

Dedication

To Father and Mother,

To My Family and Tribe,

To My Teachers,

To My Colleagues,

**I dedicate this humble search of the mawla hope to find
acceptance and success.**

Acknowledgements

Thanks first and last to Allah who enabled me to conduct this work by grace of him and devoted strength and patience.

Thanks to my supervisor Dr. **Emad Eldeen Abdallah Abdel Rahim** for his advices and help.

Also thanks are due to all teachers in the department of mathematics.

Abstract

In this work we discuss groups of matrices and Differential equations in matrices via Quaternion's and Clifford algebras. Also we present a homogeneous space as manifolds and connectivity of manifolds with examples and applications. Also we describe some results on the structure of compact connected Lie groups, focusing on the important notation of a maximal torus which is central to the classification of simple compact connected Lie groups, with some results.

Also we discuss the Morse theory with some applications to the topology of Lie groups.

الخلاصة

في هذا العمل ونحن نناقش مجموعة من المصفوفات والمعادلات التفاضلية في المصفوفات من خلال المركب المتعدد وكليفورد الجبر. كما نقدم مساحات متجانسة بإسم الفتحات والتواصل بين الفتحات مع أمثلة وتطبيقات. كما وصفنا بعض النتائج على بنية المجموعات تكمن اتصال المدمجة، مع التركيز على تدوين مهم من الحيد القسوى وهو أمر أساسي لتصنيف المجموعات تكمن اتصال مدمجة بسيطة، مع بعض النتائج.

أيضا نناقش نظرية مورس مع بعض التطبيقات الطوبولوجيا.

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Introduction

Today the theory of matrix groups becomes one of the important mathematical structure of study the theoretical and applied science. In this research we discuss some properties of this field and it is organized as follows.

In chapter (1) we discuss the groups of matrices, groups of matrices as metric space, and also matrix groups. Also we discuss some examples of matrix groups, and complex matrix groups as real matrix groups. Also we study continuous homomorphisms of matrix groups, and continuous group actions. Also we study the exponential and logarithm functions, and differential equations in matrices, one parameter subgroups. Also we discuss curves, tangent spaces, Lie algebras. and some Lie algebras of matrix groups. Also we present $SO(3)$, $SU(2)$, $SL_2(\mathbb{C})$ and the Lorentz group.

In chapter (2) we discuss algebras, Linear algebra over a division algebra, and quaternion's. Also we study quaternionic matrix groups and the real Clifford algebras. Also we discuss the spinor groups, the centers of spinor groups, and finite subgroups of spinor groups. Also we introduce smooth manifolds, tangent spaces, derivatives, and Lie groups. Also we discuss some examples of Lie groups, and some useful formula in matrix groups.

In chapter (3) we study homogeneous spaces as manifolds, and as orbits. Also we discuss projective spaces, grassmannians, and connectivity of manifolds. Also we present examples of path connected matrix groups and the path components of a Lie groups. Also we discuss another connectivity results.

In chapter (4) we study tori and maximal's tori in compact Lie groups, Also we discuss the normalize and weyl group of a maximal torus, with some applications.