

ABSTRACT:

Shell structures now days are widely used in constructing hall roofs with long spans, concrete roofs, pipe lines (gas and oil) and large water tanks. So the analysis of these shell structures become of great interest for designing purposes. For this research lamination technique is adopted as one of the well known type of composite material. This research presents a finite element concept for geometrically linear and nonlinear analysis of laminated shell structures. Only static analysis is performed. At the beginning the theory of linear analysis is presented and then extended to include geometrically nonlinear analysis for large rotations and large displacements. The finite element formulation employed the 8-node degenerated curved shell element of (parabolic) shape. The element has five degrees of freedom per node, (three global translations X , Y & Z and two rotations α and β). Shear locking as one of the famous problem facing the application finite element method is discussed and some solutions are suggested. The nonlinear finite element formulation is based on Total Lagrangian approach using both Green and Geometric strains. The nonlinear equilibrium equations are solved using the Incremental and Newton-Raphson Method. Different seven numerical examples are performed to obtain the geometrically linear and nonlinear behaviour of laminated shell structures. The structures analysed are flat plate, cylindrical shell and spherical shell for different applied load types and various boundary condition. A FORTRAN computer programs are developed and implemented to analyse some laminated shell structures. Results obtained using Green's strain is compared with those obtained when using Geometric strain. Good agreement was observed between the

results obtained by the present formulation and those available in the literature.

TABLE OF CONTENTS

Title	Page
Abstract	i
Table of Contents	iii
List of Tables	viii
List of Figures	ix
List of Symbols	xi
List of Abbreviations	xiii
Chapter One: Introduction	1
1.1 Introduction	1
1.2 Objectives	3
1.3 Methodology.....	3
1.4 Research Outlines	4
Chapter Two: Literature Review	5
2.1 Composites Materials	5
2.2 The Composite Lamina	7
2.3 Constitutive Relation of Lamina	8
2.4 Laminate Theory	14
2.4.1 Classical Laminated Plate Theory	15
2.4.2 First order Shear Deformation Theory	19
2.4.3 Third order Shear Deformation Theory	20
2.4.4 Layer wise Theories	20
2.5 Finite Element Method	21

2.6 Finite Elements for Analysis of Laminated Shell Structures	22
2.7 Locking Problems	23
2.7.1 Membrane Locking	24
2.7.2 Shear Locking	24
2.7.3 Volumetric and Thickness Locking	26
2.8 Geometrically Nonlinear Formulation	27
2.9 Conclusion	28
Chapter Three: Linear Finite Element Analysis of Laminated Shells.....	29
3.1 Introduction	29
3.2 Finite Element Method	29
3.3 Historical background	30
3.4 Types of Analyses of Structures	31
3.5 Degeneration Method	32
3.5.1 Geometry of the Element	33
3.5.2 Displacement Field.....	34
3.5.3 Strain - displacement Relations	35
3.5.4 Constitutive Relations	36
3.5.5 Stiffness Matrix	37
3.5.6 Transformation Matrix	37
3.5.7 Element Load Vector	40
3.5.7.1 Gravity load	40
3.5.7.2 Uniform Surface Load	41
3.5.7.3 Pressure normal to surface	42
3.6 Modelling of Laminated Shells	42

3.6.1 Numerical Integration	42
3.6.2 Numerical Thickness Integration of Laminates	41
3.6.3 Constitutive Relation	46
3.6.4 Mixed Interpolation of Torsorial Components – MITC	50
Chapter Four: Geometrically Non-linear Finite Element	55
4.1 Introduction	55
4.2 The Basic Problem	55
4.3 Solution Process	56
4.4 The stress- strain Relations	58
4.5 Strain – displacement Relations	58
4.6 Derivation of \mathbf{B}_L matrix	59
4.7 Derivation of Tangent Stiffness Matrix \mathbf{K}_T	61
4.8 Geometrically Non-linear Formulation of Shell Element	62
4.8.1 Stresses and strains	62
4.8.2 Strain – Displacement Relationship	63
4.8.3 Tangent Stiffness Matrix due to Geometric Strains	68
4.8.4 Tangent Stiffness Matrix due to Green's Strains	70
Chapter Five: Description of the Computer Program	72

5.1	General	72
5.2	The Main Program	73
5.3	Subroutine INPUT.....	75
5.4	Subroutine INITIAL	79
5.5	Subroutine LOD	79
5.6	Subroutine INCLUD	80
5.7	Subroutine ASSEMBLE	80
5.8	Subroutine GREduc & BKSBSTN	80
5.9	Subroutine REEORC	81
5.10	Subroutine CONVERGE	81
5.11	Subroutine RESULTS & STRES	81
5.12	Subroutine DMATX	81
Chapter Six: Application to Numerical Examples and Discussion of Results		83
6.1	Application to Numerical Examples	83
6.1.1	Convergence of displacement of simply supported square plate.....	83
6.1.2	Orthotropic square plate under uniform load	84
6.1.3	Simply square laminated plate under sinusoidal loads.....	88
6.1.4	Barrel vault	90

6.1.5 Pinched cylinder	94
6.1.6 Laminated cylindrical panel	94
6.1.7 Doubly curved shell panel	99
6.2 Discussion of Results	100
Chapter Seven: Conclusions and Recommendations	103
8.1 Conclusions	103
8.2 Recommendations for further studies	105
References	106
Appendix I.....	110
Appendix II.....	145

LIST OF TABLES

Table	Page
Table 5.1 Control and geometric data	76
Table 5.2 Material property data.....	77
Table 5.3 Co-ordinate data.....	78
Table 5.4 Boundary conditions data.....	78
Table 5.5 Element properties data	79
Table 5.6 Point load data.....	79
Table 6.1 Maximum transverse deflection of cross-ply laminated cylindrical shell roof under its own weight	86
Table 6.2 Displacement at point (a) of the laminated pinched circular cylindrical problem.....	87
Table 6.3 Vertical displacement (mm) vs. load (KN)	87
Table 6.4 Maximum radial deflection (-w×10 in.) of a simply supported shell panel under central point load.....	89

LIST OF FIGURES

Figure	Page
<hr/>	
Figure 2.1 Positive notation of principal material axes1-2 from x-y axes.....	9
Figure 2.2 Results for Graphite Epoxy	13
Figure 2.3 Results for Graphite Epoxy	14
Figure 2.4 Lamina and Laminate.....	15
Figure 2.5 Geometry of deformation in the x-z plane.....	16
Figure 2.6 Symmetric angle- ply geometry and stresses.....	18
Figure 2.7 Geometry of an N-Layered laminate.....	19
Figure 2.8 Edge view of (a) deformed linear element (b) correct Geometry in pure bending.....	25
Figure 3.1 Eight node curved element.....	32
Figure 3.2 Node director.....	34
Figure 3.3 Transformation from global to local axis.....	37
Figure 3.4 Schematic representation of the integration scheme for laminated elements where (a) shows the entire element (b) the 1,th sub-layer where the transformed coordinate t_1 runs from -1 to +1.....	45
Figure 3.5 Location of Typing points for a MITC4 element.....	52
Figure 5.1 Main program flow chart.....	74

Figure 5.2 Schematic representation of the determination of the stiffness matrix.....	82
Figure 6.1 Convergence of vertical displacement.....	84
Figure 6.2 simply square plate under uniform load	85
Figure 6.3 transverse displacement vs, pressure load	86
Figure 6.4 load vs. moments at the centre of the plate	88
Figure 6.5 modular ratios vs. central deflections.....	89
Figure 6.6 vertical deflections vs. side-to- thickness ratio.....	90
Figure 6.7 cylindrical shell roof under its own weight	91
Figure 6.8 vertical deflection at centre of free edge	92
Figure 6.9 horizontal displacement	93
Figure 6.10 convergence of vertical deflection	93
Figure 6.11 Pinched cylindrical shell.....	95
Figure 6.12 cylindrical laminated shell	96
Figure 6.13 Results of nonlinear analysis of cylindrical laminated shell.....	97
Figure 6.14 simply supported spherical shell panel under central point load.....	100

LIST OF SYMBOLS

Symbol	Description
a	Nodal displacement vector
a^e	Nodal element displacement vector
b	Vector of body force per unit volume
b_i	Nodal element body force
B_L	Strain-displacement vector
B_o	Linear strain displacement vector
C	Elasticity matrix
E	Young's modulus
f	Vector of external force
G	Matrix containing shape functions derivatives w.r.t. r, s & t
J	Jacobian matrix
K_o	large displacement stiffness matrix
K_L	Small displacement stiffness matrix
K_σ	Initial stress stiffness matrix
N	Shape function displacement matrix
N_i	Shape function at node i
T	Transformation matrix
u, v and w	Global displacements
v₁ , v₂ and v₃	Components of transformation matrix
x, y and z	Cartesian coordinates

σ	Stress vector
μ	Poisson's ratio
ψ	Residual force vector
$r, s \text{ \& } t$	Element natural coordinate
h	Element thickness

Note: other symbols are defined in text where necessary

LIST OF ABBREVIATIONS

SAP	Structural Analysis Program
STAAD	Structural Analysis and Design Program
MITC	Mixed Interpolation Tensorial Components
FORTTRAN	Formula Translation
CLPT	Classical Laminate Plate Theory
FSDT	First Order Shear Deformation Theory
EAS	Enhanced Assumed Strain
ESL	Equivalent Single Layer
ANS	Assumed Natural Strain
FEM	Finite Element Method
NEFAP	Nonlinear Finite Element Analysis Program