Optimal Speed Control for Direct Current Motors Using Linear Quadratic Regulator

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Abstract: Direct Current (DC) motors have been extensively used in many industrial applications. Therefore, the control of the speed of a DC motor is an important issue and has been studied since the early decades in the last century. This paper presents a comparison of time response specification between conventional Proportional-Integral-Derivatives (PID) controller and Linear Quadratic Regulator (LQR) for a speed control of a separately excited DC motor. The goal is to determine which control strategy delivers better performance with respect to DC motor's speed. Performance of these controllers has been verified through simulation using MATLAB/SIMULINK software package. According to the simulation results, liner quadratic regulator method gives the better performance, such as settling time, steady state error and overshoot compared to conventional PID controller. This shows the superiority of liner quadratic regulator method over conventional PID controller.

Keywords: *Optimal Control, Linear Quadratic Regulator, Proportional-Integral-Derivative Controller, Direct Current Motors, Speed Control.*

المستخلص: محركات التيار المستمر (DC) تستخدم على نطاق واسع فى العديد من التطبيقات الصناعية. لذلك فإن التحكم فى سرعة محركات التيار المستمر قضية مهمة تمت دراستها منذ العقود الأولى للقرن الماضى. هذه الورقة تقدم مقارنة بين مواصفات الإستجابة الزمنية بين الحاكمة التناسبية-التكاملية-التفاضلية (PID) و المنظم التربيعى الخطى (LQR) للتحكم فى سرعة محرك التيار المستمر ذو التغذية المنفصلة. الهدف من ذلك تحديد إستراتيجية التحكم التى تعطى أداء أفضل فيما يتعلق بالتحكم فى سرعة محرك التيار المستمر. وقد تم التحقق من أداء هذه المتحكمات من خلال المحاكاة بإستخدام حزمة برنامج MATLAB/SIMULINK. وفقاً لنتائج المحاكاة نجد أن طريقة المنظم التربيعى الخطى تعطى أداء أفضل مثل زمن الترسيخ، الخطأ عند حالة الإستقرار وتجاوز الهدف مقارنة مع الحاكمة التقليدية PID. وهذا يدل على تفوق طريقة المنظم التربيعى الخطى على الحاكمة التقليدية PID.

ntroduction:

Electrical derives involving various types of DC motors turn the wheel of industry. The main reason for their popularity is the ability to control their torque and flux easily and independently. Therefore, DC motors are comprehensively used in various industrial applications such as electrical equipment, computer peripherals, robotic manipulators, actuators, steel rolling mills, electrical vehicles, and home appliances. Its applications spread from low horse power to the multi-mega watt due to its wide power, torque, speed ranges, high efficiency, fast response, and simple and continuous control characteristics [1-4]. Controlling the speed of a DC motor is a pivotal issue. The speed of DC motor can be changed by controlling the armature and field voltages. In this paper, the controller is designed to control the armature voltage while the field voltage is fixed as a constant. Over the past decades, many techniques have been developed for the DC motor control. Some of these methods were based on classical and also intelligent approaches [5-10]. For DC motors, factors such as unknown load characteristic and parameter variation influence seriously the controlling effect of speed controller. The most commonly used controller for the speed control of DC motors is conventional PID controller. Traditional PID controllers have been successfully used in control applications since 1940s and are the most often used industrial controller today. Conventional PID controllers have several important features. The reason is that the conventional PID controller is easy to implement either by hardware or by software. No deep mathematical theory is necessary to understand how the conventional PID controller works, so everybody is able to imagine what is happening inside the controller during the control process. Furthermore, it has the ability to eliminate steady state offset trough integral action and it can anticipate the changes through derivative action. In addition to this, traditional PID controllers have very simple control structure and inexpensive cost. In spite of the major features of the fixed PID controller, it has some disadvantages such as the high starting overshoot in speed, the sensitivity to controller gains and the sluggish response due to sudden change in load torque disturbance. Therefore, a great deal of attention has been focused on adaptive or self-tuning of conventional PID controller gains. Tuning PID controller parameters is very difficult, poor robustness; therefore, it's difficult to achieve the optimal state under field conditions in the actual production. In order to overcome some problems that faced by conventional PID controller and achieve accurate control performance of speed control of a DC motor, the other type of control methods can be developed such as linear quadratic regulator [11-16].

Linear quadratic regulator design technique is well known in modern optimal control theory and has been widely used in many applications. It has a very nice robustness property. This attractive property appeals to the practicing engineers. Thus, the linear quadratic regulator theory has received considerable attention since 1950s. The liner quadratic regulator technique seeks to find the optimal controller that minimizes a given cost function (performance index). This cost function is parameterized by two matrices, Q and R, that weight the state vector and the system input respectively. These weighting matrices regulate the penalties on the excursion of state variables and control signal. One practical method is to Q and R to be diagonal matrix. The value of the elements in Q and R is related to its contribution to the cost function. To find the control law, Algebraic Riccati Equation (ARE) is first solved, and an optimal feedback gain matrix, which will lead to optimal results evaluating from the defined cost function is obtained [17-20].

In this paper, to achieve accurate control performance of speed control of DC motor, optimal linear quadratic regulator technique is presented. The remainder of the paper is organized as follows: at first the dynamic model of the separately excited DC motor is briefly reviewed for the purpose of speed control. The next section the basic concept and design of linear quadratic regulator controller is briefly reviewed. Then the simulation results are presented. Finally, the last section states the main conclusion.

Dynamic Model of DC Motor:

Direct current motors are widely used for various industrial and domestic applications. Examples are as robotic and actuator for automation process, mechanical motion, and others. Accurate speed control of the DC motor is the basic requirement in such applications. There are two main ways of controlling a DC motor: The first one named armature control consists of maintaining the stator magnetic flux constant, and varying the armature current. Its main advantage is a good torque at high speeds and its disadvantage is high energy losses. The second way is called field control, and has a constant voltage to set up the armature current, while a variable voltage applied to the stator induces a variable magnetic flux. Its advantages are energy efficiency, inexpensive controllers and its disadvantages are a torque that decreases at high speeds. In this paper, the separately excited DC motor model is chosen according to its good electrical and mechanical performances more than other DC motor models. The electric circuit of the separately excited DC motor is shown in figure 1. Objective is to control the speed of the separately excited DC motor by armature voltage control [1-4].

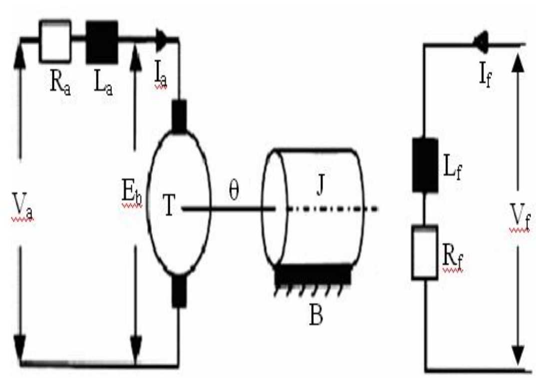

Figure 1: A separately excited DC motor model

Assuming constant field excitation the armature circuit electrical equation is written as:

$$V_a = R_a i_a + L_a \frac{di_a}{dt} + E_b$$
$$V_a = R_a i_a + L_a \frac{di_a}{dt} + K_b \omega \qquad (1)$$

Where V_a is the input terminal voltage (armature voltage) in volt, E_b is the back emf in volt, R_a is the armature resistance in ohm, L_a is the armature inductance in H. K_b is the back emf constant in Vs/rad, ω is represents angular speed in rad/s, and i_a is the armature current in A. The dynamics of the mechanical system is given by the following torque balance equation:

$$T = K_T i_a = J \frac{d\omega}{dt} + B\omega$$
(2)

Where J is the moment of inertia of the motor in kgm^2/s^2, T is the motor torque in Nm, B is the viscous friction coefficient in Nms, and K_T is the torque factor constant in Nm/A. Equation (1) and equation (2) are rearranged to obtain:

$$\frac{di_a}{dt} = -\frac{R_a}{L_a} i_a - \frac{K_b}{L_a}\omega + \frac{V_a}{L_a} \qquad (3)$$

$$\frac{d\omega}{dt} = \frac{K_T}{J} i_a - \frac{B}{J}\omega \qquad (4)$$

To design a desired controller using the linear quadratic regulator technique, the system must first be expressed in the state space form. In the state space model of a separately excited DC motor, the equation (3) and equation (4) can be expressed by choosing the angular speed (ω) and armature current (i_a) as state variables and the armature voltage (V_a) as an input. The output is chosen to be the angular speed [1-4].

$$\begin{bmatrix} \frac{di_a}{dt} \\ \frac{d\omega}{dt} \end{bmatrix} = \begin{bmatrix} \frac{-R_a}{L_a} & \frac{-K_b}{L_a} \\ \frac{K_T}{J} & \frac{-B}{J} \end{bmatrix} \begin{bmatrix} i_a \\ \omega \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{L_a} \end{bmatrix} V_a \qquad (5)$$

$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} i_a \\ \omega \end{bmatrix}$$

The physical and functional parameters of the separately excited DC motor used for simulation testing are given in Table1.

Table 1: Parameters of the separately excited DC motor

Parameters	Values
Armature Resistance, R_a	1Ω
Armature Inductance, L_a	0.05H
Moment of Inertia, J	$0.01kgm^2/s^2$
Viscous Friction Coefficient, B	0.0000 3Nms
The Back EMF Constant, K_b	0.023Vs/rad
The Torque Factor Constant, K_T	0.023Nm/A

Design of the LQR Controller:

Linear quadratic regulator design technique is well known in modern optimal control theory and has been widely used in many applications. The standard theory of the optimal control is presented in [17-20]. Under the assumption that all state variables are available for feedback, the LQR controller design method starts with a defined set of states which are to be controlled. In general, the system model can be written in state space equation as follows:

$$\dot{x} = Ax + Bu \qquad (6)$$

Where $x \in R^n$ and $u \in R^m$ denote the state variable, and control input vector, respectively. A is the state matrix of order n×n; B is the control matrix of order n×m. Also, the pair (A, B) is assumed to be such that the system is controllable. The linear quadratic regulator controller design is a method of reducing the performance index to a minimize value. The minimization of it is just the means to the end of achieving acceptable performance of the system. For the design of a linear quadratic regulator controller, the performance index (J) is given by:

$$J = \int_0^\infty \left(x^T Q x + u^T R u \right) dt \qquad (7)$$

Where Q is symmetric positive semi-definite (≥ 0) state weighting matrix of order n×n, and R is symmetric positive definite (> 0) control weighting matrix of order m×m. The choice of the element Q and R allows the relative weighting of individual state variables and individual control inputs as well as relative weighting state vector and control vector against each other. The weighting matrices Q and R are important components of an LQR optimization process. The compositions of Q and R elements have great influences of system performance. The designer is free to choose the matrices Q and R, but the selection of matrices Q and R is normally based on an iterative procedure using experience and physical understanding of the problems involved. Commonly, a trial and error method has been used to construct the matrices Q and R elements. This method is very simple and very familiar in linear quadratic regulator application. However, it takes long time to choose the best values for matrices Q and R. The number of matrices Q and R elements are dependent on the number of state variable (n) and the number of input variable (m), respectively. The diagonal-off elements of these matrices are zero for simplicity. If diagonal matrices are selected, the quadratic performance index is simply a weighted integral of the squared error of the states and inputs. The term in the brackets in equation (7) above are called quadratic forms and are quite common in matrix algebra. Also, the performance index will always be a scalar quantity, whatever the size of Q and R matrices [21-25]. The conventional linear quadratic regulator problem is to find the optimal control input law u^* that minimizes the performance index under the constraints of Q and R matrices. The closed loop optimal control law is defined as:

$$u^* = -Kx \qquad (8)$$

Where K is the optimal feedback gain matrix, and determines the proper placement of closed loop poles to minimize the performance index in equation (7). The feedback gain matrix K depends on the matrices A, B, Q, and R. There are two main equations which have to be calculated to achieve the feedback gain matrix K. Where P is a symmetric and positive definite matrix obtained by solution of the ARE is defined as:

$$A^T P + PA - PBR^{-1}B^T P + Q = 0 \qquad (9)$$

Then the feedback gain matrix K is given by:

$$K = R^{-1}B^T P \qquad (10)$$

Substituting the above equation (8) into equation (6) gives:

$$\dot{x} = Ax - BKx = (A - BK)x \qquad (11)$$

If the eigenvalues of the matrix (A-BK) have negative real parts, such a positive definite solution P always exits.

Simulation Results:

In order to verify the validity of the linear quadratic regulator controller, several simulation tests are carried out using MATLAB/SIMULINK software package. The performance of linear quadratic regulator controller has been investigated and compared with the conventional PID controller. Simulation tests are based on the facts that whether the linear quadratic regulator controller is better and more robust than the traditional PID controller or not. For the comparison, simulation tests of the speed response were performed according to the nominal condition, moment of inertia variation, and armature inductance variation of the separately excited DC motor. To determine the feedback gain matrix K, the elements of the weighting matrices Q and

R are chosen as: Q = [0.2 0;0 0.028] and R = 0.2, respectively. After solving the ARE and substituting into equation (10), the optimal values of control feedback gain matrix K are obtained as K= [0.9742 1.3709]. Figure 2 shows the step responses of speed control of the separately excited DC motor at nominal condition by two controllers. According to the simulation results, linear quadratic regulator method give the better performance compared to traditional PID controller.

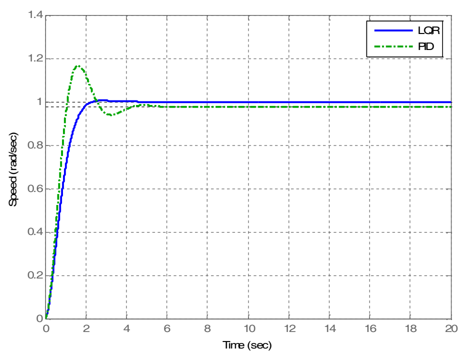

Figure 2: Comparison of output speed responses among LQR and conventional PID controllers

The time response specifications of the conventional PID controller and linear quadratic regulator technique obtained from the simulation of the separately excited DC motor speed control is shown in Table 2. Based on the Table 2, linear quadratic regulator technique has the fastest settling time of 2s while traditional PID controller has the slowest settling time of 4.5s. For the percent overshoot, linear quadratic regulator technique does not have overshoot and conventional PID controller has the greatest value of percent overshoot of 17%. Furthermore, there is no steady state error using linear quadratic regulator controller. However, the rise time for traditional PID controller is smallest value than for linear quadratic regulator controller.

Table 2: Performances metrics for LQR and PID controllers

Time Response Specifications	LQR	PID
Settling Time (T_s)	2s	4.5s
Rise Time (T_r)	2.5s	1.1s
Overshoot %	0	17
Steady State Error (e_{ss})	0	0.03

For high performance applications the proposed linear quadratic regulator scheme should be robust to parameter variations. Changes in the moment of inertia and the armature inductance are investigated through simulations. The simulation studies are undertaken by changing one parameter at a time while keeping other parameters unchanged. The separately excited DC motor is commanded to accelerate from rest to reference speed under no torque load. Figure 3 shows the separately excited DC motor responses of optimal linear quadratic regulator approach and conventional PID controller when the moment of inertia is increased by 100% of its original value, whilst figure 4 depicts the speed response when the armature inductance increased by 100% of its original value.

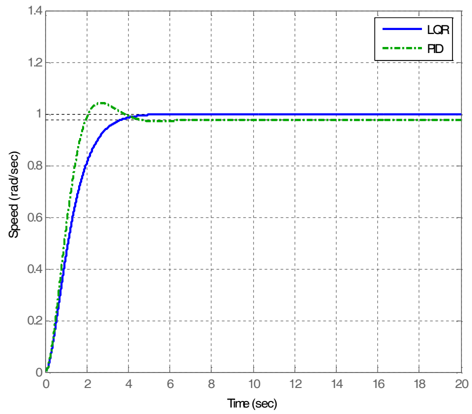

Figure 3: Responses of the DC motor using two controllers with variation in the moment of inertia

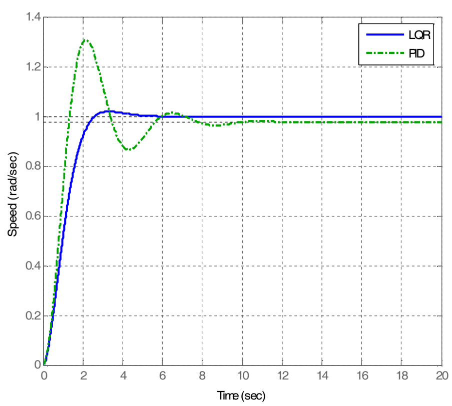

Figure 4: Responses of the DC motor using two controllers with variation in the armature inductance

From figure 3, it can be seen that the increment of the moment of inertia does not impose any significant effect on the performance of the linear quadratic regulator technique but only affects the rise time. A comparison is illustrated in Table 3 between LQR and PID controller quantitatively.

Table 3: Performances of two controllers under increased J

Time Response Specifications	LQR	PID
Settling Time (T_s)	3.6s	7.8s
Rise Time (T_r)	5.1s	2.1s
Overshoot %	0	4
Steady State Error (e_{ss})	0	0.03

It is very much clear from figure 4 that the proposed linear quadratic regulator controller is less sensitive to parametric variations and a robust tracking performance is achieved in presence of the uncertain parameters. Furthermore, it can be noted that the increase in armature inductance causes greatest value of percent overshoot, settling time, and steady state error in classical PID controller than optimal linear quadratic regulator controller which is affected only by slowest rise time. The time response parameters percent overshoot; settling time, rise time, and steady state error for LQR and PID controller are presented in Table 4.

Table 4: Performances of two controllers under increased L_a

Time Response Specifications	LQR	PID
Settling Time (T_s)	3.3s	10.3s
Rise Time (T_r)	2.6s	1.4s
Overshoot %	2	31
Steady State Error (e_{ss})	0	0.03

Conclusions:

Optimal LQR strategy and conventional PID controller have been considered in this paper for controlling the speed of a separately excited DC motor. The performance of the two controllers is validated through simulations. A number of simulation results are presented for comparison. Based on the comparative simulation results, one can conclude that the linear quadratic regulator controller realises a good dynamic behaviour of the separately excited DC motor with a rapid settling time, no overshoot, and zero steady state error compared to conventional PID controller under nominal condition. But the comparison between the speed control of the separately excited DC motor by linear quadratic regulator technique and conventional PID controller shows clearly that the linear quadratic regulator technique gives better performances than conventional PID controller against parameter variations. Furthermore, the simulation results so obtained show that the conventional PID controller gives greatest value of percent overshoot and longer settling time.

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