

Chapter one

1.1 Introduction:

Superconductivity was discovered by Heike Kamerlingh Onnes in Holland in 1911 as a result of his investigations leading to the liquefaction of helium gas. Two years later he got the Nobel prize 1913. In Onnes' time superconductors were simple metals like mercury, lead, bismuth etc. [1]. These elements become superconductors only at the very low temperatures of liquid helium. During the 75 years that followed, great studies were made in the understanding of how superconductors work. Over that time, various alloys were found that show superconductivity at somewhat higher temperatures. Unfortunately, none of these alloy superconductors worked at temperatures much more than 23 K. Thus, liquid helium remained the only convenient refrigerant that could be employed with these superconductors. The transition of a normal metal into the superconducting state is revealed by the total disappearance of the electrical resistance at low temperatures. Indeed, the current in a closed superconducting circuit can circulate forever without damping.

Another fundamental property of the superconducting state was discovered in 1933 when Walther Meissner and his Ph.D. student Robert Ochsenfeld demonstrated that superconductors expel any residual magnetic field [2]. Similarly, superconductivity can be destroyed by applying a magnetic field that exceeds a critical value. Superconductivity and magnetism usually try to avoid each other this feature can be exploited to, for example, levitate a magnet above a superconductor. The recent discovery of compounds that are both ferromagnetic and superconducting at the same time came as a surprise to experimental and theoretical, condensed matter physicists. The microscopic theory of superconductivity was created by John Bardeen, Leon Cooper and Robert Schrieffer in 1957 [3]. According to this so-called BCS-theory, the electrons form pairs, known as Cooper-pairs, due to

interactions with the crystal lattice at low temperatures. Electrons in these Cooper-pairs have opposite values of momentum, meaning that the pairs themselves generally have zero orbital angular momentum. Additionally, the angular momenta add up to zero. The formation of Cooper-pairs leads to a superconducting energy gap, which means that single electrons cannot occupy states near the Fermi surface. Such energy gaps which are essentially equal to the energy needed to break up the Cooper-pairs show up clearly as jumps in the specific heat and thermal conductivity at what is known as the critical temperature.

Another significant theoretical advancement came in 1962 when Brian Josephson, a graduate student at Cambridge University, predicted that electrical current would flow between two superconducting materials, even when they are separated by a non- superconductor or an insulator. His prediction was later confirmed and won him a shared of the 1973 Nobel Prize in Physics with Leo Esaki and Ivar Giaever. This tunnelling phenomenon is today known as the “Josephson effect” and has been applied to electronic devices such as the Superconducting Quantum Interference Device (SQUID), an instrument capable of detecting even the weakest magnetic fields. Then, in 1986, a truly breakthrough discovery was made in the field of superconductivity. Georbednorz and Alexander Muller researchers at the IBM Research Laboratory in Rüslikon, Switzerland, created a brittle ceramic compound that showed superconductivity at the highest then temperature known 30 K. What made this discovery so remarkable was that ceramics are normally insulators. They do not conduct electricity well at all. So, researchers had not considered them as possible high- temperature superconductor candidates. The Lanthanum, Barium, Copper and Oxygen ($\text{La}_{1.85}\text{Ba}_{0.15}\text{CuO}_4$) compound that Muller and bednorz and synthesized, behaved in a not-as-yet-understood way. The discovery of this first of the superconducting copper oxides (cuprates) won the 2 men a

Nobel Prize the following year. It was later found that tiny amounts of this material were actually superconducting at 58 K, due to a small amount of lead having been added as a calibration standard making the discovery even more noteworthy.

The BCS theory is quite successful at explaining the properties of most classical superconducting materials. But the discovery in 1986 of a new class of materials that are superconducting at high temperatures remains a challenge to the theoreticians, and there is still no unambiguous theoretical explanation for this phenomenon.

The observation of superconductivity in organic conductors, heavy-fermion systems, the ruthenates and, most recently, the new ferromagnetic superconductors provides strong arguments for the existence of more exotic types of superconductivity. Indeed, pairing in ferromagnets must result from a different type of electron-pairing. In these materials, electrons with spins of the same direction are paired up with each other to form Cooper-pairs with one unit of spin, resulting in so-called *triplet superconductivity*. In contrast, conventional superconductivity, also known as *s-wave* singlet superconductivity, occurs when electrons with opposite spins bind together to form Cooper-pairs with zero momentum and spin.

A magnetic field can destroy singlet superconductivity in two ways. The first of those effects is known as the *orbital effect* and is simply a manifestation of the Lorentz force. Since the electrons in the Cooper-pair have opposite momenta, the Lorentz force acts in opposite directions and the pairs break up. The second phenomenon, known as the *paramagnetic effect*, occurs when a strong magnetic field attempts to align the spins of both the electrons along the field direction. Such fields, however, do not wreck triplet superconductivity because the spins of both electrons may point in the same direction as the field. This means that triplet superconductivity can only be destroyed by the orbital effect.

Ferromagnetism arises when a large number of atoms or electrons align their spins in the same direction. There are actually two sources of magnetism in metals, localized magnetic moments and the “sea” of conduction electrons. Local magnetism occurs in rare-earth metals (such as gadolinium) due to the incomplete filling of electrons in the inner atomic shells. This leads to a well defined magnetic moment at every fixed atomic site, which in turn produces long-range magnetic coupling due to the exchange of conduction electrons.

The second type of magnetism known as band magnetism, (such as ruthenium) arises from the magnetic moments of the conduction electrons themselves. In a metal, the electrons are “itinerant”, that is they are free to move from one atomic site to another, and they tend to align their magnetic moments in the direction of an applied field. Ferromagnets only have a net magnetic moment at low temperatures; the internal magnetic field spontaneously appears at the so called *Curie temperature*, which is typically in the range 10–1000 K. At higher temperatures, however, the magnetic moments of the atoms continually change their direction so that the net moment is zero. A similar magnetic transition occurs in antiferromagnetic materials in which the spins of neighboring atoms point in opposite directions. This transition takes place at the *Néel temperature* and leads to the disappearance of the internal magnetic field.

Although superconductivity and magnetism seem to be antagonistic phenomena, could they co-exist in the same compound? This question was first posed by the Russian theorist Vitaly Ginzburg in 1957, but rities, even a few percent, was enough to completely destroy superconductivity when ferromagnetic ordering was present.

The origin of this destructive phenomenon is a quantum mechanical interaction between the spins of the electrons and the atomic magnetic moments. Below the superconducting transition temperature, this “exchange interaction” attempts to align the Cooper-pairs . Exchange

interactions therefore place stringent limits on the existence of superconductivity.

1.2 Research Problem

That become superconductors materials at High temperature have some problems by ordinary BCS theory.

1.3 Literature Review

Many attempts were made to cure and reweedy some of BCS defects. BCS theory explain several superconductor phenomena specially at low critical temperature T_c , but there are many set backs associated with Bardeen, Cooper and Schrifir BCS theory for high critical temperature $T_c \geq 135$, which was observed in some compounds specially CuO and Fe compounds.

There are many problems need to solved. First of all, one observes that till now, there is no well established theoretical expression in most celebrated SC models which show the resistance drops abruptly to zero below the critical temperature. The existence of an energy gap well above T_c with pressure and the substitution of O^{16} by isotope O^{18} affect T_c also. The effects of pressure and isotope on T_c are called pressure and isotope effect respectively. In the work done by Dirar temperature dependent Schorodinger equation was used to show that the magnetic field can destroy superconductor about critical value [4]. Rasha. A.E.Taha also shows that superconductor resistance vanishes below certain critical value, where she used tight binding approximation [5]. In the work done by Ahmed Zakaria temperature dependent Schorodinger equation was used to show that the resistance of superconductor vanishes beyond critical temperature [6].

1.4 Aims of the work

The aim of the work is to construct simple theory bases on Schorodinger equation temperature dependent quantum equation to explain isotope effect.

1.5 Thesis Lay out

The thesis consists of three chapters, chapter one is the introduction, while chapter two is concerned with the theoretical background, where as chapter three is devoted for contribution.

Chapter two

Theoretical background

2.1 Introduction:

Conventional superconductors are metals in their normal states, and their transport properties are well described by Fermi liquid theory. The cuprate high temperature superconductors (HTSC) [6] are “bad metals” with strong electron correlation with a normal state that can not be described by Fermi liquid theory. There is compelling evidence that they are better thought of as doped Mott-insulators, rather than as high temperature strongly interacting versions of conventional metals. The cuprate superconductors exhibit numerous types of low temperature ordering which interact strongly with the superconductivity, the most prominent being antiferro-magnetism and the unidirectional charge and spin density wave “stripe” order. These orders can compete or coexist with superconductivity. Furthermore, whereas phase fluctuations of the superconducting order parameter are negligibly small in conventional superconductors, fluctuation effects exist in the high- T_c superconductors because of their much smaller superfluid stiffness.

Apparently, this does not influence the fundamental character of the superconducting order parameter. It is still a charge $2e$ scalar field, although it transforms according to a nontrivial representation of the point group or symmetry of the crystal. It is a “ d -wave superconductor”. At low temperature and energy there is every reason to expect that the physics is dominated by nodal quasi-particles that are similar to those that one might find in BCS-superconductors of the same symmetry. Indeed, there is considerable direct experimental evidence that this expectation is realized. However, the Fermi liquid theory fails to describe the normal state. Especially around the so-called *pseudogap* temperature T^* competing

orders are present. This fact requires an entirely different approach to understand most of the physics. It is the purpose of this Chapter to address the basic physics of classical and high-temperature superconductivity.

2.2 What is a superconductor?

Superconducting materials have two fundamental properties:

- **No dc-resistivity** ($\rho = 0$ for all $T \leq T_c$): **Zero resistivity** $\rho = 0$, i.e. infinite conductivity, is observed in a superconductor at all temperatures below the critical temperature T_c , as depicted in Fig. 2.1. However, if the passing current is higher than the critical current j_c , superconductivity disappears. Why is the resistivity of a superconductor zero? If a superconducting metal like Al or Hg is cooled below the critical temperature T_c , the gas of repulsive individual electrons that characterizes the normal state transform itself into a different type of fluid, a quantum fluid of highly correlated pairs of electrons. A conduction electron of a given momentum and spin gets weakly coupled with another electron of the opposite momentum and spin. These pairs are called Cooper pairs. The coupling energy is provided by lattice elastic waves, called phonons. The behavior of such a fluid of correlated Cooper pairs is different from the normal electron gas. They all move in a single coherent motion. A local perturbation, like an impurity, which in the normal state would scatter conduction electrons (and cause resistivity), cannot do so in the superconducting state without immediately affecting the Cooper pairs that participate in the collective superconducting state. Once this collective, highly coordinated, state of coherent “super-electrons” (Cooper pairs) is set into motion (like the super-current induced around the loop), its flow is without any dissipation. There is no scattering of ‘individual’ pairs of the coherent fluid, and therefore no resistivity.
- **No magnetic induction** ($B = 0$ inside the superconductor): In magnetic fields lower the critical field B_c the magnetic inductance

becomes zero inside the superconductor when it is cooled below T_c . The magnetic flux is expelled from the interior of the superconductor (see Fig. 2.1). This effect is called the Meissner- Ochsenfeld effect after its discoverers [2]. To test whatever a material is superconducting both properties $\rho = 0$ and $B = 0$ must be present simultaneously.

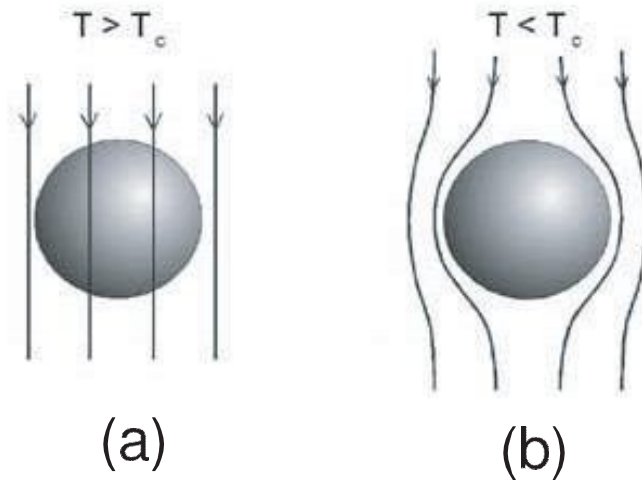


Figure 2.1: Expulsion of a weak external magnetic field from the interior of the superconducting material.

2.2.1 Normal metal vs. superconductor:

In this section a discussion is presented about the origin of electrical resistivity in the normal metal and contrast it with the absence of resistivity in the superconductor. An introduction to the basic concept of the superconducting wave function is given which will be used throughout the whole Chapter. Then showing that the Meissner effect can be described by the London model and is another distinct characteristic property of the superconducting state.

2.2.1.1 Description of the normal state:

A normal metal consists of a regular crystalline lattice of positively charged ions and a gas of free, non-interacting conduction electrons that fill the space between the ions. If there is typically one electron per ion, this means 10^{23} electrons/cm³. As the electrons are of opposite charge as the ions, the total charge is balanced and at equilibrium, the metal is electrically neutral. If we apply an electric field as an external perturbation to the gas of free electrons within the metal, the external force will accelerate the electrons and create a current flow of “free” electrons. As the ions are arranged in perfectly regular array, they do not scatter conduction electrons at $T = 0$. The scattering of electrons at $T = 0$ is actually caused by deviations from the ideal periodic potential of the lattice, i.e., by impurities, imperfections in periodicity like dislocations. Since every real metal contains some imperfections and impurities, one observes some finite resistivity at very low temperatures. This resistivity, extrapolated to $T = 0$, is called residual resistivity, ρ_i . As we increase the temperature, the electrons also get scattered by thermal vibrations of the lattice (called phonons) so the resistivity rises with temperature. This contribution is called phonon resistivity, ρ_{ph} . Therefore the temperature dependence of resistivity of a good metal can be described as : $\rho(T) = \rho_i + \rho_{ph}$. This is empirical Matthiessen’s rule and provides a basis for understanding the resistivity of metal at low temperature.

In order to derive a simple expression for the residual resistivity of the metal, first some characteristic quantities of the normal state should be considered. At $T = 0$ the maximum kinetic energy of an electron inside the metal is called Fermi energy (E_F). It is related to the number of carriers per unit volume, n , by the simple relation: $E_F = \frac{\hbar^2}{2m} (2\pi^2 n)^{2/3}$ where \hbar is the Planck constant and m is the mass of the electron. The Fermi energy of a typical metal is of the order of electron volts . Conduction electrons of maximum energy, E_F , propagate with the Fermi

velocity v_F related to the Fermi momentum P_F by $P_F = mv_F$. We have $E_F = \frac{1}{2}P_F v_F$. We also define the Fermi wave vector, k_F ; as in quantum mechanics a wave is always associated with a particle by de-Broglie relation $P_F = \hbar k_F$. The conduction electrons that propagate through the crystal with a characteristic Fermi velocity v_F are scattered by impurities or lattice imperfections. This gives rise to resistivity. Between two scattering events an electron covers on average a characteristic distance A_e , called the electron mean free path. The resistivity ρ_i of a metal, according to the Drude model, is given by:

$$\rho_i = \frac{mv_F}{nA_e e^2} \quad (2.2.1)$$

where e and m represent the charge and mass of the electron. In isotropic metals, the conductivity is equal to the inverse of the resistivity; both quantities are tensors in the anisotropic case. Equation (2.1) shows that in the normal state of a given metal the resistivity is inversely proportional to the electron mean free path. The shorter the average distance between the scattering events the higher is the resistivity. The introduction of impurities into a metal obviously reduces A_e and increases ρ_i .

2.2.2 The superconducting state:

The electrical dc-resistivity in superconductors is zero for temperatures below the critical temperature T_c . So, one can apply a dc electrical current (super-current) without energy dissipation. Let us see what happens in a superconducting state and what are its characteristic properties compared with the normal state taken Al as an example for classical superconductor material. In the normal state above the critical temperature ($T_c = 1.1\text{ K}$) Al is a good conductor and behaves just like an ideal metal or like copper which exhibits no superconducting behavior down to the lowest temperature. Its conduction electrons behave like a gas of nearly free electrons that are scattered by lattice vibrations, lattice imperfections, etc.

which contributes to the resistivity. However, when Al is cooled below T_c its dc-resistance abruptly vanishes, the resistivity is zero. One natural question is, what happens to the scattering of conduction electrons which contributed to the resistivity in the normal state? Why does it disappear? A satisfactory explanation to these questions can be given only within the rather involved quantum mechanical description of the microscopic BCS-theory, which shall be briefly discussed in next section.

When Al is cooled below the critical temperature T_c , the gas of the “repulsive” individual electrons that characterizes the normal state transforms itself into a different type of fluid. A quantum fluid of highly correlated pairs of electrons (in the reciprocal, momentum space, not in a real space). Below T_c a conduction electron of a given momentum and spin gets weakly coupled with another electron of exactly the opposite momentum and spin. These pairs are called *Cooper pairs*. The glue is provided by the elastic waves of the lattice, called *phonons*. One can visualize this attraction by a real-space picture. As the lattice consists of positive ions, the moving electron creates a lattice distortion. Due to the heavy mass of lattice ions, this positively charged distortion relaxes slowly and is therefore able to attract another electron. The ‘distance’ between the two electrons of the Cooper pair, called the *coherence length*, ξ , is large in classical superconductor materials. It has a value $\xi = 1600$ nm in pure Al, $\xi = 38$ nm in pure Nb, for example. The coherence length ξ is very small in high- T_c superconductors, it has a value of $\xi \approx 16$ nm, and $\xi_c \approx 0.3$ nm in $L_{1.85}Sr_{0.15}CuO_4$ and in $YBa_2Cu_3O_{7-\delta}$. So while the “partners” in the Cooper pair are far apart, the other nearest electrons (belonging to other Cooper pairs of the collective state) are only a few nanometer away. The behavior of such a fluid of correlated Cooper pairs is different from the normal electron “gas”. The electrons which form the pair have opposite momenta (and opposite spins), so the net momentum of the pair is zero.

2.3 London theory:

The London brother proposed a simple theory to explain the Meissner effect. The London equation provided an early simple model for describing experimental results [8].

The 1935 theory of London brothers provides the first and second London equations, which relate the electric and magnetic field E and B respectively, inside a superconductor to the current density J :

$$E = \mu_0 \lambda_L^2 \frac{dJ}{dt} \quad (2.3.1)$$

$$\mathbf{B} = -\mu_0 \lambda_L^2 \nabla \times \mathbf{J} \quad (2.3.2)$$

Where μ_0 is the permeability of vacuum.

The constant of proportionality in these expression is the London penetration depth λ_L .

$$\lambda_L = \left(\frac{m}{\mu_0 n_s e^2} \right)^{1/2} \quad (2.3.3)$$

Where n_s is the density of superconducting electron, m is the electron mass.

2.3.1 Derivation of First London Equation:

A potential difference applied along a conducting wire produces an electric field E , and hence the force F on any electron is given by:

$$F = Ee - m \frac{dv}{dt} \quad (2.3.4)$$

Where v stand for its velocity, electron undergo successive periods of acceleration interrupted by collision and during the average time (relaxation time t (scattering on defects)) between collision. The velocity is given by:

$$v = -\frac{eE}{m} t \quad (2.3.5)$$

Which called the drift velocity. The negative sign means that the electrons move in a direction opposite to that of the electric field.

When the electron is assumed to move in a resistive medium, which have frictional force proportional to the velocity, the electron equation of motion is given by:

$$m \frac{dv}{dt} = eE - \frac{mv}{t} \quad (2.3.6)$$

Where the friction force (Fr) is given by:

$$Fr = ma, \quad v = v_0 + at = 0 + at = at$$

$$F = \frac{mv}{t} \quad (2.3.7)$$

For steady state in normal metal, no acceleration exists. i.e.

$$\frac{dv}{dt} = 0$$

Therefore

$$v = \frac{eE}{m} t \quad (2.3.8)$$

Hence the current density is given by:

$$J = nev = \frac{ne^2 t}{m} E = \sigma E \quad (2.3.9)$$

Where n is the density of electron, σ is the electrical conductivity.

In the two – fluid model one has the temperature dependent expression for the super n_s and normal n_n electron densities respectively, $n_n(T) + n_s(T) = n$. The total electron density n is independent of temperature and at $T = 0$, one has $n_n(0) = 0$ and $n_s(0) = n$, and the simple theory predicts the following temperature dependences.

$$n_s(T) = n \left(\frac{T}{T_c} \right)^4 \quad (2.3.10)$$

Where T_c is the critical temperature.

For a superconductor below T_c the resistive force is zero, in this case equation (2.3.8) becomes:

$$\frac{dv}{dt} = \frac{eE}{m}$$

Taking the derivation of J in equation (2.3.9) with respect to time:

$$\frac{dJ}{dt} = n_s e \frac{dv}{dt} = \frac{n_s e^2}{m} E \quad (2.3.11)$$

The term $\frac{m}{n_s e^2} = \Lambda$ is a phenomenological parameter.

Equation (2.3.11) can thus be rewritten as:

$$E = \frac{d}{dt}(\Lambda J) = \Lambda \frac{dJ}{dt} \quad (2.3.12)$$

This equation is known as the first London equation.

2.3.2 Derivation of Second London Equation:

This equation is concerned with time-dependent field, and is important for Meissner effect. The electric current density is given by:

$$J = nev \quad (2.3.13)$$

Where n is concentration of carriers of charge q . In the presence of a magnetic field described by the vector potential A , the velocity v is related to the total momentum p by:

$$p = mv + \frac{q}{c}A; \quad v = \frac{1}{m}\left(p - \frac{q}{c}A\right) \quad (2.3.14)$$

Where m is the mass, c the speed of light in vacuum. Thus equation (2.3.14) can be written as:

$$J = \frac{nq}{m}p - \frac{nq^2}{mc}\Lambda \quad (2.3.15)$$

In this superconducting state, the total momentum p is zero, although it not equal to zero in normal state. i.e. $p = 0$, and equation (2.3.14) reduces to:

$$J = \frac{nq^2}{mc}A$$

For electrons, $q = e$, $n = n_s$ then:

$$J = -\frac{n_s e^2}{mc} A \quad (2.3.16)$$

The vector potential is related to the magnetic field by:

$$B = \nabla \times A \quad (2.3.17)$$

Equation (2.3.16) can be rewritten with the aid of (2-3) to be:

$$J = -\frac{c}{4\pi\lambda_L^2}\Lambda \quad (2.3.18)$$

This equation is known as the second London equation. Equation (2.3.19) can be expressed in another way by taking the curl of the both sides and using

equation (2.3.18) to obtain:

$$\nabla \times J = -\frac{c}{4\pi\lambda_L^2}(\nabla \times \Lambda) = -\frac{c}{4\pi\lambda_L^2}B \quad (2.3.19)$$

$$B = -c\Lambda\nabla \times J \quad (2.3.20)$$

Where

$$\Lambda = \frac{m}{n_s e^2} = \frac{4\pi\lambda_L^2}{c^2}$$

is a phenomenological parameter. Equation (2.3.20) is another form of the second equation of London.

2.4 Types of superconducting materials:

Below T_c , the superconducting state has a lower free energy than in the normal state but it requires the expulsion of the flux. This costs some magnetic energy which has to be smaller than the condensation energy gained in undergoing the phase transition into the superconducting state (i.e., by forming the coherent ensemble of Cooper pairs from the “random” electron gas). Obviously, if we begin to increase the external magnetic field it will reach the point where the cost in magnetic energy will outweigh the gain in condensation energy and the superconductor will become partially (in a particular sample geometry) or totally normal. Superconductivity disappears and the material returns to the normal state if one applies an external magnetic field of strength greater than some critical value B_c , called the critical thermodynamic field. The superconducting state can also be destroyed by passing an excessive current through the material, which creates a magnetic field at the surface of strength equal to or greater than B_c . This limits the maximum current that the material can sustain and is an important problem for applications of superconducting materials.

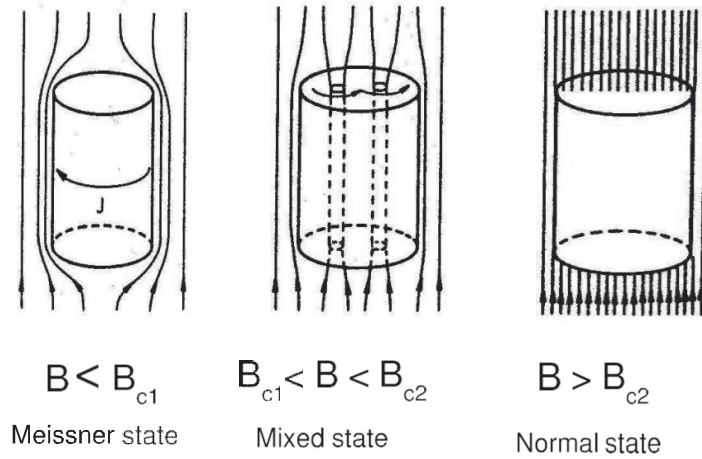


Figure 2.2: Flux penetration in the mixed state.

2.4.1 Type-I superconductors:

Superconducting materials that completely expel magnetic flux until they become completely normal are called *type-I* superconductors. With the exception of V and Nb, all superconducting elements and most of their alloys in the “dilute limit”, are type-I superconductors. The strength of the applied magnetic field required to completely destroy the state of perfect diamagnetism in the interior of the superconducting specimen is called the thermodynamic critical field B_c . As schematically shown in Fig. 2.4, the variation of the critical field B_c with temperature for type-I superconductor is approximately parabolic:

$$B_c = B_0 \left(1 - \frac{T}{T_c}\right) \quad (2.4.1)$$

where B_0 is the extrapolated value of $T = 0$. $B = \mu_0 (H + M)$ where M is the magnetization and $\mu_0 = 4\pi \times 10^{-7}$. The Meissner effect, $\mathbf{B} = \mathbf{0}$, corresponds to $\mathbf{M} = -\mathbf{H}$. Above the critical field B_c , the material becomes normal, so $\mathbf{M} = \mathbf{0}$. The negative sign shows that the sample becomes a perfect diamagnetic that excludes the flux from its interior by means of surface currents.

2.4.2 Type-II superconductors:

For a type-II superconductor there are two critical fields. The lower B_{c1} and the upper B_{c2} . The flux is completely expelled only up to the field B_{c1} .

So, in applied fields smaller than B_{c1} , the type-II superconductor behaves just like a type-I superconductor below B_c . Above B_{c1} the flux partially penetrates into the material until the upper critical field, B_{c2} , was reached. Above B_{c2} the material returns to the normal state (see Fig. 2.3). Between B_{c1} and B_{c2} the superconductor is said to be in the mixed state. The Meissner effect is only.

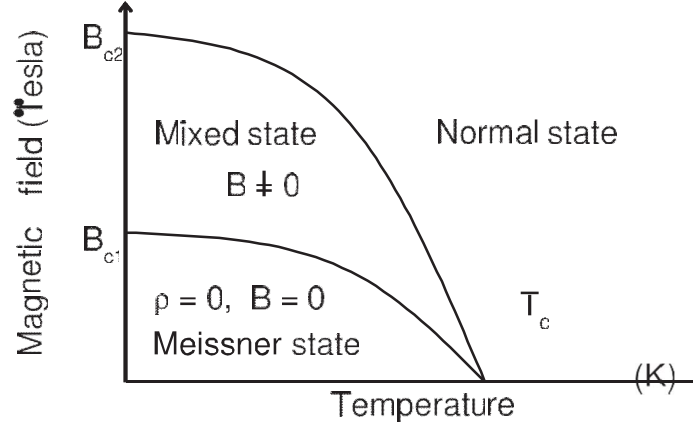


Figure 2.3: Variation of critical fields, B_{c1} and B_{c2} as a function of temperature.

The upper critical field B_{c2} can be very high above 100 T in case of high- T_c superconducting material. partial. For all applied fields $B_{c1} < B < B_{c2}$, magnetic flux partially penetrates the super- conducting specimen in the form of tiny microscopic filaments called vortices Fig. 2.3. The diameter of a vortex in conventional superconductors is typically 100 nm. It consists of a normal core, in which the magnetic field is large, surrounded by a superconducting region in which flows a persistent super-current which maintains the field within the core Each vortex carries a magnetic flux. $\Phi_0 = \frac{h}{2e} = 2.067 \times 10^{-15} T^2 m$ where h is Planck constant and e is the electron charge. The magnetic induction B is directly related to n , the number of vortices per $B = n\Phi_0$. Superconductivity can and does persist in the mixed state up to the upper critical fields of B_{c2} which is sometimes higher than 60 Tesla or even 150 Tesla in high T_c superconductors.

2.4.3 Flux Quantization:

In semi classical approximate [8].

$$\frac{E^*(r)E(r)}{4\pi} = n(r)\hbar\omega \quad (2.4.2)$$

Where:

$n(r)$ = photon number density.

$\hbar\omega$ = energy of one photon.

$E(r)$ = electric field intensity.

According to equation (2.4.2) the E can be written in the form:

$$E = n^{1/2}e^{i\theta} \quad (2.4.3)$$

$$E^* = n^{1/2}e^{-i\theta} \quad (2.4.4)$$

The wave function φ satisfies

$$n = |\varphi|^2 = \varphi\varphi^*$$

Thus one can write :

$$\varphi = n^{1/2}e^{i\theta}, \quad \varphi^* = n^{1/2}e^{-i\theta} \quad (2.4.5)$$

The momentum P for particle of mass m , velocity v , moving in electromagnetic field of potential A satisfies

$$P = mv + \frac{qA}{c} \quad (2.4.6)$$

Where:

q = Charge of electron = e

c = Speed of light.

Thus from (2.4.6) yields:

$$\begin{aligned} m v &= p - \frac{qA}{c} \\ v &= \frac{1}{m} \left(p - \frac{qA}{c} \right) \end{aligned} \quad (2.4.7)$$

Thus the current density takes the from:

$$\begin{aligned} J &= n q v \\ &= \varphi\varphi^* q v \\ &= q \varphi^* v \varphi \end{aligned} \quad (2.4.8)$$

From equation (2.4.7) and (2.4.8) yields:

$$J = \frac{q\varphi^*}{m} \left(p - \frac{qA}{C} \right) \varphi \quad (2.4.9)$$

From quantum laws

$$P \rightarrow \hat{P} = \frac{\hbar}{i} \nabla$$

Thus

$$J = \frac{q\varphi^*}{m} \left(\nabla - \frac{qA}{C} \right) \varphi$$

From equation (2.4.5) yields:

$$J = \frac{n^{1/2} e^{-i\theta}}{m} \left(\frac{\hbar}{i} \nabla - \frac{qA}{C} \right) n^{1/2} e^{i\theta}$$

For: $n = \text{Constant} \rightarrow \theta = \theta(r)$

$$\begin{aligned} \frac{\hbar}{i} \nabla &= \frac{\hbar}{i} \frac{\partial}{\partial r} n^{1/2} e^{i\theta(r)} = \frac{\hbar}{i} n^{1/2} \frac{\partial}{\partial r} e^{i\theta(r)} \\ &= \frac{\hbar}{i} n^{1/2} \frac{\partial}{\partial r} e^{i\theta(r)} \frac{\partial \theta}{\partial r} = \frac{\hbar}{i} n^{1/2} (i) \frac{\partial}{\partial r} e^{i\theta} \frac{\partial \theta}{\partial r} \\ &= \hbar n^{1/2} e^{i\theta} \frac{d\theta}{dr} = \hbar \varphi \frac{\partial \theta}{\partial r} \end{aligned} \quad (2.4.10)$$

Thus

$$\begin{aligned} J &= \frac{n^{1/2}}{m} e^{-i\theta} \left(\frac{\hbar}{i} \nabla - \frac{qA}{C} \right) n^{1/2} e^{i\theta} - \frac{q}{C} \frac{n^{\frac{1}{2}}}{m} e^{-i\theta} A n^{1/2} e^{i\theta} \\ &= \frac{n^{\frac{1}{2}}}{m} e^{-i\theta} (\hbar n^{1/2} e^{i\theta}) \frac{d\theta}{dr} - \frac{qn}{Cm} A \\ J &= \frac{\hbar n}{m} \frac{d\theta}{dr} - \frac{qn}{Cm} A \end{aligned} \quad (2.4.11)$$

2.4.4 The Meissner-Ochsenfeld Effect:

In addition to zero resistivity (i.e., infinite conductivity), the superconductor exhibits another striking property: it expels the magnetic field from its interior. This is not a consequence of infinite conductivity, it is another intrinsic characteristic property of the superconducting state which shall now be discussed in some detail. As already illustrated in Fig. 2.1, in the normal state at temperatures above T_c the field lines pass through the metallic specimen. Upon cooling below T_c , a phase transition into the superconducting state takes place and the magnetic flux gets expelled out of the interior of the metallic sample. The Meissner-Ochsenfeld effect [2] cannot be deduced from the infinite conductivity of a superconductor. The exclusion of the magnetic field from the interior of a superconducting specimen is a direct evidence that the superconducting state is not simply one of zero resistance. If it were so, then a superconductor cooled in the magnetic field through T_c would have trapped the field in its interior. When the external field is removed, the induced persistent eddy currents would nevertheless preserve the trapped field in the interior of the specimen.

The expulsion of the flux therefore implies that this new superconducting state is a true thermodynamic equilibrium state. The above argument can be proved by a few elementary formulae of electrodynamics. Consider Ohm's law, $V = RI$, written as $E = \rho j$, where E represents the electric field, ρ the resistivity and j the electrical current density in the sample. Zero resistivity implies zero electric field. So, if we take the Maxwell equation.

$$\nabla E = -\frac{\partial B}{\partial t} \quad (2.4.12)$$

We have

$$\frac{\partial B}{\partial t} = 0 \quad (2.4.13)$$

We see that the magnetic induction in the interior of the sample has to be constant as a function of time. The final state of the sample would have been different if it were cooled under an applied external field or if the field were applied after the sample has been cooled below T_c . In the former case the field would have remained within the sample, while in the latter it would have been zero. For the specimen to be in the same thermodynamic state, independent of the precise sequence that one uses in cooling or in applying the field, the superconducting metal always expels the field from its interior, and has $B = 0$ in its interior. So the expulsion of the magnetic field ensures that the superconducting state is a true thermodynamic state.

2.5 Bardeen-Cooper-Schrieffer theory:

In 1957, BARDEEN, COOPER, and SCHRIEFFER (BCS) proposed a general microscopic theory of superconductivity that quantitatively predicts many properties of super-conductors and is now widely accepted as providing a satisfactory explanation of the phenomenon [3]. There are various levels of approximation in which the BCS theory has been applied. The mathematical underpinning of the BCS-theory is so complex that it will not be of much benefit to summarize its general formulation, so this section will emphasize predictions that are often compared with experiments. These predictions arise mainly from the homogeneous, isotropic, phonon-mediated, square well, s -wave coupling simplification of the BCS-theory, and many superconductors, to a greater or lesser extent see Figure 2.4.

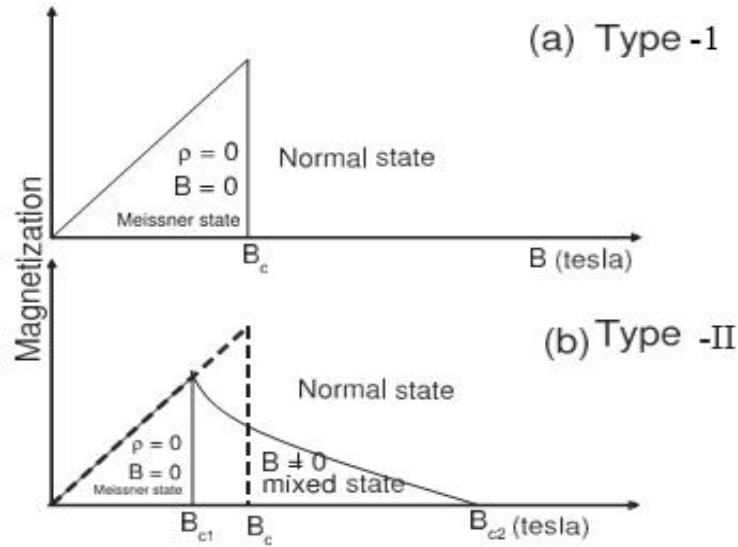


Figure 2.4: Variation of magnetization as a function of the magnetic field for type-I and type-II superconductor .

Have been found to satisfy these predictions. Some of them are as follows:
 The isotope effect involves the claim that for a particular element the transition temperature T_c depends on the mass M of the isotope as follows:

$$M^\alpha T_c = \text{const} \quad (2.5.1)$$

The weak coupling BCS limit gives the value $\alpha = 1/2$, which has been observed in some superconducting elements, but not in all of them. A superconductor has an energy gap $E_g = 2\Delta(k)$, which is assumed to be independent of wave vector k , and for this assumption the energies in the normal and superconducting states are $E(\varepsilon) = \varepsilon$ normal state and:

$$E(\varepsilon) = (\varepsilon^2 + \Delta^2)^{\frac{1}{2}} \quad (2.5.2)$$

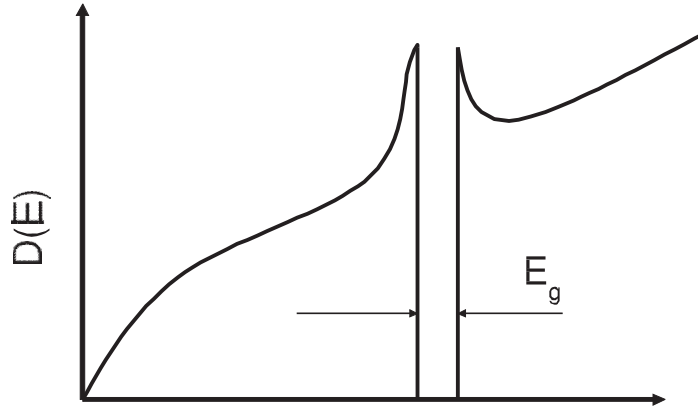
superconducting state where ε is the energy in the absence of a gap measured relative to the chemical potential:

$$\varepsilon = \frac{\hbar^2 k^2}{2m} - \mu \quad (2.5.3)$$

The density of states $D(E)$ given by (with $E = 0$ in the center of the gap)

$$D(E) = \frac{D_n(E)}{(E^2 - \Delta^2)^{\frac{1}{2}}} \quad ; \quad E > \Delta \quad (2.5.4)$$

$$= D_\varepsilon(E) = 0 \quad ; \quad -\nabla < E < \Delta$$



$$\nabla E = -\frac{D_n(E)}{(E^2 - \Delta^2)^{\frac{1}{2}}} \quad , \quad E < \Delta$$

Figure 2.5: Energy dependence of the density of states $D(E)$ in the presence of an energy gap.

Is shown plotted in Fig. 2.5, where the normal electron density of states $D_n(E)$ is assumed to have the constant value $D_n(0)$ in the neighborhood of the gap.

Consider a square-well electron-electron potential V_0 and an energy gap $\Delta(k)$ that is equal to Δ_0 in the neighborhood of the Fermi surface $\Delta(k) = \Delta_0$, $-\hbar\omega_D \leq \varepsilon(k) \leq \hbar\omega_D$ and is zero elsewhere. The Debye frequency ω_D determines the range of s because it is assumed that Cooper pair formation is mediated by phonons. The energy gap Δ_0 in this approximation is given by:

$$\Delta_0 = \frac{\hbar\omega_D}{\sinh \left[\frac{1}{V_0 D_n(0)} \right]} \quad (2.5.5)$$

In the weak coupling (small V_0) limit,

$$V_0 D_n(0) \ll 1, \quad k_B T_c \ll \hbar \omega_D$$

we obtain the dimensionless ratios

$$\frac{E_g}{k_B T_c} = \frac{2\Delta_0}{k_B T_c} = \frac{2\pi}{\exp \gamma} = 3.52, \quad (2.5.6)$$

where $\gamma = 0.5772$ is the Euler-Mascheroni constant. This ratio approximates experimental measurements that have been made on many superconductors. The dimensionless electron-phonon coupling constant λ is related to the phonon density of states $D_{ph}(\omega)$ through the Eliashberg expression [7]:

$$\lambda = 2 \int_0^\infty \frac{\alpha(\omega) D_{ph}(\omega)}{\omega} d\omega \quad (2.5.7)$$

Superconductors are characterized as having weak ($\lambda \ll 1$), intermediate ($\lambda \approx 1$), and strong ($\lambda \gg 1$) coupling. The electron-electron interaction potential V_0 for Cooper pair bonding has an attractive electron-phonon part measured by λ and a repulsive screened Coulomb part μ_c to give $V_0 D_n(0) = \lambda - \mu_c^*$ and this provides the well known formula for the critical temperature T_c :

$$T_c 1.13 \theta_D \exp \left[\frac{-1}{\lambda - \mu_c^*} \right] \quad (2.5.8)$$

where θ_D is the Debye temperature related to ω_D by $\hbar \omega_D = k_B \theta_D$. θ_D ranges from 100 K to 500 K. This range of θ_D (and $\lambda - \mu_c^* \approx 0.3$) implies a maximum BCS value of $T_c \sim 25$ K. A number of related formulae for the dependence of T_c on λ and μ_c^* have appeared in the literature, e.g., the McMillan equation [8].

$$T_c = \frac{\theta_D}{1.45} \exp \left[\frac{-1.04(1 + \lambda)}{\lambda - \mu_c^*(1 + 0.62\lambda)} \right] \quad (2.5.9)$$

The BCS theory predicts that at T_c there is a jump in the electronic specific heat from its normal state value $C_e = \gamma T$ to its superconducting

state value C_s given by:

$$\frac{C_s - \gamma T_c}{\gamma T_c} = 1.43 \quad (2.5.10)$$

In the free electron approximation the electronic specific heat coefficient γ depends on the Fermi temperature T_F and the gas constant R through the expression $\gamma = \frac{\pi R}{2T_F}$. Below T_c , the BCS-theory predicts that the specific heat $C_s(T)$ depends exponentially on the inverse temperature,

$$C_s(T) = a \exp\left[\frac{-\Delta}{k_B T}\right] \quad (2.5.11)$$

Where $\Delta = 1.76k_B T_c$, and a is a constant.

2.5.1 Cooper-pairs and T_c :

There are three levels of explanation of the nature of superconductivity that are commonly called BCS. One is the general formulation that does not specify particular interactions. The second is the phonon-mediated version of the theory, in which phonons play the role of bringing about the coupling together of two electrons to form Cooper pairs. The third level, which was described in the original formulation of the theory and provided the results summarized previously. Further it assumes the simplification of an isotropic, homogeneous material with a square-well electron-electron interaction potential involving a phonon coupling mechanism and s -wave singlet-state pairing. The superconducting elements, which are almost all type-I, as well as many classical type-II superconductors, are looked upon as phonon-mediated s -state types. Pairing mechanisms involving the exchange of particles other than phonons, such as excitons or antiferromagnetic spin fluctuations, have been proposed for non-classical superconductors. In particular, no consensus exists yet about the pairing mechanism of the cuprates, although d -wave pairing seems to be favored, another very recently results appears to support the phonon mediated in high- T_c superconductivity [9].

2.6 The Ginzburg-Landau theory:

The London theory is not applicable to situations in which the number of super-electrons, n_s , varies; it does not link n_s with the applied field or current. Therefore we need a more general framework which relates n_s to the external parameters. This is the approach of the Ginzburg-Landau theory [10] which uses the general (Landau) theory of second order phase transitions by introducing the corresponding an order parameter.

2.6.1 Ginzburg-Landau free energy:

The basis of this description is the intuitive idea that a superconductor contains superconducting electrons with density n_s and non-superconducting electrons with density $n - n_s$, where n is the total density of electrons in the metal. Ginzburg and Landau have chosen to use a kind of a wave function, to describe the superconducting electrons. This function is a complex scalar:

$$\psi(r) = |\psi(r)| \exp i\varphi(r) \quad (2.6.1)$$

and is called the order parameter. It has the following properties:

- Its modulus $|\psi^*\psi|$ can be interpreted as the number of superconducting electrons n_s at a point r .
- As in quantum mechanics, the phase $\varphi(r)$ is related to the supercurrent that flows through the material below T_c .
- $\psi \neq 0$ in the superconducting state, but $\psi = 0$ in the normal state.

Furthermore, Ginzburg and Landau have used the following form of the Helmholtz function:

$$F_s(r, T) = F_n(r, T) + \alpha |\psi(r)|^2 + \frac{\beta}{2} |\psi(r)|^4 + \frac{1}{2m} |(i\hbar\nabla - 2e\bar{A})\psi|^2 + \frac{\mu_0 \bar{H}^2}{2} \quad (2.6.2)$$
$$F_s = \int_v F_s(r, T) d^3r$$

where s and n denote the superconducting and normal state, while k is Planck's constant and V is the volume of the sample.

In order to see the advantage of using complex functions in describing superconductivity we rewrite equation (2.6.2) using the modulus and phase of the order parameter; hence we get:

$$F_s(r, T) = F_n(r, T) + \alpha |\psi(r)|^2 + \frac{B}{2} |\psi(r)|^4 + \frac{\hbar}{2m} (\nabla |\psi|)^2 + \frac{1}{2} |\psi|^2 m \bar{V}_s^2 \frac{\mu_o \bar{H}^2(r)}{2} \quad (2.6.3)$$

Where:

$$\bar{V}_s^2 = \frac{1}{m} (\hbar \nabla \phi - 2e\bar{A})$$

One can see that we have obtained the Landau expansion plus the free energy of the field and the current. If the order parameter does not vary in space, one gets back exactly to the London free energy and London equation by carrying out the minimization. If there is no magnetic field and the order parameter has no phase, one obtains the usual Landau equation. The Ginzburg-Landau free energy is thus the way to introduce the London idea in the usual second order phase transition. Equation (2.6.2) introduces two phenomenological parameters, α and β , in the free energy. The fourth term in equation (2.6.2) is the energy associated with variations of ψ in space. It is written as if representing a true quantum mechanical wave function; $\bar{A}(r)$ in the vector potential at a point r and \bar{H} is the microscopic field at the same point. As it is known from electromagnetism: $\mu_o \bar{H} = \text{curl } \bar{A}$. The Helmholtz energy is the integral over the total volume of the sample of the energy density that depends on the point of consideration. As in the Landau theory one takes:

$$\alpha = a(T - T_c), \beta = \text{positive constant, independent of } T.$$

2.6.2 Ginzburg-Landau equations:

In order to determine the order parameter $\psi(r)$ and the vector-potential $\dot{A}(r)$ we minimize the Helmholtz free energy with respect to $\psi(r)$ and \dot{A} . By this double minimization one gets two equations named after their authors, Ginzburg-Landau (GL) equations:

$$\alpha\Psi + \beta|\Psi|^2\Psi + (i\hbar\nabla - 2eA)^2\Psi = 0$$

$$J = \text{curl}H = \frac{e}{m}[\Psi(i\hbar\nabla - 2eA)^2\Psi + cc] \quad (2.6.4)$$

These two equations are coupled and should therefore be solved simultaneously. The first one gives the order parameter $\psi(r)$ while the second enables one to describe the super-current that flows in the superconductor.

2.6.3 Magnetic penetration depth λ :

One of the theoretical approaches for the description of the superconducting is London equation. It relates the curl of the current density J to the magnetic field according to equation (2.3.20) [11].

However, from Maxwell equation under static conditions:

$$\nabla \times B = \frac{-4\pi J}{c} = \mu_o J \quad (2.6.5)$$

Where

$$\mu_o = \frac{4\pi}{c}$$

This equation can be expressed in terms of the magnetic flux density by taking the curl of both side of equation (2.6.5) to get:

$$\nabla \times \nabla \times B = \nabla(\nabla \cdot B) - \nabla^2 B = \frac{4\pi}{c} (\nabla \times J) \quad (2.6.6)$$

Where

$$\nabla \cdot B = 0 \implies \nabla(\nabla \cdot B) = 0$$

according to Maxwell equation. This lead to:

$$\nabla^2 B = -\frac{4\pi}{c} (\nabla \times J)$$

Thus with the aid of equation (2.3.20) one gets:

$$\nabla^2 B = \frac{1}{\lambda_L^2} B \quad (2.6.7)$$

If $B = B_a = \text{constant}$, i.e. B_a is the field outside the superconductor and at the boundary. Then $\nabla^2 B$ is always zero, but $\frac{B_a}{\lambda_L^2}$ is not zero unless B_a is zero. This

result shows that if magnetic flux density is constant inside a superconductor.

In one dimension one of the possible solutions:

$$B = B_0 \exp(-\alpha x) \quad (2.6.8)$$

$$\frac{\partial B}{\partial x} = -\alpha B_0 \exp(-\alpha x) = -\alpha B$$

$$\frac{\partial^2 B}{\partial x^2} = \alpha \frac{\partial B}{\partial x} = \alpha^2 B \quad (2.6.9)$$

Sub (2.6.9) in (2.6.7) we get:

$$\alpha^2 B = \frac{1}{\lambda_L^2} B \quad (2.6.10)$$

$$\alpha = \frac{1}{\lambda_L}$$

Hence from (2.6.8)

$$B = B_0 \exp\left(-\frac{x}{\lambda_L}\right) \quad (2.6.11)$$

2.6.4 Coherence length ξ :

The distance between cooper pairs is called coherence length λ_0 . Consider cooper electrons having two states.

$$\begin{aligned} \varphi_1 &= Ae^{ikx} \\ \varphi_2 &= Ae^{i(k+q)x} \end{aligned} \quad (2.6.12)$$

The wave function for the two states

$$\begin{aligned}\varphi &= \frac{1}{\sqrt{2}}(e^{ikx} + e^{i(k+q)x}) \\ &= \varphi_1 + \varphi_2\end{aligned}\quad (2.6.13)$$

Where normalization condition requires the probability:

$$A = \frac{1}{\sqrt{2}}$$

That the electron is at $x=| \varphi |^2 = \varphi \varphi^*$

$$| \varphi |^2 = \frac{1}{2} (e^{ikx} + e^{i(k+q)x})(e^{-ikx} + e^{-i(k+q)x}) \quad (2.6.14)$$

$$\begin{aligned}&= \frac{1}{2} (1 + e^{-iqx} + e^{iqx} + 1) \\ &= 1 + \frac{1}{2} (\cos qx - i \sin qx + \cos qx + i \sin qx) \\ P = | \varphi^2 | &= 1 + \cos qx\end{aligned}\quad (2.6.15)$$

Cooper pairs are free hence:

$$\begin{aligned}\hat{H} &= \frac{p^2}{2m} + v = \frac{p^2}{2m} \\ \hat{H} &= \frac{\hbar^2}{2m} \nabla^2\end{aligned}\quad (2.6.16)$$

Thus the average energy:

$$\langle E \rangle = \int \varphi^* \hat{H} \varphi dx \quad (2.6.17)$$

$$\langle E \rangle = \int (\varphi_1^* + \varphi_2^*) \left(-\frac{\hbar^2}{2m} \nabla^2 \right) (\varphi_1 + \varphi_2) dx \quad (2.6.18)$$

$$\begin{aligned}\nabla^2(\varphi_1 + \varphi_2) &= \frac{1}{\sqrt{2}} \left(\frac{\partial^2}{\partial x^2} (e^{ikx} + e^{i(k+q)x}) \right) \\ &= \frac{1}{\sqrt{2}} \frac{\partial}{\partial x} (ike^{ikx} + i(k+q)e^{i(k+q)x}) \\ &= \frac{1}{\sqrt{2}} - (k^2 e^{ikx} - (k+q)^2 e^{i(k+q)x})\end{aligned}\quad (2.6.19)$$

$$= -k^2\varphi_1 - (k + q)^2\varphi_2 \quad (2.6.20)$$

Sincere normalization condition requires

$$\int \varphi_i^* \varphi_j dx = \delta_{ij} \quad (2.6.21)$$

$$\delta_{ij} \begin{cases} 0 & i \neq j \\ 1 & i = j \end{cases}$$

Thus inserting (2.6.16) in (2.6.18) and using (2.6.20) yields:

$$\langle E \rangle = \frac{\hbar^2}{2m} \int (\varphi_1^* + \varphi_2^*) (k^2\varphi_1 + (k + q)^2\varphi_2) dx \quad (2.6.22)$$

$$\begin{aligned} \langle E \rangle &= \frac{\hbar^2}{2m} k^2 \int (\varphi_1^* \varphi_1 dx + (k + q)^2 \int \varphi_1 \varphi_2^* dx) + k^2 \int (\varphi_2^* \varphi_1 dx \\ &+ (k + q)^2 \int (\varphi_2^* \varphi_2 dx) = \end{aligned}$$

$$\langle E \rangle = \frac{\hbar^2}{4m} (k^2 + 0 + 0 + (k + q)^2)$$

$$\langle E \rangle = \frac{\hbar^2}{4m} (k^2 + k^2 + 2qk + q^2)$$

But $q \ll k$

Thus:

$$\langle E \rangle = \frac{\hbar^2}{2m} (k^2 + qk) \quad (2.6.23)$$

Average Energy:-

$$\langle E \rangle = \frac{\hbar^2 k^2}{2m} + \frac{\hbar^2 kq}{2m}$$

In the average for Cooper Pairs.

For single electron the average energy is:

$$\langle E \rangle_{single} = \frac{\hbar^2 k^2}{2m}$$

Where:

E_g = energy gap = binding energy for Cooper pairs.

$$= \langle E \rangle - \langle E \rangle_{single} = \frac{\hbar^2 k q}{2m} = \frac{\hbar k_F q_0}{2m} \quad (2.6.24)$$

Where:

k_F = wave number at Fermi surface.

q_0 = Critical wave number change.

$$E_g = \frac{\hbar k_F q_0}{2m} = \quad (2.6.25)$$

The Cooper Pairs are destroyed

When:

$$\begin{aligned} E &> E_g \\ \frac{\hbar k_F q}{2m} &> \frac{\hbar k_F q_0}{2m} \\ q &> q_0 \end{aligned}$$

In this case ,when Cooper Pairs are destroyed.

Since $k \rightarrow k + q$

$$\text{And: } k = \frac{2\pi}{\lambda}$$

Hence:

$$q_0 = \frac{2\pi}{\lambda_0} \quad (2.6.26)$$

But:

$$\hbar k_F = p = m v_F$$

Since:

$$\begin{aligned} \lambda_0 &= \frac{2\pi}{q_0} = \frac{2\pi \hbar^2 k_F}{2m E_g} \\ \lambda_0 &= \frac{\pi \hbar m v_F}{m E_g} = \frac{\pi \hbar v_F}{E_g} \end{aligned} \quad (2.6.27)$$

λ_0 = Intrinsic Coherence Length.

2.6.5 Ginzburg-Landau parameter:

If both characteristic lengths λ and ξ diverge at T in the same manner as $|\alpha|^{-\frac{1}{2}}$, their ratio κ , which is called the Ginzburg-Landau parameter, does not depend on temperature:

$$\kappa = \frac{\lambda}{\xi}$$

Actually κ is the only parameter that really appears in Ginzburg-Landau equation. One can distinguish two different situations for κ :

- If $\kappa < \frac{1}{\sqrt{2}}$ the superconducting material is a **type-I** superconductor.
- If $\kappa > \frac{1}{\sqrt{2}}$ the superconducting material is **type-II** superconductor. A detailed concern of this issue is presented in next the section.

Chapter Three

Temperature Dependent Schrodinger Equation

3.1 Introduction:

In this chapter one need to find T_c for high superconducting materials in general with aid of quantum mechanical treatment in which electrons are considered as harmonic oscillator beside using plasma equations. This work tries to explain the isotope effect. Isotope effect is one of the challenging problem of HTSC. Many models were proposed to explain it. But no complete satisfactory theory exists till now. This chapter is concerned with Schrodinger temperature dependent to explain.

3.2 plasma equation and Temperature Dependent Schrodinger Equation:

Schrodinger equation can be derived by using a new expression of energy obtain from the plasma equation. This new energy expression includes thermal energy beside kinetic and potential energy. According to plasma equation, a fluid of particles of mass m , number density n , velocity v , force F and acted on by potential V at pressure p is given by:

$$mn \left[\frac{\partial v}{\partial t} + v \nabla v \right] = F - \nabla p \quad (3.2.1)$$

If F is a field force then

$$F = -n \nabla V \quad (3.2.2)$$

Where V is the potential of one particle. In one dimension

$$mn \left[\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial x} \right] = -n \nabla V - \nabla p = -n \frac{dV}{dx} - \frac{dP}{dx} \quad (3.2.3)$$

$$dv = \frac{dv}{dt} dt + \frac{dv}{dx} dx$$

$$\frac{dv}{dt} = \frac{dv}{dt} + \frac{dv}{dx} \frac{dx}{dt} = \frac{dv}{dt} + v \frac{dv}{dx}$$

Thus equation according to equation (3.2.1)

$$mn \frac{dv}{dt} = -n \frac{dV}{dx} - \frac{dP}{dx} \quad (3.2.4)$$

$$mn \frac{dv}{dx} \frac{dx}{dt} = -n \frac{dV}{dx} - \frac{dP}{dx}$$

Multiplying both sides by dx and integrating yields:

$$mn \int v dv = -n \int dV - \int dP \quad (3.2.5)$$

When P is a pressure exerted by the plasma on the surrounding. For thermal motion it is given by

$$p = nkT \quad (3.2.6)$$

For thermal pressure exerted by surrounding

$$p = -nkT \quad (3.2.6)$$

in general, thus

$$\frac{1}{2} mnv^2 = -nV = p = -nV - kT \quad (3.2.8)$$

Hence

$$\frac{1}{2} mv^2 + V \pm kT = \text{const} \quad (3.2.9)$$

Thus for uniform density the quantity within the bracket is a constant of motion. On pure dimension ground, clearly this constant of motion is the energy E , i.e.

$$E = \frac{p^2}{2m} + V \pm kT \quad (3.2.10)$$

$$E = \frac{1}{2} mv^2 + V \pm kT$$

3.3 Temperature Dependent Schrodinger Equation:

To find Schrodinger equation for it, consider the ordinary wave function

$$\psi = A \exp \frac{i}{\hbar} (px - Et) \quad (3.3.1)$$

Differential both sides by t and x yields

$$\frac{\partial \psi}{\partial t} = \frac{i}{\hbar} E \psi \Rightarrow \psi = i \hbar \frac{\partial \psi}{\partial t} = E \psi \quad (3.3.2)$$

$$\frac{\partial^2 \psi}{\partial x^2} = -\frac{p^2}{\hbar^2} \psi \Rightarrow -\hbar^2 \nabla^2 \psi = p^2 \psi$$

Multiplying both sides of equation (3.2.8) by ψ yields:

$$E\psi = \frac{p^2}{2m}\psi + V\psi \pm kT\psi \quad (3.3.3)$$

Substitution equation (3.3.2), one gets:

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + V\psi \pm kT\psi \quad (3.3.4)$$

This equation represents Schrodinger equation when thermal motion is considered. The solution for time free potential can be

$$\psi = \exp\left(-\frac{i}{\hbar}Et\right)\psi \Rightarrow \frac{\partial \psi}{\partial t} = -\frac{i}{\hbar}E\psi$$

The time independent Schrodinger equation thus takes the form

$$-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi \pm kT\psi = E\psi \quad (3.3.5)$$

3.4 Critical Temperature and Isotope Effect:

From equation (3.3.5) yields:

$$-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi \pm kT\psi = E\psi$$

For constant potential, the solution can be

$$u = e^{(ikx)}, \quad V = V_0 \quad (3.4.1)$$

Inserting this solution in equation (3.3.5) yields:

$$Eu = -\frac{\hbar^2 k^2}{2m}u + V_0 u \pm kTu \quad (3.4.2)$$

$$E = -\frac{\hbar^2 k^2}{2m} + V_0 \pm kT$$

If one set the kinetic term to be $E_0 = \frac{\hbar^2 k^2}{2m}$, one can thus write the energy in the form

$$E = E_0 + V_0 \pm kT \quad (3.4.3)$$

This quantum energy expression involves a thermal term besides kinetic and potential term .Treating electron as vibrating string, the kinetic and potential

energy are given by:

$$T = \frac{1}{2}mv^2 \quad (3.4.4)$$

$$V = \frac{1}{2}kx^2 = \frac{1}{2}(m\omega^2)x^2 \quad (3.4.5)$$

Where

$$x = x_0 e^{i\omega t} \quad (3.4.6)$$

$$v = \dot{x} = i\omega x \quad (3.4.7)$$

Thus

$$T = \frac{1}{2}m|v^2| = \frac{1}{2}m\omega^2 x^2 = V \quad (3.4.8)$$

Thus

$$E = T + V = 2T = 2V \quad (3.4.9)$$

Using the definition of R as:

$$V_e = \frac{v}{e} \quad (3.4.10)$$

where V_e electronic potential, v is potential and e is charge of electron.

$$R = \frac{V_e}{I} = \frac{V}{eI} = \frac{E}{2eI} \quad (3.4.11)$$

From equation (3.4.3) and equation (3.4.11) yields:

$$R = \frac{E_0 + V_0 \pm kT}{2eI} \quad (3.4.12)$$

For pressure exerted on the surrounding:

$$R = \frac{E_0 + V_0 \pm kT}{2eI}$$

For attractive force by no ions per unit volume where

$$n_0 = \frac{N}{V} = \frac{M}{mV} = \frac{M}{V} \frac{1}{m} = \frac{\rho}{m} \quad (3.4.13)$$

Where N is number of particle, V is total volume, M is total mass and ρ is density of particles per unit volume.

$$V = -n_0 V_a = \frac{\rho}{m} V_a \quad (3.4.14)$$

Where V_a is potential of one atom.

Thus

$$R = \frac{E_0 \pm kT - \frac{\rho}{m}}{2eI} = R_+ + R_- \quad (3.4.15)$$

Where

$R_s = R_+ =$ superconductivity resistance.

Thus

$R_s = R_+ = 0$ when $R < 0$ in this case $R = R_-$

From equation (3.4.3) yields:

$$\begin{aligned} E &= E_0 + V_0 \pm kT \\ E_0 \pm kT - \frac{\rho}{m} V_a &< 0 \end{aligned} \quad (3.4.16)$$

Thus

$$\begin{aligned} kT &< \frac{\rho}{m} V_a - E_0 \\ T &< \frac{\rho}{km} V_a - \frac{E_0}{k} \end{aligned} \quad (3.4.17)$$

Thus

$$T_c < \frac{\rho}{km} V_a - \frac{E_0}{k} \quad (3.4.18)$$

Hence

$$T_c \propto \frac{1}{m} \quad (3.4.19)$$

Consider the resistance to be:

$$R = \frac{V_e}{I} = \frac{V}{eI} = \frac{V}{enAev} \quad (3.4.20)$$

Where e is the electron charge, v is the velocity of electron, n is the density of electron and V is the potential energy. Then :

For string

$$V = T = \frac{1}{2} m v^2$$

Thus

$$R = \frac{mv^2}{2e^2nAv} = \frac{mv}{2ne^2A} \quad (3.4.21)$$

$$mv = p = \hbar k = \sqrt{3mE_0} \quad (3.4.22)$$

From equation (3.4.3) yields:

$$E = E_0 + V_0 \pm kT$$

$$mv = \sqrt{E \pm kT - V}$$

For pressure exerted by the plasma:

$$mv = \sqrt{E + kT - V}$$

For repulsive force by electron gas

$$V = V_0 = n_0 V_a = \frac{\rho V_a}{m} \quad (3.4.23)$$

Thus

$$R = \frac{\sqrt{E + kT - V_0}}{2e^2nA} \quad (3.4.24)$$

R can be written as sum of real R_r and imaginary R_i

$$R = R_r + R_i$$

Where

$R_s = R_r =$ superconductivity resistances.

R become imaginary when

$$kT + E - V_0 < 0$$

$$T < \frac{V_0 - E}{k}$$

Thus

$$T_c = V_0 - E = \frac{\rho V_a}{m} E \quad (3.4.24)$$

$$T_c \propto \frac{1}{m} \quad (3.4.25)$$

3.5 Discussion:

Plasma equation (3.2.1) is used to derive useful energy equation (3.2.10). This equation has additional term representing thermal energy Schrodinger temperature dependent equation is derived in section (3.3) see equations (3.3.4) and (3.3.5). By defining resistance in terms of energy and treating electrons as string, and assuming the electron to be effected attraction op positive ions beside a pressure exerted on the surrounding T_c is found to be inversely proportional to isotope mass. An alternative derivation is made by equating kinetic and potential term of string, then defining R in terms of potential. Assuming pressure exerted by plasma beside the repulsive effect of electron gas. The critical temperature is inversely proportional to isotope mass.

3.6 Conclusion:

Temperature dependent Schrodinger equation derived from plasma equation is shown to be useful in describing isotope effect. It is shown that the critical temperature is inversely proportional to isotope mass.

3.7 Recommendations:

The relation between critical temperature and isotopic mass showed be generalized to include all superconductors compounds, the effect of pressure on superconductors temperature need to be studied also, and the phase diagram of high temperature superconductors need also to be rest on solid theoretical background.

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