

## Chapter one

### Introduction

#### (1.1) History of Quantum Mechanics:

Classical physics divide Physical laws into two main parts .Newton's laws which describe matter particles, and Maxwell's equations which take care of electromagnetic energy waves. This classification survives till the trial of the scientists to do not be black body radiation phenomena. Max plank found that the explanation of the black body radiation is impossible unless light waves behaves as describe energy quantum known as photons [1, 2].

This means that light has dual wave and particle nature .The particle behaviour of light waves motivates DE Broglie to suggest that particles like electrons can be have some times like waves. This suggestion was confirmed experimentally, by Davison and Krimer which observe electron diffraction by a solid crystal [5 ,4 ,3].

The dual nature of atomic world entitles encourages Schrodinger and independently Heisenberg to construct to describe the behaviour of the atomic world.

The an attempt to arrive at a relativistic wave equation that put time and space on an equal footing by promoting derivations with respect to time to second order .

This was done because the Schrodinger equation has second –order spatial derivatives. The equation that results is called Klein-Gordon equation [6]. The Dirac equation applies to SPIN  $-\frac{1}{2}$  field ,and puts time and space on ground in the equation by considering first- order spatial derivatives , rather than increasing the order of the time derivatives [6].

The application of quantum mechanics to spectroscopic problems involves solving the appropriate time-independent Schrodinger equation ,

$H\Psi = E\Psi$  the most appropriate formulation of quantum mechanics for spectroscopy is based upon the Heisenberg matrix mechanics approach [7].

**(1.2) research problem:**

Research problem is related to the fact that particle and wave nature of atomic world seems to be in conflict with each other .

**(1.3) literature Review:**

A relation of the Josephson current density equation is successfully derived; this is done through a new derivation of the equation of quantum by neglecting Kinetic Newtonian term in the energy expression [8].

A new quantum model that accounts for the medium friction is derived. The first advantage is the addition of natural oscillation of particle. The second advantage is the incorporation of friction effect in the Hamiltonian operator. This means that both Schrodinger and energy Eigen equation are affected by friction. The Eigen energy is not affected by friction, which is in direct conflict with experiment and common sense [9].

**(1.4) Aim of the work:**

The aim of the work is to find some link between wave and particle nature of atomic system.

**(1.5) presentation of the thesis:**

The thesis consists of four chapters, chapter one and two are concerned with the interdicted and Maxwell equation. Chapter three and four are devoted for quantum laws beside the contribution.

## Chapter Two

### Maxwell's equations

#### (2.1) Introduction:

Electromagnetic wave known long ago in physics. The behaviour of electromagnetic fields was described by Maxwell. This chapter is concerned with these equations [10].

#### (2.2) Maxwell's Equations:

Until Maxwell's work, the known basic laws of electricity and magnetism are four. One of them is concerned with generation of electric field by electric charges. This is known as Gauss law which is given by

$$\nabla \cdot D = \rho \dots\dots\dots (2.2.1)$$

D here stands for electric flux density while represents the charge density.

The electric field intensity is related to the electric permittivity according to the relation

$$D = \epsilon E \dots\dots\dots (2.2.2)$$

Due to the fact that magnetic field is generated by magnetic dipoles which form closed loops it follows that [8].

$$\nabla \cdot B = 0 \dots\dots\dots (2.2.3)$$

Where  $B$  is magnetic flux density? The magnetic field strength, is related to the magnetic permeability

$$B = \mu H \dots\dots\dots (2.2.4)$$

**Faradays low of induction shows how the electric field can be generated by time varying magnetic field according to the relation:**

$$\nabla \times E = -\frac{\partial B}{\partial t} \dots \dots \dots (2.2.5)$$

**Amper's low for magneto motive force, in to contrans, show how magnetic field can be generated by electric current according to the relation**

$$\nabla \times H = J \dots \dots \dots (2.2.6)$$

Where J are the current density equations [10].

These are general equations and are valid for static as well as dynamic field.

The fourth equation was derived from steady- state observation and we have to examine its validity for time – varying fields.

Taken the divergence of both side of (2.2.6) we have

$$\nabla \cdot (\nabla \times H) = \nabla \cdot J = 0 \dots \dots \dots (2.2.7)$$

This indeed, is true for steady – state phenomena.

However where the currents are charging with time, the result is compatible with principle of conservation of charge, reflected in the equation of the continuityi .

$$\nabla \cdot J + \frac{\partial \rho}{\partial t} = 0 \dots \dots \dots (2.2.8)$$

Using Gauss' low (2.2.1), we can write (2.2.8) as:

$$\nabla \cdot J = -\frac{\partial \rho}{\partial t} = -\frac{\partial}{\partial t}(\nabla \cdot D) = \nabla \cdot \left(-\frac{\partial D}{\partial t}\right) = \nabla \cdot \left(J + \frac{\partial D}{\partial t}\right) = 0 \dots \dots (2.2.9)$$

Maxwell replaced J in Ampers law by  $J + \frac{\partial D}{\partial t}$ . with this modification Ampere's law takes the form:

$$\nabla \times H = J + \frac{\partial D}{\partial t} \dots \dots \dots (2.2.10)$$

$J \equiv$  Conduction current density

$\frac{\partial D}{\partial t} \equiv$  displacement current density .

Displacement current, there for, are not as significant as the current due to motion of the free charge in the study of the continues circuits.

The four equations which the field vectors  $E, D, B, H$  satiety everywhere are:

(i)  $\nabla \cdot D = \rho \dots \dots \dots (2.2.10)$

(ii)  $\nabla \cdot B = 0 \dots \dots \dots (2.2.11)$

(iii)  $\nabla \times E + \frac{\partial B}{\partial t} = 0 \dots \dots \dots (2.2.12)$

(iv)  $\nabla \times H = J + \frac{\partial D}{\partial t} \dots \dots \dots (2.2.13)$

These equations are electromagnetic field and are known as Maxwell's equations [10].

**(2.3) Wave propagation in free space:**

$$-\nabla^2 E - \mu\epsilon \frac{\partial^2 E}{\partial t^2} - \mu\sigma \frac{\partial E}{\partial t} = 0 \dots \dots \dots (2.3.1)$$

In free space:

$$J = 0, \quad \epsilon = \epsilon_0 = 8.85 \times 10^{-12}$$

$$\mu = \mu_0 = 4\pi \times 10^{-7}, \quad \rho = 0, \quad \sigma = 0$$

The solution is:

$$E = E_0 \cdot e^{i(kx - \omega t)} \dots \dots \dots (2.2.3)$$

From  $\nabla \cdot E = 0$  the  $E_z = H_z = 0$

$$\frac{\partial^2 \epsilon}{\partial z^2} - \mu_0 \epsilon_0 \frac{\partial^2 \epsilon}{\partial t^2} = 0 \dots \dots \dots (2.2.3)$$

$$\frac{\partial^2 E}{\partial t^2} = \frac{\partial}{\partial t} \left( \frac{\partial E}{\partial t} \right) = \frac{\partial}{\partial t} (i \omega E) = -\omega^2 E \dots \dots \dots (2.3.4)$$

$$\frac{\partial^2 E}{\partial z^2} + \mu_0 \epsilon_0 \omega^2 E = 0 \dots \dots \dots (2.3.5)$$

$$\frac{\partial^2 E_x}{\partial z^2} - \mu_0 \epsilon_0 \frac{\partial^2 E_x}{\partial t^2} = 0 \dots \dots \dots (2.3.6)$$

The solution is:

$$E_x = E_{0x} e^{\gamma z} \dots \dots \dots (2.3.7)$$

$\gamma \equiv$  Distribution factor.

$$\frac{\partial^2 E_x}{\partial z^2} = \gamma^2 E_x \dots \dots \dots (2.3.8)$$

In (2.3.5) Sub (2.3.8)

$$\gamma^2 E_x + \mu_0 \epsilon_0 \omega^2 = 0 \dots \dots \dots (2.3.9)$$

$E_x$  Divided by

$$\gamma^2 + \mu_0 \epsilon_0 \omega^2 = 0 \dots \dots \dots (2.3.10)$$

$$\gamma \pm i \sqrt{\mu_0 \epsilon_0} \omega = \pm \frac{i \omega}{c} \dots \dots \dots (2.3.11)$$

$$C = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \quad C = 3 \times 10^8 \text{ m/sec}$$

$$\frac{W}{C} = \frac{2\pi f}{c} = \frac{2\pi c}{\lambda c} = k \dots \dots \dots (2.3.12)$$

$$\gamma = \pm i k \dots \dots \dots (2.3.13) [15].$$

**(2.4) Wave Propagation in A conductor:**

$$-\nabla^2 E - \mu\epsilon \frac{\partial^2 E}{\partial t^2} - \mu\sigma \frac{\partial E}{\partial t} = 0 \dots \dots \dots (2.4.1)$$

In conductor

$$\sigma \neq 0, J \neq 0,$$

$$\frac{\partial^2 E}{\partial t^2} = -\omega^2 E \dots \dots \dots (2.4.2)$$

$$E = E_0 e^{i(kx - \omega t)} \dots \dots \dots (2.4.3)$$

$$\frac{\partial^2 E}{\partial z^2} = -k^2 E \dots \dots \dots (2.4.4)$$

$$\sigma \frac{\partial E}{\partial t} = \text{Conductor current} \dots \dots \dots (2.4.5)$$

$$\epsilon \frac{\partial^2 E}{\partial t^2} = \text{Displacement current} \dots \dots \dots (2.4.6)$$

$$\frac{\partial^2 E}{\partial z^2} + \epsilon\mu\omega^2 E - i\mu\sigma\omega E = 0 \dots \dots \dots (2.4.7)$$

$$\frac{\partial^2 E}{\partial z^2} + \epsilon\mu\omega^2 \left(1 - \frac{i\sigma}{\epsilon\omega}\right) = 0 \dots \dots (2.4.8)$$

$$\text{Sort of medium} = \frac{\text{Conductor current}}{\text{Displacement current}}$$

$$Q = \frac{\varepsilon w}{i\sigma} \dots\dots\dots (2.4.9)$$

$$\frac{\partial^2 E}{\partial z^2} + \varepsilon\mu w^2 \left(1 + \frac{1}{\varphi^2}\right) \dots\dots\dots (2.4.10)$$

$$\frac{\partial^2 E}{\partial z^2} = -k^2 E \dots\dots\dots (2.4.11)$$

$$k = \frac{2\pi}{\lambda} \quad , \quad w = \frac{2\pi c}{\lambda}$$

$$V = \frac{\omega}{k}, \quad n = \frac{c}{v} \quad [15].$$



## Chapter three

### Quantum law

#### (3.1) Introduction:

The theoretical exploitation of De Broglie idea about the wave nature was immediately verified after the publication of the hypothesis, by Davison and Crimer Erwin Schrodinger in 1926 proposed that the behavior of De Broglie waves associated materials particles is governed by a certain differential equation for the wave function [11].

This chapter is devoted for deriving quantum equations.

#### (3.2) Black Body Radiation:

When a body is heated, it is seen to radiate .In equilibrium the light emitted ranges over the map define a quantity  $E(\lambda, T)$ , the emissive power, as the energy emitted at wavelength  $\lambda$  per unit area, per unit time. Theoretical research in the field of thermal radiation began in 1859 with the work of Gustav Kirchhoff, who showed that for a given  $\lambda$ , the ratio of the emissive power  $E$  to the absorptivity  $A$ , defined as the fraction of incident radiation of wavelength  $\lambda$  that is absorbed by the body , is the same for all bodies .Kirchhoff considered two emitting and absorbing parallel plates and showed from the equilibrium condition that the energy emitted was equal to the energy absorbed (for each  $\lambda$ ),that the ratios  $E/A$  must be the same for all two plates soon thereafter , he observed that for a blackbody ,defined as a surface that totally absorbs all radiation that falls on it, so that  $A = 1$ , the function  $E(\lambda, T)$  is universal function.

To study this function, it is necessary to obtain the best possible source of blackbody radiation. A practical solution to this problem is to consider the radiation emerging from a small hole in an in enclosure heated to a temperature  $T$ . Given the imperfection in the surface of the inside of the cavity, it is clear that any radiation

falling on the hole will have no chance of emerging again. Thus the surface presented by the hole is very nearly "totally absorbing ".and consequently the radiation coming from it is indeed "blackbody radiation". Provided the hole is small enough, this radiation will be the same as that which falls on the walls of the cavity. It is therefore necessary to understand the distribution of radiation inside a cavity whose walls are at a temperature T in Kelvin (*k*).

Kirchhoff showed that the second law of thermodynamics requires that the radiation in the cavity be isotropic, that is, that the flux be independent of direction ,that it homogeneous, that is ,the same at all points ;and that it be the same in all cavities at the same temperature all of this for each wavelength .The emissive power may ,by simple geometric arguments ,shown to be connected with the energy density  $u(\lambda, T)$  inside the cavity. The relation is

$$u(\lambda, T) = \frac{4E(\lambda, T)}{c} \dots\dots\dots(3.2.1)$$

The energy density is the quantity of theoretical interest, and further understanding of it came in 1894 from the work of Wilhelm Wien, who, again using very general arguments, showed that the energy density had to be the form

$$u(\lambda, T) = \lambda^{-5} f(\lambda T) \dots\dots\dots (3.2.2)$$

With  $f$  still an unknown function of a single variable. If, as is convenient, one deals instead with the energy density as a function of frequency,  $u(\nu, T)$  then it flows from the fact that, with  $\lambda = c/\nu$

$$\begin{aligned} u(\nu, T) &= u(\lambda, T) \left| \frac{d\lambda}{d\nu} \right| \\ &= \frac{c}{\nu^2} u(\lambda, T) \dots\dots\dots \end{aligned} \quad (3.2.3)$$

$$u(\nu, T) = \nu^3 g\left(\frac{\nu}{T}\right) \dots\dots\dots (3.2.4)$$

The implications of this law, which was confirmed experimentally, are twofold:

1. Given the spectral distribution of black body radiation at one temperature, the distribution at any other temperature can be found with help of the expression just given.
2. If the function  $f(x)$  or has maximum for some value of  $x > 0$ , then the wavelength  $\lambda_{max}$  at which the energy density, and hence emissive power, has it maximum value, has the form

$$\lambda_{max} = \frac{b}{T} \dots\dots\dots (3.2.5)$$

Where  $b$  is universal constant? Its value as determined from the experiments O.lummer and E.pringsheim (1897) is  $b = 0.2898cm. k[8]$ .

Wien used a model (of no interest, except to the historian) to predict a form for  $g(\nu/T)$ .The form was

$$g\left(\frac{\nu}{T}\right) = C e^{-\beta\nu/T} \dots\dots\dots (3.2.6)$$

and remarkably enough, this form ,containing two adjustable  $u(\nu, T)$  parameters , fit the high frequency (low wavelength )data very well .The formula is not however , in accord with some very general notions of classical physics .J.W.S.Rayleigh, in 1900, derived the result

$$u(\nu, T) = \frac{8\pi\nu^2}{c^3} kT \dots\dots\dots (3.2.7)$$

**(3.3)The plank distribution and the Quantum energy:**

In 1900, Max plank found a formula by an ingenious interpolation between the high frequency Wien formula and the low frequency Rayleigh-Jeans law .The formula is

$$u(\nu, T) = \frac{8\pi h}{c^3} \frac{\nu^3}{e^{h\nu/kT}-1} \dots\dots\dots (3.3.1)$$

Where h plank constant, is an adjustable parameter whose numerical value was found to be  $h = 6,63 * 10^{-27}$  erg sec .This law approaches the Rayleigh-Jeans form when  $h\nu/kt \ll 1$ ,and reduces to

$$u(\nu, T) = \frac{8\pi h}{c^3} \nu^3 e^{-h\nu/kT} \left(1 - e^{-\frac{h\nu}{kT}}\right)^{-1} \cong \frac{8\pi h}{c^3} \nu^3 e^{-h\nu/kT} \dots\dots\dots (3.3.2)$$

When  $h\nu \gg kT$ . If we rewrite the formula as a product of the number of modes [we obtain this from (3.2.7) by dividing the energy density by  $kT$ ] and another factor that can be interpreted as the average energy per degree of freedom

$$u(\nu, T) = \frac{8\pi\nu^2}{c^3} \frac{h\nu}{e^{\frac{h\nu}{kT}} - 1}$$

$$= \frac{8\pi\nu^2}{c^3} kT \frac{h\nu/kT}{e^{h\nu/kT}-1} \dots\dots\dots (3.3.3)$$

We see that the classical equipartition law is altered whenever the frequencies are not small compared with  $k/h$ .This alteration in the equipartition law shows that the modes have an average energy depends on their frequency , and that the high frequency modes have a very small average energy. This effective cutoff removes the difficulty of the Rayleigh-Jeans density formula: the total energy in a cavity of unit volume is no longer infinite .We has

$$U(T) = \frac{8\pi h}{c^3} \int_0^\infty d\nu \frac{\nu^3}{e^{\frac{h\nu}{kT}}-1}$$

$$\begin{aligned}
&= \frac{8\pi h}{c^3} \left(\frac{kT}{h}\right)^4 \int_0^\infty \frac{\left(\frac{h\nu}{kT}\right)^3 d\left(\frac{h\nu}{kT}\right)}{e^{h\nu/kT}-1} \\
&= \frac{8\pi k^4}{h^3 c^3} T^4 \int_0^\infty dx \frac{x^3}{e^x-1} \dots\dots\dots(3.3.4)
\end{aligned}$$

The integral can be evaluated, and the result is the Stefan-Boltzmann expression for the total radiation energy per unit volume

$$U(T) = aT^4 \dots\dots\dots (3.3.5a)$$

With  $a = 7.56 \times 10^{-15} \text{ erg/cm}^3 \text{K}^4$ . The general form shown in (3.3.5a) was derived much earlier using thermodynamic reasoning. The result can also be written in the form of the total emissive power of a black body.

$$E(T) = \sigma(T^4) \dots\dots\dots (3.3.5b)$$

With  $\sigma = 5.42 \times 10^{-5} \text{ erg/cm}^2 \text{secK}^4$ .

A departure from the pure equipartition law not entirely unexpected: one consequence of it was the Dulong-petit law of specific heats, according to which the product of the atomic (or molecular) weight and the specific heat is a constant for all solids, yet departures from the Dulong-Petit predictions were observed as early as 1872. These departures indicated that the specific heat decreased at lower temperatures.

The unqualified success of his formula drove Planck to search for its origin and within two months he found he found that he could derive it by assuming that the energy associated with each mode of the electromagnetic field did not vary continuously (with average value  $Kt$ ) $\epsilon$ . Under these circumstances a calculation of the average energy associated with each mode, using the Boltzmann probability distribution in a system of equilibrium at temperature T,

$$P(E) = \frac{e^{-E/kT}}{\sum_E e^{-E/kT}} \dots\dots\dots (3.3.6)$$

Led to

$$\begin{aligned}
 \bar{E} &= \sum_E e EP(E) \\
 &= \frac{\sum_{n=0}^{\infty} n \epsilon e^{-n\epsilon/kT}}{\sum_{n=0}^{\infty} e^{-n\epsilon/kT}} \\
 &= \frac{-\epsilon \frac{d}{dx} \sum_{n=0}^{\infty} e^{-nx}}{\sum_{n=0}^{\infty} e^{-nx}} \Bigg|_{x=\epsilon/kT} \\
 &= \epsilon \frac{e^{-x}}{1 - e^{-x}} \Bigg|_{x=\epsilon/kT} \dots\dots\dots(3.3.7) \\
 &= \frac{\epsilon}{e^{\epsilon/kT} - 1}
 \end{aligned}$$

This agree with (3.3.3) provided we make the identification

$$\epsilon = h \nu \dots\dots\dots(3.3.8)$$

And do not change the of modes.

Plank argued that for some unknown reason the atoms in the walls of the cavity emitted radiation in "quanta" with energy  $n h \nu$  ( $n = 1, 2, 3, \dots$ ), but consistency demanded, as established by Einstein a few years later, that electromagnetic radiation behaved as if it consisted of a collection of energy quanta with energy  $h \nu$ [8].

**(3.4) Wave properties and electron diffraction:**

In 1923 De Broglie, guided by the analogy of Fermat's principle in optics, and the least action principle in mechanics, was led to suggest that the dual wave-particle nature of radiation should have its counterpart in a dual particle-wave of matter. Thus particles should have wave properties under certain

circumstances, and De Broglie suggested an expression for the wave length associated with the particle. This is given by

$$\lambda = \frac{h}{p} \dots\dots\dots (3.4.1)$$

Where  $h$  Plank's constant and  $p$  is momentum of the particle. De Broglie's work attracted much attention, and many people suggested that verification could be obtained by observing electron diffraction .The experimental observation of this effect occurred in experiments of C.J.Davission and L.H. Cermer, who found that the scattering of electrons by a crystal surface, there was preferential scattering in certain directions [8].

**(3.5) Schrodinger Equation:**

The function which represents the de Broglie's wave was proposed by schrodinger to be simi lar to the electromagnetic travelling wave one can assume this wave to be of the form:

$$\Psi(x, t) = Ae^{i(kx-\omega t)} \dots\dots\dots (3.5.1)$$

$$K = \frac{2\pi}{\lambda}$$

$\lambda$ =wave length.

$\omega = 2\pi f$  =Angular frequency

$F$  =Frequency.

To describe particle \_wave and nature one needs to replace  $K$  and  $\omega$  by momentum  $P$  and energy  $E$ .this can be done by using plank formula.

$$E = hf = \hbar\omega$$

And De Broglie Hypothesis and

$$P = \frac{h}{\lambda} = \hbar k$$

Where  $\hbar$  ( $h$  cross) =  $\frac{h}{2\pi}$

Thus one gets

$$\Psi(x, t) = Ae^{i(px-Et)/\hbar}$$

Performing partial differentiation w.r.t.  $x$  on the above equation one gets.

$$\frac{\partial \Psi(x,t)}{\partial x} = \frac{i}{\hbar} p Ae^{i(px-Et)/\hbar}$$

$$\frac{i}{\hbar} p \Psi(x, t) \dots \dots \dots (3.5.2)$$

Multiplying both sides by  $-i\hbar$ , we get

$$-i\hbar (\partial \Psi(x, t)) / \partial x = p \Psi(x, t) \dots \dots \dots (3.5.3)$$

Similarly differentiation partially Eq (3.5.1) w.r.t. time and multiplying the resulting equation by  $i\hbar$  from both the side, we get:

$$i\hbar \frac{\partial \Psi(x,t)}{\partial t} = E \Psi(x, t) \dots \dots \dots (3.5.4) [9].$$

Eqs (3-2-3) and (3-2-4) imply that energy and momentum can be represented by differential operators which act the wave function  $\Psi(x, t)$

$$E = i\hbar \frac{\partial}{\partial t}, P = -i\hbar \frac{\partial}{\partial x} \dots \dots \dots (3.5.5)$$

Now the energy of the non-relativistic free particles given by:

$$E = \frac{p^2}{2m} \dots \dots \dots (3.5.6)$$

By operating on  $\Psi(x, t)$  gives



$$E \Psi (x, t) = \left( p^2 / 2m \right) \Psi (x, t) \dots\dots\dots(3.5.7)$$

Using the operator equivalence represented by Eq (3.5.5) one gets

$$i \hbar \frac{\partial \Psi(x,t)}{\partial x} = - \hbar^2 / 2m \cdot \frac{\partial^2 \Psi(x,t)}{\partial x^2} \dots\dots\dots(3.5.8)$$

this is time dependent one dimensional shrodinger equation for a free particle at mass.

If the particle is not free but is in a field characterized by the potential energy function  $v(x, t)$ .

The total energy of the particle then is given by :

$$E = p^2 / 2m + v(x, t) \dots\dots\dots(3.5.9)$$

By operating on  $\Psi(x, t)$ , leads to

$$E \Psi(x, t) = [p^2 / 2m + v(x, t)] \Psi(x, t) \dots\dots\dots(3.5.10)$$

And then using eq (3.5.5) that is the operator equivalence of  $E$  and  $P$ , one gets:

$$i \hbar \frac{\partial \Psi(x, t)}{\partial t} = - \hbar^2 / 2m \cdot \frac{\partial^2 \Psi(x, t)}{\partial x^2} + v(x, t) \Psi(x, t) \dots\dots\dots(3.5.11)$$

This is time dependent shrodinger equation in one elimination for a particle of mass in moving in field characterized by a potential energy function  $v(x, t)$  [12].

**(3.6) Time \_ independent schrodinger equation:**

To solve the schrodinger equation,

$$i \hbar \frac{\partial \Psi(x,t)}{\partial x} = - \hbar^2 / 2m \cdot \frac{\partial^2 \Psi(x,t)}{\partial x^2} + v \Psi \dots\dots\dots(3.6.1)$$

For special potential  $v$  which depends on  $r$  only, one was separation of variables in this case the shrodinger equation can be solved by separating  $\Psi$  to spatial and time dependent part, i.e.

$$\Psi(x,t) = \Psi(x) \cdot \phi(t) \dots \dots \dots (3.6.2)$$

Where  $\Psi$  is function of  $x$  alone  $\Psi$  is a function of  $t$  alone.

For separable solution we have

$$\frac{\partial \Psi}{\partial t} = \phi \frac{\partial \Psi}{\partial t}, \frac{\partial^2 \Psi}{\partial x^2} = \frac{\partial^2 \Psi}{\partial x^2} \phi \dots \dots \dots (3.6.3)$$

(Ordinary derivatives, now) and shrodinger equation reads:

$$i \hbar \Psi \frac{\partial \phi}{\partial t} = - \frac{\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial x^2} \phi + v \Psi \phi \dots \dots \dots (3.6.4)$$

or dividing through by  $\Psi \phi$  :

$$i \hbar \frac{1}{\phi} \frac{\partial \phi}{\partial t} = - \frac{\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial x^2} \frac{1}{\Psi} + v \dots \dots \dots (3.6.5)$$

Now the left side is a function of  $t$  alone, and the right side is a function of  $x$  alone.

For reasons that will appear in moment, we shall call the ration constant  $E$ . Then

$$i \hbar \frac{1}{\phi} \frac{d\phi}{dt} = E \dots \dots \dots (3.6.6)$$

Or

$$\frac{\partial \phi}{\partial t} = - \frac{iE}{\hbar} \phi \dots \dots \dots (3.6.7)$$

Inserting (3-4-7) in (3-4-5) gives

$$- \frac{\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial x^2} \frac{1}{\Psi} + v = E \dots \dots \dots (3.6.8)$$

Separation of variable has turned a partial differential equation (eq (3.6.7) and (3.6.8)).

The general solution is  $C \exp(-i \frac{Et}{\hbar})$ , but we might as well absorb the constant  $C$  into  $\Psi$ . then.  $\varphi = E^{-i \frac{Et}{\hbar}}$

Equation (3.6.8) is called the time \_independent Schrodinger equation [12].

### (3.7) The Harmonic oscillator

The paradigm for a classical harmonic oscillator is a mass  $m$  attached to a spring of force constant  $k$ . The motion is governed by Hook's law.

$$F = -kx = m \frac{d^2x}{dt^2} \dots\dots\dots (3.7.1)$$

(As always, we ignore friction), and the solution is

$$x(t) = A \sin(\omega t) + \beta \cos(\omega t) .$$

Where

$$\omega = \sqrt{\frac{k}{m}} \dots\dots\dots (3.7.2)$$

Is the (angular) frequency of oscillation? The potential energy is

$$V(x) = \frac{1}{2} kx^2 \dots\dots\dots (3.7.3)$$

Its graph is a parabola.

Of course, there's no such thing as a perfect simple harmonic oscillator-if you stretch it too far the spring is going to break, and typically Hooke's law fails long before that point is reached. But practically any potential is approximately parabolize. In the neighborhood of a local minimum (figure 2.3). Formally. If we expand  $V(x)$  in a Taylor series about the minimum:

$$V(x) = V(x_\theta) + (x - x_\theta) + \frac{1}{2}V''(x_\theta)(x - x_\theta)^2 + \dots$$

Subtract  $V(x_\theta)$  [you can add a constant to  $V(x)$  with impunity, since that doesn't change the force], recognize that  $V'(x_\theta) = 0$  (since  $x_\theta$  is a minimum), and drop the higher-order terms [which are negligible as long as  $(x - x_\theta)$  stays small], the potential becomes.

$$V(x) \cong \frac{1}{2}V''(x_\theta)(x - x_\theta)^2$$

Which describes simple harmonic oscillation (about the point  $x_\theta$ ), with an effective spring constant  $k = V''(x_\theta)$ <sup>10</sup>. That's why the simple harmonic oscillator is so important: Virtually any oscillatory motion is approximately simple harmonic, as long as the amplitude is small.

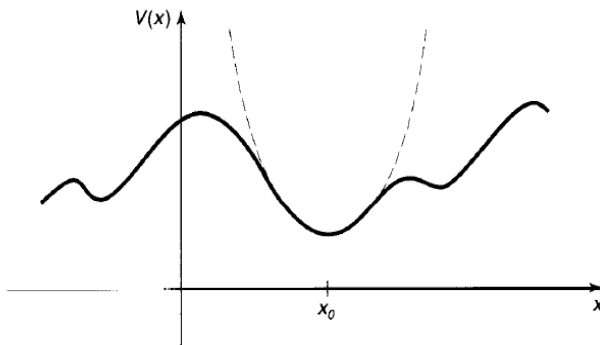


Figure (3.7.1): parabolic approximation (dashed curve) to an arbitrary potential, in the neighborhood of a local minimum.

The quantum problem is to solve the Schrodinger equation for the potential

$$V(x) = \frac{1}{2}m\omega^2x^2 \dots\dots\dots (3.7.4)$$

(It is customary to eliminate the spring constant in favor of the classical frequency, using Equation (3.4.6)). As we have seen, it suffices to solve the time-independent Schrodinger equation:

$$-\frac{\hbar^2}{2m} \frac{d^2\psi}{dx^2} + \frac{1}{2} m\omega^2 x^2 \psi = E\psi \dots\dots\dots (3.7.5)$$

In the literature you will find two entirely different approaches to this problem. The first is a straightforward “brute force” solution to the different equation, using method of power series expansion: it has the virtue that the same strategy can be applied to much other potential (in fact, we’ll use it in Chapter four to treat the Coulomb potential). The second is a diabolically clever algebraic technique, using so-called ladder operators. I’ll show you the algebraic method first, because it is quicker and simpler (and more fun): if you want to skip the analytic method for now, that’s fine, but you should certainly plan to study it at some stage.

### 3.7.1 Algebraic Method

To begin with, let’s rewrite equation (3.7.5) in a more suggestive form:

$$\frac{1}{2m} \left[ \left( \frac{\hbar}{i} \frac{d}{dx} \right)^2 + (m\omega x)^2 \right] \psi = E\psi \dots\dots\dots (3.7.6)$$

The idea is to factor the term in square brackets. If these were numbers, it would be easy:

$$u^2 + v^2 = (u - iv)(u + iv).$$

Here, however, it’s not quite so simple, because  $u$  and  $v$  are operators, and operators do not, in general. Commute ( $uv$  is not the same as  $vu$ ). Still this does invite us to take a look at the expressions.

$$a_{\pm} \equiv \frac{1}{\sqrt{2m}} \left( \frac{\hbar}{i} \frac{d}{dx} \pm im\omega x \right) \dots\dots\dots (3.7.7)$$

What is their product?  $a - a$ ? Warning: operators can be slippery with in the abstract, and you are bound to make mistakes unless you give them a “test

function",  $f(x)$ , to act on. At the end you can throw away the test function, and you'll be left with an equation involving the operator's case, we have

$$\begin{aligned}
 a - a_+ f(x) &= \frac{1}{2m} \left( \frac{\hbar}{i} \frac{d}{dx} - im\omega x \right) \left( \frac{\hbar}{i} \frac{d}{dx} + im\omega x \right) f(x) \\
 &= \frac{1}{2m} \left( \frac{\hbar}{i} \frac{d}{dx} - im\omega x \right) \left( \frac{\hbar}{i} \frac{d}{dx} + im\omega x f \right) \\
 &= \frac{1}{2m} \left[ -\hbar^2 \frac{d^2 f}{dx^2} + \hbar m \omega \frac{d}{dx} (xf) - \hbar m \omega x \frac{df}{dx} + (m\omega x)^2 f \right] \\
 &= \frac{1}{2m} \left[ \left( \frac{\hbar}{i} \frac{d}{dx} \right)^2 + (m\omega x)^2 + \hbar m \omega \right] f(x).
 \end{aligned}$$

[I used  $\frac{d(xf)}{dx} = x \left( \frac{df}{dx} \right) + f$  in the last step]. Discarding the test function. We conclude that

$$a - a_+ = \frac{1}{2m} \left[ \left( \frac{\hbar}{i} \frac{d}{dx} \right)^2 + (m\omega x)^2 \right] + \frac{1}{2} \hbar m \dots \dots \dots (3.7.8)$$

Evidently Equation (3.7.6) does not factor perfectly-there's an extra term  $(1/2)\hbar\omega$ . However, if we pull this over to the other side, the Schrodinger equation's becomes

$$\left( a - a_+ - \frac{1}{2} \hbar \omega \right) \psi = E\psi \dots \dots \dots (3.7.9)$$

Notice that the ordering of the factors  $a_+$  and  $a_-$  is important here: the same argument, with  $a_+$  on the left, yields.

Thus

$$a - a_+ - a - a_- = \hbar \omega \dots \dots \dots (3.7.11)$$

And the Schrodinger equation can also be written

$$\left(a_+a_- + \frac{1}{2}\hbar\omega\right) = E\psi\dots\dots\dots(3.7.12)$$

Now here comes the crucial step: I claim that *if*  $\psi$  satisfies the Schrodinger equation, with energy  $E$ , then  $a_+\psi$  satisfies the Schrodinger equation with energy  $(E + \hbar\omega)$ .proof.

$$\begin{aligned} \left(a_-a_+ + \frac{1}{2}\hbar\omega\right)(a_+\psi) &= (a_+a_-a_+ + \frac{1}{2}\hbar\omega a_+)\psi \\ &= a_+\left(a_-a_+ + \frac{1}{2}\hbar\omega\right)\psi = a_+\left[\left(a_-a_+ - \frac{1}{2}\hbar\omega\right)\psi + \hbar\omega\psi\right] \\ &= a_+(E\psi + \hbar\omega\psi) = (E + \hbar\omega)(a_+\psi). QED \end{aligned}$$

[Notice that whereas the ordering of  $a_+$  and  $a_-$  does matter the ordering of  $a_+$  and any constants (such as  $\hbar, \omega$  and  $E$ )doesn't] by the same token  $a_-\psi$  is a solution with energy  $(E - \hbar\omega)$ :

$$\begin{aligned} \left(a_-a_+ - \frac{1}{2}\hbar\omega\right)(a_-\psi) &= a_-(a_+a_- - \frac{1}{2}\hbar\omega)\psi \\ &= a_-\left[\left(a_+a_- + \frac{1}{2}\hbar\omega\right)\psi - \hbar\omega\psi\right] = a_-(E\psi - \hbar\omega\psi) \\ &= (E - \hbar\omega)(a_-\psi) \end{aligned}$$

Here then is a wonderful machine for grinding out new solution, with higher and lower energies-if we can just find one solution to get started. We call  $a_+$  ladder operators because they allow us to climb up and down in energy;  $a_+$  is called the raising operator, and  $a_-$  the lowering operator. The ‘ladder’ of states is illustrated in figure (3.7.2)

But wait! What if I apply the lowering operator repeatedly/ eventually I'm going to reach a state with energy less than zero, which does not exist at some point the machine, must fail. How can that happen? We know that  $a_-\psi$  is a new

solution to the Schrodinger equation, but there is no guarantee that it will be normalizable-it might be zero, or its square integral might be infinite. Problem 2.11 rules out the latter possibility. Conclusion: there must occur a “lowest rung” (let’s call it  $\psi_0$ ) such that

$$a_- \psi_0 = 0$$

That is to say,

$$\frac{1}{\sqrt{2m}} \left( \frac{\hbar}{i} \frac{d\psi_0}{dx} - im\omega x \psi_0 \right) = 0$$

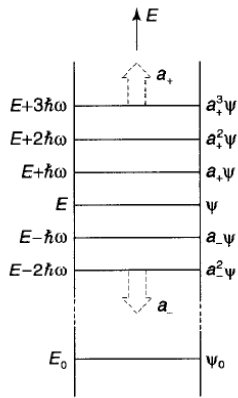


Figure (3.7.2): the ladder of stationary states for the simple harmonic oscillator.

Or

$$\frac{d\psi_0}{dx} = m\omega x \psi_0$$

This differential equation for  $\psi_0$  is easy to solve:

$$\int \frac{d\psi_0}{\psi_0} = \frac{-m\omega}{\hbar} \int x dx \Rightarrow \ln \psi_0 = -\frac{m\omega}{2\hbar} x^2 + constant$$

So

$$\psi_0(x) = A_0 e^{-\frac{m\omega}{2\hbar} x^2} \dots\dots\dots (3.7.14)$$



To determine the energy of this state, we plug it into the Schrodinger equation (in the form of equation (3.7.12)),  $(a_+a_- + (1/2)\hbar\omega)\psi_0 = E_0\psi_0$ . And exploit the fact that  $a_-\psi_0 = 0$ . evidently.

$$E_0 = \frac{1}{2}\hbar\omega \dots\dots\dots(3.7.15)$$

With our foot now securely planted on the bottom rung (the ground state of the quantum oscillator), we simply apply the raising operator to the excited states:

$$\psi_n(x) = A_0(a_+)^n e^{-\frac{m\omega}{2\hbar}x^2}, \text{ with } E_n = (n + \frac{1}{2})\hbar\omega \dots\dots\dots (3.7.16)$$

(This method does not immediately determine the normalization factor  $A_n$ : I'll let you work that out for yourself in problem) for example,

$$\begin{aligned} \psi_1 &= A_1 a_+ e^{-\frac{m\omega}{2\hbar}x^2} = A_1 \frac{1}{\sqrt{2m}} \left( \frac{\hbar}{i} \frac{d}{dx} + im\omega x \right) e^{-\frac{m\omega}{2\hbar}x^2} \\ &= \frac{A_1}{\sqrt{2m}} \left[ \frac{\hbar}{i} \left( -\frac{m\omega}{\hbar} x \right) e^{-\frac{m\omega}{2\hbar}x^2} + im\omega x e^{-\frac{m\omega}{2\hbar}x^2} \right] \end{aligned}$$

Which simplifies to?

$$\psi_1(x) = (iA_1\omega\sqrt{2m})xe^{-\frac{m\omega}{2\hbar}x^2} \dots\dots\dots (3.7.17)$$

I wouldn't want to calculate  $\psi_{50}$  in this way, but never mind: we have found all the allowed energies, and in principle we have determined the stationary states- the rest is just computation [12, 13].

**(3.8) Dirac equation:**

The Dirac equation can be thought of as a type of Schrodinger equation if we just change what the Hamiltonian operator  $H$  is, applied to the wave function [6].

Assuming that the particle in equation has rest mass  $m$ , the form of the Hamiltonian operator used in the Dirac equation is:

$$H = c \vec{\alpha} \cdot (-i\hbar\nabla) + \beta mc^2 \dots\dots\dots (3.8.1)$$

From Schrodinger equation:

$$i\hbar \frac{\partial\Psi}{\partial t} = \frac{\hbar^2}{2m} \nabla^2 \Psi + V \Psi \dots\dots\dots (3.8.2)$$

but

$$i\hbar \frac{\partial\Psi}{\partial t} = H \Psi \dots\dots\dots (3.8.3)$$

Sub Equation (3.8.1) in (3.8.3)

$$i\hbar \frac{\partial\Psi}{\partial t} = [c \vec{\alpha} \cdot (-i\hbar\nabla) + \beta mc^2] \Psi \dots\dots\dots (3-5-4)$$

The is relativistically covariant equation.

$\alpha, \beta$  are actually  $4 \times 4$  matrices.

$$\vec{\nabla} = \frac{\partial}{\partial x} \hat{x} + \frac{\partial}{\partial y} \hat{y} + \frac{\partial}{\partial z} \hat{z} \dots\dots\dots (3.8.5)$$

And

$$\vec{\alpha} = \alpha_1 \hat{e}_1 + \alpha_2 \hat{e}_2 + \alpha_3 \hat{e}_3 \dots\dots\dots (3.8.6)$$

From Dirac matrices in terms of  $\vec{\alpha}$  and  $\beta$  in the following way:

$$\gamma^0 = \beta \dots\dots\dots (3.8.7)$$

$$\gamma^i = \beta \alpha_i \dots\dots\dots (3.8.8)$$

Using the Dirac matrices:

$$i\hbar \gamma^\mu \frac{\partial\Psi}{\partial x^\mu} - mc \Psi = 0 \dots\dots\dots (3.8.9)$$

When we set  $\hbar=c=1$  and use  $\frac{\partial\Psi}{\partial s^\mu}=\partial_\mu$  to write the Dirac equation in the compact form:

$$i \gamma^\mu \partial_\mu \Psi - m\Psi = 0 \dots\dots\dots (3.8.10)$$

The correct way to interpret this equation , when the equation theory is brought in , is that it applies to the Dirac field whose quanta are spin  $-\frac{1}{2}$  particles – electrons[6].

**(3.9) Klein\_Gorden Equation:**

The Klein\_Gorden Equation is very easy to drive in two steps. We start by writing down the fundamental relation, between energy, momentum, and mass used in special relativity [6].

$$E^2=P^2C^2+M^2C^4 \dots\dots\dots (3.9.1)$$

You remember that the time – independent version of this equation is given by:

$$E \Psi = - \frac{\hbar^2}{2m} \frac{\partial^2\Psi}{\partial x^2} +V \Psi \dots\dots\dots (3.9.2)$$

So it might occur to you that the Schrodinger equation can be thought of as a statement of the non-relativistic definition of energy.

Hence we make the following substitution for energy.

$$E \rightarrow i\hbar \frac{\partial}{\partial t} \dots\dots\dots (3.9.3)$$

And the momentum is given by a spatial derivative, that is,

$$p \rightarrow -i\hbar \frac{\partial}{\partial x} \dots\dots\dots (3.9.4)$$

Generalizing to three dimensions, the relation is

$$\vec{p} \rightarrow -i\hbar \nabla \dots\dots\dots (3.9.5)$$

To derive the Klein\_Gorden Equation all we do is put the substitutions (3.9.1) and (3.9.5) into the Einstein relation for energy, momentum and mass Equation (3.9.1) and apply it to a wave function  $\Psi$ . Using eq (3.9.3) we see that:

$$E^2 \rightarrow -\hbar^2 \frac{\partial^2}{\partial t^2} \dots\dots\dots (3.9.6)$$

Now, using Equation (2-6-5) we have:

$$p^2 = -\hbar^2 \nabla^2 \dots\dots\dots (3.9.7)$$

There form, in terms of operators, the Einstein relation between energy, momentum and mass Equation (3.9.1) can be written as:

$$-\hbar \frac{\partial^2}{\partial t^2} = -\hbar^2 c^2 \nabla^2 + m^2 c^4 \dots\dots\dots (3.9.8)$$

So well apply this operator to a function of space and time  $\Psi = \Psi(x, t)$ . Doing these rearranging terms a little gives us the Klein- Gorden equation:

$$\hbar^2 \frac{\partial^2 \Psi}{\partial t^2} = -\hbar^2 c^2 \nabla^2 \Psi + m^2 c^4 \Psi = 0 \dots\dots\dots (3.9.9)$$

But  $\hbar = c = 1$  (natural units) so this becomes:

$$\frac{\partial^2 \Psi}{\partial t^2} - \nabla^2 \Psi + m^2 \Psi = 0 \dots\dots\dots (3.9.10)$$

We can simplify the appearance of the equation a little further by using different notation. In fact well write it in to two different ways. The first is to recall the D Alembertion operator in Min Rowski space as:

$$\square^2 = \frac{\partial^2}{\partial t^2} - \nabla^2 \dots\dots\dots (3.9.11)$$

This allows us to write *Equation* (3.9.10)

In the flowing simplified way:

$$(\square + m^2) \Psi = 0 \dots\dots\dots (3.9.12)$$

We can also write Equation (3.9.9) in a nice, compact style using the notation developed:

Using  $\partial_\mu \partial^\mu = \frac{\partial^2}{\partial t^2} - \nabla^2$  it becomes:

$$(\partial_\mu \partial^\mu + m^2) \Psi = 0 \dots\dots\dots (3.9.13)$$

The free particle solution is given by

$$\Psi(\vec{x}, t) = e^{-ip \cdot x} \dots\dots\dots (3.9.14)$$

The scalar product in the exponent is

$$p \cdot x = p_\mu x^\mu = E t - \vec{p} \cdot \vec{x} \dots\dots\dots (3.9.15)$$

That means:

$$\Psi(\vec{x}, t) = e^{-i(Et - px)} \dots\dots\dots (3.9.16)$$

To derive Eq (3.9.16):

$$\frac{\partial \Psi}{\partial t} = \frac{\partial}{\partial t} e^{-i(Et - px)} = -iE e^{-i(Et - px)} = -iE \Psi \dots\dots\dots (3.9.17)$$

And

$$\frac{\partial \Psi}{\partial x} = \frac{\partial}{\partial x} e^{-i(Et - px)} = -ip e^{-i(Et - px)} = -ip \Psi \dots\dots\dots (3.9.18)$$

Therefore, we have

$$\frac{\partial^2 \Psi}{\partial t^2} - \frac{\partial^2 \Psi}{\partial x^2} = -E^2 \Psi + P^2 \Psi \dots\dots\dots (3.9.19)$$

Hence, applying the full Klein-Gordon equation (3.9.10) we have:

$$(E^2 - P^2) \Psi = m^2 \Psi \dots\dots\dots (3.9.20)$$

Canceling the wave function and rearranging terms gives

$E^2 = P^2 + m^2$  , the desired result solving the energy ,we take the square root ,being care full to include both positive and negative square roots

$$E = \pm \sqrt{P^2 + M^2} \dots\dots\dots (3.9.21)$$

This dramatic result which is one reason Schrodinger discarded the Klein \_Gorden equation [6].

## Chapter Four

### Derivation of plank Quantum Theory from Classical Electromagnetic Theory

#### (4.1) Introduction:

The discovery of max plank of the particle nature of electromagnetic waves and light mode radical modification in the mechanics of atomic world. Where it leads to the appearance of new mechanics (q. m) to describe atomic word [8, 9]. Many attempts were to reconcile particle (photon) concept of (e. m. w) with electromagnetic theory [3, 4, 5] but they fail to do this completely. In this chapter attempt to find some connection between particle and wave nature of (e. m. w) x is mode.

#### (4.2) Photon Energy and displacement current:

The displacement current  $J$  is given by

$$J = \frac{\partial D}{\partial t} = \epsilon \frac{\partial E}{\partial t} \dots\dots\dots (4.2.1)$$

Where:

$$D = \epsilon E = \text{Electric flux density}$$

Thus:

$$J = \epsilon \frac{\partial E}{\partial t} \dots\dots\dots (4.2.2)$$

where  $\epsilon$  is the electric permittivity

$E$  is the electric field intensity [15, 16].

The electric travelling wave is given by:

$$E = E_0 e^{i(kx - \omega t)} \dots \dots \dots (4.2.3)$$

$E_0 \equiv$  maximum electric field

$k \equiv$  wave number

$\omega \equiv$  angular frequency

$$k = \frac{2\pi}{\lambda}, \quad \omega = 2\pi f \dots \dots \dots (4.2.4)$$

From equation (4.2.2) the current density and (4.2.3) is given by:

$$J = \frac{\partial(\epsilon E)}{\partial t} \dots \dots \dots (4.2.5)$$

$$J = \epsilon \frac{\partial E}{\partial t} = \epsilon \frac{\partial(E_0 e^{i(\Re x - \omega t)})}{\partial t} \dots \dots \dots (4.2.6)$$

Thus the displacement current is

$$J = \epsilon (-i\omega E) = -i\omega\epsilon E \dots \dots \dots (4.2.7)$$

On the other hand the electric potential V can be written as:

$$v = E \cdot L \dots \dots \dots (4.2.8)$$

$$\text{And } I = JA \dots \dots \dots (4.2.9)$$

Where L is the conductor length is given by the electric power (Pr):

$$P_r = VI \dots \dots \dots (4.2.10)$$

Substituting equation (4.2.8) and equation (4.2.9) in equation (4.2.10) yield :

$$P_r = (EL)(JA) \dots \dots \dots (4.2.11)$$

$$P_r = (EL)(J \omega \epsilon E A) \dots \dots \dots (4.2.12)$$

$$P_r = -J \omega \epsilon E^2 (AL) = -J \omega \epsilon E^2 (V) \dots \dots (4.2.13)$$



Thus the power energed per unit volume is given by:

$$\frac{P_r}{V} = -i \omega \epsilon E^2 = \omega \epsilon E^2 \dots \dots \dots (4.2.14)$$

This expression is found by considering (e .m) wave can be considered also as photon. But the density is malted to E according to

$$n = c_0 E^2 \dots \dots \dots (4.2.15)$$

The relation combining (4.2.14) and (4.2.15) yield

$$= n h f = c_0 h f E^2 = \omega \epsilon E^2 = 2\pi \epsilon f E^2 \dots \dots (4.2.16)$$

Thus the blank constant is given by :

$$c_0 h = 2\pi \epsilon$$

$$h = \frac{2\pi \epsilon}{c_0} \dots \dots \dots (4.2.17)$$

**(4.3) photon energy and Harmonic oscillator:**

According to wave mechanic the number of particles  $n$  are proportional to the square of Amplitude A, According to the relation:

$$n = c_0 A^2 \dots \dots \dots (4.3.1)$$

The photon energy is given by:

$$E = \hbar \omega \dots \dots \dots (4.3.2)$$

Thus the total photon energy is having different frequencies is given by:

$$E = (\hbar \int \omega d\omega) n = (\frac{1}{2} \hbar \omega^2) n \dots \dots \dots (4.3.3)$$

But according to Newtonian mechanics for harmonic motion

$$E = m \omega^2 A^2 \dots \dots \dots (4.3.4)$$

hence from eqns (4.3.1) & (4.3.4)

$$m\omega^2 A^2 = \left[ \left( \frac{1}{2} \right) (hc_0) \right] \omega^2 A^2 \dots\dots\dots (4.3.5)$$

This means that the photon mass is

$$m_p = m = \frac{1}{2} hc_0 \dots\dots\dots (4.3.6)$$

**(4.4) Discussion:**

It is very interesting to note that by using the concept of displacement current found from Maxwell's equation's the current density is frequency dependent see equation (4.2.7). Using the ordinary expression of electric power equations (4.3. 4) and (4.3. 5) shows that the power is frequency dependant. Using these relations with the power relation found from photon concept.

It is clear that the energy of a single photon can be found from Maxwell's equations which describe the wave behaviour of e.m.ws. The Plank constant can be obtained by adjecting the free parameter  $c_0$ .

Section (3) shows also that the wave and particle nature of photons is co related. It shows that using plank photon energy one can find the classical expression for harmonic oscillator as show by eqns (4.3.3, 4.5).

**(4.5) Conclusion:**

The wave and particle nature of electromagnetic waves can be shows to be co related by using simple arguments based on the notion of displacement current and classical harmonic oscillator energy.

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