SUDAN UNIVERSITY OF SCINCE & TECHNOLOGY

COLLEGE OF POST-GRADUATE STUDIES

FACULTY OF SCIENCES

Modelling and Forecasting Estimation Exchange Rate Volatility in the Sudan

النمذجه وتقدير التنبؤ بتقلبات اسعار الصرف في السودان

ATHESIS SUBMITTED IN FULFILLMENT OF THE

REQUAIREMENTS FOR THE PH.D IN STATISTICS

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November 2016

Dedication

To my Mother, Brothers, sister and Father Soul, Their constant love and support have guided me to where I am. My gratitude to them could never be expressed through words.

Acknowledgements

The successful completion would be impossible without the assistance and guidance of many individuals who have provided invaluable help to me throughout my whole research. I would like to express my gratitude to every individual who has contributed to this research.

First and foremost, a very special thanks and appreciation to my supervisor Dr. Ahmed Mohammed Abdullah Hamdi for being the most understanding, helpful and patient lecturer I have come to know. Also special thanks to my Co supervisor Dr.Amal Alsir for her help, effort and guidance

Last but not least, I would like to thank my parents and siblings for their love and support. I would like to thank my friends especially Dr.Abdul-Aziz Gibreel and Dr.Fath-Alrhman Idriss who have assisted me to complete this research.

Abstract:

The exchange rate is one of the macro-economic variables that have an impact on macroeconomic with its different sectors and the Exchange rate policy is one the most important policies that adopted by some countries to solve some of the economic problems.

Therefore we must stand on timeline impact of the exchange rate in Sudan and build a predictive model for predicting exchange rates in Sudan. It requires finding suitable models to the nature of commercial time-series data. We must judge these models that they can represent the data، in this research we apply the Autoregressive conditional models conditioned by non- Homogenization on the exchange rates in Sudan to provide a predictive model

Data collection was based on the monthly readings of exchange rates in Sudan in the period from 1/1/1999 to 31/12/2013

Issued by the Central System of Statistics and the Bank of Sudan

Where he used a form of GARCH symmetric and asymmetric models to predict the best model in addition to the ARIMA and Autoregressive conditional Heteroskedasticity models conditioned by non -Homogenization Using the normal distribution and distribution of (t-student).

1- The summary statistics indicate that the returns series have monthly positive mean (0.0051) while the volatility is (0.013) without loss of generality the mean grows at linear rate while the volatility grows approximately at square root rate.

2- The returns series of the exchange rate shows positive skewness this implies that the series of exchange rate is flatter to the right
3- The kurtosis value is the higher than the normal and this suggest that the kurtosis curve of the exchange rate return series is leptokurtic.
4- The coefficient in the condition variance equation GARCH(1,1) the α significant and β not significant and the $(\alpha+\beta)$ is greater than one suggesting that the condition variance process is explosive.
5- The coefficient (risk premium) of in the mean equation is positive of the market which indicate the mean of the return sequence depend on past innovation and the past conditional variance.
6- The estimation of EGARCH(1,1) model for return series of exchange rate the γ is negative and significant meaning that return series have asymmetry and has greater impact of negative shocks indicate that the conditional variance has leverage effect and asymmetry of negative shocks.
7- The result indicate that the forecasting performance of the GJR-GARCH(1,1) and DGE-GARCH(1,1) models especially when fat-tailed asymmetric conditional distribution are taken into account in the conditional volatility is better than the GARCH(1,1) model.
8- However the comparison between the models with normal and student-t distribution shows that according to the different measures used for evaluating the performance of volatility forecasts the DGE-GARCH(1,1) model provides the best forecasts.
9- It is a found that the student-t distribution is more appropriates for modeling and forecasting exchange rate return volatility.

المستخلص

ان سعر الصرف من المتغيرات الاقتصادية الكلية والتي لها تأثير علي الاقتصاد الكلي بقطاعاته المختلفه وسياسة سعر الصرف من اهم السياسات التي تتبناها الدول لعلاج بعض المشاكل الاقتصادية وبالتالي كان لابد من الوقوف علي الاثر الزمني علي سعر الصرف في السودان وبناء نموذج تنبؤئي وللتنبؤ باسعار الصرف في السودان يتطلب ايجاد نماذج مناسبه لطبيعة بيانات السلاسل الزمنية التجارية وهذه النماذج لابد ان نحكم عليها بانها يمكن ان تمثل البيانات. وفي هذه البحث نطبق نماذج الانحدار الذاتي المشروطه بعدم التجانس علي اسعار الصرف في السودان لتقديم نموذج تنبوئي.

تم جمع البيانات بالاعتماد علي القراءات الشهرية لاسعار الصرف في السودان في الفتره من 1999/1/1 حتي 2013/12/31م الصادره من الجهاز المركزي للاحصاء وبنك السودان

حيث استخدمت نماذج GARCH المتماثله والغير متماثله للتنبؤ بأفضل نموذج بالاضافة لنماذج اريما والانحدار الذاتي المشروط بعدم التجانس باستخدام التوزيع الطبيعي وتوزيع (ت-ستيودنت)

النتائج :

1- نتائج الاحصاءات الوصفيه تشير الي ان سلسلة عوائد اسعار الصرف الشهريه موجبه ب(0.0051) بينما التقلبات (0.013) وان المتوسط ينمو بمعدل خطي اما التقلبات تنمو تقريباً بمعدل الجذر التربيعي
2- من خلال قيمة الالتواء ان سلسلة عوائد اسعار الصرف تميل الي الجهه اليمني وان قيمة التفرطح اكبر من القيمه الاعتياديه للتوزيع الطبيعي مما يدل علي ان شكل السلسله محدب
3- معامل معادلة التباين الشرطي GARCH(1,1) للمعلمة α معنوية احصائياً بينما β غير معنوية ومجموع المعلمتان اكبر من الواحد و ذلك يدل علي ان عملية التباين الشرطي قابله للانفجار
4- معاملات (الخطر الابتدائي) في معادلة المتوسط موجبة لسوق اسعار الصرف وتشير الي ان سلسلة عوائد سعر الصرف تعتمد علي ابتكار الماضي والتباين الشرطي
5- بتقدير نموذج EGARCH(1,1) نجد ان المعلمة γ قيمتها سالبه ومعنوية مما يدل علي وجود الصدمات السالبه لسوق اسعار الصرف في السودان

6- النتائج تشير الي ان التنبؤ بنموذجي GJR-GARCH(1,1) و DEG-GARCH(1,1)خاصةً بتوزيع سميك الزيل غير المتماثل افضل من نموذج GARCH(1,1)

7- عند المقارنة بين نماذج GARCH للتوزيع الطبيعي وتوزيع ت- ستيودنت للتنبؤ بتقلبات الاسعار نجد ان نموذج DEG-GARCH(1,1) افضل نموذج للتنبؤ

8- ان توزيع ت- ستيودنت هو افضل توزيع لنماذج التنبؤ بتقلبات اسعار الصرف في السودان

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LIST OF ABBREVIATIONS

ACF - Autocorrelation functions

ADF - Augmented Dickey-Fuller

AIC - Akaike Information Criterion

AR - Autoregression

ARCH - Autoregressive Conditional Heteroskedasticity

ARIMA - Autoregressive Integrated Moving Average

ARMA - Autoregressive Moving Average

CGARCH - Component GARCH

EGARCH - Exponential GARCH

Eviews - Econometric Views

GARCH - Generalized Autoregressive Conditional Heteroskedasticity

IGARCH - Integrated GARCH

JB - Jarque-Bera

MA - Moving Average

MAE - Mean Absolute Error

MAPE - Mean Absolute Percentage Error

MSFE - Mean Squared Forecast Error

PACF - Partial Autocorrelation Functions

SIC - Schwarz Information Criterion

TAR - Asymmetric Threshold Autoregressive

LIST OF SYMBOLS

R^{-2} Adjusted R-squared

$\hat{\varepsilon}$ Estimated residual

$\hat{\varepsilon}^2$ Sum-of-squared residuals

ε_t Residuals

ε_t^2 Residuals squared

ζ_t White noise process

Ω_{t-1} Measurable function of time information set

H_0 Null hypothesis

R^2 R-squared

l_t Likelihood of ε_t

α_i Coefficients for ARCH

γ_0 Consistent estimate of the error variance

ρ_k Autocorrelation

σ^2 Unconditional variance

σ_t^2 Conditional variance

x^2 Chi-squared

$\emptyset_k$ Partial autocorrelation

Δ Difference linear operator

B Backshift operator

F F-statistic

Q Q-statistic

d Amount of differencing

n Number of observations

p Order of the autoregressive part

q Order of the moving average part

t time

CHAPTER ONE

INTRODUCTION

1-1 Introduction:

The economic crisis has had a differentiated impact on the world economies and on their trade, thereby changing trade patterns significantly in some cases. In the context of low employment related to recession, some policy makers are wanting to stimulate their exports, thereby hoping to improve their trade and current account balances Policy makers interested in implementing such policies have taken a closer look at exchange rate movements. Simply stated, depreciation of a country's currency makes its exports cheaper and its imports more costly. In the reality of a globalised economy, however, industries are vertically integrated, and exported products contain a large proportion of imported components. Imported components therefore become more costly for any given exporter and are not necessarily substitutable with domestically-produced products.

In addition, exchange rate levels have important implications for debt servicing and foreign investment flows. A depreciation in a country's currency implies that the nominal value of debt denominated in foreign currencies increases relative to the country's resources in local currency whereas its local-currency denominated debt decreases in value for foreign creditors. Capital investments become cheaper to foreign investors when the currency is depreciated, which is particularly important for large conomies that attract capital investments like the

United States and, to a lesser extent, the European Union? If depreciation is the result of a loss of confidence in the economy, however, foreign investors may be more hesitant to invest. Exchange rate changes affect firms within a given country differently.

Firms face a number of risks when engaging in international trade, in particular economic and commercial risks that are determined by macroeconomic conditions over which they have little control, such as exchange rates and their volatility. Risk management tools are available to help firms mitigate the impact of such risks, especially in the short term.These techniques for securing exchange rate risk are sometimes complex, however, and do not cover all commercial and financial operations. Besides, such tools may not be available to all firms, and the cost of using them may be significant, especially for small firms and in situations of high volatility.

There has been considerable volatility (and uncertainty) in the past few years in mature and emerging financial markets worldwide. Most investors and financial analysts are concerned about the uncertainty of the returns on their investment assets, caused by the variability in speculative market prices (and market risk) and the instability of business performance (Alexander, 1999). Recent developments in financial econometrics require the use of quantitative models that are able to explain the attitude of investors not only towards expected returns and risks, but towards volatility as well. Hence, market participants should be aware of the need to manage risks associated with volatility. This requires models that are capable of dealing with the volatility of the market (and the series). Due to unexpected events, uncertainties in prices (and returns) and the non-constant variance in the financial markets, financial analysts started to model and explain the behavior of exchange rate returns and volatility using time series econometric models. One of the most prominent tools for capturing such changing variance was the Autoregressive Conditional Heteroskedasticity (ARCH) process is based on the assumption that the recent past gives information about one period forecast variance. In (1982) Engle proposed a volatility process with time varying conditional variance, which is Autoregressive Conditional Heteroskedasticity (ARCH) process. Four years after Engel's introduced the ARCH process, Bollerslev 1986, proposed the Generalized ARCH (GARCH) models as a natural solution to the problem with the high ARCH orders, these models are based on an infinite ARCH specification and it allows to dramatically reducing the number of estimated parameters from an infinite number to just a few. In ARCH / GARCH models the conditional variance is expressed as a linear function of past squared innovations and earlier calculated conditional variances.

The usual assumptions of linear models are the disturbance terms ε_t distributed as a normal distribution with mean zero, constant variance and $\varepsilon_t's$ are uncorrelated, i.e.$\varepsilon_t \sim N(0,\sigma^2)$, $\mathrm{E}(\varepsilon_t) = 0$, $E(\varepsilon_t^2) = \sigma^2, E(\varepsilon_i\varepsilon_j) = 0 \; for \; i \neq j$). This research will briefly consider the case when the disturbance terms ε_t are vary over time, which means the errors $\varepsilon_{t's}$ doesn't have an equal Variance (Heteroskedasticity) which, can be caused by incorrect specification or use of the wrong functional form. Many economic time series exhibit periods of unusually large volatility followed by periods of relative tranquility, common examples of these such as a series include stock prices, foreign exchange rates and other prices determined in financial markets are known as their variance is seems to be vary over time.

This research aims at modelling and forecasting exchange rate volatility in the Sudan using Generalized Autoregressive Conditional Heteroskedasticity GARCH models as well as understanding exchange rates behavior to monetary policy and international trade.

1-2 Exchange Rate in the Sudan:

Since independence Sudan has experienced poor economic performance attributed to external as well as domestic factors particularly Policy failure and resource mismanagement. However the economic performance has improved since early 1990s when the government initiated the three -year national economic salvation (NESP 1990-1992) and the comprehensive national strategy (CNS 1992-2000) programs. The programs focused on key issue such as liberalization of trade and foreign exchange regimes sound monetary and fiscal policies phrasing out of price controls and privatization of public corporations (UN 2003)

Exchange rate is defined as the rate at which one native currency unit exchanges for one unit of internationality traded currency.

Exchange rate policy is one of the most important price policy tools and it is directly linked to the current account situation of the country.

In 1990 Sudan adopted a policy of a floating exchange rate the multiple and highly over valued exchange rate was replaced by a unified exchange rate.

In 1992-1993 the exchange rate began to revalue and the government re introduced the multiple exchange rate system.

Accordingly there were three exchange rates an exchange rate for exports determined by the Bank of Sudan and exchange rate for imports determined by committee of government banks representatives an exchange rate for individual foreign accounts determined by three markets

1-3 Real Exchange Rate:

The real exchange rate is the critical variable (along with the rate of interest) in determining the capital account. As we shall see, this is because the real exchange rate is the relative price of goods across countries. Hence changes in the real exchange rate affect the competitiveness of traded goods.

The nominal exchange rate is refered to the SDG price of foreign exchange. As with most variables in economics we distinguish between the nominal and real values, the real exchange rate measures the cost of foreign goods relative to domestic goods. It gives a measure of competitiveness, and it is a useful variable for explaining trade behavior and national income.

1-3-1 Definition

The real exchange rate Q can be divided by:

$$Q = \frac{SP^*}{P}$$

Where P^* is the price level in the foreign country. An appreciation of the real exchange rate indicates that the foreign price (in Sudanese pound SDG) of a bundle of goods has risen relative to the domestic price. If the real exchange rate appreciates it means that the real value of the SDG has depreciated; that is, the purchasing power of the SDG has fallen in relative terms. Notice that to define the real exchange rate we need to specify the price levels. If the baskets of goods in the domestic and foreign countries were the same this would be straightforward; in practice, they are not. We typically use some broad measure of the price level, such as the GDP deflator or the CPI. It should be noted that this means that P will place a relatively heavy weight on goods produced and consumed domestically, while P* will likewise place a relatively heavier weight on goods produced in the foreign country.

1-4 Research Problems:

Forecasting exchange rate in the Sudan requires finding models that reasonably represents it. In the literature several methods for constructing a financial time series models were suggested. However the suitability of any of these methods to a given time - series data has to be judged on the basis of its fit to that data.

1-5 Objectives Research:

The primary objective of the study is to fit appropriate GARCH model to estimate volatility of exchange rate in the Sudan. The study aims at:

• To investigate the volatility pattern of emerging Sudan stock market using symmetric and asymmetric models

• To identify the presence of leverage effect in monthly return series of stock market using asymmetric models

• To analyse the appropriateness of Generalized Autoregressive Conditional Heteroskedastic (GARCH) family models that capture the important facts about the index returns and fits more appropriate

1-6 **Research Hypotheses:**

This research examines the relative ability of various ARCH / GARCH models to construct accurate predictions for exchange rate volatility in the Sudan.

The null hypotheses that there are no significant difference when using Autoregressive Conditional Heteroskedasticity models such as ARCH, GARCH, IGARCH, GARCH-M, EGARCH,PGARCH and TGARCH models when each is used to forecast the exchange rate volatility in the Sudan.

Against the alternative hypotheses that there are a significant difference when using Autoregressive Conditional Heteroskedasticity models such as ARCH, GARCH, IGARCH, GARCH-M, EGARCH,PGARCH and TGARCH models when each is used to forecast the exchange rate volatility in the Sudan.

1-7 Research Data:

Monthly readings of Exchange rate in the Sudan covered the period from 01/01/1999 to 31/12/2013 will use in the analysis of this research. The data obtained from Central Bureau of Statistics, Bank of Sudan and Khartoum stock market.

1-8 Research Methodology:

In this study we briefly present the models specification, conditional distributions and forecasting criteria's as well as data set we use to model the SDG/US Dollars Exchange rate returns volatility in the Sudan economy. This article analyses the volatility of the Sudan exchange rate using various volatility models such as Autoregressive Integrated Moving Average (ARIMA), GARCH (1,1), GARCH-M (1,1), which will be used for testing symmetric volatility and EGARCH(1,1), TGARCH(1,1) and PGARCH (1,1) for modelling asymmetric volatility these models will be shortly discussed and GARCH, the Glosten, Jagannathan and Runkle (GJR) GARCH, Asymmetric Power Autoregressive conditional Heteroskedasticity APARCH model of Ding et.al (1993) as well as the conditional distributions such as normal and Student-t distributions. In this study three different criteria's, Mean Squared Error (MSE), Mean Absolute

Error (MAE) and Adjusted Mean Absolute Percentage Error (AMAPE) are used to evaluate the forecasting performance for the conditional Heteroskedasticity models.

1-9 Research Organization:

This research will be organized as follows: chapter one devoted to presenting problem, objective and the organization of the research. Chapter two devoted to review the basic concept of time series models and some other statistical methods. Chapter three devoted to review the characteristics of validity, structure of a model, volatility models that includes Autoregressive Conditional Heteroskedasticity models family for instance ARCH, GARCH, IGARCH, and GARCH-M, EGARCH models, describing the estimation methods of volatility models such as maximum likelihood estimation and models evaluation criteria. Chapter four will tackle the analysis and evaluate the data so as to estimate, test and forecasting the future values of Exchange rate. And finally the last chapter will sum up the findings of the research and point out some assumptions of future research in forecasting volatility of Exchange rate.

1-10 Literature Review:

To capture the volatility in financial time series, a comprehensive empirical analysis of the returns and conditional variance of the financial time series have been carried out using autoregressive conditional Heteroskedasticity models. Bellow a literature review of these studies:

Sharaf Obaid, Abdalla Suliman(2013).Estimating Stock Returns Volatility of Khartoum Stock Exchange through GARCH Models this study modeled and estimated stock returns volatility of Khartoum Stock Exchange (KSE) index using symmetric and asymmetric GARCH family models namelyGARCH (1,1) GARCH-M (1,1) EGARCH (1,1) and GJR-GRACH (1,1) models, they carried out that based on daily closing prices over the period from Jan 2006 to Aug 2010 ،that high volatility processing present in KSE index return series. The results also provided evidence on the existence of risk premium and indicate the presence of leverage effect in the KSE index returns series our findings indicate the student-t is the most favored distribution for all models estimated.

Mohd Aminal Islam (2013) Estimating Volatility of Stock Index Returns by using Symmetric GARCH Models, this study was utilize Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models to estimate volatility of financial asset returns of three Asian markets namely kualalampur composite index (KLCI) of Malaysia Jakarta Stock Exchange Composite Index (JKSE) of Indonesia and straits Times index (STI) of Singapore. Two symmetric GARCH models with imposing names such as the GARCH (1,1) and the GARCH-in-Mean or GARCH-M (1,1) are considered in this study. They were cover the period 2007-2012 comprising daily Observations of 1477 for

KLCI. 1461 for JKSE and 1493 for STI excluding the public holidays we choose to apply GARCH models as they are especially suitable for high frequency financial market data such as stock returns which has a time-varying variance unlike the linear structural models.

GARCH models are found useful in explaining a number of important features commonly observed in most financial time series.

Ahmed El sheikh M. Ahmed and Suliman Zakaria (2013)

Modeling stock Market volatility using GARCH Models Evidence from Sudan, they used the Generalized Autoregressive conditional Heteroskedasticity Models to estimate volatility (conditional variance).

In the daily returns of the principal stock exchange of Sudan namely Khartoum stock Exchange (KSE) over the period from 2006 to 2010 daily Observations of 1326 for (KSE). The models include both symmetric and asymmetric models that capture the most common stylized facts about index returns such as volatility clustering and leverage effect the empirical result show that the conditional variance process is highly persistent (explosive process) and provide evidence on the existence of risk premium of the KSE index return series which support the positive correlation hypothesis between volatility and the expected stock returns was findings also show that the asymmetric models provide better fit than the symmetric models; which confirms .The presence of leverage effect.

These results in general explain that high volatility of index return series is present in Sudanese stock market over the sample period.

CHAPTER TWO

TIME SERIES

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2-1 Introduction:

Analysis of time series enables us to build a mathematical model that helps in explaining past and present behavior of the series. Also it helps in forecasting future values of the series. The analysis of time series is used in many applications, for instance on the field of economics, business, engineering, medicine, agriculture, sales, export, import, stock market analysis, quality control, census analysis and environment. There are two types of time series models, firstly, univariate time series models, such as, univariate Box-Jenkins models and exponential smoothing models, secondly, multivariate time series models, such as transfer function and intervention analysis models. Time series models have been widely used in the construction of forecasting models, to achieve accurate forecasting that helps in development, planning and decision making.

The analysis of time series is of value in many applications such as economic forecasting, financial forecasting, sales forecasting, stock market analysis, quality control, census analysis and many more.

2-2 Time series:

The time series can be defined as a set of observations that are generated sequentially among the time for a specific phenomena any time series is associated with an ordered data through ordered times that is data is correlated .

we denote X_t is the time series observations where t = 1,2,3, …, n (n is number of observation)

2-3 Time series objectives:

There are many objectives of time series analysis, the most representative of these are:

1-To get precise description for the process which generates the time series data.

2-To build a mathematical explanation, demonstration and presentation the behavior of the series according to the previous observation.

3-To use the results of the estimated model of the previous data for speculating and forecasting the future values of the series.

4-To control the process which generates the time series by checking what can happens if the model parameters can be changing

2-4 Time series models:

According to the number of variables, the time series models are classified as follows:

1- Univariate Time series models, in these kinds of models, the present and past values of one time series values are used to construct the model.

2-Multivariate Time series models, these kinds of models contains more than one variable in order to explain the dynamic relationship among the variables including in the model, examples of these models are transfer function models. Multivariate time series models and intervention analysis models, these are models similar to regression models that consist of dependent variable and more than one independent variables.

2-5 Analysis of time series:

A time series x_t has four basic components, these are:

General Trend, Seasonal Variations, Cyclical components and Irregular components. Any time series can have some or all of the following components:

1. Trend component (T)

2. Cyclical component (C)

3. Seasonal component (S)

4. Irregular component (I)

These components may be combined in different ways. It is usually assumed that they are either multiplicative or additive models i.e.

$$x_t = T * S * C * I \qquad \text{...(2-1)}$$

$$x_t = T + S + C + I \qquad \text{...(2-2)}$$

To correct for the trend in the first case one divides the first expression by the trend (T). In the second case it is subtracted.

Below is a brief review about each component.

2-5-1 Trend component:

The trend is the long term pattern of a time series. A trend can be positive or negative depending on whether the time series exhibits an increasing long term pattern or a decreasing long term pattern. If a time series does not show an increasing or decreasing pattern then the series is stationary in the mean. The general trend of time series is sometimes expressed as a linear, nonlinear and exponential equation.

2-5-1-1 Modeling trend:

The simple linear function of trend equation of a time series $x_1, x_2 \dots, x_T$ can be expressed as simple linear function as follows:

$T_t = \beta_0 + \beta_1 time_t + \varepsilon_t$,……………………..……………………………(2-3)

which provides a good description of the trend. The variable t is constructed artificially and is called time trend. β_0 is the intercept, it is the value of the trend at time; (time = 0), β_1 is the slope; it is positive if the trend is increasing and negative if the trend is decreeing.

In business, finance and economics, linear trends are typically increasing, corresponding to growth.

Sometimes trend appears nonlinear, or curved like the quadratic trend equation which takes the form:

$T_t = \beta_0 + \beta_1 time + \beta_2 time^2 + \varepsilon_t$,…………….………………..…. (2-4)

A variety of different nonlinear quadratic trend shapes are possible, depending on the signs of the coefficients.

Other types of nonlinear trend is the exponential trend or log linear trend, this type of trend is very common in business, finance and economics because economic variables often display roughly constant growth rates, if the trend is characterized by constant growth at rate β_1, then the trend equation takes the form:

$T_t = \beta_0 e^{\beta_1 time_t}\, \varepsilon_t$,…………………….…...……………………….(2-5)

The above equation can be written as an exponential form as follows:

$\ln T_t = \ln \beta_0 + \beta_1 time_t + \ln \varepsilon_t$,…………………….……..……...……….(2-6)

2-5-1-2 Estimating trend models:

To estimates the various trend models to the data on a time series x_t by the least square regression using statistical software to find out:

$\hat{\theta} = \underset{\theta}{argmin} \sum_{t=1}^{T} [x_t - T_t(\theta)]^2$,……………………….……....……………(2-7)

Where θ denotes the set of parameters to be estimated. A linear trend for instance, has

$T_t(\theta) = \beta_0 + \beta_1 time_t$,…………………..…….…………………………(2-8)

And

$\theta = (\beta_0, \beta_1)$

In which the computer finds:

$(\widehat{\beta_0}, \widehat{\beta_1}) = \underset{\beta_0, \beta_1}{argmin} \sum_{t=1}^{T} [x_t - \beta_0 - \beta_1 time_t]^2$,……………...…..……..(2-9)

Similarly, in the quadratic trend form, the computer find out:

$(\widehat{\beta_0}, \widehat{\beta_1}, \widehat{\beta_2}) = \underset{\beta_0, \beta_1, \beta_2}{argmin} \sum_{t=1}^{T} [x_t - \beta_0 - \beta_1 time_t - \beta_2 time^2]^2$,...…….(2-10)

Moreover the exponential trend can be estimated in two ways. Firstly, estimate directly from the exponential representation as follows:

$(\widehat{\beta_0}, \widehat{\beta_1}) = \underset{\beta_0, \beta_1}{argmin} \sum_{t=1}^{T} [x_t - \beta_0 e^{\beta_1 time_t}]^2$,…………………...……..(2-11)

Alternatively, because the nonlinear exponential trend is nevertheless linear in logs, it can be estimated by regressing $\ln x_t$ on an intercept and time, thus to find out:

$(\widehat{\beta_0}, \widehat{\beta_1}) = \underset{\beta_0, \beta_1}{argmin} \sum_{t=1}^{T} [\ln x_t - \ln \beta_0 - \beta_1 time_t]^2$,……………….….(2-12)

The fitted values from the above regression are the fitted values of $\ln x_t$, so they must be transfer to antilog to get the fitted value of x_t.

2-5-1-3 Forecasting trend:

Given the linear trend model, which holds for any time t is expressed as:

$$T_t = \beta_0 + \beta_1 time_t + \varepsilon_t \text{ ,} \quad \text{(2-13)}$$

The future values of trend at time t+h, are given from the prediction equation:

$$\widehat{T_{t+h}} = \widehat{\beta_0} + \widehat{\beta_1} time_{t+h} \text{ ,} \quad \text{(2-14)}$$

To form confidence intervals, the trend regression error terms are assumed to be normally distributed random variable, in which case 95% confidence intervals are obtained from the equations:

$$\widehat{x_{T+h}} = \pm 1.96\sigma \text{ ,} \quad \text{(2-15)}$$

Where σ is the standard deviation of the disturbance in the trend regression.

2-5-2 Seasonal component:

Seasonality occurs when the time series exhibits regular fluctuations during the same month (or months) every year, or during the same quarter every year.

2-5-2-1 Modeling seasonality:

A key technique for modeling seasonality is regression on seasonal dummies. Lets be the number of seasons in a year, then s=4 for quarterly data, s=12 for monthly data and so forth.

To construct s seasonal dummy variables, each of which indicates the season of interest. If there are four seasons i.e. s=4, if $D_{i\ ,i=1,2,3,4}$ then:

D_1=(1,0,0,0; 1,0,0,0; 1,0,0,0; .,.,.,)

D_2=(0,1,0,0; 0,1,0,0; 0,1,0,0; .,.,.,.,)

D_3=(0,0,1,0; 0,0,1,0; 0,0,1,0; .,.,.,.,)

D_4=(0,0,0,1; 0,0,0,1; 0,0,0,1; .,.,.,.,)

D_1indicates the first quarter, (it is 1 in the first quarter and 0 otherwise), D_2indicates the second quarter, (it is 1 in the second quarter and 0 otherwise), and so on.

The pure seasonal dummy model is expressed as follows:

$x_t = \sum_{i=1}^{s} \gamma_i \, D_{it} + \varepsilon_t$,……………………….………………………..(2-16)

where γ_i's are the seasonal factors, they summarize the seasonal pattern over the year. In the absence of seasonality, the $\gamma_i{}'s$ are all the same, so the seasonal dummies drop from the model, and instead simply an intercept in the usual way.

Trend may be included as well, in which case the model is takes the form:

$$x_t = \beta_1 time_t + \sum_{i=1}^{s} \gamma_i D_{it} + \varepsilon_t \text{ ,……………..……………………(2-17)}$$

2-5-2-2 Forecasting seasonality:

As pure trend models discussed earlier, the construction of h step ahead forecast is expressed as follows:

$$x_{t+h} = \beta_1 time_{t+h} + \sum_{i=1}^{s} \gamma_i D_{it+h} + \varepsilon_{t+h} \text{ ,……………..………..….(2-18)}$$

The confidence intervals of the forecast values are given by:

$$\widehat{x_{T+h}} = \pm 1.96\sigma \text{ ,………..…..……………………………………..(2-19)}$$

where σ is the standard error of the regression.

2-5-3 Cyclical component:

Any pattern showing an up and down movement around a given trend is considered as a cyclical pattern. A cyclical variations is one of the time series component, it exist when the data are influenced by the long term variation such as economic fluctuates, business cycles, growth periods, drought periods….etc

2-5-4 Irregular component:

This component is unpredictable. Every time-series has some unpredictable component that makes it a random variable. In prediction, the objective is to model all the components to the point that the only component that remains unexplained is the random component.

2-6 Stationarity:

Stationarity plays a central part in time series analysis, because it replaces in a natural way the hypothesis of independent and identically distributed (iid) observations in standard statistics.

To analyze and forecast from time series the series must be stationary when it satisfies the following conditions:

1-The mean is fixed that means E $(x_t) = \mu$ where E (x_t) is the expected value of x_t and μ is the mean of observation of x_t

2- The variance is fixed that means σ_x^2= Var$(x_t) = E(x-\mu)^2$

where σ_x^2 is the variance of x_t

3- The auto-covariance function depends only on the time difference lag that means

γ_k = cov (x_t, x_{t+k}) = E$(x_t - \mu)(x_{t+k} - \mu)$ where γ_k is the covariance between

x_t and x_{t+k} and k = 1,2,3,…,$\frac{n}{2}$.

The conditions (1) and (2) mean that the mean and variance of series x_t are not change with passing time while condition (3) means that if we divide the series into two parts, and $\hat{\gamma}_k$calculated from the first part then this value is not different from which is calculated from the second part this means $\hat{\gamma}_k$ are independent on k and dependent only on the different time between the corresponding two lags.

If X_1, X_2 ,X_3 , …,X_n is a value of time series x_t and $\bar{X}$, $\hat{\sigma}_x^2$, $\hat{\gamma}_k$ are estimations of μ , σ_x^2 ,and γ_k as respectively that

$$\bar{X} = \frac{1}{n}\sum_{t=1}^{n} X_t$$

$$\hat{\gamma}_0 = \sigma_x^2 = \frac{1}{n-1}\sum_{t=1}^{T}(x_t - \bar{x})^2$$

$$\hat{\gamma}_k = \frac{1}{n-1}\sum_{t=1}^{n}(x_t - \bar{x})(x_{t+k} - \bar{x}) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2\text{-}20)$$

2-7 The Autocorrelation Function (ACF):

An important tool that helps in detecting stationarity and identifying models for time-series data is the autocorrelation coefficient .The autocorrelation coefficient is used to measure the strong relation between the value of series in different time period the mathematical syntax for autocorrelation function is

$$\rho_k = \frac{\boldsymbol{cov}\ (x_t, x_{t+k}),}{\sqrt{\boldsymbol{var}(x_t)}\sqrt{\boldsymbol{var}(x_{t+k})}} = \frac{\gamma_k}{\gamma_0} \qquad k = 1,2,3,\ldots,\frac{n}{2} \ \ldots\ldots\ldots\ldots\ldots (2\text{-}21)$$

So the variance of stationary series is fixed and equal for all different time period and estimated as:

$$r_k = \frac{\hat{\gamma}_k}{\hat{\gamma}_0} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2\text{-}22)$$

Where $\hat{\gamma}_0$ is variance of series observation x_t .

2-8 The partial Autocorrelation Function (PACF):

A second important tool used in identifying models of time series is the partial autocorrelation .The partial autocorrelation coefficient of order k measures the correlation between values k periods apart when the effect of time lags $1,2,\dots,k-1$ is kept constant. When the partial autocorrelation coefficient is looked at as a function of k it is called the partial autocorrelation function (PACF) to estimate the partial autocorrelation of order k, fit an autoregressive model of order k $i.e.$ $AR(k)$. The last coefficient of the independent variable in the fitted model is an estimate of the partial autocorrelation coefficient of lag k, the coefficient is computed as follows:

$$\phi_{kk} = \frac{r_k - \sum_{j=1}^{k-1} \phi_{k-1,j} r_{k-j}}{1 - \sum_{j=1}^{k-1} \phi_{k-1,j} r_j} \quad ,\dots\dots(2\text{-}23)$$

$$\phi_{k,j} = \phi_{k-1,j} - \phi_{kk}\phi_{k-1,k-j} \text{ , j=2,3,…,k-1 ,}\dots\dots(2\text{-}24)$$

2-9 Test of Stationarity:

When n is large it is found that r_k is distributed as normal with mean zero and variance $\frac{1}{n}$, so that confidence limits of r_k are:

$$-z_{1-\frac{\alpha}{2}} \frac{1}{\sqrt{n}} \le \hat{\rho}(k) \le +z_{1-\frac{\alpha}{2}} \frac{1}{\sqrt{n}} \dots\dots(2\text{-}25)$$

Where SE(r_k) = $\frac{1}{\sqrt{n}}$ (is the standard deviation of r_k) this above equation becomes

$$-\frac{1.96}{\sqrt{n}} \leq \hat{\rho}(k) \leq \frac{1.96}{\sqrt{n}} \quad \text{..........(2-26)}$$

With 95% confidence level when all r_k lies between two limits or at most the first and second autocorrelation coefficient (r_1, r_2) lies outside the above two limits then the original series is stationary , also another test of stationarity is Box-Pierce Q statistic which given

$$Q = (n\text{-}d)\sum_{k=1}^{m} {r^2}_k \quad \text{..........(2-27)}$$

Where d is the number of differences and m is the maximum number of autocorrelation coefficients (r_k) at lag k.

Then Q will be compared with tabulated chi square χ^2 with m –p –q degree of freedom and significance level (α) where q is the order of moving average model and p is the order of autoregressive model hence the hypothesis is :

H_0 : series is not stationary or is not random if Q $\leq {\chi^2}_{m-p-q,\alpha}$ then H_0 will be accepted and if Q $> {\chi^2}_{m-p-q,\alpha}$ then H_0 will be rejected

2-10 Achieving of the Stationarity:

They are many methods which are used to transform the non-stationary series to stationary series which are:

1- Logarithmic and square root transformation:

We can use two methods when the variance of series changes with processing time, the logarithmic transformation can be used efficiency when the variance of series is associated with a mean of series and the mean of the series is increased and decreased by fixed rate so in the logarithmic transformation the observation X_t(original date of series) can be represent as $Z_t Log X_t$ and also we represent the square root transformation as $Z_t \sqrt{X_t}$

2- Differencing Method

In this method we can use the symbol ∇ the back shift operator so we can define the first differences of the series $X_t$as $X_t' = \nabla\, x_t = (1 - B)x = x_t \, - Bx_t$,= $x_t \, - x_{t-1}$ Then we deal with the new series X_1' which has (n-1) values compute the autocorrelation coefficient of the series X_t and compute Q-statistic to test the stationarity if the new series still are not stationary we compute the second differences as

$X_t'' = (1 - B)^2 x_t \, = (1 - 2B + B^2)x_t \, = x_t \, - 2x_{t-1} \, + x_{t-2}$ ………..(2-28)

Then we deal with the series X_t'' which has (n-2) values compute the autocorrelation coefficient of the series $X_t''$and compute Q-statistic to test stationary after taking the first or the second differences

2-11 Box- Jenkins Models:

Box and Jenkins (1976) first introduced are very important to analyze the time series and used to forecast for specific phenomena in future these models are divided to seasonal and non-seasonal models. The non-seasonal models used to represent two types of series stationary and non-stationary series and some of these models are:

1. Autoregressive models

2. Moving average models

3. Autoregressive and moving average models

4. Autoregressive integrated moving average models

In the above models the random error (say a_t) must satisfy the following conditions:

a. E $(a_t) = 0$ for all t=1,2,3,…,n

b. Var $(\boldsymbol{a_t})$ = E$(\boldsymbol{a_t})^2 = \boldsymbol{\sigma_a}^2$ for all t=1,2,3,…,n

c. $\boldsymbol{a_t}$ is distributed as normal with mean zero and variance $\boldsymbol{\sigma_a}^2$

d. E$((\boldsymbol{a_t}, \boldsymbol{a_{t-k}}) = 0$ for all k=1,2,3,…,$\frac{n}{2}$ (means that the current error is independent from the previous error)

e. E$((\boldsymbol{Z_{t-k}}, \boldsymbol{a_t}) = 0$ k=1,2,3,…,$\frac{n}{2}$ (means that the current error is independent from the previous observations)

2-11-1 Autoregressive Model:

In the autoregressive model the current value Z_t in the time series is expressed as a linear combination of the previous values, and an unexplained portion $a_t$we assume that the value of X_t is taken as the deviation from its mean i.e we can define Z_t as $X_t - u$.The Autoregressive model of order p is denoted byAR(P).

A typical autoregressive model of order P takes the form:

$Z_t = \phi_1 Z_{t-1} + \phi_2 Z_{t-2} + \cdots + \phi_P Z_{t-P} + a_t \ , t = 1,2, \dots , P$,……..…(2-29)

where the $\phi_j \ _{(j=1,2,\dots,P)}$ is the jth autoregressive parameter and a_t is the error term at time t.

The $a_t^{'} s$ are assumed to be independently normally distributed random variable with mean zero and constant variance σ_t^2.We can also write the above model in term of B as follows:

$(1 - \phi_1 B - \phi_2 B^2 - \cdots - \phi_P B^P) Z_t = a_t$,…………………………...……(2-30)

For example of AR(p) models we take model for order one which is called AR(1) and its can be written as :

$$z_t = \phi_1 \ z_{t\text{-}1} + a_t \quad \text{……………………………………(2-31)}$$

Properties of AR(1) model

a- Mean

$E(Z_t) = \mu_t = E(\phi_1 Z_{t-1} + a_t)$

$= \phi_1 E(Z_{t-1}) + E(a_t)$

$= \phi_1(0) + E(0) = 0$

Where

$E(Z_{t-1}) = E(X_t - \mu) = \mu - \mu = 0$ ……………………………..……(2-32)

b- Variance

$\gamma_0 = E(Z_t - \mu_t)^2 = E(Z_t^2)$

$= E(\phi_1 Z_{t-1} + a_t)^2$

$= E({\phi_1}^2 {Z_{t-1}}^2 + 2\phi_1 Z_{t-1} a_t + a_t^2)$

$= {\phi_1}^2 E({Z_{t-1}}^2) + 2\phi_1 E(Z_{t-1} a_t) + E(a_t^2)$

$= {\phi_1}^2 \gamma_0 + 2\phi_1(0) + \sigma_t^2)$……………………………………..…..(2-34)

if the series is stationary the variance (Z_t)= variance (Z_{t-1}) then

$$\text{var}(z_t) = \gamma_0 = \frac{\sigma_a^2}{1-\phi_1^2},$$ ………………..……………….……….(2-34)

since the variance is positive so $\emptyset_1^2 < 1$ or $|\emptyset_1| < 1$ is the stationary condition

if $\emptyset_1 = \pm 1$ the variance (Z_t) become infinite so the series is not stationary c- Covariance

$\gamma_k = cov(Z_t + Z_{t+k}) = E(Z_t + Z_{t+k})$ k = 1,2,…, $\frac{n}{2}$

$\gamma_1 = E(Z_t + Z_{t+k}) = E[(\emptyset_1 Z_{t+1} + a_{t+1})Z_t)]$

$= \emptyset_1 \mathrm{E}(Z_t^2) + \mathrm{E}(Z_t a_{t+1})$

$= \emptyset_1 \gamma_0 + 0$

$= \emptyset_1 \gamma_0$ ……………………………………………..…………..(2-35)

$\gamma_2 = E(Z_t + Z_{t+2}) = E[(\emptyset_1 Z_{t+1} + a_{t+2})Z_t)]$

$=\emptyset_1 E(Z_t Z_{t+1}) E(Z_t a_{t+2})$

$= \emptyset_1^2 \gamma_0$ ………………………………………………..………(2-36)

In general $\gamma_k = \emptyset_1^k \gamma_0$ k = 1,2,…, $\frac{n}{2}$ ……………..…………..………(2-37)

d- Autocorrelation

$\boldsymbol{\rho_k} = \frac{\boldsymbol{\gamma_k}}{\boldsymbol{\gamma_0}} = \frac{\emptyset_1^k \gamma_0}{\boldsymbol{\gamma_0}} = \emptyset_1^k$ k = 1,2,…, $\frac{n}{2}$ ………………………..….……..(2-38)

2-11-2 Moving Average Model:

In these models the first current value of the time series is expressed as a linear combination of the current and previous errors in the moving average model of order q denoted by $MA(q)$ the current observation Z_t is expressed as a linear combination of the random disturbances going back q periods, its equation can be written as follows:

$Z_t = a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2} - \cdots - \theta_q a_{t-q} \quad t = 1,2,\ldots,q$,…..….......(2-39)

Where$\theta_1 \theta_2, \ldots, \theta_q$are the moving average parameters, it may be positive or negative. The random disturbances $a_t, a_{t-1}, a_{t-2} \ldots, a_{t-q}$ are assumed to be independently normally distributed random variables with mean zero and constant variance σ_t^2. We can also write the model in term of B in deviation from the mean as follows:

$Z_t = (1 - \theta_1 B - \theta_2 B^2 - \cdots - \theta_q B^q) a_t$,……………………………….(2-40)

Or simply

$\widetilde{Z}_t = \theta(B) a_t$,……………………………………………………….(2-41)

Properties of MA(1) model

a- Mean

$\mathrm{E}(Z_t) = \mu_z = E(-\theta_1 a_{t-1} + a_t)$

$= -\theta_1 E(a_{t-1}) + E(a_t)$ ……………………………………………….(2-42)

$= \theta_1(0) + E(0) = 0$

b- Variance

$$Var(Z_t) = E(Z_t - \mu_t)^2 = E(Z_t^2) = E(-\theta_1 a_{t-1} + a_t)^2$$

$= \text{E}(\theta_1{}^2 a^2{}_{t-1} \quad - 2\theta_1 a_t a_{t-1} + a_t^2)$

$= \ \theta_t^2 \ \text{E}(a_{t-1}^2) - 2\phi_1 E(a_t a_{t-1}) + E(a_t^2)$

$= \theta_1^2 \sigma_a^2 + 2\theta_1(0) + \sigma_a^2)$

$= (1+\theta_1^2)\sigma_a^2$ ……………………………………..………(2-43)

c- Covariance

$\gamma_1 = E(Z_t + Z_{t+1}) \ = \ E[(-\theta_1 a_{t-1} + a_t)(-\theta_1 a_t + a_{t+1})]$

$= \text{E}(\theta_1^2 a_t a_{t+1} - \theta_1 a_{t-1} a_{t+1} \ - \theta_1 a_t^2 + a_t a_{t+1})$

$= 0\text{-}0\text{-}\theta_1 \sigma_a^2 + 0$

$= -\theta_1 \sigma_a^2$ …………………………………..………...……..……(2-44)

$\gamma_2 = E(Z_t + Z_{t+2}) \ = \ E[(-\theta_1 a_{t-1} + a_t)(-\theta_1 a_{t+1} + a_{t+2})]$

$= \text{E}(\theta_1^2 a_{t-1} a_{t+1} - \theta_1 a_{t-1} a_{t+2} \ - \theta_1 a_t a_{t+1})$

$= \theta_1^2 E(a_{t-1} a_{t+1}) - \theta_1 E(a_{t-1} a_{t+2}) - \theta_1 E(a_t a_{t+1})$ ……..……(2-45)

0 - 0 - 0 = 0

In general

$\gamma_k = \begin{cases} -\theta_1 \sigma_a^2, & k = 1 \\ 0, & k > 1 \end{cases}$ …………………………………..……....……(2-46) d- Autocorrelation

$$\boldsymbol{\rho_k} = \frac{\gamma_k}{\gamma_0} = \begin{cases} \frac{-\theta_1}{1+\theta_1^2}, & k = 1 \\ 0, & k > 1 \end{cases} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2\text{-}47)$$

2-11-3 Autoregressive Moving Average Model:

A natural extension of the autoregressive and moving average models to combine both models such as mixed process are referred as autoregressive models for order p and q and these denote by ARMA(p,q) which are expressed by

$$Z_t = \phi_1 Z_{t-1} + \cdots + \phi_P Z_{t-P} + e_t - \theta_1 a_{t-1} - \cdots - \theta_q a_{t-q} \quad ,\ldots\ldots\ldots(2\text{-}48)$$

where $\phi_{i\ (i=1,2,\ldots,P)}$ are the autoregressive parameters, $\theta_{j\ (j=1,2,\ldots,q)}$ and a_t is the error term at time t.

Simply the above model can be expressed as follows:

$$\phi(B)\tilde{Z}_t = \theta(B)a_t \ ,\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2\text{-}49)$$

The above model is briefly referred to as $ARMA(p,q)$.

Properties of ARMA(1,1) model

a- Mean

$$\mathrm{E}(Z_t) = E(\phi_1 Z_{t-1} - \theta_1 a_{t-1} + a_t)$$

$$= \phi_1 E(Z_{t-1}) - \theta_1 E(a_{t-1}) + E(a_t) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2\text{-}50)$$

= 0 - 0 + 0 = 0

b- Variance

$$Var(Z_t) = E(Z_t - u_t)^2 = E(Z_t^2) = E(\phi_1 Z_{t-1} - \theta_1 a_{t-1} + a_t)^2$$

$$= \phi_1^2 E(Z_{t-1}^2) + \theta_1^2 \,\mathrm{E}(a_{t-1}^2) + E(a_t^2) - 2\phi_1\theta_1 E(Z_{t-1}a_{t-1}) + 2\phi_1 E(Z_t a_t)$$

$$- 2\theta_1 E(a_t a_{t-1})$$

$$= \phi_1^2\gamma_0 + \theta_1^2\sigma_a^2 + \sigma_a^2 - 2\phi_1\theta_1\sigma_a^2 + 2\phi_1(0) - 2\theta_1(0)$$

$$\gamma_0(1 - \phi_1^2) = (1+\theta_1^2 - 2\phi_1\theta_1)\sigma_a^2$$

$$\gamma_0 = \frac{(1+\theta_1^2-2\phi_1\theta_1)\sigma_a^2}{(1-\phi_1^2)} \quad \text{…………………………………………………..(2-51)}$$

c- Covariance

$$\gamma_0 = E(Z_t Z_{t-1}) = E[(\phi_1 Z_{t-1} - \theta_1 a_{t-1} + a_t)(Z_{t-1})]$$

$$= \phi_1 E(Z_{t-1}^2) - \theta_1 E(Z_{t-1}a_{t-1}) + E(a_t Z_{t-1})$$

$$= \phi_1\gamma_0 - \theta_1\sigma_a^2 + 0$$

$$\gamma_0 = \frac{\phi_1\left(1+\theta_1^2-2\phi_1\theta_1\right)\sigma_a^2-\theta_1(1-\phi_1^2)\ \sigma_a^2}{(1-\phi_1^2)} \text{………………..………………..(2-52)}$$

$$\gamma_1 = \frac{[\phi_1+\phi_1\theta_a^2-2\theta_1\phi_1^2 - \phi_1+\theta_1\phi_1^2)\ \sigma_a^2}{(1-\phi_1^2)}$$

$$= \frac{[(\phi_1-\theta_1)(1-\theta_1\phi_1)]\ \sigma_a^2}{(1-\phi_1^2)} \quad \text{……………………..……………………….(2-53)}$$

$$\gamma_2 = E(Z_t Z_{t-2}) = E[(\phi_1 Z_{t-1} - \theta_1 a_{t-1} + a_t)(Z_{t-2})]$$

$= (\phi_1 E(Z_t Z_{t-2} - \theta_1 E(a_{t-1} Z_{t-2} + E(a_t Z_{t-2})$

$= \phi_1 \gamma_1 - \theta_1 (0) + 0$

$= \phi_1 \gamma_1$ ……………………………......………………………(2-54)

In general

$\gamma_k = \phi_1 \gamma_{k-1}$ k = 2,3,… $\frac{n}{2}$, ……………………………..……..…(2-55)

d- Autocorrelation

$$\boldsymbol{\rho_k} = \frac{\boldsymbol{\gamma_k}}{\boldsymbol{\gamma_0}} = \begin{cases} \frac{(\phi_1-\theta_1)(1-\theta_1\phi_1)}{\mathbf{1}+\theta_1^2-2\phi_1\theta_1}, & k = 1 \\ \phi_1 \boldsymbol{\rho_{k-1}}, & k = 2,3,\ldots\frac{n}{2} \end{cases}$$ ……………..……..(2-56)

2-11-4 Autoregressive Integrated Moving Average Model

The AR, MA and ARMA models assume stationary series. If the time series is nonstationary we can have a model which reflects this fact. This model which is called an ARIMA model and written as Autoregressive Integrated- Moving Average and denoted by $ARIMA(p,d,q)$ represents ARMA model with nonstationarity. In general it takes the form:

$\phi(B)(1-B)^d Z_t = \theta(B) a_t$,………………………………….……(2-57)

where $(1-B)^d$ is the dth order difference.

$\phi(B) \nabla Z_t = \theta(B) a_t$,………………...………………………..…..(2-58)

This is the model that calls for the dth order difference of the time series in order to make it stationary. in ARIMA (p,d,q)

p = order of the autoregressive process,

d= degree of differencing employed,

q= order of moving average process .

In practice the value of p, d and q rarely exceed 2 (they are usually 0 or 1).

2-12 Box-Jenkins Methodology:

In general Box-Jenkins popularized a three-stage method aimed at selecting an appropriate (parsimonious) ARIMA model for the purpose of estimating and forecasting a univariate time series.

Three stages are: (a) identification, (b) estimation, and (c) diagnostic checking..

2-12-1 Identification of the Model:

A comparison of the sample ACF and PACF to those of various theoretical ARIMA processes may suggest several plausible models. If the series is non-stationary the ACF of the series will not die down or show signs of decay at all.

A common stationarity inducing transformation is to take logarithms and then first difference of the series.

Once we have achieved stationarity, the next step is identify the p and q orders of the ARIMA model

Table (2-1)

ARIMA(p,d,q) model	ACF	PACF
AR(1)	Declines gradually	Cuts off to zero after lag 1
AR(2)	Declines gradually	Cuts off to zero after lag 2
MA(1)	Cuts off to zero after lag 1	Declines gradually
MA(2)	Cuts off to zero after lag 2	Declines gradually
ARMA(p,q)	Declines gradually	Declines gradually
AR(p)	Declines gradually	Cuts off to zero after lag p
MA(q)	Cuts off to zero after lag q	Declines gradually

Some time's the autocorrelation and the partial autocorrelation function does not give clear patterns to identify suitable model to time series data, in this case, it is necessary to guess different numbers of ARIMA models and compare them in order to select a suitable and parsimonious model to fit the data using model selecting criteria such as MAE, RMSE, AIC, BIC, and Std Error

2-12-2 Parameters Estimation:

After we identify the model and it is degree then we estimate their parameters they are many methods used in estimation and the most important of them is the maximum likelihood method which is used to estimate the parameters of mixed model ARIMA(p,d,q) here we can describe the cumulative function with stable data is

$L((\phi_1,\theta_1,\sigma_a^2) = (2\pi)^{-\frac{n}{2}}\exp(-\frac{(s(\theta,\phi)^2}{2\sigma_a^2})$ ……………………………..…(2-59)

Where

$S(\theta,\phi)$ = SSE $= \sum_{t=1}^{n} \widehat{a_t}^{\,2} = \sum_{t=1}^{n}(Z_t - \widetilde{Z}_t)^2$

is the sum squares of residuals $\widehat{a_t}$ and $\sigma_a^2 = \frac{SSE}{n}$

The mean sum squares of residuals if we take Ln for two sides of above equation then the equation become as

$LnL((\phi_1,\theta_1,\sigma_a^2) = \frac{-n}{2}Ln(2\pi\sigma_a^2) - \frac{s(\theta,\phi)}{2\sigma_a^2}$ ……………………..………………..(2-60)

When we take the partial derivation for the equation above with respect to σ_a^2,θ,ϕ and the equaling the derivative with zero we can obtain the estimator $\widehat{\sigma_a^2},\widehat{\theta},\widehat{\phi}$

2-12-3 Diagnostic Checking:

In the diagnostic checking stage we examine the goodness of fit of model.

We must be careful here to avoid over fitting. (the procedure of adding another coefficient in appropriate)

Before we using the model to calculate the future forecasting we must check the validity and performance of the model that is done by two methods

a- Akaike's Information Criteria (AIC)

In 1973 Akaike introduced and information criteria which used to identify the best model to the data this criteria define as

AIC(k) = n Lnσ_a^2+2k ……………………………………………….(2-61)

Where k is full order of model (k = p+d+q), σ_a^2 is the error variance of the model and n is the number of original observations, then compute AIC for each model and select the best model which has the minimum AIC(k)

b- Residual Analysis

In this method firstly we must compute the estimated error from the ARIMA model after identification and estimation parameters i.e$\widehat{a_t} = Z_t - \widehat{Z_t}$ then we compute the autocorrelations coefficients for the residuals as

$r_k = \frac{\sum_{t-1}^{n} \hat{a}_t \hat{a}_{t-1}}{\sum_{t-1}^{n} a_t^2}$ ………………………………………………………(2-62)

The special statistic that we use here are the Box-Piece statistic (BP)were proved that in 1970 the autocorrelation coefficient for residuals is normally distributed with mean zero and variance $\frac{1}{n}$ where n is the size of sample then

$$Q_{BP} = n\sum_{k=1}^{m} {r_t}^2(\hat{a_t}) \quad \text{……………………………………………..……..(2-63)}$$

The value of Q_{BP} is compared with the value of $\mathcal{X}^2$ that is proceeded by an area $(1-\alpha\ )$ under $\mathcal{X}^2$ distribution with m degrees of freedom. We conclude randomness if Q_{BP} is less than this value.

Also there is a possibility to use Ljung – Box test statistic which takes the form

$$Q_{LB} = n(n+2)\sum_{k=1}^{m} \frac{{r_t}^2(\hat{a_t})}{(n-m)} \text{……………………………………………(2-64)}$$

Under the null hypothesis of no autocorrelation, $Q_{LB} \sim \mathcal{X}^2_{m,\alpha}$distribution. Using this distribution, the null hypothesis is rejected if the calculated $Q_{LB} > \mathcal{X}^2_{m,\alpha}$ at α significant level, which implies that the model is insufficient or could perhaps, be miss specified or not suitable for the data. In general if an estimate seems to be not significantly different from zero its corresponding parameter may be dropped from the model. Small standard errors are indication of stability and precision of the estimate.

2-13 Forecasting:

Forecasting is very important in time series so after reaching the fit model of series process, we can use this model to forecast the series observation in the future in this section we study how to use ARIMA models in forecasting we assume that n refers to current period of time about which we compute the forecasting we want to forecast the value Z_{n+h} that does not happen yet where h is called forecasting horizon Z_n(h) refers to forecasting value we get in n time period forZ_{n+1} observation which happens after h time periods we try to find a method of forecasting with point and explain how to construct forecasting interval about this point as we said before Z_n(h) refers to forecasting value we get in n time period for Z_{n+1}observation which happens at n+h time period so we can define the forecasting error by

a_n(h) = $Z_{n+h} - Z_n$(h) ……………………………………………………..(2-65)

And the requirement is to find small value for expected square errors therefore

E(a_n(h))² =E($Z_{n+h} - Z_n$(h)$)^2$ …………………..……..………………(2.66)

The above equation checks the good forecasting which has minimum expected square error.

2-13-1 Forecasting with Minimum Mean Square Errors:

We explain that the arithmetic mean for forecasting distribution makes the expected value of mean square errors as minimum, that means there is not other forecasting leads to minimum expected of mean square errors than arithmetic mean we assume that m_h is the expected value Z_{n+h}that we are forecasting n period i.e

$m_h = E(Z_{n+h})$…………………………………………………..…(2.67).

Suppose that m is another forecasting to Z_{n+h} such that m $= m_h + d$ where d refers to the difference between m and m_h by using forecasting point m , we find the expected value of forecasting square errors as:

$E(Z_{n+h} - m)^2 = E(Z_{n+h} - m_h + d)^2$ ……………………………..….(2-68).

We can rewrite the right side of above equation then the equation becomes

$E(Z_{n+h} - m)^2 = E(Z_{n+h} - m_h)^2 - 2d\,E(Z_{n+h} - m_h) + d^2$……..….(2-69).

From the above equation the quantity $2d\ E(Z_{n+h} - m_h)$ equals to zero according to equation (2.60), d^2 the non-negative quantity and when d=0 the above equation should be minimum and the quantity $E(Z_{n+h} - m_h)^2$ is the mean of forecasting of square errors and m = $m_h = E(Z_{n+h})$ is a good forecasting with Z_{n+h} value because the mean of forecasting square error corresponding to it minimum and enough we can compute the mean to distribute forecasting which is $E(a_{n+j})$ as follows: Assume that Z_t is ARMA(p,q) stationary process and we describe this process according equation in the time period t=n+h has the flowing

$$\mathbf{Z_{n+h} = \phi_1 Z_{n+h-1} + \cdots + \phi_P Z_{n+h-P} + e_{n+h} - \theta_1 a_{n+h-1} - \cdots - \theta_q a_{n+h-q}} \text{ ,……}(2\text{-}70)$$

So we can estimate the expected value for the variable Z_{n+h} in above equation by using the variable information until n period as below:

1- Replace the previous and current error a_{n-j} for all value j≥0 by real residuals i.e. E $(a_{n+j}) = a_{n-j}$ j = 0,1,2,…

2- Replace the coming error a_{n+j} where $0< j \leq h$ which does not happen by expected value $(a_{n+j}) = 0$

3- Replace the coming observation Z_{n+j} where $0 < j < h$ with it forecasting value $Z_n(j)$ i.e. $E(Z_{n-j}) = Z_n(j)$ j = 1,2,…

4- Replace the previous and current observations Z_{n-j} for all values j≥0 with real value E $(Z_{n-j}) = Z_{n-j}$ j = 1,2,…

2-13-2 Forecasting and Model forecasting Intervals:

In addition to finding a good forecasting point we may want to measure uncertainly about this point so we find the standard error for forecasting error and construct forecasting interval. To compute the standard errors for forecasting error we firstly express ARMA process with respect to random variables in the period t=n+h then

$$\mathbf{Z_{n+h}} = \mathbf{a_{n+h}} + \boldsymbol{\varphi}_1 \mathbf{a_{n+h-1}} + \cdots + \boldsymbol{\varphi}_{\mathbf{h-1}} \mathbf{a_{n+1}} + \boldsymbol{\varphi}_h \mathbf{a_h} + \boldsymbol{\varphi}_{\mathbf{h+1}} \mathbf{a_{h-1}} \quad ,\ldots\ldots\ldots\ldots\ldots(2\text{-}71)$$

Where $\boldsymbol{\varphi_1 \varphi_2}$… are the memory coefficients which its value depends on the kind of ARIMA models used .We can use the equation (2.64) to fit a good forecasting $Z_n(h)$ using previous and current residuals that for

$$Z_n(h) = \varphi_h a_h + \varphi_{h+1} a_{h+1} + \cdots , \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2\text{-}71)$$

From equation (2.64) and depending on equation (2.70) and equation (2.71) the forecasting error for h coming period is expressed by:

$\mathbf{a_n(h) = a_{n+h} + \varphi_1 a_{n+h-1} + \cdots + \varphi_{h-1} a_{n+1}}$ ……………………………………(2-72)

From above equation we conclude that $\mathbf{a_n(h)} \sim MA(h-1)$ and regardless conclude that the forecasting errors for one period (First step) is

$a_n(h) = a_{n+1}$ ………………………………………………………......…..(2-73)

The expected value of forecasting error is

$\mathbf{E[a_n(h)] = E[a_{n+h} + \varphi_1 a_{n+h-1} + \cdots + \varphi_{h-1} a_{n+1}]} = 0$ ……………………….….(2-74)

And the variance of forecasting errors is

$\mathbf{Var(a_n(h)) = E[a_n(h)] = E[a^2_{n+h} + \varphi^2_1 a^2_{n+h-1} + \cdots + \varphi^2_{h-1} a^2_{n+1}]}$ ………..(2-75)

$= (1 + \varphi_1^2 + \varphi_2^2 + \cdots + \varphi_{n-1}^2)\ \sigma_a^2$

$= \sigma_a^2 (1 + \sum_{j=1}^{h-1} \varphi_j^2)$

$= \sigma_a^2 \sum_{j=0}^{h-1} \varphi_j^2$ …………………………………………(2-78)

From above equation we observe that the variance of forecasting error did not decreasing by increasing of forecasting horizon h where

$\mathbf{Var(a_n(h)) - Var(a_n(h-1))} = \sigma_a^2 \sum_{j=0}^{h-1} \varphi_j^2 \geq 0$ …………………………..(2-79)

If we assume that the random variable a_t distributed normally we can determine that Z_{n+h} is distributed normally with mean $Z_n(h)$ and var$[a_n(h)]$. The forecasting interval for the value Z_{n+h} with 95% confidence level for the large sample

$Z_n(h) \pm 1.96$ SE$[a_n(h)]$……………………………………………...….(2-80)

Where SE$[a_n(h)]$is the standard error of forecasting error

2-13-3 Forecasting of MA (1) model:

To forecast for MA(1) model we can use as:

$X_{n+h} = u + a_{n+h} - \theta_1 a_{n+h-1}$………………………………..…………..(2-81)

If h=1 $X_{n+1} = u + a_{n+1} - \theta_1 a_n$

We observe that the value of a_{n+1}which happen is n+1 period is unknown in n periods so we replaced $a_{n+1}$by it mean zero when used in n periods.

The forecasting for one period (h=1)

$X_n(1) = E(X_{n+1}) = E(u + a_{n+1} - \theta_1 a_n) = u - \theta_1 a_n$ ……….………(2-82)

The forecast for two periods (h=2) is

$X_n(2) = E(X_{n+2}) = E(u + a_{n+2} - \theta_1 a_{n+1}) = u$ ………………….…(2-83)

In general the forecasting for MA(1) is

$$X_n(h) = \begin{cases} u - \theta_1 a_n, & h = 1 \\ u, & h \geq 2 \end{cases} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2\text{-}84)$$

The variance for forecasting errors interval of MA(1) express as:

$$X_n(h) \pm 1.96\, \sigma_a \sqrt{1 + \theta^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2\text{-}85)$$

2-13-4 Forecasting of AR (1) model:

The forecasting of AR(1)model we can use as:

$$X_{n+h} = \delta + \phi_1 X_{n+h-1} + a_{n+h} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2\text{-}86)$$

The forecast for one period is

$$X_n(1) = E(X_{n+1}) = E(\delta + \phi_1 X_n + a_{n+1}) = \delta + \phi_1 X_n \ldots\ldots\ldots\ldots (2\text{-}87)$$

The forecast for two periods is

$$X_n(2) = E(X_{n+2}) = E(\delta + \phi_1 X_{n+1} + a_{n+2}) = \delta + X_n(1) \ldots\ldots\ldots (2\text{-}88)$$

In general the forecasting for AR(1) is

$$X_n(h) = \begin{cases} \delta + \phi_1 X_n, & h = 1 \\ \delta + \phi_1 X_n(h-1), & h \geq 2 \end{cases} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2\text{-}89)$$

The forecasting interval of AR(1) model is express as:

$$\boldsymbol{X_n(h) \pm 1.96\, \sigma_a \sqrt{1 + \phi_1{}^2 + \phi_1^4 + \cdots \phi_1^{2(h-1)}}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2\text{-}90)$$

2-13-5 Forecasting of ARMA (1,1) model:

The forecasting of ARMA(1,1)model is express as:

$$X_{n+h} = \delta + \phi_1 X_{n+h-1} - \theta_1 a_{n+h-1} + a_{n+h} \text{..............................(2.91)}$$

The forecast for one period is

$$X_n(1) = E(X_{n+1}) = E(\delta + \phi_1 X_{n+1} - \theta_1 a_{n+1} + a_{n+1} \text{................(2.92)}$$

$$= \delta + \phi_1 X_n - \theta_1 a_n$$

The forecast for two periods is

$$X_n(2) = E(X_{n+2}) = E(\delta + \phi_1 X_{n+2} - \theta_1 a_{n+2} + a_{n+2} \text{.................(2.93)}$$

$$= \delta + \phi_1 X_n(1)$$

In general the forecasting for ARMA(1,1) model is

$$X_n(h) = \begin{cases} \delta + \phi_1 X_n - \theta_1 a_n, & h = 1 \\ \delta + \phi_1 X_n(h-1), & h \geq 2 \end{cases} \quad \text{...................................(2.94)}$$

We note that $X_n(1)$which is directly influenced by previous errors.

The forecasting interval of ARMA(1,1) express as:

$$\boldsymbol{X_n(h) \pm 1.96\, \sigma_a \sqrt{1 + (\phi_1 - \theta_1)^2)(\phi_1^2 + \phi_1^4 + \cdots \phi_1^{2(h-1)})}} \text{..................(2.95)}$$

CHAPTER THREE
ARCH/GARCH MODELS

3-1 Introduction

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3-1 Introduction:

The volatility is the measurement of variation among the prices of financial time series data. Accurate forecasting volatility is an important tool for investors to make the right investment decisions and also helps researchers to better understand the change in the financial market. Time series models are the main methodologies for forecasting volatility in financial data. Some conventional time series models are based on the assumption of homoscedasticity, which means the variance of error terms of expected values remain the same at any given time. However, under the real circumstances, the variance of error terms actually varies all the time, which implies that heteroskedasticity, exists in the data. In order to capture more accurate forecasting results, Robert F Engle (1982) proposed the Autoregressive Conditional Heteroskedasticity (ARCH) model which states that the variance in the data at time t depends on the previous time t-1. Tim Bollerslev (1986) generalized the ARCH model and named the model as the Generalized ARCH (GARCH) model which allows for a more flexible lag structure and it bears much resemblance to the extension of the standard times series autoregressive (AR) process to the general autoregressive moving average (ARMA) process.

During the last two decades, the ARCH and GARCH models have been the most popular methods for the researchers, analysts, and investors to forecast volatility.

Moreover, some scholars have also developed variant forms of the GARCH model. For example, Nelson (1991) proposed the exponential GARCH (EGARCH) model, and Engle and Bollerslev (1986) proposed the Integrated GARCH (IGARCH) model. Ding, Ganger, and Engle (1993) first mentioned the Power GARCH (PGARCH) model, while Glosten, Jaganathan and Runkle (1993) and Zakoian (1994) and developed the Threshold GARHC (TGARCH) model. Lastly, Engle and Ng (1993) first proposed the Quadratic GARCH (QGARCH) model.

Estimation of the parameters in the above models is mostly based on the Likelihood approach.Ardia (2007) used the Bayesians approach to address parameter estimation in the GARCH model.

In this research, we consider the traditional GARCH model, as well as other extensions of the GARCH model. Both the likelihood and the Bayesian approaches for model fitting are considered. In what follows, we briefly present these models and discuss the estimation methods using both likelihood and Bayesians approaches

3-2 Financial Time Series Characteristics:

3-2-1 Volatility

Volatility is a measure of the dispersion in a probability density. The variance is a measure of the dispersion of the density function around its mean. The standard deviation, σ, which is the square root of the variance, is the most common measure of dispersion for a random variable (Alexander, 2001), as it is measured in the same units as the original data (Sheppard, 2009a).

Volatility is a key parameter used in many financial applications. It measures the size of the errors made in modeling returns and other financial variables. It is very hard to predict it correctly and consistently. Forecasting volatility is an important area of research in financial markets. ARCH, GARCH and stochastic volatility models are the main tools to model and forecast volatility. There were a lot of effort exerted to improve volatility models, since better forecasts is translated in better pricing of options and better risk management.

3-2-2 A platykurtic

A platykurtic means that the distribution has a kurtosis value less than that of a standard normal distribution. This type of distribution has a fat midrange on either side of the mean and a low peak.

3-2-3 A leptokurtic

A leptokurtic means that the distribution has a kurtosis value greater than that of a standard, normal distribution which gives the distribution a high peak, a thin midrange and fat (heavy) tails.

3-2-4 Mesokurtic Distributions

Mesokurtic distributions means that the distribution has a kurtosis value equals to that of standard normal distribution.

3-2-5 Volatility Clustering

It is as well known fact that financial market volatility tends to cluster. This means that volatile periods tend to persist for some time before the market returns to the normality (Poon, 2005). Mandelbrot (1963, p.418) for example points out that “large changes tend to be followed by large changes - of either sign - and small changes tend to be followed by small changes, This effect can visually be seen when plotting a series of returns through time. A plot of the returns, together with statistical tests, show that financial returns are not independently identically distributed through time (Bollerslev et al., 1993). The positive and negative disturbances given by the time-to-time changes become a part of the information set used to construct variance forecasts for the coming period. This means that large shocks of either sign can have an influence on the forecasts for several periods to come. When the clustering is significant, the time series is said to display autoregressive conditional heteroskedasticity (Alexander, 2001). The effect becomes more pronounced the higher the frequency of the sample data is. The consequence of volatility clustering is that future volatility can be predicted by past and current volatility.

Rob Engle’s (1982) ARCH model, which will be described later, captures this kind of volatility persistence. There is a close relationship between clustering and thick tails. The volatility clustering is a type of heteroskedasticity and accounts for some of the excess kurtosis typically observed in the distribution of a financial time series. Another part of the excess kurtosis can be due to the presence of a non-normal asset distribution, e.g. the Student's t distribution, which happens to have fat tails.

3-2-6 Leverage Effects

The leverage effect refers to the tendency of volatility to increase if the previous days returns are negative. (Bollerslev et al., 1993) indicated that, changes in stock prices are negatively correlated with changes in stock volatility. A fall in stock price causes leverage and financial risk of a firm with outstanding debt and equity to increase. For time series exhibiting leverage effects, asymmetric GARCH models should be applied because the asymmetry cannot be captured by symmetric GARCH models. Asymmetric GARCH models will be presented later.

3-2-7 Long Memory

Long memory in volatility occurs when the effects of volatility shocks decay slowly, which is often detected by the autocorrelation of measures of volatility. The practical explanation is that historical event has a long and lasting effect. Fama & French (1988) and Poterba & Summers (1988) discovered positive correlation in short term and negative correlation in long term of stock returns. The significance of the phenomenon is that the existence of "long memory" enables to predict the returns.

3-2-8 Thick Tails

Mandelbrot (1963) and Fama (1965) both document the fact that asset returns tend to be leptokurtic, i.e. the time series of returns exhibit fatter tails than a normal (Gaussian) distribution. A normal distribution has a skewness equal to zero and a kurtosis equal to three. Mandelbrot (1963, p.394) finds that "the empirical distributions of price changes are usually too 'peaked' to be relative to samples from Gaussian populations". The kurtosis of a time-series measures the tail thickness. Excess kurtosis, that is kurtosis above 3, implies that the distribution has a sharper peak and fatter tails than a normal distribution. On the other hand, a low kurtosis implies that the distribution has a rounder peak and shorter, thinner tails.

A negative skewness, for instance, tells us that the distribution will have a longer left tail than a right tail. In other words, a negative skewness indicates extreme losses, while a positive skewness indicates extreme gains. The kurtosis and skewness are very sensitive to outliers in the time-series. By removing extreme outliers, both the kurtosis and the skewness will drop significantly (Poon, 2005).

On being accurate about forecasting asset price return volatility.

3-3 Returns Model:

Since the first decades of the 20th century, asset returns have been assumed to form an independently and identically distributed (i.i.d) random process with zero mean and constant variance. Bachelier (1900) was the first to contribute to the theoretical random walk model for the analysis of speculative prices. For x_t $(t = 1,2,3, \dots.)$ denoting discrete time series and r_t $(t = 1,2,3, \dots \dots)$ denoting the process of the continuously compounded returns, defined by

$$r_t = \log\left(\frac{x_t}{x_{t-1}}\right) = \log x_t - \log x_{t-1}, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.1)$$

the early literature viewed the system that generates the asset price process as a fully unpredictable random walk process:

$$x_t = x_{t-1} + \varepsilon_t \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.2)$$

$\varepsilon_t \sim N(0, \sigma^2)$ Where ε_t has a zero-mean and i.i.d. normal distribution. However, the assumptions of normality, independence and homoscedasticity do not always hold with real data.

It is assumes that for the return indexes which follow a martingale process, given by the following equation:

$$r_t = \mu + \varepsilon_t \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.3)$$

Where μ the mean value of the return ε_t is a random component of the model, not autocorrelation in time, with zero mean value. Further more ε_t may be considerd as stochastic process. To sum up, the return in the present will be equal to the mean value of r_t i.e. the expected value of r_t based on past information, plus the veriance of the error term.

3-4 Measures of Skewness and Kurtosis:

3-4-1 Skewness

Observations of the empirical distribution of $x_t$often show that the distribution is leptokurtic. Another property that deviates from the so often assumed Gaussian distribution is that the empirical distribution is not symmetric. Skewness defines the degree of asymmetry of a distribution and several types of skewness are defined. The Fisher skewness(the most common type of skewness, usually referred to simply as skewness) is defined by:

Let ε_t, $t = 1,2, \dots \dots, T$, be a set of independent and identically distributed random samples with mean μ, median M and variance σ^2. The classical estimates of skewness SK, and Kurtosis KR are given as follows:

$$sk = \frac{1}{T}\sum_{t=1}^{T}(\frac{\varepsilon_t-\mu}{\sigma})^3 \quad \text{…(3.4)}$$

Positive skewness indicates a long right tail, negative skewness indicates long left tail and zero skewness indicates a symmetry around the mean.

3-4-2 Kurtosis

The observations of the time series p_t have a distribution, which often is assumed to be normal (Gaussian) distribution. However, empirical studies of practically any financial time series show that this is not quite correct. One way to quantify this property is to look at the kurtosis of the distributions. Kurtosis is a measure of the extent to which observed data fall near the centre of a distribution or in the tails:

$$KR = \frac{1}{T}\sum_{T}^{T}(\frac{\varepsilon_t-\mu}{\sigma})^4 \quad \text{…(3.5)}$$

The kurtosis for the normal distribution is three, positive excess kurtosis indicates flatness (long, fat tails) and negative excess kurtosis indicates peakedness

where

$$\mu = \frac{1}{T}\sum_{t=1}^{T}\varepsilon_i \quad \text{…(3.6)}$$

and

$\sigma^2 = \frac{1}{T}\sum_{t=1}^{T}(\varepsilon_t - \mu)^2$……………………………………………………(3.7)

3-4-3 The Jarque–Bera (JB*)* Test Statistic

Let $\varepsilon_t, t = 1,2, \ldots \ldots, T,$ be samples randomly selected from a Gaussian distribution. Using the above notation for skewness and kurtosis, the Jarque-Bera (JB) test statistic is expressed as follows:

$\mathrm{JB} = \frac{\mathrm{T}}{6}\mathrm{SK}^2 + \frac{\mathrm{T}}{24}\mathrm{KR}^2$…………………………………………………..(3.8)

where T is the number of observations. Under the null hypothesis of independent normally distributed random variable, the Jarque–Bera (JB) test statistic is distributed as a chi-square distribution with 2 degrees of freedom in large samples.

3-5 Mean and Variance Equation:

The mean equation can be written as a function of exogenous variables with an error term. Since σ_t^2 the one-period ahead forecast variance based on past information, it is called the conditional variance. The conditional variance equation specified as a function of three terms these are:

A constant term ω, news about volatility from the previous period, measured as the lag of the squared residual from ε_{t-1}^2 the mean equation (the ARCH term) and last period's forecast variance σ_{t-1}^2 (the GARCH term).

The mean equation can be written as a function of exogenous variables with an error term. For the univariate time series data x_t the mean equation can be described by the process:

$p_t = E\ (x_t|\Omega_{t-1}) + \varepsilon_t$,…………………………………………………..(3.9)

Where$E(.|.)$ denote the conditional expectation operator, Ω_{t-1} the information set at time $t-1$ and ε_t the residuals of time series, it describes uncorrelated disturbances with zero mean and plays the role of the unpredictable part of the time series. In this research the mean equation can be model as one of the above discussed time series ARIMA model:

3-6 The volatility Models: Generalized Autoregressive Conditional Heteroskedasticity Models Family:

Over the years, the GARCH family has become more efficient in fitting the volatility data. They consist of the second order moment that measures the time-variant of the volatility data. The initial studies by Engle (1982) and Bollerslev (1986) turn out to be the better models for volatility (financial) data as the residuals of the data form fatter tailed. The maximum likelihood estimation (MLE), is a natural approach to employ, when the standardized residual is normal distributed Bollerslev and Wooldridge (1992), Horvath and Liese (2004) and many more advocated that the linear model of the conditional variance has its limitation and the GARCH itself may fail to fit some financial data especially in high frequency data. This leads to empirical findings that indicate the weakness of imposing ordinary GARCH model; subsequent development and modification of GARCH include the following: Nelson (1990) found that EGARCH the conditional variance being exponentially distributed, Engle et al (1987) with their ARCH-M, Engle and Rivera (1991) with semi parametric ARCH, Engle and Bollerslev (1986) with Integrated GARCH (IGARCH), Engle et al (1990) with factor-ARCH, Baillie et al (1996) with Fractionally GARCH (FIGARCH) and Bollerslev and Ghysels (1996) with Periodic ARCH. All these found that GARCH family has good fit for many econometric data and this tool is now widely used to explain some current economic situation. The most popular financial economic data that have been considered in various studies are inflation uncertainty, stock returns, and exchange rates. Several models with various assumption of distributions and techniques of estimates of parameters have been introduced.

The properties of ordinary linear GARCH family models and its method of parameter estimation are discussed. The properties of GARCH models can be found in Engle (1982), Bollerslev (1986), Weiss (1986) and Hamilton (1994). The use of these models in analyzing volatility in time series data can be referred to Zivot and Wang (2001).

Engle (1982) and Bollerslev (1986) provide a detail account on the method of maximum likelihood of estimation (MLE) for ordinary ARCH and GARCH parameters respectively. Bollerslev (1986) and Fiorentini et al (1996) employ the Berndt, Hall, Hall, and Hausman (BHHH) algorithm introduced by Berndt et al (1974), to speed up the iterative part so that convergence of the objective function can be achieved in less iteration.

Empirically, a wide range of financial and economic phenomena exhibit the clustering of volatilities. A variety of volatility models are used in financial time series models, among these ARCH / GARCH framework proved to be very successful in predicting volatility changes. The ARCH-type models used in this research are defined in terms of the distribution of a dynamic linear regression model.

In this section a brief review of heteroskedasticity models will be considered.

3-6-1 Autoregressive Conditional Heteroskedasticity Models (ARCH models):

One of the earliest time series models for heteroskedasticity is the Autoregressive Conditional Heteroskedasticity (ARCH) models. ARCH models are specifically designed to model and forecast conditional variances. To generate the autoregressive conditional heteroskedasticity process the conditional variance of the error term is expressed as a function of its past values squared as follows:

$$\varepsilon_t|\Omega_{t-1} \sim N(0, h_t) \quad \text{…(3.10)}$$

$$\varepsilon_t = \eta_t\sqrt{h_t} \quad \text{…(3.11)}$$

$$h_t^2 = \delta + \sum_{i=1}^{p} \alpha_i\, \varepsilon_{t-i}^2 \quad \text{…(3.12)}$$

Where ε_t is the unconditional shock, η_t is an independently identically distribution random variable (conditional) shock with mean zero and variance 1, and h_t^2 denotes the conditional variance of the information set Ω_{t-1},and $\boldsymbol{\delta} > 0, \alpha_i \geq 0$ for all $i = 2,3, \dots \dots, p$ and $\alpha_1 + \alpha_2 + \cdots + \alpha_p < 1$ are necessary to make ε_t^2 positive and covariance stationary.

3-6-1-1 Properties of ARCH Models

A simple form of autoregressive conditional heteroskedasticity model is ARCH which takes the form:

$\sigma^2 = \omega + \alpha_1 \varepsilon_{t-1}^2$,…………………………..…………………………..……(3.13)

Where $\omega > 0$, $\alpha_1 \geq 0$

First, the unconditional mean:

E (ε_t) = E [E ($\varepsilon_t | \Omega_{t-1}$)] = E [$\sigma_t$E($\eta_t$)] = 0,………………..…..……..…..(3.14)

Secondly, the unconditional variance obtained as:

Var (ε_t**)** = E (ε_t^2) = E [E ($\varepsilon_t^2 | \Omega_{t-1}$)] ,…………………....…………..……(3.15)

= E ($\omega + \alpha_1 \varepsilon_{t-1}^2$) =$\omega + \alpha_1$E($\varepsilon_{t-1}^2$). ,……………………..…….(3.16)

Thirdly, the unconditional kurtosis:

In some applications, higher order moments of ε_t is needed, for instance, to study its tail behavior, the fourth moment of ε_t is required. To obtain that:

$E(\varepsilon_t^4 | \Omega_{t-1}) = 3[E(\varepsilon_t^2 | \Omega_{t-1})]^2$,……………………………………….(3.17)

Therefore,

$E(\varepsilon_t^4) = E[E(\varepsilon_t^4|\Omega_{t-1})] = 3E(\omega + \alpha_1\varepsilon_{t-1}^2)^2 = 3E[\omega^2 + 2\omega\alpha_1\varepsilon_{t-1}^2 + \omega + \alpha_1^2\varepsilon_{t-1}^4]$,…………………..…………………………………………(3.18)

By substituting $m_4 = E(\varepsilon_t^4)$ in the above equation gives:

$m_4 = 3[\omega^2 + 2\omega\alpha_1 \text{var}(\varepsilon_t) + \alpha_1^2 m_4]$,………………..……………..…(3.19)

$= 3\omega^2\left(1 + 2\frac{\alpha_1}{1-\alpha_1}\right) + 3\alpha_1^2 m_4$,……………………………....……...…(3.20)

Consequently,

$m_4 = \frac{3\omega^2(1+\alpha_1)}{(1-\alpha_1)(1-3\alpha_1^2)}$,…………………………..…………………….………(3.21)

Since the fourth moment of ε_t is positive, so α_1 must satisfy the condition:

$1 - 3\alpha_1^2 > 0$,………………………………..………..……….…………(3.22)

that is: $0 \leq \alpha_1^2 < 1/3$.

The unconditional kurtosis of ε_t is then:

$\frac{E(\varepsilon_t^4)}{[\text{var}(\varepsilon_t)]^2} = 3\frac{\omega^2(1+\alpha_1)}{(1-\alpha_1)(1-3\alpha_1^2)} * \frac{(1-\alpha_1)^2}{\omega^2} = 3\frac{1-\alpha_1^2}{1-3\alpha_1^2} > 3$,………………..……(3.23)

Thus, the excess kurtosis of ε_t is positive and the tail distribution of ε_t is heavier than that of normal distribution.

3-6-2 Generalized Autoregressive Conditional Heteroskedasticity Models (GARCH models):

Bollerslev (1986) proposed a useful extension known as generalized ARCH (GARCH) process. In GARCH model the conditional variance of return series is expressed as a function of constant, past news about volatility (ε_{t-i}^2) terms and past forecast variance (h_{t-i}^2) terms. In the GARCH (p,q) the conditional variance is expressed as follows:

$\varepsilon_t = \eta_t\sqrt{h_t}$,……………………………..……………………..(3.24)

$h_t^2 = \delta + \sum_{i=1}^{p} \alpha_i\, \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j\, h_{t-j}^2$,……………………..………………..…(3.25)

Where η_t is independently identicaly distrebuted random variable with mean zero and variance 1, $\omega > 0$, $\alpha_i \geq 0$, $\beta_j \geq 0$ and $\sum_{i=1}^{\max(p,q)}(\alpha_i + \beta_j) < 1$

3-6-2-1 Properties of GARCH Models

Firstly, the unconditional mean:

E (ε_t) = E [E ($\varepsilon_t|\Omega_{t-1}$)] = E [$\sigma_t E(\eta_t)$] = 0 ,……………….…………….…(3.26)

Secondly, the unconditional variance obtained as:

Var ($\boldsymbol{\varepsilon_t}$) = E ($\varepsilon_t^2$) = E [E ($\varepsilon_t^2|\Omega_{t-1}$)] ,……………………..……..……..(3.27)

= E ($\omega + \alpha_1\varepsilon_{t-1}^2$) =$\omega + \alpha_1 E(\varepsilon_{t-1}^2)$,……………………..……..(3.28)

Since Var ($\boldsymbol{\varepsilon_t}$) = E ($\varepsilon_t^2$)

Therefore

In the GARCH (1.1) model it's found that:

$\text{Var}(\varepsilon_t) = \frac{\omega}{1-\alpha_1-\beta_1}$,………………………..……………………..……….(3.29)

Thirdly, the unconditional kurtosis:

In some applications, higher order moments of ε_t is needed, for instance, to study its tail behavior, the fourth moment of ε_t is required. To obtain that:

$\frac{E(\varepsilon_t^4)}{[\text{var}(\varepsilon_t)]^2} = \frac{3[1-(\alpha_1+\beta_1)^2]}{1-(\alpha_1+\beta_1)^2-2\alpha_1^2} > 3$,…………………………….……..…….(3.30) consequently, similar to ARCH models, the tail distribution of a GARCH (1,1) model is heavier than that of a normal distribution.

The above properties continue to hold for all ARCH/GARCH family models however, the formulas become more complicated for higher order of these models.

3-6-3 The Threshold GARCH (TGARCH) Model:

Another volatility model commonly used to handle leverage effects in the TARCH or Threshold ARCH and Threshold GARCH were introduced independently by Zakoïan (1994) and Glosten, Jaganathan, and Runkle (1993). The generalized specification for the conditional variance can be express as:

$$h_t = \delta + \sum_{i=1}^{p} \alpha_i \, \varepsilon_{t-i}^2 + \sum_{i=1}^{p} \gamma_i \, d_{t-i} \varepsilon_{t-1}^2 + \sum_{i=1}^{q} \beta_i \, h_{t-j} \text{ ,……..………(3.31)}$$

Where $d_i = 1$ if $\varepsilon_t < 0$ and $d_i = 0$ otherwise.

Adverse market conditions and bad news ($\varepsilon_{t-1}^2 < 0$) such as frost, drought, or political instability has an impact of (α+γ). Good news about the demand and supply conditions in the commodity market ($\varepsilon_{t-1}^2 > 0$) has an impact of α.

3-6-4 The Exponential GARCH (EGARCH) Model:

EGARCH model is one of the asymmetric models which is developed by Nelson (1991). The EGARCH (*p, q*) models the effect of recent residuals is exponential rather than quadratic. The variance equation of this model can be expressed as follows:

$$\text{Log}\ (h_t^2) = \delta + \pi_1 \left| \frac{\varepsilon_t}{\sqrt{h_{t-i}^2}} \right| + \pi_2 \frac{\varepsilon_{t-1}}{h_{t-1}^2} + \beta\ \log\ (h_{t-1}^2)\ ,\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.32)$$

A symmetry is achived when $\pi_2 \neq 0$. The impact of good news such as new market infrastructure is captured by ($\frac{\pi_1+\pi_2}{\sqrt{h_{t-1}^2}}$) while the impact of bad news such as political stabilities or unfavorable weather is expressed by ($\frac{\pi_1-\pi_2}{\sqrt{h_{t-1}^2}}$). A negative and significant π_2 is an evidence of a symmetry and greater impact of negative shocks on price volatility.

3-6-5 The Integrated GARCH (IGARCH) Models:

If the polynomial of the GARCH model has a unit root, then there is an IGARCH models. A key feature of IGARCH is that the impact of past squared shocks

$$\eta_t = \varepsilon_{t-i}^2 - h_{t-j}^2 \text{for i > 0.,}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.33)$$

The IGARCH phenomenon might be caused by occasional level shifts in volatility. The actual cause of persistence in volatility deserves a careful investigation.

3-6-6 GARCH-in-Mean (GARCH-M) Model:

In finance, the return of a securaty may depend on its volatility. To model such a phenomenon, GARCH-M model developed by Engle,

Lilien, and Robins (1987), where "M" stands for GARCH in the mean. This model is an extension of the basic GARCH framework which allows the conditional mean of a sequence to depends on its conditional variance or standard deviation. A simple GARCH (1,1) -M model can be written as:

$r_t = \mu + ch_t^2 + \varepsilon_t$,……………..…………………………………..…….(3.34)

$h_t^2 = \delta + \boldsymbol{\alpha}\varepsilon_{t-1}^2 + \beta h_{t-1}^2$,……………………..……………..…………(3.35)

Where μ, c and δ are constants. The parameter c is called the risk premium parameter. Apositive c indicate that the return is positively related to its volatility. Other specification of risk premium have also been used in the literature, including

$r_t = \mu + ch_t^2 + \varepsilon_t$,…………………………..…………………….……..(3.36)

And also

$r_t = \mu + c\ \ln h_t^2 + \varepsilon_t$,…………………………………………………..……(3.37)

The formulation of GARCH-M model in the above equation implies that there are serial correlations in the return series r_t. These serial correlations are introduced by those in the volatility process $\{h_t^2\}$.

3-6-7 Glaston, Jagannathan and Runkle Generalized Autoregressive Conditional Heteroskedasticity (GJR models):

This model is known as GJR GARCH models, proposed by Glaston, Jagannathan &Runkle (1993), are capable of capturing the symmetric effect in regard to the conditional volatility. The variance equation in the GJR (p,q) model is specified as follows:

$h_t^2 = \omega + \sum_{i=1}^{p} \alpha_i\, \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j\, h_{t-j}^2 + \gamma \varepsilon_{t-i}^2 I_{t-i}$,…………..……….(3.38)

where $\omega > 0, \quad \alpha_i \geq 0,\ \beta_j \geq 0,$ i==1,2,….p,j=1.2……q,

$I_t$is an indicator dummy variable that takes the value 1 if (ε_{t-1}< 0) and zero otherwise.

The impact of ε_t^2 on the conditional variance h_t^2 in this model is different when ε_t is positive or negative. The negative innovations (bad news) have a higher impact than positive ones. When ε_{t-1} is positive, the total contribution to the volatility of innovation is $\alpha\varepsilon_{t-1}^2$; when ε_{t-1} is negative, the total contribution to the volatility of innovation is $(\alpha + \gamma)\varepsilon_{t-1}^2$.

γwould expect to be positive, so that the (bad news) has larger impact, in this case there is a leverage effect.

3-6-8 The Power ARCH (PARCH) Model:

The Asymmetric Power Autoregressive Conditional Heteroskedasticity (APARCH) model proposed by Ding, Granger and Engle (1993) is a model that nests several other popular univariate parameterizations and therefore allows the data to determine the true form of asymmetry (Harris and Sollis, 2003). It extends TARCH and GJR-GARCH models in the sense that non-linearity in the conditional variance is directly parameterized through a parameter δ. It thus gives a greater flexibility when modeling the memory of volatility, the variance equation of this model is given by:

$h_t^\delta = \omega + \sum_{i=}^{p} \alpha_i\, (|\varepsilon_{t-1}| - \boldsymbol{\gamma}_i \varepsilon_{t-1})^\delta + \sum_{j=1}^{q} \beta_j\, h_{t-j}^\delta$,….……………….…(3.39) where $\omega > 0$, $\delta \geq 0$, $\alpha_i \geq 0$, $\beta_j \geq 0, -1 < \gamma_i < 1$, i=1,2,…,p, j=1,2,….,q.

The model is couples the flexibility of varying exponent with the asymmetry coefficient, moreover The APARCH includes other ARCH extensions as special cases.

3-6-9 Component ARCH (C-GARCH) Model:

An alternative specification for the conditional volatility process is Component Autoregressive Conditional Heteroskedasticity models. The conditional variance in the CGARCH models is given by:

$$h_t^2 = \overline{\omega} + \sum_{t=1}^{P} \alpha_i \left(\varepsilon_{t-i}^2 - \overline{\omega}\right) + \sum_{t=1}^{T} \gamma_i \left(h_{t-i}^2 - \overline{\omega}\right) ,\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(3.40)$$

$$\sum_{t=1}^{P} \left(h_t^2 - q_t\right) = \sum_{t=1}^{P} \alpha_i \left(\varepsilon_{t-i}^2 - q_{t-i}\right) + \sum_{t=1}^{T} \gamma_i \left(h_{t-i}^2 - q_{t-i}\right), \ldots\ldots\ldots(3.41)$$

$$q_t = \omega + \sum_{i=1}^{P} \phi_i \left(q_{t-1} - \omega\right) + \sum_{i=1}^{P} \beta_i \left(\varepsilon_{t-i}^2 - h_{t-i}^2\right), \ldots\ldots\ldots\ldots\ldots\ldots\ldots..(3.42)$$

Where $\overline{\omega}$ the mean constant is over time, $h_t$is the validity and q_t is the time varing long run volatility.

3-7 Testing for Autoregressive Conditional Heteroskedasticity Effects:

For ease in notation, let $\varepsilon_t = r_t - \mu_t$ be the residuals of the mean equation. The squired series ε_t^2 is then used to check for conditional hertoskedastisity which is also known as ARCH effect. Two tests are available. The first test is to apply the Ljung –Box statistics Q_{BP} to the ε_t^2 series, McLeod and Li (1983). The null hypothesis is that the first m lags of autocorrelation function of the ε_t^2 series are zero. The second test for conditional hetroskedastisity is the Lagrange Multiplier test of Engle (1982).

When the ARCH effects are suspected, the null hypothesis of homoskedasticity of the model which takes the form:

$h_t^2 = \delta + \sum_{i=1}^{p} \alpha_i \varepsilon_t^2$,…………………………………………………………(3.43)

is:

$H_0: \alpha_1 = \alpha_2 = \cdots = \alpha_q = 0$,………………………………………………(3.44)

$H_1: \alpha_1 \neq \alpha_2 \neq \cdots \neq \alpha_q \neq 0$,…………………………………..…………(3.45)

i.e.

$h_t^2 = \sigma^2 = \delta$,…………………………………………………………..(3.46)

Using OLS is an appropriate estimator, based on its squared residuals, Engle (1982) showed that on the null and alternative hypothesis, that:

$H_0: \alpha_1 = \alpha_2 = \cdots = \alpha_q = 0$,……………………….………………………(3.47)

$H_1: \alpha_1 \neq \alpha_2 \neq \cdots \neq \alpha_q \neq 0$,………………………….……………………(3.48)

$\lambda = TR^2 \sim \mathcal{X}_{m,\alpha}^2$,…….……………………..………………………(3.49)

Where T is the number of squared residuals included in the regression and $R^2$is the sample multiple correlation coefficients. Under the null hypothesis, the test is asymptotically distributed as a chi-square distribution with M degrees of freedom. If the value of the test statistic is greater than the critical value from the $\mathcal{X}_{m,\alpha}^2$distribution, then reject the null hypothesis, and vice versa.

This test is equivalent to the usual F statistic for testing $\alpha_i = 0$, $i = 1, \ldots, m$ in the linear regression equation of the form:

$$\varepsilon_t^2 = \delta + \sum_{i=1}^{t-m} \alpha_i \varepsilon_t^2 + e_t \qquad t = m+1, \ldots, T, \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.50)$$

Where e_t denote the error term, m is a prespecified positive integer, and T is the sample size. Specifically, the null hypothesis is

$$H_0: \alpha_1 = \alpha_2 = \cdots = \alpha_m = 0 \ , \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.51)$$

Let

$$SSR_0 = \sum_{t=m+1}^{T} (\varepsilon_t^2 - \overline{\omega})^2 \ , \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.52)$$

where

$$\overline{\omega} = (\frac{1}{T}) \sum_{t=1}^{T} \varepsilon_t^2 \ , \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.53)$$

is the sample mean of ε_t^2 , and

$$SSR_1 = \sum_{t=m+1}^{T} \hat{e}_t^2 \ , \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.54)$$

Where $\hat{e}_t$ is the least square residual of the prior linear regression, then the statistic test:

$$F = \frac{(SSR_0 - SSR_1)/m}{SSR_1/(T-2m-1)} \ , \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.55)$$

which is asymptotically distributed as a chi-squared distribution with m degrees of freedom under the null hypothesis. The decision rule is to reject the null hypothesis if $F > x_m^2(\alpha)$ or the $\mathrm{P-value}$ of F is less than α.

3-8 Estimation of the Autoregressive Conditional Heteroskedasticity Models:

There exists more than one method for estimating parameters in GARCH models with unknown innovation distributions. The quasi maximum likelihood estimator facilitated by hypothetically assuming the innovation distribution to be Gaussian is arguably the most frequently used estimator in practice, which simply call the Gaussian maximum likelihood estimator (GMLE).

To be able to predict the volatility for a time series, one first has to fit the GARCH-model to the time series data. This is done via estimation of the parameters in the tentative model. The most common and standard method of this estimation is the maximum-likelihood estimation (MLE).

3-8-1 Maximum-Likelihood Estimation (MLE)

Under the assumptions of a conditional normal distribution of ε_t. The maximum-likelihood estimation works as follows:

Let, $\varepsilon_1, \ldots., \varepsilon_n$ assumes to be random observations from a distribution $F_{\varepsilon_t}(\varepsilon_t; \theta)$that depends on the unknown parameter θ (where $\theta =$ [ω, $\alpha_1, \ldots., \alpha_P, \beta_1, \ldots., \beta_q$] in the GARCH (*p*,*q*) case) with the parameter space. $\varepsilon_t$has the probability distribution function $f_{\varepsilon_t}(\varepsilon_t; \theta)$, where $P_{\varepsilon_t}(\varepsilon_t; \theta)$ denotes the probability that $\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}$, thus P($\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}$).

Supposing that the probability function is known (except from the unknown parameters) it is possible to estimate the unknown parameters θ's by putting up the likelihood function which is denoted by (L(θ), and takes the form:

$$f_{\varepsilon_t}(\,\varepsilon_t|\Omega_{t-1},\theta) = \frac{1}{\sqrt{2\pi h^2}}\exp(-\frac{1}{2}\frac{\varepsilon_t^2}{h_t^2}) \;,\ldots\ldots\ldots\ldots\ldots\ldots(3.56)$$

with the large number of observation T, the Likelihood function of the above distribution is written as :

$$\mathrm{L}(\theta) = \prod_{i=1}^{T} f_t\,(\,\varepsilon_t|\Omega_{t-1},\theta)\;,\ldots\ldots\ldots\ldots\ldots\ldots(3.57)$$

Therefore

$$\mathrm{L}(\theta) = (\frac{1}{\sqrt{2\pi h^2}})^T \exp\left[-\frac{1}{2}\sum_{i=1}^{T}\frac{\varepsilon_t^2}{h_t^2}\right]\;,\ldots\ldots\ldots\ldots\ldots\ldots(3.58)$$

$$\mathrm{L}(\theta) = (\frac{1}{\sqrt{2\pi}})^T (h^2)^{-\frac{T}{2}} \exp\left[-\frac{1}{2}\sum_{i=1}^{T}\frac{\varepsilon_t^2}{h_t^2}\right]\;,\ldots\ldots\ldots\ldots\ldots\ldots(3.59)$$

The logarithm of the above form is called the Log-Likelihood function, which is expressed as follows:

$$\ln \mathrm{L}(\theta) = \mathrm{K} - \frac{\mathrm{T}}{2}\ln \mathrm{h}^2 - \frac{1}{2}\sum_{\mathrm{i}=1}^{\mathrm{T}}\frac{\varepsilon_\mathrm{t}^2}{\mathrm{h}_\mathrm{t}^2}\;,\ldots\ldots\ldots\ldots\ldots\ldots(3.60)$$

Where

$$k = \ln(\frac{1}{\sqrt{2\pi}})^T,\ldots\ldots\ldots\ldots\ldots\ldots(3.61)$$

The Maximum likelihood parameter estimation is based on choosing values for θ so as to maximise the likelihood function. That is, the MLE of θ, which denoted as $\hat{\theta}$, is the solution to the maximized problem for observations $\varepsilon_{1,\ldots,}\varepsilon_T$,

i.e.:

$\hat{\theta} = argmax_{\theta\in\Theta} \ln L(\theta)$,……………………..……………….………..(3.62)

Where $\hat{\theta}$ is the value of the argument of the likelihood, selected from anywhere in the parameter space that maximizes the value of the likelihood after given the sample of observations.

Consider a simple GARCH(1,1) specification:

$r_t = \mu_t + \varepsilon_t$,……………………………………...………………(3.63)

$\varepsilon_t = h_t\eta_t$,……………………..………………………….……….…(3.64)

$h_t^2 = \omega + \alpha_1\varepsilon_{t-1}^2 + \beta_1 h_{t-1}^2$,………..…………………....……………......(3.65)

Since the errors are assumed to be conditionally i.i.d, maximum likelihood is a natural choice to estimate the unknown parameters, $\boldsymbol{\theta}$ which contain both mean and variance parameters.

The normal likelihood for T independent variables is given by the following formulation:

$f(r;\theta) = \prod_{t=1}^{T}(2\pi h_t^2)^{-\frac{1}{2}}\exp(-\frac{(r_t-\mu_t)^2}{2h_t^2})$,…………………....………..…(3.66)

and the normal log-likelihood function is given by:

L(r; θ) = $\sum_{i=1}^{T} -\frac{1}{2}\ln(2\pi) - \frac{1}{2}\ln(h_t^2) - \frac{(r_t-\mu_t)^2}{2h_t^2}$,………………..………(3.67)

If the mean is set to zero, the log-likelihood simplifies to:

L(r; θ) = $\sum_{i=1}^{T} -\frac{1}{2}\ln(2\pi) - \frac{1}{2}\ln(h_t^2) - \frac{r_t^2}{2h_t^2}$,…………..………………(3.68)

and is maximized by solving the first order conditions:

$\frac{\partial L(r;\theta)}{\partial h_t^2}$ = $\sum_{t=1}^{T}\frac{1}{2h_t^2} + \frac{r_t^4}{2h_t^2} = 0$…………………………....…………(3.69) which can be written to provide some insight into the estimation of ARCH models,

$$\frac{\partial L(r;\theta)}{\partial h_t^2} = \frac{1}{2}\sum_{t=1}^{T}\frac{1}{h_t^2}\left(\frac{r_t^2}{h_t^2} - 1\right) \quad \text{,...(3.70)}$$

This expression clarifies that the parameters of the volatility are chosen to make $\left(\frac{r_t^2}{h_t^2} - 1\right)$ as close to zero as possible.

The derivatives take forms:

$$\frac{\partial(h_t^2)}{\partial\omega} = 1 + \beta_1\frac{\partial h_{t-1}^2}{\partial\omega} \quad \text{,...(3.71)}$$

$$\frac{\partial(h_t^2)}{\partial\alpha_1} = \varepsilon_{t-1}^2 + \beta_1\frac{\partial h_{t-1}^2}{\partial\alpha_1} \quad \text{,..(3.72)}$$

$$\frac{\partial(h_t^2)}{\partial\beta_1} = h_{t-1}^2 + \beta_1\frac{\partial h_{t-1}^2}{\partial\beta_1} \quad \text{,..(3.73)}$$

The above equations provide the necessary formulas to implement the score of the log-likelihood.

3-8-2 The Gaussian Maximum-Likelihood Estimation

Let, $\varepsilon_1, \dots, \varepsilon_n$ assumes to be random observations from a distribution $F_{\varepsilon_t}(\varepsilon_t;\theta)$that depends on the unknown parameter θ (where $\theta = [$ω, $\alpha_1, \dots, \alpha_P, \beta_1, \dots, \beta_q]$ in the GARCH (*p*,*q*) case) with the parameter space. $\varepsilon_t$has the probability distribution function $f_{\varepsilon_t}(\varepsilon_t;\theta)$, where $P_{\varepsilon_t}(\varepsilon_t;\theta)$ denotes the probability that $\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}$, thus P($\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}$).The Gaussian maximum-likelihood

Estimation (QMLE) is given by:

$$L_{n(\theta)} = L_n(\theta;\ \varepsilon_1, \varepsilon_2, \dots, \varepsilon_n) = \sum_{t=1}^{n} \frac{1}{\sqrt{2\pi \widetilde{h_t^2}}} \exp(-\frac{\varepsilon_t^2}{2\widetilde{h_t^2}}) \text{ ,} \ldots\ldots\ldots\ldots\ldots\ldots(3.74)$$

Where $\widetilde{h_t^2}$ are defined recursively, for $t \geq 1$, by

$$\widetilde{h_t^2} = \widetilde{h_t^2}(\theta) = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j \widetilde{h_t^2}_{t-j}^{2} \text{ ,} \ldots\ldots\ldots\ldots\ldots\ldots(3.75)$$

For instance, the initial values can be chosen as:

$$\varepsilon_0^2 = \cdots = \varepsilon_{1-p}^2 = \widetilde{h_0^2} = \cdots = \widetilde{h_{1-q}^2} = \omega \text{ ,} \ldots\ldots\ldots\ldots\ldots\ldots(3.76)$$

or

$$\varepsilon_0^2 = \cdots = \varepsilon_{1-p}^2 = \widetilde{h_0^2} = \cdots = \widetilde{h_{1-q}^2} = \varepsilon_1^2 \text{ ,} \ldots\ldots\ldots\ldots\ldots\ldots(3.77)$$

A QMLE of θ is defined as any measurable solution $\hat{\theta}_n$ of

$$\hat{\theta}_n = \underset{\theta \in \Theta}{argmax}\, L_{n(\theta)} = \underset{\theta \in \Theta}{argmax}\, \tilde{\mathrm{I}}_n(\theta) \text{ ,} \ldots\ldots\ldots\ldots\ldots\ldots(3.78)$$

where

$$\tilde{\mathrm{I}}_n(\theta) = n^{-1} \sum_{i=1}^{n} \tilde{\ell}_t \text{ and } \tilde{\ell}_t = \tilde{\ell}_t(\theta) = \frac{\varepsilon_t^2}{\widetilde{h_t^2}} + log\widetilde{h_t^2}$$

Lee and Hansen (1994) and Lumsdaine (1996) proved that the local QMLE is consistent and asymptotically normal, assuming $E(\ln(\alpha_1 \eta_t^2 + \beta_1)) < 0$, which is the necessary and sufficient condition for strict stationarity. However, Lee and Hansen (1994) required that all the conditional expectation of $\eta_t^{2+k} < \infty$ with $k > 0$,

3-8-3 Fat-Tailed Maximum-Likelihood Estimation

An alternative way of dealing with non-Gaussian errors is to assume a distribution that reflects the features of the data better than the normal distribution, and estimate the parameters using this distribution in the likelihood function instead of the Gaussian. Thus, the problem with the calculation of unobservable values is yet present in this model. When choosing a distribution for the innovations, QQ-plots can be very helpful. In this thesis two distributions, apart from the Gaussian, are considered; the *Student-t Distribution* (t-Distribution) and the Generalized Error Distributio*n* (GED).

The likelihood functions for two distributional assumptions are:

* the log-likelihood function for the Student-t distribution

$$\mathfrak{l}_n = \sum_{i=1}^{n}\{\log\Gamma\left(\frac{v+1}{2}\right) - \log\Gamma\left(\frac{v}{2}\right) - \frac{1}{2}\log(\pi(v-2)) - \frac{1}{2}\log\sigma^2 - \left(\frac{v+1}{2}\right)\log\left(1+\frac{X_t^2}{\sigma_t^2(v-2)}\right)\} \quad \ldots\ldots(3.79)$$

*the log-likelihood-function for the GED

$$\mathfrak{l}_n = \sum_{i=1}^{n}\{\log\left(\frac{v}{\lambda}\right) - \frac{1}{2}\left|\frac{X_t}{\sigma_t\lambda}\right|^{v} - (1+v^{-1})\log(2) - \log[\Gamma\left(\frac{1}{v}\right) - \frac{1}{2}\log(\sigma^2)\} \quad \ldots(3.80)$$

Where $\Gamma(\cdot)$ is the gamma function, and

$$\boldsymbol{\lambda} = \left(\frac{2^{-\frac{2}{v}}\Gamma(\frac{1}{v})}{\Gamma(\frac{3}{v})}\right)^{\frac{1}{2}}$$

These log-likelihood functions are maximized with respect to the unknown parameters (the same procedure as in the Gaussian quasi MLE case).

3-9 Distribution Assumptions:

As discussed earlier, observations of the financial time series $\{X_t\}$ have a distribution that one often assumes to be normal (Gaussian) but, as shown in, they often tend to be leptokurtic (fat tailed). QQ-plots have been shown to be good tools when deciding what distribution to use. In this thesis the fat tailed Student-t distribution and the GED are considered. The GED can be both leptokurtic and platykurtic depending on the chosen degree of freedom.

Here follows some further information about these distributions

3-9-1 Normal Distribution

The standard GARCH (p, q) model introduced by Tim Bollerslev (1986) is with normal distributed error $\varepsilon_t = h_t z_t,\ z_t \sim\ iid(0,1)$. Use maximum log-likelihood method to estimate the parameter in the standard GARCH model, given the error following the Gaussian and we can get the log-likelihood function:

$$f_{\varepsilon_t}(\varepsilon_t|,\theta) = \ln\prod_{i=1}^{T}\frac{1}{\sqrt{2\pi h^2}}\exp(-\frac{1}{2}\frac{\varepsilon_t^2}{h_t^2}) = \ln\prod_{i=1}^{T}\frac{1}{\sqrt{2\pi h^2}}\exp-\frac{z_t^2}{2}\ldots\ldots(3.81)$$

$$= -\frac{1}{2}\sum_t[\ln(2\pi)+\ln(\mathrm{h}_t^2)+z_t^2]$$

Where $z_t^2 = \frac{\varepsilon_t^2}{h_t^2}$ is independently and identically distributed

3-9-2 Student's t-Distribution

As mentioned before, GARCH model often does not allow asymmetry and is not sufficiently fat-tailed to capture the excess kurtosis found in most financial return data. This has led to a search for more flexible conditional distribution (non-normal distributions) to replace the conditional normal assumption. Bollerslev (1987) was the first combined the GARCH models with a standardized Student's t-distribution with $v > 2$ degrees of freedom whose density is given by:

$$f(z_t \setminus v) = \frac{\Gamma((\upsilon+1)/2)}{\Gamma(\upsilon/2)\sqrt{\pi(\upsilon-2)}}(1+\frac{z_t^2}{\upsilon-2})^{-\frac{\upsilon+1}{2}} \quad \text{..........................(3.82)}$$

Where $\boldsymbol{z_t} = \varepsilon_t / h_t$ be the standardized error, $\Gamma(v)$ is the gamma function , v is the parameter that measures the tail thickness.

3-9-3 Generalized Error Distribution

Nelson (1991) suggested the use of the generalized error distribution

(GED)

$$f(\eta_t) = \frac{\upsilon \exp(-\frac{1}{2}|\eta_t / \lambda|^{\upsilon})}{2^{(1+\frac{1}{\upsilon})}\Gamma(\upsilon^{-1})\lambda} \qquad \upsilon > 0 \text{............................(3.83)}$$

Where υ is the tail-thickness parameter and $\lambda \equiv \left[2^{(-2/\upsilon)}\Gamma(1/\upsilon)/\Gamma(3/\upsilon)\right]^{1/2}$. When $\upsilon = 2$, η_t is standard normally distributed. For $\upsilon < 2$, the distribution of η_t has thicker tails than the normal distribution (e.g., for $\upsilon = 1$, η_t has double exponential distribution) while for $\upsilon > 2$ the distribution of η_t has thinner tails than the normal distribution (e.g., for $\upsilon = \infty$, η_t has a uniform distribution on the interval $(-\sqrt{3}, \sqrt{3})$,. The conditional kurtosis is given by $(\Gamma(1/\upsilon)\Gamma(5/\upsilon))/(\Gamma(1/\upsilon))^2$.

Notice that the choice of a density has a particular impact on some models, for example in EGARCH the value of $E|\eta_t|$ depends on the density function for the standard normal distribution

$$E(\eta_{t-i}) = \sqrt{\frac{2}{\pi}}, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.84)$$

for student-*t* distribution

$$E(|\eta_{t-i}|) = \frac{2\Gamma(\frac{1+\upsilon}{2})^2\sqrt{(\upsilon-2)}}{1+\sqrt{\pi}(\upsilon-1)\Gamma(\upsilon/2)}, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(3.85)$$

for GED

$$E(|\eta_{t-i}|) = \lambda 2^{1/\upsilon}\frac{\Gamma(2/\upsilon)}{\Gamma(1/\upsilon)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(3.86)$$

3-10 Forecasting:

The forecasts of the Autoregressive Conditional Heteroskedasticity models (ARCH) model can be obtained recursively as those of an Autoregressive models (AR) model. Consider an ARCH (p) model. At the forecast origin m, the one-step ahead forecast of $h_{h+1}^2$is given by:

$$h_m^2(1) = \alpha_0 + \alpha_1\varepsilon_m^2 + \cdots + \alpha_p\varepsilon_{m+1-p}^2, \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.87)$$

The two-step ahead forecast is:

$$h_m^2(2) = \alpha_0 + \alpha_1 h_m^2(1) + \cdots + \alpha_p\varepsilon_{m+2-p}^2, \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.88)$$

and the ℓ-step ahead forecast for $h_{m+\ell}^2$ is:

$$h_m^2(\ell) = \alpha_0 + \sum_{i=1}^{p} \alpha_i h_m^2(\ell - i) \text{ ,..(3.89)}$$

where $h_m^2(\ell - i)$=$\mathcal{E}_{m+\ell-i}^2$ if $\ell - i \leq 0$.

3-10-1 Evaluation of Volatility Forecasts

A fundamental concern in forecasting is the measure of forecasting error for given data set and given forecasting method. Accuracy can be defined as "goodness of fit" or how well the forecasting model is able to reproduce data that is already known (Makridakis and Wheelwright, 1989).

The forecasting ability of GARCH models has been comprehensively discusses by Poon and Granger (2001). However Anderson and Bollerslev (1997) pointed out that squired daily returns may not be the proper measure to assess the forecasting performance of the different GARCH models for the conditional variance. The objective of applied econometrics is often to find the superior forecasting model. According to Gonzales-Rivera et al. (2004) the task of comparing the relative performance of different volatility models is built on either a statistical loss function or an economic loss function. Statistical loss functions are based on moments of forecast errors, and include statistics such as the mean error (ME), the root mean square error statistics such as the mean error (ME), the root mean square error (RMSE), the mean absolute error (MAE) and the mean absolute percent error (MAPE), the following formulas are the statistical measures considered to assess forecasting ability:

3-10-2 Mean Squared Error

As a measure of desperation of forecast error, statisticians have taken the average of the squared individual errors. The smaller the MSE value, the more stable the model. However, interpreting the MSE value can be misleading, for the mean squared error will be accentuate large error terms. It can be describes as:

$MSE = \frac{1}{h+1}\sum_{t=s}^{s+h}(\hat{\sigma}_t^2 - \sigma_t^2)^2$,…………………………………………………..(3.90)

3-10-3 Mean Absolute Error

This error measurement is the average of the absolute value of the error without regard to whether the error was an overestimate or underestimates (Krajewski and Ritzman, 1993), its equation takes the form:

$MAE = \frac{1}{h+1}\sum_{t=s}^{s+h}|\hat{\sigma}_t^2 - \sigma_t^2|$,……………………………………………..(3.91)

3-10-4 Adjusted Mean Absolute Percentage Error

Mean Absolute Percentage Error is regarded as a better error measurement than MSE because it does not accentuate large errors, it can be written as:

$AMAPE = \frac{1}{h+1}\sum_{t=s}^{s+h}\left|\frac{\hat{\sigma}_t^2 - \sigma_t^2}{\hat{\sigma}_t^2 + \sigma_t^2}\right|$,……………………………………………(3.92)

where h is the number of head steps, s is the sample size, $\hat{\sigma}_t^2$ is the forecasted variance and σ_t^2 is the actual variance.

Mean:

The best model would be the one that minimizes such a function of the forecast errors.

3-10-5 Akaiake Information Criteria

The Akaiake Information Criterion or AIC is effectively an estimate of the out of sample forecast error variance, it is used to select among competing forecasting models, the model that have smallest AIC (is the best), the formula is as follows:

$AIC = \ln \hat{\sigma}^2 + \frac{2k}{T}$,…………………………………………………….(3.93)

3-10-6 Schwarz Information Criteria

The Schwarz Information Criterion, or Sic is an alternative to the AIC with the same interpretation, the formula is denoted as follows:

$\mathrm{SIC} = \ln \hat{\sigma}^2 + \frac{\mathrm{k}}{\mathrm{T}} \ln \mathrm{T}$,……………………………..……………………(3.94)

where

$\hat{\sigma}^2 = \frac{1}{T} \sum_{i=1}^{T} (\varepsilon_t - \mu)^2$,…………………………...……………………(3.95)

T is the number of observations; k is the number of parameters

CHAPTER FOUR

ANALYSIS OF EXCHANGE RATE

4-1 Introduction

4-2 Data

4-3 Examining Exchange rate and Modeling

4-4 Descriptive Statistics

4-5 Testing for Stationarity

4-6 Exchange Rate Model Identification

4-7 Testing for Heteroskedasticity

4-8 Forecasting

4-1 Introduction:

This chapter empirically examines vital characteristics of the exchange rate data in the Sudan in order to perform an appropriate model for modeling and forecasting exchange rate volatility in the Sudan.

4-2 Data:

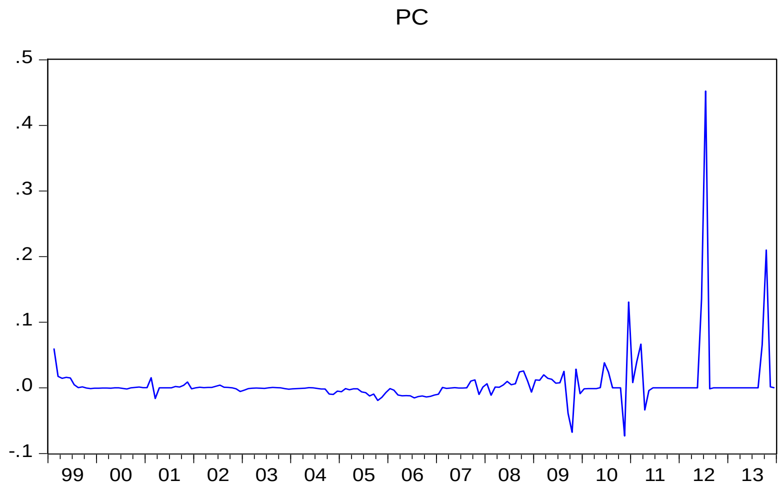
The data will be used in the analysis of this research are monthly readings of Exchange Rate in the Sudan covered the period from 01/01/1999 to 31/12/2013 obtained from Central Bureau of Statistics, Bank of Sudan and Khartoum Stock market and then transformed into logarithmic return series. The corresponding transform price series into monthly logarithmic return are calculated by using the formula: $r_t = \ln(x_t/x_{t-1})$ where x_t is the exchange rate and $r_t$denotes the returns

4-3 Examining Exchange Rate and Modeling:

This section examines empirically a vital characteristic of exchange rate prices in relation to volatility, persistence, changes in volatility and asymmetry in volatility prices of prices. Figure (1) Plots of return Exchange rate series: Monthly data (from 1/1/1999 to 31/12/2013).

Figure (4-1) illustrates monthly of return Exchange rate Monthly data plot.It can be seen that the mean of the return Exchange rate is about constant however, the variance clearly exhibit volatility clustering

Figure (4-1) The plot return Exchange prices Monthly data (from1/1/1999 to 31/12/2013).



Source:Eviews 8

4-4 Descriptive Statistics:

Table (4-2): Summary Statistics of Exchange rate Returns (SDG/ USA ($))

Sample size	Mean	S.Dev	Min.	Max.	Skew.	Kurt.	J.B	P-value
178	0.0051	0.036	-0.076	0.373	6.717	62.241	27521.52	0.000

Source :Eviews 8

The summary statistics of this study is presented in table (4-2).This indicates that the returns series have monthly positive mean of (0.0051) while the monthly volatility is (0.013),without loss of generality the mean grows at a linear rate while the volatility grows approximately at a square root rate. The lowest monthly returns correspond to (-0.076) and the best monthly exchange rate returns is (0.373). The returns series of the exchange rate shows positive skewness. This implies that the series is flatter to the right. The kurtosis value is higher than the normal value of perfectly normal distribution in which value for skewness is ‘zero’ and kurtosisis ‘three’ and this suggest that the kurtosis curve of the exchange rate return series is leptokurtic. The results of this study reveal that, the series is not normally distributed. Our empirical result is consistent with the Jarque-Bera (JB) tests obtained above which is used to assess whether the given series is normally distributed or not. Here, the null hypothesis is that the series is normally distributed. Results of JB test find that the null hypothesis is rejected for the return series and suggest that the observed series are not normally distributed

4-5 Testing for Stationarity:

To investigate whether the daily price index and its returns are stationary series, the Augmented Dickey–Fuller (ADF) test (Dickey and Fuller, 1981) has been applied. Thereby, the lag length has been selected automatically based on the Schwarz information criterion with a preset maximum lag length of 13. The results are reported in Table (4-2).

Figure (4-3) Augmented Dickey-Fuller test

Null Hypothesis: RT has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic - based on SIC, maxlag=13)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-10.60353	0.0000
Test critical values:	1% level	-3.467205	
	5% level	-2.877636	
	10% level	-2.575430	

The Augmented Dickey-fuller of unit root test (ADF) with trend, intercept and lag difference of 1 result, results conclude that exchange rate retun series has a unit root.The ADF test were also applied to the first difference of exchange rate retun series from figure (4-3) the result illustrate that the absolute value of the ADF test (10.60353)is greater than the 1%, 5% and 10% critical values in absolute terms (3.467205 ,2.877636 and 2.575430) respectively this result conclude that the transformed into logarithmic return series is stationary

The ACF and PACF plot in Figure (4-4) shows no significant peaks, also all Q-statistics shows no significant ACF, this result confirm that the first difference of exchange rate series is stationary.

Figure (4-4) Correlogram of first difference of exchange rate series

Date: 09/24/15 Time: 20:19
Sample: 1999M01 2013M12
Included observations: 179

<table>
<tr><th>Autocorrelation</th><th>Partial Correlation</th><th></th><th>AC</th><th>PAC</th><th>Q-Stat</th><th>Prob</th></tr>
<tr><td>.|** |</td><td>.|** |</td><td>1</td><td>0.216</td><td>0.216</td><td>8.5053</td><td>0.004</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>2</td><td>0.001</td><td>-0.048</td><td>8.5057</td><td>0.014</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>3</td><td>0.020</td><td>0.031</td><td>8.5758</td><td>0.035</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>4</td><td>-0.036</td><td>-0.050</td><td>8.8185</td><td>0.066</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>5</td><td>0.002</td><td>0.024</td><td>8.8194</td><td>0.116</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>6</td><td>0.009</td><td>0.001</td><td>8.8351</td><td>0.183</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>7</td><td>-0.001</td><td>-0.001</td><td>8.8355</td><td>0.265</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>8</td><td>-0.005</td><td>-0.007</td><td>8.8397</td><td>0.356</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>9</td><td>-0.010</td><td>-0.007</td><td>8.8589</td><td>0.450</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>10</td><td>-0.009</td><td>-0.005</td><td>8.8731</td><td>0.544</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>11</td><td>-0.009</td><td>-0.007</td><td>8.8901</td><td>0.632</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>12</td><td>-0.023</td><td>-0.021</td><td>8.9966</td><td>0.703</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>13</td><td>0.015</td><td>0.025</td><td>9.0395</td><td>0.770</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>14</td><td>0.045</td><td>0.037</td><td>9.4408</td><td>0.802</td></tr>
<tr><td>.|** |</td><td>.|** |</td><td>15</td><td>0.277</td><td>0.276</td><td>24.607</td><td>0.055</td></tr>
<tr><td>.|* |</td><td>.|* |</td><td>16</td><td>0.200</td><td>0.090</td><td>32.560</td><td>0.008</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>17</td><td>0.032</td><td>-0.011</td><td>32.769</td><td>0.012</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>18</td><td>0.065</td><td>0.064</td><td>33.608</td><td>0.014</td></tr>
<tr><td>.|* |</td><td>.|* |</td><td>19</td><td>0.170</td><td>0.185</td><td>39.489</td><td>0.004</td></tr>
<tr><td>*|. |</td><td>*|. |</td><td>20</td><td>-0.099</td><td>-0.183</td><td>41.499</td><td>0.003</td></tr>
<tr><td>.|. |</td><td>.|* |</td><td>21</td><td>0.007</td><td>0.076</td><td>41.508</td><td>0.005</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>22</td><td>0.006</td><td>-0.025</td><td>41.516</td><td>0.007</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>23</td><td>0.018</td><td>0.059</td><td>41.584</td><td>0.010</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>24</td><td>0.042</td><td>0.006</td><td>41.953</td><td>0.013</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>25</td><td>0.045</td><td>0.069</td><td>42.383</td><td>0.016</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>26</td><td>-0.002</td><td>-0.029</td><td>42.384</td><td>0.022</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>27</td><td>-0.003</td><td>0.039</td><td>42.386</td><td>0.030</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>28</td><td>-0.005</td><td>-0.018</td><td>42.391</td><td>0.040</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>29</td><td>-0.013</td><td>-0.014</td><td>42.429</td><td>0.051</td></tr>
<tr><td>.|. |</td><td>*|. |</td><td>30</td><td>-0.016</td><td>-0.115</td><td>42.485</td><td>0.065</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>31</td><td>0.042</td><td>0.013</td><td>42.876</td><td>0.076</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>32</td><td>0.035</td><td>-0.026</td><td>43.145</td><td>0.090</td></tr>
<tr><td>*|. |</td><td>*|. |</td><td>33</td><td>-0.079</td><td>-0.123</td><td>44.526</td><td>0.087</td></tr>
<tr><td>.|. |</td><td>*|. |</td><td>34</td><td>0.014</td><td>-0.079</td><td>44.571</td><td>0.106</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>35</td><td>-0.014</td><td>0.029</td><td>44.614</td><td>0.128</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>36</td><td>0.009</td><td>0.024</td><td>44.632</td><td>0.153</td></tr>
</table>

4-6 Exchange Rate Model Identification:

Since correlogram of retun sieres of exchange rate does not give much help in identifying an appropriate model, thus numerous ARIMA models are suggested to fit exchange rate return sieres in the Sudan. Table (4-3) bellow shows the suggested models and their corresponding AIC and BIC criteria.

Numerous statistical criterion for assessing the goodness of fit to time series models have been introduced, Akiaka's (1987) information criteria and Schwartz's (1978) Bayesian criteria are useful tools for comparing models with different parameters number, the model with smallest AIC or SBC is considered best. Several ARIMA (p,d,q) models have been suggested with the objective of identifying which of these models is adequate to fit buying exchange return series, the suggested ARIMA models and their corresponding AIC,SBC values are stated as follows:

Table (4-3) ARIMA (p,d,q)**.**

ARIMA (p,d,q).	AIC	SBC
ARIMA (1,1,0)	-3. 812939	-3.777189
ARIMA (0,1,1)	-3.811039	-3.775426
ARIMA (1,1,1)	-3.805022	-3.751397
ARIMA (1,1,2)	**-3.794413**	**-3.722912**
ARIMA (2,1,1)	-3.835265	-3.763488
ARIMA (2,1,2)	-3.824261	-3.734539

Source : return Exchange prices Monthly data (from1/1/1999 to 31/12/2013)

A closer look to table (4-3) it can be seen that ARIMA (1,1,2) model have smallest value of AIC and BSC criteria. In this model it is assumed that the exchange rate data is subject to autoregressive of order1, differing 1, and moving average of order 2.

4-7 Testing for Heteroskedasticity

Figure (4-5). ARCH-LM Test for residuals of ARIMA(1,1,2)

Heteroskedasticity Test: ARCH

F-statistic	**6.180302**	**Prob. F(1,175)**	**0.0139**
Obs*R-squared	**6.037707**	**Prob. Chi-Square(1)**	**0.0140**

Note: H_0 : There are no ARCH effects in the residual series

The ARCH-LM test results in Figure (4-5) provide strong evidence for rejecting the null hypothesis. Rejecting H_0 is an indication of the existence of ARCH effects in the residuals series of the mean equation and therefore the variance of the returns series indicates are non-constant

Table(4-4) Estimation results of different GARCH models Exchange rate Returns (SDG/ USA)

Coefficients	GARCH (1,1)	GARCH-M (1,1)	EGARCH (1,1)	TGARCH (1,1)	PGARCH (1,1)
Mu(μ)	0.038	14.53	-0.0007	0.002	0.003
	0.019	0.999	0.209	0.504	0.36
Ar1(ϕ)	0.997	1.000	0.599	0.506	0.451
	0.000	0.000	0.000	0.124	0.149
Ma(θ_1)	-0.852	-0.679	-0.605	-0.069	-0.032
	0.000	0.000	0.000	0.839	0.897
Ma(θ_2)	-0.139	-0.271	0.022	0.004	0.002
	0.005	0.000	0.319	0.982	0.99
Omega (ω)	0.00001	0.00003	-15.107	0.0002	0.0006
	0.906	_	0.000	0.000	0.863
Alpha (α_1)	5.452	0.117	0.689	1.442	0.918
	0.000	0.000	0.000	0.006	0.0007
Beta(β_1)	0.008	0.882	-0.075	-0.841	-0.197
	0.325	0.000	0.042	0.136	0.094
Gamma(γ)			-0.806	-0.027	-0.043
			0.000	0.557	0.739
Delta (δ)					1.804
					0.198
α + β	5.46	0.999	0.614	0.601	0.721
Log likelihood	506.08	507.5	468.83	467.15	464.77
ARCH-LM test	0.012	25.66	5.42	1.22	1.766
	0.912	0.000	0.0198	0.269	0.183

Source :Eviews 8

In the results for the variance equation reported in Table (4-4) provides the estimates of the GARCH (1,1) model for return series of exchange rate in Sudan. The estimation result shown that the coefficients in the conditional variance equation the α significant and β not significant at 5% significant level. The sum of ARCH and GARCH coefficients (α+β) = 5.46 (persistence coefficients) in the GARCH (1,1) model is the greater than one, suggesting that the conditional variance process is explosive. the ARCH-LM for lagged conditional variance and squared disturbance is 0.012, under $x^2_{(1)}$ the null hypothesis is accepted since the p- value is 0.912 where it has greater than 5% of significance level. Means that the Accept the null hypothesis at the same condition. Therefore the ARCH-LM test on the residuals of this model indicates that the conditional heteroskedasticity is not present.

The GARCH-M (1,1) model is estimated by allowing the mean equation of the return series to depend on a function of the conditional variance. From estimation results in Table (4- 4) the estimated coefficient (risk premium) of Rt in the mean equation is positive for the markets, which indicates that the mean of the return sequence depends on past innovations and the past conditional variance.

From Table (4-4)the estimates of the EGARCH (1,1) model for return series of exchange rate, the estimation results shows that; the estimates γ is negative and significant, meaning that returns series have asymmetry and has greater impact of negative shocks on the return series of exchange rate volatility. Moreover, the estimates β = -0.075 is significantly at 5% significant level which is an indication of not persistence of volatility. In addition the estimates α is statistically significant while γ is statistically significant, $\alpha > 0$ indicating that the conditional variance has leverage effect. Furthermore $\gamma \neq 0$; meaning that an asymmetry of negative shocks on the conditional variance is present. The ARCH-LM for lagged conditional variance and squared disturbance is 5.42, under $x^2_{(1)}$ the null hypothesis is rejected since the p- value is 0.019 where it has less than 5% of significance level the null hypothesis is rejected This result indicates that the ARCH effect occur in the residuals of EGARCH (1,1) model this model is adequate to presents return series of exchange rate.

From above table demonstrate the estimation result of the TGARCH (1,1) for return series of exchange rate model. It can be seen that the ARCH is statistical significant and GARCH term is not statistical significant. The sum of ARCH and GARCH is equal to 0.601 less than one indicating that volatility shocks is quite persistent. Moreover, the symmetry term in the TGARCH (1,1) model γ = -0.027 it is not statistically significant at 5% significant level, means that the conditional variance has not a leverage effect , Adverse market conditions and bad news such as, the effect of risk management policies and political instabilities has an impact of $\alpha + \gamma$ = 1.415. Good news about demand and supply conditions in the price of exchange rate market has an impact of α = 1.442 the ARCH-LM for one lag difference of residuals squared is 1.22, under $x^2_{(1)}$ the null hypothesis is not rejected since the p- value is 0.269where it has greater than 5% of significance level. Accept the null hypothesis Therefore the ARCH-LM test on the residuals of this model this results confirm that the model is appropriate

From table (4-4) demonstrate the estimates of the APARCH (1,1) model for returns exchange rate series, the estimated power term $\delta = 1.8$ is not statistically significant at 5%significant level. Moreover the a symmetry term γ in the APARCH (1,1) model is not statistically significant at 5%significant level. In addition $\gamma = -0.043$ is less than zero; this means that the conditional variance has not a symmetric term on the price volatility. The output on the ARCH test as shown in table(4- 4) signifies that the null hypothesis did not rejected, that there is no ARCH effect in the residuals, because of the insignificant squared residual term (p-value of 0.183 is more than 0.05 level of significance). This result confirms that the APGARCH (1,1) model for returns exchange rate series model is adequate.

Table(4- 5)
Parameter Estimation of the ARIMA (1,1, 2)-GARCH (1, 1), GJR (1, 1) and DGE (1, 1) Models with the Conditional

	GARCH		GJR-GARCH		DEG-GARCH	
Conditional Distribution	**Normal**	**Student-t**	**Normal**	**Student-t**	**Normal**	**Student-t**
Mu(μ)	0.048	-0.00011	0.002	-7.82E-05	0.003	-4.29E-05
	0.053	0.4221	0.474	0.2887	0.3602	0.99
Ar1(ϕ)	0.998	0.372	0.394	0.351	0.45	0.38
	0.000	0.000	0.466	0.000	0.14	0.000
Ma(θ_1)	-0.883	-0.042	0.056	-0.081	-0.032	-0.049
	0.000	0.336	0.991	0.011	0.89	0.042
Ma(θ_2)	-0.109	0.00048	0.009	-0.008	0.002	0.0026
	0.035	0.975	0.971	0.375	0.99	0.364
Omega (ω)	1.26E-07	2.05E-06	0.0002	6.55E-07	0.00065	0.0016
	0.908	0.007	0.000	0.0134	0.863	0.605
Alpha (α_1)	5.272	2.022	0.842	7.233	0.918	6.957
	0.000	0.0292	0.000	0.039	0.0007	0.352
Beta(β_1)	0.006	-0.0015	-0.168	-0.125	-0.197	-0.155
	0.5007	0.6735	0.101	0.192	0.094	0.127
Gamma(γ_1)			-0.028	-0.002	-0.043	0.015
			0.376	0.604	0.739	0.602
Delta (δ)			2	2	1.8	0.893
					0.198	0.0018
Shape (v)				2.253		2.025
				0.000		0.000

Source :Eviews 8

Table (4-5) presents the parameter estimation results of ARIMA (1,1,2) , GARCH (1, 1), GJR-GARCH (1, 1) and DGE-GARCH (1, 1) models with the normal and, student's-t distributions and their corresponding p-values. The results show that the parameters estimated in these three models are all

significant under the given conditional distributions except for the coefficients of Mu. Under the student's- t distribution, the sum of the GARCH parameter estimates $(\alpha_i + \beta_j)$ is greater than 1, implying that the volatility rate model is strictly stationary GARCH model is less than 1, which indicates that the model is well fitted. For the normal distribution the sum of the GARCHparameter is less than 1 for the GJR-GARCH and DGE-GARCH models with the which also show that the shocks in volatility is limited and stationary and the model is well fitted, the sum is greater than1 in case of the GARCH model. The leverage effect term (gamma) in both the GJR model and the DGE is not statistically significant but it is negative, implying that negative shocks results to a higher next period

conditional variance thanpositive shocks of the same sign, it indicates that the bad news (negative shocks) effects the volatility more than the good news. The table shows that the estimated **δ**of the DGE -GARCH model under the normal distribution is 1.8 is not significant which is significantly in student-t distribution.

Table (4-6) **Analysis of standardized residuals and fitted parameters**

	GARCH		GJR-GARCH		DEG-GARCH	
	Normal	**Student-t**	**Normal**	**Student-t**	**Normal**	**Student-t**
Log likelihood	**504.885**	**675.582**	**466.085**	**695.039**	**464.773**	**708.127**
Jarque – Bera Test	**4134.971**	**36547.94**	**7546.622**	**42675.11**	**8051.477**	**50508.8**
	0.000	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**
Ljung- Box Test R (Q10)	**12.69**	**1.2776**	**3.3783**	**0.3547**	**4.1727**	**0.0376**
	0.08	**0.989**	**0.848**	**1.000**	**0.76**	**1.000**
Ljung- Box Test R (Q15)	**20.483**	**17.563**	**16.566**	**18.276**	**17.43**	**20.51**
	0.058	**0.130**	**0.167**	**0.108**	**0.134**	**0.058**
Ljung- Box Test R (Q20)	**32.424**	**42.635**	**37.737**	**45.818**	**39.782**	**45.216**
	0.013	**0.001**	**0.003**	**0.000**	**0.001**	**0.000**
Ljung- Box Test R^2 (Q10)	**1.462**	**0.2803**	**2.9889**	**0.2483**	**2.2772**	**0.2189**
	0.999	**1.000**	**0.982**	**1.000**	**0.994**	**1.000**
Ljung- Box Test R^2 (Q15)	**3.533**	**8.3468**	**13.078**	**8.2591**	**11.666**	**9.013**
	0.999	**0.909**	**0.596**	**0.913**	**0.704**	**0.877**
Ljung- Box Test R^2 (Q20)	**41.934**	**26.456**	**35.174**	**27.357**	**33.795**	**22.171**
	0.003	**0.151**	**0.019**	**0.126**	**0.028**	**0.331**
LM Arch Test	**0.0167**	**0.0317**	**2.404**	**0.0293**	**1.7665**	**0.025**
	0.897	**0.858**	**0.1224**	**0.864**	**0.183**	**0.874**
AIC	**-5.594**	**-7.5009**	**-5.147**	**-7.7088**	**-5.121**	**-7.844**
BIC	**-5.469**	**-7.357**	**-5.004**	**-7.547**	**-4.960**	**-7.665**

Source **:**Eviews 8

The coefficients reported as shown in the table(4-6) are the maximum likelihood estimates of the parameters and the p-values are in parentheses for the ARIMA (1,1,2) - GARCH (1,1), GJR-GARCH (1, 1) and DGE-GARCH (1, 1), models. The estimation results of the models with the conditional distributions, including log-likelihood value, the Box-Pierce statistics of lags 10, 15 and 20 of the standardized and squared standardized residuals, the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC), the ARCH test and their respective p-values are listed in Table 3. Comparing the log-likelihood, the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) values among these models DGE-GARCH and GJR-GARCH models better estimate the exchange rate return series than the GARCH model with the Student t-distribution assumption gives better results. The results also show that the student's t-distribution outperforms the normal distribution, discussed in this chapter. With these models, DEG-GARCH with Student t-distribution gives the highest log-likelihood value of 708.124. The AIC and BIC values of the GARCH and DEG -GARCH models under the three conditional distribution gives the lowest values when compared to the GJR-GARCH and GARCH models and that the DEG -GARCH model with the student's t-distribution provides the smallest values of AIC (-7.884) and BIC (-7.665) respectively, this implies that DEG -GARCH model under the student's t-distribution provides a better fit for the monthly exchange rate returns according to this criterions.

The table shown the t-statistics and p-values are in parentheses for ARIMA (1,1, 2)- GARCH(1,1),GJR(1,1) and DGE(1, 1) models.(AIC) represent Akaike Information Criterion, (BIC) is Bayesian Information Criterion (BIC), Ljung-Box Test R (Standardized Residuals and Ljung-Box TestR^2 (Square

Standardized Residual) The Jarque-Bera statistic to test the null hypothesis of whether the standardized residuals are normally distributed. The results presented in table 3 show that the standardized residuals are leptokurtic and the Jarque-Bera statistic strongly rejects the hypothesis of normal distribution which means that the fat-tailed asymmetric conditional distributions outperform the normal for modeling and forecasting the exchange rates volatility returns. The Ljung Box tests for the residuals have p-values that are statistically not significant indicating that no serial correlation exists except twentieth-order. The Ljung-Box statistics for up to twentieth-order serial correlation of squared residuals are not significant suggesting that no significance correlation exist. As for the LM-ARCH test the results reveals that the conditional heteroskedasticity that existed in the exchange rate returns time series have successfully removed, indicating that no significant appearance of the ARCH effect

4-8 Forecasting:

The forecasting ability of the GARCH models has been discussed precisely by Poon and Granger (2003). We use the Eviews 8 to evaluate a five step ahead forecast using 180 observations for the monthly exchange rate returns. The forecasts are evaluated using three different measures which provide robustness in choosing the optimal predicts models for the return series

Table (4-7)
Forecasting Analysis for the Exchange rate returns with the Conditional distributions

Exchange rate returns (SDG /USA($)	**GARCH**		**GJR-GARCH**		**DEG-GARCH**	
	Normal	**Student**	**Normal**	**Student**	**Normal**	**Student**
MSE	0.00406	0.001354	0.001337	0.001354	0.001334	0.001353
MAE	0.058991	0.011101	0.012129	0.011098	0.012587	0.011080
AMAPE	8582.155	90.45945	372.7228	87.82762	491.7924	83.77597

Source:Eviews 8

The results, as shown in the table (4-7) above, indicate that the forecasting performance of the GJR-GARCH and DGE-GARCH models, especially when fat-tailed asymmetric conditional distributions are taken into account in the conditional volatility, is better than the GARCH model. However, the comparison between the models with normal and student-t distributions shows that, according to the different measures used for evaluating the performance of volatility forecasts, the DEG –GARCH model provides the best forecasts and clearly outperforms GJR-GARCH and GARCH models and the DGE-GARCH model provides less satisfactory forecast results while the poorest forecast results was registered for the GARCH model. Moreover, it is found that the Student-t distribution is more appropriate for modeling and forecasting the exchange rate returns volatility.

CHAPTER FIVE

5-1 Conclusion

5-2 Recommendations

5-1 Conclusion:

Modelling and forecasting the volatility of returns series in stock markets has become fertile field of empirical research in financial markets. This is simply because volatility is considered as an important concept in many economic and financial applications like asset pricing; risk management and portfolio allocation this thesis attempts to explore the comparative ability of different statistical and econometric volatility forecasting models in the context of exchange rate market. A total of five different models were considered in this study and comparative with normal and student-t distribution. The volatility of the exchange rate returns in Sudan have been modeled by using a Generalized Autoregressive Conditional heteroskedasticity (GARCH) models including both symmetric and asymmetric models that captures most common stylized facts about index returns such as volatility clustering and leverage effect, these models are GARCH(1,1), GARCH-M(1,1), exponential GARCH(1,1), threshold GARCH(1,1) and power GARCH(1,1). Based on the empirical results presented, the following can be concluded:

10- The summary statistics indicate that the returns series have monthly positive mean (0.0051) while the volatility is (0.013) without loss of generality the mean grows at linear rate while the volatility grows approximately at square root rate.

11- The returns series of the exchange rate shows positive skewness this implies that the series of exchange rate is flatter to the right

12- The kurtosis value is the higher than the normal and this suggest that the kurtosis curve of the exchange rate return series is leptokurtic.

13- The coefficient in the condition variance equation GARCH(1,1) the α significant and β not significant and the ($\alpha+\beta$) is greater than one suggesting that the condition variance process is explosive.

14- The coefficient (risk premium) of Rt in the mean equation is positive of the market which indicate the mean of the return sequence depend on past innovation and the past conditional variance.

15- The estimation of EGARCH(1,1) model for return series of exchange rate the γ is negative and significant meaning that return series have asymmetry and has greater impact of negative shocks indicate that the conditional variance has leverage effect and asymmetry of negative shocks.

16- The result indicate that the forecasting performance of the GJR-GARCH(1,1) and DGE-GARCH(1,1) models especially when fat-tailed asymmetric conditional distribution are taken into account in the conditional volatility is better than the GARC(1,1) model.

17- However the comparison between the models with normal and student-t distribution shows that according to the different measures used for evaluating the performance of volatility forecasts the DGE-GARCH(1,1) model provides the best forecasts.

18- It is a found that the student-t distribution is more appropriates for modeling and forecasting exchange rate return volatility.

5-2 Recommendations:

Following the analysis and conclusions presented above, somesuggestion concerning future research in the area may be made to fill thegap:

1- The study applied Generalized Autoregressive Conditional heteroskedasticity (GARCH) models including both symmetric and asymmetric modelsfor only monthly returnsof exchangerates, and revealed the presence of time-varyingMore research is needed to see whether this appliesalso to other time periods e.g. daily, weekly, quartly.

2- There is the possibility that a wide range of factors may relevant inexplaining the stock returns volatility such as good prices, money supply,

real activity, political risks,……..,etc. To find the effects ofthese factors on stock return volatility further research is require

3- It should be noted that this research was concerned on suitability of Generalized Autoregressive Conditional Heteroskedasticity (GARCH) approaches to model and forecastExchange rate in the Sudan. In terms of limitations, future research should be done for a hybrid method, specifically combines ARIMA with GARCH, non linear time series approaches for instance, artificial neural network (ANN) models as well as multivariate GARCH models. More over GARCH models could be applied to other types of economic sectors.

The above points are just a few interesting fields for further research. Volatility forecasts and its related subjects will most certainly continue to attract a lot of empirical work in future.

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Annex

Monthly readings of Exchange-Rate covered the period from 1/1/ 1999 to 31/12/2013

Month \ Year	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013
January	2.2649	2.569	2.5684	2.6069	2.609	2.5932	2.4999	2.2986	2.0004	2.0416	2.2163	2.2330	2.5004	2.6702	4.398
February	2.4011	2.5684	2.6085	2.6096	2.6063	2.5904	2.4973	2.2907	2.0018	2.0188	2.2423	2.2306	2.6015	2.6702	4.398
March	2.4431	2.5677	2.5661	2.6103	2.6049	2.5883	2.4904	2.2658	2.0005	2.0214	2.2868	2.2281	2.7747	2.6702	4.398
April	2.4791	2.5663	2.5666	2.6123	2.604	2.5873	2.4866	2.2383	2.0002	2.0235	2.3199	2.2255	2.6814	2.6702	4.398
May	2.5187	2.5667	2.5671	2.6144	2.6029	2.588	2.483	2.212	2.0007	2.0324	2.3502	2.2261	2.6702	2.6702	4.398
June	2.5572	2.567	2.5675	2.6207	2.601	2.5885	2.468	2.1852	2.0006	2.0526	2.367	2.3113	2.6702	3.0322	4.398
July	2.5699	2.5652	2.5675	2.6315	2.6014	2.5864	2.4502	2.1519	2.0002	2.0623	2.3857	2.3666	2.6702	4.4037	4.398
August	2.5708	2.5608	2.5732	2.634	2.6035	2.5818	2.4197	2.1232	2.0006	2.07561	2.4462	2.3668	2.6702	4.398	4.398
September	2.5747	2.5614	2.5764	2.6355	2.6046	2.5771	2.3971	2.0973	2.0218	2.1262	2.3519	2.3668	2.6702	4.398	4.6875
October	2.5745	2.5628	2.5868	2.6359	2.6048	2.5528	2.351	2.0685	2.0466	2.18101	2.1932	2.3668	2.6702	4.398	5.6717
November	2.5718	2.5664	2.6102	2.6322	2.6022	2.5271	2.3173	2.0419	2.026	2.2041	2.2554	2.1932	2.6702	4.398	5.6814
December	2.5702	2.5674	2.6067	2.6179	2.5971	2.5147	2.301	2.0198	2.0286	2.1897	2.2359	2.4800	2.6702	4.398	5.6816

Source: Bank of Sudan and Central Bureau of Statistics

Appendix:

Figure (4-2) Augmented Dickey-Fuller Unit Root Test on Exchange rate series

Null Hypothesis: P has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic - based on SIC, maxlag=13)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		2.416686	1.0000
Test critical values:	1% level	-3.466994	
	5% level	-2.877544	
	10% level	-2.575381	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(P)
Method: Least Squares
Date: 09/24/15 Time: 23:00
Sample (adjusted): 1999M02 2013M12
Included observations: 179 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
P(-1)	0.035842	0.014831	2.416686	0.0167
C	-0.075203	0.040278	-1.867102	0.0635

R-squared	0.031942	Mean dependent var	0.019088
Adjusted R-squared	0.026473	S.D. dependent var	0.135613
S.E. of regression	0.133806	Akaike info criterion	-1.173740
Sum squared resid	3.169019	Schwarz criterion	-1.138127
Log likelihood	107.0497	Hannan-Quinn criter.	-1.159299
F-statistic	5.840370	Durbin-Watson stat	1.668076
Prob(F-statistic)	0.016678		

Figure (4-6) Augmented Dickey-Fuller Unit Root Test on Exchange returns series

Null Hypothesis: PC has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic - based on SIC, maxlag=13)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-10.70231	0.0000
Test critical values:	1% level	-3.467205	
	5% level	-2.877636	
	10% level	-2.575430	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(PC)
Method: Least Squares
Date: 09/24/15 Time: 22:24
Sample (adjusted): 1999M03 2013M12
Included observations: 178 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
PC(-1)	-0.783803	0.073237	-10.70231	0.0000
C	0.004301	0.003117	1.380220	0.1693

R-squared	0.394230	Mean dependent var	-0.000338
Adjusted R-squared	0.390788	S.D. dependent var	0.052754
S.E. of regression	0.041175	Akaike info criterion	-3.530780
Sum squared resid	0.298392	Schwarz criterion	-3.495030
Log likelihood	316.2395	Hannan-Quinn criter.	-3.516283
F-statistic	114.5393	Durbin-Watson stat	1.979214
Prob(F-statistic)	0.000000		

Figure (4-7) Parameter Estimation of an ARIMA (1,1,0)

Dependent Variable: RT
Method: Least Squares
Date: 05/29/16 Time: 11:45
Sample (adjusted): 1999M03 2013M12
Included observations: 178 after adjustments
Convergence achieved after 3 iterations

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.004743	0.003464	1.369035	0.1727
AR(1)	0.226312	0.072965	3.101638	0.0022

R-squared	0.051827	Mean dependent var	0.004839
Adjusted R-squared	0.046440	S.D. dependent var	0.036618
S.E. of regression	0.035758	Akaike info criterion	-3.812939
Sum squared resid	0.225034	Schwarz criterion	-3.777189
Log likelihood	341.3516	Hannan-Quinn criter.	-3.798441
F-statistic	9.620160	Durbin-Watson stat	1.978127
Prob(F-statistic)	0.002242		

Inverted AR Roots	.23

Figure (4-8) Parameter Estimation of an ARIMA (0,1,1)

Dependent Variable: RT
Method: Least Squares
Date: 05/29/16 Time: 11:48
Sample (adjusted): 1999M02 2013M12
Included observations: 179 after adjustments
Convergence achieved after 6 iterations
MA Backcast: 1999M01

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.005196	0.003341	1.554870	0.1218
MA(1)	0.250204	0.072798	3.436958	0.0007

R-squared	0.055919	Mean dependent var	0.005138
Adjusted R-squared	0.050585	S.D. dependent var	0.036734
S.E. of regression	0.035793	Akaike info criterion	-3.811039
Sum squared resid	0.226757	Schwarz criterion	-3.775426
Log likelihood	343.0880	Hannan-Quinn criter.	-3.796598
F-statistic	10.48382	Durbin-Watson stat	2.007965
Prob(F-statistic)	0.001438		

Inverted MA Roots	-.25

Figure (4-9) Parameter Estimation of an ARIMA (1,1,1)

Dependent Variable: RT
Method: Least Squares
Date: 05/29/16 Time: 11:47
Sample (adjusted): 1999M03 2013M12
Included observations: 178 after adjustments
Convergence achieved after 11 iterations
MA Backcast: 1999M02

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.004891	0.003289	1.487162	0.1388
AR(1)	-0.078334	0.288020	-0.271972	0.7860
MA(1)	0.322465	0.275634	1.169904	0.2436

R-squared	0.054969	Mean dependent var	0.004839
Adjusted R-squared	0.044169	S.D. dependent var	0.036618
S.E. of regression	0.035800	Akaike info criterion	-3.805022
Sum squared resid	0.224288	Schwarz criterion	-3.751397
Log likelihood	341.6470	Hannan-Quinn criter.	-3.783276
F-statistic	5.089551	Durbin-Watson stat	2.003110
Prob(F-statistic)	0.007104		

Inverted AR Roots	-.08
Inverted MA Roots	-.32

Figure (4-10) Parameter Estimation of an ARIMA (1,1,2)

Dependent Variable: RT
Method: Least Squares
Date: 05/29/16 Time: 11:50
Sample (adjusted): 1999M03 2013M12
Included observations: 178 after adjustments
Convergence achieved after 10 iterations
MA Backcast: 1999M01 1999M02

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.004722	0.003334	1.416301	0.1585
AR(1)	0.290732	0.625609	0.464719	0.6427
MA(1)	-0.047137	0.629846	-0.074839	0.9404
MA(2)	-0.079862	0.169638	-0.470780	0.6384

R-squared	0.055561	Mean dependent var	0.004839
Adjusted R-squared	0.039277	S.D. dependent var	0.036618
S.E. of regression	0.035892	Akaike info criterion	-3.794413
Sum squared resid	0.224148	Schwarz criterion	-3.722912
Log likelihood	341.7027	Hannan-Quinn criter.	-3.765417
F-statistic	3.412106	Durbin-Watson stat	2.005977
Prob(F-statistic)	0.018781		

Inverted AR Roots		.29	
Inverted MA Roots	.31		-.26

Figure (4-11) Parameter Estimation of an ARIMA (2,1,1)

Dependent Variable: RT
Method: Least Squares
Date: 05/29/16 Time: 11:52
Sample (adjusted): 1999M04 2013M12
Included observations: 177 after adjustments
Convergence achieved after 17 iterations
MA Backcast: 1999M03

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.004441	0.003407	1.303463	0.1941
AR(1)	-0.703097	0.075250	-9.343428	0.0000
AR(2)	0.157533	0.075446	2.088011	0.0383
MA(1)	0.993007	0.006458	153.7580	0.0000

R-squared	0.098091	Mean dependent var	0.004768
Adjusted R-squared	0.082451	S.D. dependent var	0.036710
S.E. of regression	0.035164	Akaike info criterion	-3.835265
Sum squared resid	0.213912	Schwarz criterion	-3.763488
Log likelihood	343.4210	Hannan-Quinn criter.	-3.806155
F-statistic	6.271804	Durbin-Watson stat	2.005670
Prob(F-statistic)	0.000459		

Inverted AR Roots	.18		-.88
Inverted MA Roots		-.99	

Figure (4-12) Parameter Estimation of an ARIMA (2,1,2)

Dependent Variable: RT
Method: Least Squares
Date: 05/29/16 Time: 11:53
Sample (adjusted): 1999M04 2013M12
Included observations: 177 after adjustments
Convergence achieved after 13 iterations
MA Backcast: 1999M02 1999M03

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.004466	0.003473	1.285832	0.2002
AR(1)	-0.608478	0.451665	-1.347190	0.1797
AR(2)	0.236453	0.378088	0.625393	0.5325
MA(1)	0.895287	0.463007	1.933635	0.0548
MA(2)	-0.097366	0.461078	-0.211170	0.8330

R-squared	0.098357	Mean dependent var	0.004768
Adjusted R-squared	0.077389	S.D. dependent var	0.036710
S.E. of regression	0.035261	Akaike info criterion	-3.824261
Sum squared resid	0.213849	Schwarz criterion	-3.734539
Log likelihood	343.4471	Hannan-Quinn criter.	-3.787873
F-statistic	4.690729	Durbin-Watson stat	1.998152
Prob(F-statistic)	0.001285		

Inverted AR Roots	.27	-.88
Inverted MA Roots	.10	-.99

Figure (4-13) Estimation parameters of GARCH (1,1)

Dependent Variable: RT
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 06/01/16 Time: 15:33
Sample (adjusted): 1999M03 2013M12
Included observations: 178 after adjustments
Convergence achieved after 49 iterations
MA Backcast: 1999M01 1999M02
Presample variance: backcast (parameter = 0.7)
GARCH = C(5) + C(6)*RESID(-1)^2 + C(7)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.038449	0.016460	2.335948	0.0195
AR(1)	0.997768	0.000881	1132.227	0.0000
MA(1)	-0.852568	0.051117	-16.67864	0.0000
MA(2)	-0.139888	0.050600	-2.764604	0.0057
		Variance Equation		
C	1.31E-07	1.12E-06	0.117030	0.9068
RESID(-1)^2	5.452500	0.437139	12.47315	0.0000
GARCH(-1)	0.008452	0.008590	0.983988	0.3251

R-squared	0.062740	Mean dependent var	0.004839
Adjusted R-squared	0.046580	S.D. dependent var	0.036618
S.E. of regression	0.035755	Akaike info criterion	-5.607740
Sum squared resid	0.222444	Schwarz criterion	-5.482613
Log likelihood	506.0888	Hannan-Quinn criter.	-5.556997
Durbin-Watson stat	1.849759		

Inverted AR Roots		1.00	
Inverted MA Roots	.99		-.14

Figure (4-14) ARCH LM test on GARCH (1,1) model

Heteroskedasticity Test: ARCH

F-statistic	0.012019	Prob. F(1,175)	0.9128
Obs*R-squared	0.012156	Prob. Chi-Square(1)	0.9122

Figure (4-15) Estimation parameters of EGARCH (1,1)

Dependent Variable: RT
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 06/01/16 Time: 15:35
Sample (adjusted): 1999M03 2013M12
Included observations: 178 after adjustments
Convergence achieved after 43 iterations
MA Backcast: 1999M01 1999M02
Presample variance: backcast (parameter = 0.7)
LOG(GARCH) = C(5) + C(6)*ABS(RESID(-1)/@SQRT(GARCH(-1))) + C(7)
*RESID(-1)/@SQRT(GARCH(-1)) + C(8)*LOG(GARCH(-1))

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	-0.000767	0.000610	-1.255623	0.2093
AR(1)	0.599041	0.030728	19.49493	0.0000
MA(1)	-0.605905	0.063558	-9.533080	0.0000
MA(2)	0.022571	0.022652	0.996437	0.3190
Variance Equation				
C(5)	-15.10780	0.124333	-121.5108	0.0000
C(6)	0.689774	0.041855	16.47991	0.0000
C(7)	-0.075748	0.037295	-2.031057	0.0422
C(8)	-0.806909	0.006544	-123.2994	0.0000

R-squared	-0.022317	Mean dependent var	0.004839
Adjusted R-squared	-0.039944	S.D. dependent var	0.036618
S.E. of regression	0.037342	Akaike info criterion	-5.177893
Sum squared resid	0.242631	Schwarz criterion	-5.034892
Log likelihood	468.8325	Hannan-Quinn criter.	-5.119902
Durbin-Watson stat	1.514683		

Inverted AR Roots		.60	
Inverted MA Roots	.57		.04

Figure (4-16) ARCH LM test on EGARCH (1,1) model

Heteroskedasticity Test: ARCH

F-statistic	5.535514	Prob. F(1,175)	0.0197
Obs*R-squared	5.427110	Prob. Chi-Square(1)	0.0198

Figure (4-17) Estimation parameters of APARCH (1,1) model

Dependent Variable: RT
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 06/01/16 Time: 15:37
Sample (adjusted): 1999M03 2013M12
Included observations: 178 after adjustments
Failure to improve Likelihood after 33 iterations
MA Backcast: 1999M01 1999M02
Presample variance: backcast (parameter = 0.7)
@SQRT(GARCH)^C(9) = C(5) + C(6)*(ABS(RESID(-1)) - C(7)*RESID(-1))^C(9) + C(8)*@SQRT(GARCH(-1))^C(9)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.003082	0.003369	0.914933	0.3602
AR(1)	0.451778	0.313039	1.443203	0.1490
MA(1)	-0.032673	0.253721	-0.128776	0.8975
MA(2)	0.002733	0.232577	0.011753	0.9906
		Variance Equation		
C(5)	0.000655	0.003809	0.172055	0.8634
C(6)	0.918552	0.270347	3.397678	0.0007
C(7)	-0.197651	0.118333	-1.670297	0.0949
C(8)	-0.043835	0.131930	-0.332262	0.7397
C(9)	1.804122	1.403332	1.285599	0.1986

R-squared	0.010270	Mean dependent var	0.004839
Adjusted R-squared	-0.006795	S.D. dependent var	0.036618
S.E. of regression	0.036742	Akaike info criterion	-5.121045
Sum squared resid	0.234897	Schwarz criterion	-4.960168
Log likelihood	464.7730	Hannan-Quinn criter.	-5.055805
Durbin-Watson stat	2.305904		

Inverted AR Roots	.45	
Inverted MA Roots	.02-.05i	.02+.05i

Figure (4-18) ARCH LM test on APARCH (1,1) model

Heteroskedasticity Test: ARCH

F-statistic	1.764225	Prob. F(1,175)	0.1858
Obs*R-squared	1.766579	Prob. Chi-Square(1)	0.1838

Figure (4-19) Estimation parameters of TGARCH (1,1) model

Dependent Variable: RT
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 06/01/16 Time: 15:41
Sample (adjusted): 1999M03 2013M12
Included observations: 178 after adjustments
Convergence achieved after 45 iterations
MA Backcast: 1999M01 1999M02
Presample variance: backcast (parameter = 0.7)
GARCH = C(5) + C(6)*RESID(-1)^2 + C(7)*RESID(-1)^2*(RESID(-1)<0) + C(8)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.002201	0.003298	0.667380	0.5045
AR(1)	0.506841	0.329913	1.536287	0.1245
MA(1)	-0.069281	0.342482	-0.202292	0.8397
MA(2)	0.004778	0.217589	0.021960	0.9825
Variance Equation				
C	0.000268	1.71E-05	15.70029	0.0000
RESID(-1)^2	1.442378	0.531462	2.713983	0.0066
RESID(-1)^2*(RESID(-1)<0)	-0.841730	0.564719	-1.490529	0.1361
GARCH(-1)	-0.027821	0.047401	-0.586926	0.5573

R-squared	-0.002686	Mean dependent var	0.004839
Adjusted R-squared	-0.019974	S.D. dependent var	0.036618
S.E. of regression	0.036982	Akaike info criterion	-5.159068
Sum squared resid	0.237972	Schwarz criterion	-5.016067
Log likelihood	467.1571	Hannan-Quinn criter.	-5.101077
Durbin-Watson stat	2.321562		

Inverted AR Roots	.51	
Inverted MA Roots	.03+.06i	.03-.06i

Figure (4-20) ARCH LM test on TGARCH (1,1) model

Heteroskedasticity Test: ARCH

F-statistic	1.215661	Prob. F(1,175)	0.2717
Obs*R-squared	1.221072	Prob. Chi-Square(1)	0.2692

Figure (4-21) Estimation parameters of Component ARCH (1,1) model

Dependent Variable: RT
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 06/01/16 Time: 15:46
Sample (adjusted): 1999M03 2013M12
Included observations: 178 after adjustments
Convergence achieved after 26 iterations
MA Backcast: 1999M01 1999M02
Presample variance: backcast (parameter = 0.7)
Q = C(5) + C(6)*(Q(-1) - C(5)) + C(7)*(RESID(-1)^2 - GARCH(-1))
GARCH = Q + C(8) * (RESID(-1)^2 - Q(-1)) + C(9)*(GARCH(-1) - Q(-1))

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.002531	0.003323	0.761525	0.4463
AR(1)	0.119622	0.961306	0.124436	0.9010
MA(1)	0.093069	0.974823	0.095473	0.9239
MA(2)	0.022932	0.202190	0.113417	0.9097
Variance Equation				
C(5)	0.000564	3.82E-05	14.76191	0.0000
C(6)	0.221740	0.300145	0.738777	0.4600
C(7)	0.080495	0.011658	6.904767	0.0000
C(8)	0.073633	0.005013	14.68846	0.0000
C(9)	-0.179860	0.360903	-0.498360	0.6182

R-squared	0.048483	Mean dependent var	0.004839
Adjusted R-squared	0.032077	S.D. dependent var	0.036618
S.E. of regression	0.036026	Akaike info criterion	-4.637040
Sum squared resid	0.225828	Schwarz criterion	-4.476163
Log likelihood	421.6966	Hannan-Quinn criter.	-4.571800
Durbin-Watson stat	1.947993		

Inverted AR Roots	.12	
Inverted MA Roots	-.05+.14i	-.05-.14i

Figure (4-22) ARCH LM test on Component ARCH (1,1) model

Heteroskedasticity Test: ARCH

F-statistic	38.10034	Prob. F(1,175)	0.0000
Obs*R-squared	31.64594	Prob. Chi-Square(1)	0.0000

Figure (4-23) Estimation parameters of GARCH-M(1.1)) model

Dependent Variable: RT
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 06/01/16 Time: 16:00
Sample (adjusted): 1999M03 2013M12
Included observations: 178 after adjustments
Convergence achieved after 41 iterations
MA Backcast: 1999M01 1999M02
Presample variance: backcast (parameter = 0.7)
GARCH = 0.0012750136671*(1 - C(5) - C(6)) + C(5)*RESID(-1)^2 + C(6)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	14.53483	23352.47	0.000622	0.9995
AR(1)	1.000007	0.010631	94.06838	0.0000
MA(1)	-0.679863	0.030529	-22.26975	0.0000
MA(2)	-0.271186	0.029126	-9.310915	0.0000
		Variance Equation		
C	3.87E-07	--	--	--
RESID(-1)^2	0.117539	0.000629	186.7393	0.0000
GARCH(-1)	0.882158	0.000536	1646.798	0.0000

R-squared	0.043743	Mean dependent var	0.004839
Adjusted R-squared	0.027256	S.D. dependent var	0.036618
S.E. of regression	0.036115	Akaike info criterion	-5.634926
Sum squared resid	0.226952	Schwarz criterion	-5.527674
Log likelihood	507.5084	Hannan-Quinn criter.	-5.591432
Durbin-Watson stat	2.139980		

Inverted AR Roots	1.00	
	Estimated AR process is nonstationary	
Inverted MA Roots	.96	-.28

Figure (4-24) ARCH LM test on GARCH-M (1,1) model

Heteroskedasticity Test: ARCH

F-statistic	29.67396	Prob. F(1,175)	0.0000
Obs*R-squared	25.66174	Prob. Chi-Square(1)	0.0000

Figure (4-25) Estimation parameters of GARCH(1,1)) model
Student's t distribution

Dependent Variable: RT
Method: ML - ARCH (Marquardt) - **Student's t distribution**
Date: 02/06/16 Time: 15:07
Sample (adjusted): 1999M03 2013M12
Included observations: 178 after adjustments
Convergence achieved after 27 iterations
MA Backcast: 1999M01 1999M02
Presample variance: backcast (parameter = 0.7)
GARCH = C(5) + C(6)*RESID(-1)^2 + C(7)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	-0.000115	0.000143	-0.802774	0.4221
AR(1)	0.372570	0.026692	13.95811	0.0000
MA(1)	-0.042051	0.043731	-0.961580	0.3363
MA(2)	0.000480	0.015722	0.030556	0.9756
		Variance Equation		
C	2.05E-06	7.59E-07	2.694730	0.0070
RESID(-1)^2	2.022671	0.927461	2.180870	0.0292
GARCH(-1)	-0.001590	0.003774	-0.421288	0.6735
T-DIST. DOF	2.287487	0.153527	14.89959	0.0000

R-squared	0.031229	Mean dependent var	0.004839
Adjusted R-squared	0.014526	S.D. dependent var	0.036618
S.E. of regression	0.036351	Akaike info criterion	-7.500926
Sum squared resid	0.229922	Schwarz criterion	-7.357925
Log likelihood	675.5824	Hannan-Quinn criter.	-7.442935
Durbin-Watson stat	2.149576		

Inverted AR Roots		.37	
Inverted MA Roots	.02		.02

Figure (4-26) ARCH LM test on GARCH (1,1) model

Heteroskedasticity Test: ARCH

F-statistic	0.031420	Prob. F(1,175)	0.8595
Obs*R-squared	0.031773	Prob. Chi-Square(1)	0.8585

Figure (4-27) Estimation parameters of APARCH(1,1)) model

Student's t distribution

Dependent Variable: RT
Method: ML - ARCH (Marquardt) - **Student's t distribution**
Date: 02/06/16 Time: 15:24
Sample (adjusted): 1999M03 2013M12
Included observations: 178 after adjustments
Convergence achieved after 52 iterations
MA Backcast: 1999M01 1999M02
Presample variance: backcast (parameter = 0.7)
@SQRT(GARCH)^C(9) = C(5) + C(6)*(ABS(RESID(-1)) - C(7)*RESID(-1))^C(9) + C(8)*@SQRT(GARCH(-1))^C(9)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	-4.29E-08	1.57E-05	-0.002728	0.9978
AR(1)	0.380958	0.024603	15.48400	0.0000
MA(1)	-0.049849	0.024573	-2.028573	0.0425
MA(2)	0.002635	0.002904	0.907564	0.3641
		Variance Equation		
C(5)	0.001671	0.003233	0.516923	0.6052
C(6)	6.957140	7.479854	0.930117	0.3523
C(7)	-0.155892	0.102206	-1.525280	0.1272
C(8)	0.015149	0.029121	0.520222	0.6029
C(9)	0.893790	0.286698	3.117533	0.0018
T-DIST. DOF	2.025636	0.041077	49.31374	0.0000

R-squared	0.030704	Mean dependent var	0.004839
Adjusted R-squared	0.013992	S.D. dependent var	0.036618
S.E. of regression	0.036361	Akaike info criterion	-7.844127
Sum squared resid	0.230047	Schwarz criterion	-7.665376
Log likelihood	708.1273	Hannan-Quinn criter.	-7.771639
Durbin-Watson stat	2.150631		

Inverted AR Roots	.38	
Inverted MA Roots	.02-.04i	.02+.04i

Figure (4-28) ARCH LM test on APARCH (1,1) model

Heteroskedasticity Test: ARCH

F-statistic	0.024801	Prob. F(1,175)	0.8750
Obs*R-squared	0.025081	Prob. Chi-Square(1)	0.8742

Figure (4-29) Estimation parameters of GJR- GARCH(1,1)) model

Student's t distribution

Dependent Variable: RT
Method: ML - ARCH (Marquardt) - **Student's t distribution**
Date: 02/06/16 Time: 15:42
Sample (adjusted): 1999M03 2013M12
Included observations: 178 after adjustments
Convergence achieved after 40 iterations
MA Backcast: 1999M01 1999M02
Presample variance: backcast (parameter = 0.7)
GARCH = C(5) + C(6)*(ABS(RESID(-1)) - C(7)*RESID(-1))^2 + C(8)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	-7.82E-05	7.37E-05	-1.060885	0.2887
AR(1)	0.351250	0.018432	19.05652	0.0000
MA(1)	-0.081387	0.032262	-2.522711	0.0116
MA(2)	-0.008677	0.009796	-0.885829	0.3757
		Variance Equation		
C(5)	6.55E-07	2.65E-07	2.473207	0.0134
C(6)	7.233805	3.518592	2.055880	0.0398
C(7)	-0.125300	0.096085	-1.304058	0.1922
C(8)	-0.002121	0.004093	-0.518284	0.6043
T-DIST. DOF	2.253724	0.139430	16.16389	0.0000

R-squared	0.039633	Mean dependent var	0.004839
Adjusted R-squared	0.023074	S.D. dependent var	0.036618
S.E. of regression	0.036193	Akaike info criterion	-7.708311
Sum squared resid	0.227928	Schwarz criterion	-7.547435
Log likelihood	695.0397	Hannan-Quinn criter.	-7.643071
Durbin-Watson stat	2.039945		

Inverted AR Roots		.35	
Inverted MA Roots	.14		-.06

Figure (4-30) ARCH LM test onGJR- GARCH (1,1) model

Heteroskedasticity Test: ARCH

F-statistic	0.029022	Prob. F(1,175)	0.8649
Obs*R-squared	0.029349	Prob. Chi-Square(1)	0.8640

Figure (4-31) Forecast with GARCH(1.1)model

Normal distribution

Forecast: RTF
Actual: RT
Forecast sample: 1999M01 2013M12
Adjusted sample: 1999M03 2013M12
Included observations: 178

Root Mean Squared Error	0.063775
Mean Absolute Error	0.058991
Mean Absolute Percentage Error	8582.155
Theil Inequality Coefficient	0.679456
Bias Proportion	0.669529
Variance Proportion	0.314371
Covariance Proportion	0.016100

Figure (4-32) Forecast with GARCH(1.1)model

Student's t distribution

Forecast: RTF
Actual: RT
Forecast sample: 1999M01 2013M12
Adjusted sample: 1999M03 2013M12
Included observations: 178

Root Mean Squared Error	0.036805
Mean Absolute Error	0.011101
Mean Absolute Percentage Error	90.45945
Theil Inequality Coefficient	0.953848
Bias Proportion	0.016715
Variance Proportion	0.892241
Covariance Proportion	0.091044

Figure (4-33) Forecast with APARCH(1.1)model

Normal distribution

Forecast: RTF
Actual: RT
Forecast sample: 1999M01 2013M12
Adjusted sample: 1999M03 2013M12
Included observations: 178

Root Mean Squared Error	0.036528
Mean Absolute Error	0.012587
Mean Absolute Percentage Error	491.7924
Theil Inequality Coefficient	0.895979
Bias Proportion	0.001687
Variance Proportion	0.888489
Covariance Proportion	0.109824

Figure (4-34) Forecast with APARCH(1.1)model

Student's t distribution

Forecast: RTF
Actual: RT
Forecast sample: 1999M01 2013M12
Adjusted sample: 1999M03 2013M12
Included observations: 178

Root Mean Squared Error	0.036789
Mean Absolute Error	0.011080
Mean Absolute Percentage Error	83.77597
Theil Inequality Coefficient	0.952184
Bias Proportion	0.015886
Variance Proportion	0.890835
Covariance Proportion	0.093279

Figure (4-35) Forecast with GJR-GARCH(1.1)model

Normal distribution

Forecast: RTF
Actual: RT
Forecast sample: 1999M01 2013M12
Adjusted sample: 1999M03 2013M12
Included observations: 178

Root Mean Squared Error	0.036572
Mean Absolute Error	0.012129
Mean Absolute Percentage Error	372.7228
Theil Inequality Coefficient	0.916753
Bias Proportion	0.004167
Variance Proportion	0.901391
Covariance Proportion	0.094442

Figure (4-36) Forecast with GJR-GARCH(1.1)model

Student's t distribution

Forecast: RTF
Actual: RT
Forecast sample: 1999M01 2013M12
Adjusted sample: 1999M03 2013M12
Included observations: 178

Root Mean Squared Error	0.036802
Mean Absolute Error	0.011098
Mean Absolute Percentage Error	87.82762
Theil Inequality Coefficient	0.956605
Bias Proportion	0.016582
Variance Proportion	0.898314
Covariance Proportion	0.085104

Figure (4-37) Correlogram of first difference of Exchange rate series

Date: 09/24/15 Time: 20:19
Sample: 1999M01 2013M12
Included observations: 179

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
.|** |	.|**	|1	0.216	0.216	8.5053	0.004
.|. |	.|.	|2	0.001	-0.048	8.5057	0.014
.|. |	.|.	|3	0.020	0.031	8.5758	0.035
.|. |	.|.	|4	-0.036	-0.050	8.8185	0.066
.|. |	.|.	|5	0.002	0.024	8.8194	0.116
.|. |	.|.	|6	0.009	0.001	8.8351	0.183
.|. |	.|.	|7	-0.001	-0.001	8.8355	0.265
.|. |	.|.	|8	-0.005	-0.007	8.8397	0.356
.|. |	.|.	|9	-0.010	-0.007	8.8589	0.450
.|. |	.|.	|10	-0.009	-0.005	8.8731	0.544
.|. |	.|.	|11	-0.009	-0.007	8.8901	0.632
.|. |	.|.	|12	-0.023	-0.021	8.9966	0.703
.|. |	.|.	|13	0.015	0.025	9.0395	0.770
.|. |	.|.	|14	0.045	0.037	9.4408	0.802
.|** |	.|**	|15	0.277	0.276	24.607	0.055
.|* |	.|*	|16	0.200	0.090	32.560	0.008
.|. |	.|.	|17	0.032	-0.011	32.769	0.012
.|. |	.|.	|18	0.065	0.064	33.608	0.014
.|* |	.|*	|19	0.170	0.185	39.489	0.004
*|. |	*|.	|20	-0.099	-0.183	41.499	0.003
.|. |	.|*	|21	0.007	0.076	41.508	0.005
.|. |	.|.	|22	0.006	-0.025	41.516	0.007
.|. |	.|.	|23	0.018	0.059	41.584	0.010
.|. |	.|.	|24	0.042	0.006	41.953	0.013
.|. |	.|.	|25	0.045	0.069	42.383	0.016
.|. |	.|.	|26	-0.002	-0.029	42.384	0.022
.|. |	.|.	|27	-0.003	0.039	42.386	0.030
.|. |	.|.	|28	-0.005	-0.018	42.391	0.040
.|. |	.|.	|29	-0.013	-0.014	42.429	0.051
.|. |	*|.	|30	-0.016	-0.115	42.485	0.065
.|. |	.|.	|31	0.042	0.013	42.876	0.076
.|. |	.|.	|32	0.035	-0.026	43.145	0.090
*|. |	*|.	|33	-0.079	-0.123	44.526	0.087
.|. |	*|.	|34	0.014	-0.079	44.571	0.106
.|. |	.|.	|35	-0.014	0.029	44.614	0.128
.|. |	.|.	|36	0.009	0.024	44.632	0.153

Figure (4-38) Residuals Correlogram of GARCH (1,1)

Date: 02/06/16 Time: 15:05
Sample: 1999M01 2013M12
Included observations: 178
Q-statistic probabilities adjusted for 3 ARMA terms

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
.|* |	.|*	|1	0.170	0.170	5.2587	
.|. |	.|.	|2	0.066	0.038	6.0538	
.|. |	.|.	|3	0.051	0.035	6.5372	
.|. |	.|.	|4	0.002	-0.015	6.5377	0.011
.|. |	.|.	|5	-0.025	-0.028	6.6573	0.036
.|. |	.|.	|6	0.047	0.056	7.0680	0.070
.|. |	.|.	|7	0.050	0.038	7.5296	0.110
.|* |	.|*	|8	0.097	0.084	9.3131	0.097
.|* |	.|.	|9	0.089	0.054	10.826	0.094
.|* |	.|.	|10	0.099	0.067	12.690	0.080
.|* |	.|*	|11	0.128	0.098	15.840	0.045
.|. |	.|.	|12	0.071	0.029	16.805	0.052
.|. |	.|.	|13	0.060	0.037	17.498	0.064
.|* |	.|.	|14	0.089	0.066	19.058	0.060
.|* |	.|.	|15	0.085	0.058	20.483	0.058
.|* |	.|*	|16	0.132	0.105	23.944	0.032
.|. |	.|.	|17	0.054	-0.004	24.521	0.040
.|. |	.|.	|18	0.019	-0.018	24.591	0.056
*|. |	**|.	|19	-0.191	-0.240	31.979	0.010
.|. |	.|.	|20	-0.047	-0.021	32.424	0.013
.|. |	.|.	|21	0.045	0.042	32.842	0.017
.|. |	.|.	|22	0.021	-0.021	32.932	0.024
.|* |	.|.	|23	0.082	0.038	34.326	0.024
.|* |	.|*	|24	0.160	0.083	39.642	0.008
.|. |	*|.	|25	-0.006	-0.082	39.649	0.012
*|. |	*|.	|26	-0.130	-0.181	43.209	0.007
*|. |	*|.	|27	-0.084	-0.076	44.716	0.006
.|. |	.|.	|28	-0.066	-0.028	45.634	0.007
.|. |	.|.	|29	-0.050	-0.009	46.168	0.009
.|. |	.|.	|30	0.012	0.055	46.199	0.012
.|. |	.|.	|31	0.021	0.010	46.294	0.016
.|. |	.|.	|32	-0.018	-0.062	46.365	0.022
.|. |	.|.	|33	0.030	0.041	46.561	0.027
*|. |	*|.	|34	-0.067	-0.067	47.574	0.029
.|. |	.|.	|35	-0.060	0.009	48.393	0.032
.|. |	.|.	|36	-0.059	0.004	49.175	0.035

*Probabilities may not be valid for this equation specification.

Figure (4-39) The correlogram of standardized residuals squared for GARCH (1,1)

Date: 02/22/16 Time: 14:02
Sample: 1999M01 2013M12
Included observations: 178

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
.|. |	.|.	|1	-0.010	-0.010	0.0171	0.896
.|. |	.|.	|2	-0.033	-0.033	0.2128	0.899
.|. |	.|.	|3	-0.023	-0.024	0.3096	0.958
.|. |	.|.	|4	-0.024	-0.026	0.4159	0.981
.|. |	.|.	|5	0.052	0.050	0.9111	0.969
.|. |	.|.	|6	-0.027	-0.029	1.0499	0.984
.|. |	.|.	|7	-0.026	-0.025	1.1780	0.991
.|. |	.|.	|8	-0.024	-0.024	1.2831	0.996
.|. |	.|.	|9	-0.025	-0.027	1.4062	0.998
.|. |	.|.	|10	-0.017	-0.025	1.4623	0.999
.|. |	.|.	|11	-0.019	-0.021	1.5288	1.000
.|. |	.|.	|12	-0.025	-0.028	1.6503	1.000
.|. |	.|.	|13	-0.030	-0.033	1.8209	1.000
.|. |	.|.	|14	-0.020	-0.025	1.9025	1.000
.|* |	.|*	|15	0.091	0.086	3.5339	0.999
.|. |	.|.	|16	-0.005	-0.009	3.5387	0.999
.|. |	.|.	|17	-0.034	-0.032	3.7696	1.000
.|. |	.|.	|18	-0.023	-0.023	3.8780	1.000
.|*** |	.|***	|19	0.435	0.442	41.925	0.002
.|. |	.|.	|20	-0.007	-0.023	41.934	0.003
.|. |	.|.	|21	-0.009	0.014	41.949	0.004
.|. |	.|.	|22	-0.016	0.004	42.001	0.006
.|. |	.|.	|23	-0.013	0.036	42.036	0.009
.|* |	.|.	|24	0.095	0.057	43.929	0.008
.|. |	.|.	|25	-0.019	0.003	44.004	0.011
.|. |	.|.	|26	0.034	0.067	44.243	0.014
.|. |	.|.	|27	-0.007	0.019	44.252	0.019
.|. |	.|.	|28	-0.015	0.019	44.301	0.026
.|. |	.|.	|29	0.003	0.024	44.304	0.034
.|. |	.|.	|30	-0.019	-0.009	44.384	0.044
.|. |	.|.	|31	-0.018	-0.003	44.457	0.056
.|. |	.|.	|32	-0.018	0.012	44.529	0.069
.|. |	.|.	|33	0.012	0.064	44.560	0.086
.|* |	.|.	|34	0.108	0.031	47.139	0.066
.|. |	.|.	|35	-0.006	0.004	47.146	0.082
.|. |	.|.	|36	-0.022	0.013	47.254	0.099

*Probabilities may not be valid for this equation specification.

Figure (4-40) Residuals Correlogram of GARCH(1,1)

- **Student's t distribution**

Date: 02/06/16 Time: 15:08
Sample: 1999M01 2013M12
Included observations: 178
Q-statistic probabilities adjusted for 3 ARMA terms

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
.|. |	.|.	|1	0.011	0.011	0.0224	
.|. |	.|.	|2	-0.007	-0.007	0.0316	
.|. |	.|.	|3	-0.005	-0.005	0.0358	
.|. |	.|.	|4	-0.007	-0.007	0.0441	0.834
*|. |	*|.	|5	-0.081	-0.081	1.2734	0.529
.|. |	.|.	|6	-0.002	0.000	1.2739	0.735
.|. |	.|.	|7	-0.000	-0.002	1.2739	0.866
.|. |	.|.	|8	0.003	0.003	1.2760	0.937
.|. |	.|.	|9	0.002	0.001	1.2765	0.973
.|. |	.|.	|10	0.002	-0.004	1.2776	0.989
.|. |	.|.	|11	0.002	0.002	1.2783	0.996
.|. |	.|.	|12	-0.000	-0.001	1.2784	0.998
.|. |	.|.	|13	0.001	0.002	1.2788	0.999
.|. |	.|.	|14	0.005	0.005	1.2829	1.000
.|** |	.|**	|15	0.288	0.290	17.563	0.130
.|. |	.|.	|16	0.011	0.007	17.588	0.174
.|. |	.|.	|17	-0.004	0.000	17.592	0.226
.|. |	.|.	|18	-0.005	-0.001	17.596	0.285
***|. |	***|.	|19	-0.352	-0.383	42.617	0.000
.|. |	.|.	|20	-0.009	0.055	42.635	0.001
.|. |	.|.	|21	0.005	0.000	42.641	0.001
.|. |	.|.	|22	-0.003	-0.006	42.643	0.001
.|. |	.|.	|23	0.018	0.064	42.708	0.002
.|* |	.|*	|24	0.142	0.095	46.897	0.001
.|. |	.|.	|25	0.007	-0.002	46.906	0.002
.|. |	.|.	|26	-0.016	-0.020	46.957	0.002
.|. |	.|.	|27	-0.011	-0.020	46.985	0.003
.|. |	.|.	|28	-0.004	-0.028	46.989	0.005
.|. |	.|.	|29	-0.006	0.034	46.995	0.007
.|. |	*|.	|30	-0.006	-0.110	47.004	0.010
.|. |	.|.	|31	0.004	0.006	47.008	0.014
.|. |	.|.	|32	-0.006	0.000	47.017	0.019
.|. |	.|.	|33	0.012	-0.001	47.048	0.025
*|. |	.|*	|34	-0.162	0.077	52.877	0.008
.|. |	.|.	|35	-0.003	-0.019	52.879	0.012
.|. |	.|.	|36	0.005	0.002	52.885	0.015

*Probabilities may not be valid for this equation specification.

Figure (4-41) The correlogram of standardized residuals squared for GARCH(1,1)

- **Student's t distribution**

Date: 02/22/16 Time: 14:04
Sample: 1999M01 2013M12
Included observations: 178

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
.|. |	.|.	|1	-0.013	-0.013	0.0325	0.857
.|. |	.|.	|2	-0.014	-0.014	0.0681	0.967
.|. |	.|.	|3	-0.014	-0.014	0.1042	0.991
.|. |	.|.	|4	-0.012	-0.013	0.1310	0.998
.|. |	.|.	|5	0.005	0.004	0.1356	1.000
.|. |	.|.	|6	-0.012	-0.013	0.1635	1.000
.|. |	.|.	|7	-0.012	-0.013	0.1920	1.000
.|. |	.|.	|8	-0.012	-0.013	0.2210	1.000
.|. |	.|.	|9	-0.012	-0.013	0.2504	1.000
.|. |	.|.	|10	-0.013	-0.014	0.2803	1.000
.|. |	.|.	|11	-0.013	-0.014	0.3110	1.000
.|. |	.|.	|12	-0.013	-0.014	0.3421	1.000
.|. |	.|.	|13	-0.013	-0.015	0.3738	1.000
.|. |	.|.	|14	-0.013	-0.015	0.4050	1.000
.|* |	.|*	|15	0.201	0.199	8.3468	0.909
.|. |	.|.	|16	-0.013	-0.009	8.3778	0.937
.|. |	.|.	|17	-0.013	-0.010	8.4120	0.957
.|. |	.|.	|18	-0.013	-0.010	8.4452	0.971
.|** |	.|**	|19	0.299	0.315	26.451	0.118
.|. |	.|.	|20	-0.005	-0.004	26.456	0.151
.|. |	.|.	|21	-0.005	0.004	26.461	0.189
.|. |	.|.	|22	-0.005	0.004	26.467	0.232
.|. |	.|.	|23	-0.005	0.028	26.472	0.279
.|. |	.|.	|24	0.042	0.047	26.848	0.312
.|. |	.|.	|25	-0.005	0.006	26.854	0.363
.|. |	.|.	|26	-0.005	0.006	26.860	0.417
.|. |	.|.	|27	-0.005	0.008	26.866	0.471
.|. |	.|.	|28	-0.006	0.012	26.873	0.525
.|. |	.|.	|29	-0.006	0.008	26.880	0.578
.|. |	.|.	|30	-0.006	-0.046	26.887	0.629
.|. |	.|.	|31	-0.006	0.005	26.895	0.677
.|. |	.|.	|32	-0.006	0.006	26.902	0.722
.|. |	.|.	|33	-0.005	0.008	26.909	0.764
.|. |	.|.	|34	0.071	-0.065	28.039	0.754
.|. |	.|.	|35	-0.006	0.002	28.047	0.792
.|. |	.|.	|36	-0.006	0.001	28.056	0.825

*Probabilities may not be valid for this equation specification.

Figure (4-42) Residuals Correlogram of APARCH(1,1)

Normal distribution

Date: 02/06/16 Time: 15:23
Sample: 1999M01 2013M12
Included observations: 178
Q-statistic probabilities adjusted for 3 ARMA terms

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
.|* |	.|*	|1	0.092	0.092	1.5238	
.|. |	.|.	|2	-0.032	-0.041	1.7122	
.|. |	.|.	|3	0.009	0.016	1.7267	
*|. |	*|.	|4	-0.103	-0.108	3.6816	0.055
.|. |	.|.	|5	-0.011	0.010	3.7047	0.157
.|. |	.|.	|6	0.032	0.024	3.8948	0.273
.|. |	.|.	|7	0.017	0.015	3.9475	0.413
.|. |	.|.	|8	0.021	0.009	4.0293	0.545
.|. |	.|.	|9	0.004	0.001	4.0330	0.672
.|. |	.|.	|10	0.027	0.034	4.1727	0.760
.|. |	.|.	|11	0.006	0.003	4.1786	0.841
.|. |	.|.	|12	-0.056	-0.053	4.7782	0.853
.|* |	.|*	|13	0.078	0.090	5.9583	0.819
.|. |	.|.	|14	0.046	0.031	6.3684	0.848
.|** |	.|**	|15	0.237	0.248	17.430	0.134
.|** |	.|*	|16	0.245	0.205	29.296	0.006
.|. |	.|.	|17	-0.028	-0.025	29.455	0.009
.|* |	.|*	|18	0.111	0.160	31.943	0.007
*|. |	*|.	|19	-0.172	-0.179	37.914	0.002
*|. |	.|.	|20	-0.096	-0.012	39.782	0.001
.|. |	.|.	|21	0.021	-0.021	39.873	0.002
.|. |	.|.	|22	-0.048	-0.060	40.349	0.003
.|. |	.|.	|23	0.066	0.054	41.251	0.003
.|* |	.|.	|24	0.108	0.067	43.680	0.003
.|. |	.|.	|25	0.026	0.035	43.817	0.004
.|. |	*|.	|26	-0.044	-0.074	44.234	0.005
.|. |	.|.	|27	-0.005	0.025	44.238	0.007
.|. |	.|.	|28	-0.008	-0.032	44.254	0.010
.|. |	*|.	|29	-0.021	-0.091	44.350	0.014
.|. |	.|.	|30	0.014	-0.037	44.394	0.019
.|* |	.|.	|31	0.099	-0.059	46.528	0.015
.|. |	.|.	|32	-0.030	-0.061	46.731	0.020
.|. |	.|.	|33	-0.001	-0.017	46.731	0.026
*|. |	*|.	|34	-0.075	-0.102	47.998	0.026
*|. |	.|*	|35	-0.067	0.090	49.010	0.028
.|. |	.|.	|36	0.034	0.011	49.265	0.034

*Probabilities may not be valid for this equation specification.

Figure (4-43) The correlogram of standardized residuals squared for APARCH(1,1)

Normal distribution

Date: 02/22/16 Time: 14:08
Sample: 1999M01 2013M12
Included observations: 178

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
.|* |	.|*	|1	0.100	0.100	1.8063	0.179
.|. |	.|.	|2	-0.017	-0.028	1.8620	0.394
.|. |	.|.	|3	-0.007	-0.003	1.8716	0.599
.|. |	.|.	|4	-0.005	-0.004	1.8757	0.759
.|. |	.|.	|5	0.008	0.009	1.8888	0.864
.|. |	.|.	|6	-0.017	-0.019	1.9435	0.925
.|. |	.|.	|7	-0.023	-0.019	2.0410	0.958
.|. |	.|.	|8	-0.020	-0.016	2.1146	0.977
.|. |	.|.	|9	-0.020	-0.017	2.1870	0.988
.|. |	.|.	|10	-0.022	-0.020	2.2772	0.994
.|. |	.|.	|11	-0.022	-0.019	2.3674	0.997
.|. |	.|.	|12	-0.015	-0.013	2.4131	0.998
.|. |	.|.	|13	-0.013	-0.012	2.4463	0.999
.|. |	.|.	|14	0.032	0.033	2.6412	1.000
.|** |	.|*	|15	0.214	0.209	11.666	0.704
.|* |	.|.	|16	0.086	0.048	13.136	0.663
.|. |	.|.	|17	-0.017	-0.022	13.192	0.723
.|. |	.|*	|18	0.066	0.077	14.067	0.725
.|** |	.|**	|19	0.312	0.320	33.732	0.020
.|. |	.|.	|20	0.018	-0.037	33.795	0.028
.|. |	.|.	|21	-0.011	0.007	33.818	0.038
.|. |	.|.	|22	-0.003	0.027	33.820	0.051
.|. |	.|.	|23	-0.005	0.024	33.826	0.068
.|. |	.|.	|24	0.054	0.063	34.424	0.077
.|. |	.|.	|25	-0.008	0.006	34.436	0.099
.|. |	.|.	|26	-0.007	0.022	34.445	0.124
.|. |	.|.	|27	-0.013	0.007	34.481	0.153
.|. |	.|.	|28	-0.013	0.011	34.520	0.184
.|. |	.|.	|29	-0.010	-0.003	34.540	0.220
.|. |	.|.	|30	-0.008	-0.047	34.553	0.259
.|. |	.|.	|31	0.018	0.002	34.621	0.299
.|. |	.|.	|32	0.013	0.024	34.659	0.342
.|. |	.|.	|33	0.059	0.026	35.423	0.355
.|* |	.|.	|34	0.094	-0.063	37.408	0.315
.|. |	.|.	|35	0.015	-0.026	37.461	0.357
.|. |	.|.	|36	-0.009	0.011	37.479	0.401

*Probabilities may not be valid for this equation specification.

Figure (4-44) Residuals Correlogram of APARCH(1,1)

Student's t distribution

Date: 02/06/16 Time: 15:26
Sample: 1999M01 2013M12
Included observations: 178
Q-statistic probabilities adjusted for 3 ARMA terms

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
.|. |	.|. |	1	-0.003	-0.003	0.0013	
.|. |	.|. |	2	-0.004	-0.004	0.0042	
.|. |	.|. |	3	-0.004	-0.004	0.0070	
.|. |	.|. |	4	-0.002	-0.002	0.0080	0.929
.|. |	.|. |	5	-0.012	-0.012	0.0342	0.983
.|. |	.|. |	6	-0.002	-0.002	0.0349	0.998
.|. |	.|. |	7	-0.002	-0.002	0.0356	1.000
.|. |	.|. |	8	-0.002	-0.002	0.0361	1.000
.|. |	.|. |	9	-0.002	-0.002	0.0368	1.000
.|. |	.|. |	10	-0.002	-0.002	0.0376	1.000
.|. |	.|. |	11	-0.002	-0.002	0.0383	1.000
.|. |	.|. |	12	-0.002	-0.002	0.0393	1.000
.|. |	.|. |	13	-0.002	-0.002	0.0400	1.000
.|. |	.|. |	14	-0.002	-0.002	0.0405	1.000
.|** |	.|** |	15	0.323	0.323	20.510	0.058
.|. |	.|. |	16	-0.001	0.001	20.510	0.083
.|. |	.|. |	17	-0.003	-0.000	20.512	0.115
.|. |	.|. |	18	-0.002	0.001	20.512	0.153
***|. |	***|. |	19	-0.350	-0.390	45.216	0.000
.|. |	.|. |	20	0.000	0.008	45.216	0.000
.|. |	.|. |	21	0.001	-0.000	45.216	0.000
.|. |	.|. |	22	0.001	-0.001	45.216	0.001
.|. |	.|. |	23	0.003	0.061	45.218	0.001
.|. |	.|. |	24	0.020	0.015	45.302	0.002
.|. |	.|. |	25	0.002	0.001	45.303	0.002
.|. |	.|. |	26	-0.000	0.000	45.303	0.004
.|. |	.|. |	27	0.000	-0.009	45.303	0.005
.|. |	.|. |	28	0.001	-0.004	45.303	0.008
.|. |	.|. |	29	0.001	0.003	45.303	0.011
.|. |	*|. |	30	0.001	-0.138	45.303	0.015
.|. |	.|. |	31	0.002	0.004	45.304	0.021
.|. |	.|. |	32	0.000	0.001	45.304	0.027
.|. |	.|. |	33	0.002	-0.002	45.305	0.036
*|. |	.|* |	34	-0.174	0.112	52.013	0.010
.|. |	.|. |	35	0.001	-0.004	52.013	0.014
.|. |	.|. |	36	0.002	-0.001	52.014	0.019

*Probabilities may not be valid for this equation specification.

Figure (4-45) The correlogram of standardized residuals squared for APARCH(1,1)

Student's t distribution

Date: 02/22/16 Time: 14:11
Sample: 1999M01 2013M12
Included observations: 178

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
.|. |	.|.	|1	-0.012	-0.012	0.0256	0.873
.|. |	.|.	|2	-0.012	-0.012	0.0517	0.974
.|. |	.|.	|3	-0.012	-0.012	0.0783	0.994
.|. |	.|.	|4	-0.010	-0.011	0.0974	0.999
.|. |	.|.	|5	-0.010	-0.011	0.1160	1.000
.|. |	.|.	|6	-0.010	-0.011	0.1358	1.000
.|. |	.|.	|7	-0.010	-0.011	0.1560	1.000
.|. |	.|.	|8	-0.010	-0.011	0.1766	1.000
.|. |	.|.	|9	-0.011	-0.012	0.1975	1.000
.|. |	.|.	|10	-0.011	-0.012	0.2189	1.000
.|. |	.|.	|11	-0.011	-0.012	0.2406	1.000
.|. |	.|.	|12	-0.011	-0.012	0.2628	1.000
.|. |	.|.	|13	-0.011	-0.012	0.2854	1.000
.|. |	.|.	|14	-0.011	-0.013	0.3083	1.000
.|* |	.|*	|15	0.210	0.209	9.0138	0.877
.|. |	.|.	|16	-0.011	-0.008	9.0377	0.912
.|. |	.|.	|17	-0.011	-0.008	9.0619	0.938
.|. |	.|.	|18	-0.011	-0.008	9.0867	0.958
.|** |	.|**	|19	0.255	0.270	22.168	0.276
.|. |	.|.	|20	-0.004	0.004	22.171	0.331
.|. |	.|.	|21	-0.004	0.004	22.174	0.390
.|. |	.|.	|22	-0.004	0.004	22.177	0.449
.|. |	.|.	|23	-0.004	0.021	22.180	0.509
.|. |	.|.	|24	-0.003	0.005	22.182	0.568
.|. |	.|.	|25	-0.004	0.004	22.185	0.625
.|. |	.|.	|26	-0.004	0.005	22.188	0.678
.|. |	.|.	|27	-0.004	0.006	22.192	0.728
.|. |	.|.	|28	-0.004	0.005	22.195	0.772
.|. |	.|.	|29	-0.004	0.005	22.199	0.812
.|. |	.|.	|30	-0.004	-0.050	22.203	0.847
.|. |	.|.	|31	-0.004	0.004	22.207	0.876
.|. |	.|.	|32	-0.004	0.004	22.211	0.902
.|. |	.|.	|33	-0.004	0.004	22.216	0.923
.|. |	.|.	|34	0.060	-0.061	23.028	0.923
.|. |	.|.	|35	-0.005	-0.001	23.033	0.940
.|. |	.|.	|36	-0.005	-0.001	23.038	0.954

*Probabilities may not be valid for this equation specification.

Figure (4-46) Residuals Correlogram of GJR-GARCH (1.1)

Normal distribution

Date: 02/06/16 Time: 15:34
Sample: 1999M01 2013M12
Included observations: 178
Q-statistic probabilities adjusted for 3 ARMA terms

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
.|* |	.|*	|1	0.103	0.103	1.9191	
.|. |	.|.	|2	-0.031	-0.042	2.0985	
.|. |	.|.	|3	0.031	0.040	2.2790	
.|. |	*|.	|4	-0.057	-0.067	2.8802	0.090
.|. |	.|.	|5	-0.008	0.009	2.8914	0.236
.|. |	.|.	|6	0.034	0.028	3.1026	0.376
.|. |	.|.	|7	0.018	0.016	3.1601	0.531
.|. |	.|.	|8	0.019	0.014	3.2247	0.665
.|. |	.|.	|9	0.005	0.001	3.2304	0.779
.|. |	.|.	|10	0.028	0.032	3.3783	0.848
.|. |	.|.	|11	0.029	0.024	3.5446	0.896
.|. |	.|.	|12	-0.041	-0.044	3.8683	0.920
.|* |	.|*	|13	0.077	0.088	5.0225	0.890
.|. |	.|.	|14	0.047	0.026	5.4603	0.907
.|** |	.|**	|15	0.238	0.250	16.566	0.167
.|** |	.|*	|16	0.250	0.206	28.916	0.007
.|. |	.|.	|17	-0.030	-0.043	29.099	0.010
.|* |	.|*	|18	0.105	0.142	31.306	0.008
*|. |	*|.	|19	-0.154	-0.195	36.116	0.003
*|. |	.|.	|20	-0.089	-0.018	37.737	0.003
.|. |	.|.	|21	0.022	-0.024	37.833	0.004
.|. |	.|.	|22	-0.027	-0.045	37.987	0.006
.|. |	.|.	|23	0.039	0.042	38.302	0.008
.|* |	.|.	|24	0.104	0.070	40.572	0.006
.|. |	.|.	|25	0.026	0.031	40.715	0.009
.|. |	*|.	|26	-0.047	-0.083	41.187	0.011
.|. |	.|.	|27	-0.005	0.007	41.192	0.016
.|. |	.|.	|28	0.004	-0.025	41.195	0.022
.|. |	*|.	|29	-0.023	-0.098	41.314	0.029
.|. |	.|.	|30	0.010	-0.023	41.335	0.038
.|* |	.|.	|31	0.095	-0.063	43.286	0.033
.|. |	.|.	|32	-0.035	-0.059	43.560	0.040
.|. |	.|.	|33	-0.010	-0.009	43.581	0.052
*|. |	*|.	|34	-0.075	-0.097	44.835	0.052
.|. |	.|*	|35	-0.061	0.097	45.676	0.055
.|. |	.|.	|36	0.029	0.011	45.866	0.068

*Probabilities may not be valid for this equation specification.

Figure (4-47) The correlogram of standardized residuals squared for of GJR-GARCH (1.1)

Normal distribution

Date: 02/22/16 Time: 14:13
Sample: 1999M01 2013M12
Included observations: 178

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
.|* |	.|*	|1	0.117	0.117	2.4580	0.117
.|. |	.|.	|2	-0.018	-0.032	2.5197	0.284
.|. |	.|.	|3	-0.011	-0.005	2.5403	0.468
.|. |	.|.	|4	-0.014	-0.013	2.5776	0.631
.|. |	.|.	|5	0.008	0.011	2.5891	0.763
.|. |	.|.	|6	-0.017	-0.020	2.6441	0.852
.|. |	.|.	|7	-0.023	-0.019	2.7452	0.908
.|. |	.|.	|8	-0.020	-0.017	2.8229	0.945
.|. |	.|.	|9	-0.020	-0.016	2.8952	0.968
.|. |	.|.	|10	-0.022	-0.020	2.9889	0.982
.|. |	.|.	|11	-0.025	-0.021	3.1046	0.989
.|. |	.|.	|12	-0.017	-0.013	3.1588	0.994
.|. |	.|.	|13	-0.013	-0.012	3.1895	0.997
.|. |	.|.	|14	0.036	0.037	3.4477	0.998
.|** |	.|**	|15	0.221	0.214	13.078	0.596
.|* |	.|.	|16	0.096	0.050	14.899	0.532
.|. |	.|.	|17	-0.017	-0.025	14.956	0.599
.|. |	.|*	|18	0.069	0.084	15.921	0.598
.|** |	.|**	|19	0.308	0.319	35.080	0.014
.|. |	.|.	|20	0.022	-0.041	35.174	0.019
.|. |	.|.	|21	-0.011	0.009	35.198	0.027
.|. |	.|.	|22	-0.006	0.028	35.205	0.037
.|. |	.|.	|23	-0.010	0.026	35.226	0.049
.|. |	.|.	|24	0.054	0.063	35.826	0.057
.|. |	.|.	|25	-0.007	0.006	35.835	0.074
.|. |	.|.	|26	-0.007	0.024	35.844	0.095
.|. |	.|.	|27	-0.013	0.008	35.881	0.118
.|. |	.|.	|28	-0.015	0.011	35.927	0.144
.|. |	.|.	|29	-0.010	-0.004	35.948	0.175
.|. |	.|.	|30	-0.006	-0.048	35.957	0.209
.|. |	.|.	|31	0.017	-0.003	36.020	0.245
.|. |	.|.	|32	0.016	0.026	36.074	0.284
.|. |	.|.	|33	0.064	0.027	36.966	0.291
.|* |	*|.	|34	0.097	-0.068	39.077	0.252
.|. |	.|.	|35	0.020	-0.025	39.167	0.288
.|. |	.|.	|36	-0.010	0.012	39.190	0.329

*Probabilities may not be valid for this equation specification.

Figure (4-48) Residuals Correlogram of GJR-GARCH (1.1)

Student's t distribution

Date: 02/06/16 Time: 15:43
Sample: 1999M01 2013M12
Included observations: 178
Q-statistic probabilities adjusted for 3 ARMA terms

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
.|. |	.|.	|1	0.000	0.000	3.E-05	
.|. |	.|.	|2	-0.004	-0.004	0.0032	
.|. |	.|.	|3	-0.005	-0.005	0.0074	
.|. |	.|.	|4	-0.004	-0.004	0.0105	0.918
.|. |	.|.	|5	-0.043	-0.043	0.3534	0.838
.|. |	.|.	|6	-0.002	-0.002	0.3539	0.950
.|. |	.|.	|7	-0.001	-0.001	0.3540	0.986
.|. |	.|.	|8	0.001	0.000	0.3541	0.997
.|. |	.|.	|9	-0.001	-0.001	0.3542	0.999
.|. |	.|.	|10	-0.002	-0.004	0.3547	1.000
.|. |	.|.	|11	-0.002	-0.002	0.3554	1.000
.|. |	.|.	|12	-0.002	-0.002	0.3562	1.000
.|. |	.|.	|13	-0.001	-0.001	0.3564	1.000
.|. |	.|.	|14	-0.000	-0.000	0.3564	1.000
.|** |	.|**	|15	0.302	0.302	18.276	0.108
.|. |	.|.	|16	0.003	0.003	18.277	0.147
.|. |	.|.	|17	-0.003	-0.001	18.279	0.194
.|. |	.|.	|18	-0.001	0.002	18.279	0.248
***|. |	***|.	|19	-0.370	-0.405	45.817	0.000
.|. |	.|.	|20	-0.002	0.031	45.818	0.000
.|. |	.|.	|21	0.001	-0.001	45.819	0.000
.|. |	.|.	|22	-0.001	-0.004	45.819	0.001
.|. |	.|.	|23	0.005	0.062	45.824	0.001
.|. |	.|.	|24	0.071	0.044	46.860	0.001
.|. |	.|.	|25	0.003	-0.000	46.862	0.002
.|. |	.|.	|26	-0.004	-0.004	46.865	0.002
.|. |	.|.	|27	-0.003	-0.012	46.867	0.003
.|. |	.|.	|28	-0.001	-0.014	46.868	0.005
.|. |	.|.	|29	-0.001	0.015	46.868	0.007
.|. |	*|.	|30	-0.002	-0.124	46.869	0.010
.|. |	.|.	|31	0.003	0.006	46.871	0.014
.|. |	.|.	|32	-0.002	0.002	46.872	0.019
.|. |	.|.	|33	0.005	-0.006	46.878	0.026
*|. |	.|*	|34	-0.181	0.092	54.167	0.006
.|. |	.|.	|35	-0.000	-0.013	54.167	0.008
.|. |	.|.	|36	0.003	0.001	54.169	0.012

*Probabilities may not be valid for this equation specification.

Figure (4-49)The correlogram of standardized residuals squared for GJR-GARCH (1.1)

Student's t distribution

Date: 02/22/16 Time: 14:14
Sample: 1999M01 2013M12
Included observations: 178

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
.|. |	.|.	1	-0.013	-0.013	0.0300	0.862
.|. |	.|.	2	-0.013	-0.013	0.0607	0.970
.|. |	.|.	3	-0.013	-0.013	0.0919	0.993
.|. |	.|.	4	-0.011	-0.012	0.1149	0.998
.|. |	.|.	5	-0.007	-0.008	0.1246	1.000
.|. |	.|.	6	-0.011	-0.012	0.1484	1.000
.|. |	.|.	7	-0.011	-0.012	0.1727	1.000
.|. |	.|.	8	-0.011	-0.012	0.1974	1.000
.|. |	.|.	9	-0.012	-0.013	0.2226	1.000
.|. |	.|.	10	-0.012	-0.013	0.2483	1.000
.|. |	.|.	11	-0.012	-0.013	0.2745	1.000
.|. |	.|.	12	-0.012	-0.013	0.3011	1.000
.|. |	.|.	13	-0.012	-0.014	0.3282	1.000
.|. |	.|.	14	-0.012	-0.014	0.3557	1.000
.|* |	.|*	15	0.201	0.199	8.2591	0.913
.|. |	.|.	16	-0.012	-0.009	8.2875	0.940
.|. |	.|.	17	-0.012	-0.009	8.3166	0.959
.|. |	.|.	18	-0.012	-0.009	8.3462	0.973
.|** |	.|**	19	0.307	0.323	27.353	0.097
.|. |	.|.	20	-0.005	0.002	27.357	0.126
.|. |	.|.	21	-0.005	0.004	27.361	0.159
.|. |	.|.	22	-0.005	0.005	27.365	0.198
.|. |	.|.	23	-0.005	0.029	27.370	0.241
.|. |	.|.	24	0.006	0.014	27.378	0.287
.|. |	.|.	25	-0.005	0.005	27.383	0.337
.|. |	.|.	26	-0.005	0.005	27.388	0.389
.|. |	.|.	27	-0.005	0.007	27.393	0.443
.|. |	.|.	28	-0.005	0.007	27.398	0.497
.|. |	.|.	29	-0.005	0.006	27.404	0.550
.|. |	.|.	30	-0.005	-0.046	27.410	0.602
.|. |	.|.	31	-0.005	0.005	27.416	0.651
.|. |	.|.	32	-0.005	0.005	27.422	0.698
.|. |	.|.	33	-0.005	0.005	27.428	0.741
.|. |	*|.	34	0.073	-0.068	28.614	0.729
.|. |	.|.	35	-0.006	0.000	28.621	0.768
.|. |	.|.	36	-0.006	-0.000	28.628	0.804

*Probabilities may not be valid for this equation specification.