

## Chapter One

### Introduction

#### (1.1) Electromagnetic field:

The universe building blocks are atoms. These atoms consist of electrons, protons and neutrons. The recent elementary particle models, like electroweak standard model, say that even these particles are consist of smaller elementary particles called quarks and lepton [1]. This particles have certain masses. The origin of these masses are explained by the so called Higgs mechanism [2].

Recently Higgs particles which are thought to be responsible for mass generation was discovered experimentally in CERN[3]. This discovery comes as un-ultimate reword confirming the key predictions of the electroweak standard model (ESM). This mechanism states that when symmetry is broken at minimum potential masses are generated.

The electron charge is responsible for electric field generation. The motion of electric field generate magnetic field [1]. Electromagnetic field shown to be generated by time changing electric and magnetic field as shown By Maxwell [2]. Maxwell equations predicts that electromagnetic wave (E.M.W) can be produced by oscillating charges [3]. These (E.M.W) are shown to carry information, there for they are widely used in telecommunication systems [4]. According to Maxwell equations (M.E) electromagnetic field and light behaves as waves. But the black body relation and other related phenomena, shows that (E.M.W) behaves some times as particles called photons describe behavior of (E.M.W) and the atomic scale [5]. The theory which describes the behavior of photons is quantum electrodynamics [6]. This theory succeeded in describing a wide

meaty of physical phenomena can caring (E.M.W) an the atomic scale. However there are some setbacks associated with quantum electrodynamics one of these problems is quantization of electron charge and electron self energy [7,8,9].

**(1.2) Research problems:**

Although Higgs mechanism and electrodynamics are responsible for appearance of mass and charge terms in the lagrangian, but still unable to explain some of mass and charge phenomena. For example it cannot explain why the masses and charges of elementary particles assume their present values. The masses and charges themselves are not quantized. Moreover Higgs model is mathematically complex. Higgs mechanism does not also provide the mass to some particles like neutrino. The gravity force is not also incorporated in the standard model [4, 5].

**(1.3) Literature review:**

Different attempts were made to go beyond Higgs model. In some of them the masses of elementary particles and their generation is explained by using simple mathematics [6]. In other attempts the masses are subjected to quantization and unification with gravity force [7,8,9].

**(1.4) Aim of the work:**

The aim of the work is to construct simple model which can unify gravity and ESM. This model should link mass generation with gravity and charge generation with electromagnetic field. This model should be based on generalized lagrangian [GL] and Einstein generalized special relativity [EGSR].

**(1.5) Thesis layout:**

The Thesis consists of five chapters. Chapter one and two are concerned with introduction and theoretical back ground respectively, while chapter three is the basic concept, the literature review is in chapter four and chapter five is devoted for contribution.

## **Chapter Two**

### **Maxwell's Equations**

#### **(2.1) Introduction:**

The basic electrodynamics equations are usually driven from laws of a general course of electricity and magnetism [10,11]. These laws can describe the behavior of electromagnetic fields inside matter as well as free space [12]. In this chapter M.E are derived by utilizing the basic laws of electricity and magnetism [13].

#### **(2.2) Electric and magnetic Field Intensity:**

It is well known that, the electromagnetic field in a medium is described by four vectors quantities the electromagnetic field, the electric induction , the magnetic field and the magnetic induction. The force acting on unit electric charge at a given point in space is called the electric field intensity [14].

In future, instead of the field intensity one can simply speak of the field at a given point in space. The magnetic field intensity or, for short the magnetic field is defined analogously, separate magnetic charges, unlike electric charges, don't exist in nature, however if we make a long permanent magnet in the form of a needle, then the magnetic force acting at its ends will be the same as if there existed point charges at the end[14].

A rigorous definition of the electric and magnetic induction vectors. Where the field equations in a medium will be derived from the equations for point charges in free space. It need only be recalled that in free space there is no

need to use four vectors for a description of the electromagnetic field, only two vectors being sufficient: The electric and magnetic fields [14].

**(2.3) Electromotive Force:**

One can recall the definition for electromotive force in circuit: this is the work performed by the forces of the electric field when unit charge is taken along the given closed circuit. It is filled with a conductor or whether it is merely a closed line drawn in space. One can thus write down the expression for electromotive force (e.m.f). The force acting on unit charge at a given point is the electric field E. the work done by this force on an element of path  $d1$  is the scalar product  $E.d1$ . Then, the work done on the whole closed circuit. Or the e.m.f is equal to the integral [14].

$$V = \text{e.m.f} = \int E \cdot d1 \tag{2.3.1}$$

Where V is the induction potential

**(2.4) Magnetic field Flux Across Surface:**

Let us suppose that some surface is bounded by the given circuit. We shall denote the magnetic field by the letter H. the magnetic filed flux through an element of the chosen surface  $dS$  is given by  $d\emptyset = H.ds$

The magnetic field flux through the whole surface , bounded by the circuit, is given by:

$$\emptyset = \int H \cdot ds = \int B \cdot ds \tag{2.4.1}$$

Where B is the magnetic field density

One can consider a section of the surface through which unit flux  $\Delta\emptyset = 1$  passes. If one draws through this section of the surface a line tangential to the direction of the filed at some point on the surface. A line which is tangential to the direction of the field at its point is called a magnetic line of force. For this reason, the total flux  $\emptyset$  is equal, by definition, to the number of magnetic line of force crossing the surface [14].

Magnetic line of force are either closed or extended to infinity. Indeed, a magnetic line force may being or end only at a single charge, but separate magnetic charges do not exist in nature. In a permanent magnet the lines of force are completed inside the magnet, from this it follows that a magnet flux through any surface, bounded by circuit, is the same at a given instant. Otherwise, a number of the magnetic lines of force would have to begin or end in the space between the surfaces through which different fluxes pass. Consequently, at a given instant a constant number of magnetic lines force, i.e. a constant magnetic field flux passes across any surface bounded by the circuit. Therefore, the flux can be a scribed to the circuit itself, irrespective of the surface for which it is calculated [14].

**(2.5) Faraday’s induction law:**

Faraday’s induction law is written in the form of the following equation

$$V = \text{emf} = - \frac{1}{C} \frac{\partial \phi}{\partial t} \tag{2.5.1}$$

The constant of proportionality C is a universal with the dimensions usually, Faraday is law applied to circuits of conductors, however, (e.m.f) is simply the quantity of work performed by unit charge in moving along the circuit, and for a given field value through the circuit, cannot depend upon the form of the circuit. The (emf) is simply equal to the integral  $\int E \cdot dl$  . in a conducting circuit, this work can be dissipated in the generation of Joule heat (an ohmic load). However it is completely justifiable to consider the circuit in a vacuum also. In this case, the work performed on the charge us a pent in increasing the kinetic energy of the charged particles, as for instance in the case in an induction accelerator, the betatron[14].

## (2.6) Maxwell's Equations:

Equation (2.3) refers to any arbitrary closed circuit closed circuit. We substitute the definitions (2.3.1) and (2.4.1) into this equation we get [15].

$$\int E \cdot dl = \frac{-1}{c} \frac{\partial}{\partial t} \int B \cdot ds \quad (2.6.1)$$

The left hand side of the equation can be transformed by the stokes theorem: Which state that; the line integral of the tangential component of a vector A taken around a simple closed curve C is equal to the surface integral of the normal component of the curl of A taken over any surface having C as it's boundary [15] i.e.

$$\int A \cdot dl = - \int (\nabla \times A) \cdot ds \quad (2.6.2)$$

When one takes  $A = E$ , Then

$$\int E \cdot dL = - \int (\nabla \times A) \cdot ds \quad (2.6.3)$$

Thus equation (2.4) becomes

$$\int (\nabla \times A) \cdot ds = \frac{-1}{c} \frac{\partial}{\partial t} \int B \cdot ds = - \frac{1}{c} \int \frac{\partial B}{\partial t} \cdot ds \quad (2.6.4)$$

Where, on the right hand side, the order of the time differentiation and surface integration is interchanged. Thus taking this integral over to the left hand side, one obtains.

$$\int \left( (\nabla \times E) + \frac{1}{c} \frac{\partial B}{\partial t} \right) \cdot ds = 0 \quad (2.6.5)$$

But the initial circuit is completely arbitrary. i.e. it can have arbitrary magnitude and shape. Let us assume that the integrand, in parentheses, of equation (2.6.5) is not equal to zero. Then one can choose the surface and the circuit that bounds it so that the integral (2.6.5) does not become zero. Thus, in all cases, the following equation must be satisfied.

$$\nabla \times E + \frac{1}{c} \frac{\partial B}{\partial t} = 0 \quad (2.6.6)$$

This is one of the Maxwell's equations relating to electric and magnetic fields in a differential form. In many applications the differential form is more convenient than the integral form.

Magnetic field lines of force are either closed or go off to infinity. Hence, in any closed surface, the same number of magnetic field lines enters as leave. The magnetic field flux, in free space, across any closed surface [15], is equal to.

$$\oint \mathbf{B} \cdot d\mathbf{s} = 0 \quad (2.6.7)$$

Transforming this integral to a volume integral according to the Gauss-Ostrogradsky theorem [5]:

$$\oint \mathbf{A} \cdot d\mathbf{s} = \int \left( \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} \right) dv = \int (\nabla \cdot \mathbf{A}) dv \quad (2.6.8)$$

One obtains

$$\oint \mathbf{B} \cdot d\mathbf{s} = \int (\nabla \cdot \mathbf{B}) dv = 0 \quad (2.6.9)$$

Due to the fact that the surface bounding the volume is completely arbitrary, we can always choose this volume to be so small that the integral is taken over the region in which  $\nabla \cdot \mathbf{B}$  has constant sign if it is not equal to zero. But, then in spite of (2.6.7) and (2.6.8),  $\int \nabla \cdot \mathbf{B} ds$  will not be equal to zero. For this reason, the divergence of a vector is the density of source of a vector field. The sources of the field are free charges from which the vector (force) magnetic field lines originate. Thus (2.6.9) are together called the first pair of Maxwell's equations. The electric field flux through a closed surface is not equal to zero. But the total electric charge  $q$  inside the surface multiplied by  $4\pi$ [6].

$$\oint \mathbf{D} \cdot d\mathbf{s} = 4\pi q \quad (2.6.10)$$

Where,  $D$  is the electric flux density, the field due to a point charge  $q$  is expressed by the following equation



$$E = q/r^2 \quad (2.6.11)$$

Then the field is inversely proportional to  $r^2$ . If one surround the charge by a spherical surface centered on the charge. The element of the surface for the sphere  $ds$  is  $r^2 d\Omega$  where an elementary solid angle.

The flux of the field across the surface element is given by [4].

$$D \cdot ds = \frac{q}{r^2} \cdot r^2 d\Omega = qd\Omega \quad (2.6.12)$$

The flux across the whole surface of the sphere is thus given by

$$\int q \cdot d\Omega = q \int d\Omega = 4\pi q \quad (2.6.13)$$

But since lines of force begin only at a charge the flux will be the same through the sphere as through any closed surface around the charge. Therefore, if there is an arbitrary charge, distribution  $q$  inside a closed surface, then equation (2.6.10) holds. In order to rewrite this equation, in differential form, we introduce the concept of charge density. The charge density  $\rho$  is the charge contained in unit volume, so that the total charge in the volume is related to the density by the following equation.

$$q = \int \rho dv \quad (2.6.14)$$

introducing the charge density in (2.6.10), and utilizing the relation

$$\begin{aligned} \int D \cdot ds &= \int \nabla \cdot D dv \\ \int (\nabla \cdot D - 4\pi\rho) dv &= 0 \end{aligned} \quad (2.6.15)$$

Repeating the same argument for this integral as used (2.6.8) one have

$$\nabla \cdot D = 4\pi\rho \quad (2.6.13)$$

According to (2.6.8) one can that the density of sources of an electric field is equal to the electric charge density multiplied by  $4\pi$ . [14]

## (2.7) Electromagnetic potentials:

One can introduce new unknown quantities such that each equation will contain only one unknown. In this way we overall number of equations

is reduced. These new quantities are called electromagnetic potential. Thus for the magnetic field one can define by  $B = \nabla \times A$

Where  $A$  is a vector called the vector potential and for the electric field the electric potential is define to satisfy  $E = -\frac{1}{c} \frac{\partial A}{\partial t} - \nabla \phi$ .

Where  $\phi$  is also called the scalar potential[4].

**(2.8) Magneto motive Force:**

By analogy with electromotive force  $\int E \cdot dL$ . one can define

The magneto motive force  $\int H \cdot dL$ , where the integration is performed over a closed circuit. Using Ampere’s law, it may be shown that the integration of  $H$  in a closed circuit is equal to the summation of the electric current  $I$  surrounded by the magnetic loop. In other word.

$$\int H \cdot dl = \sum I$$

$$\sum J \cdot ds = \int J \cdot ds \tag{2.8.1}$$

But according to vector algebra

$$\int H \cdot dl = \int (\nabla \times H) \cdot ds,$$

Hence

$$\int (\nabla \times H) \cdot ds = \int J \cdot ds$$

$$\int (\nabla \times H - J) \cdot ds = 0$$

This relation can be satisfied if

$$\nabla \times H - J = 0 \tag{2.8.2}$$

$$\nabla \times H = J$$

This relation holds for static magnetic field and constant current which doesn’t vary with time. But it is no larger hold for time dependent, current and field. To verify this take divergence of both sides of equation (2.8.2) one gets.

$$\nabla \cdot (\nabla \times H) = \nabla \cdot J \tag{2.8.3}$$

But for vector algebra

$$\nabla \cdot (\nabla \times H) = |\nabla||\nabla \times H| \cos 90 = 0 \quad (2.8.4)$$

$$\text{Hence } \nabla \cdot J = 0 \quad (2.8.5)$$

But from continuity equation

## **Chapter Three**

### **Euler – Lagrange Equation**

#### **(3.1) Introduction:**

The notion of energy plays an important role in physics. Since energy is a scalar quantity, thus one can expect any mathematical framework based on it to be very simple if it is utilized to describe the state of physical systems. Due to this very important feature at Euler and Lagrange equation it succeeded to construct a very simple and elegant formalism known as Euler-Lagrangian formalism. In this chapter the basic concept associated with this formalism is presented.

#### **(3.2) Frame of reference:**

In order to describe the motion of a mechanical system, it is necessary to specify it's position in space as a function of time. Obviously, it is only meaningful to speak of the relative position of any point. For instance the position of a flying aircraft is given relative to some coordinate system fixed with respect to the earth. The motion of a charged particle in an accelerator is given relative to the accelerator, and so on. The system, relative to which the motion is described, is called a frame of reference[14].

#### **(3.3) Specification Of Time:**

Specification of time in the general case is also connected with defining the frame of reference in which it is given. Newton's laws permit a determination of the position of a mechanical system at an arbitrary instant of time, if the position and velocities of all points of the system are known at the some initial instant and also if the forces acting in the system are known[14].

### **(3.4) Degree of Freedom:**

The number of independent parameters definition the position of a mechanical system in space is termed the number of it's degree of freedom. The position of a particle in space relative to other bodies is defined with the aid of three independent parameters for example it's Cartesian coordinates. The position of a system consisting of N particles is determined by 3N independent parameters[14].

### **(3.5) Generalized coordinates:**

It is not always convenient to describe the position of a system in Cartesian coordinates. Cartesian coordinate must satisfy supplementary equations. In addition, the choice of coordinate system is a arbitrary and should be determined primarily on the basis of expediency for instant, if the forces depend only on the distance between particles, it is reasonable to introduce these distances into dynamical equations explicitly and not by means of Cartesian coordinates. That means a mechanical system can be describes by coordinates whose number is equal to the number of degrees of freedom of the system. These coordinates may sometimes coincide with the Cartesian coordinates for some of the particles. The independent parameters which define the position of a mechanical system in space are called it's generalized coordinates. They are represented by symbols  $q_\alpha$  , where the subscript  $\alpha$  signifies the number of the degree of freedom. As in the case of Cartesian coordinates, the choice of generalized coordinates is to a considerable extent arbitrary. It must be chosen so that the dynamical laws of motion of the system can be formulated as conveniently as possible[15].

### **(3.6) Lagrange's Equation:**

Equation of motion can be obtained in terms of arbitrary generalized coordinates in such form to be convenient for theoretical physicals[15]. The

system of lagrangian equations can be obtained not only from Newton's second law, but also from a very simple assertion about the value of the integral of the Lagrangian taken with respect to time. The basic laws of mechanics thus formulated are usually called Integral principles or Principles of least action. The particular importance of these principles is that they allow us to understand, in a unified manner, the laws relating to various areas of theoretical physical (mechanics and electrodynamics), thus opening up a field for broad generalization[16,17,18].

**(3.7) Action Principle:**

For a certain mechanical system let it be possible to define the lagrangian:

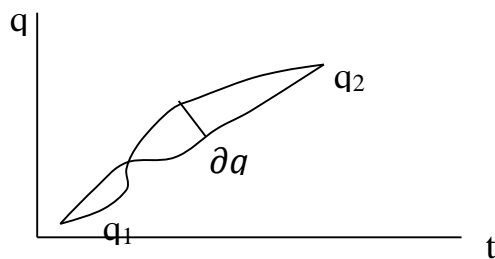
$$L = L(q, \dot{q}, t) \tag{3.7.1}$$

Where q stand for generalized coordinates,  $\dot{q}$  stands for velocities and t for time. Which are all assumed to be independent. The only requirement imposed on q(t) is that the functions should be smooth. The time integral of the lagrangian is called the action of the system.

$$I = \int_{t_0}^{t_1} L (q, \dot{q}, t) dt \tag{3.7.2}$$

**(3.8) The Calculus of Variation:**

From figure (3.1) both paths pass through the same points at the initial and final instants of time. The difference between the two paths at some instant of time is called the variation of q and is denoted by  $\delta q$ [16].



(Variation of two paths)

**Figure (3.1)** both paths pass through the same points at the initial and final instants of time

### **(3.9) Principles of Least Action:**

The principle of least action can be tackled by asserting that closed to the actual path passing between the given initial and final position of the system the increment of the action is equal to zero. This principle of least action can be utilized to derive Lagrange's equations. The action of an actual path is minimal, and therefore the assertion we have made is called the principle of least Action [16]. It is important now to find the variation of the lagrangian that means the difference of the function for two adjacent paths.

Since[17]:  $L = L (q , q^{\cdot} , t)$ .

And the variation is taken at the same instant of time i.e.  $\delta t = 0$

One obtains:

$$\delta L = \frac{\partial L}{\partial q} \delta q + \frac{\partial L}{\partial q^{\cdot}} \delta q^{\cdot} \quad (3.9.1)$$

$$\text{Since : } \delta q^{\cdot} = \frac{d}{dt} \delta q .$$

Then:

$$\frac{\partial L}{\partial q^{\cdot}} \delta q^{\cdot} = \frac{\partial L}{\partial q^{\cdot}} \frac{d}{dt} \delta q = \frac{d}{dt} \left( \frac{\partial L}{\partial q^{\cdot}} \delta q \right) + \delta q \frac{d}{dt} \left( \frac{\partial L}{\partial q^{\cdot}} \right) \quad (3.9.2)$$

Substituting this into (3.9.1) one gets:

$$\delta L = \frac{d}{dt} \left( \frac{\partial L}{\partial q^{\cdot}} \delta q \right) + \delta q \frac{\partial L}{\partial q} - \frac{d}{dt} \left( \frac{\partial L}{\partial q^{\cdot}} \right) \delta q \quad (3.9.3)$$

The integral of variation of L is equal to the variation of action  $\delta I$  , since the difference between integrals taken between the same limits is equal to the difference between the integrals[18].

$$\delta L = \int \delta L dt = \int D \left( \frac{\partial L}{\partial q^{\cdot}} \delta q \right) + \int \left[ \frac{\partial L}{\partial q} - \frac{d}{dt} \left( \frac{\partial L}{\partial q^{\cdot}} \right) \right] \delta q dt \quad (3.9.4)$$

Then the equation becomes:

$$\delta L = \frac{\partial L}{\partial q^*} \delta q \int_{t_0}^{t_1} \delta q \left[ \left( \frac{\partial L}{\partial q} \right) - \frac{d}{dt} \left( \frac{\partial L}{\partial q^*} \right) \right] dt \quad (3.9.5)$$

Then the equation becomes:

$$\delta L = \frac{\partial L}{\partial q^*} \delta q \left. \vphantom{\int} \right|_{t_0}^{t_1} + \int_{t_0}^{t_1} \delta q \left[ \left( \frac{\partial L}{\partial q} \right) - \frac{d}{dt} \left( \frac{\partial L}{\partial q^*} \right) \right] dt \quad (3.9.6)$$

$$\delta L = \frac{\partial L}{\partial q^*} \left. \vphantom{\int} \right|_{t=t_1} \delta q_{t_1} - \frac{\partial L}{\partial q^*} \left. \vphantom{\int} \right|_{t=t_0} \delta q_{t_0} + \int_{t_0}^{t_1} \left[ \left( \frac{\partial L}{\partial q} \right) - \frac{d}{dt} \left( \frac{\partial L}{\partial q^*} \right) \right] \delta q dt \quad (3.9.7)$$

If one consider only those oaths which pass through the same points  $q_0$  and  $q_1$  at the initial and final instants of time. Hence at these instants the variation. [12] .  $\delta q = 0$  .i.e.

$$\left. \begin{aligned} \delta q_{(t_0)} &= \hat{q}_{(t_0)} - q_{(t_0)} = 0 \\ \delta q_{(t_1)} &= \hat{q}_{(t_1)} - q_{(t_1)} = 0 \end{aligned} \right\} \quad (3.9.8)$$

As a result the first term disappears and equation (3.9.6) becomes:

$$\delta l = \int_{t_0}^{t_1} \delta q \left( \frac{\partial L}{\partial q} - \frac{d}{dt} \frac{\partial L}{\partial q^*} \right) dt \quad (3.9.9)$$

The extremism condition requires:

$$\delta l = 0 \quad (3.9.10)$$

This is known as the principle of least action

The equation (3.7) can be satisfied if:

$$\frac{d}{dt} \left( \frac{\partial L}{\partial q^*} \right) - \frac{\partial L}{\partial q} = 0 \quad (3.9.11)$$

Thus one gets the ordinary Euler – Lagrange's equation.

### (3.10) The generalized Euler – Lagrange Equation:

If the Lagrange depends on  $q$  beside  $q$  and  $q^*$  i.e. [19].

$$L = L(t, q, q^*, \hat{q}) \quad (3.10.1)$$



The variation of L can thus given by:

$$\delta L = \frac{\partial L}{\partial t} \delta t + \frac{\partial L}{\partial q} \delta q + \frac{\partial L}{\partial q^*} \delta q^* + \frac{\partial L}{\partial \ddot{q}} \delta \ddot{q} - \left( \frac{\partial L}{\partial \dot{q}} \right) \delta \dot{q} \quad (3.10.2)$$

Using the ordinary calculus one gets:

$$\left( \frac{\partial L}{\partial \dot{q}} \right) \delta \dot{q} = \frac{\partial L}{\partial \dot{q}} \frac{d}{dt} \delta q^* = \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}} \delta q^* \right) - \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}} \right) \delta q^* \quad (3.10.3)$$

$$\begin{aligned} \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}} \right) \delta q^* &= \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}} \right) \frac{d}{dt} \delta q \\ &= \frac{d}{dt} \left[ \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}} \right) \delta q \right] - \frac{d^2}{dt^2} \left( \frac{\partial L}{\partial \dot{q}} \right) \delta q \end{aligned} \quad (3.10.4)$$

Thus:

$$\frac{\partial L}{\partial \ddot{q}} = \frac{d}{dt} \left[ \frac{\partial L}{\partial \dot{q}} \delta q^* \right] - \frac{d}{dt} \left[ \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}} \right) \delta q \right] + \frac{d^2}{dt^2} \left( \frac{\partial L}{\partial \dot{q}} \right) \delta q \quad (3.10.5)$$

With the aid of equation (3.9.3) the variation of the action becomes:

$$\delta I = \int_{t_0}^{t_1} \delta L = \int_{t_0}^{t_1} \frac{\partial L}{\partial q} \delta q + \frac{\partial L}{\partial q^*} \delta q^* + \frac{\partial L}{\partial \ddot{q}} \delta \ddot{q} \quad (3.10.6)$$

$$\begin{aligned} \delta I &= \int_{t_0}^{t_1} \left[ \frac{\partial L}{\partial q} - \frac{d}{dt} \left( \frac{\partial L}{\partial q^*} \right) + \frac{d^2}{dt^2} \left( \frac{\partial L}{\partial \dot{q}} \right) \right] \delta q + \int_{t_0}^{t_1} \frac{d}{dt} \left[ \frac{\partial L}{\partial q^*} \delta q^* + \frac{\partial L}{\partial \dot{q}} \delta q - \right. \\ &\quad \left. \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}} \right) \delta q \right] = 0 \end{aligned} \quad (3.10.7)$$

The second term ( $S_2$ ) vanishes i.e.

$$S_2 = \int_{t_0}^{t_1} \frac{d}{dt} \left[ \frac{\partial L}{\partial q^*} \delta q + \frac{\partial L}{\partial \dot{q}} \delta q - \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}} \right) \delta q \right] = 0 \quad (3.10.8)$$

By the same arguments as in equation (3.9.8). as a result equation (3.10.7) reduces to:

$$\delta I = \int_{t_0}^{t_1} \left[ \frac{\partial L}{\partial q} - \frac{d}{dt} \left( \frac{\partial L}{\partial q^*} \right) + \frac{d^2}{dt^2} \left( \frac{\partial L}{\partial \dot{q}} \right) \right] \delta q = 0 \quad (3.10.9)$$

Which we can be satisfied if:

$$\left[ \frac{\partial L}{\partial q} - \frac{d}{dt} \left( \frac{\partial L}{\partial q^*} \right) + \frac{d^2}{dt^2} \left( \frac{\partial L}{\partial \dot{q}} \right) \right] = 0 \quad (3.10.10)$$

When one replaces time by the ordinary coordinates  $x_1$ ,  $x_2$ ,  $x_3$  and  $x_4$

Where:

$$X_1 = x_1, x_2 = y, x_3 = z, x_4 = iCt \quad (3.10.11)$$

Equation (3.10.10) can thus be generalized to becomes:

$$\left[ \frac{\partial L}{\partial q} - \partial_\mu \left( \frac{\partial L}{\partial \partial_{\lambda q}} \right) + \partial_{\lambda y q}^2 \left( \frac{\partial L}{\partial \partial_{\lambda q}} \right) \right] = 0 \quad (3.10.12)$$

Where one makes use of the following short hand notation

$$\delta_\lambda = \frac{\partial}{\partial x^\lambda}, \quad \delta_{\lambda y} = \frac{\partial^2}{\partial x^\lambda \partial x^y} \quad (3.10.13)$$

When L is independent of  $\delta_{\lambda y q}$  i.e.

$$L = L(q, \delta_{\lambda q}) \quad (3.10.14)$$

This equation takes the form:

$$\frac{\partial L}{\partial q} - \partial_\lambda \left( \frac{\partial L}{\partial \partial_{\lambda q}} \right) = 0 \quad (3.10.15)$$

This is the ordinary Euler – Lagrange equation.

### (3.11) Lagrangian Formalism of Maxwell's Equations

To express M.E in terms of Euler- Lagrange, it is important to choose suitable generalized coordinates to perform this purpose. The most suitable choice is to set.

$$q = A_\mu \quad \text{where } \mu = 1, 2, 3, 4$$

$$A_1 = A_x, \quad A_2 = A_y, \quad A_3 = A_z, \quad A_4 = iQ \quad (3.11.1)$$

Where  $A_x$ ,  $A_y$ ,  $A_z$  stand for the components of the magnetic vector potential, while  $\phi$  represents the electric scalar potential. With this choice one can define a second rank tensor F [20] as

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \quad (3.11.2)$$

The Lagrangian density for electromagnetic system is given by [20].

$$L = \frac{1}{16\mu} F_{\lambda\rho} F_{\lambda\rho} + J_\mu A_\mu \quad (3.11.3)$$

To find Maxwell's equation one utilized Euler- Lagrange Equation (3.10.15) which for  $q = A_\mu$  takes the form.

$$\frac{\partial L}{\partial A_\mu} - \partial_\mu \left[ \frac{\partial L}{\partial \partial_\mu A_\mu} \right] = 0 \quad (3.11.4)$$

Where the use of (3.11.3) leads to

$$\frac{\partial L}{\partial A_\mu} = \frac{J_\mu}{c} \quad (3.11.5)$$

And

$$\frac{\partial L}{\partial \partial_\nu A_\mu} = - \frac{1}{16\mu} F_{\lambda\rho} \frac{\partial F_{\lambda\rho}}{\partial \partial_\nu A_\mu} - \frac{\partial F_{\lambda\rho}}{\partial \partial_\mu A_\mu} \frac{F_{\lambda\rho}}{16\mu} = - \frac{F_{\lambda\rho}}{8\mu} \frac{\partial F_{\lambda\rho}}{\partial \partial_\nu A_\mu} \quad (3.11.6)$$

From the definition of equation (3.11.2), the derivatives of  $F_{\lambda\rho}$  vanishes except:  $\lambda = \mu$ ,  $\rho = \nu$  and  $\lambda = \nu$ ,  $\rho = \mu$

Hence:

$$\frac{\partial L}{\partial \partial_\nu A_\mu} = - \frac{F_{\mu\nu}}{8\pi} \frac{\partial F_{\mu\nu}}{\partial \partial_\nu A_\mu} - \frac{F_{\nu\mu}}{8\mu} \frac{\partial F_{\nu\mu}}{\partial \partial_\nu A_\mu} \quad (3.11.7)$$

Where :

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu = -(\partial_\nu A_\mu - \partial_\mu A_\nu) = -F_{\nu\mu} \quad (3.11.8)$$

$$\frac{\partial F_{\mu\nu}}{\partial_\nu A_\mu} = -1, \quad \frac{\partial F_{\nu\mu}}{\partial_\nu A_\mu} = 1 \quad (3.11.9)$$

$$\frac{\partial L}{\partial_\nu A_\mu} = -\frac{F_{\mu\nu}}{8\pi}(-1) - \frac{-(F_{\mu\nu})}{8\pi}(1) = \frac{F_{\mu\nu}}{8\pi} + \frac{F_{\mu\nu}}{8\pi} = \frac{F_{\mu\nu}}{4\pi} \quad (3.11.10)$$

Using (3.11.6) and (3.11.5) in (3.11.4) yields:

$$\frac{1}{4\pi} \partial_\nu F_{\mu\nu} - \frac{J_\mu}{c} = 0 \quad (3.11.11)$$

Or:

$$\partial_\nu F_{\mu\nu} = \partial_{\mu\nu} A_\nu - \partial_{\nu\nu} A_\mu = \frac{4\pi}{c} J_\mu \quad (3.11.12)$$

The magnetic flux density is related to the magnetic vector potential according to the relation [21].

$$\mathbf{B} = \nabla \times \mathbf{A} = \begin{bmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{bmatrix} \quad (3.11.13)$$

Where :

$$\mathbf{B} = B_x \mathbf{i} + B_y \mathbf{j} + B_z \mathbf{k} \quad (3.11.14)$$

In view of equation (3.11.8) and (3.11.9) one gets:

$$\mathbf{B} = \left( \frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) \mathbf{i} + \left( \frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \right) \mathbf{j} + \left( \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right) \mathbf{k}$$

$$B_x = \left( \frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) = \partial_2 A_3 - \partial_3 A_2 \quad (3.11.15)$$

$$\left. \begin{aligned} B_x &= \left( \frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) = \partial_3 A_1 - \partial_1 A_3 \\ B_y &= \left( \frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \right) = \partial_3 A_1 - \partial_1 A_3 \\ B_z &= \left( \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right) = \partial_1 A_2 - \partial_2 A_1 \end{aligned} \right\}$$

Where :

$$x_1 = x, x_2 = y, x_3 = z, x_4 = iCt$$

From which one utilize the relation (3.11.2) i.e.

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \quad (3.11.16)$$

To get the M.E

The components of the electric field intensity are also related to the vector and scalar potentials as follows:

$$F_{41} = \partial_4 A_1 - \partial_1 A_4 = iE_x = -\frac{\partial A_x}{iC \partial t} - \frac{i\partial\phi}{\partial x} \quad (3.11.17)$$

Thus:

$$E_x = -\frac{1}{C} \frac{\partial A_x}{\partial t} - \frac{\partial\phi}{\partial x} \quad (3.11.18)$$

Where:

$$X_4 = iCt, \quad (3.11.19)$$

$$F_{41} = E_x, F_{42} = E_y, F_{43} = E_z \quad (3.11.20)$$

$$iE_y = F_{42} = \partial_4 A_2 - \partial_2 A_4 = \frac{\partial A_y}{iC \partial t} - \frac{i\partial\phi}{\partial y}$$

$$E_y = \frac{1}{C} \frac{\partial A_y}{\partial t} - \frac{\partial\phi}{\partial y}$$

$$iE_z = F_{43} = \partial_4 A_3 - \partial_3 A_4 = \frac{\partial A_z}{iC \partial t} - \frac{i\partial\phi}{\partial z}, \quad (3.11.21)$$

$$E_z = -\frac{1}{C} \frac{\partial A_z}{\partial t} - \frac{\partial\phi}{\partial z}$$

Then we have [11]:

$$F_{\mu\nu} \begin{bmatrix} 0 & B_3 & -B_2 & -iE_1 \\ -B_3 & 0 & B_1 & -iE_2 \\ B_2 & -B_1 & 0 & -iE_3 \\ iB_1 & iB_2 & iE_3 & 0 \end{bmatrix} \quad (4.11.22)$$

Equation (3.11.7) can be utilized to get Maxwell's equations if one let  $\mu = 1$

Then :

$$\frac{\partial F_{1\nu}}{\partial x_\nu} = \frac{J_x}{C},$$

$$\frac{\partial F_{11}}{\partial x_1} + \frac{\partial F_{12}}{\partial x_2} + \frac{\partial F_{13}}{\partial x_3} + \frac{\partial F_{14}}{\partial x_4} = \frac{J_x}{c}.$$

$$0 + \frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z} - \frac{i \partial E_x}{\partial i c t} = \frac{J_x}{c},$$

$$\frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z} - \frac{1}{c} \frac{\partial E_x}{\partial t} = \frac{J_x}{c}$$

For  $\mu = 2$

$$\frac{\partial F_{2v}}{\partial y_v} = \frac{J_y}{c},$$

$$\frac{\partial F_{21}}{\partial x_1} + \frac{\partial F_{22}}{\partial x_2} + \frac{\partial F_{23}}{\partial x_3} + \frac{\partial F_{24}}{\partial x_4} = \frac{J_x}{c},$$

$$\frac{\partial B_z}{\partial z} + 0 + \frac{\partial B_z}{\partial z} - \frac{i \partial E_y}{\partial i c t} = \frac{J_x}{c},$$

$$\frac{\partial B_z}{\partial z} - \frac{\partial B_z}{\partial x} - \frac{1}{c} \frac{\partial B_y}{\partial t} = \frac{J_y}{c}$$

(3.11.23)

For  $\mu = 3$

$$\frac{\partial F_{30}}{\partial x_v} = \frac{J_z}{c},$$

$$\frac{\partial F_{31}}{\partial x_1} + \frac{\partial F_{32}}{\partial x_2} + \frac{\partial F_{33}}{\partial x_3} + \frac{\partial F_{34}}{\partial x_4} = \frac{J_z}{c},$$

$$\frac{\partial B_y}{\partial x} + \frac{\partial B_x}{\partial y} + 0 - \frac{i \partial E_z}{\partial i c t} = \frac{J_z}{c},$$

(3.11.24)

$$\frac{\partial B_y}{\partial x} - \frac{\partial B_z}{\partial y} - \frac{1}{c} \frac{\partial E_z}{\partial t} = \frac{J_z}{c},$$

For  $\mu = 4$ :

$$\frac{\partial F_{40}}{\partial x_v} = \frac{J_4}{c} = \frac{i c \rho}{c}$$

$$\frac{\partial F_{41}}{\partial x_1} + \frac{\partial F_{42}}{\partial x_2} + \frac{\partial F_{43}}{\partial x_3} + \frac{\partial F_{44}}{\partial x_4} = i \rho.$$

$$\frac{i \partial E_x}{\partial x} + \frac{i \partial E_y}{\partial y} + \frac{i \partial E_z}{\partial z} + 0 = i \rho,$$

(3.11.25)

$$\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = \rho$$

With the aid of (3.11.17) , (3.11.18) and (3.11.19) one gets:

$$\nabla \times \mathbf{B} = \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t} + \frac{\mathbf{J}}{c} \quad (3.11.26)$$

Using also relation (3.11.20) one obtains:

$$\nabla \cdot \mathbf{E} = \rho \quad (3.11.27)$$

Equation (3.11.21) can be rewritten in a matrix form to be:

$$\nabla \times \mathbf{B} - \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t} = \begin{bmatrix} i & j & k & t \\ \frac{\partial}{\partial x_1} & \frac{\partial}{\partial x_2} & \frac{\partial}{\partial x_3} & \frac{\partial}{\partial x_4} \\ B_1 & B_2 & B_3 & B_4 \end{bmatrix} - \frac{1}{c} \frac{\partial}{\partial t} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \\ E_4 \end{bmatrix} \quad (3.11.28)$$

Equation (3.11.21) and (3.11.22) represents the first pair of M.Es.

For the second pair of the M.Es one can use the previous tensor with a little change to be

$$F_{\mu\nu} = \begin{bmatrix} 0 & B_3 & -E_2 & -iB_1 \\ -E_3 & 0 & E_1 & -iB_2 \\ E_2 & -E_1 & 0 & -iB_3 \\ iB_1 & iB_2 & iB_3 & 0 \end{bmatrix} \quad (3.11.29)$$

By setting  $J_\mu = 0$  in equation (3.11.7) one get

$$\frac{\partial F_{\mu\nu}}{\partial x_\nu} = 0$$

For  $\mu = 1$

$$\frac{\partial F_{1\nu}}{\partial x_\nu} = 0,$$

$$\frac{\partial F_{11}}{\partial x_1} + \frac{\partial F_{12}}{\partial x_2} + \frac{\partial F_{13}}{\partial x_3} + \frac{\partial F_{14}}{\partial x_4} = 0,$$

$$0 + \frac{\partial E_z}{\partial y} + \frac{\partial E_y}{\partial z} + \frac{\partial iB_x}{\partial iCt} = 0.$$

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} - \frac{1}{c} \frac{\partial B_x}{\partial t} = 0 \quad (3.11.29)$$

And for :  $\mu = 2$

$$\frac{\partial F_{2\nu}}{\partial x_\nu} = 0$$

$$\frac{\partial F_{21}}{\partial x_1} + \frac{\partial F_{22}}{\partial x_2} + \frac{\partial F_{23}}{\partial x_3} + \frac{\partial F_{24}}{\partial x_4} = 0$$

$$0 - \frac{\partial E_z}{\partial x} + \frac{\partial E_x}{\partial z} + \frac{\partial iB_y}{\partial iCt} = 0,$$

$$\frac{\partial E_z}{\partial z} - \frac{\partial E_x}{\partial x} + \frac{1}{c} \frac{\partial B_y}{\partial t} = 0 \quad (3.11.30)$$

While for  $\mu = 3$

$$\frac{\partial F_{3v}}{\partial x_v} = 0,$$

$$\frac{\partial F_{31}}{\partial x_1} + \frac{\partial F_{32}}{\partial x_2} + \frac{\partial F_{33}}{\partial x_3} + \frac{\partial F_{34}}{\partial x_4} = 0$$

$$\frac{\partial E_y}{\partial x} + \frac{\partial E_x}{\partial y} + 0 + \frac{\partial iB_z}{\partial iCt} = 0,$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} + \frac{1}{c} \frac{\partial B_z}{\partial t} = 0$$

Also for  $\mu = 4$ .

$$\frac{\partial F_{4v}}{\partial x_v} = 0,$$

$$\frac{\partial F_{41}}{\partial x_1} + \frac{\partial F_{42}}{\partial x_2} + \frac{\partial F_{43}}{\partial x_3} + \frac{\partial F_{44}}{\partial x_4} = 0,$$

$$\frac{i \partial B_x}{\partial x} + \frac{i \partial B_y}{\partial y} + \frac{i \partial B_z}{\partial z} + 0 = 0,$$

$$\frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} = 0 \quad (3.11.31)$$

Equations (3.11.25) , (3.11.26), (3.11.27) thus yield:

$$\nabla \times \mathbf{E} + \frac{1}{c} \frac{\partial \mathbf{B}}{\partial t} = 0 \quad (3.11.32)$$

While equation (3.11.28) also yields:

$$\nabla \cdot \mathbf{B} = 0 \quad (3.11.33)$$

Therefore equations (3.11.29) and (3.11.30) represent the second pair of M.Es.

Equation (3.11.32) can be rewritten in matrix form to be:

$$\nabla \times \mathbf{E} + \frac{1}{c} \frac{\partial \mathbf{B}}{\partial t} = \begin{bmatrix} i & j & k & t \\ \frac{\partial}{\partial x_1} & \frac{\partial}{\partial x_2} & \frac{\partial}{\partial x_3} & \frac{\partial}{\partial x_4} \\ E_1 & E_2 & E_3 & E_4 \end{bmatrix} + \frac{1}{c} \frac{\partial \mathbf{B}}{\partial t} \begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \end{bmatrix} \quad (3.11.34)$$



### (3.12) Klein-Gordon Wave Equation and Hamiltonian:

Lagrangian of the scalar Klein-Gordon field the familiar form of the lagrangian equation of the Klein-Gordon field

$$\mathcal{L} = \frac{1}{2} \dot{\Psi}^2 - \frac{1}{2} \nabla^2 \Psi^2 - \frac{1}{2} m^2 \Psi^2 \quad (3.12.1)$$

We have derived the lagrangian equation for scalar field. By assuming that  $\Psi$  is a Lorentz we obtained the above lagrangian. We now use the lagrangian equation.

$$\partial_t \left( \frac{\partial \mathcal{L}}{\partial \dot{\Psi}} \right) - \partial_x \left( \frac{\partial \mathcal{L}}{\partial \partial_x \Psi} \right) - \frac{\partial \mathcal{L}}{\partial \Psi} = 0 \quad (3.12.2)$$

Differentiation the lagrangian with respect to  $\Psi$  yields

$$\frac{\partial \mathcal{L}}{\partial \Psi} = m^2 \Psi \quad (3.12.3)$$

Differentiation the lagrangian with respect to the time derivative with  $\Psi$  yields

$$\partial_t \left( \frac{\partial \mathcal{L}}{\partial \dot{\Psi}} \right) = \partial_t (\partial_t \Psi) = \partial_t^2 \Psi \quad (3.12.4)$$

The same procedure can be done for x to get

$$\partial_x \left( \frac{\partial \mathcal{L}}{\partial \partial_x \Psi} \right) = \partial_x (\partial_x \Psi) = \partial_x^2 \Psi \quad (3.12.5)$$

Thus the direct substituting of equations (3.12.3,4,5) in equation (3.12.2) yields

$$\partial_t^2 \Psi - \partial_x^2 \Psi + m^2 \Psi = 0 \quad (3.12.6)$$

In order to obtain the equation the motion, what we get is the Klein-Gordon.

The scalar quantum field representation derived directly from the classical relativistic particle lagrangian.

The Hamiltonian equation of the classical particle, we now using the lagrange equation,

$$H = L - \left( \frac{\partial L}{\partial \partial_t \Psi} \right) \partial_t \Psi - \frac{\partial L}{\partial \Psi} \quad (3.12.7)$$

Differentiation the lagrangian with respect to  $\Psi$  yields

$$\frac{\partial L}{\partial \Psi} \Psi = (m^2 \Psi) \Psi = -m^2 \Psi^2 \quad (3.12.8)$$

Differentiation the lagrangian with respect to the time derivative of  $\Psi$  yields

$$\left( \frac{\partial L}{\partial \partial_t \Psi} \right) \partial_t \Psi = (\partial_t \Psi) \partial_t \Psi = \partial_t^2 \Psi \quad (3.12.9)$$

Substituting equation (3.12.1,8,9) in equation (3.3.7) yields

$$H = \frac{1}{2} \Psi'^2 - \frac{1}{2} \nabla^2 \Psi^2 + \frac{1}{2} m^2 \Psi^2 - \Psi^2 + m^2 \Psi^2 \quad (3.12.10)$$

Or

$$H = -\frac{1}{2} \Psi'^2 - \frac{1}{2} \nabla^2 \Psi^2 + \frac{1}{2} m^2 \Psi^2 \quad (3.12.11)$$

This corresponds with the following Hamiltonian equation for the Klein-Gordon equation.

### (3.13) Dirac Equation and Hamiltonian:

The Dirac lagrangin equation

$$L = i\hbar \psi \partial_t \psi^* + i\hbar \psi \nabla \Psi^* + \beta m c^2 \psi \psi^* \quad (3.13.1)$$

We would now like to quantize Dirac lagrangian , using equation

$$\frac{\partial L}{\partial \Psi^*} - \partial_t \frac{\partial L}{\partial \partial_t \Psi^*} - \partial_x \left( \frac{\partial L}{\partial \partial_x \Psi^*} \right) = 0 \quad (3.13.2)$$

Differentiation lagrangian to  $\Psi^*$  give

$$\frac{\partial L}{\partial \Psi^*} = \beta m c^2 \Psi \quad (3.13.3)$$

The Differentiation the lagrangian to x derivative of  $\Psi^*$  yields

$$\partial_t \left( \frac{\partial L}{\partial \partial_t \Psi^*} \right) = \partial_t (i\hbar c \psi) = i\hbar \partial_t \psi$$

$$\partial_x \left( \frac{\partial L}{\partial \partial_x \Psi^*} \right) = i\hbar \partial_x \psi \quad (3.13.4)$$

Substituting equations (3.13.3,4) in equation (3.13.2) yields

$$i\hbar c \partial_t \psi - i\hbar c \partial_x \psi + \beta m c^2 \psi \quad (3.13.5)$$

Or

$$i\hbar \partial_t \psi = -i\hbar c \partial_x \psi + \beta m c \psi \quad (3.13.6)$$

This is the Dirac equation

The Hamiltonian of the lagrangian can found with aid of equation (3.13.7)

$$H = L - \left( \frac{\partial L}{\partial \partial_t \Psi^*} \right) \partial_t \Psi^* \quad (3.13.7)$$

The differentiation to time derivative of  $\Psi^*$  and time yields

$$\left( \frac{\partial L}{\partial \partial_t \Psi^*} \right) \partial_t \Psi^* = (i\hbar c \psi) \partial_t \Psi^* = i\hbar c \psi \partial_t \Psi^* \quad (3.13.8)$$

Using to find Hmiltonian in equation (3.13.7) for the quantum system by inserting equation (3.13.1,8) in equation (3.4.7) to get

$$H = i\hbar c \psi \partial_t \Psi^* + i\hbar c \psi \nabla \Psi^* + \beta m c^2 \psi \Psi^* - i\hbar c \psi \partial_t \Psi^* \quad (3.13.9)$$

And

$$H = i\hbar c \psi \nabla \Psi^* + \beta m c^2 \psi \Psi^* \quad (3.13.9)$$

This is the Dirac Hamiltonian equation.

### (3.14) Electrodynamic and electroweak lagrangian:

The electroweak lagrangian is that describe the behavior of the electronic charges and the electromagnetic field generated by them. It combine Dirac lagrangian

$$L = -i\hbar\Psi \partial_t\Psi^* + i\hbar c\Psi\nabla\Psi^* + \beta mc^2\Psi\Psi^* \quad (3.14.1)$$

with electromagnetic lagrangian

$$L = \frac{1}{16\pi} F_{\mu\nu}F^{\mu\nu} + J^\mu A_\mu \quad (3.14.2)$$

With the first term standing for the free electromagnetic field, while the second one describe the interaction between charges and electromagnetic field. Thus the complete electrodynamic lagrangian which describe charges and electromagnetic field beside their interactions becomes

$$L = -i\hbar\Psi \partial_t\Psi^* + i\hbar c\Psi\nabla\Psi^* + \beta mc^2\Psi\Psi^* + \frac{1}{16\pi} F_{\mu\nu}F^{\mu\nu} + J^\mu A_\mu \quad (3.14.3)$$

For electroweak interaction which unifies electromagnetic interaction one needs to add the weak interaction lagrangian

$$L = L = C\bar{\Psi}_w (i\hbar\gamma^\mu \partial_\mu - Mc)\Psi_w c g\bar{\Psi}_w \gamma^\mu (\vec{W}_\mu \cdot \vec{\sigma})\Psi_w \quad (3.14.4)$$

### (3.15 ) Electroweak interaction and poson masses

Physical world consists of matter particles beside field quanta (particles) like photons  $w^+$ ,  $w^-$  and  $z^0$  bosons matter particles are called fermions , while field quanta (particles) are called bosons.

The lagrangian of the matter field is described by

$$L = \bar{\Psi}(i\gamma^\mu \partial_\mu - m)\Psi \quad (3.15.1)$$

In physics one needs can served quantities like energy and momentum .

This can be achieved by space-time translation which leads to 4- momentum energy + 3-momentum components. This leads to action and lagrangian invariance. Noether's theorem is the generalization of this symmetry by introducing continuous group of transformation of the fields.

The corresponding lagrangian determine a conserved tensor and an associated time – independent observable

The lagrangian in equation (3.15.1) can be made invariant under local dependent gauge

$$\begin{aligned} \psi &\rightarrow \psi' = e^{iq\Lambda(x)}\psi(x) \\ \bar{\psi} &\rightarrow \bar{\psi}' (x) = e^{-iq\Lambda(x)}\bar{\psi}(x) \end{aligned} \quad (3.15.2)$$

Under this transformation the first term of lagrangian (3.15.1) transforms as

$$\begin{aligned} \psi' i\gamma^\mu \partial_\mu \psi' &= e^{-iq\hat{\Lambda}} i\gamma^\mu \partial_\mu (e^{iq\Lambda(x)}\psi(x)) \\ &= e^{-iq\hat{\Lambda}} (i\gamma^\mu (iq \partial_\mu \Lambda) \psi + \partial_\mu \psi) e^{iq\Lambda} \\ &= \bar{\psi}' (i\gamma^\mu \partial_\mu \psi + i^2 q \gamma^\mu \psi \partial_\mu \Lambda) \end{aligned} \quad (3.15.3)$$

Clearly this term is not invariant under the gauge transformation the second term in (3.15.1) is given by

$$\psi' m \psi' = e^{-iq\hat{\Lambda}} \psi e^{iq\Lambda} = \psi' m \psi' \quad (3.15.4)$$

The mass term is thus invariant . the invariant of the first term and the lagrangian can be restored by replacing ordinary derivative

$$D_\mu \psi = \partial_\mu \psi + iqA_\mu(x) \quad (3.15.5)$$

Where  $A_\mu$  is a field boson quanta , which transform as  $A_\mu(x) \rightarrow A_\mu^1(x) =$

$$A_\mu(x) - \partial_\mu \Lambda(x) \quad (3.15.6)$$

Under these constraints the lagrangian is invariant ,

$$L^1 = \bar{\psi}' i\gamma^\mu D_\mu \psi' - m\psi' = L = \bar{\psi} (i\gamma^\mu D_\mu \psi - m\psi) - \quad (3.15.7)$$

This lagrangian describe free matter particles and interaction of them with electromagnetic field  $A_\mu$ . the complete lagrangian requires adding free electromagnetic (e.m) of the form

$$L_{(e.m)} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \quad (3.15.8)$$

The complete electroweak lagrangian is

$$L = \bar{\Psi}(i\gamma^\mu D_\mu - m)\Psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \quad (3.15.9)$$

$$\bar{\Psi}(i\gamma^\mu \partial_\mu - m)\Psi - j^\mu A_\mu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

It is important to note that the lagrangian invariance required massless field quanta (bosons) ,since the mass term

$$m_\gamma A_\mu A^\mu \neq m_\gamma A_\mu^\lambda A^{\lambda\mu} = m_\gamma (A_\mu - \partial_\mu \Lambda)(A^\mu - \partial^\mu \Lambda)$$

(3.15.10) is not invariant

### (3.16) Higgs Mechanism and Mass Generation

The field quanta (particles) of electroweak interaction particles are not all mass less.

Some of these quanta (bosons) are massive. According to equation (3.15.10) the guage invariance of L requires mass less quanta. Thus one needs a certain mechanism (Higgs mechanism) for mass generation. According to Higgs mechanism (H.M) field quanta masses and generated at vacuum state ,which can be found by considering the scalar (spin less) lagrangian ,which describes Klein-Gordon equation, where

$$L = (\partial_\mu \Phi)^*(\partial^\mu \Phi) - V(\varphi) = (\partial_\mu \varphi)^*(\partial^\mu \varphi) - \frac{m^2}{2}\varphi^*\varphi - \frac{\lambda}{4}(\varphi^*\varphi)^2$$

(3.16.1)

For real scalar field the potential V is given by

$$V = \frac{m^2}{2}\varphi^2 + \frac{\lambda}{4}\varphi^4 \quad (3.16.2)$$

The potential is minimum , where

$$\frac{dV}{dQ} = \frac{2m^2}{2} \varphi + \frac{\lambda(4\varphi^2)}{4} = 0$$

$$\varphi^3 = \frac{m^2}{\lambda} \varphi \quad \varphi = \sqrt{\frac{-m^2}{\lambda}} = v \quad V=\text{vacuum}$$

Thus the vacuum state potential is describe by (3.16.3).

Now assume existence of new Higgs field H which fluctuates around vacuum. Thus

$$\varphi = v + H(x) \quad (3.16.4)$$

Where v is constant . clearly the lagrangian (3.16.2) is not invariant under mirror reflection (parity) , i.e  $H \rightarrow -H$

In this case the term

$$\begin{aligned} \frac{m^2}{2} \varphi^* \varphi &= \frac{m^2}{2} (v + H)(v + H) = \frac{m^2}{2} (v^2 + 2vH + H^2) \\ &\neq \frac{m^2}{2} (v - H)(v - H) = \frac{m^2}{2} (v^2 - 2vH + H^2) \end{aligned}$$

Thus the symmetry is spontaneously broken , since parity is conserved or reflection symmetry is hidden . now consider the scalar field to be

$$\varphi = \frac{1}{\sqrt{2}} [v + H] e^{\frac{i\theta}{v}}$$

Where

$$\theta = \theta(x) \quad H = H(x) \quad (3.16.5)$$

The field can transform as

$$\varphi^1(x) = \varphi^1 e^{iq\Lambda(x)} \varphi(x) = e^{iq\Lambda} \varphi \quad (3.16.6)$$

(1.2.6) from (1.25)

$$\varphi^1 = \frac{1}{\sqrt{2}} [v + H^1] e^{\frac{i\theta^1}{v}} \quad (3.16.7)$$

But from (3.16.5) and (3.16.7)

$$\varphi^1 = \frac{1}{\sqrt{2}} [v + H] e^{iq\Lambda + \frac{i\theta}{v}} \quad (3.16.8)$$

Comparing (3.16.7) and (3.16.8) requires

$$H^1 = H$$

$$\theta^1 = q\Lambda + \frac{\theta}{v}$$

$$\theta^1 = qv\Lambda + \theta \quad (3.16.10)$$

By choosing

$$\Lambda = -\frac{1}{qv} \theta \quad (3.16.11)$$

The mass less gold sane boson can be removed. The field Q transforms as

$$\begin{aligned} \varphi &= [v + H]e^{\frac{i\theta}{v}} \rightarrow \varphi^1 = [v + H^1]e^{\frac{i\theta^1}{v}} \\ \varphi^1 &= [v + H] \end{aligned} \quad (3.16.12)$$

Where

$$\varphi^1 = [v + H]$$

in equation( 3.16.10)

vanishes according to equation (3.16.11)

$$\theta^1 = \theta \quad (3.16.13)$$

According to this choice the Lagrangian

$$L = D_\mu^* \varphi^* D_\mu \varphi - \frac{m^2}{2} \varphi^* \varphi - \frac{\lambda}{4} (\varphi^* \varphi)^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \quad (3.16.14)$$

With the covariant derivative

$$\begin{aligned} D_\mu &= \partial_\mu + iqA_\mu \\ D_\mu^* &= \partial_\mu - iqA_\mu \end{aligned} \quad (3.16.15)$$

And

$$A_\mu^1 = A_\mu + \partial_\mu \Lambda = A_\mu + \frac{1}{qv} \partial_\mu \theta \quad (3.16.16)$$

$$D_\mu^1 = \partial_\mu + iqA_\mu^1 = \partial_\mu + iqA_\mu + \frac{i}{v} \partial_\mu \theta \quad (3.16.17)$$

Is invariant , i.e

$$L^1 = D_\mu^1 \varphi^{1*} D_\mu^1 \varphi^1 - \frac{m^2}{2} \varphi^{1*} \varphi^1 - \frac{\lambda}{4} (\varphi^{1*} \varphi^1)^2 - \frac{1}{4} F_{\mu\nu}^1 F^{\mu\nu 1} = L \quad (3.16.18)$$



Using equation (3.16.17)

$$L = \left[ \frac{1}{2} (\partial_\mu H)^2 - \lambda v^2 H^2 \right] + \frac{1}{2} q^2 v^2 A_\mu A^\mu + \frac{1}{2} q^2 A_\mu A^\mu H^2 + q^2 v A_\mu A^\mu H - \lambda v H^3 - \frac{\lambda}{4} H^4 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{\lambda}{4} v^4$$

Thus the field quanta bosons acquire mass, where the mass term is

$$\frac{1}{2} q^2 v^2 A_\mu A^\mu = \frac{1}{2} m_A^2 A_\mu A^\mu$$

Their mass is given to be

$$m_A = qv$$

The Higgs quanta bosons mass term is given by

$$\lambda v^2 H^2 = \frac{1}{2} m_H^2 H^2$$

Thus the Higgs particles masses are

$$m_H = \sqrt{2\lambda v^2}$$

- the Lagrangian invariance and form can be easily found knowing that

$$F_{\nu\mu}^1 = \partial_\mu A_\nu - \partial_\nu A_\mu = \partial_\mu \left( A_\nu + \frac{\partial_\nu \theta}{qv} \right) - \partial_\nu \left( A_\mu + \frac{\partial_\mu \theta}{qv} \right) = \partial_\mu A_\nu - \partial_\nu A_\mu + \frac{\partial_{\mu\nu} \theta - \partial_{\nu\mu} \theta}{qv} = \partial_\mu A_\nu - \partial_\nu A_\mu = F_{\mu\nu}$$

Where

$$\partial_{\mu\nu} \theta = \partial_{\nu\mu} \theta$$

### (3.17) CP violation and CPT Theorem

A parity transformation in quantum physics involves the change in sign on spatial coordinate, where parity inversion transforms a chiral phenomenon into its mirror image. All the laws of physics are believed to be exactly the same when looked at in a mirror image of the world, Thus when watching physics experiment reflected in a mirror, observers believe that they are seen reality, this referred to as parity invariance. Three of

fundamental forces of nature, namely – the electromagnetic, the gravitational, and the strong nuclear forces are found to obey parity invariance. The weak force, on the other hand, was found to be an exception, until it was realized that replacing matter with antimatter, also called charge reversal, will establish charge –parity (CP) invariance. Further investigation revealed an anomaly in the decay of the K-meson which was fingerprint of the violation of the CP invariance principle. Subsequently, other mesons were discovered to violate the CP theorem. CP violation can be removed by introducing time reversal (T) to form a coherent CPT symmetry can be visualized as a mirror image of our universe with all matter replaced by antimatter (charge inversion or conjugation), all objects having their position reflected by an imaginary plane (parity inversion), and all momenta reversed (time reversal). CPT implies that particles and antiparticles have the same inertial masses and lifetimes, and equal but opposite charges and magnetic moments. Therefore, one of the important aspects to be considered is the ratios of the inertial masses of  $e^+/e^-$  and  $P^-/P^+$ . The difference in masses was found to be  $|m(e^+) - m(e^-)| < 4 \times 10^{-8} m(e^-)$  at the 90% confidence level and  $|m(P^-) - m(P^+)| < 4 \times 10^{-8} m(P^-)$  to one standard deviation. It was shown that CPT violation implies Lorentz symmetry breaking. The overwhelming majority of the experimental work to look for Lorentz violation yielded negative results however one has to remember that CPT invariance is not necessarily deeply rooted in the physical world the way, for instance, the conservation of energy is. In fact, Oksak and Todorov and Stoyanov and Todorov have shown that the CPT theory can be violated when non-finite –dimensional representations of the Lorentz group are allowed.

Wald has argued that the CPT theorem may face obstacles when applied in curved space time. In reality, many models have been proposed that predict a small CPT violation in curved space time. In the next section, the CPT theorem will be shown to govern our understanding of baryogenesis in the early universe.

## Chapter Four

### Literature review

#### (4.1) introduction

Different attempts were made to utilize second order lagrangian in fields in general [22] . also attempts were made to quantize electron charge [23]. Here we caught some of them.

#### (4.2) structure and self energy of the electron

In the work of S.M. Blinder one [24]. Assume that the electron rest mass ( $0.511 \text{ MeV}/c^2$ ) is otally electromagnetic, which was the original idea of Lorentz and Abra-ham. This is consistent with the (nearly, if not exactly) zero mass of the electrons uncharged weak isodoublet partner-the neutrino- and with ore r of magnitude of the neutron-proton mass difference ( $1.29 \text{ MeV}/c^2$ ). There is no need to invoke any nonelectromagnatic forces within the electron collectively known as Poincare stresses.

The energy of an electromagnetic field in a rest frame is given by

$$w = \frac{1}{8\pi} \int (E \cdot D + B \cdot H) d^3r \quad (4.2.1)$$

The field produced by a point charge e in vacuum is represented by

$$D = E = \frac{e\hat{r}}{r^2} \quad B = H = 0$$

$$\text{And } w = \frac{1}{8\pi} \int \frac{e^2}{r^2} 4\pi r^2 dr = \infty \quad (4.2.2)$$

Unless a lower cutoff is introduced.

It was suggested a long time ago by furry and Oppenheimer[25] that quantum electrodynamic (QED) effects could give the vacuum some characteristics of a polarizable medium , which Weiskopf[26] represented phenomenologically by an inhomogeneous dielectric constant, viz

$$D(r) = \epsilon(r)E(r). \quad (4.2.3)$$

Constitutive relations in classical electrodynamics describe properties of matter which must be determined experimentally computation. In the same sense, Eq. (4) represents a constitutive relation for the vacuum, as implied by quantum electrodynamics. Using (4.2.3) in (4.2.1)

$$W = \frac{1}{8\pi} \int_0^\alpha \frac{1}{\epsilon(r)} \frac{e^2}{r^4} 4\pi r^2 dr \quad (4.2.4)$$

And equating this to the self-energy of the electron

$$W = \frac{e^2}{2} \int_0^\infty \frac{dr}{r^2 \epsilon(r)} = mc^2 \quad (4.2.5)$$

Remarkably, the functional form of  $\epsilon(r)$  need not be further specified, provided only that it satisfies the limiting conditions

$$\epsilon(\infty) = 1 \text{ and } \epsilon(0) = \infty \quad (4.2.6)$$

Maxwell's first equation  $\nabla \cdot E = 4\pi\rho$  applied to the electric field

$$E = \frac{er}{\epsilon(r)r^3} \quad (4.2.7)$$

Determines the charge density

$$\rho(r) = -\frac{e\epsilon'}{4\pi r^2 [\epsilon(r)]^2} \quad (4.2.8)$$

Note that this represents the net or total charge density, the sum of the free and polarization densities.

This function is appropriately normalized since

$$\begin{aligned} \int_0^\infty \rho(r) 4\pi r^2 dr &= - \int_0^\infty \frac{\epsilon'(r) dr}{[\epsilon(r)]^2} \\ &= e \left[ \frac{1}{\epsilon(\infty)} - \frac{1}{\epsilon(0)} \right] = e \end{aligned} \quad (4.2.9)$$

An explicit functional form for  $\epsilon(r)$  does follow if it is conjectured that the net charge density (4.2.8) is proportional to field energy density from (4.2.5) for then,

$$\frac{\epsilon'(r)}{\epsilon(r)} = \frac{e^2}{2mc^2 r^2} \quad (4.2.10)$$

With the solution

$$\epsilon(r) = \exp\left(\frac{e^2}{2mc^2 r}\right) = \exp\left(\frac{r_0}{2r}\right) \quad (4.2.11)$$

It should be emphasized for the benefit of the QED theorists who might be reading this that our use of them “vacuum polarization” is intended only in a classical phenomenological context. The leading contribution to vacuum polarization in real life comes from the interaction of the electron with the transverse radiation field, which does not enter in our model. We are thereby overlooking additional self-energy contributions arising from fluctuations in the vacuum radiation field. Accordingly, our representation of vacuum polarization is not to be compared with QED computations.

Somewhat of a rationalization for functional form of  $\epsilon(r)$  is suggested by Debye-Huckel theory for ionic solution and plasmas. The dielectric constant depends on a Boltzmann factor  $e^{-\mathcal{E}/KT}$ . If in place of the average thermal energy  $KT$ , we substitute the relativistic energy of pair formation  $2mc^2$ , regarding the vacuum as an effective thermal reservoir, then Eq.(2.4.11) follows with  $\mathcal{E} = e^2/r$ .

An explicit expression for the charge density follows by substituting (4.2.11) into (4.2.8):

$$\rho(r) = \frac{er_0}{8\pi r^4} e^{-r_0/2r} \quad (4.2.12)$$

Since  $\rho_{free}(r) = e\delta(r)$ , the density from vacuum polarization must equal

$$\rho_{vp}(r) = \frac{er_0}{8\pi r^4} e^{-r_0/2r} - e\delta(r), \quad (4.2.13)$$

According to this model, the free point charge is exactly canceled by the delta function term of the polarization charge.

The electrostatic potential corresponding to (4.2.13) is given by

$$\Phi(r) = \frac{2e}{r_0} (1 - e^{-r_0/2r}) \quad (4.2.14)$$

This implies a deviation from Coulombs law of the same magnitude as the fine structure in atoms, but totally negligible on a macroscopic scale reality such as the lamb shift. Note that (4.2.14) reduces to  $e/r$  when either  $r_0 \rightarrow 0$  Or  $r \rightarrow \infty$ .

An alternative valuation of the electromagnetic self-energy follows from transformation of Eq. (4.2.1)

$$W = \frac{1}{8\pi} \int E \cdot D d^3r = \frac{1}{2} \int \Phi_{\text{free}} \rho d^3r \quad (4.2.15)$$

Using

$$D = -\nabla\Phi_{\text{free}} = \frac{er}{r^3} \quad (4.2.16)$$

And assuming the requisite vanishing of integrands at infinity. Thus

$$\begin{aligned} W &= \frac{1}{2} \int_0^\infty \Phi_{\text{free}}(r) \rho(r) 4\pi r^2 dr \\ &= \frac{e^2 r_0}{4} \int_0^\infty \frac{e^{-r_0/2r}}{r^3} dr = mc^2 \end{aligned} \quad (4.2.17)$$

In agreement with the previous result, and further justification for the conjectured functional form of  $\epsilon(r)$ .

We have also been able to obtain the result of this study by an alternative derivation from the view-point of general relativity.[27] A modification of the Reissner-Nordstrom solution to the Einstein-Maxwell equations has been shown to give a finite electron self-energy.

### (4.3) Comment on “On the Electric Charge Quantization from the Aharonov-Bohm Potential”

On the paper [28], of Barone and Halayel-Neto (BH) the work of R-Mackenzie is based on claim that charge quantization in quantum mechanics can be proven without the need for the existence of magnetic monopoles. The argument relies on a re-analysis of the Aharonov-Bohm (AB) effect, as follows. The magnetic field of an infinitely long solenoid of radius  $R$  lying along the  $z$  axis is

$$B_{sol}(r) = \hat{z}B\Theta(R - \rho), \quad (4.3.1)$$

where  $\Theta(x)$  is the Heaviside step function, equal to 0 or 1 for  $x$  negative or positive, respectively. This is described in [28] by the vector potential

$$A(r) = \hat{\phi} \left\{ \frac{B\rho}{2} \Theta(R - \rho) + \frac{\gamma}{2\rho} \Theta(\rho - R) \right\} \quad (4.3.2)$$

where  $\gamma$  is, a priori, a free constant.

Were  $\gamma = BR^2/2$  (the standard choice in the literature, written  $\gamma_{AB}$  in [18]), this vector potential indeed describes  $B_{sol}$ ; with any other value of there is (as noted in [28]) in addition a magnetic field localized at  $\rho = R$  whose integrated flux is  $\Phi' = 2\pi\gamma - \Phi_{sol}$ , where  $\Phi_{sol} = B\pi R^2$  is the flux of the solenoid. BH describe the cylinder  $\rho = R$  as “non-physical,” possibly because it is a “singularity region for the field strength, since there is a surface charge density,” which apparently justifies their use of a vector potential describing a magnetic flux (as measured by  $\int A \cdot dl$  integrated around a curve encircling the solenoid) unequal to  $\Phi_{sol}$ . BH parameterize their choice of vector potential by  $k \equiv \gamma_{AB}$ , so that  $\Phi' = 2\pi\gamma$ .



It is then argued that the solenoid will exhibit the usual AB effect (with the correct magnetic flux,  $\Phi_{sol}$ ) for particles of charge  $q_i$  only if  $q_i k \in Z$ , which is to be viewed as a condition on  $q_i$ . In order for this to occur for a particle of charge  $q_1$ , we must therefore have  $k = n_1/q_1$ , where  $n_1$  is some integer. A second charged particle, of charge  $q_2$ , must then obey  $q_2 k = n_2, n_2 \in Z$ , so that

$$q_2 = \frac{n_2}{n_1} q_1. \quad (4.3.3)$$

This is the charge quantization condition as derived by BH.

The Mackenzie Objections can be put into two categories. Firstly, and most bluntly, the vector potential (2) is simply wrong. Rather than declaring the position of the solenoid to be “unphysical”, since indeed  $B_{sol}$  is discontinuous there, and so ignoring the flux in that region, one should recognize that (1) is the field of an ideal solenoid made of infinitely thin wires infinitely close to one another. A better treatment of this “singularity region” would be to smooth out the discontinuity in the magnetic field (equivalent to considering finite-thickness wires) so that it is equal to  $B$  inside a certain radius and zero outside a second slightly larger radius, with a smooth interpolation between these two values. Such a magnetic field can easily be described unambiguously (up to gauge transformation) by a smooth vector potential without invoking fictitious magnetic fluxes which must subsequently be rendered unobservable by insisting on a charge quantization condition.

One can also put aside this argument and examine the reasoning used in [28], and its consequences. Starting with the consequences, their main result (3) is an exceedingly weak charge quantization condition, indeed. It

merely states that all charges must be related to one another by rational factors. To illustrate the weakness of (3), note that while it must be admitted that charges  $e$  and  $\sqrt{2}e$  are not compatible,  $e$  and  $1.4142136e$  are. (The reader may replace  $\sqrt{2}$  by her/his favorite irrational number, and  $1.4142136$  by an arbitrarily accurate rational approximation to it.)

What of the reasoning itself? Essentially, the authors let the value of one charge determine the possible values of  $k$ , which then determines the possible values of all other charges in the problem, by insisting that the flux  $\Phi'$  causes no AB effect for any particle.

(While somewhat peripheral to the discussion at hand, note that this seems to select one charge [the one which determines the allowed values of  $k$  – that is, the allowed strengths of the fictitious delta-function magnetic field which can be added] as having a special role. Indeed, the allowed strengths of the fictitious magnetic field would be different if this initial charge were  $e$  or  $1.4142136e$ . One might argue that this is not a serious problem, since, after all, it is arranged that the fictitious magnetic field is unobservable. The more important point, perhaps, is that if the initial charge were  $e$ , then  $1.4142136e$  is allowed, and vice versa. Essentially, the choice of with which charge one begins is immaterial in so far as charge quantization is concerned, because (3) is symmetric.)

However, why not turn the argument around and argue that the additional delta-function field must be constrained by insisting that, whatever charges exist, this additional field must yield no AB effect? In other words, rather than having one charge constrain  $k$  and then having the allowed values of  $k$  constrain subsequent charges, why not let the pre-existing charges (which obey no a priori quantization condition) constrain  $k$ ?

For instance, if  $e$  and  $1.4142136e$  existed, certain magnetic fields could exist without having any observable effects. (To be specific, writing  $1.4142136 = m/n$  with  $m, n$  relatively prime integers, one finds that the allowed values of  $k$  are integer multiples of  $n/e$ .) However, let us apply this same reasoning to a situation where charges  $e$  and  $\sqrt{2}e$  exist. In order for neither charge to experience an AB effect, it is necessary that  $k$  is an integer multiple of both  $1/e$  and  $1/(\sqrt{2}e)$ . This has only one possible solution:  $k = 0$ . Thus, the nonexistence of an AB effect arising from the delta-function magnetic field considered in [18] might be used to eliminate charge non-quantization, but it is equally true that with only a minor change of logic the existence of charge non-quantization provides a second reason for eliminating the fictitious delta-function magnetic field. Since neither reasoning seems advantageous over the other, we prefer to invoke the physical reasoning outlined in our first objection above, wherein the added delta-function magnetic field is eliminated by making the standard choice of gauge potential.

In summary, unless there is an a priori reason independent of whatever charges are present for restricting the allowed values of the parameter  $k$ , the derivation of a charge quantization condition by insisting that  $k$  be unobservable is firstly without teeth (since the charge quantization condition so derived is so weak), and secondly, not even necessary. Furthermore, a much stronger objection can be raised, in that the addition of an ad hoc delta-function magnetic field, which is at the heart of the charge quantization condition derived in [28], is completely without physical motivation: if the discontinuous magnetic field  $B_{sol}$  is to be avoided, one need merely smooth it out, as would be the case in any case with a real

solenoid. No fictitious magnetic field arises,  $k = 0$ , and the quantization condition – weak as it is – never sees the light of day.

#### **(4.4) Tunneling-induced self-energy shift of energy levels of a quantum dot**

On the work of R.kh Gainutdino is based one the method which allows one to solve the problem nonperturbatively. This method is based on the generalized dynamical equation (GDE) that has been derived in [29] as the most general dynamical equation consistent with the current concepts of quantum physics. Being equivalent to the Schrodinger equation in the case when the interaction in a quantum system is instantaneous, GDE allows one to extend dynamics to the case of nonlocal-in-time interactions. This equation provides a new insight into many problems in atomic physics [30], nuclear physics [31] and quantum optics [32]. The method allows one to take into account from the every beginning that the contribution to the Green operator  $G(z)$ , which comes from the processes associated with the self-interaction of particles, has the same structure as the free Green operator  $G_0(z)$ . For this reason it is natural to replace  $G_0(z)$  by the operator  $G_0^{(v)}(z)$ , which describes the evolution of the system when particles propagate freely or interact with vacuum, and, hence, has the structure

$$\langle m' | G_0^{(v)}(z) | m \rangle = \langle m' | m \rangle (z - E_m - C_m(z))^{-1} ,$$

(4.4.1)

with  $|m\rangle$  being the eigenvectors of the free Hamiltonian ( $H_0 |m\rangle = E_m |m\rangle$ ).

Other contributions are described by the operator  $G^{(1)}(z) = G_0^{(v)}(z)M(z)G_0^{(v)}(z)$ :

$$G(z) = G_0^{(v)}(z) + G^{(I)}(z) \equiv G_0^{(v)}(z) + G_0^{(v)}(z)M(z)G_0^{(v)}(z);$$

where the operator  $M(z)$  describes the processes in which some particles interact each with other. The equations for  $C(z)$  and  $M(z)$  are derived from GDE. The equation for the function  $C_m(z)$  referred to as the self-energy function reads

$$\frac{dC_m(z)}{dz} = \langle m|M(z) \left(G_0^{(v)}(z)\right)^2 M(z)|m \rangle, \langle m|m \rangle = 1, \quad (4.4.2)$$

and the condition

$$z - E_m^{(0)} - C_m(z) = 0 \quad (4.4.3)$$

determines the physical masses of particles. In the case when we deal with an atom and  $|m\rangle$  describes an atomic state, equation (3) determines the self-energy correction (the Lamb shift) to the energy  $E_m$  of the state  $|m\rangle$ . An approximative solution of this equation is  $E_m \equiv E_m^{(0)} + C_m^{(0)}(E_m^{(0)}) \equiv E_m^{(0)} + \nabla E_m^L - \frac{i}{2} \Gamma_m$ , with  $\nabla E_m^L$  and  $\Gamma_m$  being the Lamb shift and the natural width of the energy level of the state  $|m\rangle$  respectively. For this approximation to be valid the variation of  $C_m(z)$  in the energy interval between  $E_m^{(0)}$  and  $E_m$  must be negligible. This is the case for atoms in free space. In fact, at leading order in  $\alpha$  the equation for  $C_m(z)$  is reduced to the equation [32, 33]

$$\frac{dC_m^{(0)}(z)}{dz} = \langle m|H_I \left(G_0^{(v)}(z)\right)^2 m|H_I \rangle, \langle m|m \rangle = 1, \quad (4.4.4)$$

with  $H_I$  being the interaction Hamiltonian. By solving this equation with an appropriate boundary condition we arrive at the ordinary expressions for the self-energy shifts and widths of energy levels. However, in the case of

quantum dots the variation of the self-energy function in the relevant vicinity of the point  $z = E_m^{(0)}$  can be very significant and, as a result, the above approximation is invalid. In this case the self-interaction function cannot be parameterized by a shift and a width, and one has to derive the self-energy function from a nonperturbative solution of the equations for  $C_m(z)$  and  $M(z)$ .

There are several different kinds of quantum dots which differ in both their method of manufacture and their electronic and optical properties. In this work we investigate the self interaction of a single-level quantum dot with arbitrary strong one-site Coulomb interaction tunnel coupled to two noninteracting leads. This model was used in [33] for analyzing tunneling-induced quantum fluctuation in such quantum dots. The Hamiltonian of system consisting of the quantum dot and leads is described by  $H = H_{\text{qd}} + H_{\text{tun}} + H_{\text{lead}}$ , where  $H_{\text{qd}}$  is the Hamiltonian of the quantum dot,  $H_{\text{tun}}$  is the Hamiltonian of tunneling process between the leads and the quantum dot and  $H_{\text{lead}}$  is the Hamiltonian of the leads. The quantum dot can be described by the single-level Anderson impurity model,  $H_{\text{qd}} = \sum_{\sigma=\uparrow,\downarrow} \epsilon d_{\sigma}^{\dagger} d_{\sigma} + U n_{\uparrow} n_{\downarrow}$ , where the creation (annihilation) operator for an electron with spin  $\sigma$  on the dot is given by  $d_{\sigma}^{\dagger}$  and  $d_{\sigma}$ ,  $n_{\sigma} = d_{\sigma}^{\dagger} d_{\sigma}$  is the corresponding number operator. The on-site repulsion  $U$  describes the energy cost for double occupation (when we have two electrons in the quantum dot). The Hamiltonian  $H_{\text{tun}}$  is given by  $H_{\text{tun}} = \sum_{\alpha,k,\sigma} V_{\alpha} c_{\alpha,k,\sigma}^{\dagger} d_{\sigma} + \text{H.c.}$ , where  $V_{\alpha}$  is the momentum and spin-independent tunnel matrix element,  $c_{\alpha,k,\sigma}^{\dagger}$  ( $c_{\alpha,k,\sigma}$ ) is the creation (annihilation) operator

for electrons with spin  $\sigma$  and momentum  $k$  in lead  $\alpha = l, r$ . The Hamiltonian  $H_{lead}$  is given by  $H_{lead} = \sum_{\alpha, k, \sigma} \mathcal{E}_{\alpha} c_{\alpha, k, \sigma}^{\dagger} c_{\alpha, k, \sigma}$

The chemical potentials of the two leads differ by the applied bias  $\mu_l - \mu_r = eV$ . We assume that the density of states  $\rho_{\alpha}$  in the leads is constant for transport and defines the tunnel coupling strength  $R_{\alpha}$  as  $R_{\alpha} = 2\pi\rho_{\alpha}|V_{\alpha}|^2$ , where  $V_{\alpha}$  is the tunnel matrix element and  $R = R_l + R_r$ . We will denote the states of quantum dots as  $|\Psi_0\rangle$  for an empty dot,  $|\Psi_{\sigma}\rangle$  for a singly occupied dot with spin  $\sigma = \uparrow, \downarrow$  and  $|\Psi_d\rangle$  for a doubly occupied dot. In this model we assume that reservoirs are in equilibrium and we average over the reservoir part of the initial states according to the Fermi distribution

$$f_{\sigma}(E) = \frac{1}{1 + \exp\left(\frac{E - \mu_{\alpha}}{k_B T}\right)}$$

solving the leading-order equation (4) yields the following expressions for the self-energy functions  $C_0(z), C_{\sigma}(z)$  and  $C_d(z)$  of the quantum-dot states  $|\Psi_0\rangle, |\Psi_{\sigma}\rangle$  and  $|\Psi_d\rangle$  respectively:

$$C_0(z) = 2 \sum_{\alpha} \frac{R_{\alpha}}{2\pi} \int d\omega \frac{f_{\alpha}(\omega)}{z - \varepsilon + \omega} \quad (4.4.5)$$

$$C_{\sigma}(z) = \sum_{\alpha} \frac{R_{\alpha}}{2\pi} \int d\omega \left( \frac{1 - f_{\alpha}(\omega)}{z - \omega} + \frac{f_{\alpha}(\omega)}{z - 2\varepsilon - U + \omega} \right) \quad (4.4.6)$$

$$C_d(z) = \sum_{\alpha} \frac{R_{\alpha}}{2\pi} \int d\omega \frac{1 - f_{\alpha}(\omega)}{z - \varepsilon - \omega} \quad (4.4.7)$$

Let us assume that the variations of these self-energy functions with  $z$  are weak. In this case  $\nabla E_0^{(ap)} = C_0(E_0), \nabla E_{\sigma}^{(ap)} = C_{\sigma}(E_{\sigma})$  and  $\nabla E_d^{(ap)} = C_d(E_d)$  can be considered as the approximative energy shifts of the corresponding energy levels of the quantum dot. The imaginary parts of these shifts  $iI_m E_0^{(ap)}, iI_m E_{\sigma}^{(ap)}$  and  $iI_m E_d^{(ap)}$  describe dissipative

phenomena. Note that the expressions for the energy shifts that follow from equations (5)-(7), are just the same that have been derived in Ref. [34] in second order in the tunnel matrix element  $V_{nm}$ . However, as it follows from the results of our calculations the variations with  $z$  of the self-energy functions of the quantum-dot states are strong, and one has to solve equation (3) exactly. Figure 1 where the results of calculations of the self-energy correction

$$\delta_\varepsilon = \Delta E_\sigma - \Delta E_0 \quad (4.4.8)$$

are depicted, shows that the energy shifts obtained in this way can differ dramatically from their approximative values. In our calculations we have used the fact that the part that is independent of the Fermi distribution function of  $C_\sigma(z)$  given by equation (6) can be included into the correction to the energy of empty level. In this way from equations (5), (6) and (8) we get

$$\delta_\varepsilon = \sum_\alpha \frac{R_\alpha}{2\pi} \int d\omega \left( \frac{1 - f_\alpha(\omega)}{z_\sigma - \omega} + \frac{f_\alpha(\omega)}{z_\sigma - 2\varepsilon - U + \omega} - 2 \frac{f_\alpha(\omega)}{z_\sigma - \varepsilon + \omega} \right) \quad (4.4.9)$$

where  $z_0$  and  $z_\sigma$  are the solutions of equation (3) both for empty and for single occupation energy levels. If we do not take into account the variation of the self-energy function, and put in equations (5) and (6)  $z = 0$  and  $z = \varepsilon$  respectively we arrive at the expression derived in [33]:

$$\delta\varepsilon^{(ap)} = \sum_\alpha \frac{R_\alpha}{2\pi} \int d\omega \left( \frac{f_\alpha(\omega)}{\omega - \varepsilon} + \frac{f_\alpha(\omega)}{\varepsilon - U + \omega} \right). \quad (4.4.10)$$



#### (4.5) Self-energy function of quantum-dot states and resonance fluorescence

In the work of R Kh Gainutdinov [34].The strong interaction of an atom with the laser field resonant with an atomic transition is successfully treated within the quantum optics formalism [35]. When the laser mode frequency  $\omega_L$  is close to the frequency  $\omega_R = \omega_e - \omega_g$  of the transition between two bound states  $|e\rangle$  and  $|g\rangle$  the atom-laser interaction is strong. In quantum optics this interaction is usually described by using the two-level model and the rotating wave approximation (RWA). In this approximation the laser-dressed states are the eigenstates of the system Hamiltonian HRWA [35] and are given by

$$|+, n\rangle = \cos \theta_n |e, n\rangle + \sin \theta_n |g, n + 1\rangle, |-, n\rangle = -\sin \theta_n |e, n\rangle + \cos \theta_n |g, n + 1\rangle. \quad (4.5.1)$$

Here  $|e, n\rangle$  ( $|g, n\rangle$ ) denotes the state of the combined laser-atom system containing the atom in the bare state  $|e\rangle$  ( $|g\rangle$ ) and  $n$  photons in the laser mode, and  $\theta_n$  is the mixing angle defined by  $\tan(2\theta_n) = -\Omega_n/\Delta$  with  $\Omega_n$  and  $\Delta$  being the Rabi frequency  $\Omega_n = 2gL\sqrt{n+1}$  and detuning, respectively. The energies of the dressed states are given by  $E_{\pm, n} = (n + 1/2)\omega_L + (\omega_e - \omega_g)/2 \pm \Omega_R^{(n)}/2$  with  $\Omega_R^{(n)}$  being the generalized Rabi frequency  $\Omega_R^{(n)} = \sqrt{\Omega_n^2 + \Delta^2}$ . The states dressed by the interaction of the atom with the resonance laser field are not dressed by its interaction with the vacuum modes, and in our investigation of the problem we do not assume a priori that this interaction is weak in any case and use nonperturbative methods of its description based on the generalized dynamical equation (GDE), which in [36] has been derived as a direct consequence of the first

principles of quantum physics. Being equivalent to the Schrödinger equation in the case when the interaction in a quantum system is instantaneous, GDE allows one to extend quantum dynamics to the case of nonlocal-in-time interactions. This equation has been proven to provide a new insight of many problems in nuclear physics [37-38], atomic physics [39-40] and quantum optics. Within this approach the QED bound state problem is solved by using the formalism of the self-energy function  $C_n(z)$  (see for details Refs. [41]), that describes the interaction of the particles in the state  $|n\rangle$  with the vacuum. The selfenergy QED corrections to the energies  $E_n^{(0)}$  of the bare states are determined by the equation  $z - E_n^{(0)} - C_n(z) = 0$  where  $C_n(z)$  in turn are determined by GDE. In the first approximation the solution of this equation is  $E_n \equiv E_n^{(0)} + C_n^{(0)}(E_n^{(0)}) \equiv E_n^{(0)} + \Delta E_n^L - \frac{i}{2}\Gamma_n$ , where  $E_n$  is the energy of the real dressed state,  $E_n^L$  is the Lamb shift,  $\Gamma_n$  is the natural level width, and  $C_n^{(0)}(z)$  is determined by the equation

$$\frac{dC_n^{(0)}(z)}{dz} = -\langle n|H_I\tilde{G}_0^2(z)H_I|n\rangle, \quad (4.5.2)$$

with  $H_I$  being the QED Hamiltonian. For this *approximation* to be valid, probability amplitudes of the nonradiative transitions between the states with the same total angular momentum  $J$ , its projection  $J_z$  and parity must be much smaller than the energy distance between these states. In addition the variation of the function  $C_n(z)$  with the energy must be negligible on the energy interval of order of the value of the Lamb shift. This takes place in the case when the atom is in free space. In fact, in this case the energy distance between the states with the same  $J, J_z$  and parity are of order of  $\alpha^2 m_e$ , while the probability amplitudes of nonradiative transitions between

them are of order of  $\alpha^5 m_e$ , and the self-energy function varies significantly with the energy only on energy intervals of order  $m_e$ . The situation is dramatically changed in the case of an emitter subject to resonance laser field because the energy distance between dressed states  $|+, n\rangle$  and  $|-, n\rangle$  having the same  $J, J_z$  and parity equals the Rabi frequency. As a result the nonradiative transitions between these states begin to play an important role. At leading order the probability amplitudes  $\langle \pm, n | \Sigma(z) | \mp, n \rangle$  of these transitions

are given by the formula  $\langle \pm, n | \Sigma(z) | \mp, n \rangle = \cos \theta \sin \theta \left( C_g^{(0)}(z) - C_e^{(0)}(z) \right)$ , where  $C_g^{(0)}(z)$  and  $C_e^{(0)}(z)$  are the self-energy functions of the bare states  $|g\rangle$  and  $|e\rangle$  respectively. At the same time, in the case of an atom being in free space the energy interval of order  $\Omega R$  is too small for the variation with energy of  $\langle \pm, n | \Sigma(z) | \mp, n \rangle$  to be noticeable. In this case the energy dependence of  $\langle \pm, n | \Sigma(z) | \mp, n \rangle$  can be neglected, and they can be regarded as matrix elements of an additional term in the Hamiltonian describing the interaction of the atom with resonance laser field. Adding such a term to the RWA Hamiltonian will lead only to a change in the mixing angle  $\theta_n$ , and, as a consequence, will not give rise to asymmetry of the Mollow spectrum. However, as we show below, in the case when a quantum dot acts as an emitter the variation of the self-energy functions  $C_g(z)$  and  $C_e(z)$  on an energy interval compared to the energy difference between the dressed states  $|+, n\rangle$  and  $|-, n\rangle$  can be very significant, and hence the vacuum modes are involved into the strong atom-laser interaction.

Let us consider the model of a quantum dot interacting with two reservoirs in which the Hamiltonian of a system consisting of quantum dot and leads is

described by  $H = H_{\text{QD}} + H_T + H_{\text{lead}}$ , where  $H_{\text{QD}}$  is the Hamiltonian describing the quantum dot,  $H_{\text{lead}}$  is the lead Hamiltonian,  $H_T$  is the Hamiltonian which describes tunneling processes between dots and leads. The singleparticle level spacing in the dot is assumed to be larger than any other energy scale (temperature, Coulomb interaction, and transport voltage) so that only one energy level needs to be taken into account. The quantum dot can be described by the single-level Anderson impurity model, in which  $H_{\text{QD}}$  is of the form  $H_{\text{QD}} = \sum_{\sigma=\uparrow,\downarrow} E d_{\sigma}^{\dagger} d_{\sigma} + U n_{\uparrow} n_{\downarrow}$ , where the creation (annihilation) operator for an electron with spin  $\sigma$  on the dot is given by  $d_{\sigma}^{\dagger}$  and  $d_{\sigma}$ , and  $n_{\sigma} = d_{\sigma}^{\dagger} d_{\sigma}$  is the corresponding number operator. The on-site repulsion  $U$  describes the energy cost for double occupation (when we have two electrons in quantum dot) and stems from Coulomb interaction. The Hamiltonian  $H_T$  is given by  $H_T = \sum_{s,k,\sigma} V_s C_{s,k,\sigma}^{\dagger} d_{\sigma} + H.C.$ , where  $V_s$  is the momentum- and spin-independent tunnel matrix element,  $C_{s,k,\sigma}^{\dagger}$  ( $C_{s,k,\sigma}$ ) is creation (annihilation) operators for electrons with spin  $\sigma$  and momentum  $k$  in lead and  $s = L; R$ . The Hamiltonian  $H_{\text{lead}}$  is given by  $H_{\text{lead}} = \sum_{s,k,\sigma} E_{s,k} C_{s,k,\sigma}^{\dagger} C_{s,k,\sigma}$ . The chemical potentials of the two leads differ by the applied bias  $\mu_L - \mu_R = -eV$ .

We assume that the density of states  $\rho_s$  in the leads is constant for transport and determines the tunnel coupling strength  $R_s$  as  $R_s = 2\pi |V_s|^2 \rho_s$  and  $R = R_L + R_R$ . The Hilbert space for the quantum dot is spanned by the states  $|0\rangle$  for an empty dot,  $|\sigma\rangle$  for a singly occupied dot with spin  $\sigma = \uparrow, \downarrow$  and  $|d\rangle$  for a doubly occupied dot. The corresponding energies are  $E_0, E_{\sigma}$ , and  $E_d$ . By generalizing the equations derived in [42] for the self-energy function of quantum dot we get

$$C(z) = -\sum_s \frac{R_\alpha}{2\pi} \int d\omega \left( \frac{fs(\omega)}{\omega-z} + \frac{fs(\omega)}{z+U-\omega} \right) \quad (4.5.3)$$

where  $fs(\omega) = (1 + \exp(\frac{\omega-\mu_s}{K_B T}))^{-1}$  is the Fermi distribution function.

As it follows from the results of calculations presented in Fig. 1 the variation of the self-energy function is very significant in the vicinity of the point  $z = 2 eV$ .

#### (4.6) second order field dependent lagrangian & its effect on Higgs field

The work done by Mubarak [43] devoted to describe the general fields beside investigating its direct impact on Higgs field [44] and its role in generating mass, by using generalized general relativity (GGR) the work is based on second order dependent lagrangian.

The lagrangian of (EGGR) is in the form

$$L = L(xy, \phi, \partial_\mu \phi), \partial_{\mu\nu} \phi$$

Where

$$X_\gamma = x_0, x_1, x_2, x_3 \quad (4.6.1)$$

$$x_0 = t, x_1 = x, x_2 = y, x_3 = z \quad (4.6.2)$$

Thus the lagrangian variation takes the form:

$$\delta L = \frac{\partial L}{\partial x_\mu} \delta x_\mu + \frac{\partial L}{\partial \phi} \delta \phi + \frac{\partial L}{\partial \partial_\mu \phi} \delta \partial_\mu \phi + \frac{\partial L}{\partial \partial_{\mu\nu} \phi} \delta \partial_{\mu\nu} \phi \quad (4.6.3)$$

Where

$$\delta x_\mu = 0 \quad \delta \partial_\mu = \partial_\mu \phi(x) - \partial_\mu \phi(x) = \partial_\mu [\phi(x) - \phi(x)] = \partial_\mu \delta \phi$$

$$\delta \partial_{\mu\nu} \phi = \partial_{\mu\nu} \phi(x_1) - \partial_{\mu\nu} \phi(x) = \partial_{\mu\nu} [\phi(x) - \phi(x)]$$

$$= \partial_{\mu\nu} \delta \phi$$

Thus:

$$\frac{\partial L}{\partial \partial_\mu \phi} \delta \partial_\mu \phi = \frac{\partial L}{\partial \partial_\mu \phi} \partial_\mu \delta \phi = \partial_\mu \left[ \frac{\partial L}{\partial \partial_\mu \phi} \delta \phi \right] - \partial_\mu \frac{\partial L}{\partial \partial_\mu \phi} \delta \phi$$

Similarly:

$$\begin{aligned} \frac{\partial L}{\partial \partial_\mu \phi} \delta \partial_{\mu\nu} \phi &= \frac{\partial L \partial_{\mu\nu}}{\partial \partial_{\mu\nu} \phi} \delta \phi = \frac{\partial L \delta_\mu}{\partial \partial_{\mu\nu}} - \partial_\mu (\partial_\nu \delta \phi) \\ &= \partial_\mu \frac{[\partial L \partial \phi]}{\partial \partial_{\mu\nu} \phi} - \partial_\mu \frac{[\partial L \delta \phi]}{\partial \partial_{\mu\nu} \phi} - \partial_\mu [\partial L] \partial_\nu \delta \end{aligned} \quad (4.6.4)$$

The lagrangian of the electroweak field takes the form:

$$L = i\gamma\Psi \partial_\mu\Psi - m\Psi\Psi - j_\mu A_\mu - \frac{1}{4} F_{\mu\nu} F \quad (4.6.5)$$

Disappearance of mass term in the lagrangian:

$$m_{\Psi\Psi} = \rho \quad (4.6.6)$$

According to poisson equation:

$$\Phi = \partial_{\mu\nu} = -c_1 \rho \quad (4.6.7)$$

Thus the mass term in L can be replaced by (4.5.5) to get

$$\begin{aligned} L = L &= i\gamma\mu\Psi \partial_\mu\Psi + C_0 \partial_{\mu\nu} \Phi - j^\mu A_\mu - \frac{1}{4} F_{\mu\nu} F \\ C_0 &= 1/c_1 \end{aligned} \quad (4.6.8)$$

It is clear that the mass term which prevents invariance disappear.

According to equation (4.6.8) the mass term appears to be.

$$\delta L = i\gamma\mu\Psi \partial_\mu\Psi + C_0 \partial_{\mu\nu} \Phi \quad (4.6.9)$$

Thus the need to Higgs fields variables to generate mass need to be revised.

#### **(4.7) summary and critique**

All attempts made to find second order lagrangian electrodynamic equations [45,46,47] or electron self energy [48,49,50] or charge quantization [51,52,53] are incomplete or complex. Thus one needs complete self consistent simple theory that can solve these problems.

## Chapter Five

### Second order lagrangian electromagnetic Hamiltonian mass generation and charge quantization

#### (5.1) Introduction:

This work is concerned with deriving second order electromagnetic lagrangian [54, 55, 56] and discussing mass generation also[57, 58, 59]. It is also concerned with charge quantization [60, 61, 62]. The mathematical framework is based on Lagrangian formalism, Klein-Gordon equation and generalized special relativity.

#### (5.2) Ordinary Lagrangian of Electromagnetic field:

The electromagnetic field lagrangian is given by [63, 64]

$$\begin{aligned} L &= \frac{1}{8\pi} \left( \frac{1}{c} \frac{\partial A}{\partial t} + \nabla \phi \right)^2 - \frac{1}{8\pi} (\nabla \times A)^2 \\ &= \frac{1}{8\pi} (\partial_0 A_i - \partial_i A_0)^2 - \frac{1}{8\pi} (\partial_i A_0 - \partial_i A_0)^2 \end{aligned} \quad (5.2.1)$$

$$P_x = \frac{\partial L}{\partial \partial_x q}$$

$$P_x = \frac{\partial L}{\partial \partial_x A} = \frac{1}{4\pi c} \left( \frac{1}{c} \frac{\partial A_x}{\partial t} + \frac{\partial Q}{\partial x} \right) \quad (5.2.2)$$

$$P = \frac{1}{4\pi c} \left( \frac{1}{c} \frac{\partial A}{\partial t} + \nabla \phi \right) = \frac{1}{4\pi c} (\partial_0 A_i - \partial_i A_0)$$

Where  $A_i$ 's stands for magnetic potential, while  $\phi$  represents the electric potential. The corresponding Hamiltonian is given by

$$\begin{aligned}
H &= P \cdot \frac{\partial A}{\partial t} - L = \frac{1}{4\pi} \left( \frac{1}{c} \frac{\partial A}{\partial t} + \nabla \phi \right) \left( \frac{1}{c} \frac{\partial A}{\partial t} \right) - L \\
&= \frac{1}{4\pi} \left( \frac{1}{c} \frac{\partial A}{\partial t} + \nabla \phi \right) \left( \frac{1}{c} \frac{\partial A}{\partial t} \right) - \frac{1}{8\pi} \left( \frac{1}{c} \frac{\partial A}{\partial t} + \nabla \phi \right) \left( \frac{1}{c} \frac{\partial A}{\partial t} \right) \\
&\quad - \frac{1}{8\pi} \left( \frac{1}{c} \frac{\partial A}{\partial t} + \nabla \phi \right) \cdot \nabla \phi + \frac{1}{8\pi} (\nabla \times A)^2 \\
&= \frac{1}{8\pi} \left( \frac{1}{c} \frac{\partial A}{\partial t} + \nabla \phi \right) \cdot \left( \frac{1}{c} \frac{\partial A}{\partial t} \right) - \frac{1}{8\pi} \left( \frac{1}{c} \frac{\partial A}{\partial t} + \nabla \phi \right) \cdot \nabla \phi + \frac{1}{8\pi} (\nabla \times A)^2 \\
&= \frac{1}{8\pi} \left( \frac{1}{c} \frac{\partial A}{\partial t} + \nabla \phi \right) \cdot \left( \frac{1}{c} \frac{\partial A}{\partial t} + \nabla \phi \right) - \frac{1}{4\pi} \left( \frac{1}{c} \frac{\partial A}{\partial t} + \nabla \phi \right) \cdot \nabla \phi + \frac{1}{8\pi} (\nabla \times A)^2 \\
&= 2\pi C^2 P^2 - CP \cdot \nabla \phi + \frac{1}{8\pi} (\nabla \times A)^2 \tag{5.2.3}
\end{aligned}$$

The equation of motion and the Hamiltonians are derived from lagrangian dependent on the field variables and their first derivatives.

### (5.3) New second order lagrangian:

The equation of motion for second order Lagrangian takes the form [65, 66, 67]

$$\frac{\partial L}{\partial A_u} - \partial_\mu \left[ \frac{\partial L}{\partial \partial_\mu A_u} \right] + \partial_{\sigma\mu} \left[ \frac{\partial L}{\partial \partial_{\sigma\mu} A} \right] = 0 \tag{5.3.1}$$

The corresponding Hamiltonian is also given by [68, 69, 70]

$$T_0^0 = H = \frac{\partial L}{\partial \partial_o A_i} \partial_o A_i - \partial_\sigma \left[ \frac{\partial L}{\partial \partial_{\sigma o} A_i} \right] + \frac{\partial L}{\partial \partial_{\sigma\sigma} A_i} \partial_{\sigma\sigma} A_i - L \tag{5.3.2}$$

Where  $i$  is a dummy index, beside  $\sigma$ .



The appropriate second order lagrangian that can give Maxwell's equations and the correct Hamiltonian given by:

$$L = c_1 \eta^{\rho\sigma^2} [\partial_\rho A_\sigma - \partial_\sigma A_\rho]^2 - c_2 A_\lambda [J_\psi^\lambda + J_v^\lambda] - J_\psi - J_A \quad (5.3.3)$$

$$J_\psi^\lambda = q \bar{\Psi} \gamma^\lambda \Psi = \text{charge current density}$$

$$J_A = c_3 (\partial_{\lambda\rho} A_\rho - \partial_{\rho\rho} A_\lambda) = \text{source generating or absorbing field}$$

$$J_u^\mu \text{ vacuum current density} \quad (5.3.4)$$

thus the system of matter have charge current density corresponding to rest mass energy which at the same time acts as a source emitting or absorbing field mediators. to find the equation of motion of electromagnetic field (E. M. F) one differentiate w. r. t A to get:

$$\frac{\partial L}{\partial A_v} = -c_2 (J_\psi^\lambda + J_v^\lambda) \quad (5.3.5)$$

Also

$$\begin{aligned} \frac{\partial L}{\partial \partial_\mu A_v} &= c_1 \frac{\partial}{\partial \partial_\mu A_v} \left[ \eta^{\mu\nu^2} (\partial_\mu A_\nu - \partial_\nu A_\mu)^2 + \eta^{\mu\nu^2} (\partial_\nu A_\mu - \partial_\mu A_\nu)^2 \right] \\ &= c_1 [2\eta^{\mu\nu^2} (\partial_\mu A_\nu - \partial_\nu A_\mu)(1) + 2\eta^{\mu\nu^2} (\partial_\nu A_\mu - \partial_\mu A_\nu)(-1)] \\ &= 4c_1 \eta^{\mu\nu^2} (\partial_\mu A_\nu - \partial_\nu A_\mu) \end{aligned} \quad (5.3.6)$$

Equations (5.3.5) and (5.3.6) can help in finding the equation of motion. To find the Hamiltonian, one needs to differentiate L w. r. t. to time derivative of magnetic potential  $A_i$  to get.

$$\begin{aligned}
& \frac{\partial L}{\partial \partial_o A_i} \partial_o A_i = \\
& = \frac{\partial L}{\partial \partial_o A_i} [c_1 \eta^{oi^2} (\partial_o A_i - \partial_i A_o)^2 + c_1 \eta^{io^2} (\partial_i A_o - \partial_o A_i)^2] \partial_o A_i \\
& \quad c_1 \eta^{oi^2} [(\partial_o A_i - \partial_i A_o) + 2(\partial_i A_o - \partial_o A_i)(-1)] \partial_o A_i \\
& \quad = 2c_1 (\partial_o A_i - \partial_i A_o) \partial_o A_i
\end{aligned} \tag{5.3.7}$$

By choosing

$$\begin{aligned}
\partial_o &= \frac{\partial}{\partial x_0} & x_0 &= ict & A_0 &= i\phi \\
x_1 &= x, & x_2 &= y & x_3 &= z \\
A_1 &= A_x & A_2 &= A_y & A_3 &= A_z
\end{aligned} \tag{5.3.8}$$

$$i = 1, 2, 3$$

One gets

$$\begin{aligned}
\frac{\partial L}{\partial \partial_o A_i} \cdot \partial_o A_i &= 2c_1 \left[ -\frac{j}{c} \frac{\partial A_x}{\partial t} - j \frac{\partial \phi}{\partial x} \right] \left[ -\frac{j}{c} \frac{\partial A_x}{\partial t} \right] + \dots \\
&= -2c_1 \left[ \frac{1}{c} \frac{\partial A_x}{\partial t} + \frac{\partial \phi}{\partial x} \right] \left[ \frac{1}{c} \frac{\partial A_x}{\partial t} \right]
\end{aligned}$$

$$= -2c_1 \left[ \frac{1}{c} \frac{\partial A}{\partial t} + \nabla \phi \right] \left[ -\frac{1}{c} \frac{\partial A}{\partial t} \right]$$

The terms differentiated with respect to the second order derivatives are given according to equation (5.3.3) and (5.3.4) to be.

$$\frac{\partial L}{\partial \partial_{\mu\sigma} A_v} = \frac{\partial}{\partial \partial_{\mu\sigma} A_v} [c^{\lambda g g} (\partial_{\lambda g} A_g - \partial_{\lambda g} A_\lambda)]$$

$$\text{Let : } \mu = \lambda = o, \sigma = v = g = i, \quad \mu = \sigma = g = 0 \quad v = \lambda = i$$

$$= c^{\mu\sigma\sigma} \frac{\partial}{\partial \partial_{\mu\sigma} A_v} [(\partial_{\mu\sigma} A_v) - \partial_{\sigma\sigma} A_\mu]$$

$$- c^{\mu\sigma\sigma} \frac{\partial}{\partial \partial_{\mu\sigma} A_v} [\partial_{v\mu} A_\mu - \partial_{\mu\sigma} A_v]$$

$$= c^{\mu\sigma\sigma} - c^{\nu\sigma\sigma} = c_3 - c_3 = 0 \quad (5.3.9)$$

Thus according to equations (5.3.1), (5.3.4) , (5.3.5) and (5.3.6) and (5.3.9) the equation of motion is given by

$$- \partial_\mu [4c_1 \eta^{\mu\nu^2} (\partial_\mu A_\nu - \partial_\nu A_\mu)] - c_2 (J_\psi^v + J_v^v) = 0$$

$$\text{By setting} \quad (5.3.10)$$

$$c_1 = -1/8\pi \quad c_2 = -\frac{2}{c}$$

One gets the ordinary equation of motion [71,72]

$$\partial_{\mu\nu} A_\mu - \partial_{\mu\mu} A_\nu = \frac{4\pi}{c} (J_\psi + J_\nu) \quad (5.3.11)$$

The Hamiltonian can be found by a direct substitution of equations (3) , (3.7) and (3.8) in equation (3.2) to get

$$H = -2c_1 \left( \frac{1}{c} \frac{\partial A}{\partial t} + \nabla \phi \right) \left( \frac{1}{c} \frac{\partial A}{\partial t} \right) - c_1 \eta^{oi^2} (\partial_o A_i - \partial_i A_o)^2$$

$$\begin{aligned}
& -c_1 \eta^{ij} (\partial_i A_j - \partial_j A_i)^2 \\
& + J_\psi + J_A + c_2 A_v (J_\psi^v + J_v^v)
\end{aligned}$$

According to equations (3.8) the Hamiltonian is given by

$$\begin{aligned}
H &= -2c_1 \left( \frac{1}{c} \frac{\partial A}{\partial t} + \nabla \phi \right) \left( \frac{1}{c} \frac{\partial A}{\partial t} \right) \\
&= -c_1 \left( -\frac{i}{c} \frac{\partial A_x}{\partial t} - i \frac{\partial \phi}{\partial x} \right) \left( -\frac{i}{c} \frac{\partial A_x}{\partial t} - i \frac{\partial \phi}{\partial x} \right) \\
&\quad - c_1 (\vec{\nabla} \times \underline{A}) + J_\psi + J_A + c_2 A_v (J_\psi^v + J_v^v) \\
&\quad - 2c_1 \left( \frac{1}{c} \frac{\partial A}{\partial t} + \nabla \phi \right) \left( \frac{1}{c} \frac{\partial A}{\partial t} \right) \\
&+ c_1 \left( \frac{1}{c} \frac{\partial A}{\partial t} + \nabla \phi \right)^2 - c_1 (\vec{\nabla} \times \underline{A})^2 + J_\psi + J_A + c_2 A_v (J_\psi^v + J_v^v)
\end{aligned} \tag{5.3.12}$$

$$\begin{aligned}
&= -2c_1 \left[ \left( \frac{1}{c} \frac{\partial A}{\partial t} + \nabla \phi \right)^2 - \frac{1}{2} \left( \frac{1}{c} \frac{\partial A}{\partial t} + \nabla \phi \right)^2 - \left( \frac{1}{c} \frac{\partial A}{\partial t} + \nabla \phi \right) (\nabla \phi) \right. \\
&\quad \left. + \frac{1}{2} (\nabla \times A)^2 \right] \\
&\quad + J_\psi + J_A + c_2 A_v (J_\psi^v + J_v^v) \\
&= -2c_1 \left[ \frac{1}{2} \left( \frac{1}{c} \frac{\partial A}{\partial t} + \nabla \phi \right)^2 - \left( \frac{1}{c} \frac{\partial A}{\partial t} + \nabla \phi \right) \cdot \nabla \phi + \frac{1}{2} (\nabla \times A)^2 \right] \\
&\quad + J_\psi + J_A + c_2 A_v (J_\psi^v + J_v^v) \\
&= \frac{1}{8\pi} \left[ \left( \frac{1}{c} \frac{\partial A}{\partial t} + \nabla \phi \right)^2 - 2 \left( \frac{1}{c} \frac{\partial A}{\partial t} + \nabla \phi \right) \cdot \nabla \phi + (\nabla \times A)^2 \right]
\end{aligned}$$

$$\frac{1}{8\pi} \left[ 6\pi^2 c^2 p^2 - 8\pi c p \cdot \nabla \phi + (\nabla \times \underline{A})^2 \right] + J_\psi + J_A + c_2 A_v (J_\psi^v + J_v^v)$$

$$H = 2\pi c^2 p^2 - c p \cdot \nabla \phi + \frac{1}{8\pi} (\nabla \times \underline{A})^2 + J_\psi + J_A + c_2 A_v (J_\psi^v + J_v^v) \quad (5.3.13)$$

There

$$c_1 = -\frac{1}{8\pi}$$

This Hamiltonian is the usual electromagnetic field Hamiltonian.

#### **(5.4) Electromagnetic Hamiltonian in a curved space time and vacuum energy.**

According to general relativity (GR) [73]. Any energy form cause space to be curved. Thus electromagnetic field can casus space to be curved. According to GR the time – time component of the metric is gives by [74, 75, 76]

$$g_{00} = -\left(1 + 2\frac{\phi_g}{c^2}\right) \quad (5.4.1)$$

Where  $\phi_g$  is the gravity potential per unit mass and is related to electric potential  $\phi$  and electric charge e thought the relation [77, 78]

$$\phi_g = \frac{V}{m} = \frac{e\phi}{m} \quad (5.4.2)$$

Thus equation (2.1) becomes

$$g_{00} = -\left(1 + 2\frac{e\phi}{mc^2}\right) \quad (5.4.3)$$

At early stages of the universe electric charge is generated due to the electromagnetic ( $e.m$ ) field at vacuum stage[51]. This requires minimizing the Hamiltonian  $H$  w.r.t electric potential  $\phi$  to find the electric charge and see how it is generated. Since the Hamiltonian part representing charge itself can be neglected as for as they are independent of  $\phi$ . The charge field interactions are neglected for simplicity. One also assumes electric charge to be at rest. This means that the magnetic field is not generated. Therefore

$$A_o = \phi \quad A_i = 0 \quad i = 1,2,3 \quad (5.4.4)$$

To find the Hamiltonian in curved space, one generalizes the space one

$$H = \eta^{oo2} \epsilon_i (\partial_i A_o - \partial_o A_i)^2 \quad (5.4.5)$$

To be written in a curved space in the form

$$\begin{aligned} H &= g^{oo2} \epsilon_i (\partial_i A_o - \partial_o A_i)^2 \\ &= \left(1 + 2 \frac{\phi_g}{mc^2}\right)^2 (\nabla\phi)^2 \end{aligned} \quad (5.4.6)$$

From  $\phi_g$  (2.3) one gets

$$H = \left(1 + 2 \frac{e\phi}{mc^2}\right)^2 (\nabla\phi)^2 \quad (5.4.7)$$

Thus minimization condition requires:

$$\frac{dH}{d\phi} = 2 \left(1 + 2 \frac{e\phi}{mc^2}\right) \left(\frac{2e}{mc^2}\right) (\nabla\phi) = 0$$

$$1 + 2 \frac{e\phi}{mc^2} = 0 \quad \phi = -\frac{mc^2}{2e} \quad (5.4.8)$$

A summing the mass energy to be resulting from electric field energy density

$$E_d = \epsilon_0 E^2$$

Inside electron of radius  $r_o$ , one gets

$$mc^2 = E_n = E_d \times V = \epsilon_0 E^2 \left( \frac{4\pi}{3} r_o^3 \right) = \frac{\epsilon_0 e^2}{8\pi^2 \epsilon_0^2 r_o} \left( \frac{4\pi}{3} r_o^3 \right)$$

$$\frac{e^2}{6\pi\epsilon_0 r_o} \quad (5.4.9)$$

The vacuum potential which results from electric electric charge becomes

$$V_v = -e\phi = + \frac{e^2}{12\pi\epsilon_0 r_o} \quad (5.4.10)$$

But according to muntasir model [79]

Vacuum energy potential takes the form

$$V_v = \rho_v \left[ \frac{\pi^2 n^2}{x_0^2 n_0^2} + \omega^2 \right]^{-3} \quad (5.4.11)$$

Thus combining (2.10) and (2.11) yield

$$\frac{e^2}{12\pi \epsilon_0 r_o} = \left[ \frac{\pi^2 n^2}{x_0^2 n_0^2} + \omega^2 \right]^{-3}$$

Thus the electric charge is give by

$$e = 2 \left[ \frac{\pi^2 n^2}{x_0^2 n_0^2} + \omega^2 \right]^{\frac{-3}{2}} [12\pi \epsilon_0 r_o]^{\frac{1}{2}} \quad (5.4.12)$$

Setting  $w$  to be equal to zero, for simplification. The electric charge is given by

$$e = (12\pi \epsilon_0 r_o)^{\frac{1}{2}} \left( \frac{x_o n_o}{n\pi} \right)^3 \quad (5.4.13)$$

Where  $n$  and  $n_o$  are quantum numbers

$r_o$  = electron radius

$x_o$  = universe radius

Thus

The electron radius Can be found by assuming that the electron energy results from its spinning, where the spin angular momentum is given by

$$L_s = \hbar \sqrt{s(s+1)} = \frac{\sqrt{3}}{2} \hbar, \frac{\hbar}{2} \quad (5.4.14)$$

Where for electrons [80,81]

$$S = \mp \frac{1}{2} \quad (5.4.15)$$

At vacuum stage we choose minimums lower value.

$$L_s = \frac{1}{2} \hbar \quad (5.4.16)$$

Assume that rest mass is neglected in relativistic expression to get

$$mc^2 = E = cp \quad (5.4.17)$$

$$mc = p \quad (5.4.18)$$

The same relation can hold for Newtonian mechanics by considering wave nature of electrons, where the maximum velocity  $V_m$  is related to the effective value  $v$  through the relations [82,83]

$$V = \frac{V_m}{\sqrt{2}} \quad (5.4.19)$$



By assuming the momentum to be

$$P = mV_m$$

Thus the Newtonian expression for free particle takes the form

$$E = \frac{1}{2}mV_m^2 = \frac{m^2V_m^2}{2} = mV^2 = \frac{m^2V^2}{m} = \frac{P^2}{m} \quad (5.4.20)$$

If can believe in relativistic energy mass relation, one gets [84, 85]

$$mc^2 = E = \frac{p^2}{m}$$

Thus one gets:

$$m^2c^2 = p^2 \quad mc = p \quad (5.4.21)$$

Since the momentum p is related to L according to the

$$p = mV = \frac{mVV}{r_o} = \frac{L_s}{r_o} \quad (5.4.22)$$

It follows from equation (2.21) that

$$\frac{L_s}{r_o} = mc$$

Using equation (2.16) one gets

$$r_o = \frac{L_s}{mc} = \frac{\hbar}{2mc} \quad (5.4.23)$$

Substituting the values of h ,  $\mu$  and c, the electron radius becomes the electric charge is assumed to be born at very early stages of the universe where vacuum exist and the minimum radius is  $x_o$  where  $x_o = 26 * 635 \times 10^{-3}$

The electric charge see equation (2013)

$$e = 1.6 \times 10^{-19}$$

$$1.6 \times 10^{-20}$$

Can be obtained by adjusting the quantum number  $h$  and  $n_o$  to be

$$\frac{n_o}{n} = \left( \frac{e}{(12\pi\epsilon_o r_o)^2} \right)^{\frac{1}{3}} \frac{\pi}{x_o} \quad (5.4.24)$$

Equation (5.4.10) shows that vacuum energy is repulsive due to the existence of positive sign. This can forms with cosmological models, which suggests repulsive vacuum energy. Inflation models suggest also very large vacuum energy. If one believe in that such that

$$\phi_g \sim \frac{V_v}{m_o} \rightarrow \frac{c^2}{2} \quad (5.4.25)$$

in this case according to generalized special relativity model the electron mass is given by

$$m = m_o \left( 1 - 2 \frac{\phi_g}{c^2} \right) \rightarrow large \quad (5.4.26)$$

Assume for simplicity

$$m = 10^{13} m_o \simeq 9 \times 10^{-13} \times 10^{13} \quad (5.4.27)$$

Hence the electron radius can be given to be

$$r_o = \frac{\hbar}{mc} = \frac{66 \times 10^{-35}}{2 \times 3.14 \times 9 \times 10^{-13} \times 10^{13} \times 3 \times 10^8}$$

$$r_o = \frac{11}{9 \times 3.14} \times 10^{-25} \sim 3 \times 10^{-24} m \quad (5.4.28)$$

Which is quite reasonable as far as nucleus or proton radius for very light atoms are

$$r_b \sim 10^{-14} m \quad r_p \sim 10^{-16} m \quad (5.4.29)$$

The electron charge can also be obtained from power and intensity definition where the electric power is given by

$$\begin{aligned} P_r &= VI = (EL)(nevA) \\ &= (EL)JA = (EL)(\sigma E)A \end{aligned} \quad (5.4.30)$$

The intensity is given by [86, 87]

$$I = \frac{P_r}{A} = \sigma LE^2 \quad (5.4.31)$$

The conductivity is given by

$$\sigma = \frac{ne^2t}{m} \quad (5.4.32)$$

$$\text{Thus } I = \frac{ne^2tLE^2}{m} \quad (5.4.33)$$

But the intensity is also given by

$$I = \text{energy density} \times c = \epsilon E^2 C \quad (5.4.34)$$

Thus combining equations (2.33) and (2.34) one gets

$$V_v = e^2 \frac{(nlt)}{mc} \quad (5.4.35)$$

$$\frac{ne^2lt}{m} E^2 = \epsilon E^2 C \quad (5.4.36)$$

For single electron (n=1) thus

$$e^2 = \frac{C\epsilon m}{lt} \quad e = \left( \frac{C\epsilon m}{xt} \right)^{\frac{1}{2}} = C \quad (5.4.37)$$

Where X = L

### **(5.5) Electron charge from self energy and mass energy**

The charge e can also be found by equating electron mass energy to electron self energy. The electron mass energy is given by

$$E = \left(1 + \frac{2\phi}{c^2}\right) m_0 \left(1 + \frac{2\phi}{c^2} - \frac{V^2}{c^2}\right)^{-\frac{1}{2}} c^2 \quad (5.5.1)$$

The vacuum state can be found by minimizing E to get

$$\frac{dE}{d\phi} = \left[ \frac{2}{c^2} \left(1 + \frac{2\phi}{c^2} - \frac{V^2}{c^2}\right)^{-\frac{1}{2}} - \frac{2}{c^2} \left(1 + \frac{2\phi}{c^2}\right) m_0 \left(1 + \frac{2\phi}{c^2} - \frac{v^2}{c^2}\right)^{-\frac{3}{2}} \right] m_0 c^2$$

$$\left[ \left(1 + \frac{2\phi}{c^2} - \frac{V^2}{c^2}\right) \right]^{-\frac{3}{2}} \left[ +2 \frac{4\phi}{c^2} - 1 - \frac{2\phi}{c^2} - 2 \frac{V^2}{c^2} \right] m_0 = 0 \quad (5.5.2)$$

$$\frac{2\phi}{c^2} - \frac{2V^2}{c^2} - 1 \quad (5.5.3)$$

If vacuum consists of photons which produces electron position pairs

$$V = C \quad (5.5.4)$$

$$\phi = \frac{c^2}{2} \quad (5.5.5)$$

By inserting equation (2.10) in equation (3.5) one

$$\frac{e^2}{12\pi \epsilon_0 r_0} = m_0 \phi = \frac{m_0 c^2}{2}$$

Thus electron charge is given by

$$e = (6\pi \epsilon_0 r_0 m_0)^{\frac{1}{2}} c \quad (5.5.6)$$

### (5.6) String Oscillation and Electron Charge

String theory recently pays attention of many researches [88,89,90] Charge quantization for oscillation string which act as harmonic oscillator can be found energy relation

$$E_H = n\hbar\omega + \frac{1}{2}\hbar\omega \quad (5.6.1)$$

Since energy cannot be measured. Thus one deducts the difference between vacuum energy  $E_v$  and the actual value  $E_H$  thus our devices measure the energy to be

$$\begin{aligned}
 E &= E_H - E_v. \\
 &= \left(n + \frac{1}{2}\right) \hbar\omega - \frac{1}{2} \hbar\omega \\
 &= n\hbar\omega
 \end{aligned}
 \tag{5.6.2}$$

Where the energy is given

$$E_v = \frac{1}{2} \hbar\omega \tag{5.6.3}$$

$$E = n\hbar\omega = n\hbar f = \frac{nh}{T} \tag{5.6.4}$$

From Ibrahim model and the works time is quantized at early time when charge and mass and time are quantized, and when charge is generated. Thus vacuum energy given by:

$$E_v = \frac{nh}{T} = \frac{nhc}{n_o x_o} \tag{5.6.5}$$

Where

$$CT = x_o$$

And

$$x = n_o x_o = CT$$

$$n = 1,2,3 \dots\dots\dots$$

$$n_o = 1,2,3 \dots\dots\dots$$

Thus from (5.4.9) and (5.6.5)

$$\frac{e^2}{12\pi\epsilon_0 r_0} = \frac{nhc}{n_0 x_0} \quad (5.6.6)$$

Thus the charge is quantized and is given by:

$$e = \left( \frac{2\pi\epsilon_0 r_0 hc}{x_0} \right)^{\frac{1}{2}} \left( \frac{n}{n_0} \right)^{\frac{1}{2}}$$

Using equation (5.4.23) and (5.6.5)

$$e = \left( \frac{mc}{nLt} \right)^{\frac{1}{2}} \left( \frac{nhc}{n_0 x_0} \right)^{\frac{1}{2}} \quad (5.6.7)$$

Using the expression for the minimum radius of the universe  $r_0$ , plank constant, the speed of light in vacuum and electronic charge one get

$$\begin{aligned} x_0 &= 26635 \times 10^{-6} & c &= 3 \times 10^8 & h &= 6.626 \times 10^{-34} \\ r_0 &= 3 \times 10^{-24} & &= 6626 \times 10^{-37} & & \end{aligned} \quad (5.6.8)$$

$$\frac{n}{n_0} = \frac{e^2 x_0}{2\pi\epsilon_0 v_0 hc} \quad (5.6.9)$$

### (5.7) Kline – Gordon Equation and Electron charge Quantization

Also one can use Klein – Gordon equation to predict electron charge, where [91,92]

$$\begin{aligned} -\hbar^2 \frac{\partial^2 \Psi}{\partial t^2} &= c^2 \hat{p}^2 \Psi + m_0^2 c^4 \Psi & (5.7.1) \\ &= -c^2 \hbar^2 \nabla^2 \Psi + m_0^2 c^4 \Psi \end{aligned}$$

Where for fast particles

$$\mathcal{V} \mapsto c \quad m = m_0 \left( 1 - \mathcal{V}^2/c^2 \right)^{-\frac{1}{2}} \mapsto \infty \quad (5.7.2)$$

Thus the relativistic energy is given by

$$E^2 \Psi^2 = c^4 p^2 \Psi$$

Using the wave function:  $\Psi = \frac{i}{\hbar}(pr - Et)$

$$i\hbar \frac{\partial \Psi}{\partial t} = E\Psi \quad \frac{i}{\hbar} \nabla \Psi = P\Psi \quad -\hbar^2 \nabla^2 \Psi = P^2\Psi \quad (5.7.3)$$

Thus the reduced Klein garden equation reads

$$-\hbar^2 \frac{\partial^2 \Psi}{\partial t^2} = c^2 \hbar^2 \nabla^2 \Psi \quad (5.7.4)$$

Try solution

$$\Psi = u e^{i\omega t} = u e^{i\omega t/\hbar} \quad (5.7.5)$$

Substituting (5.7.5) in (5.7.4) one gets

$$-i^2 \omega^2 \hbar^2 u = -c^2 \hbar^2 \nabla^2 u \quad (5.7.6)$$

Inserting  $E = \hbar\omega$

One gets

$$c^2 \hbar^2 \nabla^2 u = -E^2 u \quad (5.7.7)$$

Consider now the solution

$$u = \text{as in } kx \quad \nabla u = kA \cos kx \quad (5.7.8)$$

$$\nabla^2 u = -k^2 A \sin kx = -k^2 u$$

Thus substituting (5.7.8) (5.7.7) yields

$$c^2 \hbar^2 k u^2 = E^2 u$$

$$E = \pm c\hbar k \quad (5.7.9)$$

In the early universe the radius it only few millimeters thus one can treat particles as if they are in a box of length this means that their probability of existence outside the box is zero

$$u(x = L) = \sin kL = 0$$

$$kL = 2n\pi \quad (5.7.10)$$

$$k = \frac{2n\pi}{L} = \frac{2n\pi}{x}$$

i.e at vacuum stage the distance is quantized, i.e.

$$x = n_0 x_0 = n_0 \lambda_0 \quad (5.7.11)$$

Thus from equation (5.7.9,10,11) the vacuum energy is given by

$$E_v = \pm c\hbar \left( \frac{2\pi n}{n_0 x_0} \right) \quad (5.7.12)$$

Using the vacuum energy relation (5.4.9) and equating it with equation (5.7.12) one gets;

$$\frac{e^2}{12\pi \epsilon_0 r_0} = \frac{2hc}{x_0} \left( \frac{n}{n_0} \right)$$

$$e = \pm \left( \frac{24\pi h \epsilon_0 r_0}{x_0} \right)^{\frac{1}{2}} \left( \frac{n}{n_0} \right)^{\frac{1}{2}} \quad (5.7.13)$$

It is very interesting to note that we have positive and negative signs, which indicate existence of positive and negative charges, which are electrons and positrons. Thus this secures charge conservation of vacuum and universe

Thus from (5.7.13) one can eerily adjust the quantum numbers  $n$  and  $n_0$ , such that

$$\frac{n}{n_0} = \frac{x_0}{24\epsilon_0 r_0} e^2 \quad (5.7.14)$$

To find the electron charge

$$e = 1.6 \times 10^{-19} \text{ c} \quad (5.7.15)$$



This means that electron charge is quantized, and its value can be found to be as in equation (5.7.15) by adjusting the quantum numbers  $n$  and  $n_0$ .

### (5.8) new mass generation and Higgs mechanism within the framework of GSR

The Lagrangian of matter field within the framework of SR is given by [93, 94]

$$L = i\bar{\Psi}\gamma^\mu D_\mu \Psi - m\bar{\Psi}\Psi \quad (5.8.1)$$

With

$$D_\mu \Psi = \partial_\mu \Psi + iqA_\mu \Psi \quad (5.8.2)$$

is invariant due to the fact that the mass is not a function of the field potential thus

$$m^1\bar{\Psi}^1\Psi^1 = m\bar{\Psi}\Psi \quad (5.8.3)$$

However, in GSR  $m$  is dependent on field potential, where

$$m = m_0 \left(1 + \frac{2\Phi}{c^2} - v^2/c^2\right)^{-\frac{1}{2}} = m_0 \left(1 + \frac{2A_\mu}{c^2}\right)^{-\frac{1}{2}} = m(A_\mu) \quad (5.8.4)$$

This means that for the transformation

$$A_\mu \rightarrow A^1_\mu$$

$$A^1_\mu = A_\mu - \partial_\mu \Lambda \quad (5.8.5)$$

$$m^1 \neq m \quad (5.8.6)$$

$$\Psi \rightarrow \Psi^1 = e^{i\Lambda q} \Psi$$

$$m^1\bar{\Psi}^1\Psi^1 = \bar{\Psi}^1 e^{-\frac{i\Lambda q}{\Psi}} e^{i\Lambda q} \Psi = m^1\bar{\Psi}\Psi \neq m\bar{\Psi}\Psi \quad (5.8.7)$$

This equation redefining (5.8.5) to be

$$A^1_\mu = A_\mu - \partial_\mu \Lambda + b \quad (5.8.8)$$

Inserting (5.8.8, 6, 5, 2) in (5.8.1) with  $L$  replaced by

$$L = i\bar{\Psi}^1\gamma^\mu D^1_\mu \Psi^1 - m^1\bar{\Psi}^1\Psi^1 = i\bar{\Psi}^1\gamma^\mu (\partial_\mu \Psi^1 + iqA^1_\mu \Psi^1) - m^1\bar{\Psi}^1\Psi^1$$

$$\begin{aligned}
&= ie^{\frac{-i\Lambda q}{\Psi}} \gamma_{\mu} [(\partial_{\mu} \Psi + iq\Psi \partial_{\mu} \Lambda)e^{i\Lambda q} + (iqA_{\mu} - iq \partial_{\mu} \Lambda)e^{iq\Lambda\Psi}] + \\
&bi\bar{\Psi}\gamma_{\mu}(iq\Psi) - m^l e^{-iq\Lambda}\bar{\Psi}e^{iq\Lambda\Psi} \\
&= i[\bar{\Psi}\gamma_{\mu} \partial_{\mu} \Psi + iqA_{\mu}\Psi] - m^l \bar{\Psi}\Psi - bq\bar{\Psi}\gamma^{\mu}\Psi \\
&= i\Psi\gamma^{\mu}D_{\mu}\Psi - m\bar{\Psi}\Psi + (m - m^l)\bar{\Psi}\Psi - bq\bar{\Psi}\gamma^{\mu}\Psi \\
L^1 &= L + \bar{\Psi}(m - m^l)\Psi - \bar{\Psi}(bq\gamma^{\mu})\Psi \tag{5.8.9}
\end{aligned}$$

The invariance takes place, when

$$\bar{\Psi}(m - m^l)\Psi = \bar{\Psi}(bq\gamma^{\mu}) \tag{5.8.10}$$

i.e. when

$$m - m^l = bq\gamma^{\mu} \tag{5.8.11}$$

In this case

$$L^1 = L \tag{5.8.12}$$

Thus the gauge transformation that preserves mass term need to be charged to in the form (5.8.8).

### (5.9) Discussion:

This second order lagrangian was shown to be successful in describing the gravitational phenomena by the so called generalized general relativity. This motivates us to try to construct new second order lagrangian for the electromagnetic field, as shown by equation (5.3.3). In this lagrangian matter energy manifests is self through two terms. The first term  $J_{\Psi}$  represents energy stored in a charge which equals charge energy per particles  $\gamma^{\mu}$  multiplied by the number of particles  $|\Psi|^2$ . The second terms represents the contribution of charged system in generating or absorbing photons. This term,  $J_A$ , which represents the a field source manifests its role

in absorbing or generating field through the equation of motion (5.3.10). Which shows dependence of  $J_A$  on second order derivative of  $A_\mu$ .

It is very interesting to note that this terms gives no contribution to the equation of motion according to equations (5.3.11) and (5.3.1). At the same time the Hamiltonian of equation (5.3.13) shows the appearance of the terms  $J_A$  which describes absorption or emission of photons energy by charged systems, beside the terms  $J_\psi$  which represent the energy stored in electric charges.

It is also important to note that equation (5.3.10) shows that both electric charges and vacuum energy can generate electromagnetic field. The generation of e. m. field was proved by Cassimar effect. The Hamiltonian in equations (5.3.13) too also have a term recognizing the vacuum energy. This is also experimentally verified by Cassimar effect.

In section the Hamiltonian of free electromagnetic field in a curved space time is minimized to get  $\phi$  in terms of  $c$ ,  $m$  and  $c$  at vacuum stage as shows by equation (5.4.8). vacuum energy obtained by minimizing  $\phi$  is equated with that obtained from electric energy density according to equation (5.4.9). The expression for classical angular momentum and quantum spin angular momentum are used to find electron radius. The electron charge is shown to be quantized according to equation (5.4.13) and (5.4.25), due to the existence of two quantum numbers  $n$  and  $n_0$  which can be adjusted early to find the value of  $e$ .

In section of string vibrating model is used to find electric charge which is shown to be quantized according to equation (5.6.7). Again by adjusting the quantum numbers  $n$  and  $n_0$  in equation (5.6.9) one can find easily Klein – Gordon equation for particle in a box is used to quantize electron charge

where vacuum here is the early inverse stage which has finite radius and can be treated as particle in a box.

This leads to finding vacuum quantized energy as shown by equation (5.7.12). The charge is quantized and can be found by adjusting  $n$  and  $n_0$  as equations (5.7.13) and (5.7.14) shown.

### **(5.10) Conclusion:**

The lagrangian depending on second order field derivatives shows that vacuum energy can generate electromagnetic field as well as electric charges. It shows also that the Hamiltonian consists of terms recognizing charge energy, source energy, and vacuum energy.

The electromagnetic Hamiltonian in a curved space beside generalized special relativity. In addition to quantum theory can successfully show, that electron charge is quantized and take its known proper values. Even quark charges can be obtained and shown to be quantized.

### **(5.11) Recommendation:**

1. The use of second order lagrangian in electromagnetic theory and even weak and strong nuclear field need more investigation
2. The generation of mass and its effect on transformation and conservation laws needs more study.
3. The problem of charge self energy and the relativity between electromagnetic and gravitational mass needs intensive research.

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