Chapter 3

3.1 Introduction

OFDM signals are in general bipolar signals and have both negative and positive amplitudes. Since VLC uses IM, a real and unipolar valued signal needs to be produced. Therefore, the conventional OFDM scheme used in RF communications should be modified. To achieve a real valued output signal, Hermitian symmetry is used on the parallel data streams into the IFFT input. This comes at the cost of losing half the available bandwidth [6]. The resulting real signal becomes bipolar, but it should be unipolar. For this purpose, direct current biased optical OFDM (DCO-OFDM) utilizes addition of direct current (DC)-bias to the bipolar signal to convert it to a unipolar signal. As in RF communications, the addition of DC-bias introduces high peak-to-average power ratio (PAPR) and high DC-bias can adversely affect the communication performance because LEDs do not have a linear relationship between optical power and current.

Another technique to avoid the DC bias is asymmetrically clipped optical OFDM (ACO-OFDM) which utilizes the properties of OFDM without requiring DC-biasing. In order to create a symmetric time domain signal, only the odd sub-carriers are used in ACO-OFDM. In this way, negative values in signals are set to zero without altering carried information. Half of the spectrum is wasted in ACO-OFDM [8].

3.2 ACO-OFDM system model

The system model is illustrated in Fig 3.1. In ACO-OFDM, \(N/4\) symbols in \(X_{(l)}, \ l = 0, ..., N/4 - 1\), are mapped onto half of the odd subcarriers, \(X_{\text{frame}}(m), m = 1, 3, 5, ..., N/2 - 1\), whereas the even subcarriers are set to zero. Hermitian symmetry of the subcarriers is imposed on the second half
of the OFDM frame, $X_{\text{frame}}(m)$, $m = N/2, ..., N - 1$, in order to ensure a real-valued time domain signal, $x(k)$, $k = 0, ..., N - 1$, at the expense of 50% reduction in spectral efficiency. The utilization factor for the double-sided bandwidth $B$ of the OFDM frame is denoted by $G_B$. In general, in the OFDM framework there exists the flexibility to employ QAM symbols from different modulation orders, $M$, across the OFDM frame. Square $M$-QAM constellations, e.g. {4-QAM, 16-QAM, 64-QAM, ...}, are considered. The $G_B N M$-QAM symbols on the enabled subcarriers have an average electrical power of

$$P_{s(elec)} = P_{b(elec)} \cdot \log_2(M) \quad (3.1)$$

where

$P_{b(elec)}$ is the average electrical power of $G_BN$ bits

Because of the fact that the clipping noise is added equally to each symbol in the OFDM frame, irrespectively of the modulation order, and for the sake of simplicity, the same modulation order, $M$, is chosen for the symbols in the OFDM frame.

Conventionally, the average optical power of the transmitted signal, $x_{\text{time}}(k)$, is defined in the time domain as $E [x_{\text{time}}(k)]$. Here, $E[ \cdot ]$ stands for the expectation operator. The average electrical power of the signal, however, is defined in the frequency domain as $E [|X_{\text{frame}}(m)|^2]$. The mean electrical signal power is proportional to the variance of $x(k)$, $\sigma_x^2(k)$. Therefore, in order to fix a certain electrical SNR, $\sigma_x^2(k)$ needs to be specified accordingly [13].
For this purpose, the subcarriers at the transmitter require a prescaling by a factor, \( \alpha \), to obtain \( X_{\text{scaled}}(m) \). Following the Parseval theorem and using an unbiased estimator for the variance of \( x(k) \), the factor, \( \alpha \), is expressed as follows:

\[
\alpha = \sigma_x(k) \sqrt{\frac{N-1}{\sum_{m=0}^{N-1} |X_{\text{frame}}(m)|^2}} 
\]

\( E[\alpha^2] = \sigma_x^2(k)/G_B \)
where
\[ \sigma^2_x(k) \] is the variance of \( x(k) \), \( N \) is IFFT/FFT sizes, \( N > 64 \), \( |X_{\text{frame}}(m)|^2 \) is the transmitted signal in the frequency domain, \( G_B \) is the bandwidth utilization factor.

Before the scaling clock, the average electrical power of the enabled subcarriers, \( X_{\text{frame,info}}(m) \), amounts to \( P_{s(elec)} = 1 \). Therefore, a time domain signal, \( x(k) \), with a variance of \( \sigma^2_x(k) \) is obtained when the power of the enabled subcarriers is scaled to \( P_{s(elec)}/G_B \), where \( P_{s(elec)} = \sigma^2_x(k) \). As a result, the average bit energy, \( E_{b(elec)} \), can be expressed as
\[
E_{b(elec)} = \frac{\sigma^2_x(k)}{(\log_2(M)G_B B)}.
\]
(3.4)

Next, the scaled subcarriers are passed through an IFFT block. Without loss of generality
\[
x(k) = \frac{1}{\sqrt{N}} \sum_{m=0}^{N-1} X_{\text{scaled}}(m) \exp\left(\frac{j2\pi km}{N}\right)
\]
(3.5)

where

- \( B \) is the double-side bandwidth,
- \( M \) modulation constellation order,
- \( X_{\text{scaled}} \) is the scaled subcarriers,
- \( N \) is IFFT/FFT sizes.

In general, a cyclic prefix (CP) is included in OFDM based systems to combat ISI and inter-carrier interference (ICI). In addition, the CP transforms the dispersive optical wireless channel into a flat fading channel over the subcarrier bandwidth. However, the CP is shown to have a negligible impact on the electrical SNR requirement and the spectral efficiency. Therefore, for simplicity, it is omitted in the derivations [13].
In order to efficiently utilize the dynamic range of the digital-to-analog (D/A) converter, any structure-specific signal clipping needs to be performed in the digital signal processor (DSP). Moreover, in order to facilitate a power efficient D/A conversion, further DC biasing is performed in the analog circuitry. Therefore, the constraints imposed by the emitter front-end need to be pre-set in the DSP signal shaping block which results in signal pre-clipping. In general, the LED is biased by a constant current source which supports the entire range of forward voltages across the LED. The bias current is added to the data-carrying current, yielding the forward current through the LED. Since the radiated optical power is directly proportional to the forward current, the signal and the constraints imposed by the transmitter front-end are described in terms of optical power. The non-linear I-V characteristic of the LED can be compensated by pre-distortion. A linear characteristic is obtainable, however, only over a limited range between $i_{\text{min}}$ and $i_{\text{max}}$. Therefore, a linear dynamic range of the LED is assumed between a corresponding point of minimum optical power, $P_{Tx,\text{min}}$, and a point of maximum optical power, $P_{Tx,\text{max}}$. The amount of optical power needed to bias the time domain signal is denoted as $P_{Tx,\text{bias}}$ [14].

Because of the time domain signal structure in ACO-OFDM, different combinations of frontend biasing parameters, \textit{i.e.} $\sigma_x(k)$ and $P_{Tx,\text{bias}}$, are chosen for a particular linear dynamic range, $P_{Tx,\text{min}}$ to $P_{Tx,\text{max}}$. Therefore, in the systems the time domain signal, $x(k)$, is pre-clipped at different bottom and top levels, $\varepsilon_{\text{bottom}}$ and $\varepsilon_{\text{top}}$. In the case of insufficient forward biasing, \textit{i.e.} $P_{Tx,\text{bias}} < P_{Tx,\text{min}}$, the signal is pre-clipped at a positive bottom level defined as

$$
\varepsilon_{\text{bottom}} = P_{Tx,\text{min}} - P_{Tx,\text{bias}}
$$

(3.6)
In the opposite case, i.e. $P_{Tx,bias} \geq P_{Tx,min}$, $\varepsilon_{bottom}$ is kept at zero, in order to facilitate the structure-specific asymmetric zero-level signal clipping in ACO-OFDM. Therefore, the signal is pre-clipped at a positive bottom level defined as

$$\varepsilon_{bottom} = \max(P_{Tx,min} - P_{Tx,bias}, 0). \quad (3.7)$$

In addition, the signal is pre-clipped at a top level, $\varepsilon_{top} = P_{Tx,max} - P_{Tx,bias}$. Since the plausible clipping levels satisfy the inequality $\varepsilon_{bottom} < \varepsilon_{top}$, the clipping levels in ACO-OFDM assume only non-negative values. Here, the scenario with the least signal clipping is defined as: $\varepsilon_{bottom} = 0$ and $\varepsilon_{top} = +\infty$.

As a result of the signal pre-clipping in the DSP, the discrete signal $x_{DSP}(k)$ is obtained. After parallel-to-serial (P/S) and D/A conversion and addition of the biasing optical power, $P_{Tx,bias}$, the signal is passed to the optical emitter. The DC bias is employed to the unipolar ACO-OFDM signal to overcome the minimum required optical power, $P_{Tx,min}$.

Furthermore, it is important to mention that the addition of the DC bias influences the useful electrical power of the biased time domain signal, $x_{time}(k) = x_{DSP}(k) + P_{Tx,bias}$, to be transmitted. The total electrical power, $E[x_{time}(k)^2]$, is a summation of the useful electrical alternating current (AC) power and the electrical DC power. Therefore, for a fixed total electrical power, the addition of the DC bias reduces the useful electrical AC power of the signal.

The biased time domain signal, $x_{time}(k)$, represents the OFDM symbol to be transmitted. After passing through the optical wireless channel, it is received by the optical detector, a combination of a PD and a trans-impedance amplifier (TIA). The impulse response of the optical wireless
channel can be modeled by a rapidly decaying exponential function with root mean-square (RMS) delay spreads between 1.3 ns and 13 ns for line-of-sight (LOS) and non-line-of-sight (NLOS) links [13].

In ACO-OFDM, the ISI from maximum delay spreads of up to 100 ns can be compensated by a CP of 2 samples at a sampling rate of 20 MHz for a negligible reduction of the electrical SNR requirement and the spectral efficiency. Therefore, the channel can be safely considered as flat fading over the entire OFDM frame for bandwidths up to 20 MHz, and it can be primarily characterized by the optical path gain coefficient, $g_{h(\text{opt})}$.

$$g_{h(\text{opt})} = I_{\text{PD}} S_{\text{PD}} \rho_{\text{PD}} G_{\text{TIA}} / (E [x_{\text{time}}(k)] \sqrt{r_{\text{load}}})$$  \hspace{1cm} (3.8)

where

$I_{\text{PD}}$ denotes the average irradiance of the PD, $S_{\text{PD}}$ is the photosensitive area of the PD, $\rho_{\text{PD}}$ is the responsivity of the PD, $G_{\text{TIA}}$ is the gain of the transimpedance amplifier (TIA), $E [x_{\text{time}}(k)]$ is the average transmitted optical power, $r_{\text{load}}$ is the load resistance over which the received current is measured.

Furthermore, it is assumed that the upside clipping occurs only at the transmitter. At the detector, the signal is distorted by a zero-mean real-valued bipolar additive white Gaussian noise (AWGN), $n_{\text{AWGN}} (k)$. It accounts for the shot noise and the thermal noise at the receiver. After optical-to-electrical conversion at the received unequalized constellation, it can be modeled as a zero-mean complex-valued AWGN with a two sided power spectral density of $N0/2$ per complex dimension and a variance of $\sigma_{\text{AWGN}}^2 = BN_0$ [15].
After serial-to-parallel (S/P) and analogue-to-digital (A/D) conversion the signal is passed through a unitary FFT block back to the frequency domain

\[ \bar{X}_{\text{scaled}}(m) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} \bar{x}(k) \exp\left(\frac{-j2\pi km}{N}\right) \]  

(3.9)

where

\( \bar{x}(k) \) is the received signal after (S/P) and (A/D), \( N \) is IFFT/FFT sizes.

The asymmetric clipping at the transmitter results in halving of the amplitude of the odd subcarriers at the receiver. Therefore, further clipping at the front-ends introduces further attenuation. In general, in the OFDM framework, pilot tones are used for channel estimation and equalization. Thus, by the use of pilot tones within the OFDM frame, the equalization block is able to compensate for the effect of the optical wireless channel and the attenuation due to the signal clipping. Finally, the data-carrying symbols in \( \bar{X}_{\text{frame}}(m) \) can be extracted due to the known frame structure, and the symbols are demodulated using a maximum likelihood (ML) detector [17].

The clipping of an OFDM time domain signal modifies its mean and consequently its average optical power. Since the non-distorted signal follows a close to Gaussian distribution, the modified mean can be derived based on the statistics of a truncated Gaussian distribution. Thus, the average optical power of the transmitted signal after front-end-induced clipping, \( E[x_{\text{time}}(k)] \), can be expressed as

\[
E[x_{\text{time}}(k)] = \sigma_x(k) \left( \phi(\lambda_{\text{bottom}}) - \phi(\lambda_{\text{top}}) + \lambda_{\text{top}} Q(\lambda_{\text{top}}) - \lambda_{\text{bottom}} Q(\lambda_{\text{bottom}}) \right)
+ P_{\text{bottom}}
\]

(3.10)

\[
\lambda_{\text{bottom}} = \frac{\varepsilon_{\text{bottom}}}{\sigma_x(k)}
\]

(3.11)
\[ \lambda_{\text{top}} = \frac{\varepsilon_{\text{top}}}{\sigma_x(k)} \]  
(3.12)

\[ \phi(u) = \frac{1}{\sqrt{2\pi}} \exp\left(\frac{-u^2}{2}\right) \]  
(3.13)

\[ Q(u) = \frac{1}{\sqrt{2\pi}} \int_{u}^{\infty} \exp\left(\frac{-u^2}{2}\right) \, du \]  
(3.14)

where 
\[ \varepsilon_{\text{bottom}} \] is positive bottom level, \( \varepsilon_{\text{top}} \) is positive top level, 
\[ \sigma_x(k) \] is the signal variance.

Because of the default zero-level clipping in the DSP, \( P_{\text{bottom}} \) can be expressed as
\[ P_{\text{bottom}} = \max (p_{\text{Tx,min}}, p_{\text{Tx,max}}) \]  
(3.15)

In general, the eye safety regulations and/or the design requirements constrain the level of radiated average optical power to \( P_{\text{Tx,mean}} \). Therefore, \( E[x_{\text{time}}(k)] \leq P_{\text{Tx,mean}} \). Here, \( \lambda_{\text{bottom}} \) and \( \lambda_{\text{top}} \) are the normalized bottom and top clipping levels relative to a standard normal distribution. In addition, \( \phi(\cdot) \) and \( Q(\cdot) \) are the respective probability density function (PDF) and complementary cumulative distribution function (CCDF). Plausible clipping levels satisfy the inequality \( \lambda_{\text{bottom}} < \lambda_{\text{top}} \). In addition, lower \( \lambda_{\text{top}} \) results in larger signal clipping, whereas the opposite holds for \( \lambda_{\text{bottom}} \). The scenario with the least signal clipping is defined in ACO-OFDM for \( \lambda_{\text{bottom}} = 0 \) and \( \lambda_{\text{top}} = +\infty \). For a given dynamic range of the transmitter, \( P_{\text{Tx,min}} \) to \( P_{\text{Tx,max}} \), the variables in (3.10) depend only on \( P_{\text{tx,bias}} \) and \( \sigma_x(k) \). It can, therefore, be ascertained that the average optical power of the transmitted signal, \( E[x_{\text{time}}(k)] \), is only a function of the front-end biasing parameters, \( \sigma_x(k) \) and \( P_{\text{tx,bias}} \). In addition, because of the fact that the time domain signal is clipped, the resulting average optical power, \( E[x_{\text{time}}(k)] \), differs from the
undistorted optical power of the OFDM symbol, $P_{s\text{(opt)}}$. In general, $P_{s\text{(opt)}}$, is defined for the least signal clipping scenario. In ACO-OFDM,

$$P_{s\text{(opt)}} = (P_{Tx,\text{bias}} + \sigma_x(k)\sqrt{2\pi})$$

where

$$\sigma_x^2(k)$$ is the variance of $x(k)$, $P_{Tx,\text{bias}}$ is electrical power of the biased time domain signal.

Combined with (3.12), these equations can be used to obtain the relation between $E[x_{\text{time}}(k)]$ and the undistorted optical power of the OFDM symbol, $P_{s\text{(opt)}}$. Therefore, for a given set of front-end optical power constraints, $P_{Tx,\text{min}}$, $P_{Tx,\text{mean}}$ and $P_{Tx,\text{max}}$, one can obtain the signal scaling factor, $\alpha$, for a target signal variance, $\sigma_x^2(k)$, and the required DC bias, $P_{Tx,\text{bias}}$, from (3.2) and (3.10) [18].

In addition to the modification of the average optical signal power, the front-end-induced signal clipping distorts the data carrying symbols. An expression for the distorted scaled data carrying subcarriers at the receiver, $\tilde{x}_{\text{scaled,info}}(m)$, is derived by the use of the Bussgang theorem and the CLT. First, the Bussgang theorem is used to obtain $\tilde{x}(k)$ as

$$\tilde{x}(k) = U(x(k))g_{h\text{(opt)}}Ax(k) + g_{h\text{(opt)}}n_c(k) + g_{h\text{(opt)}}P_{Tx,\text{bias}} + n_{\text{AWGN}}(k)$$

where

$U(\cdot)$ stands for the unit step function which is used to denote the default zero-level clipping of the time domain signal, $g_{h\text{(opt)}}$ is the optical path gain coefficient, $A$ is the attenuated by a factor, $n_c(k)$ is the uncorrelated non-Gaussian clipping noise.
The theorem states that after the nonlinear clipping distortion the signal is attenuated by a factor, $A$, and an uncorrelated non-Gaussian clipping noise, is added. $n_c(k)$ is non-negative and it has a unipolar distribution. Therefore, in the presence of double-sided clipping in ACO-OFDM, the effective attenuation factor at the received odd subcarriers, $K$, is related to $A$ as $K = A/2$, where $A = 1$ in the least signal clipping scenario. Further on, $\tilde{x}(k)$ is passed through an FFT. Applying the CLT, the distorted scaled data-carrying subcarriers, $\tilde{X}_{\text{scaled,info}}(m)$, can be expressed as a function of the transmitted data-carrying subcarriers, $X_{\text{frame,info}}(m)$ as

$$\tilde{X}_{\text{scaled,info}}(m) = \alpha g_{h(\text{opt})} K X_{\text{frame,info}}(m) + g_{h(\text{opt})} \sigma_{\text{clip}} N_{\text{CN}}(m) + \sigma_{\text{AWGN}} N_{\text{CN}}(m)$$

(3.18)

where

$\sigma_{\text{clip}}$ is the standard deviation of the complex-valued Gaussian clipping noise at the data-carrying subcarriers, $N_{\text{CN}}(m)$ is a sample of a complex valued Gaussian distribution with zero mean and unity variance, $\alpha$ is the signal scaling factor, $\sigma_{\text{AWGN}}$ is the AWGN variance.

And in order to recover $X_{\text{frame,info}}(m)$, a zero forcing (ZF) equalizer is employed. Even though ZF is an equalization technique widely used in OFDM-based systems, it results in AWGN amplification when the path gain decreases.

Applying the assumption of Gaussianity of $x(k)$, the attenuation factor, $K$, is expressed as:

$$K = \frac{\text{Cov} [x(k), x_{DSP}(k)]}{\sigma_x^2(k)} = Q(\lambda_{\text{bottom}}) - Q(\lambda_{\text{top}})$$

(3.19)

where
Cov[ · ] stands for the covariance operator, $x_{DSP}(k)$ is the time domain signal in DSP.

Since $x_{time}(k)$ is real in ACO-OFDM, $K$ is a real-valued function. It essentially represents the likelihood of samples not being clipped. In addition, it proves to be independent of the modulation scheme, $M$-QAM, and the IFFT/FFT size, $N$.

Figure 3.2: Attenuation factor of the clipping noise as a function of the normalized clipping levels in ACO-OFDM [13]

The attenuation factor as a function of the normalized bottom and top clipping levels is illustrated in Figure3.2. The attenuation factor approaches
0.5 when the least signal clipping is present. Furthermore, the ACO-OFDM symbol suffers larger attenuation for downside clipping as compared to upside clipping [13].

According to (3.19) the time domain clipping noise, $n_c(k)$, is an independent component and can be estimated separately. The time domain signal, subject only to double sided clipping, $x_{DSP}(k)$, can be written as

$$x_{DSP}(k) = U(x(k)) x(k) - \Delta x(k) = U(x(k)) 2K x(k) + n_c(k)$$  \hspace{1cm} (3.20)$$

$$n_c(k) = U(x(k)) (1 - 2K) x(k) - \Delta x(k)$$  \hspace{1cm} (3.21)$$

where, $\Delta x(k)$ is the clipped signal portion.

Because of the unitary FFT at the receiver, $n_c(k)$ is transformed into a zero-mean Gaussian noise component at the data-carrying subcarriers according to the CLT.

![Figure 3.3](image.png)

**Figure 3.3:** Time domain signal in DSP, $x_{DSP}(k)$, in ACO-OFDM [13]
The time domain signal in the DSP, $x_{DSP}(k)$, is depicted in Figure 3.3. The symmetries allow the unfolding of the truncated half Gaussian distribution of $x_{DSP}(k)$, the mirroring of the clipping levels around the origin and the redistribution of the signal samples. The resulting signal is denoted as $\tilde{x}_{DSP}(k)$, and it is depicted in Figure 3.4. It is a symmetric signal with respect to the origin, and it follows a close to Gaussian distribution with zero mean and variance of $P_{s(elec)}/2$ when the least signal clipping is present. However, $\tilde{x}_{DSP}(k)$ has a bias of $-\sigma_x(k)\lambda_{bottom}/\sqrt{2}$ on the negative samples and a bias of $\sigma_x(k)\lambda_{bottom}/\sqrt{2}$ on the positive ones. Since these biases are to be mounted on the first subcarrier in the ACO-OFDM frame after the FFT, they are irrelevant to the clipping noise variance on the data carrying subcarriers [13].

The variance of the clipping noise, $\sigma_{clip}^2$ can be expressed as [17]

![Figure 3.4: Unfolded time domain signal in DSP, $\tilde{x}_{DSP}(k)$, in ACO-OFDM [13]](image-url)
\[ \sigma_{\text{clip}}^2 = p_{\text{s(elec)}}(K \lambda_{\text{bottom}}^2 + 1) - 2K^2 - \lambda_{\text{bottom}}(\varnothing(\lambda_{\text{bottom}}) - \varnothing(\lambda_{\text{top}})) \\
- \varnothing(\lambda_{\text{top}})(\lambda_{\text{top}} - \lambda_{\text{bottom}}) + Q(\lambda_{\text{top}})(\lambda_{\text{top}} - \lambda_{\text{pottom}})^2 \]  
(3.22)

where

\( p_{\text{s(elec)}} \) is the average electrical power, \( K \) is the attenuation factor, \( \lambda_{\text{top}}, \lambda_{\text{pottom}} \) are bottom and top clipping levels.

And similarly to the attenuation factor, \( K \), the clipping noise variance \( \sigma_{\text{clip}}^2 \) is independent of the modulation order, \( M \), and the IFFT/FFT size, \( N \). Since \( K \) and \( \sigma_{\text{clip}}^2 \) are independent of \( M \), they remain constant across the modulation orders for a particular choice of normalized bottom and top clipping levels.

An analytical expression for the effective electrical SNR per bit in ACO-OFDM, \( \Gamma_{b(elec)} \), can be expressed for ZF as a function of the undistorted electrical SNR per bit, \( \gamma_{b(elec)} \)

\[ \Gamma_{b(elec)} = \frac{K^2 P_{b(elec)}/G_B}{\sigma_{\text{clip}}^2 + \frac{G_B \sigma_{\text{AWGN}}^2}{g_{h\text{(opt)}}^2 G_{\text{DC}}}} = \frac{K^2}{p_{b(elec)}} \frac{G_B}{\sigma_{\text{clip}}^2 + \frac{G_B \gamma_{b(elec)}}{G_{\text{DC}}}} \]  
(3.23)

\[ \gamma_{b(elec)} = E_{b(elec)}/N_0 \]  
(3.24)

where

\( K \) is the attenuation factor, \( P_{b(elec)} \) is the average electrical power of \( G_B \) and \( N \) is the AWGN variance, \( G_B \) is the utilization factor of the double side bandwidth, \( g_{h\text{(opt)}} \) is the optical path gain coefficient, \( G_{\text{DC}} \) denotes the attenuation of the useful electrical signal power of \( x_{\text{time}}(k) \) due to the biasing of the transmitter front-end by \( P_{\text{Tx,bias}} \) in the least signal clipping scenario.

\[ G_{\text{DC}} = \frac{\sqrt{2\pi} \sigma_{x(k)}^2}{\sqrt{2\pi} \sigma_{x(k)}^2 + 4\sigma_{x(k)} P_{\text{Tx,bias}} + 2\sqrt{2\pi} P_{\text{Tx,bias}}^2} \]  
(3.25)

where
\( \sigma_x^2(k) \) is the variance of \( x(k) \), \( P_{\text{Tx,bias}} \) is electrical power of the biased time domain signal.

In addition, because of the bias added to the ACO-OFDM signal, \( P_{\text{Tx,bias}} \), the electrical-to-optical conversion has to be generalized as:

\[
P_{s(\text{opt})} = \frac{2\pi P_{\text{Tx,bias}}^2 + 2\sigma_x(k) P_{\text{Tx,bias}} \sqrt{2\pi + \pi \sigma_x^2(k)}}{2\pi P_{\text{Tx,bias}}^2 + 2\sigma_x(k) P_{\text{Tx,bias}} \sqrt{2\pi + \pi \sigma_x^2(k)}} P_{s(\text{elec})}
\]  

(3.26)

Where

\( P_{s(\text{elec})} \) is the average electrical subcarrier power, \( \sigma_x(k) \) is the signal variance, \( P_{\text{Tx,bias}} \) is the biasing power.