

Sudan university of Science and Technology

College of Gradute Studies



**Asymptotic Analysis with Deferential Inverse
and Variational inequality with Gap function**

التحليل التقاربي مع المعكوس التفاضلي ومتباينة التغيرات مع دالة الفجوة

A Thesis Submitted in Fulfillment Requirement for the Degree
Master of in Mathematics

By

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DEDICATION

To my Parents,

Brother,

And

Sisters

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Thank to ALLAH who gave me patience to conduct this study. I would like to thank my supervisor prof. Shawgy Hussein Abdalla for his advices and discussions during the preparation of the study of this dissertation. I would also like to thank my Family for the great support and for their patience before getting this work done. Finally I would like to thank my friends .

Abstract

We present a unified approach for studying vector variational inequality problems in finite dimensional spaces. We introduce a class of weak normal mapping by virtue of the vector-valued indicator function. A new differential inverse variational inequality is introduced and studied in finite dimensional Euclidean spaces. Some results concerned with the linear growth solution set for the differential inverse variational inequalities are obtained under different conditions. We develop a theory of variational inequality for demicontinuous S -contractive maps in reflexive smooth Banach spaces. We obtain some gap functions for generalized mixed quasi-variational inequality problems in terms of regularized gap function and D-Gap function.

الخلاصة

تم تقديم الأقتراب الموحد لأجل دراسة مسائل متباينة تغاير المتجه في الفضاءات منتهية البعد. تم ادخال عائلة للراسم الناظم الضعيف بواسطة تأثير الدالة المميزه قيمة-المتجه. أدخلت ودرست متباينة التغير العكسي التفاضلي الجديدة في الفضاءات الإقليدية منتهية البعد. تم إعطاء بعض النتائج المعينة مع النمو الخطي لفئة الحل لمتباينات التغير العكسي التفاضلي تحت شروطاً مختلفة. تم تطوير نظرية متباينة التغير لرواسم الإنكماش-S نصف المستمرة في فضاءات باناخ الملساء الإنعكاسية. تم تحصيل بعض دوال الفجوة لمسائل متباينة شبه التغير المختلطة المعمم بدلالات دالة الفجوة المنتظمة ودالة الفجوة-D

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List of Symbols

Symbol	Mains	Page
PPA	Proximal point algorithm	1
VVI	Vector Variational inequality	4
WVI	Weak Variational inequality	4
Int	Integer	4
dom	Domain	8
Sup	Supremum	8
Osc	Outer semi continuous	8
Ptm	Proximal-type method	9
Vip	Variational inequality problem	12
Inf	Infimum	26
l^2	Hilbert space	39
a.e	Almost everywhere	43
$W_o^{1,p}$	Sobolev space	54
Max	Maximality	54
l^p	Lebesgue space	54
l^q	Dual Lebesgue space	55
l^1	Lebesgue on the real line	64
l^∞	Essential Lebesgue space	65
meas	measure	72

References

- [1] ZheChen^{a,b} :Asymptotic Analysis for Proximal-type Methods in Vector Variational Inequality problems,Operations Research Letters,February 17,2015,page1-14.
- [2] Wei LI,XingWANG,Nanjing HUANG :Differetial Inverse VariationalInequalities in Finite Dimensional Spaces,Acta mathematica2015,35B(2):Page407-422.
- [3] KunquanLan :Avariational inequality theory in reflexive smooth Banach spaces and applications to p-Laplacian Elliptic Inequalities,Nonlinear Analysis 113(2015) page 71-86.
- [4] SuhelAhmed Khan^{a,*},Jia-Wei Chen^b :Gap Function and Global Error Bounds for Generalized Mixed Quasi variationalInequalities,Applied Mathematics Computation 260(2015) page 71-81.
- [5] Ginchev,1;Guerraggio;Rocca,M.(2006).”From Scalar to Vector Optimization”.Application of Mathematics 51:5.doi:10.1007/S10492-006-0002-1.
- [6] Horn, Roger A.;Johnson,Charles R.(1990).Matrix Analysis.Cambridge University Press.ISBN 978-0-521-38632-6.