left(T_{2}-T_{1}\right)}{T_{2}} \Delta \omega-\frac{\left(T_{2}-T_{1}\right)}{T_{2}} \Delta v_{l}-\Delta v_{2}\right]  \tag{3.77}\\
& \Delta \dot{v_{s}}=\frac{1}{T_{4}}\left[\mathrm{~K}_{\mathrm{PSS}} \frac{T_{1}}{T_{2}} \frac{\left(T_{4}-T_{3}\right)}{T_{4}} \Delta \omega-\frac{T_{1}}{T_{2}} \frac{\left(T_{4}-T_{3}\right)}{T_{4}} \Delta v_{l}+\frac{\left(T_{4}-T_{3}\right)}{T_{4}} \Delta v_{2}-\Delta v_{s}\right]  \tag{3.78}\\
& \Delta E_{f d}^{\cdot}=-\frac{1}{T_{r}} \Delta E_{f d}+\frac{K_{r}}{T_{r}} \Delta v_{r e f}-\frac{K_{r}}{T_{r}} \Delta v_{k}+\frac{K_{r}}{T_{r}} \Delta v_{s} \tag{3.79}
\end{align*}
$$

### 3.7 State space modal:



## CHAPTER FOUR SIMULATION AND RESULTS

### 4.1 Background

In the previous chapter the power system stabilizer (PSS) was designed. Eigenvalues analysis and time-domain simulation have been represented for the multi-machines. The results are obtained through simulations of the system response for different operating conditions: the system with static AVR and the system with PSS for the following different cases:
i. Switch off the Load at bus 3 for part of second and then return to service.
ii. A three-phase short circuit is applied at bus 16 and, then cleared after specific time.
iii. Tripping line5-6 and a three phase short circuit is applied at bus 16 at the same time.

### 4.2 Case study:

IEEE system with 10 machines, 39-bus-bars power system has been used to demonstrate the modal analysis of power system. Generator (2) is a slack bus the other bus-bars are PV bus-bars and the total generation is 6140.811 MW , 198.2518 Mvar. Each M/C had rated generation 10 kV . The network had total load of 6097.1MW, 1408.9 Mvar, and the total number of transformers are twelve 3- phases transformers.

The single line diagram of the system is shown in Figure (4.1). Analysis of the third order model of the $10 \mathrm{M} / \mathrm{Cs}$ under normal condition is carried out by NEPLAN software.


Figure 4.1: IEEE 39 - 10 machins bus.

### 4.2.1 The system without PSS:

## a. eigenvalue analysis:

The stability in small nonlinear system is given by roots of the characteristic equation of the eigen values $\lambda=\sigma+j \omega$.

The system is stable if all eign values have negative real parts, if any of eignvalues have postive real part that mean the system is un stable and if the eignvalues have real part equal to zero it is not possible to judge.

Simulation are carried on case study without PSS ,result of shown in table (4.1) .

Table (4.1): The Eigenvalues of The System With Out PSS

|  | Eigenvalue Real Part | Eigenvalue <br> Imaginary Part | Damping <br> Ratio | Frequency | Nature of Mode |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1/s | 1/s | - | Hz | - |
| 1 | -6.650589e-005 | 0 | 1 | 0 | Non swing |
| 2 | -0.055 | 0.063 | 0.455 | 0.2 | Inter area |
| 3 | -0.076 | 0 | 1 | 0 | Non swing |
| 4 | -0.125 | 0 | 1 | 0 | Non swing |
| 5 | -0.167 | 0 | 1 | 0 | Non swing |
| 6 | -0.176 | 0 | 1 | 0 | Non swing |
| 7 | -0.199 | 0 | 1 | 0 | Non swing |
| 8 | -0.208 | 0 | 1 | 0 | Non swing |
| 9 | -0.23 | 0 | 1 | 0 | Non swing |
| 10 | -0.238 | 7.449 | 0.032 | 1.186 | Local |
| 11 | -0.244 | 6.297 | 0.039 | 1.002 | Local |
| 12 | -0.245 | 0 | 1 | 0 | Non swing |
| 13 | -0.275 | 0 | 1 | 0 | Non swing |
| 14 | -0.28 | 3.801 | 0.073 | 0.605 | Inter area |
| 15 | -0.288 | 5.894 | 0.049 | 0.938 | Local |
| 16 | -0.293 | 6.876 | 0.043 | 1.094 | Local |
| 17 | -0.358 | 8.614 | 0.042 | 1.371 | Local |
| 18 | -0.411 | 8.841 | 0.046 | 1.407 | Local |
| 19 | -0.736 | 8.666 | 0.085 | 1.379 | Local |
| 20 | -0.906 | 0 | 1 | 0 | Non swing |
| 21 | -0.964 | 0 | 1 | 0 | Non swing |
| 22 | -0.978 | 0 | 1 | 0 | Non swing |


| 23 | -0.979 | 7.47 | 0.13 | 1.189 | Local |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 24 | -1.079 | 0 | 1 | 0 | Non swing |
| 25 | -1.346 | 0 | 1 | 0 | Non swing |
| 26 | -1.483 | 0 | 1 | 0 | Non swing |
| 27 | -5.103 | 0 | 1 | 0 | Non swing |
| 28 | -5.586 | 0 | 1 | 0 | Non swing |
| 29 | -6.8 | 0 | 1 | 0 | Non swing |
| 30 | -21.208 | 0 | 1 | 0 | Non swing |

From table (4.1), All of the real parts are negative. The system is poorly damped there are two modes (inter area mode of oscillation having damping ratio less than 5\%, and local mode of oscillations)

Two states have an inter area nature of mode and 8 states have a local nature of mode. Figure (4.2), show the eigenvalues from table (4.1) which previously in s plane.


Figure 4.2: Eignvalues real part( $\sigma$ ) and imagenary part (j $\omega$ ).

From figur (4.2), describe that most of eigenvalues are located close to Y axis in s plane which mean that the system is poorly damped.

## b. Time domain simulation

To verify the EVs analysis, following cases are considered:

## i. small disturbance:

A switch off load at bus 3 (load out), at $\mathrm{t}=0.5 \mathrm{sec}$. and load return to serves at $\mathrm{t}=0.6$.

## ii. large disturbance:

1. A three phase short circuit is applied at bus 16 , at $\mathrm{t}=0.5$ sec. and cleared at $\mathrm{t}=0.6$.
2. A three phase short circuit is applied at bus 16, tripping line $5-6$ at $\mathrm{t}=0.5 \mathrm{sec}$. and the fault cleared at $\mathrm{t}=0.6 \mathrm{sec}$.

First: small disturbance (switch off bus 3 load):
The time domain simulation represented as follow:


Figure 4.3: response of field voltage.


Figure 4.4: response of rotor speed.


Figure 4.5: response of output active power.


Figure 4.6: response of rotor angle.
From Figures (4.3) (4.4) (4.5) (4.6) observed that the effect of small disturbance (switch off load at bus 3) make small changes of oscillation and then the system return back to stable condition after few cycles.

Second: large disturbance

1. Three -phase to ground short circuit fault at bus 16 at 0.5 sec and then the fault removing at 0.6 sec .

The time domain simulation represented as follow:


Figure 4.7: response of field voltage.


Figure 4.8: response of rotor speed.


Figure 4.9: response of output active power.


Figure 4.10: response of rotor angle.
From figures (4.7) (4.8) (4.9) (4.10) are observed that the effect of large disturbance (Three -phase to ground short circuit fault at bus 16 at 0.5 sec and then the fault cleared at 0.6 sec ) make the system oscillatory.
2. A three phase short circuit is applied at bus 16 at $\mathrm{t}=0.5 \mathrm{sec}$, and tripping line 5-6 at $\mathrm{t}=0.55 \mathrm{sec}$ the fault cleared at $\mathrm{t}=0.6 \mathrm{sec}$.

The time domain simulation represented as follow:


Figure 4.11: response of field voltage.

$\longrightarrow W$ G08 W G10 $\rightleftharpoons W$ G06 W G09 $\rightleftharpoons W$ G02 $\rightleftharpoons W$ G01
$\longrightarrow W$ G03 $\rightleftharpoons W$ G05 $\rightleftharpoons W$ G07
Figure 4.12: response of rotor speed.


Figure 4.13: response of output active power


Figure 4.14: response of rotor angle.
From figures (4.11) (4.12) (4.13) (4.14) observed that the effect of large disturbance (Three -phase to ground short circuit fault at bus 16 at 0.5 sec and tripping line 5-6 at 0.55 sec ). Increases the oscillations for the system and the system may be unstable.

In power system, to Selection optimal location of PSS participation factors give good indications for power system stabilizers placement, discussed in chapter (3).

There are different oscillations frequencies but we interested here in a swing modes (local modes and inter area mode of oscillations (0.2-2 HZ)). The eigenvectors for swing modes have been determined by using slip-participation matrix; which represent the nature of oscillatory modes.

Table (4.2): the contribution of generators in inter area mode.

| Participated generator | $\Delta \delta$ | $\Delta \omega$ |
| :---: | :---: | :---: |
| G01 | 0.747 | 0.879 |
| G09 | 0.245 | 0.236 |
| G05 | 0.154 | 0.16 |
| G04 | 0.132 | 0.136 |
| G06 | 0.059 | 0.059 |

From table (4.2) the contributed generators in inter area mode are arranged in a sending form, the contribution greater than $5 \%$ has been assumed that it high contribution in the modes, then PSS's are added.

### 4.2.2 system with PSS:

## a. eigenvalue analysis:

the eigenvalues of the system with PSS shown below
Table (4.3): The Eigenvalues of The System With PSS

|  | eigenvalue <br> real part | eigenvalue <br> imaginary part | damping <br> ratio | frequency | Nature of <br> Mode |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathbf{1 / s}$ | $\mathbf{1 / s}$ | - | $\mathbf{H z}$ | - |
| 1 | -0.0001198 | 0.001 | 0.108 | 0 | Non swing |
| 2 | -0.1 | 0 | 1 | 0 | Non swing |
| 3 | -0.1 | 0 | 1 | 0 | Non swing |
| 4 | -0.1 | 0 | 1 | 0 | Non swing |
| 5 | -0.1 | 0 | 1 | 0 | Non swing |
| 6 | -0.147 | 7.422 | 0.05 | 1.181 | Local |
| 7 | -0.156 | 0 | 1 | 0 | Non swing |
| 8 | -0.359 | 6.428 | 0.056 | 1.023 | Local |
| 9 | -0.413 | 3.811 | 0.108 | 0.607 | Inter area |
| 10 | -0.458 | 0.675 | 0.562 | 0.107 | Non swing |
| 11 | -0.482 | 0.609 | 0.62 | 0.097 | Non swing |
| 12 | -0.486 | 0.639 | 0.605 | 0.102 | Non swing |
| 13 | -0.588 | 0.662 | 0.664 | 0.105 | Non swing |
| 14 | -0.69 | 0.808 | 0.65 | 0.129 | Non swing |
| 15 | -0.722 | 8.734 | 0.082 | 1.39 | Local |
| 16 | -0.809 | 0.821 | 0.702 | 0.131 | Non swing |
| 17 | -0.846 | 0.772 | 0.738 | 0.123 | Non swing |
| 18 | -0.906 | 0 | 1 | 0 | Non swing |


| 19 | -0.959 | 0 | 1 | 0 | Non swing |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 20 | -0.976 | 0 | 1 | 0 | Non swing |
| 21 | -1.024 | 8.587 | 0.118 | 1.367 | Local |
| 22 | -1.046 | 0.227 | 0.977 | 0.036 | Non swing |
| 23 | -1.099 | 0 | 1 | 0 | Non swing |
| 24 | -1.149 | 7.299 | 0.155 | 1.162 | Local |
| 25 | -1.173 | 6.563 | 0.176 | 1.045 | Local |
| 26 | -1.22 | 0.909 | 0.802 | 0.145 | Non swing |
| 27 | -1.459 | 0 | 1 | 0 | Non swing |
| 28 | -1.479 | 0.511 | 0.945 | 0.081 | Non swing |
| 29 | -1.516 | 0 | 1 | 0 | Non swing |
| 30 | -1.89 | 9.989 | 0.186 | 1.59 | Local |
| 31 | -2.093 | 1.705 | 0.775 | 0.271 | Inter area |
| 32 | -2.287 | 7.274 | 0.3 | 1.158 | Non swing |
| 33 | -3.49 | 3.383 | 0.718 | 0.538 | Inter area |
| 34 | -4.327 | 1.6 | 0.938 | 0.255 | Inter area |
| 35 | -5.531 | 0 | 1 | 0 | Non swing |
| 36 | -5.792 | 0 | 1 | 0 | Non swing |
| 37 | -5.877 | 3.742 | 0.844 | 0.596 | Inter area |
| 38 | -7.182 | 0 | 1 | 0 | Non swing |
| 39 | -12.673 | 6.678 | 0.885 | 1.063 | Local |
| 40 | -21.154 | 0 | 1 | 0 | Non swing |
| 41 | -59.421 | 0 | 1 | 0 | Non swing |
| 42 | -61.092 | 0 | 1 | 0 | Non swing |
| 43 | -63.074 | 0 | 1 | 0 | Non swing |
| 44 | -63.398 | 0 | 1 | 0 | Non swing |
| 45 | -63.864 | 0 | 1 | 0 | Non swing |
| 46 | -63.924 | 0 | 1 | 0 | Non swing |
| 47 | -64.655 | 0 | 1 | 0 | Non swing |


| 48 | -64.675 | 0 | 1 | 0 | Non swing |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 49 | -65.975 | 0 | 1 | 0 | Non swing |
| 50 | -66.407 | 0 | 1 | 0 | Non swing |
| 51 | -100.902 | 0 | 1 | 0 | Non swing |
| 52 | -101.293 | 0 | 1 | 0 | Non swing |
| 53 | -101.309 | 0 | 1 | 0 | Non swing |
| 54 | -101.316 | 0 | 1 | 0 | Non swing |
| 55 | -101.898 | 0 | 1 | 0 | Non swing |
| 56 | -102.118 | 0 | 1 | 0 | Non swing |
| 57 | -102.209 | 0 | 1 | 0 | Non swing |
| 58 | -102.655 | 0 | 1 | 0 | Non swing |
| 59 | -103.738 | 0 | 1 | 0 | Non swing |
| 60 | -104.568 | 0 | 1 | 0 | Non swing |

From table (4.2) note that eigenvalues has been ranking, the PSS's raise the damping ratio of inter area and local modes above the $5 \%$.

From table (4.2), damping ratios has been increased for local as well inter-area modes, and eigenvalues shifted to the left of s plane.


Figure 4.15: Eignvalues real part( $\sigma$ ) and imagenary part (j $\omega$ ).

## b. Time domain simulation

First: small disturbance (switch off bus 3 load):
The time domain simulation represented as follow:


Figure 4.16: response of field voltage.


Figure 4.17: response of rotor speed.


Figure 4.18: response of output active power.


Figure 4.19: response of rotor angle.
From figures (4.16) (4.17) (4.18) (4.19), note that the effect of small disturbance (switch off load at bus 3) with PSS the oscillations decreases also maximum over shot decreases and then the system return back to stable condition after few cycles.

Second: large disturbance:

1. Three -phase to ground short circuit fault at bus 16 at 0.5 sec and then the fault removing at 0.6 sec .

The time domain simulation represented as follow:


Figure 4.20: response of field voltage.


Figure 4.21: response of rotor speed.


Figure 4.22: response of output active power.


Figure 4.23: response of rotor angle.
From figures (4.20) (4.21) (4.22) (4.23), note that the effect of PSS at large disturbance (Three -phase to ground short circuit fault at bus 16 at 0.5 sec and then the fault removing at 0.6 sec ) with PSS the oscillations decreases also maximum over shot decreases and the system return to stable condition after few cycles.
2. A three phase short circuit is applied at bus 16 , at $\mathrm{t}=0.5 \mathrm{sec}$. and tripping line $5-6$ at 0.55 sec and clear fault at $\mathrm{t}=0.6 \mathrm{sec}$.

The time domain simulation represented as follow:


Figure 4.24: response of field voltage.


Figure 4.25: response of rotor speed.


Figure 4.26: response of output active power.


Figure 4.27: response of rotor angle.
From figures (4.24) (4.25) (4.26) (4.27), note that the effect of PSS at large disturbance (Three -phase to ground short circuit fault at bus 16 at 0.5 sec and then the fault removing at 0.6 sec and tripping line $5-6$ at 0.55 sec ) the oscillations decreases, also maximum over shot decreases and the system return to stable conditions after few cycles.

The settling time for all cases with and without PSS show in table (4.4) below.

Table (4.4): The settling time for cases.

| No | the case | Variables | settling time (sec) |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  |  | without PSS | with PSS |
| 1 | A switch off bus element at bus 3 (load out) | $\Delta \delta$ | 9 | 7 |
|  |  | $\Delta \omega$ | 11.5 | 9 |
|  |  | $\mathrm{E}_{\mathrm{fd}}$ | 7.5 | 6.5 |
|  |  | P | 11.5 | 7 |
| 2 | A three phase short circuit at bus 16 | $\Delta \delta$ | oscillatory | 13 |
|  |  | $\Delta \omega$ | oscillatory | 9 |
|  |  | $\mathrm{E}_{\mathrm{fd}}$ | oscillatory | 11 |
|  |  | P | oscillatory | 13 |
| 3 | A three phase short circuit at bus 16 . and tripping line 5-6 | $\Delta \delta$ | oscillatory | 14 |
|  |  | $\Delta \omega$ | oscillatory | 13 |
|  |  | $\mathrm{E}_{\mathrm{fd}}$ | oscillatory | 14 |
|  |  | P | oscillatory | 13 |

Note that the settling time with out PSS at the small distubance state the time was decrease ,at large disturbance without PSS the sysem has been oscilltory and at case 3 the system would be unstalble after using PSS the settling time was decrease and the system return to stable condition after few cycles as shown as upper table(4.4).

## CHAPTER FIVE CONCLUSION AND RECOMMENDATIONS

### 5.1 CONCLUSION:

The problem of the power system stability (rotor angle stability) of multimachines system ( $10 \mathrm{~m} / \mathrm{c}-39$ bus) has been addressed in this thesis by using eigenvalue method, and time domain simulation. The system in base case (the system without PSS) is a stable at normal condition (no disturbance occurs) but have poor damping ratio. To provide adequate damping ratio, the power system stabilizer has been designed by using phase compensation technique.

Before design of PSS the optimal location of PSS has been specified according to participation factor, which represent the contribution of machines in modes. After that the effect of PSS has been discussed for small and large disturbances.

Finally, the power system stabilizer (PSS) is a cost effective way of improving the damping of electromechanical oscillations of rotor and return the stability to the system. Also it improves the power transfer capability of transmission lines.

### 5.2 RECOMMENDATIONS:

In this thesis design and optimal location of PSS to improve dynamic stability were determined, speed deviation was taken as the input signal; other investigations:

- Change the input signal to PSS such as terminal bus frequency or electrical power output.
- Improve the stability by using another controller addition to PSS such as flexible AC transmission system (FACTS) family.
- Use other methods to specify the optimal location of PSS such as Genetic Algorithm (GA) method.
- Use other methods to appropriate tuning of PSS parameters such as Particle Swarm Optimization (PSO) technique or MultiObjective Honey Bee Mating Optimization (MOHBMO).


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## APPENDIX ( A )

## A. 1 Generator inertia data

| Unit No. | $\mathrm{M}=2 * \mathrm{H}$ |
| :---: | :---: |
| 1 | 2 *500.0/(120 ${ }^{\text {r }}$ |
| 2 | 2* $30.3 /(120 \pi)$ |
| 3 | 2* $35.8 /(120 \pi)$ |
| 4 | 2 * $28.6 /(120 \pi)$ |
| 5 | 2* $26.0 /(120 \pi)$ |
| 6 | 2* $34.8 /(120 \pi)$ |
| 7 | 2 * $26.4 /(120 \pi)$ |
| 8 | 2* $24.3 /(120 \pi)$ |
| 9 | 2* $34.5 /(120 \pi)$ |
| 10 | 2* $42.0 /(120 \pi)$ |

## A. 2 Generator data

| Unit No. | H | Ra | x́d | x́q | xd | xq | T́do | T́qo | x |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 500 | 0 | 0.006 | 0.008 | 0.02 | 0.019 | 7 | 0.7 | 0.003 |
| 2 | 30.3 | 0 | 0.0697 | 0.17 | 0.295 | 0.282 | 6.56 | 1.5 | 0.035 |
| 3 | 35.8 | 0 | 0.0531 | 0.0876 | 0.2495 | 0.237 | 5.7 | 1.5 | 0.0304 |
| 4 | 28.6 | 0 | 0.0436 | 0.166 | 0.262 | 0.258 | 5.69 | 1.5 | 0.0295 |
| 5 | 26 | 0 | 0.132 | 0.166 | 0.67 | 0.62 | 5.4 | 0.44 | 0.054 |
| 6 | 34.8 | 0 | 0.05 | 0.0814 | 0.254 | 0.241 | 7.3 | 0.4 | 0.0224 |
| 7 | 26.4 | 0 | 0.049 | 0.186 | 0.295 | 0.292 | 5.66 | 1.5 | 0.0322 |
| 8 | 24.3 | 0 | 0.057 | 0.0911 | 0.29 | 0.28 | 6.7 | 0.41 | 0.028 |
| 9 | 34.5 | 0 | 0.057 | 0.0587 | 0.2106 | 0.205 | 4.79 | 1.96 | 0.0298 |
| 10 | 42 | 0 | 0.031 | 0.008 | 0.1 | 0.069 | 10.2 | 0 | 0.0125 |

## A. 3 Bus Load

Active and reactive power draws for all loads at initial voltage

| Bus no | $\mathrm{P}[\mathrm{pu}]$ | Q [pu] |
| :---: | :---: | :---: |
| 1 | 0.000 | 0.000 |
| 2 | 0.000 | 0.000 |
| 3 | 3.220 | 0.024 |
| 4 | 5.000 | 1.840 |
| 5 | 0.000 | 0.000 |
| 6 | 0.000 | 0.000 |
| 7 | 2.338 | 0.840 |
| 8 | 5.220 | 1.760 |
| 9 | 0.000 | 0.000 |
| 10 | 0.000 | 0.000 |
| 11 | 0.000 | 0.000 |
| 12 | 0.075 | 0.880 |
| 13 | 0.000 | 0.000 |
| 14 | 0.000 | 0.000 |
| 15 | 3.200 | 1.530 |
| 16 | 3.290 | 0.323 |
| 17 | 0.000 | 0.000 |
| 18 | 1.580 | 0.300 |
| 19 | 0.000 | 0.000 |
| 20 | 6.280 | 1.030 |
| 21 | 2.740 | 1.150 |
| 22 | 0.000 | 0.000 |
| 23 | 2.475 | 0.846 |
| 24 | 3.086 | -0.920 |
| 25 | 2.240 | 0.472 |
| 26 | 1.390 | 0.170 |
| 27 | 2.810 | 0.755 |
| 28 | 2.060 | 0.276 |
| 29 | 2.835 | 0.269 |
| 31 | 0.092 | 0.046 |
| 39 | 11.040 | 2.500 |

## A. 4 Network data:

| Line Data |  |  |  |  | Transformer Tap |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| From Bus | To Bus | R | X | B | Magnitude | Angle |
| 1 | 2 | 0.0035 | 0.0411 | 0.6987 | - | - |
| 1 | 39 | 0.001 | 0.025 | 0.75 | - | - |
| 2 | 3 | 0.0013 | 0.0151 | 0.2572 | - | - |
| 2 | 25 | 0.007 | 0.0086 | 0.146 | - | - |
| 3 | 4 | 0.0013 | 0.0213 | 0.2214 | - | - |
| 3 | 18 | 0.0011 | 0.0133 | 0.2138 | - | - |
| 4 | 5 | 0.0008 | 0.0128 | 0.1342 | - | - |
| 4 | 14 | 0.0008 | 0.0129 | 0.1382 | - | - |
| 5 | 6 | 0.0002 | 0.0026 | 0.0434 | - | - |
| 5 | 8 | 0.0008 | 0.0112 | 0.1476 | - | - |
| 6 | 7 | 0.0006 | 0.0092 | 0.113 | - | - |
| 6 | 11 | 0.0007 | 0.0082 | 0.1389 | - | - |
| 7 | 8 | 0.0004 | 0.0046 | 0.078 | - | - |
| 8 | 9 | 0.0023 | 0.0363 | 0.3804 | - | - |
| 9 | 39 | 0.001 | 0.025 | 1.2 | - | - |
| 10 | 11 | 0.0004 | 0.0043 | 0.0729 | - | - |
| 10 | 13 | 0.0004 | 0.0043 | 0.0729 | - | - |
| 13 | 14 | 0.0009 | 0.0101 | 0.1723 | - | - |
| 14 | 15 | 0.0018 | 0.0217 | 0.366 | - | - |
| 15 | 16 | 0.0009 | 0.0094 | 0.171 | - | - |
| 16 | 17 | 0.0007 | 0.0089 | 0.1342 | - | - |
| 16 | 19 | 0.0016 | 0.0195 | 0.304 | - | - |
| 16 | 21 | 0.0008 | 0.0135 | 0.2548 | - | - |
| 16 | 24 | 0.0003 | 0.0059 | 0.068 | - | - |
| 17 | 18 | 0.0007 | 0.0082 | 0.1319 | - | - |
| 17 | 27 | 0.0013 | 0.0173 | 0.3216 | - | - |
| 21 | 22 | 0.0008 | 0.014 | 0.2565 | - | - |
| 22 | 23 | 0.0006 | 0.0096 | 0.1846 | - | - |
| 23 | 24 | 0.0022 | 0.035 | 0.361 | - | - |
| 25 | 26 | 0.0032 | 0.0323 | 0.513 | - | - |
| 26 | 27 | 0.0014 | 0.0147 | 0.2396 | - | - |
| 26 | 28 | 0.0043 | 0.0474 | 0.7802 | - | - |
| 26 | 29 | 0.0057 | 0.0625 | 1.029 | - | - |
| 28 | 29 | 0.0014 | 0.0151 | 0.249 | - | - |
| 12 | 11 | 0.0016 | 0.0435 | 0 | 1.006 | 0 |
| 12 | 13 | 0.0016 | 0.0435 | 0 | 1.006 | 0 |
| 6 | 31 | 0 | 0.025 | 0 | 1.07 | 0 |
| 10 | 32 | 0 | 0.02 | 0 | 1.07 | 0 |
| 19 | 33 | 0.0007 | 0.0142 | 0 | 1.07 | 0 |
| 20 | 34 | 0.0009 | 0.018 | 0 | 1.009 | 0 |
| 22 | 35 | 0 | 0.0143 | 0 | 1.025 | 0 |
| 23 | 36 | 0.0005 | 0.0272 | 0 | 1 | 0 |
| 25 | 37 | 0.0006 | 0.0232 | 0 | 1.025 | 0 |
| 2 | 30 | 0 | 0.0181 | 0 | 1.025 | 0 |
| 29 | 38 | 0.0008 | 0.0156 | 0 | 1.025 | 0 |
| 19 | 20 | 0.0007 | 0.0138 | 0 | 1.06 | 0 |

## APPENDIX ( B )

## NEPLAN Data

## B. 1 Single-time constant exciter with simple limits:

| Gen.no. | TR | KA | TA | TB | TC | EFDMIN | EFDMAX |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0.1 | 200 | 0.015 | 10 | 1 | -5.0 | 5.0 |
| 2 | 0.1 | 200 | 0.015 | 10 | 1 | -5.0 | 5.0 |
| 3 | 0.1 | 200 | 0.015 | 10 | 1 | -5.0 | 5.0 |
| 4 | 0.1 | 200 | 0.015 | 10 | 1 | -5.0 | 5.0 |
| 5 | 0.1 | 200 | 0.015 | 10 | 1 | -5.0 | 5.0 |
| 6 | 0.1 | 200 | 0.015 | 10 | 1 | -5.0 | 5.0 |
| 7 | 0.1 | 200 | 0.015 | 10 | 1 | -5.0 | 5.0 |
| 8 | 0.1 | 200 | 0.015 | 10 | 1 | -5.0 | 5.0 |
| 9 | 0.1 | 200 | 0.015 | 10 | 1 | -5.0 | 5.0 |
| 10 | 0.1 | 200 | 0.015 | 10 | 1 | -5.0 | 5.0 |

## B.2 System Stabilizer conventional PSS:

| Gen.no. | KPSS | TW | T1 | T2 | VPSS MAX | VPSSMIN |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 1 | 10 | 5 | 0.6 | 2.0 | -2.0 |
| 4 | 2 | 10 | 1 | 0.1 | 2.0 | -2.0 |
| 5 | 1 | 10 | 1.5 | 0.2 | 2.0 | -2.0 |
| 6 | 4 | 10 | 0.5 | 0.1 | 2.0 | -2.0 |
| 9 | 2 | 10 | 1 | 0.5 | 2.0 | -2.0 |

APPENDEX(C)


Figure: IEEE 39-10 machins bus with PSS

