

CHAPTER THREE

DC MOTOR OVERVIEW AND MATHEMATICAL MODEL

3.1 Introduction

Almost every mechanical movement that we see around us is accomplished by an electric motor. Electric machines are a means of converting energy. Motors take electrical energy and produce mechanical energy.

Electric motors are used to power hundreds of devices we use in everyday life. Motors come in various sizes.

Huge motors that can take loads of 1000's of Horsepower are typically used in the industry. Some examples of large motor applications include elevators, electric trains, hoists, and heavy metal rolling mills. Examples of small motor applications include motors used in automobiles, robots, hand power tools and food blenders.

Micro-machines are electric machines with parts the size of red blood cells, and find many applications in medicine.

Electric motors are broadly classified into two different categories: DC (Direct Current) and AC (Alternating Current). Within these categories are numerous types, each offering unique abilities that suit them well for specific applications.

In most cases, regardless of type, electric motors consist of a stator (stationary field) and a rotor (the rotating field or armature) and operate through the interaction of magnetic flux and electric current to produce rotational speed and torque. DC motors are distinguished by their ability to operate from direct current.

There are different kinds of D.C. motors, but they all work on the same principles. In this chapter, we will study their basic principle of operation and their characteristics. It's important to understand motor characteristics so we can choose the right one for our application requirement [7].

3.2 Principle of Operation

The DC motor works on the principle of passing a current in a conductor inside a magnetic field. A force is developed on the conductor. When a group of these conductors are fixed on a rotating armature, a resultant torque is produced from all the forces on the individual conductors.

A diagram of this principle is shown in Figure 3.1.

Most power supplies are AC (except for special applications, where the supply is DC, e.g. railways). Thus, in order to operate a DC motor, the AC supply has to be rectified (and possibly controlled) to provide the necessary DC voltage to feed the motor.

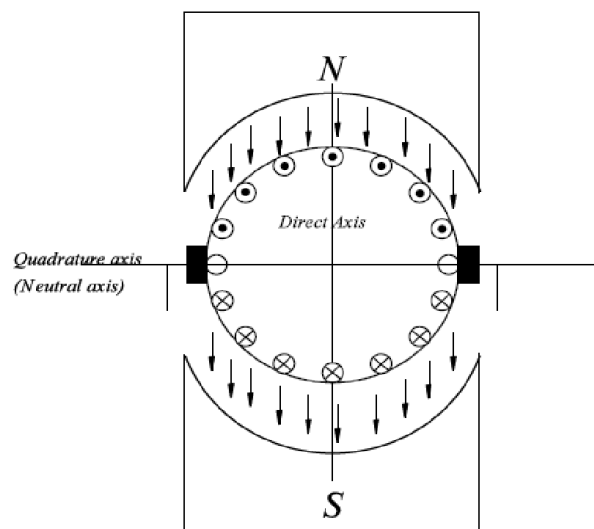


Figure 3.1: Principle of operation of the DC motor (the position of the brushes is not correct in the figure: the brushes should be positioned in the mid-point of each pole).

On Figure 3.1 two axes are shown: the direct axis, which is the direction of the

magnetic field and the quadrature axis, which is perpendicular to it. The latter is sometimes referred to as the neutral axis. The neutral axis is the position at which the direction of current flow in the conductors reverses. All the conductors are connected to the commutator segments on the end of the armature. Fixed sets of brushes make electrical contact with the commutator segments. The purpose of the commutator is to ensure that the direction of current flow in the conductors reverses when passing through the neutral axis. The position of the brushes has to coincide with the neutral axis.

The windings on the armature are implemented as *distributed* windings. The armature has a specific number of slots, and each of these slots can usually accommodate two layers of conductors. It is customary to insert each coil, with one conductor in the top of one slot, and the other conductor in the bottom of an opposite slot. Thus, for each coil, one side of it will be under the north pole of the field winding, while the other side will be under the south pole. As the current will flow in opposite direction in each conductor, the forces on each conductor lead to a torque, as shown in Figure 3.2.

The left hand rule can be used to find the direction of the resultant force on the conductor. The left hand rule states that if the index of the left hand points in the direction of the magnetic flux and the middle finger points in the direction of the electric current, the thumb point in the direction of the resultant force (this is shown Figure 3.3).

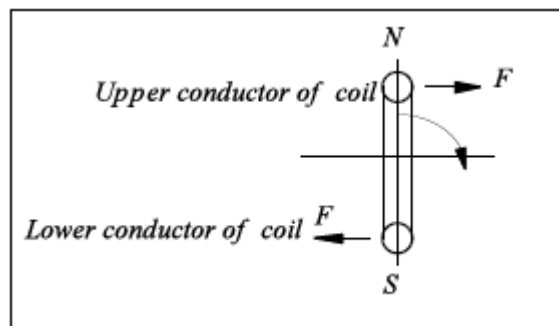


Figure 3.2: Forces on two opposite conductors of the same coil.

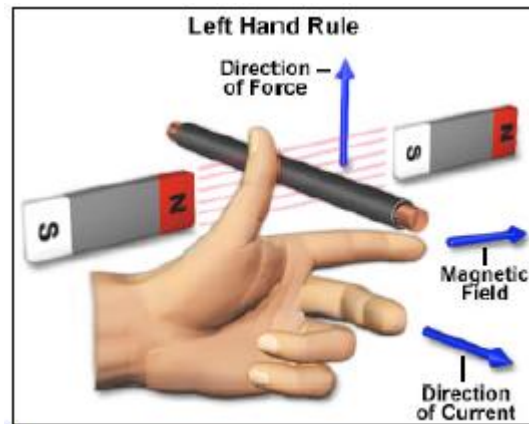


Figure 3.3: The left hand rule for finding the direction of force.

In order to be able to visualise the connections on the armature, it is customary to “open and spread” the armature flat. This type of connection is called lap winding, because coils actually overlap. The Figure shows a 12 slot armature, and 12 coils inserted in it. As each slot takes two conductors, each coil has one conductor in the top of one slot, and the second conductor in the bottom of the opposite slot. The magnetic fields are shown by two rectangles, labelled North and South. Notice, that for the machine to work properly, all conductor under the North pole should have their currents flowing in the same direction, while all conductors under the South pole should have their current flowing in the opposite direction to the those under the North pole. This leads to the addition of the torques from all conductors. The purpose of the commutator segment is to ensure that the current reverses in the relevant conductors at the correct point in time. The brushes complete the connection between the terminals and the segments [8].

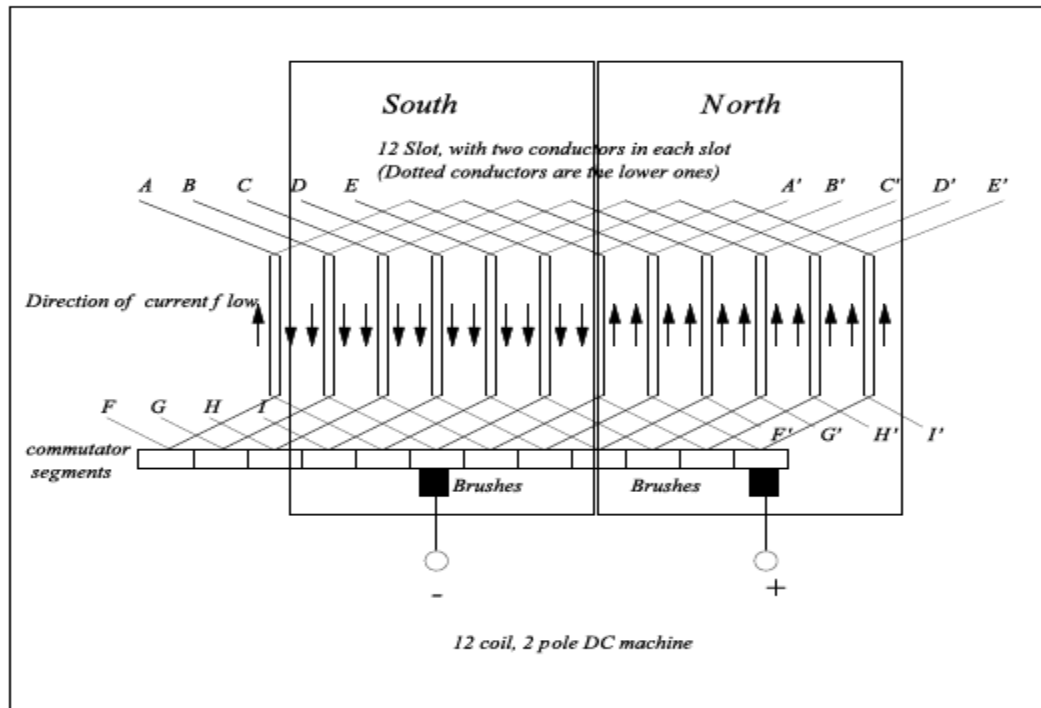


Figure 3.4: Winding diagram for a 12 coil, 12 slot 2 pole machine.

3.3 Construction

DC motors consist of one set of coils, called armature winding, inside another set of coils or a set of permanent magnets, called the stator. Applying a voltage to the coils produces a torque in the armature, resulting in motion.

3.3.1 Stator

- The stator is the stationary outside part of a motor.
- The stator of a permanent magnet dc motor is composed of two or more permanent magnet pole pieces.
- The magnetic field can alternatively be created by an *electromagnet*. In this case, a DC coil (field winding) is wound around a magnetic material that forms part of the stator.

3.3.2 Rotor

- The *rotor* is the inner part which rotates.
- The rotor is composed of windings (called armature windings) which are connected to the external circuit through a mechanical commutator.

- Both stator and rotor are made of ferromagnetic materials. The two are separated by air-gap.

3.3.3 Winding

A winding is made up of series or parallel connection of coils.

- Armature winding - The winding through which the voltage is applied or induced.
- Field winding - The winding through which a current is passed to produce flux (for the electromagnet).
- Windings are usually made of copper.[7]

3.4 Types of DC Motors

Depending on the method of field excitation, three types of dc motors are possible.

These are discussed here.

3.4.1 Shunt motor

As the name implies, this motor has a field winding which is electrically connected in parallel to the motor, and thus is rated for the full voltage of the motor. This leads to constant field excitation, if the terminal voltage is constant. The resistance of the shunt field is usually high. Sometimes, series resistors are used to limit or control the value of field current.

The diagram for a shunt dc motor is shown in Figure (3.5)[8].

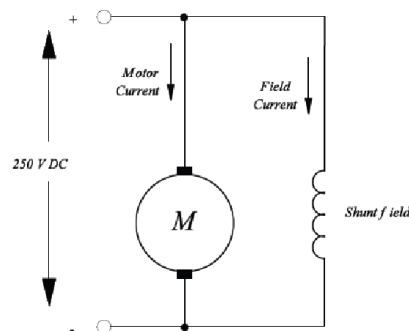


Figure 3.5: Set-up of a shunt motor.

3.4.2 Series motor

The field excitation in a series motor is produced by a field connected in series with the armature. This is usually made of a thick wire and a few turns, as it has to carry the full armature current. At low speeds, this motor draws large current, because the back emf is small, and thus it can produce large starting torques. A diagram of this motor is shown in Figure (3.6).

This motor is only used in special applications.

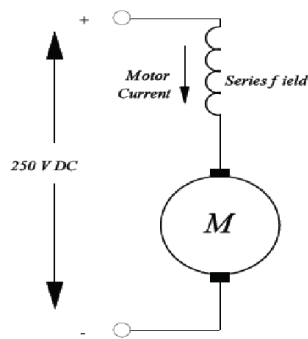


Figure 3.6: Set-up of a series DC motor.

3.4.3 Cumulative compound motor

The most widely used motor in lift applications is the cumulative compound motor.

Its field excitation is provided by both a shunt field and a series field. The term cumulative refers to the fact that the two fields aid each other. The alternative would have been a differentially compound motor, in which the two fields oppose each other. However, a differentially compound DC motor can become unstable.

This is explained as follows:

When the motor is running at rated speed and no load, it will be drawing a small armature current, and thus the main excitation will be provided by the shunt field. As the load starts increasing on the motor, the speed will slowdown slightly. This leads to an increase in armature current, which increases the

amount of series field excitation. As this acts in opposition to the shunt field, the total field will tend to reduce. This leads to further increases in armature current and so on until the motor either runs away or stalls [8].

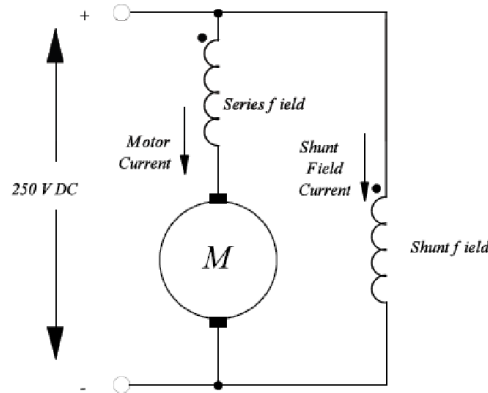


Figure 3.7: Set-up of a cumulative compound DC motor.

The DC motors have been popular in the industry control area for a long time, because they have many good characteristics, for example: high start torque characteristic, high response performance, easier to be linear control...etc.

The speed of a DC motor is given by the relationship

$$N = \frac{V - I_a R_a}{k\Phi} \quad (3.1)$$

This Equation shows that the speed is dependent on the supply voltage V , the armature circuit resistance R_a , and field flux Φ , which is produced by the field current. This paper describes the MATLAB/ SIMULINK of the DC motor speed control method namely field resistance, armature voltage, armature resistance control method and feedback control system for DC motor drives.

When speed control over a wide range is required, combination of armature voltage control and field flux control is used. This combination permits the ratio of maximum to minimum speed to be 20 to 40. With closed loop control, this range can be extended up to 200. The parameters of the PID controller k_p , k_i and k_d (or k_p , T_i and T_d) can be manipulated to produce various response Curves from a given process as we will see later.

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3.5 Speed Control of D.C Motors

In the case of speed control, armature voltage control and flux control method sare available. The voltage control can be from a variable voltage source like Ward-Leonard arrangement or by the use of series armature resistance. Unlike the starting conditions the series resistance has to be in the circuit throughout in the case of speed control. That means considerable energy is lost in these resistors. Further these resistors must be adequately cooled for continuous operation. The variable voltage source on the other hand gives the motor the voltage just needed by it and the losses in the control gear is a minimum. This method is commonly used when the speed ratio required is large, as also the power rating.

Field control or flux control is also used for speed control purposes. Normally field weakening is used. This causes operation at higher speeds than the nominal speed.

Strengthening the field has little scope for speed control as the machines are already in a state of saturation and large field mmf is needed for small increase

in the flux. Even though flux weakening gives higher speeds of operation it reduces the torque produced by the machine for a given armature current and hence the power delivered does not increase at any armature current. The machine is said to be in constant power mode under field weakening mode of control. Above the nominal speed of operation, constant flux mode with increased applied voltage can be used; but this is never done as the stress on the commutator insulation increases.

Thus operation below nominal speed is done by voltage control. Above the nominal speed field weakening is adopted. For weakening the field, series resistances are used for shunt as well as compound motors. In the case of series motors however field weakening is done by the use of 'diverters'. Diverters are resistances that are connected in parallel to the series winding to reduce the field current without affecting the armature current [9].

3.6 SPEED TORQUE CHARACTERISTICS

Having developed the equations for the models of both types of motors, we now turn to examine the speed torque characteristic for both types of machines.

A set of speed torque characteristics for a shunt motor is shown in Figure 3.8.

Several lines are possible depending on the value of the terminal voltage. As discussed earlier, the lines are negatively sloping, and their intercept and slope are proportional to the square of the voltage. This forms the basis for speed control in DC motors. On the diagram, a constant torque line at 450 N·m has been drawn. Its intersection with the various speed torque curves shows the possible running speeds at various voltages.

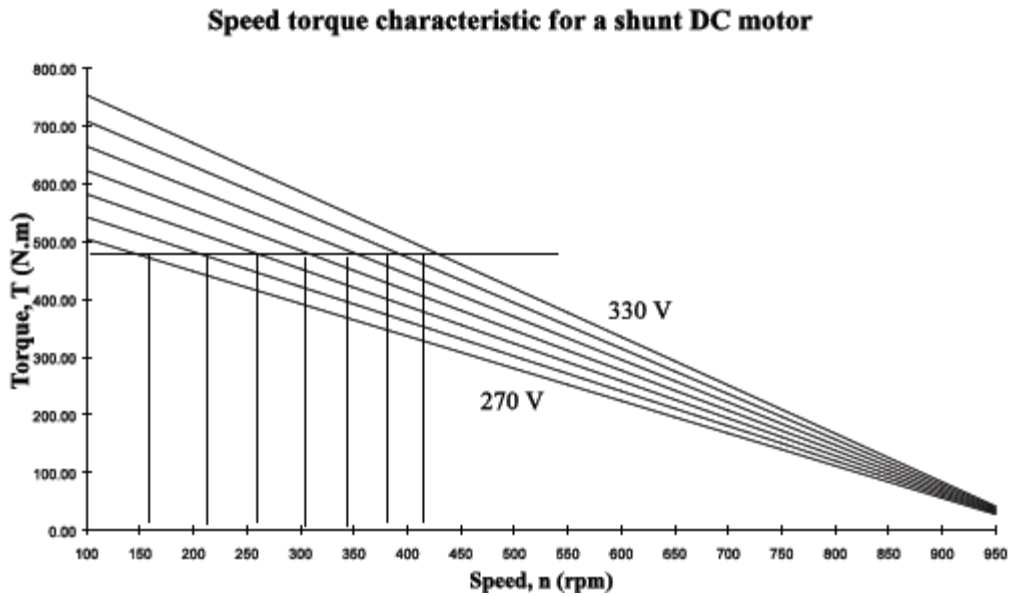


Figure 3.8: Speed torque characteristics for a shunt DC motor.

A set of speed torque curves for a series motor are shown in Figure 3.9. As discussed earlier, they take a hyperbolic shape, and are also proportional to the square of the voltage. The high torques at low speed, give the series motor its advantage of high starting torques, especially in traction applications.

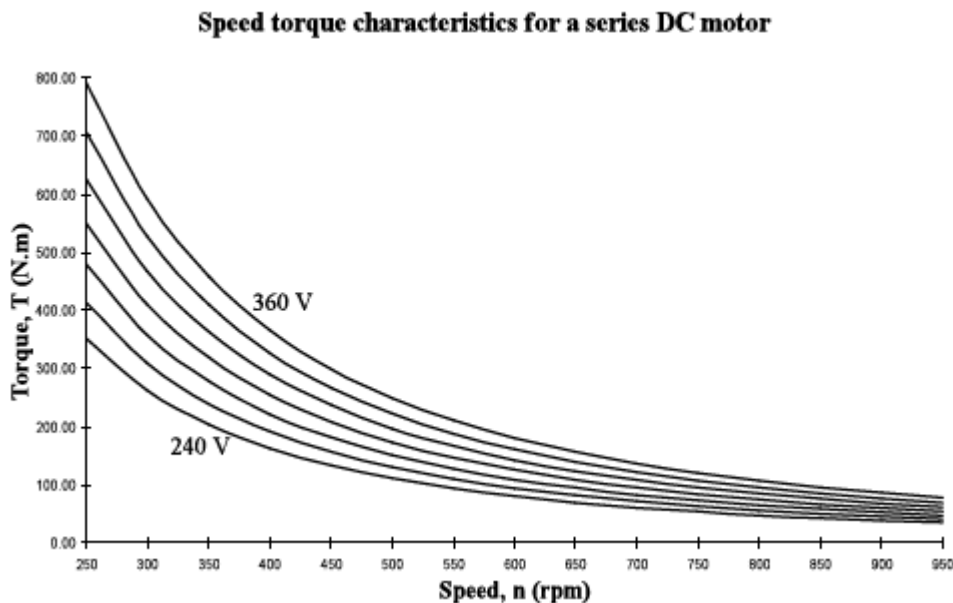


Figure 3.9: Speed torque characteristic for a series DC motor.

As the compound machine is a combination of both shunt and series fields, its speed torque characteristic lies in between the two, depending on the relative

strength of the shunt and series fields. A comparative diagram of the three speed-torque characteristics is shown in Figure 3.10.

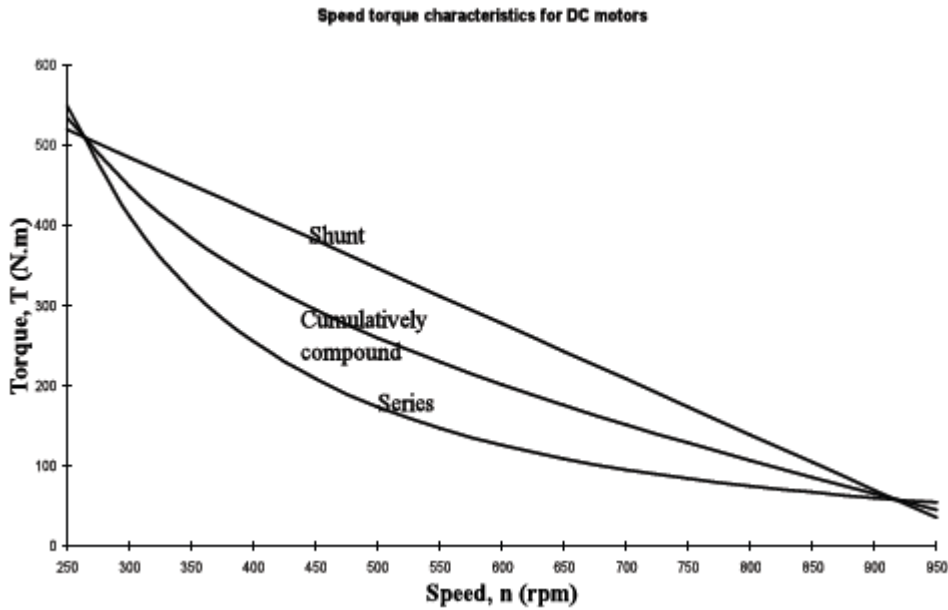


Figure 3.10: Comparison of speed torque curves for the three connection methods of a DC motor[8].

3.7 DC Motor Mathematic Model and the Control Theory

DC motors have speed-control capability, which means that speed, torque and even direction of rotation can be changed at any time to meet new conditions. The electric circuit of the armature and the free body diagram of the rotor are shown in the following fig- 2

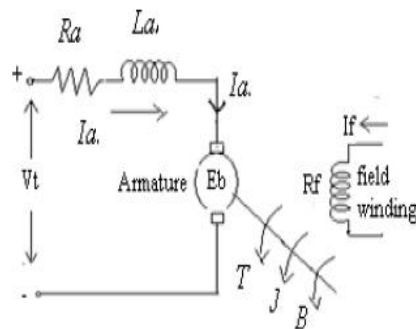


Fig 3.11: The electric circuit of the armature and the free body diagram of the rotor.

The DC motors are generally used in the linear range of the magnetization curve. Therefore, air gap flux Φ is proportional of the field current i.e.

$$\Phi \propto i_f$$

$$\Phi \propto k_f i_f$$

Where K_f is a constant

The torque T developed by the motor is proportional to the armature current and air gap flux i.e.

$$T \propto \Phi i_a$$

$$T = k_a \Phi i_a$$

$$T = k_a k_f \Phi i_a$$

$$T = k i_a \tag{3.2}$$

Where K_T = motor torque constant

The motor back EMF being proportional to speed is given as

$$E_b \propto \omega \Phi$$

$$E_b = k_b \omega$$

$$E_b = k_b \frac{d\theta}{dt} \tag{3.3}$$

Where K_b = back emf constant

Applying KVL in the armature circuit

$$v = R_a i_a + L_a \frac{di_a}{dt} + E_b \tag{3.4}$$

And the dynamic equation with moment of inertia & coefficient of friction will be

$$T = J \frac{d^2\theta}{dt^2} + B \frac{d\theta}{dt}$$

And with load torque

$$T = J \frac{d^2\theta}{dt^2} + B \frac{d\theta}{dt} + T_L \tag{3.5}$$

Take the Laplace transform of equation of (3.2), (3.3), (3.4) and (3.5)

$$T(s) = KI_{a(s)}$$

$$E_{b(s)} = kb s \theta(s)$$

$$v(s) = I_{a(s)}(R_a + sL_a) + E_{b(s)}$$

$$v(s) - E_{b(s)} = I_{a(s)}(R_a + sL_a)$$

$$T(s) = (Js^2 + Bs)\theta(s)$$

$$T(s) = (Js + B)s\theta(s)$$

Or

$$T(s) = (Js + B)\omega(s)$$

$$T(s) = KI_{a(s)} \quad (3.6)$$

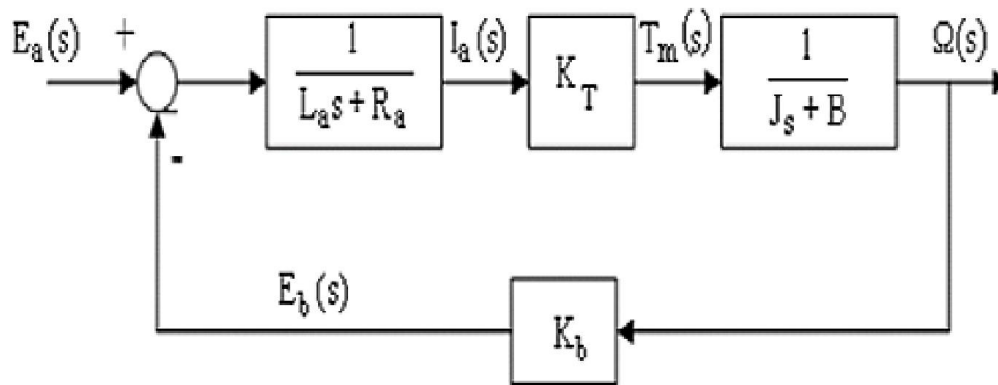


Figure 3.12: Block diagram of armature controlled d. c. motor.

The transfer function of DC motor speed with respect to the input voltage can be written as follows:

$$G(s) = \frac{\omega(s)}{V(s)} = \frac{K_T}{(R_a + sL_a)(Js + B) + K_b K_T} \quad (3.7) \quad \text{From}$$

equation (4.7) the armature inductance is very small in practices, hence, the transfer function of DC motor speed to the input voltage can be simplified as Follows,

$$G(s) = \frac{\omega(s)}{V(s)} = \frac{K_m}{\tau s + 1}$$

$$K_m = \frac{K_T}{R_a B + K_b K_T}$$

$$\tau = \frac{R_a J}{R_a B + K_b K_T} \quad (3.8)$$

From equation (3.8), the transfer function can be drawn the DC motor system block diagram which is shown in.

Key point; Field controlled dc motor is open loop while armature controlled is closed loop system. Hence armature controlled dc motor are preferred over field controlled system.

For small size motor field control is advantageous because only a low power servo amplifier is required while the armature current which is not large can be supplied from an expensive constant current amplifier.

For large size motor it is on the whole cheaper to use armature control scheme. Further in armature controlled motor, back emf contributes additional damping over and above that provided by load friction [6].