CHAPTER ONE

INTRODUCTION

1.1 Preface

Wireless technology has become the primary enabler of mobility and ubiquitous network access over the past decade. The demand for higher peak data rates and anytime, anywhere connectivity has been driving the rapid developments in cellular technology. There is a need to serve a host of data-intensive applications dominated by video streaming, real time services like navigation, and graphics-heavy social media interfaces on hand-held devices.

Multiple-In Multiple-Out (MIMO) technology is now being introduced in modern wireless broadband standards e.g., Long Term Evolution Advanced (LTE-Advanced). According to 3GPP LTE standard, LTE permits for up to 8 antennas at the base station [4]. The goal of wireless communication improvement is to provide a high data rate for each user. At present, the latest wireless technology uses MU-MIMO system for that reason. Theoretically, increasing the number of antennas at the transmitter or receiver can improve the performance of the system in terms of data throughput and link reliability. Besides improving the data throughput and link reliability, MU-MIMO enables to save the transmitter energy, owing to the array gain [15]. In multiuser systems, the benefits are more attractive because such systems offer the possibility to transmit simultaneously to several users [4]. With a multiuser MIMO (MU-MIMO) system, the base station is equipped with multiple antennas and serves several users. Commonly, the base station communicates with many users through orthogonal channels. More precisely, the base station communicates with each user in a separate time and frequency resource [15]. However, the higher data rate can be achieved if the base station communicates with the user in the same time-frequency resources. The main challenge of this system is inter user interference, which significantly reduces the system performance. In the downlink, dirty-paper coding can be used to reduce the effect of inter user interference [9], [20].When the number of antennas increases, the random channel vectors between the users and base station become nearly orthogonal [4].Other important advantage of massive MU-MIMO systems is that they enable us to reduce the transmitted power. On the uplink, reducing the power of the terminals will save their battery life. On the downlink, much of the electrical base station power is spent by power amplifiers and associated circuits and cooling systems [24]. Hence reducing the RF power would help in cutting the electricity power consumption of the base station.

Here, I study the analysis of two linear Pre-coding techniques for single cell downlink massive MIMO downlink system. By consider the system performance when the number of antennas are large than number of users. I study the system performance in terms of achievable sum rate at perfect channel. In addition give the performance comparisons among linear pre-coders: Zero Forcing (ZF), and Matched Filter (MF) pre-coding.

1.2 Problem Statement

The main problem facing wireless communications systems is interference .In downlink channel (broadcast channel) for massive MIMO system; the problem of Multi user Interference (MUI) received widespread attention. MUI is interference result from other user in same cell, which lead to reduce achievable sum rate for cellular communication system.

1.3 Proposed Solution

This research will evaluate and analyze the performance of Matched Filter (MF) and Zero Forcing (ZF) pre-coding technique , through comparing the sum rate of these methods under different configureuration scenarios.

1.4 Aim and Objectives

The overall aim of this study is to simulate and analyze two linear pre-coding performance (ZF and MF) for massive MIMO system in downlink specific objection includes :

- Simulate the ZF and MF technique .
- Compare the achievable sum rate versus number of transmitting antenna .
- Compare the achievable sum rate versus number of users.

1.5 Scope of Work

This thesis considering the comparison and analysis of two linear pre-coding technique (ZF and MF) for massive MIMO downlink system .

1.6 Methodology

The study considered four different scenario by using the mathematical equations to calculating the achievable sum rate of each pre-coding technique in a single-cell downlink massive MIMO system at different value of power under the same assumptions (perfect channel state information), some of scenarios accomplished by taking different numbers of antennas, the other scenarios assumed a different number of users. All this is done with respect to the normalization methods. Finally a comparative analysis of results for the two scenarios of normalization methods presented in term of achievable sum rate overall pre-coding techniques by implementing software MATLAB code of ZF and MF pre-coding techniques

1.7 Thesis Outline

The remaining part of the thesis is gives organized as follow :

- Chapter one is composed introduction to discuss the research preface, problem statement, aim and objectives, and the scope work of research.
- Chapter two is literature review is divided into background and related works.
- Chapter three explains the (ZF and MF) linear pre-coding techniques , also simulation is done by using Mat lab software and the important parameters that have been used in system design and simulation are clearly stated in this Chapter .
- Chapter four presents the simulation and analysis of results,
- Chapter five layout the conclusions for the whole thesis ,It also provides suggestion for future work where the proposed system can be modified to enable the simulation to be more practical and continuously.

CHAPTER TWO
LITERATURE REVIEW

2.1 Overview

This chapter constituting of two parts, first part is background which covers the MIMO technology and advantages of MIMO technology , also it cover MIMO system model and massive MIMO system , part two cross the related work .

2.2 Background

In recent decades wireless networks have changed dramatically in personal communications. Now, many people can connect to the Internet and share large amounts of data through the cellular network. Popular applications (social media, video streaming and exchange applications) produce increased traffic volume and advanced mobile terminals are able to handle data from the most complex applications. The increase in demand for high data transmission is expected to accelerate in the coming years. In the late 1990s, researchers proposed the use of multi-input multi-output (MIMO) systems as a means of providing wireless systems with more spectral efficiency. Multi-input and multi-output (MIMO) systems is considered as the main component of future wireless communications systems, because of their promising performance and bandwidth , MIMO systems have multiple antennas both in the transmitter and on the receiver side [31][40].The MIMO channel is linearly increased capacity with minimum antennas at the transmitter and receiver when channel knowledge is available on both sides of the communication link. In addition, MIMO systems provide improved diversification leading to more reliable communication systems [25]. The LTE Release 10, also known as LTE-Advanced, targets the achievement of 1 Gb/s for downlink and 500 Mb/s for uplink. One of the key enabling features to meet this requirement is the downlink Multiuser (MU-MIMO). MU-MIMO systems are particularly important because they have the potential to combine the high throughput achievable with the MIMO processor with the benefits of space division multiple access (SDMA).In the downlink scenario, Base Station is equipped with multiple antennas and is simultaneously transmitting to a group of users . The performance of a given user may significantly degrade due to the interference from other users. To tackle this problem, interference reduction or cancellation techniques should be used. These techniques are complicated and have high computational complexity .To achieve a high spatial multiplexing gain, the BS needs to process the received signals coherently. This requires accurate and timely acquisition of channel state information (CSI). This can be challenging, especially in high mobility scenarios.The advantages offered by MIMO systems are based on two fundamental gains ; spatial diversity (SD) and spatial multiplicity (SM).

2.2.1 Advantages of MIMO Technology

MIMO technology have a number of advantage like spatial diversity gain, spatial multiplexing, array gain, interference reduction and avoidance

2.2.1.1 Array Gain

Array gain indicates improvement of SNR at the receiver compared to traditional systems with one transmit and one receive antenna. The said improvement can be achieved with correct processing of the signals at the transmit or at the receive side, so the transmitted signals are coherently combined at the receiver. To achieve array gain at the transmitter antenna array, the channel state information (CSI) has to be known at the transmit side whereas for the exploitation of antenna array gain at the receiver, the channel has to be known at the receive side. Receive array gain is achieved regardless of the correlation between the antennas.

2.2.1.2 Spatial Diversity

Spatial diversity used in this narrower sense often refers to transmit and receive diversity. These two methodologies are used to provide improvements in the signal to noise ratio and they are characterized by improving the reliability of the system with respect to the various forms of fading. As a result of use of multiple antennas, MIMO wireless technology is able to considerably increase the capacity of a given channel while still obeying Shannon's law. By increasing the number of receive and transmit antennas it is possible to linearly increase the throughput of the channel with every pair of antennas added to the system. This makes MIMO wireless technology one of the most important wireless techniques to be employed in recent years. As spectral bandwidth is becoming an ever more valuable commodity for radio communications systems, techniques are needed to use the available bandwidth more efficiently.

2.2.1.3 Spatial Multiplexing

Spatial multiplexing is not intended to make the transmission more robust; rather it increases the data rate. To do this, data is divided into separate streams; the streams are transmitted independently via separate antennas. Because MIMO transmits via the same channel In spatial multiplexing MIMO systems, independent data streams are transmitted through different antennas which maximize the data throughput of the MIMO systems. Divide input data stream into as many independent data streams as there are transmit antennas. Then the signals are modulated and simultaneously sent through all M transmit antennas. In the case of spatial multiplexing, the processing at the transmitter is quite simple but the processing at the receiver can be very complex, depending on the complexity of the receiver decoding algorithm. The performance of an MIMO spatial multiplexing system depends highly on the receiver quality, since all N receive antennas receive signals from all M transmit antennas and they have to be separated sufficiently.

2.2.1.4 Interference Reduction and Avoidance

Interference in the wireless channel appears due to frequency reuse. It decreases the performance of the communication systems. Using multiple antennas, it is possible to separate the signals with different spatial signature and thus decrease inter-channel interference. When traveling through wireless medium, each signal is marked with the path that it has traveled. For the interference reduction, it is necessary to know the CSI. At the transmit side, the transmitted signal can be directed to the chosen users. With this, the interferences to the other users are decreased, more efficient frequency planning is thus possible, which, in turn, increases the capacity of cellular systems. This technique is also called beam forming and is a very common spatial processing technique. A beam former can be seen as a spatial filter that separates the desired signal from interfering signals given that all the signals share the same frequency band and originate from different spatial locations. It essentially weighs and sums the signals from different antennas in the antenna array to optimize the quality of the desired signal. In addition to interference rejection and multi-path fading mitigation, a beam former also increases the antenna gain in the direction of the desired user. Common beam forming criteria are minimum mean square error (MMSE), maximum signal to interference and noise ratio (MSINR), maximum SNR (MSNR), constant modulus (CMA), and maximum likelihood (ML).

2.2.2 MIMO Channel Systems Model

This section describe the types of wireless communication systems , traditional wireless communication systems with one transmit and one receive antenna are denoted as single input single output (SISO) systems, whereas systems with one transmit and multiple receive antennas are denoted as single input multiple output (SIMO) systems, systems with multiple transmit and one receive antenna are called multiple input single output (MISO) systems , single user multiple input and multiple output (SU-MIMO),multiuser multiple input and multiple output(MU-MIMO) and massive multiple input and multiple output (massive MIMO) systems.

2.2.2.1 SISO System

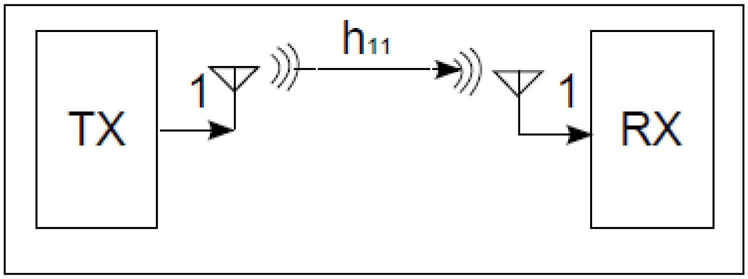


Figure 2.1: A SISO system.[39]

SISO Systems or the single input, single output communication systems is the simplest form of the communication system out of all four in which there is single transmitting antenna at the source and a single receiving antenna at the destination. Figure: 2.1 illustrate a SISO wireless system . SISO system is cheaper to build with fewer components and does not require the use of any special coding schemes at the transmitter and at the receiver . SISO are advantageous in terms of the simplicity. It does not require processing in terms of diversity schemes. The throughput of the system depends upon the channel bandwidth and signal to noise ratio. In some conditions, these systems are exposed to the issues like multipath effects. When an electromagnetic wave interacts with hills, buildings and other obstacles, waveform get scatter and takes many paths to reach the destination. Such issues are known as multipath. This causes several issues like fading, losses and attenuation also the reduction in data speed, packet loss and errors are increased.

2.2.2.2 SIMO System

SIMO or the Single input and multiple output form of wireless communication scheme in which there are multiple antennas are present at the receiver and there is single transmitting antenna at the source. In order to optimize the data scheme, various receive diversity schemes are employed at the receiver like selection diversity, maximum gain combining and equal gain combining schemes. SIMO systems were used for short waves listening and receiving stations to counter the effects of ionosphere fading. The SIMO systems are acceptable in many applications but where the receiving system is located in the mobile device like mobile phone, the performance me be limited by size, cost and battery.

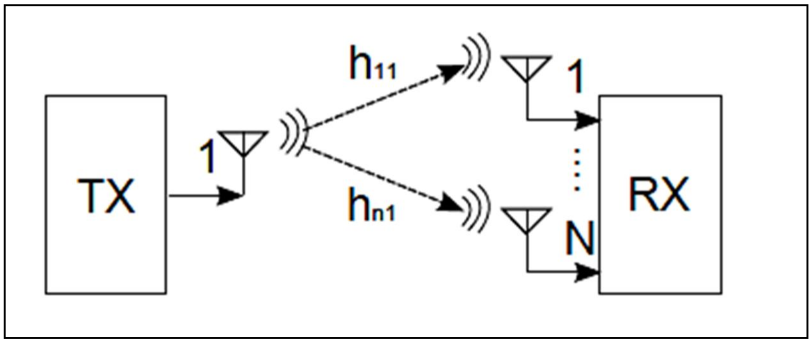


Figure 2.2: A SIMO system.[39]

2.2.2.3 MISO System

MISO or the multiple input and single output is a scheme of RF wireless communication system in which there are multiple transmitting antennas at the source and single receiving antenna at the system like SIMO but at the destination, receiver has a single antenna. When we use two or more antenna at the receiving end or at destination, the effects of multipath wave propagation, delay, packet loss etc can be reduced. This scheme has various applications. MISO systems are advantageous because the redundancy and coding has been shifted from receiving end towards the transmitting end and hence say in examples of mobile phones, less power and processing is required at the user end or the receiver end.

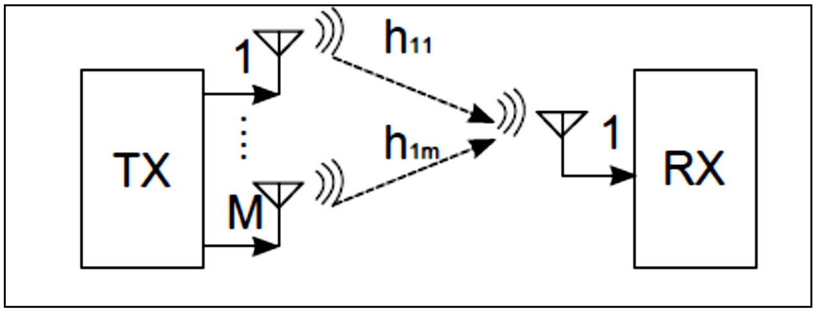


Figure 2.3: A MISO system.[39]

2.2.2.4 MIMO System

MIMO systems or the multiple input and multiple output systems are the one with multiple antennas at transmitting end and multiple antennas at receiving end as well .Between a transmitter and receiver, signal can go through many paths and if we move the antenna with a small distance, With the use of MIMO technology, the different paths available can be used for an advantage. By using MIMO, these additional paths can be used to advantage. They can be used to provide additional robustness to the radio link by improving the signal to noise ratio, or by increasing the link data capacity. MIMO systems often employs Spatial Multiplexing which enable signal to be transmitted across different spatial domains. A MIMO system can employ a transmit diversity scheme at the transmitter and a receive diversity at the receiver at the same time, allowing it to combine all the advantages offered by SIMO and MISO systems. Figure: 2.4 depict a MIMO system.

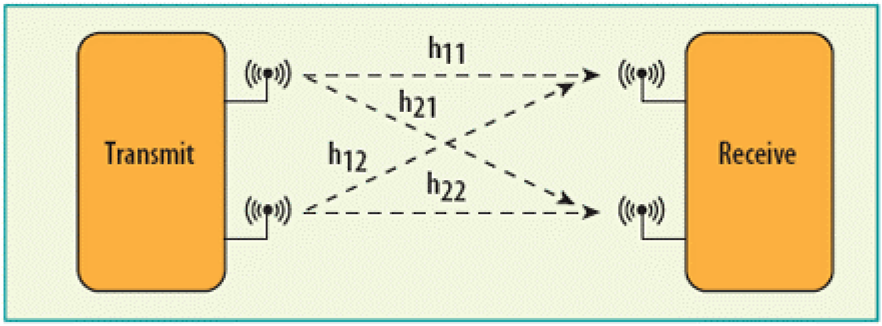
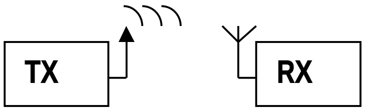
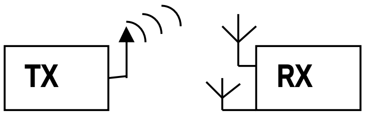
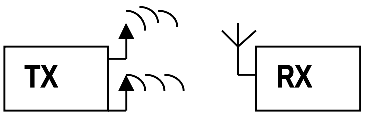
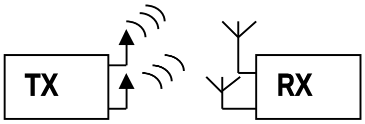


Figure 2.4: MIMO system.[39]

MIMO systems have a highest throughput when compared to the other types of wireless systems .

Table 2.1 illustrates of various antennas types.

Multi-antenna Types		
SISO	Single-Input-Single-Output means that the transmitter and receiver of the radio system have only one antenna	
SIMO	Single-Input-Multiple-Output the receiver has multiple antenna while transmitter has one antenna	
MISO	Multi-Input-Single-Output means that the transmitter has multiple antennas while the receiver has one antenna	
MIMO	Multi-Input-Multiple-Output means that the both the transmitter and receiver have multiple antenna	

2.2.2.5 SU-MIMO System

In SU-MIMO transmission only one user is served on a given time-frequency resource within a cell, possibly over multiple streams. With the simplifying assumption that out-of-cell interference is treated as additional Gaussian noise by the base station and users, the channel model reduces to a (distributed) point-to-point MIMO channel .Model for MIMO systems with detailed parameters are given in 3GPP standards [34]. The typical SU-MIMO channel is shown as follows in Figure:2.5, where N_b is the total number of transmit streams, the vector s is the transmit data, y denotes the received signal, and $\hat{s}$ is the estimated data.

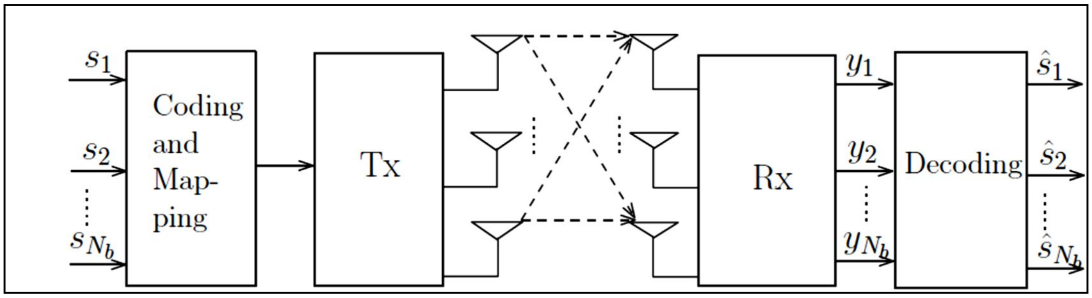


Figure 2.5: SU- MIMO system.[39]

2.2.2.6 MU-MIMO System

With MU-MIMO, multiple users are served in parallel over a given time-frequency resource by means of spatial multiplexing. While in SU-MIMO the multiplexing gain is limited by the minimum of the number of transmit and receive antennas, in MU-MIMO the multiplexing gain scales with the number of transmit antennas, provided there are enough users in the cell. Although multiple streams per user are possible in MU-MIMO, it has been shown that single stream transmission per user is asymptotically optimal in the number of user and that for finite number of users mostly only one stream is activated per selected user

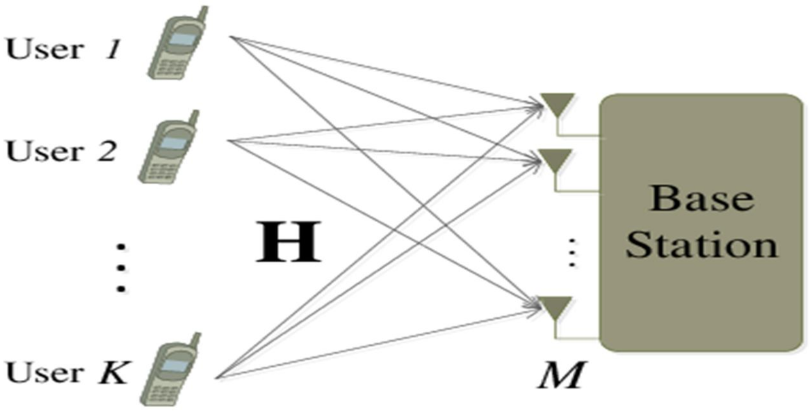


Figure 2.6: MU- MIMO system.[39]

Multiuser MIMO techniques and performance have begun to be intensely investigated because of several key advantages over single user MIMO communications:

- MU-MIMO schemes allow for a direct gain in multiple access capacity (proportional to the number of base station (BS) antennas) thanks to so-called multiuser multiplexing schemes.
- MU-MIMO appears more immune to most of propagation limitations plaguing single user MIMO communications such as channel rank loss or antenna correlation. Although increased correlation still affects per-user diversity, this may not be a major issue if multiuser diversity [18] can be extracted by the scheduler instead. Additionally, line of sight propagation, which causes severe degradation in single user spatial multiplexing schemes, is no longer a problem in multiuser setting.
- MU-MIMO allows the spatial multiplexing gain at the base station to be obtained without the need for multiple antenna terminals, thereby allowing the development of small and cheap terminals while intelligence and cost is kept on the infrastructure side.
- MU-MIMO in cellular communication systems brings improvements on four fronts:
    - ***increased data rate,*** because the more antennas, the more independent data streams can be sent out and the more terminals can be served simultaneously;
    - ***enhance reliability***, because the more antennas the more distinct paths that the radio signal can propagate over.
    - ***improved energy efficiency,*** because the base station can focus its emitted energy into the spatial directions where it knows that the terminals are located; and
    - ***reduced interference***, because the base station can purposely avoid transmitting into directions where spreading interference would be harmful.

2.2.2.7 Massive MIMO System

The MU-MIMO technology has been incorporated in LTE Release 8 and further standards, with a maximum of 8 BS antennas expected to serve roughly an equal number of terminals with FDD operation [8]. Massive MIMO (also known as Large-Scale Antenna Systems, Very Large MIMO, Hyper MIMO, Full-Dimension MIMO and ARGOS) makes a clean break with current practice through the use of a very large number of service antennas (e.g., hundreds or thousands) that are operated fully coherently and adaptively. Extra antennas help by focusing the transmission and reception of signal energy into ever-smaller regions of space. This brings huge improvements in throughput and energy efficiency, in particularly when combined with simultaneous scheduling of a large number of user terminals (e.g., tens or hundreds). Massive MIMO was originally envisioned for time division duplex (TDD) operation, but can potentially be applied also in frequency division duplex (FDD) operation. Other benefits of massive MIMO include the extensive use of inexpensive low-power components, reduced latency, simplification of the media access control (MAC) layer, and robustness to interference and intentional jamming. The anticipated throughput depends on the propagation environment providing asymptotically orthogonal channels to the terminals, and experiments have so far not disclosed any limitations in this regard. While massive MIMO renders many traditional research problems irrelevant, it uncovers entirely new problems that urgently need attention; for example, the challenge of making many low-cost low-precision components work effectively together, the need for efficient acquisition scheme for channel state information, resource allocation for newly-joined terminals, the exploitation of extra degrees of freedom provided by an excess of service antennas, reducing internal power consumption to achieve total energy efficiency reductions, and finding new deployment scenarios. Figure: 2.7 represent the massive MIMO system.

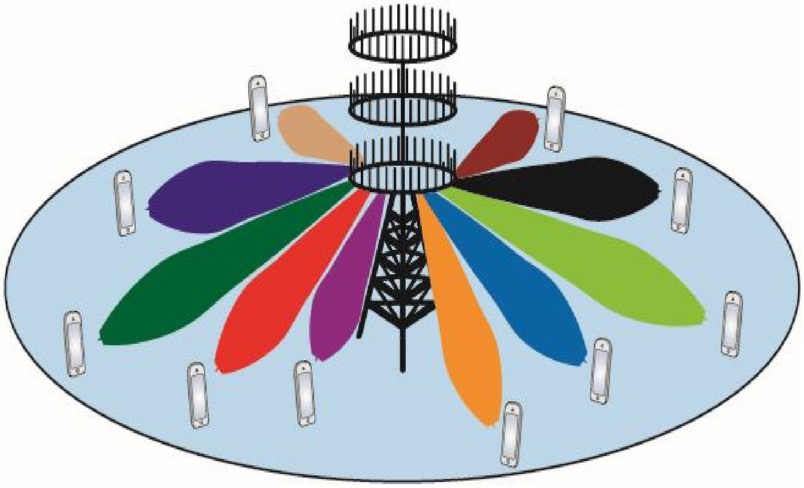


Figure2.7: massive MIMO system model.[39]

2.3 Related Works

One of the applications of point-to-point communications is in cellular networks where a serviced area is partitioned into several cells; each typically has one BS serving some mobile terminals (MTs). The very earlier forms of cellular communications consist of single-antenna BSs and single-antenna MTs. For this scenario, since each BS has one antenna, it can only transmit to one user at each time. Since there are several users expecting to be serviced by the BS, they need to be spread across time or frequency. In this case, time division multiple access (TDMA) and frequency division multiple access (FDMA) are prevalent techniques which provide fair access to all MTs within a cell. For example, one of the earliest standards for cellular communications is Global System for Mobile (GSM) where users within each cell are serviced via TDMA [39]. However even for this case, if there are two or more cells and the adjacent cells share the same set of radio frequencies, the transmission of each BS can cause interference to the other active MTs in nearby cells. Therefore, one approach to suppress this inter-cell interference is allocating distinct radio frequency bands to adjacent cells. In the following, we first consider the scenario where different sets of frequencies have been assigned to nearby cells. In this case, each cell can be analyzed separately and we therefore turn our focus on the transmission schemes which provide reasonable performance in downlink of single-cell scenario. We will next consider the scenarios wherein the nearby cells share the same radio frequency bands and we introduce the relevant state of the art interference management techniques to overcome the inter-cell interference in this case. Note that in the cellular downlink, since BS can have access to partial or perfect channel state information (CSI), it is more appropriate to shift the major signal processing enhancements to transmit side to keep MTs simple and low-cost. In downlink of cellular networks, it has been shown that if instead of merely one antenna, multiple antennas are deployed at BS, significant throughput gains can be achieved [45]. Now, all the aforementioned performance gains of MIMO systems can be gleaned. However, instead of assuming one user with multiple antennas, it is more reasonable to consider several MTs with a single antenna. This is due to the fact that deploying more than one antenna at each MT results in larger handsets and also leads to more power consumption, which is not practical from the user’s perspective. Therefore, in the case of multi-antenna BS and single-antenna MTs, each transmit antenna can be used to serve one single-antenna MT, and consequently several MTs can be simultaneously serviced within each cell.

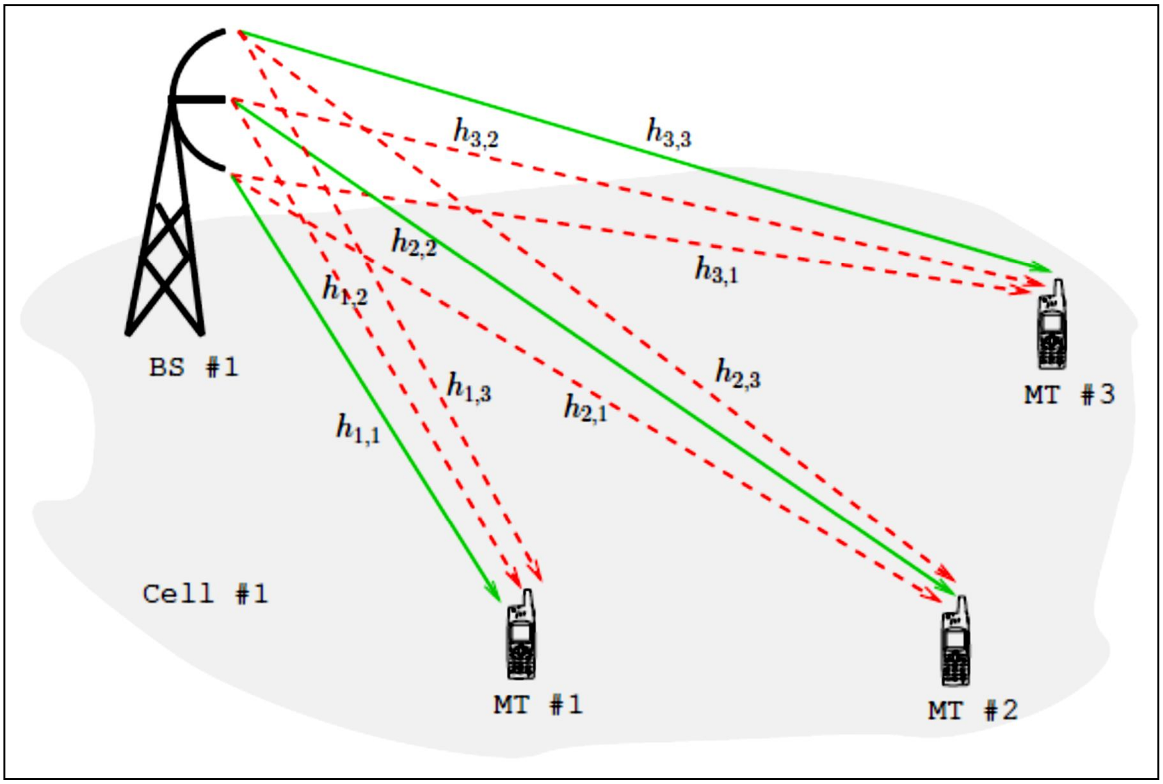


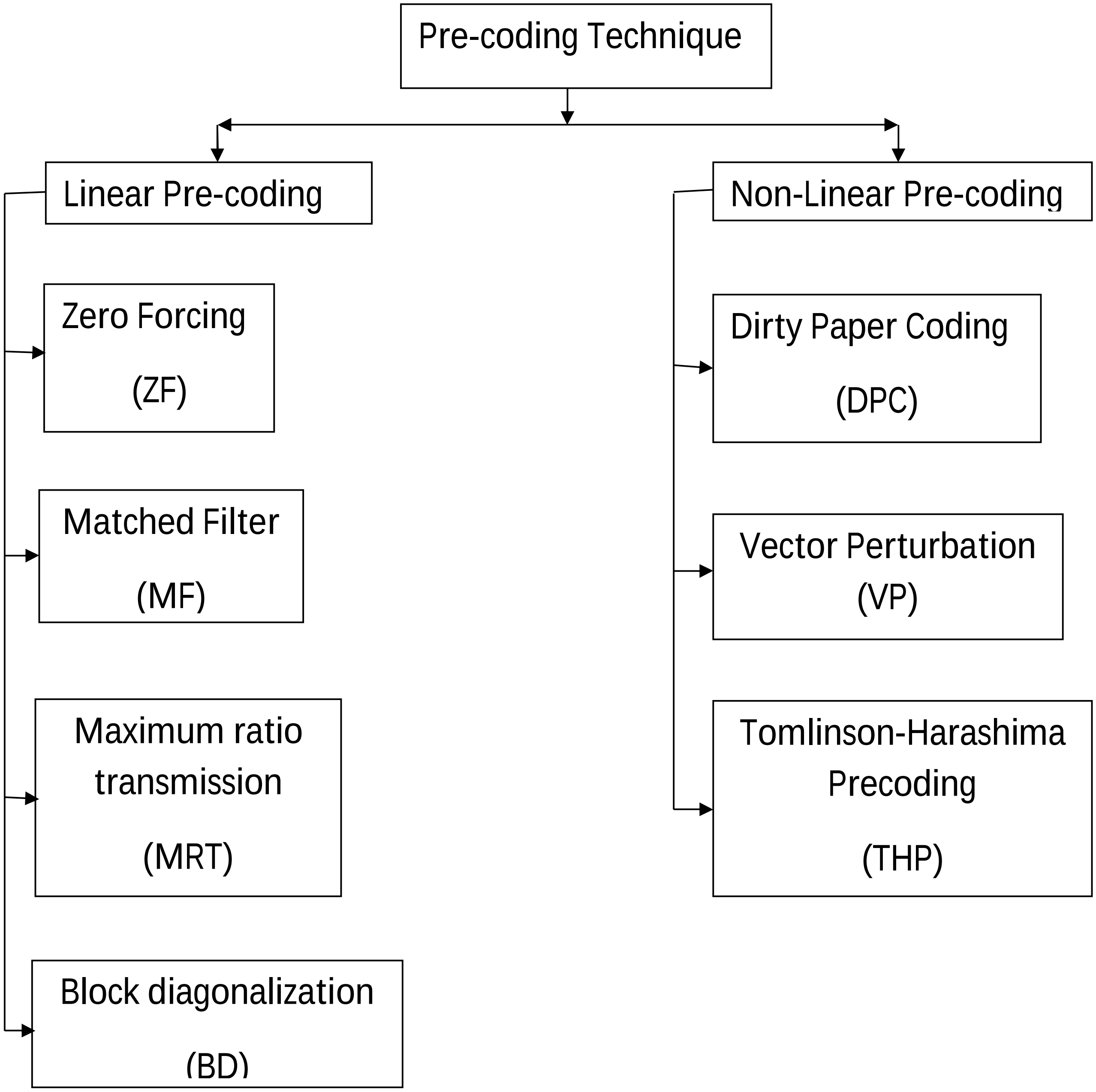
Figure 2.8: Single-cell broadcast .*[39]*

This is the multiplexing gain of MIMO systems, discussed in the previous section, and results in higher sum rates, i.e., multiple MTs can now be provided with higher data rates. Although using multiple antennas at BS results in higher multiplexing gains, it causes intra-cell interference (channel where dash red arrows represent intra-cell interference while solid green arrows denote desired links)(Figure 2.8), i.e., while each transmitted signal from one specific antenna at the BS is intended for just one specific single-antenna MT, it causes interference to the other receiving MTs within the same cell. Consequently, the downlink transmission strategy tries to design the beam patterns such that each MT receives its intended signal interference free. The more judicious transmission schemes try to increase the received SNR by having the received signals from the various transmit antennas add up in-phase (coherently) and/or by allocating more power to the transmit antenna with the better gain. This strategy, i.e., aligning the transmit signal in the direction of the transmit antenna array pattern, is called transmit beam forming (or hereafter we call it “pre-coding”).

2.3.1 Pre-coding Techniques

Flow chart (2.1) illustrate the types of pre-coding technique :

(2.1) Types of pre-coding technique



There are various pre-coding schemes, each of which has been designed to meet a certain criterion. Based on how the transmitted signals are related to the input data streams, the pre-coding techniques are categorized as linear and nonlinear. For example, it has been shown that the dirty paper coding (DPC), which is a nonlinear pre-coding, is capable of achieving the downlink capacity [20]. Nevertheless, due to its very high complexity, some less complex nonlinear pre-coders like vector perturbation [7 ,21] and Tomlinson-Harashima [12, 41] are also of particular interest. The simplest transmission scheme for multi-antenna downlink is channel inversion [38], which is linear, such that the intra-cell interferences are pre-cancelled at BS in order to enable each MT to receive its intended signal interference free. Zero-forcing (ZF) pre-coding is one technique of linear pre-coding in which the inter user interference can be cancelled out at each user [42] ,Maximum ratio transmission (MRT) pre-coding MRT is one technique of linear pre-coding which maximizes the signal gain at the intended user [42]. Block diagonalization (BD) is a linear pre-coding technique for the downlink of MU MIMO systems [11]. It decomposes a MUMIMO downlink channel into multiple parallel independent single-user MIMO downlink channels. Another technique also proposed in [11], named successive optimization (SO), addresses the power minimization and the near-far problem and it can yield better results in some situations but its performance depends on the power allocation and the order in which the users' signals are pre-processed. The zero MUI constraint is relaxed and a certain amount of interference is allowed. For example, as illustrated in Figure:2.8, it has been assumed that BS has 3 antennas and is therefore able to send 3 independent data streams simultaneously to 3 single-antenna MTs. Therefore, each transmitted data stream acts as an intra-cell interference to the other 2 unintended MTs.

A brief comparison between linear and nonlinear pre-coding techniques: Linear pre-coding techniques often have lower computational complexity than nonlinear ones. However nonlinear techniques often come with the advantage of improved performance by high computational complexity. Shown in Figure 2.9 .

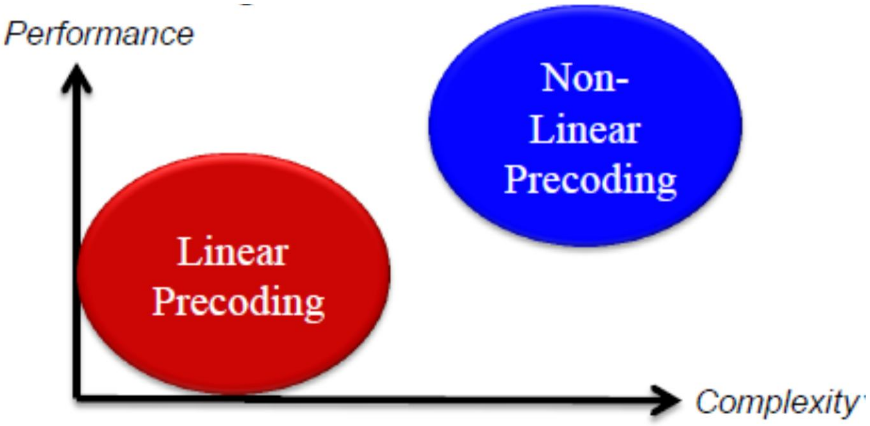


Figure.2.9 : Relation between linear and non-linear pre-coding techniques

As stated earlier, even by deploying multiple antennas at BS and using one of the aforementioned linear or nonlinear pre-coding and in order to avoid inter-cell interference, we still have to assign different radio frequency bands to adjacent cells. Since spectrum allocation is extremely conservative and expensive, in order to increase the spectral efficiency of the entire network, it is more desirable to use the same set of radio frequencies for two or more nearby cells. To meet this demand, several advanced pre-coding techniques of increased complexity have recently emerged for wireless access, which are inherently cooperative schemes. One of the promising techniques is network MIMO [35] which enables BSs to share the same frequency bands by the combined use of multiple antennas in several neighbouring cell sites. However, the BSs further need to share all the transmitted data streams through, for example, low-latency high-capacity backhaul links like the optical Fiber , as illustrated in Figure: 2.10. This way, the inter-cell interference gets cancelled and each user receives its intended data interference free. However, these backhaul links request for additional infrastructures which may not be readily implementable due to the excessive needs of data sharing between different sites and the need for additional antennas, which is impractical due to many hardware and cost constraints.

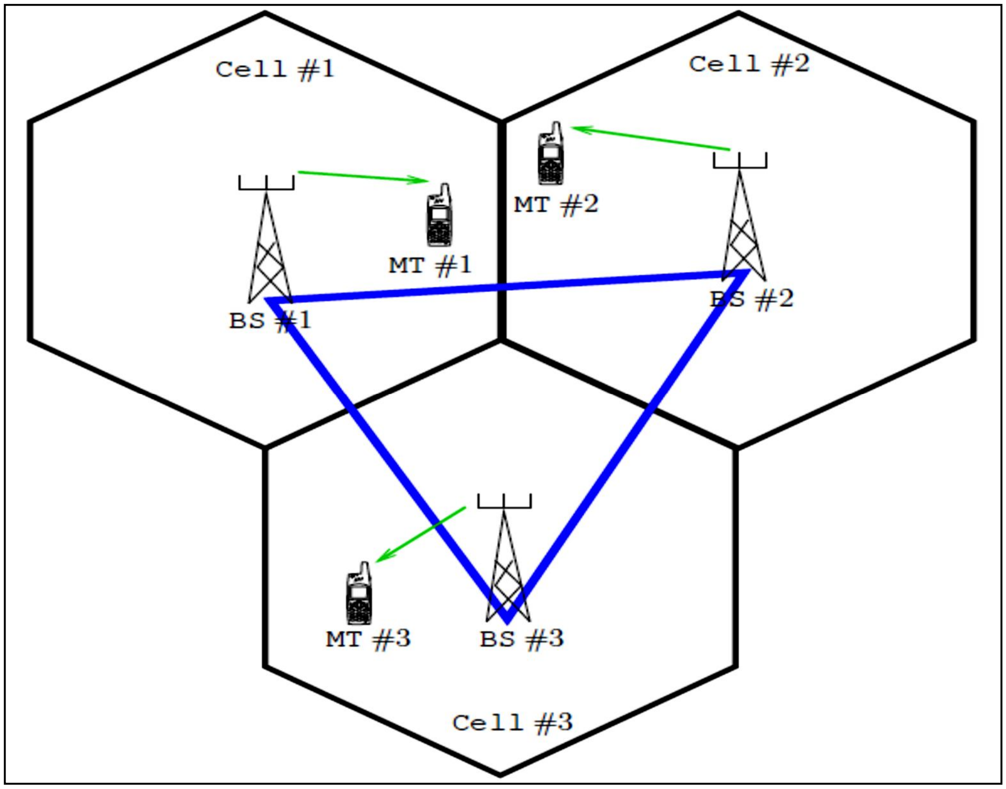


Figure 2.10: Network MIMO for cellular communications.[39]

Apart from network MIMO, another interesting approach towards higher spectral efficiency is massive MIMO where unlike network MIMO; there is no need to share the data streams between BSs [11] (where all the BSs share the transmitted data streams via backhaul links which are denoted as thick blue lines) showing in Figure 2.10. However as illustrated in Figure: 2.11, large number of antennas is needed to be deployed. This way, the transmitted signals are beam formed towards the intended MTs without causing interference to the unintended MTs in nearby cells. Massive MIMO communications heavily depend on the two emerging technologies such as : [14]

- Remote radio heads (RRHs) which allow for geographically distributed access via radio-over-fibre connections to a BS.
- Electronically steerable passive array radiators (ESPARs) which provide multi antenna-like functionality with a single active radio frequency chain only.

Although massive MIMO seems to be a part of the future wireless networks, it also needs extra infrastructure like fiber connections between each BS and the RRHs, and the spread out installation of large number of antennas across a wide area which may result in huge implementation costs. Therefore, there is a need for a different interference management technique which removes the extra overhead of network MIMO and massive MIMO systems.

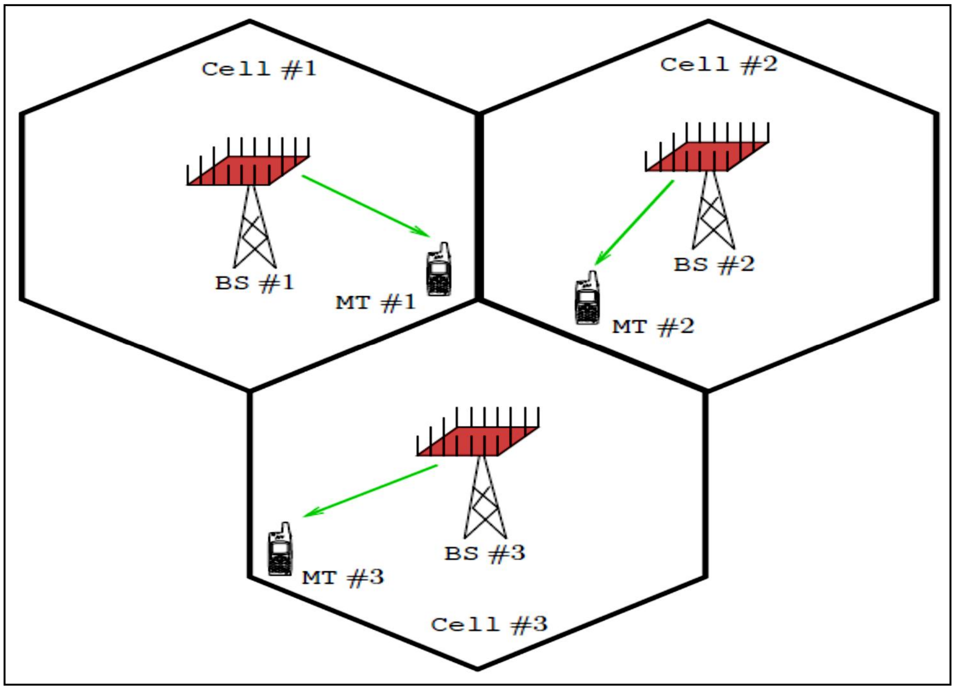


Figure 2.11: Massive MIMO for cellular communications.[39]

The massive MIMO assistive technology to the attention of many scientists with a lot of focus on the spectral efficiency, power efficiency and signal processing for cellular communications systems. Linear pre-coding schemes play an important role in the massive MIMO signal processing .The following subsections show the relevant techniques for linear pre-coding massive MIMO system.

In [3] , the other give the performances of zero-forcing(ZF) and maximum ratio transmission (MRT) are analysed and compared in a downlink massive multiple-input multiple output system. Simulation results are found to coincide with the theoretical results, and show that ZF performs better than MRT under the same conditions.

In [33] , the work propose network , massive multiple input multiple-output (MIMO) systems, where three radio units (RUs) connected via one digital unit (DU) support multiple user equipment (UEs) at a cell-boundary through the same radio resource, i.e., the same frequency/time band. For pre-codingdesigns, zero-forcing (ZF) and matched filter

(MF) with vector or matrix normalization are considered. We also derive the formulae of the lower and upper bounds of the achievable sum rate for each pre-coding. Based on our analytical results, we observe that vector normalization is better for ZF while matrix normalization is better for MF. Given antenna conFigureurations, we also derive the optimal switching point as a function of the number of active users in a network. Numerical simulations confirm our analytical results.

In [16] , The other give the performances of zero-forcing(ZF) and maximum ratio transmission (MRT) are analysed and compared in a downlink massive multiple-input multiple output system. The system employs a large number of base station antennas serving multiple user terminals within the same cell. The achievable sum rate and the required downlink transmit power using each of the pre-coding schemes are derived, analysed and compared under the same conditions and assumptions. Simulation results are found to coincide with the theoretical results, and show that ZF performs better than MRT under the same conditions.

In [46], Marzetta gives that the use of an excessive number of BS antennas compared with the number of active users makes simple linear processing nearly optimal.

In 2012, the authors consider the downlink of a time-division duplexing Multicell multiuser MIMO system where the base stations (BSs) are equipped with a very large number of antennas. Assuming channel estimation through uplink pilots, arbitrary antenna correlation and user distributions, they derive approximations of achievable rates with linear pre-coding techniques, namely Eigen-beamforming (BF) and regularized zero-forcing (RZF). The approximations are tight in the large system limit with an infinitely large number of antennas and user terminals (UTs), but match their simulations for realistic system dimensions. they further show that a simple RZF pre-coding scheme can achieve the same performance as BF with one order of magnitude fewer antennas in both uncorrelated and correlated fading channels [29].

To further maximize the network capacity, several network MIMO algorithms with multiple receive antennas have been proposed [47]. These systems assume, however, that the network supports maximums of three users through a relatively small number of transmit antennas. The assumption of an infinite number of antennas at the transmitter, however, is not, in practice, really feasible. This issue was studied in [48].

In [49], the authors showed theoretically and numerically the impact of pilot contamination, proposing a multi-cell minimum mean square error (MMSE) based pre-coding algorithm to reduce both intra- and inter-cell interference. The importance of resource allocation for massive MIMO was described in [50], where initial guidelines were given.

In May 2012, Hien Quoc Ngo, Erik G. Larsson, and Thomas L. Marzetta in paper” Energy and Spectral Efficiency of Very Large Multiuser MIMO Systems” give the trade off between the energy efficiency (as measured in bits/J) and spectral efficiency (as measured in bits/channel use/terminal). It is shown that the use of moderately large antenna arrays can improve the spectral and energy efficiency with orders of magnitude compared to a single-antenna system [15].

In 10th Annual IEEE (CCNC), 2013,Jinkyu Kang, Joonhyuk Kang, Namjeong Lee, Byung Moo Lee and Jongho Bang in paper ”Minimizing Transmit Power for Cooperative Multi cell System with Massive MIMO” the authors take problem of designing transmit beam former and power fo downlink cooperative base-station (BS) system with a large antenna arrays. Since the design of the beam forming vector at the transmitter requires high computational complexity, in a large antenna arrays, they utilize the zero-forcing transmit beam former, their paper focuses on the design of power allocation with fixed transmit beam former for minimizing the transmit power while meeting target signal-to-interference-and-noise-ratio (SINR) of each user and power constraints. They consider two scenarios according to the power constraints of cooperative BSs. One scenario is the sum power constraint on the cooperative base-stations. In this case, the cooperative BSs share the total available transmits power. However, each BS exists a maximum available transmit power in practical implementations. Thus, they consider a more realistic per BS power constraints. they proposed the solution strategies for both scenarios: For the sum power constraint case, a simple intuitive solution, where the power is allocated without regard to the power constraint until the SINR constraints is satisfied, is presented. For the per BS power constraints case, they use the properties of a large antenna arrays to find the solution of closed form. they also demonstrate, via numerical simulation, the performance of proposed strategy is convergent to the optimal performance which is achieved by using the iterative algorithm[28].

In May 2013, Eakkamol Pakdeejit in thesis “Linear Pre-coding Performance of Massive MU-MIMO Downlink System “who analyzed the performance of such pre-coding schemes in terms of spectral efficiency in a single-cell downlink scenario, fixed the same value of signal-to-interference-to-noise ratio for both pre-coders; which should not be done for a good analysis[42].

In September 2013 Yeon-Geun Lim, Chan-Byoung Chae and Giuseppe Caire in paper” Performance Analysis of Massive MIMO for Cell-Boundary Users” the authors in this paper consider massive multiple-input multiple-output (MIMO) systems for both downlink and uplink scenarios, where three radio units (RUs) connected via one digital unit (DU) support multiple user equipments (UEs) at the cell-boundary through the same radio resource, i.e., the same time-frequency slot. Zero-forcing (ZF) and maximum ratio transmission (MRT) are considered as downlink transmitter options, while ZF and maximum ratio combining (MRC) are considered as uplink receiver options. they derive simple closed form formulas for the sum rate of each such technique [22].

In January 2014, authors Erik G. Larsson, Ove Edfors, Fredrik Tufvesson and Thomas L. Marzetta, in paper ”MASSIVE MIMO FOR NEXT GENERATION WIRELESS SYSTEMS” they give the overview of massive MIMO system for next generation[14].

In 2014, authors Long Zhao, Kan Zheng, Hang Long Hui Zhao and Wenbo Wang in paper” Performance Analysis for Downlink Massive MIMO System with ZF pre-coding”, this paper investigates the performance for the downlink massive multiple-input multiple-output system when the base station serves multiple user terminals (UTs) using zero-forcing pre-coding. The tight lower bound of average area spectrum efficiency (A2SE) is derived as a function of the number of transmission antennas, the number of UTs and the equivalent transmission signal-to-noise ratio. Regarding the lower bound as the approximate A2SE, the optimal number of UTs maximizing the A2SE is attained for given the number of transmission antennas and the equivalent transmission signal-to-noise ratio. The trade-off between energy efficiency (EE) and A2SE is established, and the optimal EE with respect to A2SE is deduced. Simulation results coincide to the analysis well, and they indicate that deploying more transmission antennas or multiplexing a rational number of UTs can improve the A2SE and EE, increasing the degrees of freedom will better both the outage probability and bit error ratio [5].

Although the above works provide results about the performances of the linear pre-coding schemes, they did not give the comparison of ZF and MF performances in terms of achievable data rate versus number of transmit antennas and number of users in a single-cell downlink massive MIMO systems.

This research analyzes and compares the performance of ZF and MF; in terms of achievable data rate versus number of transmit antenna and achievable data rate versus number of users, for ZF and MF pre-coding schemes in a single-cell downlink massive MIMO system at different value of power.

CHAPTER THREE

SYSTEM MODEL

This chapter demonstrate the concept of using zero forcing and matched filter linear pre-coding techniques for a downlink massive MIMO system in a single cell to evaluate the system performance (Achievable Sum Rate) by applying the vector and matrix normalization methods under the assumption of base station with perfect channel state information and channel matrix are modelled as independent complex Gaussian random variables with zero mean and unit variance. Figure:(3.1) show downlink massive MIMO system in single cell .The base station equipped with large number of M antennas and each k user equipped with one single antenna ($M \gg K$).

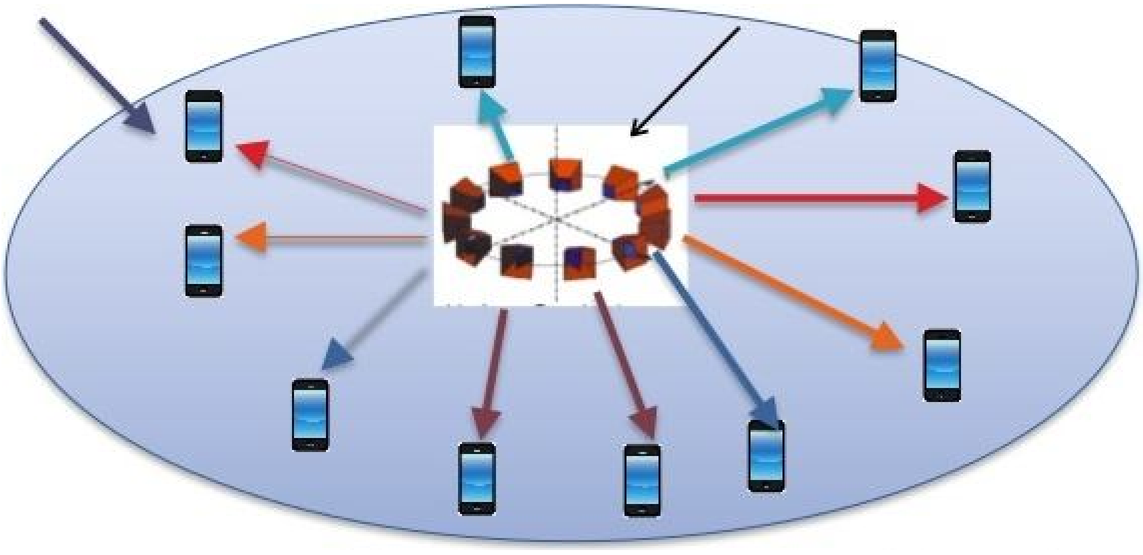


Figure 3.1: A single-cell downlink massive MIMO system.[39]

3.1 Mathematical Model

The system model of downlink massive MIMO system
in a single cell shown in Figure (3.2). This thesis; does not consider pilot contamination and assume perfect channel state information at the base station (BS).

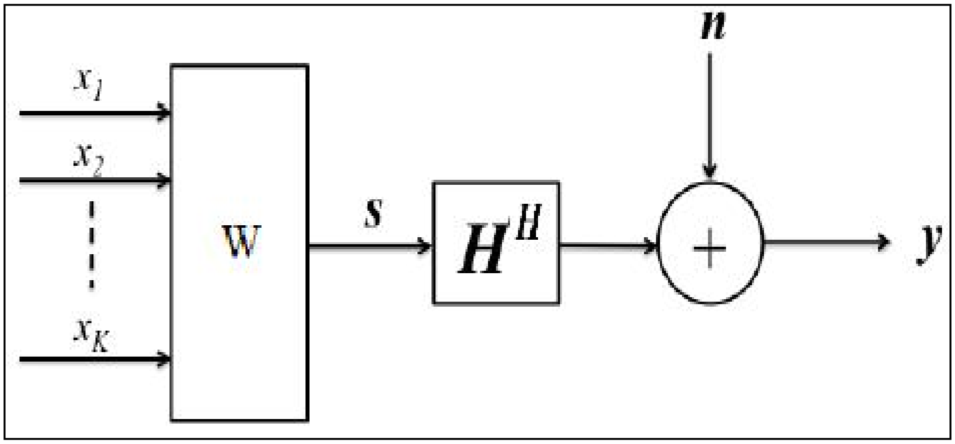


Figure 3.2: A downlink massive MIMO system model.[39]

From assumption the channel matrix is modelled as independent complex Gaussian random variables with zero mean and unit variance. The channel between the BS and the k_{th} user is denoted by a $1 \times M$ row vector h_k^T $(K = 1,2,3 \ldots \ldots .. K)$.A $M \times N$ channel matrix H between the BS and all users consists of channel vectors h_k^T . Let w_k denote the column vector of transmit pre-coding and s_k represent the transmit symbol for the k_{th} user at downlink. Similarly, let w_k denote the column vector of receive combining filter for the k_{th} user at uplink. Also, let n_k be the additive white Gaussian noise vector. Then, the receiver vector is given by equation (3.1):

$$y = \sqrt{P_d}\,Hx + n = \sqrt{P_d}\,HWx + n \qquad (3.1)$$

The received signal at the k_{th} user is expressed by:

$$y_k = \underbrace{\sqrt{P_d}\,h_k^T w_k s_k}_{\text{desired signal}} + \underbrace{\sqrt{P_d}\,\sum_{\substack{l=1 \\ l\neq k}}^{k} h_k^T w_l s_l}_{\text{interference}} + n_k \qquad (3.2)$$

Where: P_d is transmit power in a downlink.

The received signal-to-interference-plus-noise ratio of the k_{th} user can be given by equation (3.3):

$$\mathrm{SINR}_K = \frac{P_d \ |h_w \ w_k|^2}{P_d \sum_{\substack{i=1 \\ i \neq k}}^{k} |h_w \ w_k|^2 + 1} \tag{3.3}$$

Which is a function of transmit pre-coding vector w_k.

3.2 Linear Pre-coding Techniques

Equation (3.2) containing signal interference, noise and the desired signal. To mitigate signal interference we can use the linear pre-coding techniques such as (Zero forcing pre-coding (ZF) and Matched Filter (MF)).

3.2.1 Zero Forcing Pre-coding (ZF) Technique

One of simple linear pre-coding technique is ZF pre-coding in which the multiuser interference can be cancelled out at each user. This pre-coding is assumed to implement a pseudo-inverse of the channel matrix. The ZF pre-coding employed by the base station is given by equation (3.4):

$$W_{ZF} = H^H (HH^H)^{-1} = [w_1 w_2 \ \dots \ w_k] \tag{3.4}$$

Where: W is a pre-coding matrix consisting of each column vector w_k.

3.2.2 Matched Filter Pre-coding (MF) Technique

MF is one technique of linear pre-coding which mitigate the interference for multi-user by matching filters concept. The MF pre-coding employed by the BS is written as:

$$W_{MF} = H^* = [w_1 w_2 \ \dots \ w_k] \tag{3.5}$$

H^* :conjugate transpose of channel matrix

3.3 Normalization Method

We make normalize the pre-coding matrix to make satisfy the power constraint. As mentioned above, we consider two methods of normalization, i.e., vector normalizations and matrix normalizations. The normalized transmit beam forming vectors (columns of a pre-coding matrix) with vector and matrix normalizations are given as $\boldsymbol{w_k} = \boldsymbol{w_1}/(\sqrt{\boldsymbol{K}}\|\boldsymbol{w_k}\|)$ and $\boldsymbol{w_k} = \boldsymbol{w_k}/\|\boldsymbol{W}\|_{\boldsymbol{W}}$ respectively. Note that vector normalization imposes equal power per downlink stream, while matrix normalization yields streams with different power. In this thesis, to more simplify, we do not consider a power optimization that could yield a complexity in massive MIMO antenna systems.

3.3.1 ZF and MF with Vector Normalization

The received signal at the k_{th} user can be expressed as:

$$y_k = \sqrt{P_d} h_k^T \frac{w_k}{\sqrt{K}\|w_k\|} s_k + \sqrt{P_d} \sum_{\substack{l=1 \\ l \neq k}}^{k} h_k^T \frac{w_l}{\sqrt{K}\|w_l\|} s_l + n_k \tag{3.6}$$

3.3.2 ZF and MF with Matrix Normalization

The received signal can be change with matrix normalization as

$$y_k = \sqrt{P_d} h_k^T \frac{w_k}{\|W\|_W} s_k + \sqrt{P_d} \sum_{\substack{l=1 \\ l \neq k}}^{k} h_k^T \frac{w_l}{\|W\|_W} s_l + n_k \tag{3.7}$$

3.4 Achievable Sum Rate

It's one of the methods to quantify the system performance. The achievable sum rate follows the Shannon theorem. Shannon theorem gives the maximum rate at which the transmitter can transmit via the channel. This section describes the achievable sum rate with ZF and MT, with the assumption that the total downlink power is fixed and equally divided between all the users. From Shannon theorem, the channel capacity over Additive White Gaussian Noise channel is given by equation (3.8) [22]:

$$R = \log_2(1 + \mathrm{SNR})(\mathrm{bits/s/\ Hz}) \tag{3.8}$$

where , SNR : is the signal to noise ratio.

Channel status information (CSI) is a very important issue in multiuser communication systems. Typically, the transmitter sends multiple data streams for each user simultaneously and selectively with CSI . The all receivers send feedback to the transmitter on the reverse link, so the transmitter obtains CSI. Hence, the transmitter communicate with all the receivers with perfect CSI. And as shown in equation (3.2), the signal received by the each user consists of additive noise and interference between the users themselves. Then, the achievable sum rate per user in downlink massive MIMO system in a single cell, with perfect CSI is given by equation (3.9):

$$R_k = \log_2(1 + \mathrm{SINR}_k) \tag{3.9}$$

And for K number of users, the achievable sum rate is given in equation (3.10):

$$R_{sum} = k\log_2(1 + \mathrm{SINR}_k) \tag{3.10}$$

3.4.1 Achievable Sum Rate for Zero Forcing (ZF) Pre-coding:

From equation (3.6), the achievable sum rate with ZF can be written as in equation (3.11):

$$R_{sum}^{ZF} = k\log_2(1 + \mathrm{SINR}_k^{ZF}) \tag{3.11}$$

The achievable sum rate for zero forcing by using vector /matrix normalization methods is given by equation (3.12) and (3.13):

$$R_{ZF_{vec}} = k\log_2(1 + \frac{P_d(M - K + 1)}{K}) \qquad (\ 3.12)$$

$$R_{ZF_{mat}} = k\log_2(1 + \frac{P_d(M - K)}{K}) \qquad (\ 3.13)$$

3.4.2 Achievable Sum Rate for Matched Filter (MF) Pre-coding

From equation (3.7), the achievable sum rate with MF can be expressed as

$$R_{\mathrm{sum}}^{\mathrm{MF}} = k\log_2(1 + \mathrm{SINR}_{k}^{\mathrm{MF}}) \tag{3.14}$$

The achievable sum rate for match filter by using vector /matrix normalization methods [41] is given by

$$R_{\mathrm{MF}_{\mathrm{vec}}} = R_{\mathrm{MF}_{\mathrm{mat}}} = k\log_2(1 + \frac{P_d(M+1)}{P_d(K-1)+K}) \tag{3.15}$$

The following flow chart which represents the manner of applying the zero forcing and matched filter linear pre-coding techniques for single cell downlink massive MIMO system when number of base station antennas from 1 to 300 and from 1 to 600 and number of user from 1 to 200 at two value of power 0 dB and -10 dB for matrix and vector normalization methods as shown in **Figure**: (3.3) and (3.4).

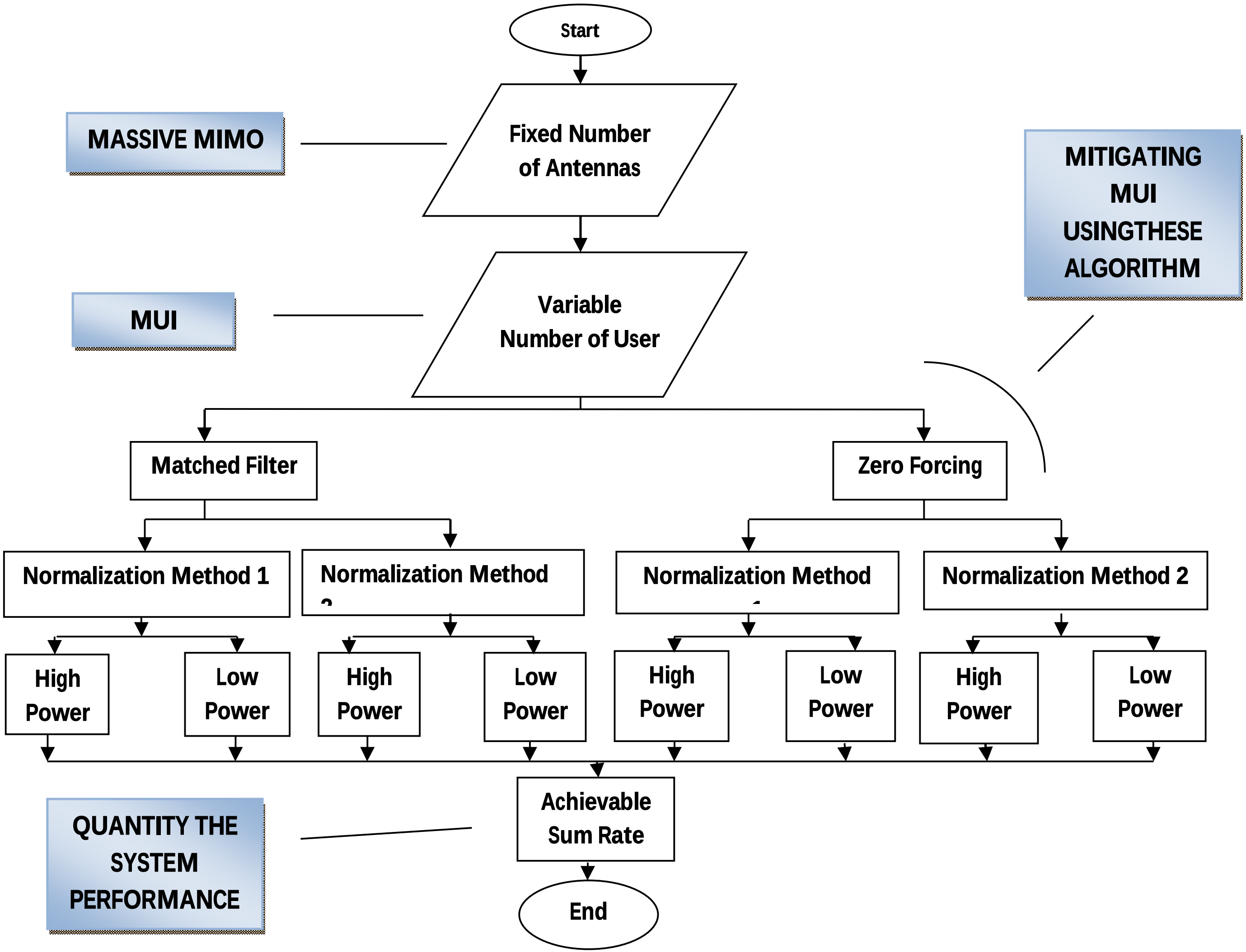


Figure3.3 Flow chart of fixed number of antennas with different number of users

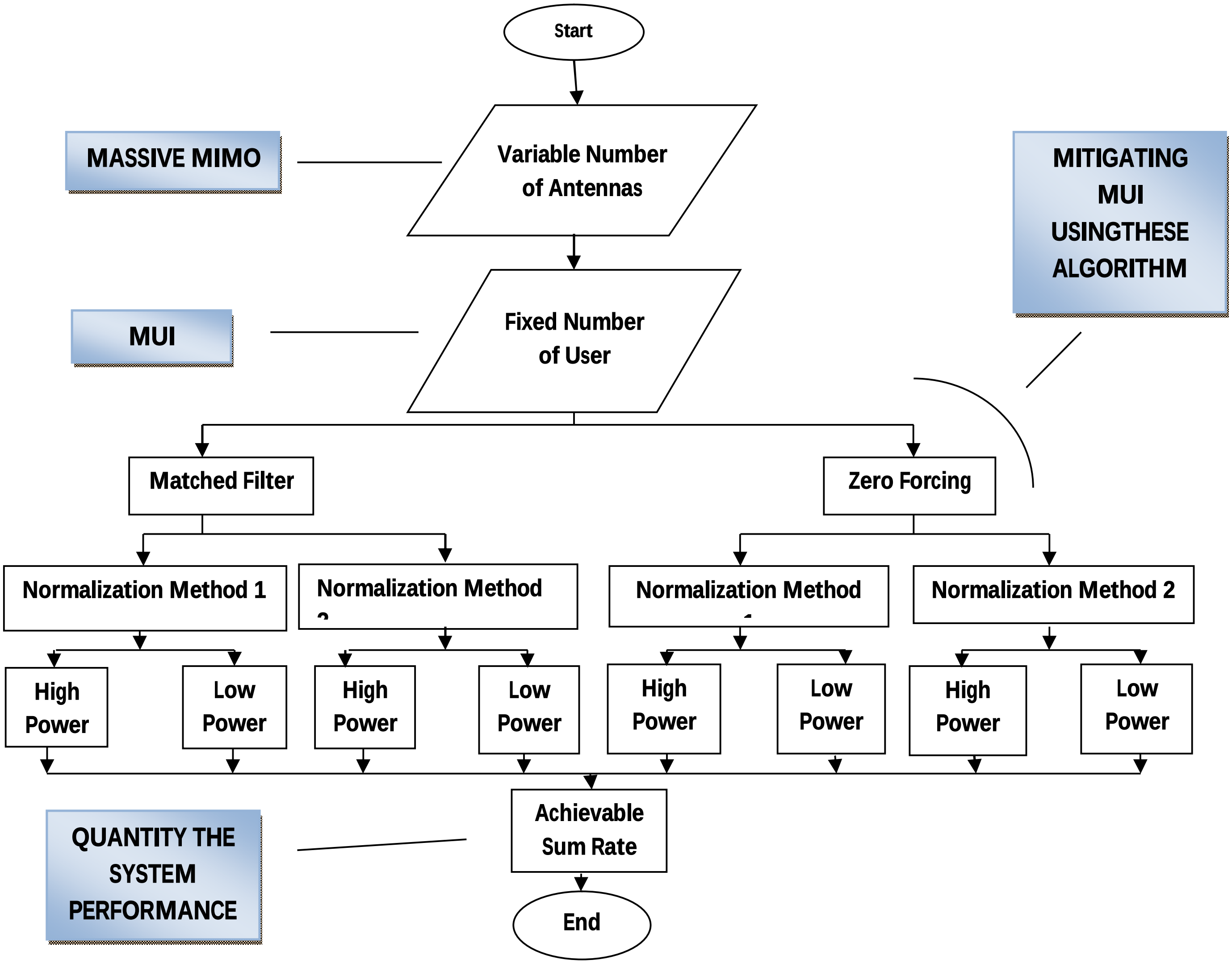


Figure3.4 Flow chart of different number of antennas with fixed number of users

CHAPTER FOUR

SIMULATION RESULTS AND DISCUSSIONS

Consider only single cell downlink massive MIMO system, There are different scenarios, which depicts the comparison performance analysis for tow linear Pre-coding techniques (Zero Forcing and Matched Filter) in a perfect channel and single antenna user , depending on vector normalization method, matrix normalization method and compare the two normalization methods. Finally a comparative analysis of results for the scenarios of normalization methods presented in term of achievable sum rate overall Pre-coding techniques. However; the limitation of models are that the number of users K is not greater than the number of antennas M ($M \gg K$).All scenarios simulate by using MATLAB software modelling tools.

4.1 Vector Normalization

Table 4.1 Simulation parameters for Vector normalization .

Low Power	-10 DB
High Power	0 DB
Number of Antennas	300 , 400 , 500 and 600
Number of Users	50 , 100 , 150 and 200

This section illustrates the performance of MF and ZF in single cell downlink massive MIMO system over perfect channel by considering the achievable sum rate depending on vector normalization method. Select the number of users K = 200 and the number of antennas is 600. Then set up the base station downlink transmitting power to 0 dB & -10dB.

4.1.1 Performance of MF and ZF Using Vector Normalization at K=50,100,150 and 200 Users

Figures (4.1 , 4.2 , 4.3 , 4.4) shows the achievable sum rate across the entire antenna range according to equations (3.12 and 3.15).This scheme consists of the number of antennas M =1:600 and the number of users K =50 , 100 , 150 and 200 .

In Figure (4.1) K = 50 , it can be seen from the result that MF generally gives slightly better performance than ZF at low value of power when number of base station antennas less than 560 .On the other hand, ZF gives better performance than MF at high power when number of base station antennas greater than 100.

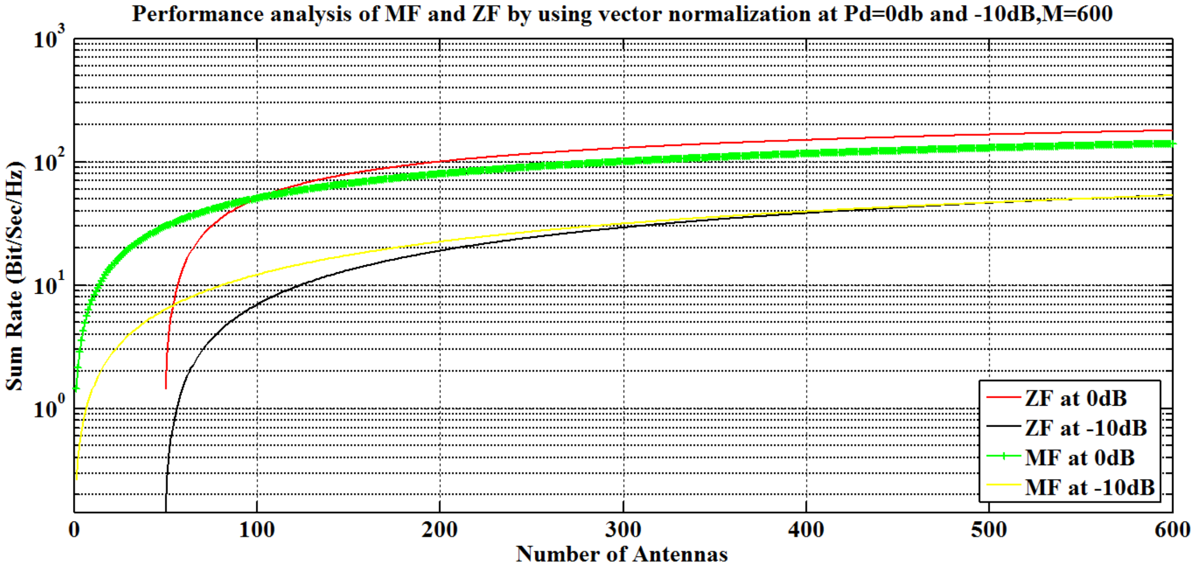


Figure 4.1: Performance of MF and ZF using vector normalization [at k=50 & M=1:600].

Figure (4.2) ; it witch (K = 100) , it can be seen from the result that MF generally gives slightly better performance than ZF at low value of power and better performance when number of base station antennas less than 201 at high power . ZF gives better performance at high power when number of base station antennas greater than 200.

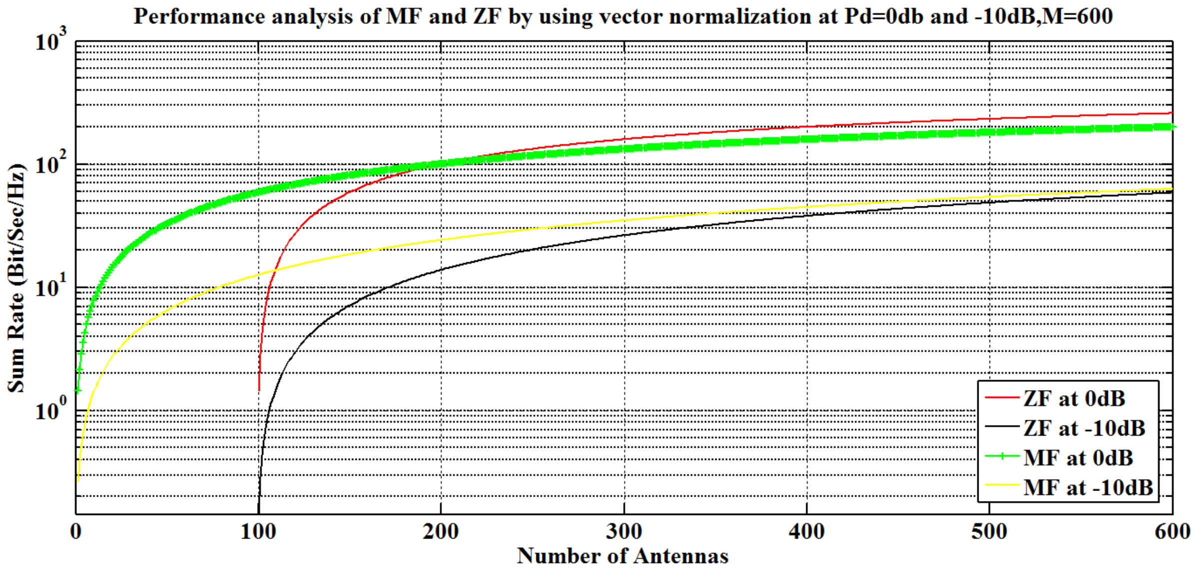


Figure 4.2: Performance of MF and ZF using vector normalization [at k=100 & M=1:600].

In Figure (4.3) shows K = 150 it can be seen from the result that MF generally gives slightly better performance than ZF at low value of power and better performance when number of base station antennas less than 301 at high power . ZF gives better performance at high power when number of base station antennas greater than 300.

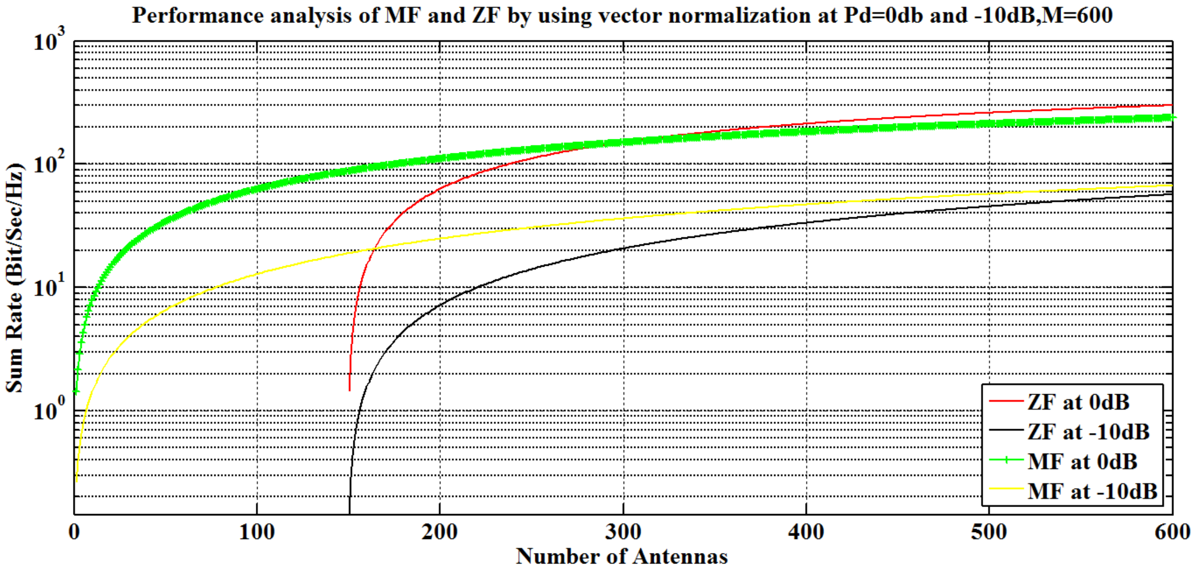


Figure 4.3: Performance of MF and ZF using vector normalization [at k=150 & M=1:600].

Figure (4.4) it witch (K = 200) , it can be seen from the result that MF generally gives slightly better performance than ZF at low value of power and better performance when number of base station antennas less than 401 at high power and ZF gives better performance at high power when number of base station antennas greater than 400.

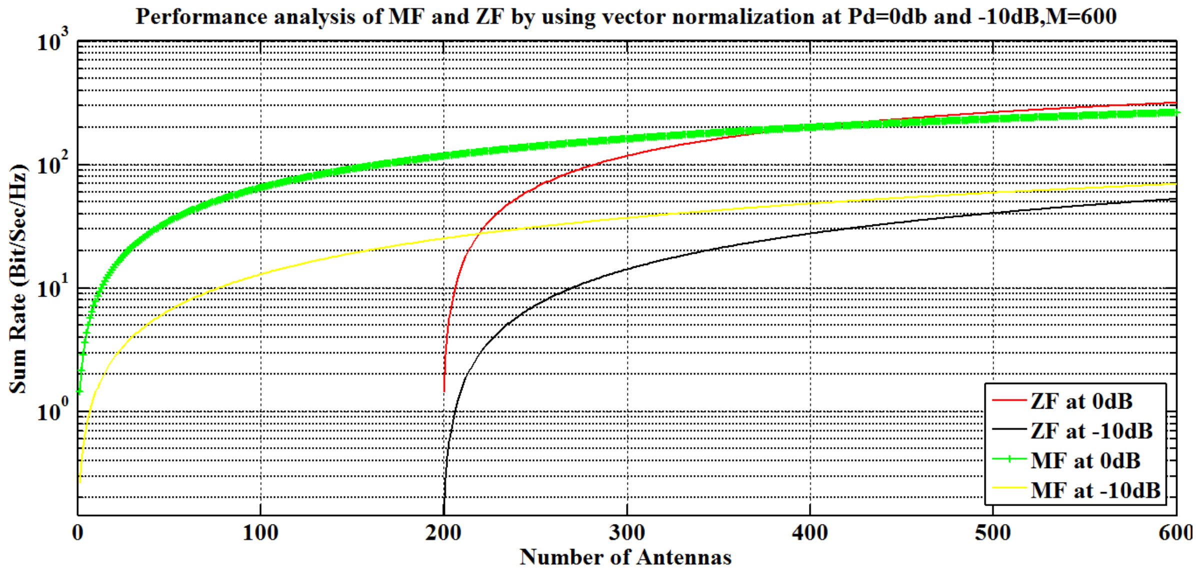


Figure 4.4: Performance of MF and ZF using vector normalization [at k=200 & M=1:600].

4.1.2 Performance of MF and ZF Using Vector Normalization at M=300, 400 , 500 and 600 Antennas

Figures : (4.5 , 4.6 , 4.7 , 4.8) shows the achievable sum rate across the whole user range according to equations (3.12 and 3.15).This scheme consists of the number of antennas M = 300 , 400 , 500 and 600 and the number of users K = 1:200.

In Figure (4.5) shows M = 300 it can be seen from the result that MF generally gives slightly better performance at low value of power and better performance when number of users less than 121 at high power .On the other sides, ZF gives better performance at high power when the number of users more than 120.

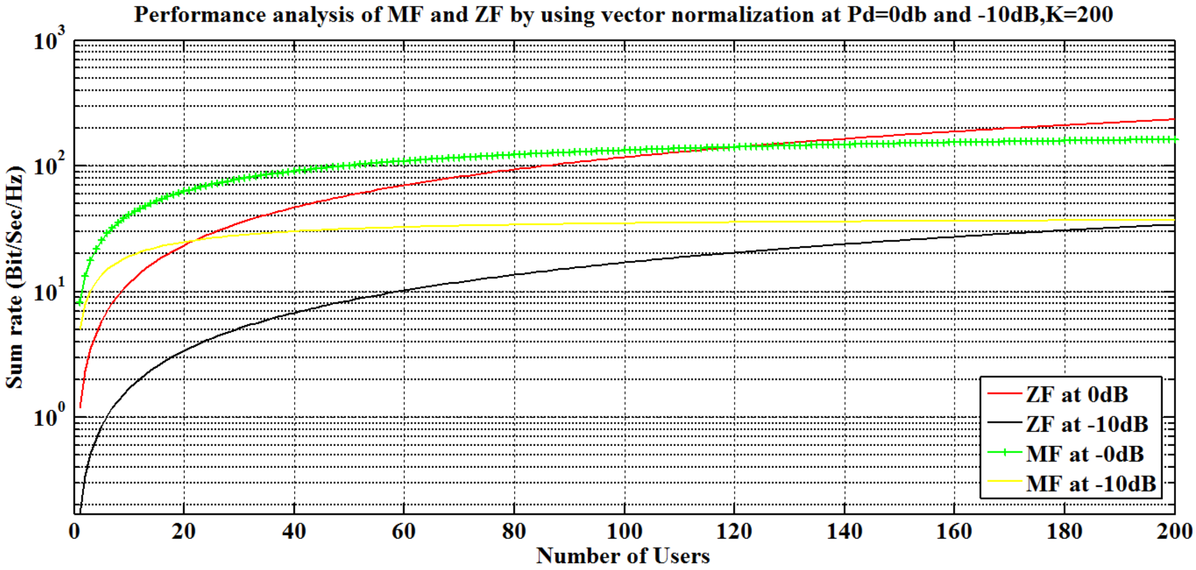


Figure 4.5: Performance of MF and ZF using vector normalization [at M=300 & k=1:200].

Figure (4.6) ; it witch (M = 400) , it can be seen from the result that MF generally gives slightly better performance at low value of power and better performance when number of users less than 101 at high power .On the other sides, ZF gives better performance at high power when the number of users more than 100 and better performance at low power when the number of users more than 182.

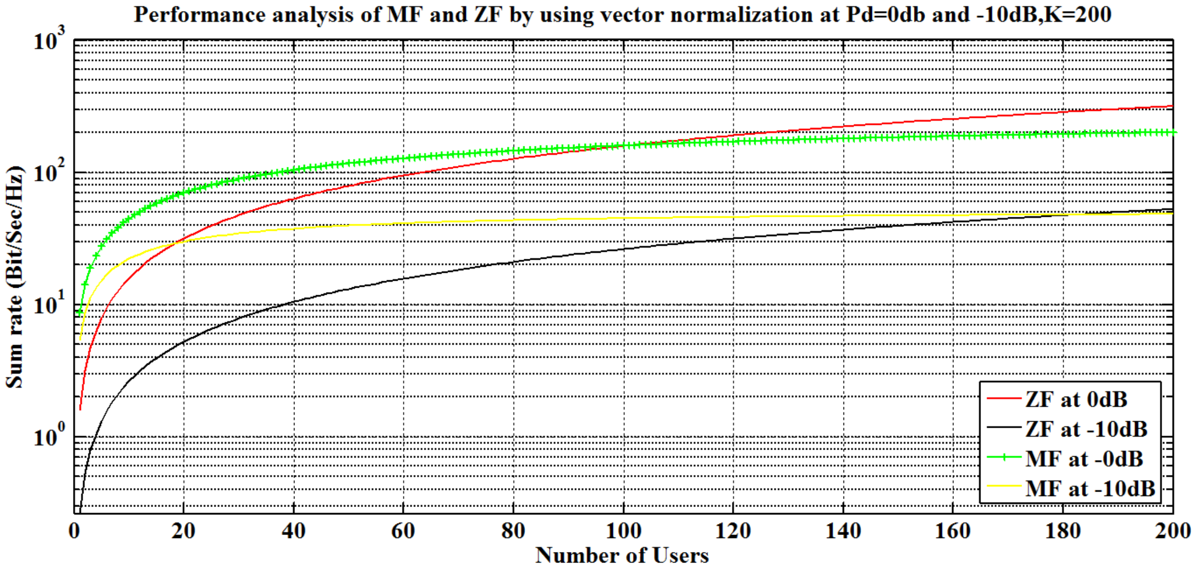


Figure 4.6: Performance of MF and ZF using vector normalization [at M=400 & k=1:200].

In Figure (4.7) shows M = 500 it can be seen from the result that MF generally gives slightly better performance at low value of power when the number of users less than 166 and better performance when number of users less than 92 at high power .On the other sides, ZF gives better performance at high power when the number of users more than 91 and better performance at low power when the number of users more than 165.

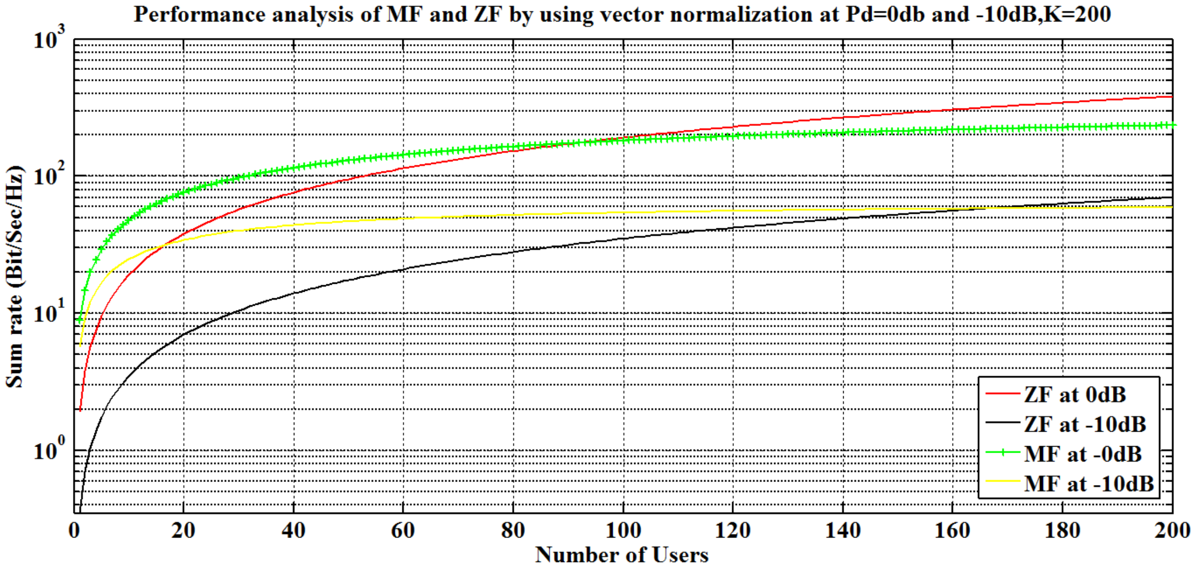


Figure 4.7: Performance of MF and ZF using vector normalization [at M=500 & k=1:200].

Figure (4.8) ; it witch (M = 600) , it can be seen from the result that MF generally gives slightly better performance at low value of power when the number of users less than 157 and better performance when number of users less than 87 at high power .On the other sides, ZF gives better performance at high power when the number of users more than 86 and better performance at low power when the number of users more than 156.

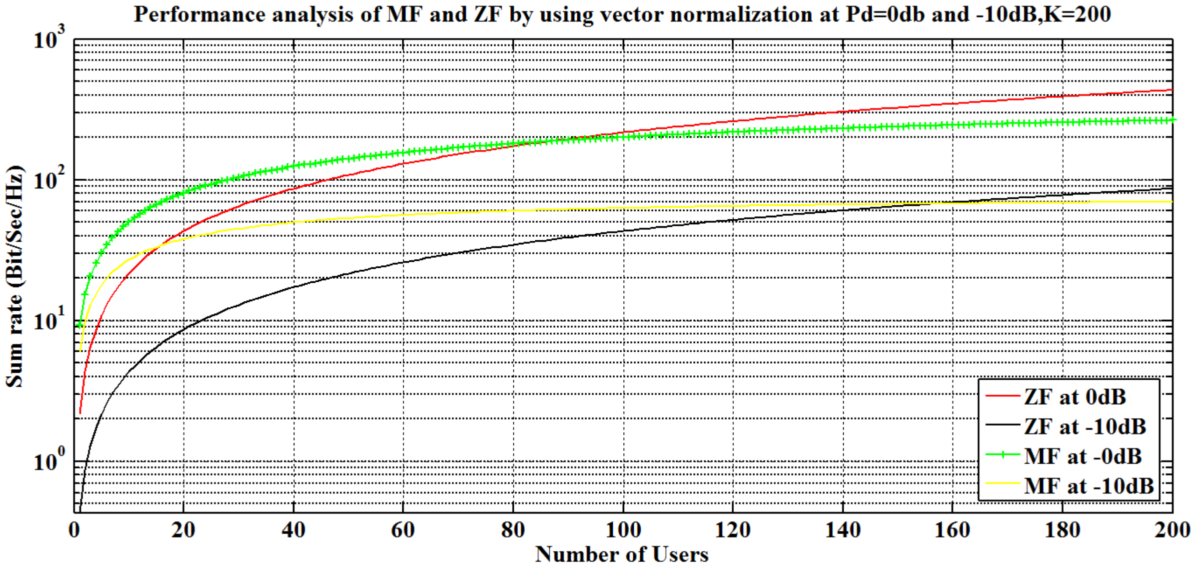


Figure 4.8: Performance of MF and ZF using vector normalization [at M=600 & k=1:200].

4.2 Matrix Normalization

Table 4.2 Simulation parameters for Matrix normalization .

Low Power	-10 DB
High Power	0 DB
Number of Antennas	300 , 400 , 500 and 600
Number of Users	50 , 100 , 150 and 200

We will study the performance of MF and ZF in single cell downlink massive MIMO system over perfect channel by considering the achievable sum rate depending on Matrix normalization method. Select the number of users K = 200 and the number of antennas is 600. Then set up the base station downlink transmitting power to 0 dB & -10 dB.

4.2.1 Performance of MF and ZF Using Matrix Normalization at K= 50 , 100 , 150 and 200 Users

Figures (4.9 , 4.10 , 4.11 and 4.12) illustrations the achievable sum rate across the whole user range according to equations (3.16, 3.19 and 3.21).This scheme consists of the number of antennas M =1:600 and the number of users K = 50 , 100 , 150 and 200 .

In Figure (4.9) shows k=50 it can be seen from the results that MF gives better performance at low power and better performance at high value of power when the number of antenna less than 571 antennas and better performance at high power when then number of antenna less than 103 .On the other hand, the ZF gives the better performance at high value of power when the number of antenna more than 570 antenna and better performance at low power when then number of users more than 102 .

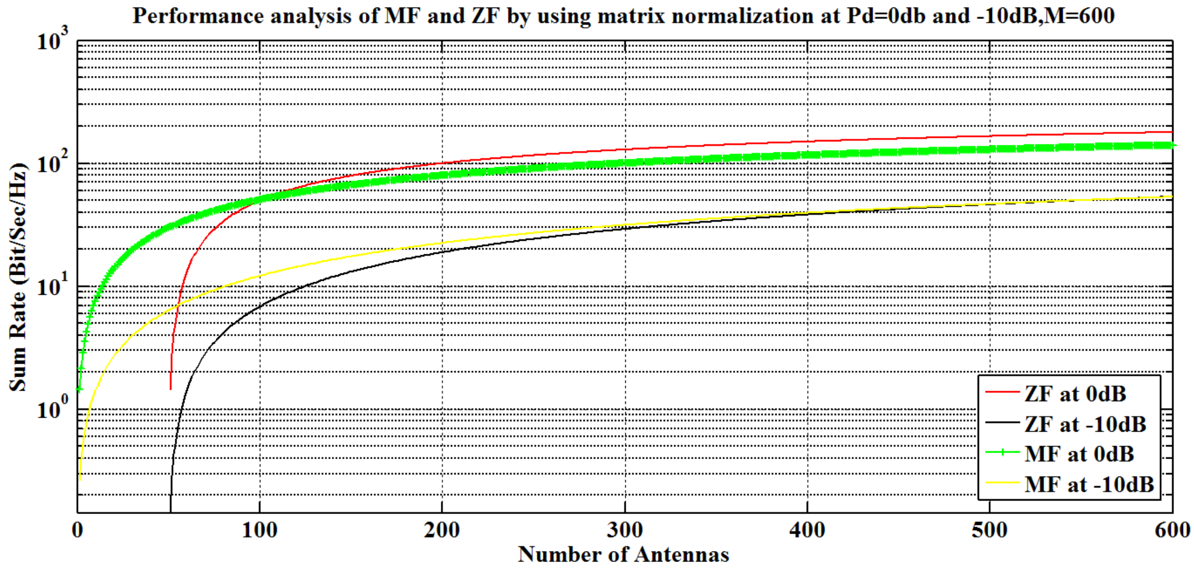


Figure4.9: Performance of MF and ZF using matrix normalization [at k=50 & M=1:600].

Figure (4.10) ; it witch (k=100) it can be seen from the results that MF gives better performance at low power and better performance at high value of power when the number of antenna less than 571 antennas and better performance at high power when then number of antenna less than 103 .On the other hand, the ZF gives the better performance at high value of power when the number of antenna more than 570 antenna and better performance at low power when then number of users more than 102 .

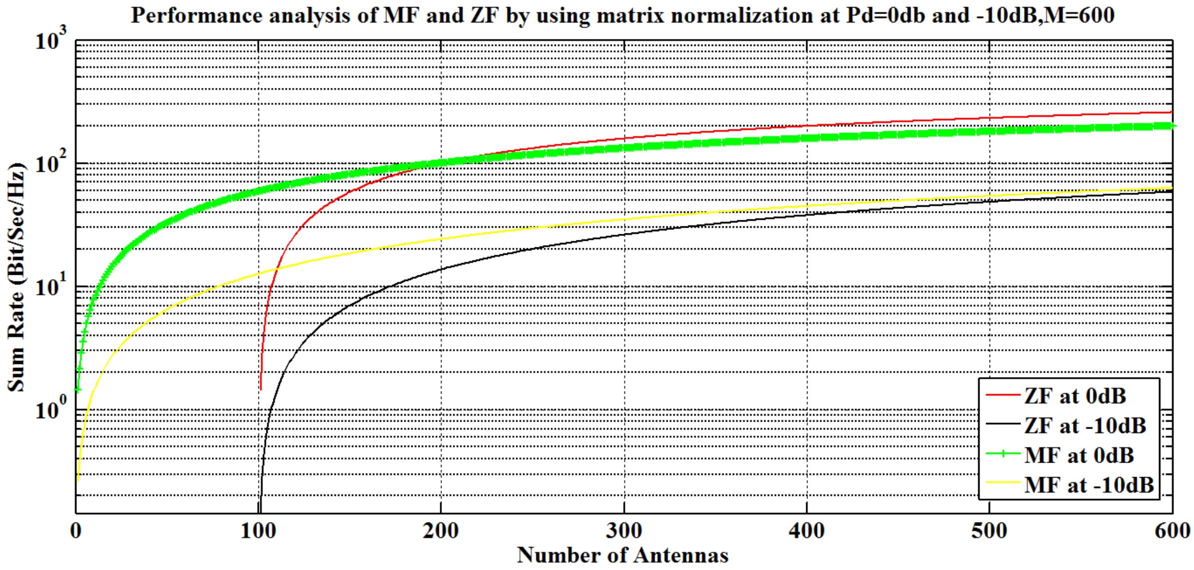


Figure 4.10: Performance of MF and ZF using matrix normalization [at k=100 & M=1:600].

In Figure (4.11) shows k=150 it can be seen from the results that MF gives better performance at low power and better performance at high value of power when the number of antenna less than 303 antennas .On the other hand, the ZF gives the better performance at high value of power when the number of antenna more than 302 antenna.

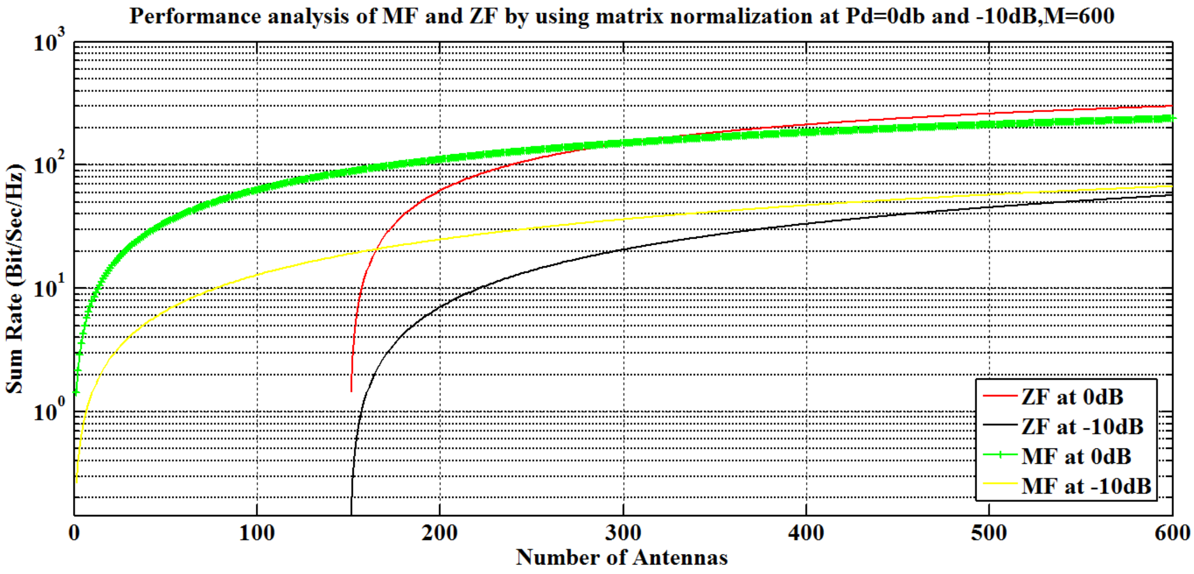


Figure 4.11: Performance of MF and ZF using matrix normalization [at k=150 & M=1:600].

In Figure (4.12) shows k=200 it can be seen from the results that MF gives better performance at low power and better performance at high value of power when the number of antenna less than 403 antennas .On the other hand, the ZF gives the better performance at high value of power when the number of antenna more than 402 antenna.

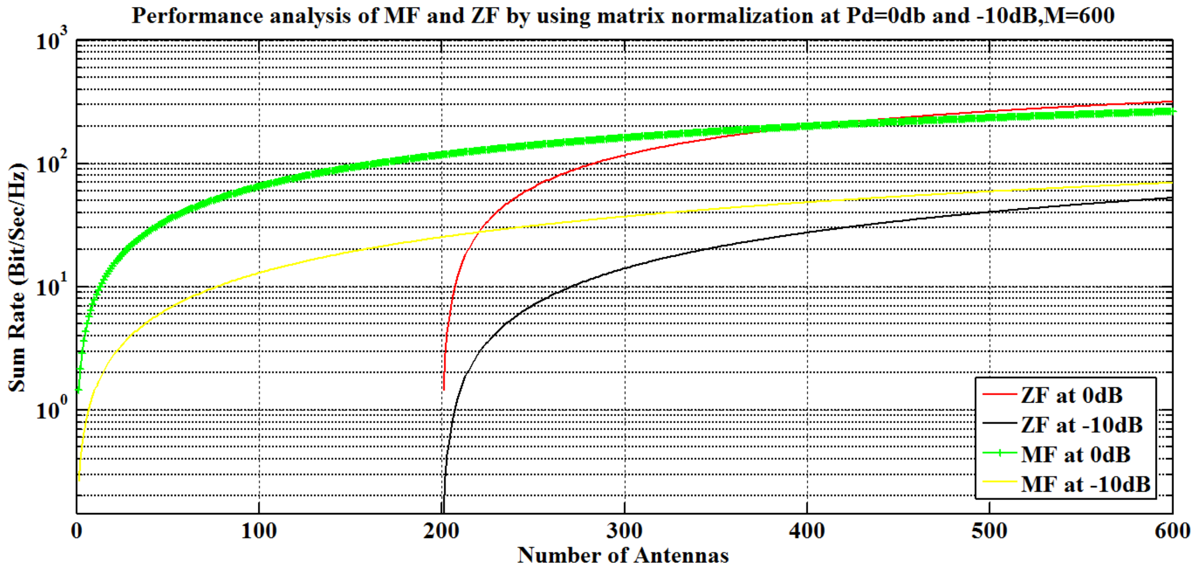


Figure 4.12: Performance of MF and ZF using matrix normalization [at k=200 & M=1:600].

4.2.2 Performance of MF and ZF Using Matrix Normalization at M=300 , 400 , 500 , 600 Antennas

Figures (4.13 , 4.14 , 4.15 , 4.16) shows the achievable sum rate across the entire user range according to equations (3.13 and 3.15).This scheme consists of the number of antennas M =300 , 400 , 500 and 600 and the number of users K = 1:200 .

Figure (4.13) ; it which (M=300) , it can be seen from the results that MF gives the better performance at low value of power and better performance at high power when the number of user less than 122 users .On the other hand, ZF gives better performance when number of user more than 121 at high power .

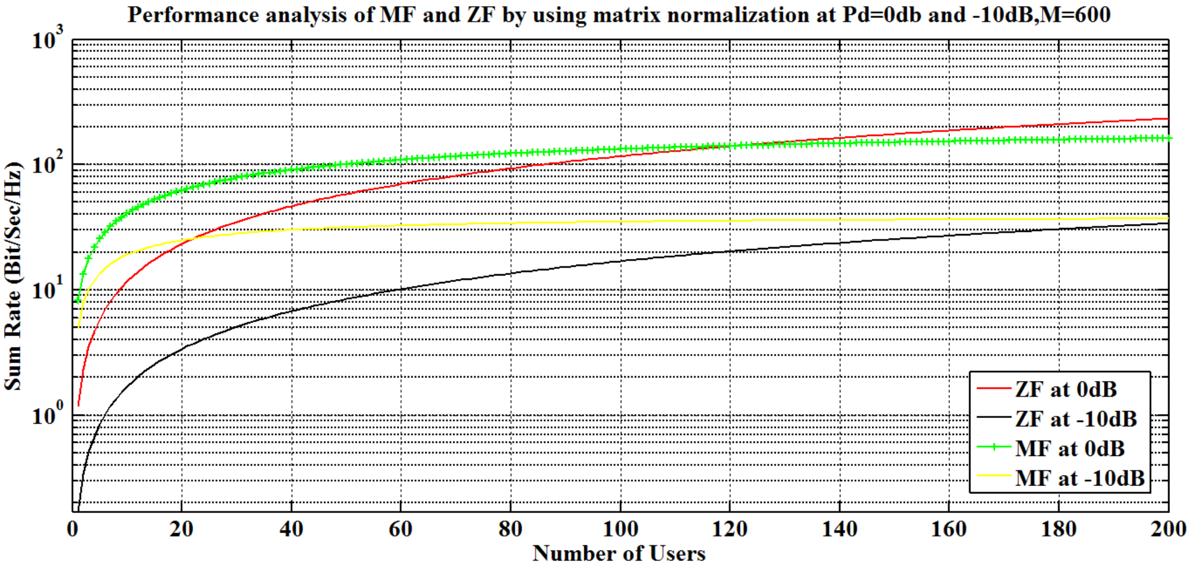


Figure 4.13: Performance of MF and ZF using matrix normalization [at k=1:200 & M=300].

Figures (4.14) shows it can be seen from the results that MF gives the better performance at low value of power when the number of user less than 184 users and less than 102 users at high power .On the other hand, ZF gives better performance when number of user more than 101 at high power and more than 183 user at low power.

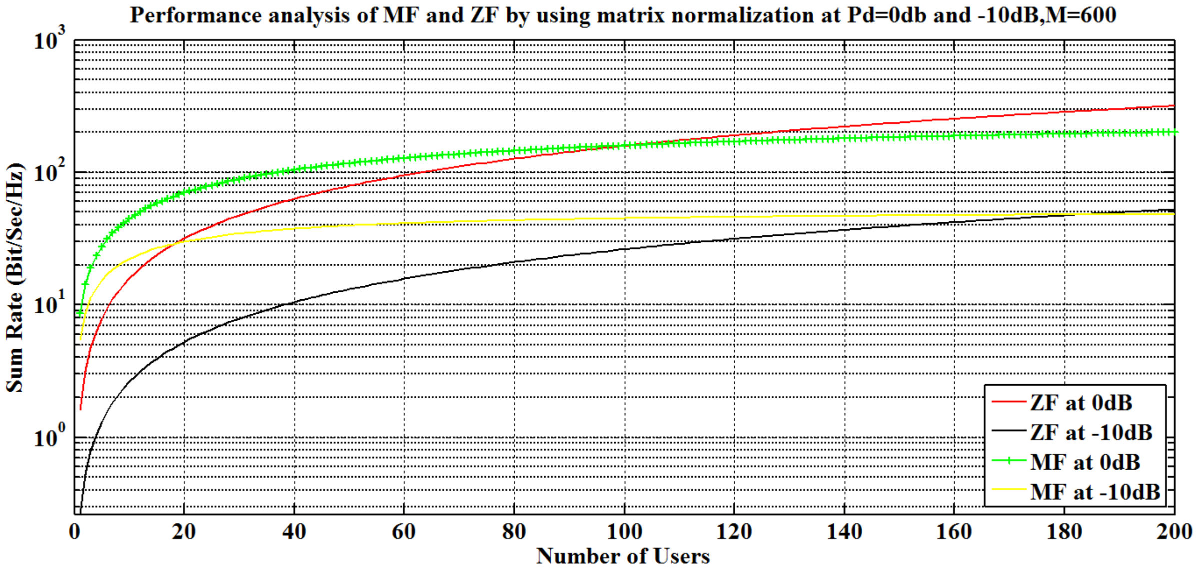


Figure 4.14: Performance of MF and ZF using matrix normalization [at k=1:200 & M=400].

In Figure (4.15) shows M=500 it can be seen from the results that MF gives the better performance at low value of power when the number of user less than 167 users and less than 92 users at high power .On the other hand, ZF gives better performance when number of user more than 91 at high power and more than 166 at low power.

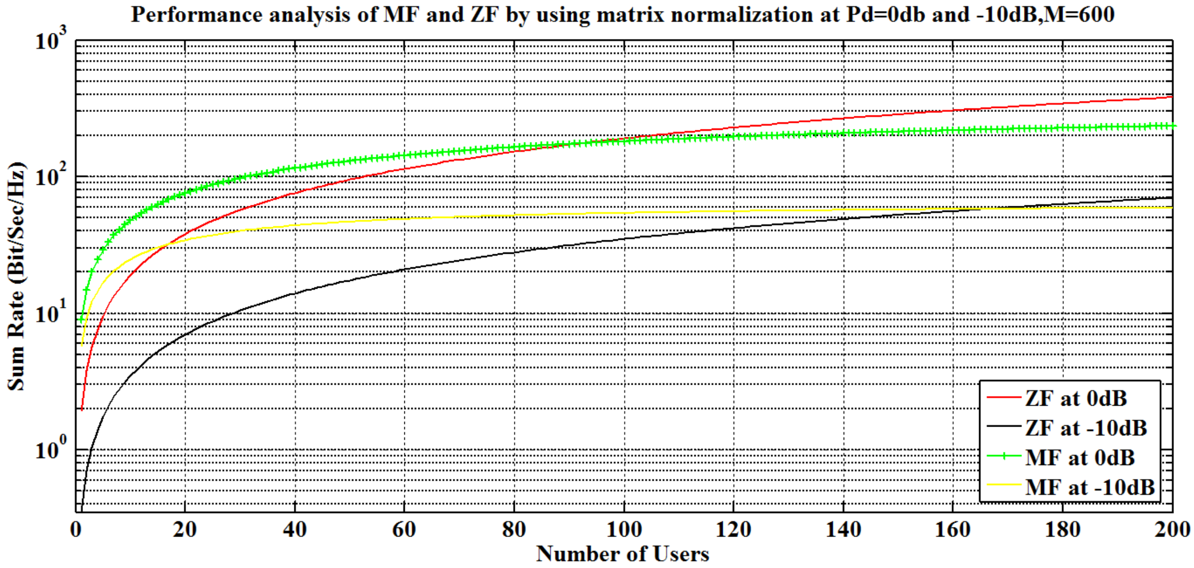


Figure 4.15: Performance of MF and ZF using matrix normalization [at k=1:200 & M=500].

Figure (4.16) ; it which (M=600) , it can be seen from the results that MF gives the better performance at low value of power when the number of user less than 157 users and less than 87 users at high power .On the other hand, ZF gives better performance when number of user more than 86 at high power and more than 156 at low power.

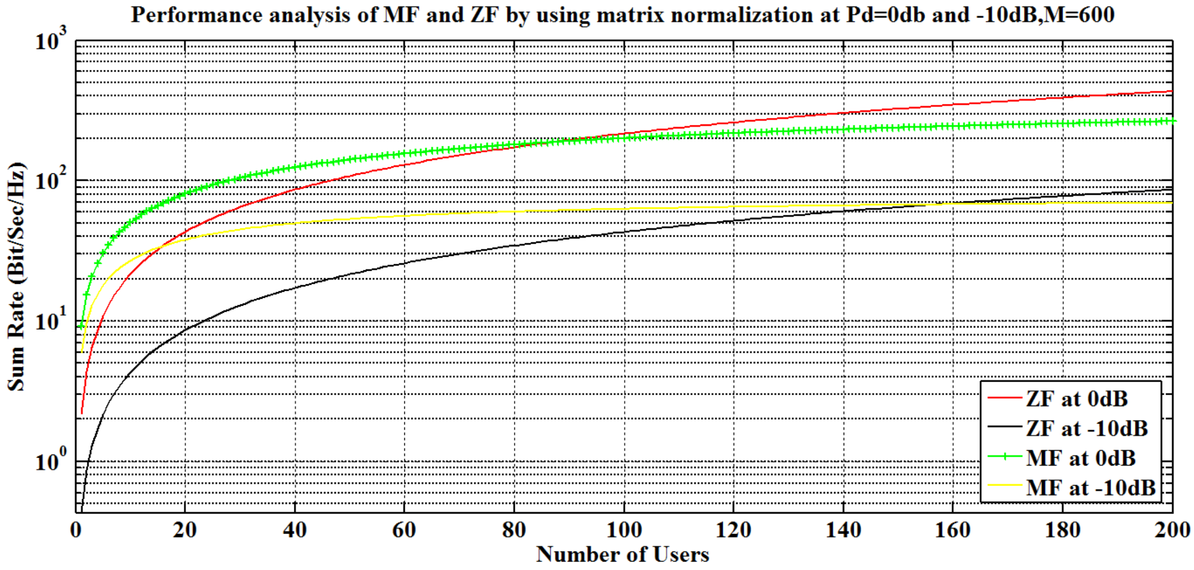


Figure 4.16: Performance of MF and ZF using matrix normalization [at k=1:200 & M=600].

4.3 Vector Normalization Versus Matrix Normalization

This part shows the comparison of Vector normalization versus Matrix normalization for match filter, zero forcing and maximum ratio transmission. Consider number of users k=200 users, number of antennas equal 300 and 600 and two value of power (0dB & -10dB).

4.3.1 Vector Normalization Versus Matrix Normalization at 600 Antennas and 0dB

Figure (4.17) shows the achievable sum rate across the whole antenna range according to equations (3.12& 3.13 for ZF and 3.15 for MF).This scheme consists of the number of antennas M =1:600, number of users K = 200 and downlink transmitted power 0dB. It can be seen from the result that vector/matrix normalization for MF gives the same and gives better performance than ZF when number of antennas less than 401 antennas at high value of power .On the other sides, vector/matrix normalization for ZF is better than MF vector/Matrix normalization when the number of antennas greater than 400 antennas .

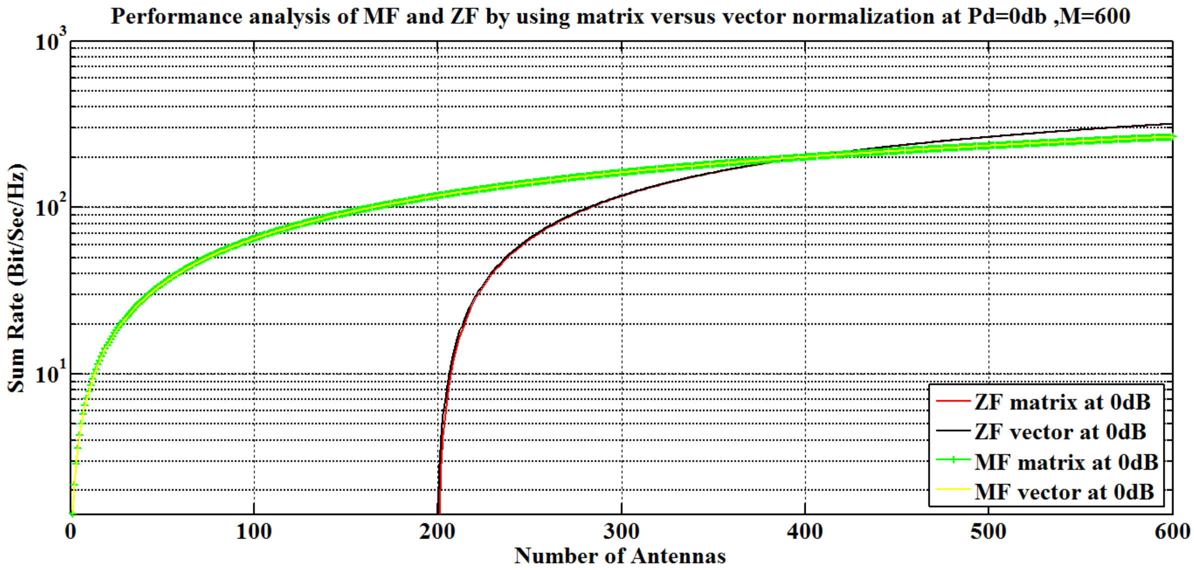


Figure 4.17: Matrix normalization vs. Vector normalization [at k=200, M=1:600 &P_{ad} =0dB].

4.3.2 Vector Normalization Versus Matrix Normalization at 600 Antennas and -10dB

Figure (4.18) shows the achievable sum rate across the whole antenna range according to equations (3.12& 3.13 for ZF and 3.15 for MF).This scheme consists of the number of antennas M =1:600, number of users K = 200 and downlink transmitted power -10 dB. It can be seen from the result that vector/matrix normalization for MF matrix normalization gives the same performance and better performance than vector/matrix normalization for ZF .

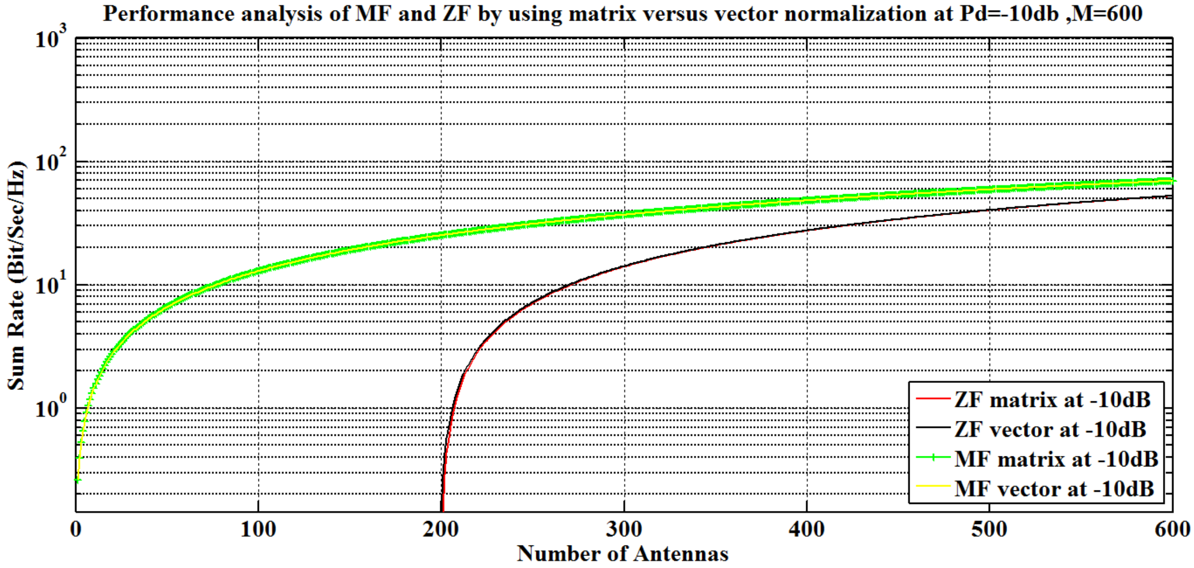


Figure 4.18: Matrix normalization vs. Vector normalization [at k=200, M=1:600 &P_{ad} =-10dB].

The Table 4.3 below gives the achievable sum rate performance of 1 to 600 numbers of antenna and fixed number of various users 50 , 100 , 150 and 200 which shows the numerical result of this study. Also Table 4.4 illustrates the achievable sum rate performance of 1 to 300 numbers of antennas and fixed number of different users 50 , 100 , 150 and 200 which shows the numerical result. This numerical result can specify the best performance of two linear pre-coding techniques in which all techniques gives better performance for different number of antennas and users only they differ in best performance of the rest.

Table 4.3 Achievable sum rate improvements (in bps/Hz) for MF and ZF at M=600antennas & K=50 , 100 , 150 and 200 .

Number of Antennas = 600																
Pre-coding Technique	Zero Forcing								Matched Filter							
Normalization Method	Matrix				Vector				Matrix				Vector			
Number of Users	50	100	150	200	50	100	150	200	50	100	150	200	50	100	150	200
Achievable sum rate at 0 dB	108.3	216.6	324.9	433.2	108.4	216.8	325.3	433.7	141.0	200.7	238.4	265.1	141.0	200.7	238.4	265.1
Achievable sum rate at - 10 dB	21.5	43.1	64.7	86.3	21.6	43.2	64.8	86.5	53.3	62.9	67.2	69.7	53.3	62.9	67.2	69.7

Table 4.4 Achievable sum rate improvements (in bps/Hz) for MF and ZF at M=300antennas & K=50 , 100 , 150 and 200 .

Number of Antennas = 300																
Pre-coding Technique	Zero Forcing								Matched Filter							
Normalization Method	Matrix				Vector				Matrix				Vector			
Number of Users	50	100	150	200	50	100	150	200	50	100	150	200	50	100	150	200
Achievable sum rate at 0 dB	108.3	216.6	174.9	233.2	58.5	117.1	175.	234.2	100.7	132.9	150.7	162.1	100.7	132.9	150.7	162.1
Achievable sum rate at - 10 dB	8.4	16.9	25.3	33.8	8.5	17.0	25.5	34.0	31.5	34.9	36.2	37.0	31.5	34.9	36.2	37.0

CHAPTER FIVE

CONCLUSION AND RECOMMENDATIONS

5.1 Conclusion

This thesis investigate the achievable sum rate of two linear pre-coding techniques: zero forcing and match filter. The analysis of these pre-coding depends on matrix normalization and vector normalization at low and high power. The analysis assumes different number of antennas with fixed number of users and different number of users with fixed number of antennas for all scenarios. The result found that in high power, the achievable sum rate improvement for ZF vector normalization and matrix normalization are (433.7451 bps/Hz, 433.2624bps/Hz) respectively and in low power (86.5119 bps/Hz 86.3519 bps/Hz) respectively, for MF vector normalization and MF matrix normalization are equal , in high power (265.1079 bps/Hz) and in low power (69.7158 bps/Hz).Simulations results show that using linear pre-coding techniques and increasing number of base station antennas enhance system performance. In conclusion ZF has better performance at vector/matrix normalization in high power (cell centre) and in low power (cell boundary) , MF has better performance in vector normalization and ZF has better performance in matrix normalization.

5.2 Recommendations

The recommendations of this study are to investigate the performance of linear pre-coding techniques by taking into account the imperfect channel state information (CSI) and to Study the nonlinear pre-coding techniques for massive MIMO system. Compare the performance the linear and non linear pre-coding techniques for massive MIMO system. Study the linear pre-coding techniques by taking into account pilot contamination phenomena.

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APPENDIX

Appendix A : MATLAB code for Matrix normalization (achievable sum_rate vs number of K users)at 0dB and -10 dB for ZF and MF.

```
%Matrix normalization (Sum rate vs. number of K users) at -0dB
%and -10 dB for ZF  and MF.

M =input('inter the number of base station antennas:');%M=1:1:600;

K = input('inter the number of users:');% K=200;

Pd_dB = -0;
Pd1_dB = -10;

Pd = 10^(Pd_dB/10);
Pd1 = 10^(Pd1_dB/10);

%Zero Forcing(ZF)

Rsum_zf = K.*log2(1 + Pd*(M-K)/K);
Rsum_zf1 = K.*log2(1 + Pd1*(M-K)/K);

% Matiching Filter (MF)

Rsum_MF=  K.* log2(1 + Pd*(M+1)./(Pd*(K-1)+K));
Rsum_MF1 =K.* log2(1 + Pd1*(M+1)./(Pd1*(K-1)+K));

figure;

P1=semilogy(M,Rsum_zf,'r-','LineWidth',2);

hold on ;

P2=semilogy(M,Rsum_zf1,'k-','LineWidth',2);
P3=semilogy(M,Rsum_MF,'g-+','LineWidth',2);
P4=semilogy(M,Rsum_MF1,'y-','LineWidth',2);
grid on
xlabel('Number of Antennas');ylabel('Sum Rate (Bit/Sec/Hz)');
sTitle = sprintf('Performance analysis of MF and ZF by using matrix
normalization at Pd=0db and -10dB,M=600');

title(sTitle);

leg1='ZF at 0dB';
leg2='ZF at -10dB';
leg3='MF at 0dB';
leg4='MF at -10dB';
legend(leg1,leg2,leg3,leg4);
axis([0,600,0,1000]);
```

Appendix B : MATLAB code for Matrix normalization (achievable sum_rate vs number of M Antennas)at 0dB and -10 dB for ZF and MF.

```
%Matrix normalization (Sum rate vs. number of M Antennas) at 0dB
%and -10 dB for ZF and MF.

M =input('inter the number of base station antennas:');%M=600;

K = input('inter the number of users:');% K=1:1:200;

Pd_dB = 0;
Pd1_dB = -10;

Pd = 10^(Pd_dB/10);
Pd1 = 10^(Pd1_dB/10);

%Zero Forcing(ZF)

Rsum_zf = K.*log2(1 + Pd*(M-K)/K);
Rsum_zf1 = K.*log2(1 + Pd1*(M-K)/K);

% Matiching Filter (MF)

Rsum_MF=  K.* log2(1 + Pd*(M+1)./(Pd*(K-1)+K));
Rsum_MF1 =K.* log2(1 + Pd1*(M+1)./(Pd1*(K-1)+K));

figure;

P1=semilogy(K,Rsum_zf,'r-','LineWidth',2);

hold on ;

P2=semilogy(K,Rsum_zf1,'k-','LineWidth',2);
P3=semilogy(K,Rsum_MF,'g-+','LineWidth',2);
P4=semilogy(K,Rsum_MF1,'y-','LineWidth',2);
grid on
xlabel('Number of Users');ylabel('Sum Rate (Bit/Sec/Hz)');
sTitle = sprintf('Performance analysis of MF and ZF by using matrix
normalization at Pd=0db and -10dB,M=600');

title(sTitle);

leg1='ZF at 0dB';
leg2='ZF at -10dB';
leg3='MF at 0dB';
leg4='MF at -10dB';
legend(leg1,leg2,leg3,leg4);

axis([0,200,0,1000]);
```

Appendix C : MATLAB code for Vector normalization (achievable sum_rate vs number of K users)at 0dB and -10 dB for ZF and MF.

```
%Vector normalization (Sum rate vs. number of K users) at 0dB
%and -10 dB for ZF and MF.

M =input('inter the number of base station antennas:');%M=1:1:600;

K = input('inter the number of users:');% K=200;

Pd_dB = 0;
Pd1_dB = -10;

Pd = 10^(Pd_dB/10);
Pd1 = 10^(Pd1_dB/10);

%Zero Forcing(ZF)

Rsum_zf = K.*log2(1 + Pd*(M-K+1)/K);
Rsum_zf1 = K.*log2(1 + Pd1*(M-K+1)/K);

% Matiching Filter (MF)

Rsum_MF=  K.* log2(1 + Pd*(M+1)./(Pd*(K-1)+K));
Rsum_MF1 =K.* log2(1 + Pd1*(M+1)./(Pd1*(K-1)+K));

figure;

P1=semilogy(M,Rsum_zf,'r-','LineWidth',2);

hold on ;

P2=semilogy(M,Rsum_zf1,'k-','LineWidth',2);
P3=semilogy(M,Rsum_MF,'g-+','LineWidth',2);
P4=semilogy(M,Rsum_MF1,'y-','LineWidth',2);
grid on
xlabel('Number of Antennas');ylabel('Sum Rate (Bit/Sec/Hz)');
sTitle = sprintf('Performance analysis of MF and ZF by using vector
normalization at Pd=0db and -10dB,M=600');

title(sTitle);

leg1='ZF at 0dB';
leg2='ZF at -10dB';
leg3='MF at 0dB';
leg4='MF at -10dB';
legend(leg1,leg2,leg3,leg4);

axis([0,600,0,1000]);
```

Appendix D : MATLAB code for Matrix normalization (achievable sum_rate vs number of M Antennas)at 0dB and -10 dB for ZF and MF.

```
%Vector normalization (Sum rate vs. number of K users) at 0dB
%and -10 dB for ZF and MF.

M =input('inter the number of base station antennas:');%M=600;

K = input('inter the number of users:');% K=1:1:200;

Pd_dB = 0;
Pd1_dB = -10;

Pd = 10^(Pd_dB/10);
Pd1 = 10^(Pd1_dB/10);

%Zero Forcing(ZF)

Rsum_zf = K.*log2(1 + Pd*(M-K+1)/K);
Rsum_zf1 = K.*log2(1 + Pd1*(M-K+1)/K);

% Matiching Filter (MF)

Rsum_MF=  K.* log2(1 + Pd*(M+1)./(Pd*(K-1)+K));
Rsum_MF1 =K.* log2(1 + Pd1*(M+1)./(Pd1*(K-1)+K));

figure;

P1=semilogy(K,Rsum_zf,'r-','LineWidth',2);

hold on ;

P2=semilogy(K,Rsum_zf1,'k-','LineWidth',2);
P3=semilogy(K,Rsum_MF,'g-+','LineWidth',2);
P4=semilogy(K,Rsum_MF1,'y-','LineWidth',2);
grid on
xlabel('Number of Users');ylabel('Sum rate (Bit/Sec/Hz)');
sTitle = sprintf('Performance analysis of MF and ZF by using vector
normalization at Pd=0db and -10dB,K=200');

title(sTitle);

leg1='ZF at 0dB';
leg2='ZF at -10dB';
leg3='MF at -0dB';
leg4='MF at -10dB';
legend(leg1,leg2,leg3,leg4);

axis([0,200,0,1000]);
```

Appendix E : MATLAB code for Vector versus Matrix normalization (achievable sum_rate vs number of M antennas)at 0dB for ZF and MF.

```
%Matrix versus vector normalization (Sum rate vs. number of M Antennas) at
0dB for ZF  and MF.

M =input('inter the number of base station antennas:');%M=1:1:600;

K = input('inter the number of users:');% K=200;

Pd_dB = 0;
Pd1_dB = 0;

Pd = 10^(Pd_dB/10);
Pd1 = 10^(Pd1_dB/10);

%Zero Forcing(ZF)

Rsum_zf = K.*log2(1 + Pd*(M-K)/K);
Rsum_zf1 = K.*log2(1 + Pd1*(M-K+1)/K);

% Matiching Filter (MF)

Rsum_MF=  K.* log2(1 + Pd*(M+1)./(Pd*(K-1)+K));
Rsum_MF1 =K.* log2(1 + Pd1*(M+1)./(Pd1*(K-1)+K));

figure;

P1=semilogy(M,Rsum_zf,'r-','LineWidth',2);

hold on ;

P2=semilogy(M,Rsum_zf1,'k-','LineWidth',2);
P3=semilogy(M,Rsum_MF,'g-+','LineWidth',2);
P4=semilogy(M,Rsum_MF1,'y-','LineWidth',2);
grid on
xlabel('Number of Antennas');ylabel('Sum Rate (Bit/Sec/Hz)');
sTitle = sprintf('Performance analysis of MF and ZF by using matrix versus
vector normalization at Pd=0db ,M=600');

title(sTitle);

leg1='ZF matrix at 0dB';
leg2='ZF vector at 0dB';
leg3='MF matrix at 0dB';
leg4='MF vector at 0dB';
legend(leg1,leg2,leg3,leg4);
axis([0,600,0,1000]);
```

Appendix F : MATLAB code for Matrix versus vector normalization (achievable sum_rate vs number of M antennas)at -10 dB for ZF and MF.

```
%Matrix versus vector normalization (Sum rate vs. number of M Antennas) at
-10dB for ZF  and MF.

M =input('inter the number of base station antennas:');%M=1:1:600;

K = input('inter the number of users:');% K=200;

Pd_dB = -10;
Pd1_dB = -10;

Pd = 10^(Pd_dB/10);
Pd1 = 10^(Pd1_dB/10);

%Zero Forcing(ZF)

Rsum_zf = K.*log2(1 + Pd*(M-K)/K);
Rsum_zf1 = K.*log2(1 + Pd1*(M-K+1)/K);

% Matiching Filter (MF)

Rsum_MF=  K.* log2(1 + Pd*(M+1)./(Pd*(K-1)+K));
Rsum_MF1 =K.* log2(1 + Pd1*(M+1)./(Pd1*(K-1)+K));

figure;

P1=semilogy(M,Rsum_zf,'r-','LineWidth',2);

hold on ;

P2=semilogy(M,Rsum_zf1,'k-','LineWidth',2);
P3=semilogy(M,Rsum_MF,'g-+','LineWidth',2);
P4=semilogy(M,Rsum_MF1,'y-','LineWidth',2);
grid on
xlabel('Number of Antennas');ylabel('Sum Rate (Bit/Sec/Hz)');
sTitle = sprintf('Performance analysis of MF and ZF by using matrix versus
vector normalization at Pd=-10db ,M=600');

title(sTitle);

leg1='ZF matrix at -10dB';
leg2='ZF vector at -10dB';
leg3='MF matrix at -10dB';
leg4='MF vector at -10dB';
legend(leg1,leg2,leg3,leg4);
axis([0,600,0,1000]);
```