Chapter one

Introduction

(1.1) Gravity and relativity :

Gravitational field plays an important role in our day life . It affect the human life to a great extent . Gravitational energy is utilized to generate electricity by waterfall . Gravity force is responsible for preventing bodies from moving away in free space . It also affect the motion of aero plains and satellites as well as cars and trains. It also forces rain water droplets to fall down from clouds so as to provide people with fresh water .

Its gravitational laws describe a number of observation but unfortunately it fails in describing some astronomical observations . These observations include , the universe expansion , the perihelion of mercury , beside the deflection of light by the sun [1,2,3] . This motivates Einstein to formulate his general relativity (GR) theory . The (GR) succeeded in explaining most of the astronomical observations . But recently the (GR) fails to explain the behavior of black holes and guasi stellar objects [4,5]. General relativity can be considered as a natural extension of special relativity (SR) . Special relativity promote Newton's laws of motion by assuming that space and time are not absolute concepts but depend on the observer motion . Space and time intervals values for a certain physical system have different values for different reference frame moving with respect to (w.r.t) each other with different uniform velocities [6,7] .

Special relativity changes also the notion of energy in Newtonian mechanics by recognizing the presence of rest mass energy . This new energy relation was used by Dirac , in his solution of Dirac relativistic equation , to propose the existence of anti matter [8,9] . This discovery changes radically our understanding of the nature of elementary particles . The serious attempt to describe the gravitational phenomenon was started by Newton . He formulated his famous inverse square law for the gravitational force [10,11] .

(1.2) Advantages and Disadvantages of Special Relativity :

The theory of SR succeeded in explaining the constancy of the speed of light . It also explains the transformation of some nuclear mass into pure energy . Moreover it can describe time dilation in the meson decay [12,13,14].It also describe production of particles and anti particles (pair production) , beside describe annihilation process [15,16] .

Despite these success SR suffers from noticeable setbacks . First of all it does not satisfy correspondence principle , since its energy expression does not reduce to that of Newton for low velocities [17,18,19]. Moreover the energy relation in SR , suffers from the lack of an expression representing the field potential [20,21] . This is in direct conflict with observations and other physical theories [22,23]. For example in quantum mechanics the nuclear potential is very important in quantum Schrödinger equation that describes the structure of energy levels and electronic configuration [24,25,26]. Even in our day life , the motion of rockets and projectiles is impossible without taking into account the potential energy concept [27,28,29].

(1.3) Research problem :

The lack of SR energy relation from potential energy terms , make it fail to satisfy Newtonian limit and to be compatible with quantum laws and other physical theories [30,31]. Relativity theory also have no explicit relation that can describe the nature of interactions between particles and anti particles [32,33].

(1.4) Literature Review :

Different attempts were made to describe gravitational phenomena by modifying SR [34,35]or by trying to predict the existence of anti gravity [36,37,38,39,40] or by relating it to vacuum energy [41,42,43].

In these attempts that modify SR the expressions used are complex and some of them describes only weak field [44,45,46,47]. Attempts are also made to describe interaction between matter and anti matter or dark matter on the basis of general relativity (GR) and curved space time

[48,49,50,51,52] . Unfortunately these attempts are complex and uses only special constraints .

(1.5) Aim of the work :

The aim of this work is to construct a theoretical model that can modify SR to cure some of its defects .

(1.6) Thesis layout :

The thesis consist of 4 chapters . Chapters one and two are concerned with introduction and theoretical back ground respectively . The literature review is in chapter three . Chapter four is devoted for contribution .

Chapter two
Special relativity and Gravity

2.1 Introduction :

Special relativity was formulated by Einstein and first published in 1895 in the article . It change Newtonian concept of space and time . The theory is called "special "because it applies the principle of relativity to the "restricted "or" special" case of inertial reference frames that moves with constant velocity w.r.t each other .

In this chapter the postulates of SR beside the expressions of time, length , mass and energy beside gravity laws are exhibited .

2.2 Special Relativity Postulates :

Einstein's special theory of relativity is based on two postulates , stated by Einstein postulates states that . The two

1- The special principle of relativity: The laws of physics are the same for all observers , regardless of their velocity.

2- The speed of light in a vacuum (c) is constant : That is , everyone will always measure the speed of light as being the same , and equal to c, regardless of their own velocity.

The important point here is that the speed of light is the same for all observers. Suppose you measure the speed of a beam of light traveling towards you and record its speed as c . According to Newtonian (or classical) physics, if someone else traveling at $3m/s$ (relative to you) were then to measure the speed of a beam of light traveling towards them, then they would measure the speed of light to be $c+3m/s$. However, this does not prove to be the case in practice . Everyone records the same speed of light regardless of their velocity relative to each other. Einstein explained this by proposing that the way you view space and time to be different from the way the other person views space and time. The mathematical description of this became the special theory of relativity (SR) [9,10,11,12] . Special relativity remained controversial for many years after its first publication. However, as experiments became more accurate, special relativity was accepted by the scientific community. Despite this, Einstein did not receive a Nobel prize for this work , he was granted that honor for his work on the photo , electric effect[12].

2.3 The Lorentz Transformation :

Lorentz transformation deals with inertial frames that moves w.r.t each other with constant velocity .

The correct transformation equations between two inertial frames in Special Relativity, is known as Lorentz transformation . Consider two inertial frames S and S' , which have a relative velocity v between them along the x-axis

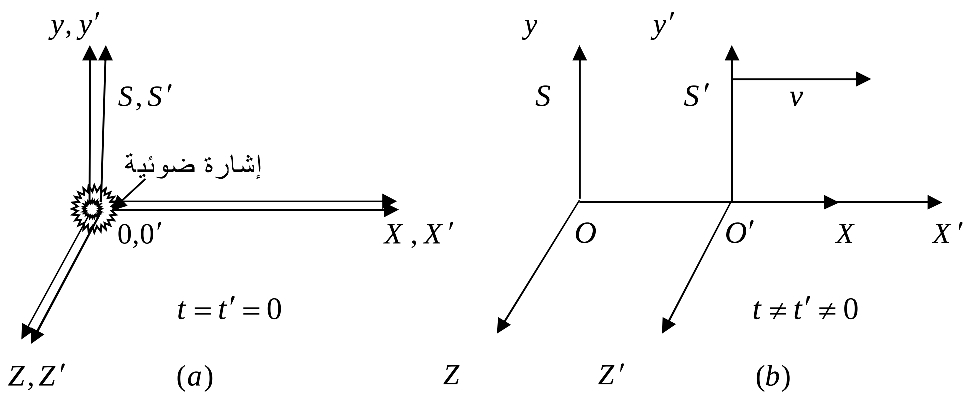


Figure (2.3.1)

Assume that there is a single flash at the origin of S and S' at time t, when the two inertial frames happen to coincide . The outgoing light wave will be spherical in shape moving outward with a velocity C in both S and S' by Einstein's Second Postulate[8,9].

$$x^2 + y^2 + z^2 = c^2t^2$$
$$x'^2 + y'^2 + z'^2 = c^2t'^2 \qquad (2.3.1)$$

One expect that the orthogonal coordinates will not be affected by the horizontal velocity

$$y' = y$$
$$z' = z \qquad (2.3.2)$$

But the x coordinates will be affected .This effect can be described by linear transformation:

$$x = \gamma\, x' + bt' \qquad (2.3.3)$$

$$x' = \gamma\, x - bt \qquad (2.3.4)$$

Where b and γ are certain parameters.

Consider the outgoing light wave along the x-axis ($y = z = 0$) [10]

$$x = ct \quad (2.3.5)$$

$$x' = ct' \quad (2.3.6)$$

Consider the case when both the origin of S and S' coincide .i.e

$$x = 0 \qquad x' = 0$$

Using (2.3.3) and (2.3.4) yields

$$v = \frac{x}{t} = -\frac{x'}{t'} = \frac{b}{\gamma}$$

Compensation value x' from equation (2.3.6) in equation (2.3.4) and replace x in equation (2.3.3) to equally (2.3.5) yields [11] :

$$ct = \gamma\, ct' + bt'$$

$$ct' = \gamma\, ct - bt$$

But $\quad b = \gamma\, v \qquad \frac{b}{\gamma} = \frac{x}{t} = v \qquad b = \gamma\, v \qquad (2.3.7)$

$$ct = \gamma\, t'(c + v)$$

$$ct' = \gamma\, t(c - v) \quad (2.3.8)$$

Using equations (2.3.7) and (2.3.8) one gets [11]:

$$ct = \gamma(\frac{\gamma}{c})(c - v)(c + v)t$$

$$c^2 = \gamma^2(c^2 - v^2) \quad (2.3.9)$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (2.3.10)$$

The mathematical problem here is to find the relationships of x' and t' in terms of x and t. The results are the, now well known , as Lorentz transformation , named after Hedrik A, Lorentz of Leyden University who had almost developed them while studying Maxwell's equations . Einstein did derive these equations independently[13].This inserting (2.3.7) and (2.3.10) in equation (2.3.4) yields

$$x' = \gamma\,(x - vt)$$

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{x - vt}{\sqrt{1 - \beta^2}} = \gamma(x - vt) \qquad (2.3.11)$$

Using equations (2.3.11) , (2.3.3) and (2.3.4) one gets

$$t' = \frac{t - \frac{v}{c^2}x}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{t - \frac{v}{c^2}x}{\sqrt{1 - \beta^2}} = \gamma\left(t - \frac{\beta}{c}x\right) \qquad (2.3.12)$$

The inverse transformations (from system $S^{'}$ to S) can be done by replacing v by $-v$ and simply interchanging primed and unprimed coordinates. This gives[14,15,16].

$$x = \frac{x' + vt'}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{x' + vt'}{\sqrt{1 - \beta^2}} = \gamma(x' + vt') \qquad (2.3.13)$$

$$t = \frac{t' + \frac{v}{c^2}x'}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{t' + \frac{v}{c^2}x'}{\sqrt{1 - \beta^2}} = \gamma\left(t' + \frac{\beta}{c}x'\right) \qquad (2.3.14)$$

These equations say that time t and displacement x are not fixed invariants, as stated in the classical Newtonian Physics, but they depend on the velocity v of the coordinate frame S' relative to S as well as coordinates and time [17].

2.4 Relativity of Time :

Consider observers S and S'. The two observers will measure time a bit differently. From Equations (2.3.11) and (2.3.12) one have

$$t_2 - t_1 = \frac{(t_2' - t_1') + (v/c^2)(x_2' - x_1')}{\sqrt{1-\beta^2}}$$

Which says that the observer on S read the clock on S' and see $(t'_2 - t'_1)$ and compares it to his own time interval $(t_2 - t_1)$ and sees

$$t_2 - t_1 = \frac{t_2' - t_1'}{\sqrt{1-\beta^2}} \qquad (2.4.1)$$

If we define $t_\circ = t'_1 - t'_2$ and $t = t_1 - t_2$ we have [19] .

$$t = \frac{t_0}{\sqrt{1-\beta^2}} = \gamma t_0 \qquad (2.4.2)$$

Thus a clock appears to slow down by the factor $\sqrt{1-\beta^2}$ as it speeds up relative to the fixed frame S.

2.5 Relativity of Length :

Consider two observers, one working on the fixed coordinate system S and the other on the moving system S' observing the length of a rod. If the two systems are initially at rest and coinciding then both will measure the same length l, or $l_\circ$. The observer on S will express the length as $l = x_2 - x_1$ and the one on S' will read it as $l_\circ = x_2' - x'_1$. As the system S' begins to move with a velocity v along the X axis, the observer on S' will continue to read the length of the rod as l_0, but the one on S will read $l = x_2 - x_1$, where x_2 and x_1 must be connected to x_2' and x_1' via the Lorentz transformations Equations (2.3.11) and (2.3.12) ,that is [19] .

$$x_2' = \gamma(x_2 - vt_2) \qquad (2.5.1)$$

$$x_1' = \gamma(x_1 - vt_1) \qquad (2.5.2)$$

$$x_2' - x_1' = \gamma[(x_2 - x_1) - v(t_2 - t_1)] \qquad (2.5.3)$$

Since the observer on S will measure both ends of the rod at the same time then $t_2 = t_1$, so Equation . (2.5.3) gives [19]

$$x_2 - x_1 = \frac{1}{\gamma}\left(x'_2 - x'_1\right) \qquad (2.5.4)$$

Where $l_\circ = x'_2 - x'_1$, $1/\gamma = \sqrt{1-\beta^2}$

And $x_2 - x_1 = L$ is

the length as measured by the observer on S. Thus, we have

$$L = \frac{L_0}{\gamma} = L_0\sqrt{1-\beta^2} \qquad (2.5.5)$$

which simply says that length appears to shrink as observed from S, as it speeds up along its length [19]

2.6 Velocity Transformation :

As a direct consequence to these new transformations, all the other mathematical operations and physical variables follow accordingly. For example, the velocity equations though still the derivatives of the displacement assume a new form, so the Lorentz form of the velocities is:

$$u'_x = \frac{u_x - v}{1 - \frac{v u_x}{c^2}} \qquad (2.6.1)$$

and the inverse is [35.36] :

$$u_x = \frac{u'_x - v}{1 - \frac{v u'_x}{c^2}} \qquad (2.6.2)$$

2.7 Relativity of Mass and energy :

Relativity effects on mass are evaluated by considering the conservation of linear momentum and connecting the variables using the Lorentz velocity transformations. From Equations. (2.4.1) and (2.4.2) we find that the relativity effects also affect the mass. Details can be found Reference S. The result is

$$m\sqrt{1-\frac{v^2}{c^2}} = m_0 \qquad (2.7.1)$$

$$m = \frac{m_0}{\sqrt{1-\beta^2}} = \gamma m_0 \qquad (2.7.2)$$

Thus, mass also appears to change, it increases, and it becomes infinite as it approaches the speed of light [20,21,22,23,29] .

Any relativistic generalization of Newtonian must satisfy two criteria:

1- Relativistic Momentum must be conserved in all frames of reference .

2- Relativistic Momentum must reduce to Newtonian Momentum at law speed.

The first criterion must be satisfied in order to satisfy Einstein's first postulate while second criterion must be satisfied as its know that Newtonian's law are correct at sufficiently low speeds . a definition for the relativistic momentum of particle moving with a velocity v as measured with respect to frame of reference S , that satisfies these criteria can be shown to take the form [22,24,30,31].

$$p = \frac{m_* v}{\sqrt{1-\frac{v^2}{c^2}}} = \gamma m v \qquad (2.7.3)$$

where v is velocity of the particle and m is mass of the particle . when v is much less than c , $\gamma = \left(1-\frac{v^2}{c^2}\right)^{-\frac{1}{2}}$.

In many s situation , it is useful to interpret equation (2.4.5) as the product of a relativistic mass, γm, and the velocity of the object using this description . one can be say that the observed mass of an object increase with the speed according to the relation [22]

$$\text{Relativistic mass} = \gamma m_{\circ} = \frac{m_{\circ}}{\sqrt{1-\frac{v^2}{c^2}}} \qquad (2.7.4)$$

$$m = \frac{m_{\circ}}{\sqrt{1-\frac{v^2}{c^2}}}$$

Where the relativistic mass is mass of the object moving at a speed υ with respect to the observer and m is the mass of the object as measured by an observer at rest with respect to the object ,the quantity m is often referred to the as the rest mass of the object .

The relativistic force F acting on a particle whose momentum is p is defined as

$$F = \frac{dp}{dt} \qquad (2.7.5)$$

Where p is given by equation (2.4.5) which is the relativistic form Newton's second law.

Now work is defined as force applied over a distance . It corresponds to the expended energy to accelerate a body . If the force and path are constant.

$$W = F.d$$

In order to derive the relativistic form of SR energy , one can use the definition of relativistic force . The work done by force F is given by

$$W = \int_{x_1}^{x_2} F.dx = \int_{x_1}^{x_2} \frac{dp}{dt} dx \qquad (2.7.6)$$

$$\left(\frac{dp}{dt}\right)dx=\left(\frac{dp}{dv}.\frac{dv}{dt}\right)dx=\frac{dp}{dv}\frac{dx}{dt}dv=\frac{dp}{dv}vdv \qquad (2.7.7)$$

Since p depend on v according to its definition

$$p = mv = \frac{m_0 v}{\sqrt{1-v^2/c^2}}$$

Therefore

$$\frac{dp}{dv}=\frac{d}{dv}\frac{m_0 v}{\sqrt{1-v^2/c^2}}=\frac{m_0}{\left(1-\frac{v^2}{c^2}\right)^{\frac{3}{2}}} \qquad (2.7.8)$$

Using this relation (2.7.8) in (2.7.6) with the aid of (2.7.7) , one gets

$$W=\int_0^v \frac{dp}{dv}vdv=\int_0^v \frac{mv}{\left(1-v^2/c^2\right)^{\frac{3}{2}}}dv$$

One assumed here that the particle is accelerated from rest to some final velocity v. Evaluating the integral . we find that

$$W=\frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}}-mc^2 \qquad (2.7.9)$$

Because we assumed that the initial speed of the particle is zero . We know that its initial kinetic energy is zero . Thus the work W in equation (2.7.9) is equal to the relativistic kinetic energy T [24,25,32,33]

$$T=\frac{m_0c^2}{\sqrt{1-v^2/c^2}}-m_0c^2=\gamma m_0c^2-m_0c^2=(\gamma-1)m_0c^2 \qquad (2.7.10)$$

The constant term m_0c^2 in equation (2.7.10) which is independent of the speed of the particle, is called the reset energy E_0 of the particle :

$$E_0=m_0c^2 \qquad (2.7.11)$$

The term $\gamma m_0 c^2$, which dose depend on the particle speed, is therefore the sum of the kinetic and rest energies. We define $\gamma m_0 c^2$ to be the total energy E:

Total energy= kinetic energy +rest energy

$$E = T + m_0 c^2 \qquad (2.7.12)$$

$$E = \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma m_0 c^2 = mc^2 \qquad (2.7.13)$$

The relation (2.7.12) shows that rest mass is a form of energy . [25,26,27,28,34,35]

Chapter three
Literature Review

(3.1) Introduction :

Recently anti gravity and anti matter pays scientists attention .

Many attempts were made for prediction of anti gravity and modify gravity theories . Here some of them are discussed .

(3.2) Negative Matter and Repulsive Force :

In the work of Yi-Fang Chang he throw light on many theories that predict existence of repulsive force .

This prediction is based in some astronomical observations .The speed of an object surrounded a galaxy is measured, which can estimate mass of the galaxy. Many results discover that the total mass of galaxies is always far larger than luminous mass of these galaxies. This shows the existence of dark matter in various galaxies. Dark matter is fundamentally different from the normal matter. It is invisible using modern telescopes because it gives off no light or heat, and appears to interact with other matter only gravitationally. In contrast, the luminous matter is everything commonly associated with the universe: the galaxies, stars, gas and planets. Further, this is confirmed that there are abundant dark matters by the mass-to-light ratio, etc., in group of galaxies and cluster of galaxies, in the universe. And the ratio between dark matter and luminous matter increases with dimensions of these systems [36]. Now investigation of dark matter is a focus of fundamental interest to astronomers, astrophysicists, cosmologists, and nuclear and particle physicists [37].

Since 1981, dark energy is proposed in order to explain the acceleration of inflation in the universe, which is produced due to dark energy as a huge repulsive force [38]. Usually assume that dark energy is a scalar field, and connects with the cosmological constant , which corresponds to a repulsive force predicted by the general relativity. It is better model that there are about 70% mass in the universe from the cosmological constant. This can explain not only the acceleration of inflation in the universe, and unify many different results of observations.

The Dirac equations of fermions can describe anti-matter . The cosmological constant Λ describes possibly the negative matter, which corresponds to the Λ term field equation in the Klein-Gorden equation the m^2 term correspond to $\pm m$, both describe bosons . In the Dirac equations $m \to -m$ may also describe the negative matter . A universal relation is

$$E^2 = m^2 c^4 + c^2 p^2 \qquad (3.2.1)$$

It may be generally applied for various positive, opposite and negative matters, and for all $\pm m$, $\pm E$ and $\pm p$ only the mass is negative in the equations described negative matter, while the charge and so on are opposite in the equations described opposite matter. For a relation:

$$E^2 = m^2 c^4 + c^2 (p - \frac{q}{c} A)^2, i.e, p - \frac{q}{c} A = \pm \frac{1}{c} \sqrt{E^2 - m^2 c^4} \qquad (3.2.2)$$

$$\therefore q = (mv \mp \frac{1}{c} \sqrt{E^2 - m^2 c^4}) \frac{c}{A} \qquad (3.2.3)$$

Such the charge may be positive or negative, and is particular distinct for $v = 0$. It corresponds to the opposite matter.

The negative matter is possibly influence on the universal gravitational laws, classical mechanics, the motion laws of planet, electrodynamics, general relativity and quantum mechanics, etc. In this case the light ray is red shift at the neighborhood of a gravitational field (positive matter), and is violet shift at the neighborhood of a repulsive field (negative matter),

$$\Delta\lambda / \lambda = -MG / rc^2 \qquad (3.2.4)$$

Of course, light emitted from the negative matter cannot be observed directly. The light ray should have repulsive deflection in a field of the negative matter,

$$\alpha = -4MG / c^2 R \qquad (3.2.5)$$

and a more general deflection should be

$$\alpha = 4G(M_1 - M_2) / c^2 R \qquad (3.2.6)$$

In which M_1 and M_2 are mass of the positive and negative matters , respectively .

For the Kepler laws of planet ,

$$F(r) = -\frac{G}{r^2}(M_1 - M_2), u = \frac{1}{r} \qquad (3.2.7)$$

So $$\frac{d^2u}{d\vartheta^2} + u = \frac{G}{H}(M_1 - M_2) \qquad (3.2.8)$$

$$r = \frac{H/G(M_1 - M_2)}{1 + CH\cos(\vartheta - \vartheta_0)/G(M_1 - M_2)} \qquad (3.2.9)$$

When $\vartheta_0 = 0$, it becomes a quadric curve ,

$$r = \frac{p}{1 + e\cos\vartheta} \qquad (3.2.10)$$

In which $e = Cp = CH/G(M_1 - M_2)$. It is ellipse for $E < 1$ and $M_1 > M_2$.it is hyperbola for $E > 1$ and $M_1 < M_2$: it is parabola for $E = 1$ and $M_1 = M_2$ this is modified Kepler first Law .

The Kepler second law should be invariable.

In the gravitational law:

$$F = -\frac{G}{r^2}m_1 m_2 \qquad (3.2.11)$$

There are two masses, but in the Newton second law $F = ma$ there is only one mass. In order to keep the consistency of natural laws, and a repulsive force between positive and negative matters, we should suppose $-F = -ma$, i.e., $F = ma$ hold always for the negative matter, so that a is still an acceleration in the negative matter, while is always deceleration between the positive and negative matters.

+ In the special relativity the mass increases still. In the four-vector change only is($\pm mv; \pm E/c$)) , the time-like interval is $-v < -c, i\,e, v > c$; the space-like interval is $-v > -c, i\,e, v < c$; both are just opposite. Therefore, the superluminal is in the time-like interval. In the general relativity there is similarly curved time-space.

In the quantum mechanics the negative matter may be still $E = h\nu$, in which $E \to -E$ and $-h \to h$.

Such the de Broglie wave length is positive. The uncertainty principle

$$(\Delta x)^2(\Delta p_x)^2 \geq \hbar^2/4 \qquad (3.2.12)$$

is invariant, but another relation

$$(\Delta x)(-\Delta p_x) \geq -\hbar/2 \qquad (3.2.13)$$

will become probably to

$$(\Delta x)(\Delta p_x) \leq \hbar/2 \qquad (3.2.14)$$

The Heisenberg equation is also invariant, mass becomes an opposite sign in the Schrödinger equation , because the energy-momentum operators are invariant. Such

$$-E = \frac{p^2}{(-2m)} + U(r) \qquad (3.2.15)$$

whose corresponding equation in quantum mechanics is:

$$i\hbar\frac{\partial\psi}{\partial t} = \frac{\hbar^2}{2m}\nabla^2\psi - U(r)\psi \qquad (3.2.16)$$

Here only $U \to -U$. The Klein-Gordon equation and the Dirac equations are invariant. But, an equation in an electromagnetic field is different:

$$\left[-E + e\phi - \alpha(-cp + eA) + \beta\mu c^2\right]\psi = 0 \qquad (3.2.17)$$

Negative matter is possibly a dark matter :-

A unique dependable method determined all mass is to study their gravitation effect, for which the easiest method is the measurement of the circular speed curves in the galaxy [39]. These curves may be measured from the Doppler shift of spectrum [40].

Dark matter self does not emit light, and does also not interact with light. In the negative matter there is the negative photon, which possesses negative energy and negative mass, and is repulsion with general matter, so the negative matter is invisible. The state equation of dark energy is different with the equation of usual matter, and at present assume that it is repulsive force each other. So this may correspond to the negative matter. Moreover, according to the mass-energy relation in Einstein relativity, dark matter and dark energy should be unified in this case.

Recently, Caldwell proposed phantom as cosmological consequences of a dark energy component with super-negative equation of state, whose cosmic energy density has negative pressure [41]. Then phantom becomes an important dark energy model [42-47], where the kinetic energy is negative. Such it must possess negative mass according to classical mechanics or relativity, and is namely the negative matter. Hong, et al., considered a higher dimensional cosmological model with a negative kinetic energy scalar phantom field and a cosmological constant [43]. Scherrer and Sen examined phantom dark energy models produced by a field with a negative kinetic term [44]. Chimento, et al., discussed the dark energy density derived from the 3-scalar phantom field, and its negative component plays the role of the negative part of a classical Dirac field [45]. Gonzalez and Guzman presented the first full nonlinear study of a phantom scalar field accreted into a black hole. Here the analysis includes that the total energy of the space-time is positive or negative [46].

The observations for luminous mass find that the velocity V is approximately constant, for example, in a range of radio $0.5kpcs < R < 20kpcs$ for our Galaxy [48]. This is an important proof of the existence of dark matter, and which exists in the galactic halo. For a galaxy, if the movement of a star round the centre of the galaxy obeys the Kepler law, and the negative matter is introduced, the equation of the star with mass m and distance R to the centre will be

$$\frac{Gm}{R^2}(M_1 - M_2) = \frac{m}{R}V^2 \qquad (3.2.18)$$

The total mass of this galaxy inside radius R is:

$$M(R) = M_1 + M_2 = \int_0^R \rho(r)dV \int_0^R \rho(r)4\pi r^2 dr \qquad (3.2.19)$$

Such $\frac{dM(r)}{dr} = 4\pi r^2 \rho(r)$. The continuity equation is:

$$\frac{\partial\rho}{\partial t} + \nabla(\rho v) = 0 \qquad (3.2.20)$$

In which $\rho = \rho_1 + \rho_2$ is a total density. The Euler equation is:

$$(\rho_1 + \rho_2)\frac{dv}{dt} = (\rho_1 + \rho_2)\left[\frac{\partial v}{\partial t} + (v.\nabla)v\right] = -\nabla(\rho_1 + \rho_2) - (\rho_1 + \rho_2)\nabla\Phi \quad (3.2.21)$$

The cosmological constant corresponds to a fictitious fluid introduced, whose density is

$\rho_\Lambda = \Lambda / 8\pi G$, and pressure is $p_\Lambda = -\Lambda c^2 / 8\pi G$. The mass-to-light ratio connects to $(\rho_1 + \rho_2)/\rho_1 = 1 + \rho_2 / \rho_1$, such more is the negative matter, and bigger is the mass-to-light ratio.

From Eq.(3.2.18), we may obtain a radial velocity

$$V = \sqrt{\frac{G(M_1 - M_2)}{R}}\ , \qquad (3.2.22)$$

Of the star, and an angular velocity

$$\Omega = \sqrt{\frac{G(M_1 - M_2)}{R^3}}\ , \qquad (3.2.23)$$

from the movement equation. Such this measurement determines only difference of positive mass and negative mass, i.e., a breaking part of symmetry between positive and negative matters.

We suppose an isolated particle system with the positive and negative matters under gravitational self interaction, whose kinetic energy:

$$T = T_1 - T_2 = \frac{1}{2}(\sum_i m_i r_i^{\cdot 2} - \sum_j m_j r_j^{\cdot 2}) \qquad (3.2.24)$$

It is simplified to

$$T = \frac{1}{2}(M_1 - M_2) < V^2 >= \frac{3}{2}(M_1 - M_2) < V_{saw}^2 >. \qquad (3.2.25)$$

For an object of spherical symmetry the potential energy is:

$$U = -\frac{G}{R}(M_1 - M_2)^2 = -\frac{G}{R}(M_1^2 + M_2^2 - 2M_1 M_2) \qquad (3.2.26)$$

Applied the virial theorem determined the mass of cluster of galaxies, the sum of kinetic energy T and potential energy U for this system is:

$$2T + U = \frac{1}{2}\frac{d^2}{dt^2}(\sum_i m_i r_i^2 - \sum_j m_j r_j^2) = 0 \qquad (3.2.27)$$

The kinetic energy of entire system in each particle as a galaxy is namely T. By above formula, the mass of this galaxy becomes:

$$M_1 - M_2 = \frac{3R}{G} < V_{saw}^2 > \qquad (3.2.28)$$

Therefore, the existence of the negative matter will derive bigger decrease of mass by this way. For example, assume that the positive matter and negative matter are 55% and 45% of total mass, respectively. We observed mass that is only its 10%. Such the negative matter is possibly an important reason produced an effect of dark matter. Moreover, the negative matter is repulsive force for photon, and negative-photon is also repulsive force for matter, both cannot be observed, and show dark matter.

The field equations of general relativity on the negative matter are:

$$G_{\mu\nu} = 8\pi k (T_{\mu\nu} - T'_{\mu\nu}) \qquad (3.2.29)$$

$$i.e \quad G_{\mu\nu} + 8\pi k T'_{\mu\nu} = 8\pi k T_{\mu\nu}) = G_{\mu\nu} + \Lambda g_{\mu\nu} \qquad (3.2.30)$$

So Λ corresponds to the negative matter. And

$$\Lambda = 8\pi k T_{\mu\nu} / g_{\mu\nu} = \left[\rho' + (p'/c^2)\right](u_\mu u_\nu / g_{\mu\nu}) - p' \qquad (3.2.31)$$

On the other hand, the gravitational field equation with the cosmological constant is extended to:

$$G_{\mu\nu} = 8\pi k T_{\mu\nu} \Rightarrow 8\pi k (T_{\mu\nu} + \Lambda g_{\mu\nu}) \qquad (3.2.32)$$

Here $\Lambda g_{\mu\nu}$ corresponds to the negative energy state and vacuum energy, i.e., Dirac sea. The Friedmann equation is:

$$\ddot{R}(t) = -\frac{4}{3}\pi G\left[(\rho_1 - \rho_2) + 3(p_1 - p_2)/c^2\right]R(t) \qquad (3.2.33)$$

In which $(\rho_1 - \rho_2) + 3(p_1 - p_2)/c^2$ is effective mass density.

$$\dot{R}^2 - \frac{8}{3}\pi G(\rho_1 - \rho_2)R^2 = 2C \qquad (3.2.34)$$

in which $\dot{R}(t_0) = H_0$ is the Hubble constant. The density parameter is:

$$\Omega_0 = \frac{8\pi G \rho_0}{3H_0^2} = \frac{\rho_0}{\rho_c}, \quad (3.2.35)$$

$\rho_0 = (\rho_1 - \rho_2)$ is an observed density. The accelerating expansion of the universe shows

$$(\rho_1 - \rho_2) + 3(p_1 - p_2)/c^2 < 0\,.\, ie \quad \rho_2 + (3p_2/c^2) > \rho_1 + (3p_1/c^2) \quad (3.2.36)$$

The negative matter is more than the positive matter.

For the negative matter there should also have the corresponding black hole, whose radius is:

$$r = -2Gm/c^2 \quad (3.2.37)$$

Various positive matter and black hole exhibit the gravitational lensing effect. While the negative matter and negative black hole will be the repulsive lensing phenomena. Both should be different in observations.

Negative matter and inflationary cosmology

In the standard model of hot big-bang cosmology there are some problems [49,50], for example, the horizon problem, the flatness problem, the antimatter problem, the structure problem and the expansion problem, etc. Such Guth proposed an inflationary universe. In the early history the universe pass through some phase transitions when the latent heat is released. A huge expansion factor results from a period of exponential growth. It is based on the grand unified models (GUM) of particle interactions [49], and the Higgs field [50]. The inflationary universe may explain some problems and the monopole problem. Further, the inflation theory is extended to the chaotic inflation models [51,52]. These inflation theories relate dark matter [38].

Since a Universe must have a zero net value for all conserved quantities, and it must consist equally of matter and anti-matter, Tryon supposed that the Universe is a quantum fluctuations in vacuum [53], and the creation of the cosmos from nothing [53,54].

We propose that under this case the positive matter and negative matter are created at the same time. It is a Planck time, whose time scale is about $10^{-43}s$, and whose length is about $10^{-33}cm$. At this very small space the positive matter and negative matter are repulsive each other, and are the very strong repulsive interaction, whose ratio with the gravitational interaction is $15/10^{-39} \sim 10^{40}$. Therefore, the Universe inflates, which is a phase transformation of the Grand Unified Theories (GUT) at the form of the strong interaction. While an exponential inflation is just a form of the strong interaction:

$$F = -g^2 \frac{e^{-kr}}{r^2} \qquad (3.2.38)$$

Here the positive matter is g, and the negative matter is $-g$, so $F > 0$ is a huge strong repulsive force for the length inside $10^{-13} cm$. When the time is $10^{-34} s$ and the length is bigger than one of the strong interaction, the inflation finishes, and the positive matter and opposite matter are created.

While the force between the positive matter and negative matter will become a usual repulsion. When the space between the positive matter and negative matter are bigger, both will form two regions of topological separation repulsed each other.

Negative matter and Higgs mechanism

The Higgs field is necessary, from this the symmetries are spontaneously broken, and the gauge bosons obtain masses [55,56]. The Higgs field equations are [56]:

$$\nabla_\mu \nabla^\mu \varphi_a + \frac{1}{2}(m_0^2 - f^2 \varphi_b \varphi_b)\varphi_a = 0 \qquad (3.2.39)$$

If $\varphi_b = 0$ or $f = 0$ so $m_0^2 < 0$ for usual field. Such Higgs boson cannot be measured, and so far various experiments do not search any Higgs boson. It is a puzzle that the Higgs bosons form various mesons and baryons whose masses are very small than huge Higgs mass along with the energy increased by those accelerators.

The Lagrangian of two scalar or pseudo-scalar fields without spin is [57]:

$$L = \frac{1}{2}(\partial_\mu \varphi_1)^2 + \frac{1}{2}(\partial_\mu \varphi_2)^2 - \frac{1}{2}\mu^2(\varphi_1^2 + \varphi_2^2) - \frac{\lambda}{4}(\varphi_1^2 + \varphi_2^2)^2 \qquad (3.2.40)$$

It is an ordinary solution for $\mu^2 > 0$. While $\mu^2 < 0$ corresponds to the Goldstone Lagrangian, which has two solutions:

$$|\varphi| = (\varphi_1^2 + \varphi_2^2)^{1/2} = \begin{cases} 0(V_s = 0) \\ V_0 = \sqrt{-\mu^2/\lambda}\ (V_s = -\mu^4/4\lambda). \end{cases} \qquad (3.2.41)$$

The non-ordinary solution obtains a minimal value $V_s - \mu^4/4\lambda$ and a non-zero vacuum expectation value, and the symmetries are spontaneously broken. From this various particles obtain masses. It is namely the Higgs mechanism [55,58].

The Lagrangian is [76]:

$$L_{\varphi,QUAD} = -\frac{1}{2}\sum_n \left[\partial_\mu \varphi_n^1 - i\sum_{m.\alpha}(t_{nm}^\alpha A_{\alpha\mu} v_m)\right]^2 = -\frac{1}{2}\sum_n (\partial_\mu \varphi_n' \partial^\mu \varphi_n') - \frac{1}{2}\sum_{\alpha\beta}(\mu_{\alpha\beta}^2 A_{\alpha\mu} A_\beta^\mu) \quad (3.2.42)$$

Here the mass matrix

$$\mu_{\alpha\beta}^2 \equiv -\sum_{nml} t_{nm}^\alpha t_{nl}^\beta v_m v_l \qquad (3.2.43)$$

It should be $\mu_{\alpha\beta}^2 < 0$. Further, this yields a ghost Lagrangian.

In Higgs field the $m_0^2 < 0$ originate possibly from a product of positive and negative matter $(m)(-m) = -m^2$, but it is not an imaginary particle. This show that when the Higgs field is decomposed, the positive matter is namely particle, and obtain spontaneously mass; the negative matter is namely dark matter. The corresponding Goldstone boson is $(+m) + (-m) = 0$, which is a symmetry. While the Higgs mechanism is spontaneously broken symmetry. If the Higgs bosons are tested in search, it will imply that the positive and negative matters are obtained. Generally, the superluminal and corresponding imaginary mass may be also explained by the same way, i.e., a product of positive mass and negative mass.

The Dirac equations of positive matter are:

$$(\gamma_\mu \partial_\mu + m)\psi = 0. \qquad (3.2.44)$$

The Dirac equations of negative matter are:

$$(\gamma_{\mu}\partial_{\mu} - m)\psi = 0. \qquad (3.2.45)$$

While $$(\gamma_{\mu}\partial_{\mu} + m)(\gamma_{\mu}\partial_{\mu} - m)\psi = (\gamma_{\mu}^{2}\partial_{\mu}^{2} - m^{2})\psi = 0 \qquad (3.2.46)$$

It is the Klein-Gordon equation. For

$$(\gamma_{\mu}\partial_{\mu} + m)(\gamma_{\mu}\partial_{\mu} + m)\psi = (\gamma_{\mu}^{2}\partial_{\mu}^{2} + 2m\gamma_{\mu}\partial_{\mu} + m^{2})\psi = 0. \, (3.2.47)$$

it is replaced by the Dirac equation, then this is still the Klein-Gordon equation. For the Dirac equations of the negative matter the Klein-Gordon equation is obtained yet by the same method. This shows that there is the same Klein-Gordon equation for the positive-positive matter, or positive-negative matter, or negative-negative matter.

For the Higgs equation and the Klein-Gordon equation only the mass has the opposite sign. Such it may be structured from a pair of the Dirac equations of the positive-negative matter. And it is simplified to the Schrödinger equation, only whose mass is an opposite sign.

In a certain extent an adjoint field of the Dirac field is namely the negative matter, whose equations are:

$$\gamma_{\mu}\overline{\psi}\partial_{\mu} - m\overline{\psi} = 0 \qquad (3.2.48)$$

Here $\overline{\psi} = \psi^{+}\gamma_{4}$, and ψ^{+} is an adjoint operation, and is a covariant operation [59]. This seems to imply that the negative matter accompanies the positive matter $\psi\overline{\psi}$. may construct the six types and 16 quantities, in which an antisymmetric tensor is [59]:

$$T_{\mu\nu} = \overline{\psi}\frac{\gamma_{\mu}\gamma_{\nu} - \gamma_{\nu}\gamma_{\mu}}{2i}\psi \qquad (3.2.49)$$

It corresponds perhaps to a repulsive force, and is not a symmetric tensor with 10 components of the gravitational field.

For two components the matrix Dirac equations are [59]:

$$c\sigma_{i}p_{i}\psi_{S} + m_{0}c^{2}\psi_{L} = E\psi_{L} \, , \qquad (3.2.50)$$

$$c\sigma_{i}p_{i}\psi_{L} - m_{0}c^{2}\psi_{S} = E\psi_{S} \, , \qquad (3.2.51)$$

For (3.2.51) there is

$$\psi_S = \frac{c\sigma_i p_i}{E + m_0 c^2}\psi_L \ , \qquad (3.2.52)$$

there are the two types and 8 quantities [59]. At low energy ψ_S is a negative mass and a negative energy. When the negative energy cannot be neglected, we derive the Schrödinger equation and the Pauli equation. From (3.2.50) there is

$$\psi_L = \frac{c\sigma_i p_i}{E - m_0 c^2}\psi_S \ , \qquad (3.2.53)$$

It is replace by Eq.(3.2.51), and obtain:

$$c^2(\sigma_i p_i)^2\psi_S + m_0^2 c^4\psi_S = E^2\psi_S \qquad (3.2.54)$$

Let $E = -m_0 c^2 + E'$, for non-relativity (low energy) $E' << m_0 c^2$, so $E^2 = m_0^2 c^4 - 2m_0 c^2 E'$.

Assume

$$(\sigma_i p_i)^2 = -\hbar^2\Delta \qquad (3.2.55)$$

so
$$-\hbar^2\Delta\psi_S = -2m_0 E'\psi_S \qquad (3.2.56)$$

It is the Schrödinger equation of the negative matter. If

$$(\sigma_i p_i)^2 = -\hbar^2\Delta - (e\hbar / c)\sigma_l H_l \qquad (3.2.57)$$

we will obtain the Pauli equation of the negative matter, whose magnetic pole will be $\mu = -e\hbar / 2m_0 c$. This is the same results in the Schrödinger equation and the Pauli equation replaced directly by the negative mass.

(3-3) **Gravitational Signature of matter – Antimatter Interaction :**

Another work was done by Shawqi and others . They propose existence of anti matter and suggest some techniques to test its existence .

The discovery of the positron, following the theoretical work of Dirac in 1932 [60] [61], marked a new era in our understanding of the world of particles. Since then, a whole range of antiparticles has been discovered. The Standard Model (SM) extends the concept of antiparticles to include all known leptons, mesons, hadrons, and force

carrier bosons. In the last few decades, the scientific community witnessed a growing interest in acquiring knowledge about the constituents of matter and antimatter and their interactions [62].
Toward achieving this goal, immense theoretical work has been conducted and large experimental setups have been constructed to identify the theoretical models that best fit the natural world [63]-[64]. In this quest, researchers tried to answer the following question: could there be an antigravity tensor? In other words, does antigravity exist? Despite dedicated efforts, certain fundamental problems in this regard remained unanswered. One of the most basic questions is the gravitational interaction between matter and antimatter. Attempts to build quantum theories of gravity have led to the emergence, in addition to the gravitational force mediated by spin-2 gravitons, of two more forces mediated by spin-0 and spin-1 partners [65] [66]. The CPT (Charge-Parity-Time reversal) theorem is at the origin of the argument describing the gravitational nature of forces arising from the exchange of graviton mediator particles, and it is almost impossible to construct a theory that violates CPT in flat space- time [67]. However, in curved spacetime, no generalization of the CPT theorem has been unequivocally demonstrated [68] [69], and the validity of the CPT theorem, in this case, is open to question.

Antigravity arising from matter-antimatter repulsion might have deep implications on current scientific theories. One of the key issues in cosmology is why the universe is composed of matter and not antimatter. A repulsive gravitational force between matter and antimatter could be at the core of a plausible theory that answers this question. Goldhaber [70] has speculated that the universe might have separated into galaxies or clusters of galaxies that are entirely composed of matter and others that are entirely composed of antimatter. Alfvén and Klein [71] and Alfvén [72] addressed this issue by proposing a cosmology based on an electromagnetic plasma separation of matter and antimatter assisted by gravitational forces. Furthermore, a repulsive gravitational force between matter and antimatter could explain certain cosmological observations that escape our understanding based on current theories. In this work, we address these issues in the framework of the available observational data, and we investigate the possible extension of certain astrophysical theories. Today, the absence of observational evidence of antimatter at the large scale is viewed as a problem of the early universe.

Over the past few decades, many experiments have been devised to measure the acceleration of antiparticles in a gravitational field. So far, none of these experiments has been successful in settling the problem of the gravitational signature of antiparticles, albeit a noticeable progress has been achieved [73]. Antihydrogen atoms are currently being used by many research groups [74] [75] in an attempt to reach a definite answer to this issue in the coming few years. In this work, we also propose a new space-borne experiment, which is an extension of the current efforts to determine the response of antiparticles to a gravitational field.

The Equivalence Principle of Antiparticles :-

The equivalence principle is a cornerstone of general relativity and all metric theories of gravity. The weak equivalence principle states that the inertial and gravitational masses are equal. Different versions of the principle have been applied to different classes of phenomena [76]. The weak equivalence principle (WEP) is restricted to mechanical quantities, whereas the Einstein equivalence principle (EQP) exhibits more general aspects that involve WEP and all non-gravitational phenomena. The inertial mass (m_i) and the gravitational mass (m_g) of a body in the Earth's gravitational field can be expressed as: $m_i g = m_g G M_E / r^2$, where G is the universal gravitational constant , M_E is the mass of the earth , and r is the distance of the body from the centre of the earth . According to Einstein's formulation of the equivalence principle [77], an observer in free fall has the right to consider his state as being one of rest and his environment as being gravitationally field-free [78].

In the 1800s, Eötvös made some geophysical exploration [79] to test the principle of equivalence using a torsion balance. He compared the relative acceleration of pairs of different materials toward the Earth. Eötvös reported null results with an accuracy of 5×10^{-9} [80] [81] . Subsequently ,Dicke [82] and Braginsky [83] verified the principle of equivalence by measuring the relative acceleration of objects toward the Sun to accuracies of 3×10^{-11} and 0.9×10^{-12}, respectively. Data from laser ranging [84] verified the principle of equivalence for the Moon in comparison to the Earth to an accuracy of 5×10^{-12} [85] . In 1986 , Fischbach *et al* . [86] published their re-analysis of the Eötvös experiment. They correlated the null results of Eötvös with the baryon number per unit mass, and concluded that this evidence was in agreement with the data of Stacey's group on α and λ (see Equation (1)). This work created a sensation when the authors further suggested that this was evidence for a vector force due to hypercharge [86] [87], a "fifth force". This announcement was at the origin of the great controversy.

CP Violation and the CPT Theorem :-

A parity transformation in quantum physics involves the change in sign of one spatial coordinate, where a parity inversion transforms a chiral phenomenon into its mirror image. All the laws of physics are believed to be exactly the same when looked at in a mirror image of the world. Thus, when watching a physics experiment reflected in a mirror, observers believe that they are seeing reality. This is referred to as parity invariance. Three of the fundamental forces of nature, namely—the electromagnetic, the gravitational, and the strong nuclear forces are found to obey parity invariance. The weak force, on the other hand, was found to be an exception, until it was realized that replacing matter with antimatter, also called charge reversal, will establish charge-parity (CP) invariance. Further investigation revealed an anomaly in the decay of the K-meson [88], which was a fingerprint of the violation of the CP invariance principle. Subsequently, other mesons were discovered to violate the CP theorem. CP violation can be removed by introducing time reversal (T) to form a coherent CPT theorem that holds for all physics phenomena [89]-[92]. The implication of CPT symmetry can be visualized as a mirror image of our universe with all matter replaced by antimatter (charge inversion or conjugation), all objects having their position reflected by an imaginary plane (parity inversion), and all momenta reversed (time reversal). CPT implies that particles and antiparticles have the same inertial masses and lifetimes, and equal but opposite charges and magnetic moments. Therefore, one of the important aspects to be considered is the ratios of the inertial masses of e^+/e^- and of p^-/p^+ The difference in masses was found to be: $\left|m(e^+)-m(e^-)\right| < 4\times10^{-8}m(e^-)$ at the 90% confidence level [93] and $\left|m(p^-)-m(p^+)\right| < 4\times10^{-8}m(p^-)$ to one standard deviation [94]. It was shown that CPT violation implies Lorentz symmetry breaking [95]. The overwhelming majority of the experimental work to look for Lorentz violation yielded negative results [67] [96]. However, one has to remember that CPT invariance is not necessarily deeply rooted in the physical world the way, for instance, the conservation of energy is. In fact, Oksak and Todorov [97] and Stoyonov and Todorov [98] have shown that the CPT theory can be violated when non-finite-dimensional representations of the Lorentz group are allowed. Wald [99] has argued that the CPT theorem may face obstacles when applied in curved spacetime. In reality, many models have been proposed that predict a small CPT violation in curved spacetime [100]-[103]. In the next section, the CPT theorem will be shown to govern our understanding of baryogenesis in the early universe.

Baryogensis, Sphalerons, and Cosmic Implications :-

In the universe we observe consists almost entirely of matter. Observational evidence reveals that there are no galaxies in the vast regions of the universe consisting of antimatter [104] [105]. Antimatter is produced only locally in processes involving high energies. This observation has its roots in the early stages of the Big Bang. Baryogenesis refers to the hypothetical processes that led to an asymmetry between baryons and antibaryons in the early universe. As outlined in the previous section, the CPT theorem implies that a particle and its corresponding antiparticle have exactly the same mass and lifetime, and that they exhibit exactly opposite charge. In 1967, Andrei Sakharov proposed a set of conditions to explain the predominance of matter over antimatter in the early universe [50]. The first condition requires a violation of the baryon number to produce an excess of baryons over antibaryons. The second condition involves C-symmetry violation so that the interaction produces more baryons than antibaryons without being reciprocally counter balanced by reversal processes. CP violation is also required because otherwise an equal number of left handed and right handed baryons will be produced. The last condition states that the interaction must be out of thermal equilibrium, since otherwise CPT symmetry would lead to compensation between processes involved in increasing or decreasing the baryon number [107]. In the Standard Model, baryogenesis requires the electroweak symmetry breaking to be a first order phase transition; otherwise, sphalerons will remove any baryon asymmetry that is produced up to the phase transition [108]. This argument concerning the CPT theorem is valid in flat spacetime.

Quantum Gravity Force Mediating Particles :-

In quantum gravity theories, the gravitational force is mediated by the graviton which is a tensor spin-2 particle. In these theories, the spin-2 graviton has spin-0 (graviscalar) and spin-1 (graviphoton) partners [65] [66]. If the masses associated with these new partners are sufficiently small, the force they generate will have enough range to produce measurable effects in the macroscopic world. In field theories, a gaviscalar couples to the trace of the energy-momentum tensor, with the result that the exchange of even-spin bosons, such as the spin-2 graviton, or its spin-0 partner, will always produce attractive forces, whereas the force produced by the exchange of odd-spin particles, such as the spin-1 partner, will produce repulsive forces between like charges and attractive forces between opposite charges [65]. Scherk [110] has emphasized the importance of the vector partner (graviphoton), and has pointed out that it couples only to the CPT-non-self-conjugate degrees of freedom, but not to gauge bosons such as the photon. Goldman et al. [111] noted that these partners of the graviton are necessary for super- symmetry, which is at the core of certain desirable features such as the cancelation of divergences in theories of super gravity [112]. From a phenomenological point of view, the static potential energy between particles and antiparticles is given by [113]

$$V = -\frac{G_\infty m}{r}\left[1+\sum_{k=1}^{n}\alpha_k e^{-r/\lambda_k}\right] \qquad (3.3.1)$$

where the range λ_k of the k^{th} component of the non-Newtonian force is linked to the mass m_k of the mediating antiparticle through the relation

$$\lambda_k = \hbar / m_k c \qquad (3.3.2)$$

Here, $\hbar = \frac{h}{2\pi}$. Where h is the Planck constant , and c is the speed of light. α_k in Equation (3.3.1) is the coupling constant. Equation (3.3.1) predicts more than one spin-0 and spin-1 particles. For convenience, we restrict the gravitational potential to the exchange of one spin-0 and one spin-1 particles only. In this case, Equation (3.3.1) becomes [114]

$$V = -\frac{G_1 G_2}{r}\left[1 \pm a e^{-r/v} + b e^{-r/s}\right] \qquad (3.3.3)$$

where a and b are the moduli of the product of the vector and scalar charges in units of Gm_1m_2 , and v and s are the inverse graviphoton and graviscalar masses(ranges).The ± sign applies to the matter-antimatter interaction .In general, these charges are some linear combination of baryon and lepton numbers, and the force will therefore be composition dependent [115]. The graviphoton and graviscalar are likely to have small masses [116] (large v and s). Macrae and Riegert argued that zero masses are not to be ruled out [117].

The Newtonian limit of gravity has only been tested to a high accuracy at laboratory distance scales, and in the solar system at distances of 106 to 1013 m. At intermediate distances, deviation from the inverse square law has not been excluded [118]. Stacey and coworkers [119] [120] identified certain anomalies when conducting geophysical experiments which are consistent with deviations from Newtonian gravity on length scales between 1 and $10^6\,m$.

They analyzed their data using only one Yukawa term of the gravitation interaction energy between two massive fermionic objects, separated by a distance r,

$$I(r) = -(G_\infty M_1 M_2 / r)\left[1+\alpha\exp(-r/\lambda)\right] \quad (3.3.4)$$

They found an effective repulsive parameter with [119] ,

$$2m \le \lambda \le 10^4\,m \ , \ and \quad \alpha = -0.010 \pm 0.005 \quad (3.3.5)$$

Equation (3.3.5) involves large uncertainties. Nevertheless, an observation of a definite repulsive component is claimed.

Antiproton experiments were originally proposed to investigate the possibility that antimatter could have a different gravitational acceleration than matter [121] [122]. In 1985, Zacho [123] observed that a vector partner would produce an attraction for antiprotons, and would cancel normal gravity for matter if the vector had zero mass. Goldman et al. [115] noted that the gravitational potential given by Equation (3.3.3) would have a very small effect in matter-antimatter interactions. This happens when $a \approx b \ and \ v \approx s$. This situation indicate an proximate symmetry between the two partners. If a difference is to arise from symmetry breaking, or higher-order gravitational corrections, then $a - b \ and \ v - s$ become truly negligible [124]. This demonstrates the need to conduct antimatter-matter experiments. In the past few decades, it was realized that theoretical work on matter-antimatter has to be subjected to accurate experimental tests by taking advantage of the latest technology.

Villata’s Theory :-

Villata argued that antigravity appears as a prediction of general relativity when CPT theory is applied [125]. Villata constructed a new equation by applying discrete operators for charge (C), parity (P), and time reversal (T) to the equation of motion of general relativity for a particle in a gravitational field. The resulting equation becomes. [125])

$$\frac{d^2x^\lambda}{d\tau^2} = \frac{m_{(g)}}{m_{(i)}} \frac{dx^\mu}{d\tau} \Gamma^\lambda_{\mu\nu} \frac{dx^p}{d\tau} \qquad (3.3.6)$$

The new constructed equation becomes (Equation (9) in Villata 2011, Ref. [125])

$$\frac{d^2x^\lambda}{d\tau^2} = -\frac{-m_{(g)}}{m_{(i)}} \frac{dx^\mu}{d\tau} \Gamma^\lambda_{\mu\nu} \frac{dx^p}{d\tau} \qquad (3.3.7)$$

This equation is then interpreted as describing the behaviour of antimatter in the gravitational field of ordinary matter, and it predicts a repulsion of matter and antimatter. Cabbolet [88] criticized the approach of Villata on the ground that quantum physics from which the CPT symmetry is taken and relativity theory are two distinct paradigms in physics that are proven to be incompatible. Therefore, it is at least an epistemologically controversial practice to add a theorem of one paradigm as an additional assumption to the other [126]. Cabbolet argued that the method employed by Villata for the theory’s construction is in itself inadmissible, and that the resulting equation cannot be reconciled with the ontological prepositions of general relativity.

Villata [127] responded to the above comment by noting that the criticisms are of a methodological and an ontological kind that arise from a misinterpretation of some concepts, perhaps due to some lack of clarity or some omissions of details in his original article. Villata provided additional explanations regarding the assumptions and results of the theory.

Matter-Antimatter Interactions: A Cosmic Perspective

Antimatter is produced locally in the universe. One important source of their creation is the appearance of virtual particles where, according to the uncertainty principle, they come into existence for a brief period of time corresponding to their energy.This phenomenon becomes important in the vicinity of the event horizon of black holes. The creation of a particle-antiparticle pair in this region may cause one of the particles to be attracted to the black hole, and thus disappears from our observable universe. The second particle has a probability to remain solitary and may contribute to what is called Hawking radiation [128]. Thus, the quantum vacuum is a source of gravitational mass that depends on the gravitational properties of antimatter. A new idea suggested by Hajdukovic [129] states that the gravitational properties of antimatter determine the properties of the quantum vacuum. The above author conjectured that if antihydrogen falls up, then super massive black holes should be emitters of antineutrinos.

In 2008, the INTEGRAL satellite discovered a giant cloud of antimatter at the centre of the Milky Way Galaxy. The cloud itself is about 10,000 light years across. Several explanations were advanced to explain the origin of the antimatter cloud. It was discovered that X-ray binary star systems are distributed in the same manner. However, it remains a mystery how these X-ray binaries are producing this huge amount of antimatter.

Possible alternative sources include supernovae where, for instance, the radioactive ^{26}Al isotope, produced in some of these supernovae, decays through positron emission and releases a magnesium atom:

$$^{26}Al \rightarrow {}^{26}Mg^{*} + e^{+} \ .$$

If matter and antimatter have different gravitational signatures, then one may speculate that the creation of say an electron-positron pair in the vicinity of the event horizon of the black hole at the centre of our Galaxy could be the cause of this antimatter halo, since the electrons (matter) will be attracted toward the black hole, whereas the positrons will feel a repulsive gravitational field and form the observed halo.

The past two decades have witnessed a revived interest in the gravitational physics of antimatter. A thorough understanding of the gravitational signature of matter-antimatter interactions is essential. Many theories have been proposed to address this issue. Several experiments have also succeeded in shedding some light on this issue. However, the behaviour of antimatter in a gravitational field remains, to a certain extent, elusive. Space-borne experiments, including investigations on the Moon’s surface, might pave the way to a better understanding of this fundamental issue.

(3.4) Antigravity and Anticurvature :

In the work done by M.I.Wanas he showed existence of repulsive gravity or antigravity, together with its attractive side. He fined two pieces of evidences for the existence of antigravity in nature. The first is on the laboratory scale, the COW experiment, and the second is on the cosmic scale, SN type Ia observation. He show how gravity theories can predict antigravity, using a new defined geometric object called Parameterized anticurvature. This shows clearly how Einstein's geometrization philosophy can solve recent gravity problems in a satisfactory and easy way. Also, it may throw some light on the mystery of physical nature of "Dark Energy."

Gravity, as we experience on the Earth's surface and in the solar and similar systems, is associated with an attraction force. Theoretical physicists take this "*fact*" into consideration when constructing gravity theories. Newton has succeeded to quantify this attractive force using his law of universal gravity. Einstein, in the context of his theory of General Relativity (GR) , has interpreted gravity as a geometric property, *space-time curvature*. It has been shown that, using Einstein's point of view, one can interpret more physical phenomena than using Newton's one.

Although GR is the most acceptable theory for gravity, so far, it suffers from several problems, especially those connected with recent observations. None of the existing theories of gravity, including GR, can interpret the results of the following observations, for instance

(1) supernova type Ia observation [130];

(2) the rotation velocities of stars in spiral galaxies [131] ;

(3) pioneer 10, 11 velocity observation, "Pioneer Anomaly" [132] ;

(4) the mass discrepancy in clusters of galaxies [133] .

Such observations indicate that our understanding of gravity is not complete enough. It seems that there is something missing in the theories describing gravity. Such theories should be modified or replaced by others, that take into account the *missing factors*, if any. Many authors have tried to tackle such problems, suggesting different solutions. The most famous candidate used is "*Dark Energy*". [134][135]), an exotic term implying the existence of an unknown force, most likely repulsive.

Assuming that *attraction* is one side of gravity and *repulsion* is its other side, many of the recent gravity problems can be analyzed, understood, and solved. We discuss briefly some experimental and observational pieces of evidence, also theoretical predictions, for the existence of the other side of gravity, the *repulsive* side. This may illuminate the road towards a more satisfactory theory for gravity and a better understanding of this interaction. Pieces of evidences for attractive gravity are popular and do not need any sophisticated equipments to discover. In contrast, lines of evidence for *repulsive* gravity are not so obvious and need sophisticated technology to be explored. In what follows, we are going to discuss, briefly, two of these lines of evidence.

The first evidence is on the very large scale, the cosmic scale, while the second evidence is on the laboratory scale.

The first evidence emerged from the analysis of the results of Supernovae type Ia observation [130] . These observations need space telescopes and equipments and would not have been carried out without the use of such sophisticated space technology. The objects associated with this type of Supernovae are considered to be standard candles, which can be used to measure long distances in the Universe with high accuracy. On the other hand, radial velocities of such objects can be easily obtained as a result of measurements of their red shifts. Knowing distances and velocities, one can get the rate of expansion of our

Universe, using Hubble's relation. It has been shown [130] that the Universe is in a phase, with an accelerating expansion rate. This result is in contradiction with all accepted theories of gravity, including GR (with vanishing cosmological constant). The increasing rate of expansion indicates very clearly that there is a large scale repulsive force driving the expansion of the Universe.

The second evidence comes from a sophisticated experiment which has been suggested and carried out from more than three decades ago, in an Earth's laboratory. This experiment is known in the literature as the "COW" experiment. It has been suggested by Colella, Overhauser, and Werner in 1974 and carried out many times starting from 1975 to

1997 [136-140] . The results of this experiment indicate clearly that there is a real discrepancy with existing theories. Before discussing this discrepancy, we give a simple account on the experiment.

The experiment studies quantum interference of thermal neutrons moving in the Earth's gravitational field. A neutron interferometer is used for this purpose . A beam of thermal neutrons A is split into two beams A_1 , A_2 at the point a. The beam A_1 is reflected at b, while A_2 is reflected at d. The two reflected beams A_1 and A_2 interfere at the point c of the interferometer.

Assuming that the path lengths ab = dc, ad = bc and that the trajectory of the neutrons is affected by the Earth's gravitational potential, a phase difference between the two beams A_1 and A_2 is expected. This is due to the difference in the Earth's gravitational potential affecting the paths ab and dc, (since ab is more close to the Earth's surface than dc) . Using the interference pattern, one can measure the phase difference, and consequently the difference in the Earth's gravitational potential.

The theories used for calculating the phase shift have been quantum mechanics and Newton's theory of gravitation (The Earth's gravitational field is a weak field. Newton's theory of gravitation is a limiting case of GR in the weak field regime. So both theories will give about the same prediction) . It has been found that the experimental results are lower than theoretical predictions by eight parts in one thousand (0.008) , while the sensitivity of Advances in High Energy Physics the interferometer used is one part in one thousand (0.001) . Consequently, there is a real discrepancy between the results of this experiment and theoretical predictions [136] . Now, the results of this experiment show clearly that the Earth gravitational potential measured is different from that predicted by Newton theory of gravity (or by GR) , even in the weak field regime. The absolute value (measured) of this potential is less than the corresponding value predicted by known theories of gravity! One probable interpretation is that there is a repulsive force reducing the value of the potential, predicted by theories that take into consideration attraction only. The above two lines of evidence give a probable indication that there is a repulsive force affecting trajectories of particles, whether long range (photons in the cosmos) or short range (neutrons in the laboratory). Now, we have two possible approaches for interpreting the above evidence. The first is that they can be considered as indicators for the existence of a new interaction, fifth force, different from those given in the introduction. The second is that one, or more, of the four known interactions is not well understood. The first possibility

has been extensively examined . [141,142]. So, let us examine the second one. Weak and strong interactions can be easily ruled out, since they are of very short range (the order of one Fermi) . Also, the electromagnetic interaction can be excluded since the two pieces of evidence considered concern trajectories of electrically neutral particles (photons or neutrons). Thus, we are left with the gravitational interaction only. Deep examination of this interaction may lead to a better understanding of gravity. If we assume that gravity has two sides as mentioned above. The first is the side that is well known on the Earth's surface and in the solar and similar systems, the side connected with attraction. The second is the side connected with repulsion which is not so obvious in the solar system. Then, two important questions emerge as follows.

(i) What geometric object (Assuming the geometrization philosophy (very successful in dealing with gravity) is being applied_ is responsible for repulsion?

(ii) Why repulsion is so small, compared with attraction, in some systems while it is relativity large in others?

In what follows we are going to discuss two theoretical (geometric) features predicting, very naturally, the existence of repulsive gravity. This will give possible answers to the above-mentioned questions and, consequently, a convincing theoretical interpretation for accelerating expansion of the Universe and for the discrepancy in the COW experiment.

In the last decade, many attempts have been done suggesting new theories, or modifying the existing theories, of gravity in order to account for the accelerating expansion of the Universe and the repulsive force driving it. These attempts can be classified into two classes: physical and geometrical. The physical class includes suggestions about the existence of types of peculiar matter, having certain equations of state, filling the Universe (e.g., Chapling gas [143] , phantom [144] , etc.) . The geometrical class comprises geometric suggestions to solve the problem (

e.g., increasing the number of space-time dimensions [145] , increasing the order of the curvature scalar R in the lagrangian , f (R) theories [146] , the use of geometries with nonvanishing torsion [147] , the increase of order of torsion (T) in the lagrangian, f(T) theories [148], etc.) . None of the above-mentioned attempts could explain, satisfactorily, the repulsive features of gravity. If one of these attempts is accepted as an interpretation for accelerating

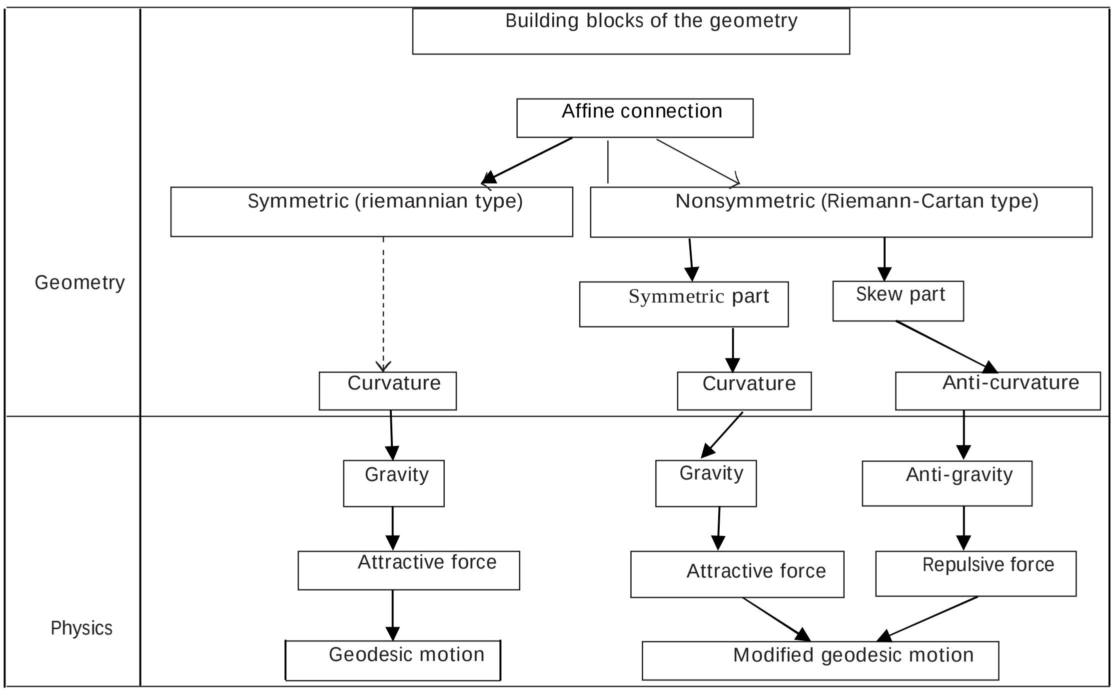


Figure (3.4.1) : Geometrization of physics using geometries of Riemann and of Riemann-Cartan type.

expansion of the Universe, it cannot account for the discrepancy of the COW experiment, discussed above.

In what follows we are going to give a brief account on an attempt, belonging to the geometric class, that can give a convincing interpretation for both large-scale and laboratory scale problems as those given in the above Section. This attempt also gives two geometric properties for the existence of repulsive gravity.

Before reviewing the attempt, we are going to give a brief idea, as simple as possible, about the underlying geometry of this attempt. The geometric structure used is called the “Parameterized Absolute Parallelism” (PAP) geometry [22]. In 4-dimensions, the structure of a PAP space is defined completely by a tetrad vector field. The general linear connection characterizing this space is written as (we are using starred symbol, to characterize an object belonging to the PAP-geometry, while the same symbol, unstarred, is used for AP objects (b = 1))

$$\overset{*}{\Gamma}{}^{\alpha}_{\cdot\,\beta\sigma} = \left\{ {\alpha \atop \beta\sigma} \right\} + b\Upsilon^{\alpha}_{\cdot\,\beta\sigma}, \tag{3.4.1}$$

Where $\left\{ {\alpha \atop \beta\sigma} \right\}$ is the ordinary Christoffel symbol of the Riemannian space (used to construct GR), is a third order nonsymmetric tensor, called contortion, defined in the PAP-space, and b is a dimensionless parameter whose importance will be discussed later. An important feature of the PAP-space is that it is more general than both Riemannian and conventional Absolute Parallelism (AP) spaces in the sense that

(i) for $b = 0$ the PAP-space covers all the Riemannian structure, without any need for a vanishing contortion;

(ii) for $b = 1$ the PAP-space reduces to the conventional AP space.

Among other things, these features facilitate comparison between a theory constructed in the PAP-space and the results of any other theory constructed in the AP space or in the Riemannian one, including GR.6 Advances in High Energy Physics. The antisymmetric part of the parameterized linear connection (3.4.1) is called the torsion $\overset{*}{\Lambda}{}^{\alpha}_{\cdot\,\beta\sigma}$ of the connection:

$$\overset{*}{\Lambda}{}^{\alpha}_{.\beta\sigma'} = b\overset{*}{\Lambda}{}^{\alpha}_{.\beta\sigma'} \tag{3.4.2}$$

Where $\overset{*}{\Lambda}{}^{\alpha}_{.\beta\sigma}$ is the torsion of the AP space. The PAP curvature tensor is defined by the fourth order tensor (20) given by

$$\overset{*}{B}{}^{\alpha}_{\mu\nu\sigma} \overset{def}{=} \overset{*}{R}{}^{\alpha}_{\mu\nu\sigma} + b\overset{*}{Q}{}^{\alpha}_{\mu\nu\sigma} \neq 0 \qquad (3.4.3)$$

Where

$$R^{\alpha}_{\mu\nu\sigma} \overset{def}{=} \left\{ \begin{matrix} \alpha \\ \mu\sigma \end{matrix} \right\}_{,\nu} - \left\{ \begin{matrix} \alpha \\ \mu\nu \end{matrix} \right\}_{,\sigma} + \left\{ \begin{matrix} \alpha \\ \varepsilon\nu \end{matrix} \right\}\left\{ \begin{matrix} \varepsilon \\ \mu\sigma \end{matrix} \right\} - \left\{ \begin{matrix} \alpha \\ \varepsilon\sigma \end{matrix} \right\}\left\{ \begin{matrix} \varepsilon \\ \mu\nu \end{matrix} \right\} \quad (3.4.4)$$

is the Riemann-Christoffel curvature tensor and

$$\overset{*}{Q}{}^{\alpha}_{.\mu\nu\sigma} \overset{def.}{=} \gamma^{\overset{\alpha}{+}}_{.\ \mu\sigma \underset{+-}{|}\nu} - \gamma^{\alpha}_{.\ \mu\nu \underset{+}{|}\sigma} + b[\gamma^{\varepsilon}_{.\ \mu\nu}\gamma^{\alpha}_{.\ \varepsilon\sigma} - \gamma^{\varepsilon}_{.\ \mu\sigma}\gamma^{\alpha}_{.\ \varepsilon\nu}], \qquad (3.4.5)$$

is a tensor of type (1, 3), purely made of $\gamma^{\alpha}_{.\ \beta\sigma}$.

The curvature (3.4.3) is, in general, nonvanishing. Consequently, the PAP-space is of the Riemann-Cartan type, that is having simultaneously non-vanishing torsion (3.4.2) and curvature (3.4.3) .

Now, the two geometric properties predicting the existence of antigravity, and consequent repulsive force, are given below.

(1) Since the PAP geometry covers, at least the domains of both Riemannian and AP geometries as limiting cases, we are going to use the advantages and properties of these two limiting cases to discuss relation (3.4.3) . The tensor $R^{\alpha}_{.\beta\sigma\delta}$, in Riemannian geometry, measures the curvature of the space, that is, the deviation of the space from being flat. It is completely made of Christoffel symbols. Its vanishing is a necessary and sufficient condition for the space to be

flat. Einstein's idea has been to use this tensor as a measure of the gravitational field of the system.

Let us now examine the curvature in the case of the AP space. It can be written, using (3.3) with $b = 1$, as

$$B^{\alpha}_{\beta\sigma\delta} = R^{\alpha}_{.\beta\sigma\delta} + Q^{\alpha}_{.\beta\sigma\delta} = 0 \qquad (3.6)$$

where $Q^{\alpha}_{.\beta\sigma\delta}$ is the limiting case of $\overset{*}{Q}{}^{\alpha}_{.\beta\sigma\delta}$ for $b = 1$. It is to be considered that neither $R^{\alpha}_{.\beta\sigma\delta}$ nor $Q^{\alpha}_{.\beta\sigma\delta}$ vanishes, while their sum, that is, the curvature $B^{\alpha}_{.\beta\sigma\delta}$ of the AP space, vanishes identically. This implies an interesting property, that is, the non-vanishing tensors $R^{\alpha}_{.\beta\sigma\delta}$ and $B^{\alpha}_{.\beta\sigma\delta}$ compensate (balance) each other in such a way that the total curvature of the space is zero. This compensation gives rise to the flatness of the AP space. Now, as $R^{\alpha}_{.\beta\sigma\delta}$ measures the curvature of the space, $Q^{\alpha}_{.\beta\sigma\delta}$ represents the additive inverse of this curvature. For this reason, we call $Q^{\alpha}_{.\beta\sigma\delta}$ the "anticurvature" tensor [150] , and consequently the tensor defined by (3.4.5) is the "parameterized anticurvature" tensor.

An interesting physical result can now be obtained. Einstein has used curvature $R^{\alpha}_{.\beta\sigma\delta}$ as a geometric object representing gravity, in his theory of GR. Similarly, we can use the anticurvature $Q^{\alpha}_{.\beta\sigma\delta}$ as a geometric object representing antigravity, in any suggested theory.

But, the complete balance between $R^{\alpha}_{.\beta\sigma\delta}$ and $Q^{\alpha}_{.\beta\sigma\delta}$ gives rise to a flat space, that is, balance of the two (basic features) sides of gravity. Observationally, this is not the case, at least in the solar and similar systems, in which gravity dominates over antigravity, that is, curvature dominates over anticurvature. Thus, one needs a certain parameter, to be adjusted, in order to fine tune the ratio between the curvature and anticurvature in any theory dealing with both sides of gravity. This is ready and clarifies the importance of the parameter b, which appears in (3.4.5) , in the PAP-geometry. The presence of this parameter in (3.4.3) makes the PAP curvature non-vanishing (in general $b \neq 1$) .

This represents the first geometric feature which shows how antigravity can be predicted in the context of the geometrization philosophy. It gives the first geometric feature predicting the existence of antigravity, on theoretical basis.

(2) In the context of the geometrization philosophy, path equations in any appropriate geometry are used to represent trajectories of test particles. For example, the geodesic equation, of Riemannian geometry, is used as equation of motion of a test particle (e.g., planet) in the solar system, in the context of GR. Now, for the PAP-geometry, the path equation can be written in the following form [145]:

$$\frac{d^2x^\mu}{d\tau^2}+\begin{Bmatrix}\mu\\ \alpha\beta\end{Bmatrix}\frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau}=-b\Lambda_{(\alpha\beta)}{}^{\mu}\frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau'} \quad (3.4.7)$$

where τ is the parameter characterizing the path. If $b=0$, (3.4.7) reduces to the ordinary geodesic of Riemannian geometry. Equation (3.4.7) can be considered as a geodesic equation modified by a torsion term. For any field theory written in the PAP-geometry, (3.4.7) can be used as an equation of motion of a neutral test particle moving in the field, in the domain of this theory.

In order to understand (3.4.7) , physically, let us analyze it using Newton's terminology. The first term of (3.4.7) can be considered as the generalized acceleration of a test particle. The other two terms can be viewed as representing two forces driving the motion of this test particle. On one hand, the first force is related to the Christoffel symbols

$\left\{ \begin{matrix} \alpha \\ \beta\sigma \end{matrix} \right\}$ which is the only geometric object forming the curvature $R^{\alpha}_{.\beta\sigma\delta}$.

This force is the gravity force, since it is connected with the curvature of the space time. On the other hand, the second force is connected with the torsion (or contortion (There are some relations between torsion and contortion [151] in such a way that the vanishing of one is a necessary and sufficient condition for the vanishing of the other. So, in principle, any function of the contortion can be easily written in terms of the torsion and vice versa.)) of space-time. Since the contortion (or torsion) is the only ingredient forming the anticurvature $Q^{\alpha}_{.\beta\sigma\delta'}$, then by similarity, we can call this force the “antigravity force”. So, this equation can be written in the following block equation:

Acceleration Gravity force + Antigravity force. (3.4.8)

Consequently, a complete balance between gravity and antigravity forces would result in the vanishing of acceleration.

In order to explore the quantitative nature of these two forces, let us examine the consequences of linearizing (3.4.7) . It has been shown [152] that the potential ϕ resulting from the existence of the two forces (two sides of gravity) is given by

$$\phi = \phi_N - b\phi_N \qquad (3.4.9)$$

The first term, on the R.H.S., is the Newtonian potential due to gravity and the second term is the potential due to antigravity, written in terms of ϕ_N , for simplicity. Recalling the classical relation between potential and force, and knowing that $b \geq 0$ appear in due course as will, then we can easily conclude that as gravity force is attractive, antigravity force is necessarily repulsive .

Now, we come to the dimensionless parameter b. As stated above, the function of this parameter is to adjust a certain ratio between gravity and antigravity (i.e., between attraction and repulsion) in a certain system. This parameter can be decomposed as follows [150]:

$$b = \frac{n}{2}\alpha\gamma = \frac{antigravity}{gravity}, \qquad (3.4.10)$$

where n is a natural number taking the values 0, 1, 2, . . . for particles with quantum spin 0, 1/2, 1, . . ., respectively; α is the fine structure constant (~1/137) and γ is a dimensionless parameter depending on the size of the system under consideration, to be fixed by experiment or observation. The vanishing of b switches off antigravity in any system and reduces any suggested theory, constructed in the PAP-geometry, to a conventional gravity theory (e.g., orthodox GR) . Also, in this case (3.4.7) reduces to the geodesic equation, in which attraction is the only force affecting the trajectory of any test particle.

The discussion given above represents the second geometric feature which shows the quantity and quality of the repulsive force predicted in the context of the geometrization philosophy.

From the discussion given in (1) and (2) , one can outlines the main features of a geometric theory predicting and dealing with the two sides of gravity. Applying such theory, a satisfactory interpretation can be achieved to the discrepancy in the COW experiment [153] and for the accelerating expansion of the Universe [154] . The values of the parameter γ are found to be of order unity for the Earth's system and which more greater than unity for the Universe.

For the two questions raised at the end of the previous Section, we have now the following answers.

(1) For the first question: the geometric object responsible for repulsion is the anticurvature tensor (or the torsion tensor) of the background geometry characterizing the field.

(2) For the second question: the strength of the repulsive force depends on the value of the parameter γ which characterizes the size of the system under consideration.

Concluding Remarks

1) Two main philosophies are used in the 20th century, to solve the emerged physical problems, quantization and geometrization .
2) Gravity problems are successfully solved using the geometrization philosophy, not the quantization one.
3) The two main objects characterizing any geometry are curvature and torsion (giving rise to anticurvature) .
4) Einstein has used the curvature in constructing the theory of General Relativity, solving the problems of attractive gravity in the Solar and comparative systems, in the context of the geometrization philosophy.
5) Some experiments and recent observations give strong evidence for the existence of repulsive gravity together with the attractive one.
6) In the present section it is shown that using both curvature and the parameterized anticurvature, one can account for both sides of gravity, attraction and repulsion, and consequently give a satisfactory interpretation for accelerating expansion of the Universe and the discrepancy of COW experiment.

7) The modified geodesic, in its linearized form, shows clearly that the force resulting from the torsion term is a repulsive force.

(3.5) The Necessity of Unifying Gravitation and Electromagnetism and the Mass-Charge Repulsive Force :-

In the work of C.Y.Lo curved space time is used to predict anti gravity

General relativity makes it explicit that the gravity generated by mass and that by the electromagnetic energy are different, as shown by the existence of repulsive effect in the Riessner-Nordstrom metric [159-161],

$$ds^2 = \left(1 - \frac{2M}{r} + \frac{q^2}{r^2}\right)dt^2 - \left(1 - \frac{2M}{r} + \frac{q^2}{r^2}\right)^{-1} dr^2 - r^2 d\Omega^2 \qquad (3.5.1)$$

where q and M are the charge and mass of a particle and r is the radial distance (in terms of the Euclidean-like structure 1) [157]) from the particle center. In this metric (1), the gravitational components generated by electricity have not only a very different radial coordinate dependence but a different sign that makes it a new repulsive gravity in general relativity.

This means that the effective mass in metric (1) is

$$M - \frac{q^2}{2r} \qquad (3.5.2)$$

(in the units, the light speed $c = 1$) since the total electric energy outside a sphere of radius r is $q^2/2r$, and thus (3.5.2) could be interpreted as supporting $m = E/c^2$ at least for electromagnetic energy. However, such a view overlooked a simple fact that the gravitational forces would not be the same. From metric (1), the gravitational force is different from the force created by the “effective mass” $M - q^2/2r$ because

$$-\frac{1}{2}\frac{\partial}{\partial r}\left(1 - \frac{2M}{r} + \frac{q^2}{r^2}\right) = \left(\frac{M}{r^2} - \frac{q^2}{r^3}\right) > -\frac{1}{r^2}\left(M - \frac{q^2}{2r}\right) \qquad (3.5.3)$$

They achieved only exposing further an inadequate understanding in the theory of relativity [159,160].

To show the repulsive effect, one needs to consider only g_{tt} in metric (1). According the geodesic equation [161],

$$\frac{d^2x^\mu}{ds^2} + \Gamma^\mu{}_{\alpha\beta}\frac{dx^\mu}{ds}\frac{dx^\nu}{ds} = 0 \qquad where \qquad ds^2 = g_{\mu\nu}dx^\mu dx^\nu \quad (3.5.4)$$

And

$$\Gamma^\mu{}_{\alpha\beta} = (\partial_\alpha g_{\nu\beta} + \partial_\beta g_{\nu\alpha} - \partial_\nu g_{\alpha\beta})g^{\mu\nu}/2 \qquad (3.5.5)$$

are defined by the space-time metric $g_{\nu\alpha}$. However, we need to consider only the case $dx/ds = dy/ds = dz/ds = 0$. Thus,

$$\frac{d^2x^\mu}{ds^2} = -\Gamma^\mu{}_{tt}\frac{dct}{ds}\frac{dct}{ds}, \quad where \quad -\Gamma^\mu{}_{tt} = -\frac{1}{2}(2\frac{\partial g_{t\nu}}{\partial ct} - \frac{\partial g_{tt}}{\partial x^\nu})g^{\mu\nu} = \frac{1}{2}\frac{\partial g_{tt}}{\partial x^\nu}g^{\mu\nu} \quad (3.5.6)$$

Since $g_{\mu\nu}$ is static. (One need not worry whether the gauge of the Reissner-Nordstrom metric is physically valid since the gauge affects only the second order approximation of g_{tt} [161].) For a particle p with mass m at r, the force on p is

$$-m\frac{M}{r^2} + m\frac{q^2}{r^3} \qquad (3.5.7)$$

in the first order approximation since $g^{rr} \cong -1$. Thus, the first term is attractive, but the second term is repulsive.

If the particles are at rest, then the force acts on the charged particle Q has the same magnitude

$$(-m\frac{M}{r^2} + m\frac{q^2}{r^3})\hat{r}, \quad \text{where} \quad \hat{r} \text{ is a unit vector} \quad (3.5.8)$$

since the action and reaction force is equal and in the opposite direction. However, if one considers the motion of the charged particle with mass M and calculates the metric according to the particle p of mass m, only the first term is obtained; but the second term is absent. Thus, the geodesic equation is inadequate for the equation of motion of the inverse problem of the Reissner-Nordstrom metric. Moreover, since the second term is proportional to p^2, it is not a Lorentz force either. In other words, it is necessary to have a repulsive force with the coupling p^2 to the charged particle Q in a gravitational field generated by masses. In conclusion, force (3.5.8) to particle Q is beyond current gravity + electromagnetism. Thus, unification is forced upon, not just desired.

$$\frac{d}{d\tau}\left(g_{5k}\frac{dx^k}{d\tau}+\frac{1}{2}g_{55}\frac{q}{KMc^2}\right)=\Gamma_{k,55}\frac{q}{KMc^2}\frac{dx^k}{d\tau}+\frac{1}{2}\frac{\partial g_{kl}}{\partial x^5}\frac{\partial x'}{\partial\tau}\frac{\partial x^k}{\partial\tau} \quad (3.5.9)$$

One may ask what the physical meaning of the fifth dimension is. Note that although the string theorists talk about space of much higher dimensional, they have no physical reason except for mathematical validity of their speculation. They claimed that those dimensions are curl up. Our position is that the physical meaning the fifth dimension is not yet very clear [1].The fifth dimension is assumed [155] as part of the physical reality, and the metric signature is (+,-,-,-,-). However, our approach is to find out the full physical meaning of the fifth dimension as our understanding get deeper. Unlike mathematics, in Physics things are not defined right at the beginning. For example, it takes us a long time to understand the physical meaning of energy-momentum conservation.

For a static case, it follows from (3.5.9) and (3.5.8), we have the forces on the charged particle Q in the ρ-direction

$$-\frac{mM}{\rho^2}\approx\frac{Mc^2}{2}\frac{\partial g_{tt}}{\partial\rho}\frac{dct}{d\tau}\frac{dct}{d\tau}g^{\rho\rho}, and \quad \frac{mq^2}{\rho^3}\approx-\Gamma_{\rho,55}\frac{1}{K^2}\frac{q^2}{Mc^2}g^{\rho\rho} \quad (3.5.10)$$

And

$$\Gamma_{k,55}\frac{q}{KMc^2}\frac{dx^k}{d\tau}=0 \text{ where } \Gamma_{k,55}\equiv\frac{\partial g_{i5}}{dx^5}-\frac{1}{2}\frac{\partial g_{55}}{\partial x^k}=-\frac{1}{2}\frac{\partial g_{55}}{\partial x^k} \quad (3.5.11)$$

In the (-r)-direction. The meaning of (3.5.10) is the energy momentum conservation. It is interesting that the same force would come from different type of metric element depending on the test particle used. Thus,

$$g_{tt}=1-\frac{2m}{\rho c^2}, \quad and \quad g_{55}=\frac{mMc^2}{\rho^2}K^2+\text{constant} \qquad (3.5.12)$$

In other words, g_{55} is a repulsive potential plus a constant. Since g_{55} depends on M, it is a function of local property, and thus is difficult to calculate. This is different from the metric element g_{tt} that depends on a distant source of mass m.

On the other hand, since g_{55} is independent of q, but $[\partial g_{55}/\partial\rho]/M$ depends only on the distant source with mass m. Thus, although this force acts on a charged particle, this force will penetrate electromagnetic screening since it is independent of an electromagnetic field. This would be a rather unique characteristic that makes such a force easier to be identified. From (3.5.12), it is possible that a charge-mass repulsive potential would exist for a metric based on the mass M of the charged particle Q. However, since p is neutral, there is no charge-mass repulsion force (from $\Gamma_{k,55}$) on p.

Experimental Verification of Mass-Charge Repulsive Force :-

The repulsive force in (3.4.6) can be detected with a neutral mass. To see the effect of repulsive gravity, one must have

$$\frac{1}{2}\frac{\partial}{\partial r}\left(1-\frac{2M}{r}+\frac{q^2}{r^2}\right)=\frac{M}{r^2}-\frac{q^2}{r^3}<0 \qquad (3.5.13)$$

Thus, repulsive gravity would be observed at

$$r<\frac{q^2}{M} \qquad (3.5.14)$$

Thus, for the electron the repulsive gravity would exist only inside the classical electron radius $r_0(=2.817\times10^{-13}cm)$. This would be a very difficult to test a single charged particle even if such a situation could be possible.

However, the existence of repulsive gravity can actually be verified with a charged metal ball. (The test object should be an isolator such as glass and china. This will greatly reduce the static electrical effect.) The reason is that the attractive effect in gravity is proportional to mass related to the number of electrons, but the repulsive effect in gravity is proportional to square of charge related to the square of the number of electrons. Thus, when the electrons are numerous enough accumulated in a metal ball, the effect of repulsive gravity will be shown in a macroscopic distance.

Now, consider the charge q and mass M is consist of N electrons, i.e.,

$$q = Ne, \quad M = Nm + M_0 \qquad (3.5.15)$$

where M_0 is the mass of the metal ball, m and e are the mass and charge of an electron. To have sufficient electrons, from equations. (3.5.9),(3.5.10) and (3.5.11), the necessary condition is

$$N > \frac{r}{r_0}, \quad where \quad r_0 = \frac{e^2}{mc^2} = 2.817 \times 10^{-13} cm \quad (3.5.16)$$

For example, if $r = 10cm$, then it requires $N > 3.550 \times 10^{13}$. Thus $q = 5.683 \times 10^{-7} coulomb$. Then, one will see the attractive and repulsive additional forces change hands. However, this experiment is difficult since, just like measuring the other small effects of general relativity.

Similarly, the mass-to-charge repulsive force in (3.5.8) can be detected with a charge particle. However, since the repulsive force is very small, the interference of electricity would be comparatively large. Thus, it would be necessary to screen the electromagnetic effects out. The modern capacitor is such a piece of simple equipment that can do this screening. When a capacitor is charged, it separates the electron from the atomic nucleus. Since the repulsive force is proportional to charge square, both the positive charge and the negative charge would receive a repulsive force.

Moreover, there is no change of mass, since there is no change on the number of charged particles (for the ideal case). Thus, the net result is that the capacitor would have less weight after being charged. One should observe also nonlinear effects towards charges. This simple experiment would confirm the mass-charge repulsive force, and thus the unification in term of a five-dimensional theory.

One may ask whether the force F_{cm} of charge to mass repulsion and the force F_{cm} of mass to charge repulsion are the same kind of force. Since they have different origin; according to Einstein's equation, the repulsive term in g_{tt} is due to the electromagnetic energy, where the term in g_{55} is due to mass alone. Since the electromagnetic energy is subjected to electromagnetic screening, the force F_{cm} would also be subjected to screening although the force F_{mc} would not.

It should be pointed out that the screening effect to the force F_{cm} is only a result of the current four-dimensional theory.

From the viewpoint of the five-dimensional theory, the charge creates an independent field to react with the mass. To test this, one should observe whether there is a repulsive force from a charged capacitor to a mass particle. For instance, one can have a large spherical capacitor to do the testing. From the view point of five-dimensional theory, an additional repulsive force on the test mass would be observed after sufficiently charging up.

In other words, the charge-mass repulsive force mq^2 / r^3 is a prediction of the five-dimensional theory and is independent of the four known forces. It should be noted also that in electrodynamics the term $-\Gamma_{k,55}(dx^5 / d\tau)^2$ is also necessary because it has been shown in 1981 that the terms $\partial g_{5k} / \partial x^5$ are related to the radiation reaction force [155]. The period from 1981-2006 shows that it is essentially the bias of the physicists that makes this separation so long.

Gravitation was generally considered as producing attractive force only, and all the coupling constants were assumed to have the same sign. Recently, it is proven that for the gravitational radiation of binary pulsars the coupling constants must have different signs [157,158]. Now it is shown that even the electromagnetic energy would produce repulsive forces. Thus, it is clear that the physical picture provided by Newton is just too simply for a phenomenon as complicated as gravity that relates to everything. Also, the notion of black holes is just a simple continuation of attractive gravity. Einstein is really a genius and the full meaning of general relativity is still emerging after 100 years of its creation.

(3.6) Theory of Dark Energy and Dark Matter :

Dark energy and dark matter phenomena :-

Dark matter and Rubin rotational curve. In astrophysics, dark matter is an unknown form of matter, which appears only participating in gravitational interaction, but does not emit nor absorb electromagnetic radiations. Dark matter was first postulated in 1932 by Holland astronomer Jan Oort, who noted that the orbital velocities of stars in the Milky Way don't match their measured masses.

Namely, the orbital velocity v and the gravity should satisfy the equilibrium relation

$$\frac{v^2}{r} = \frac{M_r G}{r^2} \qquad (3.6.1)$$

Where M_r is the total mass in the ball B_r with radius r. But the observed mass M_0 was less than the theoretic mass M_r, and the difference $M_r - M_0$ was explained as the presence of dark matter. The phenomenon was also discovered by Fritz Zwicky in 1933 for the missing mass in the orbital velocities of galaxies in clusters. Subsequently, other observations have manifested the existence of dark matter in the Universe, including the rotational velocities of galaxies, gravitational lensing, and the temperature distribution of hot gaseous.

A strong support to the existence of dark matter is the Rubin rotational curves for galactic rotational velocity. The rotational curve of a galaxy is the rotational velocity of visible stars or gases in the galaxy on their radial distance from the center of the galaxy.

The Rubin rotational curve amounts to saying that most stars in spiral galaxies orbit at roughly the same speed. If a galaxy had a mass distribution as the observed distribution of visible astronomical objects, the rotational velocity would decrease at large distances. Hence, the Rubin curve demonstrates the existence of additional gravitational effect to the gravity by the visible matter in the galaxy.

More precisely, the orbital velocity $v(r)$ of the stars located at radius r from the center of galaxies is almost a constant:

$v(r) \cong$ a constant for a given galaxy, (3.6.2)

PID cosmological model and dark energy :-

We have shown in [165] that both dark matter and dark energy are a property of gravity. Dark matter and dark energy are reflected in a) the large scale space curved structure of the Universe caused by gravity, and b) the gravitational attracting and repelling aspects of gravity.

PID cosmological model :-

The metric of a homogeneous spherical universe is of the form

$$ds^2 = -c^2 dt^2 + R^2 \left[\frac{dr^2}{1-r^2} + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2) \right] \qquad (3.6.3)$$

where $R = R(t)$ is the cosmic radius. The PID induced gravitational field equations are given by

$$R_{\mu\nu} = -\frac{8\pi G}{c^4}(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T) - (\nabla_{\mu\nu}\phi - \frac{1}{2} g_{\mu\nu}\Phi) \qquad (3.6.4)$$

where $\Phi = g^{\alpha\beta} D_{\alpha\beta}\phi$, and ϕ depends only on t.

The nonzero components of $R_{\mu\nu}$ read as

$$R_{00} = \frac{3}{c^2}\frac{1}{R} R_{tt,}$$

$$R_{kk} = -\frac{1}{c^2 R^2} g_{kk} (RR''_{tt} + 2R_t^2 + 2c^2) \qquad \textit{for } 1 \le k \le 3,$$

and by $T_{\mu\nu} = diag(c^2 p, g_{11} p, g_{22} p, g_{33} p)$, we have

$$\begin{aligned}
T_{00} - \frac{1}{2} g_{00} T &= \frac{c^2}{2}(\rho + \frac{3p}{c^2}), \\
T_{kk} - \frac{1}{2} g_{kk} T &= \frac{c^2}{2} g_{kk} (\rho - \frac{p}{c^2}), \quad \textit{for } 1 \le k \le 3, \\
\phi_{00} - \frac{1}{2} g_{00}\Phi &= \frac{1}{2c^2}(\phi_{tt} - \frac{3R_t}{R}\phi_t), \\
\phi_{kk} - \frac{1}{2} g_{kk}\Phi &= \frac{1}{2c^2} g_{kk} (\phi_{tt} + \frac{R_t}{R}\phi_t) \quad \textit{for } 1 \le k \le 3.
\end{aligned}$$

These equations lead to the solution

$$\begin{aligned}
\varphi &= -8\pi G\left(\rho + \frac{3p}{c^2} \right), \\
p &= -\frac{c^4}{8\pi G R^2}
\end{aligned} \qquad (3.6.5)$$

It is natural to postulate that the equation of state is linear. Hence, one can use the relation

$$p = \frac{c^2}{G}(\alpha_1\varphi - \alpha_2 G\rho) \qquad (3.6.6)$$

where α_1 and α_2 are non dimensional parameter, which will be determined by the observed data.

Then equations (3.6.5) and (3.6.6) are the PID cosmological model, where the cosmological significant of R, p, φ, ρ are as follows:

R the cosmic radius (of the 3D spherical universe),

p the negative pressure, generated by the repulsive aspect of gravity,

φ represents the dual gravitational potential, (3.6.7)

ρ the cosmic density, given by $\frac{3M}{4\pi R^3} = \frac{M_{total}}{\pi^2 R^3}$,

Where M and M_{total} total are the observed and total mass respectively .

Theory of dark energy :-

In the static cosmology, dark energy is defined in the following manner. Let E_{ob} be the observed energy, and R be the cosmic radius. We define the observable mass and the total mass as follows:

$$M_{0b} = \frac{E_{0b}}{c^2} \qquad (3.6.8)$$

$$M_T = \frac{Rc^2}{2G} \qquad (3.6.9)$$

If $M_T > M_{0b}$, then the difference

$$\Delta E = E_T - E_{0b} \qquad (3.6.10)$$

is called the dark energy.

The CMB measurement and the WMAP analysis indicate that the difference ΔE in (3.6.10) is positive,

$$\Delta E > 0$$

which is considered as another evidence for the presence of dark energy.

From the PID cosmological model (3.6.5)-(3.6.7), we see that the dark energy DE in (3.6.10) is essentially due to the dual gravitational potential j. In fact, we infer from (3.13) that

$$\begin{aligned} &\varphi = 0 \quad \Leftrightarrow \quad R = 2M_{0b}G/c^2 \qquad (i.e.\ \Delta E = 0) \\ &\varphi > 0 \quad \Leftrightarrow \quad \Delta E > 0 \end{aligned} \qquad (3.6.11)$$

Hence, dark energy is generated by the dual gravitational field. This fact is reflected in the PID gravitational force formula derived in sections hereafter.

PID gravitational interaction formula :-

Consider a central gravitational field generated by a ball B_{r0} with radius r_0 and mass M. It is known that the metric of the central field at $r > r_0$ can be written in the form

$$ds^2 = -e^u c^2 dt^2 + e^v dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2), \qquad (3.6.12)$$

and $u = u(r), v = v(r)$.

In the exterior of B_{r0}, the energy-momentum is zero, i.e.

$$T_{\mu\nu} = 0, \ \ for\ r > r_0$$

Hence, the PID gravitational field equation for the metric (3.6.12) is given by

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\nabla_{\mu\nu}\phi, \quad r > r_0 \qquad (3.6.13)$$

where $\phi = \phi(r)$ is a scalar function of r.

It is known that the interaction force F is given by

$$F = -m\nabla\psi, \ \ \psi = \frac{c^2}{2}(e^u - 1).$$

Then, it follows that

$$F=\frac{mc^2}{2}e^u\left[-\frac{1}{1-\frac{r}{2}\phi'}\frac{1}{r}(e^v-1)-\frac{r\phi''}{1-(\frac{r}{2}\phi')^2}\right] \qquad (3.6.14)$$

The formula (3.6.14) provides the precise gravitational interaction force exerted on an object with mass m in a spherically symmetric gravitation field.

In classical physics, the field functions u and v in (3.6.14) are taken by the Schwarzschild solution:

$$e^u=1-\frac{2GM}{c^2r}, \quad e^v=\left(1-\frac{2GM}{c^2r}\right)^{-1}, \qquad (3.6.15)$$

and $\phi'=\phi''=0,$ which leads to the Newton gravitation.

However, due to the presence of dark matter and dark energy, the field functions u,v,ϕ in (3.6.14) should be an approximation of the Schwarzschild solution (3.6.15). Hence we have

$$|r\phi'|<<1 \quad for \ r>r_0 \qquad (3.6.16)$$

Under the condition (3.6.16), formula (3.6.14) can be approximatively expressed as

$$F=\frac{mc^2}{2}e^u\left[-\frac{1}{r}(e^v-1)-r\phi''\right] \qquad (3.6.17)$$

Simplified gravitational interaction formula :-

We have shown that all four fundamental interactions are layered. Namely, each interaction has distinct attracting and repelling behaviors in different scales and levels. The dark matter and dark energy represent the layered property of gravity.

In this section ,we simplify the gravitational formula (3.6.15) to clearly exhibit the layered phenomena of gravity.

In (3.6.15) the field functions u and v can be approximatively replaced by the Schwarzschild solution . Since $2MG/c^2r$ is very small for $r>r_0$, the formula (3.6.15) can be expressed as

$$F = mMG\left[-\frac{1}{r^2} - \frac{r}{\delta r_0}\phi''\right], \quad r > r_0 \qquad (3.6.18)$$

By the field equation (3.6.13), we have

$$R = \Phi \quad for\ r > r_0 \qquad (3.6.19)$$

where R is the scalar curvature, and

$$\Phi = g^{\mu\nu} D_{\mu\nu}\phi = e^{-v}\left[-\phi'' + \frac{1}{2}(u' - v')\phi' + \frac{2}{r}\phi'\right]$$

In view of (3.6.19), we obtain that

$$\phi'' = -e^{v} R + \frac{2}{r}\phi' + \frac{1}{2}(u' - v')\phi'$$

Again by the Schwarzschild approximation, we have

$$\phi'' = \left(\frac{2}{r} + \frac{\delta r_0}{r^2}\right)\phi' - R \qquad (3.6.20)$$

Integrating (3.6.20) and omitting $e^{\pm\delta r_{0/r}}$, we derive that

$$\phi' = -r^2\left[\varepsilon + \int r^{-2} R dr\right],$$

where ε is a constant. Thus (3.6.18) can be rewritten as

$$F = mMG\left[-\frac{1}{r^2} - \frac{r}{\delta r_0}R + \left(1 + \frac{2r}{\delta r_0}\right)\left(\varepsilon r + r\int \frac{R}{r^2}dr\right)\right] \qquad (3.6.21)$$

We see that

$$u'(r) = \frac{1}{r^2}\sum_{k=0}^{\infty} a_k (r - r_0)^k$$

The gravitational force F takes the following form

$$F = \frac{1}{r^2}\sum_{k=0}^{\infty} b_k r^k\ ,\ b_0 = -mMG$$

In view of (3.6.21), it implies that R can be expanded as

$$R = \frac{\varepsilon_0}{r} - \varepsilon_1 + 0(r)$$

Where ε_0 and ε_1 are two to-be-determined , Since they are free parameters. Inserting R into (3.6.21) we obtain that

$$F = mMG\left[-\frac{1}{r^2} - \frac{k_0}{r} + \varphi(r)\right] \quad for \quad r > r_0 \qquad (3.6.22)$$

where $k_0 = \frac{1}{2}\varepsilon_0$, and

$$\varphi(r) = \varepsilon_1 + k_1 r + 0(r), \qquad k_1 = \varepsilon + \frac{\varepsilon_1}{\delta r_0}$$

The nature of dark matter and dark energy suggests that

$k_0 > 0$, $k_1 > 0$

$\varphi(r) \to 0$ *as* $r \to \infty$,, and (3.6.22) can be further simplified as in the form for $r_0 < r < r_1$

$$F = mMG\left[-\frac{1}{r^2} - \frac{k_0}{r} + k_1 r\right] \qquad (3.6.23)$$

where k_0 and k_1 will be determined by the Rubin rotational curve and the astronomical data for clusters of galaxies in the next section, where we obtain that

$$k_0 = 4 \times 10^{-18} km^{-2} \quad , \quad k_1 = 10^{-57} km^{-3} \qquad (3.6.24)$$

The formula (3.6.23) is valid only in the interval

$$r_0 < r < r_1$$

and r_1 is the distance at which F changes its sign:

$$F(r_1) = 0$$

Both observational evidence on dark energy and Theories show that the distance r_1 exists.

Attraction and repulsion of gravity :-

Gravity possesses additional attraction and repelling to the

Newtonian gravity, as shown in the revised gravitational formula:

$$F = mMG\left[-\frac{1}{r^2} - \frac{k_0}{r} + k_1 r\right] \qquad (3.6.25)$$

By using this formula we can explain the dark matter and dark energy phenomena. In particular, based on the Rubin rotational curve and astronomical data, we can determine an approximation of the parameters k_0 and k_1 in (3.6.25).

1) Dark matter: the additional attracting. Let M_r be the total mass in the ball with radius r of galaxy, and V be the constant galactic rotational velocity. By the force equilibrium, we infer from (3.6.25) that

$$\frac{V^2}{r} = M_r G\left(\frac{1}{r^2} + \frac{k_0}{r} - k_1 r\right) \qquad (3.6.26)$$

which implies that

$$M_r \frac{V^2}{G}\frac{r}{1+k_0 r - k_1 r^3} \qquad (3.6.27)$$

The mass distribution (3.6.27) is derived based on both the Rubin rotational curve and the revised formula (3.6.25). In the following we show that the mass distribution M_r fits the observed data.

We know that the theoretic rotational curve given by Figure 2.1 (b) is derived by using the observed mass M_{ob} and the Newton formula

$$F = -\frac{mM_{ob}G}{r^2}$$

Hence, to show that $M_r = M_{ob}$, we only need to calculate the rotational curve v_r by the Newton formula from the mass M_r, and to verify that v_r is consistent with the theoretic curve. To this end, we have

$$\frac{v_r^2}{r} = \frac{M_r G}{r^2}$$

which, by (3.6.27), leads to

$$v_r = \frac{V}{\sqrt{1-k_0 r - k_1 r^2}}$$

As $k_1 << k_0 << 1, v_r$ can be approximatively written as

$$v_r = V(1 - \frac{1}{2}k_0 r + \frac{1}{4}k_0^2 r^2) \qquad (3.6.28)$$

It is clear that the rotational curve described by (3.6.28) is consistent with the theoretic rotational curve as illustrated by Figure 2.1 (b). Therefore, it implies that

$$M_r = M_{ob} \qquad (3.6.29)$$

The facts of (3.6.26) and (3.6.29) are strong evidence to show that the revised formula (3.6.25) is in agreement with the astronomical observations.

We now determine the constant k_o in (3.6.25). According to astronomical data, the average mass M_{r1} and radius r_1 of galaxies are about

$$\begin{aligned} M_{r1} &= 10^{11} M_\Theta \cong 2 \times 10^{41} Kg \\ r_1 &= 10^4 \sim 10^5 pc \cong 10^{18} Km \end{aligned} \qquad (3.6.30)$$

The observations show that the constant velocity V in the Rubin rotational curve is about $V = 300 Km/s$. Then we have

$$\frac{V^2}{G} = 10^{24} Kg / Km$$

3.7 The effect of speed and potential on time , mass and energy on the basis Newton and relativity prediction

The nature of time, mass, and energy, and the effect of speed and potential field on them was experimentally tested. These experiments show that the time, mass, and energy are affected by both speed and potential [166].

Newtonian mechanics shows that only energy is affected by speed and potential. Thus it is in direct conflict with experiments that shows the effect of speed and potential on time and mass. However special relativity shows the effect of speed on time, mass, and energy but does not recognize the effect of potential on them. But generalized special relativity shows that time, mass, and energy are affected by velocity as well as field potentials. Fortunately the theoretical relations agree with the empirical ones.

Mechanical experiment :-

The verification of the laws of mechanics was made by some experiments which show clearly the viability of these laws.

For the comparison between Newton's lows (NL), Einstein special relativity (ESR) and generalized Einstein special relativity (GESR), the following experiment can clearly enable performing this task.

1.**Potential energy experiments** :

a .Motions of rockets and projectiles:

Rockets and projectiles motion can be described by using the Newton expression of energy E

$$E = E_0$$

$$T + V = T_0 + V_0$$

$$\frac{1}{2}mv^2 + V = \frac{1}{2}mv_0^2 + V_0 \qquad (3.7.1)$$

By knowing the initial velocity v_0 and the initial potential Vo, one can obtain the velocity of the rocket V at any time t, when the potential V at this time t is known. This experiment shows clearly the importance of the potential energy in describing the motion of particles in any field. Any viable mechanical theory must include potential energy in its energy expression otherwise it predicts its own break down.

2.Time dilatation experiments :

It was observed experimentally that the time is affected by speed as well as by potential of fields. Two famous experiments were made:

a. Effect of velocity (speed):

The Mesons which are at rest have life time

$$t_0 = 2\times10^{-2}s \qquad (3.7.2)$$

But μ Mesons which are produced in the atmosphere by fast cosmic-ray particles arrive at the earth from space, in profusion, travelling a distance of more than 6km. if no time dilatation exists, the speed $2.994\times10^8 m/s$ is given by

$$L_0 = vt_0 = 2.994\times10^8\times2\times10^{-6} = 600m \qquad (3.7.3)$$

This distance travelled is much less than the actual distance travelled which is more that 6km.one of the possible ways to explain this is to use time dilation relation of SR, where the life time t of μ Meson travelling

with speed

$$v = 2.994\times10^8 m/s$$

Is given by

$$t = \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}} = 31.7\times10^{-6}s \qquad (3.7.4)$$

In this case the meson travels distance

$$L = vt = 9.5Km \qquad (3.7.5)$$

Which agrees with the fact that μ meson reaches the earth after travelling more than 6km before decaying.

b. The effect of field potential :

The effect of gravitational field on periodic time of light was verified by Pound and Rebka in 1960 they allowed a -ray emitted by $14.4keV$, $0.1\mu_s$ transition in Fe^{57} to fall $22.6m$, and observed its resonant absorption by a Fe^{57} target . The difference in the gravitational potential per unit mass for $22.6m$ is

$$\Delta\Phi = -2.46\times10^{-15} \qquad (3.7.6)$$

The observed relative frequency shift was found to be

$$\frac{\Delta f}{f} = (2.57 \pm 0.26) \times 10^{-15} \qquad (3.7.7)$$

This results indicates a change of the periodic time t which is related to F according to the relation

$$T = 1/f \qquad (3.7.8)$$

This result was explained by general relativity (G) according to the relation

$$T_2 = To\left(-g_{oo}(x_2)\right)^{-1/2} = T_o\left(1 + 2\,{}^{\varphi}/{}_{c^2}\right)^{-1/2}$$
$$T_1 = T_o\left(-g_{oo}(x_1)\right)^{-1/2} = T_o\left(1 + 2\,{}^{\varphi_1}/{}_{c^2}\right)^{-1/2}$$
$$\Delta T = T_2 - T_1$$
$$\Delta f = f_2 - f_1 \qquad (3.7.9)$$

Where

$$f_1 = {}^1/{}_{T_1} \qquad , f_2 = {}^1/{}_{T_2}$$

Thus

$$\Delta f = f_2 - f_1$$
$$f = f_1$$
$$ {}^{f_2}/{}_{f_1} = \left(\frac{g_{oo}(x_2)}{g_{oo}(x_1)}\right)^{\frac{1}{2}} \qquad (3.7.10)$$

The metric g is given by

$$g_{oo} = -\left(1 + \frac{2\varphi}{c^2}\right) \qquad (3.7.11)$$

The predicted value by GR

$$ {}^{\Delta f}/{}_{f} = 2.46X10^{-15} \qquad (3.7.12)$$

Which is in excelent agreement with the experimental one .

1. Mass – Energy Relation Experiments

The Max Plank quantum theory explants' a wide variety of physical phenomena. One of these phenomenon's is the so called pair production. In pair production a highly energy etic photon of energy hf produce pair of particles and anti-particles according to the relation

$$hf = 2m_o c^2 + k_1 + k_2 \qquad (3.7.13)$$

Some of the photen energy is frozen in the form of rest mass energy with

m_o = rest mass.

k_1 = kinetic energy of the particle.

k_2 = kinetic energy of the anti-particle.

This phenomenon confirms the another phenomenon, also confirms the concept of rest mass energy.

This phenomenon is concerned with the difference of mass between the total mass of protons and neutrons constituting a certain nucleus and the mass energy.

This the difference of mass between the total mass of portend and neutrons constituting a certain nucleus and the mass of the nucleus itself. This difference is called mass defect or binding energy B where:

$$B = [n_p m_p + n_n m_n]c^2 - Mc^2 \qquad (3.7.14)$$

With

n_b = number of proteins

n_n = number of neutrons

m_n = number of a neutron

M = mass of the nucleus

The energy liberated in nuclear power stations and unclear bomb confirms this relation.

The change of energy by the potential of the gravitational was observed in the gravitational red shift phenol men on.

In this phenomenon the energy of the photon entering gravitation's field is given by:

$$hf' = hf + V$$

$$E' = E + V \tag{3.7.15}$$

Where :

$f' =$ new photon frequency after entering the gravitational field

$f =$ photon frequency in free space

$V =$ potential energy

If one believes in the Einstein SR one has energy relation for photon in the form

$$hf = mc^2 \tag{3.7.16}$$

Therefore

$$m'c^2 = m + \frac{V}{c^2} \tag{3.7.17}$$

Thus the mass is affected by the potential energy .

Another experiment indicates that the mass of electrons are affected by the crystal field according to the relations

$$m' = m\frac{(f_e)}{(F_e + F_L)} \tag{3.7.18}$$

III. THEORETICAL MODELS

For particle of mass m, velocity v moving in a field of potential V. the time, mass and energy in frames S and So

moving with respect to each other with speed v is given in Newtonian mechanics by:

$$t = t_0 \tag{3.7.19}$$

$$m = m_0 \qquad (3.7.20)$$

$$E = E_0 \qquad (3.7.21)$$

$$\frac{1}{2}mv^2 + V = \frac{1}{2}mv_0^2 + V_0 \qquad (3.7.22)$$

When the particle is at rest in S_0:

$$v_0 = 0 \qquad (3.7.23)$$

According to SR Einstein theory [165] these relations are:

$$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad (3.7.24)$$

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad (3.7.25)$$

$$E = E_0 + T \qquad (3.7.26)$$

$$Ec^2 = m_0c^2 + T \qquad (3.7.27)$$

$$E = mc^2 = \frac{m_0c^2}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad (3.7.28)$$

In the Newtonian limit , for low speed

$$E = mc^2 = m_0c^2 + T \qquad (3.7.29)$$

According to mass and energy relation in SR the mass can be converted to energy according to equations (3.7.13) and (3.7.14).

This is since NM equation (3.7.22) has no term recognizing rest mass energy.

The NM energy equation (3.7.22) cannot explain the effect of lattice crystal force F_r on mass shown by equation (3.7.18)

However, Einstein SR can explain a wide variety of physical phenomena, but not all of them. While it cannot explain the motion of macro particle in fields [compare (3.7.1) with (3.7.28)], but it can explain the effect of velocity on life time [see equations (3.7.4) and (3.7.24)]. However as shown by equation (3.7.9) cannot explained by SR equation (3.7.24),

Fortunately GSR can explain all there phenomena . These is straight forward from the comparison of all equations in the section of mechanical experiment , with GSR equations . The effect of shown experimentally to obey equations (3.7.4) and (3.7.9) can be easily explained by equations (3.7.30) by setting $\varphi = 0$ and $v = 0$ for the expressions of t respectively

$$t = \frac{t_0}{\sqrt{1 - \frac{2\varphi}{c^2} - \frac{v^2}{c^2}}} \qquad (3.7.30)$$

$$m = \frac{m_0}{\sqrt{1 + \frac{2\varphi}{c^2} - \frac{v^2}{c^2}}} \qquad (3.7.31)$$

IV. DISCUSSION

The viability of Newton, Special Relatively and Generalized Special Relatively Theories:

Consider first Newton's laws. It is clear from equations (3.7.1) and (3.7.22) that NM can explain the motion of macro particles in any field. The comparison of equations (3.7.4) with (3.7.19) shows the failure of NM in explaining time dilation experiments. This is since according to equation (3.7.4) the life time is affected by velocity. It cannot also recognize the effected potential on mass. Newton laws cannot also explain pair production and mass defect in which energy can be converted to mass, Or the mass can be converted to energy according to equations (3.7.13) and (3.7.14).

This is since NM equation (3.7.22) has no term recognizing rest mass energy.

The NM energy equation (3.7.22) cannot explain the effect of lattice crystal force F_r on mass as shown by equation (3.7.18).

However, Einstein SR can explain a wide variety of physical phenomena, but not all of them. While it cannot explain the motion of macro particle in fields [compare (3.7.1) with (3.7.28)], but it can explain the effect of velocity on life time [see equations (3.7.4) and 93.6)]. However, the effect on potential on time as shown by equation (3.7.9) cannot explain by SR equation (3.7.24).

Fortunately GSR can explain all these phenomena . This is straight forward from the comparison of all equations in sections , with the GSR equations . The effect of velocity and potential on time shown experimentally to obey equations (3.7.4) and (3.7.9) can be easily explained by equations (3.7.30) by setting $\varphi = 0$ and $v = 0$ for the expressions of t respectively . Equations (3.7.31) and (3.7.14) which states that rest mass is a form of energy can be easily explained by equation (3.7.32) , when the potential term φ vanishes .

Moreover the empirical expressions for Energy and mass in equations (3.7.15) and (3.7.17) which explains the photon frequency red shift can be easily explained on the basis of relation (3.7.32) by considering the case of weak field, where

$$m = m_0 \left(1 + \frac{2\varphi}{c^2} - \frac{v^2}{c^2}\right)^{-\frac{1}{2}}$$

$$m = m_0 \left(1 + \frac{2\varphi}{c^2} + \frac{1}{2}\frac{v^2}{c^2}\right) \qquad (3.7.33)$$

For the observer in the photon frame of reference and for negative attractive potential.

$$m = m_0 + \frac{\varphi m_0}{c^2}$$

$$m = m_0 + \frac{V}{c^2} \qquad (3.7.34)$$

Multiplying both sides by c^2, yields

$$mc^2 = m_0 c^2 + V \qquad (3.7.35)$$

It is quite obvious that equations (3.7.34) and (3.7.35) are typical to equations (3.7.17) and (3.7.15) respectively.

The effective mass of electrons in crystal, having field V_1 can also easily explained by the expression of mass in GSR, where fir a mass in an external field equation (3.7.31) gives.

$$m\left(1+\frac{2\varphi}{c^2}\right)^{\frac{1}{2}}=m_0 \qquad (3.7.36)$$

$$m\left(1+\frac{2m_0\varphi}{m_0c^2}\right)^{\frac{1}{2}}=m_0 \qquad (3.7.37)$$

$$m\left(1+\frac{m_0\varphi}{m_0c^2}\right)=m_0$$

$$m\left(1+\frac{V_0}{m_0c^2}\right)^{\frac{1}{2}}=m_0 \qquad (3.7.38)$$

Where one reflects the speed, with V_e standing for external potential.

Inside the crystal both external potential V_e and crystal potential V_L affect the mass thus the electron effective mass m can be found from equation (3.7.38) to be

$$m^*\left(1+\frac{V_e+V_L}{m_0c^2}\right)=m_0 \qquad (3.7.39)$$

Thus from (3.7.38) and (3.7.39)

$$m^*+\frac{m^*(V_e+V_L)}{m_0c^2}=m+1+\frac{mV_e}{m_0c^2}$$

It one considers an approximation

$$m^*\approx m$$

One gets

$$\frac{m^*}{m}=\frac{V_e}{V_e+V_L}=\frac{V_e/d}{V_e/d+V_L/d}=\frac{F_e}{F_e+F_L}$$

$$m^* = m\frac{F_e}{F_e + F_L} \qquad (3.7.40)$$

Where the furls is related to the potentials according to the relations.

$$\begin{aligned} F_e d &= V_e \\ F_L d &= V_L \end{aligned} \qquad (3.7.41)$$

Another approach is based on the wave nature of particles can be also used. Where the wave bucket representation of atomic particles states that a wave bucket formed from a very large number of interfering waves, having different of interfering waves having different frequencies becomes highly localized in the forms of a particle. According to this version the potential and kinetic energy of waves can be assumed as that of a harmonic oscillator where the potential is given by

$$V = -\int f\,dx = m\omega^2 \int x\,dx = \frac{m\omega^2 x^2}{2} \qquad (3.7.42)$$

$$x = x_0 e^{i\omega t}$$

$$T = \left|\frac{1}{2}mv^2\right| = \frac{1}{2}m\omega^2 x^2 \qquad (3.7.43)$$

They for harmonic as collator

$$T = V \qquad (3.7.44)$$

For low speed and weak attractive potential

$$v << c^2 \qquad \varphi \to -\varphi \qquad \varphi << c^2 \qquad (3.7.45)$$

Thus equation (3.7.32) reduced to

$$m_0 c^2 = mc^2\left(1 - 2\varphi / c^2 - \frac{v^2}{c^2}\right)^{\frac{1}{2}} \qquad (3.7.46)$$

But from (4.7.45)

$$m_0 c^2 = mc^2\left(1 - \frac{m_0\varphi}{m_0 c^2} - \frac{m_0 v^2}{2m_0 c^2}\right)$$

But from (4.7.44)

$$m_0c^2 = mc^2\left(\frac{m_0c^2 - V - T}{m_0c^2}\right) = \frac{mc^2}{m_0c^2}\left[m_0c^2 - 2V\right] \qquad (3.7.47)$$

For the effect of external field only

$$m = m_e \qquad , \quad V = V_e = F_e d$$

$$m_0^2c^4 = m_e c^2(m_0c^2 - 2F_e d) \qquad (3.7.48)$$

But when crystal field affect electrons also , the effective mass is given by

$$m_0^2c^4 = m^* c^2\left[m_0c^2 - 2(F_e + F_L)d\right] \qquad (3.7.49)$$

Since m_0c^2 cannot be measured , it can be ignored within the brackes .

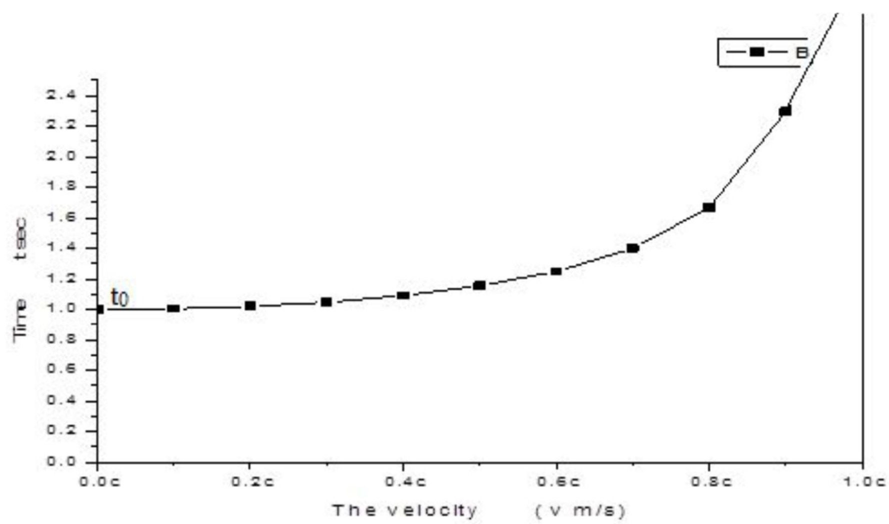
A direct substitution of (3.7.47) and (3.7.48) in (3.7.49) yields

$$m_0^2c^4 = m_e c^2\left[-2F_e d\right] = m^* c^2\left[-2(F_e + F_L)d\right]$$

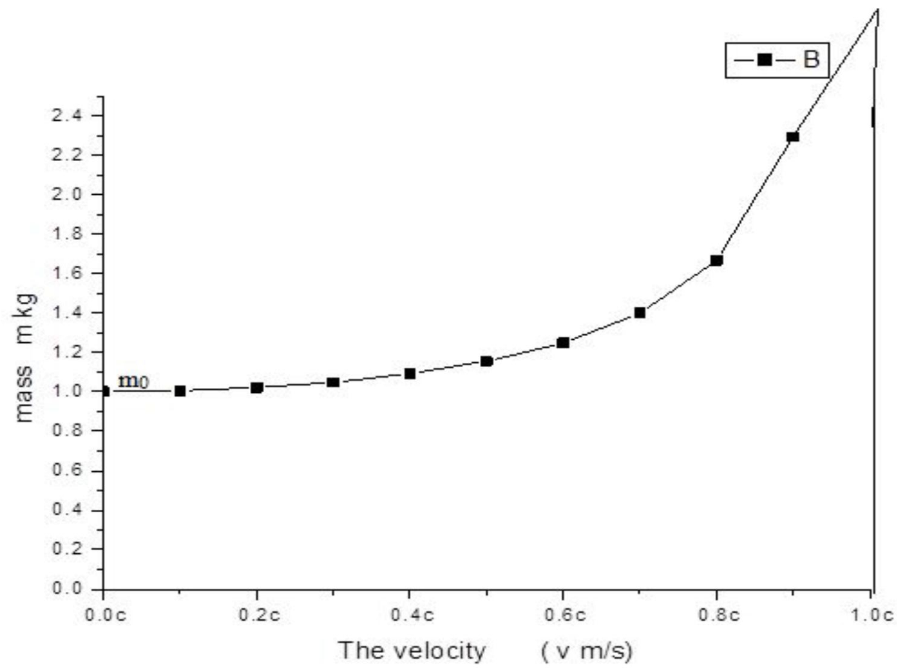
Thus

$$m^* = m_e\left(\frac{F_e}{F_e + F_L}\right) \qquad (3.7.50)$$

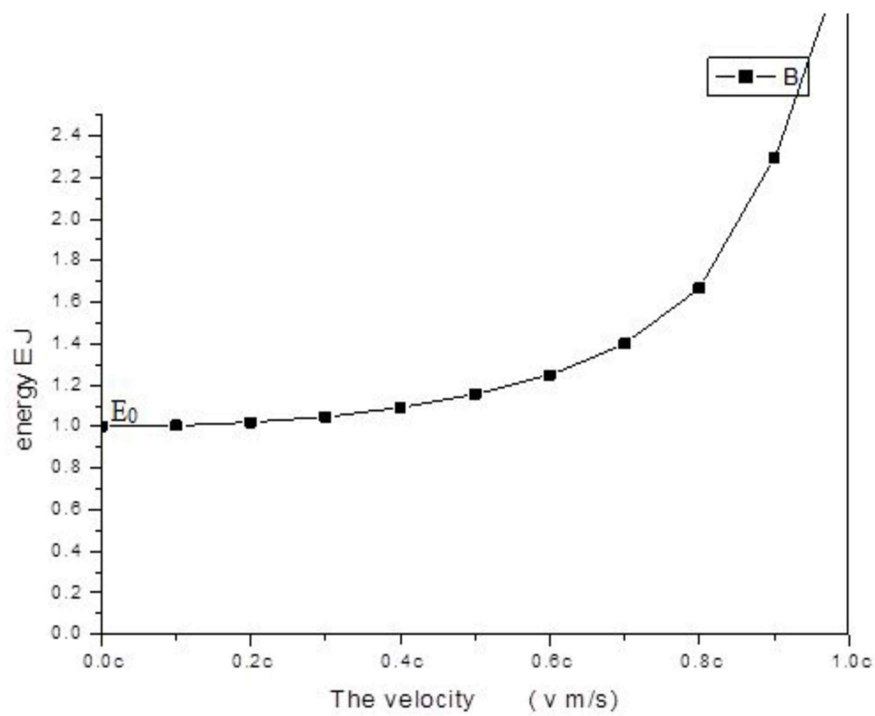
Which conforms with equation (3.7.18)



(a)

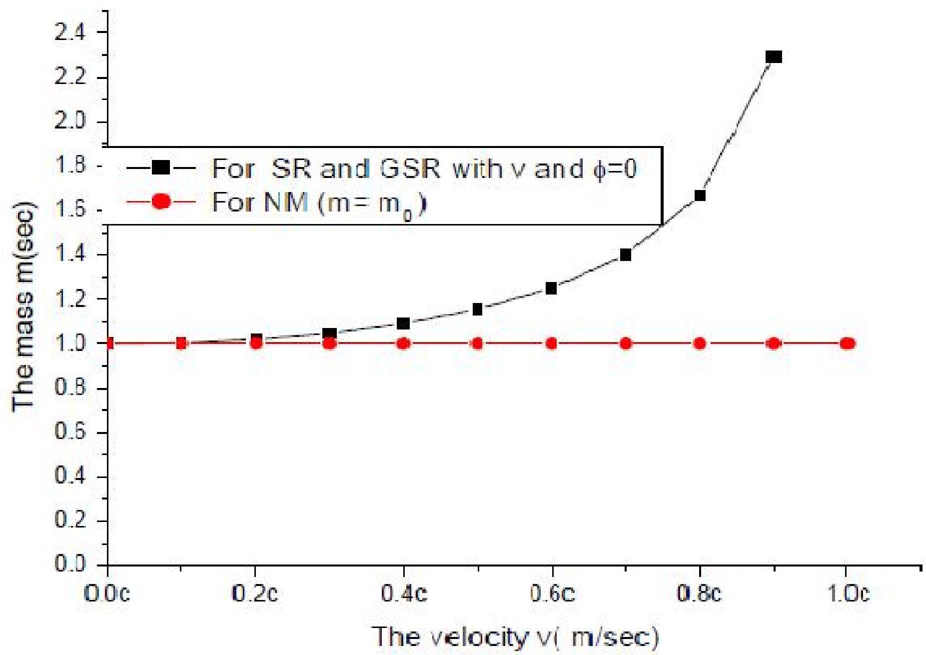


(b)

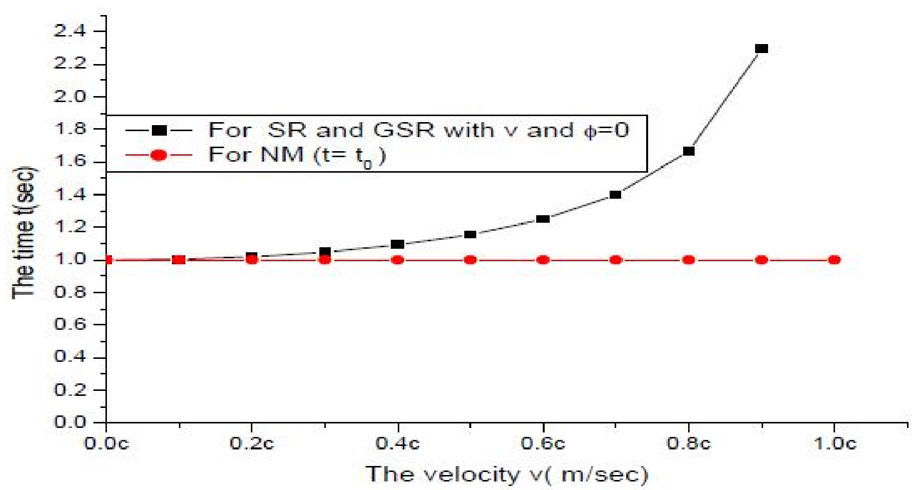


(c)

Fig 1. The empirical relation between (a) t and v, (b) m and v and (c) E and v, when $\Phi = 0$

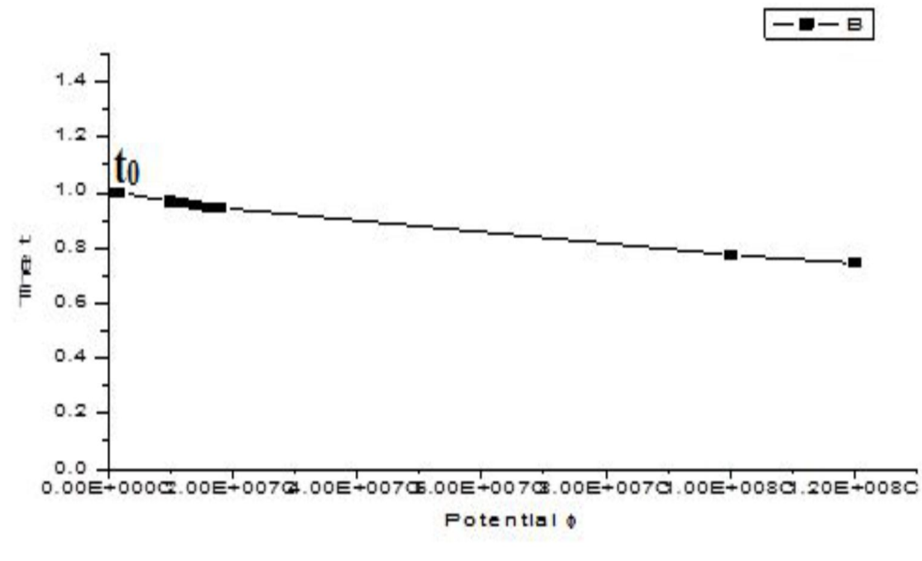


(a)

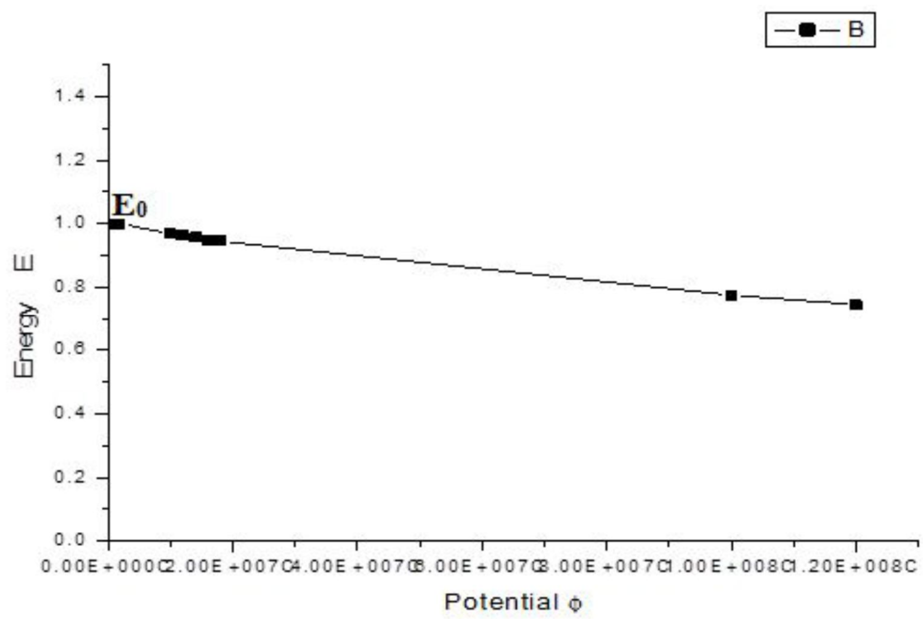


(b)

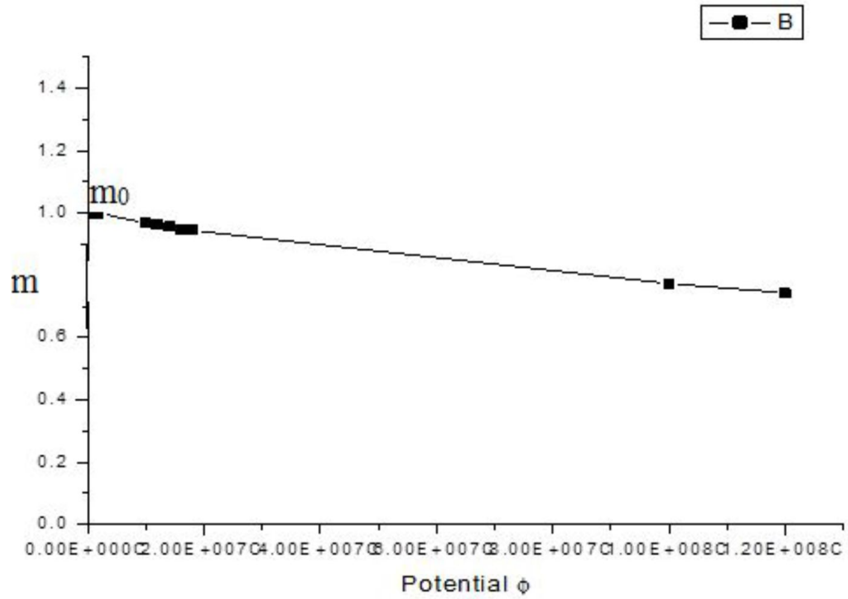
Fig 2.The empirical relation between (a) m and v and (b) t and v, when $\phi = 0$ for (NM, SR and GRS)

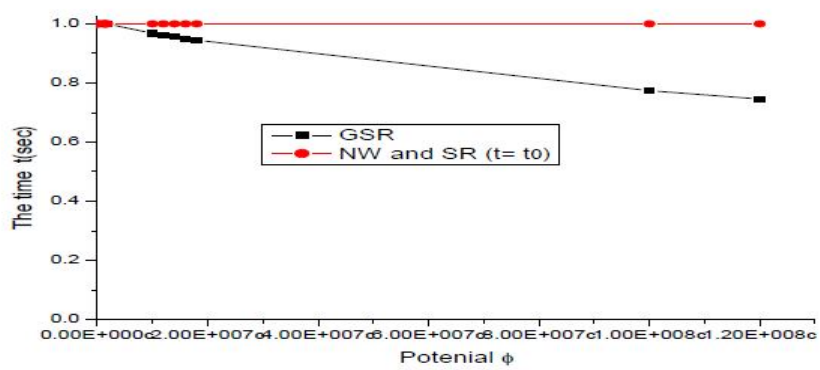

B
1.4
1.2
t0
1.0
0.8
0.6
0.4
0.2
0.0
0.00E+000
2.00E+007
4.00E+007
6.00E+007
8.00E+007
1.00E+008
1.20E+008
Potential ϕ

(a)

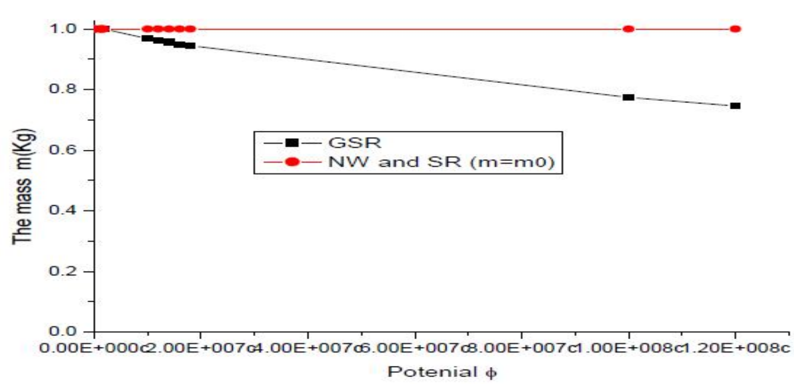

B
1.4
1.2
E0
1.0
0.8
0.6
0.4
0.2
0.0
Energy E
0.00E+000
2.00E+007
4.00E+007
6.00E+007
8.00E+007
1.00E+008
1.20E+008
Potential ϕ

(b)





(a)



(b)

Fig 2.The empirical relation between (a) m and v and (b) t and v, when v=0 for (NM, SR and GRS)

V. CONCLUSION:-

The direct comparison of NM, SR and GSR with the experiments that shows the effect of speed and potential and time, mass and energy shows that GSR is the best candidate that can describe physical phenomena concerning time - mass – energy relations.

Chapter four

Generalized general relativity and antimatter

(4.1) Introduction :

The discovery of anti matter is related to relativistic quantum Dirac equation . Thus the two issues are related to each other . This chapter is concerned with the two phenomena and their relation to each other .In this chapter Generalized non linear Lorentz transformation is utilized to derive modified special relativistic space – time equations . The equations are found for particles moving in a potential field .

Using GSR energy relation vacuum energy was found together with its role in generating matter and antimatter .

(4.2) New derivation of general special relativity :

According to Newton's second law of motion the force F can be expressed in terms of the mass m and acceleration a as

$$F = ma \qquad (4.2.1)$$

Thus the potential V is given by

$$V = m\Phi = \int F.dx = \int madx = ma\int dx = m\,a\,x$$

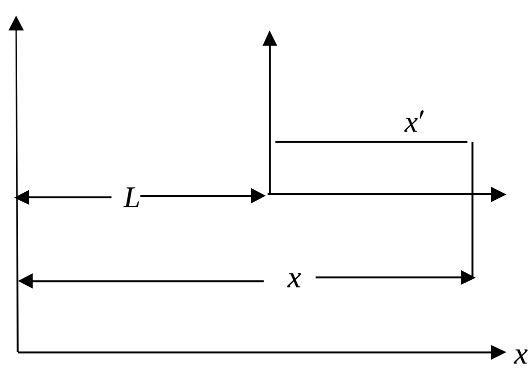
Where Φ is defined as the potential per unit mass . Thus

$$m\Phi = ma\,x$$

Hence

$$\Phi = a\,x \qquad (4.2.2)$$

Let two reference frames (x,t) and (x',t') moves with initial velocity v_0 and constant acceleration a with respect to each others .Thus the



distance between their origin at any time t is given by

$$L = v_0 t + \frac{1}{2} a t^2$$

i-e

$$L = v_0 t + \frac{1}{2} \frac{a x}{x} t^2 \qquad (4.2.3)$$

Using equation (4.2.2) one can rewrite equation (4.2.3) as

$$L = v_0 t + \frac{\Phi}{2x} t^2 \qquad (4.2.4)$$

This represent the length as measured by the observer O .assuming v_0 and Φ to be the same for all observers , the length for observer O' is given by

$$L' = v_0 t' + \frac{1}{2} a t'^2 = v_0 t' + \frac{1}{2} a \frac{x'}{x'} t'^2$$

$$L' = v_0 t' + \frac{1}{2} \frac{a \Phi}{x'} t'^2 \qquad (4.2.5)$$

The space – time coordinate in two frames can be described by Lorentz transformation . According to Lorentz transformation

$$x' = \gamma (x + L) = \gamma (x + v_0 t + \frac{\Phi}{2x} t^2) \qquad (4.2.6)$$

$$x = \gamma (x' + L') = \gamma (x' + v_0 t' + \frac{\Phi}{2x'} t'^2) \qquad (4.2.7)$$

Consider now a source of light that emits light pulse when the two frames origin coincide , i.e

$$t = t' = 0$$

The light pulse which are emitted travels distances x' and x respectively , where

$$x = ct \quad ، \quad x' = ct' \qquad (4.2.8)$$

Substituting (4.2.8) in (4.2.6) yields

$$ct' = \gamma (ct + v_0 t + \frac{\Phi}{2ct} t^2)$$

$$t' = \gamma\left[(1+\frac{v_0}{c})\,t\ +\frac{\Phi}{2c^2}\,t\,)\right]$$

$$t'\ = \gamma\left[1+\frac{v_0}{c}\ +\frac{\Phi}{2c^2}\ \right]t \qquad (4.2.9)$$

Inserting also (4.2.8) in (4.2.7) gives

$$ct = \gamma\ \left[ct' - v_0 t - \frac{\Phi}{2ct}t'^2\right]$$

$$t = \gamma\left[1 - \frac{v_0}{c}\ - \frac{\Phi}{2c^2}\ \right]t' \qquad (4.2.10)$$

From (4.2.9) and (4.2.10)

$$t = \gamma^2\left[1+\frac{v_0}{c}\ +\frac{\Phi}{2c^2}\ \right]\left[1-\frac{v_0}{c}-\frac{\Phi}{2c^2}\right]t$$

Therefore

$$\gamma = \frac{1}{\sqrt{(1+\frac{v_0}{c}+\frac{\Phi}{2c^2})(1-\frac{v_0}{c}-\frac{\Phi}{2c^2})}} \qquad (4.2.11)$$

It is very interesting to note that when no field exists

$$\Phi = 0 \qquad (4.2.12)$$

The factor γ in equation (4.2.11) reduces to

$$\gamma = \frac{1}{\sqrt{(1+\frac{v_0}{c}+\frac{\Phi}{2c^2})(1-\frac{v_0}{c}-\frac{\Phi}{2c^2})}} = \frac{1}{\sqrt{1-\frac{v_0^2}{c^2}}} \qquad (4.2.13)$$

Which is ordinary SR relation . A direct insertion of equation (4.2.11) in (4.2.6) and (4.2.7) yields

$$x' = \frac{(x+v_0\,t+\frac{\Phi}{2x}t^2)}{\sqrt{(1+\frac{v_0}{c}+\frac{\Phi}{2c^2})(1-\frac{v_0}{c}-\frac{\Phi}{2c^2})}} \qquad (4.2.14)$$

$$x = \frac{(x' - v_0 t' - \frac{\Phi}{2x'} t^2)}{\sqrt{(1 + \frac{v_0}{c} + \frac{\Phi}{2c^2})(1 - \frac{v_0}{c} - \frac{\Phi}{2c^2})}} \qquad (4.2.15)$$

In the absence of fields again (4.2.14) and (4.2.15) reduces to that of SR .

The expression for energy is given by

$$E = mc^2 = \gamma\, m_0\, c^2 \qquad (4.2.16)$$

Inserting (4.2.11) in (4.2.16) yields

$$E = \frac{m_0\, c^2}{\sqrt{(1 + \frac{v_0}{c} + \frac{\Phi}{2c^2})(1 - \frac{v_0}{c} - \frac{\Phi}{2c^2})}} \qquad (4.2.17)$$

When no field exists the energy relation reduces to

$$E = \frac{m_0\, c^2}{(1 - \frac{v_0^2}{c^2})} \qquad (4.2.18)$$

Let now

$$x = \frac{v_0}{c} + \frac{\Phi}{2c^2} \qquad (4.2.19)$$

Assuming

$$\frac{\Phi}{c^2} < \frac{v_0}{c} \qquad (4.2.20)$$

Equation (4.2.17) becomes

$$E = m_0\, c^2\, (1 - \frac{v_0^2}{c^2})^{-\frac{1}{2}} \qquad (4.2.21)$$

But for law speed

$$\frac{v_0}{c} << 1 \qquad (4.2.22)$$

Thus

$$E = m_0 c^2 \left[1 + \frac{1}{2} \frac{v_0^2}{c^2} \right] \tag{4.2.23}$$

But according to Newton's laws

$$v^2 = v_0^2 + 2\Phi \tag{4.2.24}$$

Thus

$$E = m_0 c^2 \left[1 + \frac{1}{2} \frac{v^2}{c^2} - \frac{\Phi}{c^2} \right] = m_0 c^2 + \frac{1}{2} m_0 v^2 + m_0 \Phi - m_0 c^2 + T + V \tag{4.2.25}$$

Where

$$V = -m_0 \Phi$$

$$T = \frac{1}{2} m_0 v^2 \tag{4.2.26}$$

Which is the usual Newton energy relation beside rest mass term .

(4.3) Production of particles and anti particles on the basis of Generalized special Relativity and the Repulsive nature :

The energy relation according to Einstein generalized relativity is given by

$$E = \frac{m_0 c^2 (1 + \frac{2\Phi}{c^2})}{\sqrt{1 + \frac{2\Phi}{c^2} - \frac{v^2}{c^2}}} \tag{4.3.1}$$

When m_0, Φ, v stands for rest of mass , potential and velocity respectively

Thus

$$E = m_0 c^2 (1 + \frac{2\Phi}{c^2})(1 + \frac{2\Phi}{c^2} - \frac{v^2}{c^2})^{-\frac{1}{2}} \tag{4.3.2}$$

To find vacuum minimum energy , the energy one minimizes E to get

$$\frac{dE}{d\Phi} = m_0 c^2 \left[(1 + \frac{2\Phi}{c^2}) \times \frac{-1}{2} (1 + \frac{2\Phi}{c^2} - \frac{v^2}{c^2})^{-\frac{3}{2}} \times \frac{2}{c^2} + (1 + \frac{2\Phi}{c^2} - \frac{v^2}{c^2})^{-\frac{1}{2}} \times \frac{2}{c^2} \right]$$

Thus

$$\frac{dE}{d\Phi}=\frac{2m_0c^2}{c^2}\frac{(\frac{1}{2}+\frac{\Phi}{c^2}-\frac{v^2}{c^2})}{(1+\frac{2\Phi}{c^2}-\frac{v^2}{c^2})^{\frac{3}{2}}} \tag{4.3.3}$$

$$\frac{2m_0c^2}{c^2}\frac{(\frac{1}{2}+\frac{\Phi}{c^2}-\frac{v^2}{c^2})}{(1+\frac{2\Phi}{c^2}-\frac{v^2}{c^2})^{\frac{3}{2}}}=0 \tag{4.3.4}$$

Hence

$$\frac{1}{2}+\frac{\Phi}{c^2}-\frac{v^2}{c^2}=0$$

Therefore

$$\frac{\Phi}{c^2}=\frac{v^2}{c^2}-\frac{1}{2}$$

$$\Phi=c^2(\frac{v^2}{c^2}-\frac{1}{2})$$

Thus the value of Φ which make E minimum is given by

$$\Phi=v^2-\frac{c^2}{2} \tag{4.3.5}$$

Due to the wave nature of light

$$c_e=\frac{c_m}{\sqrt{2}}$$

$$c_m=\sqrt{2}c_e \tag{4.3.6}$$

Let

$$c=c_m=\sqrt{2}c_e \tag{4.3.7}$$

$$\Phi=v^2-c_e^2 \tag{4.3.8}$$

When $v=0$

$$\Phi=-c_e^2 \tag{4.3.9}$$

Thus the potential is given by

$$V = m_0\Phi = -m_0 c_e^2 \tag{4.3.10}$$

In this case the vacuum constituents are at rest ($v = 0$) .

But when a photon , which constitute vacuum move with speed c , thus

$$v = c \tag{4.3.11}$$

Substitute in (5) to get

$$\Phi = c^2 - \frac{c^2}{2} = \frac{c^2}{2} \tag{4.3.12}$$

When inserting equation (4.3.7) , one gets

$$\Phi = c_e^2 \tag{4.3.13}$$

But the potential energy is given by

$$V = m_0\Phi$$

Thus from (4.3.13) and (4.3.1) the potential is given by

$$V = m_0 c_e^2$$

The vacuum energy can be found by inserting (4.3.11) and (4.3.12) in (4.3.1) to get

$$E = m_0 c^2 (1+1) = 2m_0 c^2 \tag{4.3.14}$$

Thus one can imagine vacuum energy levels as shown below

$$E_+ = m_m c^2 = m_0 c_e^2$$

$$E = 0$$

$$E_- = -m_0 c^2 = m_a c_e^2$$

Figure (4.3.1) : vacuum states as consisting of photons producing and destructing particles and anti particles with rest masses m_0 .

The production of pair particles can be regarded as due to electron transfer from state the lower to the upper state after absorbing a photon .

Where

$$m_m = matter\ mass = m_0$$
$$m_a = anti\ matter\ mass = -m_0 \tag{4.3.15}$$

The energy diagram is shown in figure (4.3.1)

According to Newton's laws the potential is given by

$$\Phi = -\frac{G\ m}{r} \tag{4.3.16}$$

For matter

$$m_m = m_0$$

Thus

$$\Phi_m = matter\ \ potential = -\frac{G\ m_m}{r} = -\frac{G\ m_0}{r} \tag{4.3.17}$$

This is an attractive force .

For anti matter

$$m_a = -m_0$$

$$\Phi_a = anti\ \ matter = -\frac{G\ m_a}{r} = +\frac{G\ m_0}{r} \tag{4.3.18}$$

This is a repulsive force

When matter and anti matter interact with each other the potential is given by

$$V = -\frac{G\,m\,M}{r} \tag{4.3.19}$$

Where the force is given by

$$F = -\nabla V$$
$$= -\frac{\partial V}{\partial r} = +G\,m\,M\,\frac{\partial r^{-1}}{\partial r}$$

Hence the force is given by

$$F = -\frac{G\,m\,M}{r^2} \tag{4.3.20}$$

For matter and anti matter reaction

$$m = m_m = m_0$$

$$M = m_a = -m_0 \tag{4.3.21}$$

Thus the force between matter and anti matter is given by

$$F = -\frac{G\,(m_0)(-m_0)}{r^2} = \frac{G\,m_0^2}{r^2} \tag{4.3.22}$$

Thus there is repulsive force between matter and anti matter .

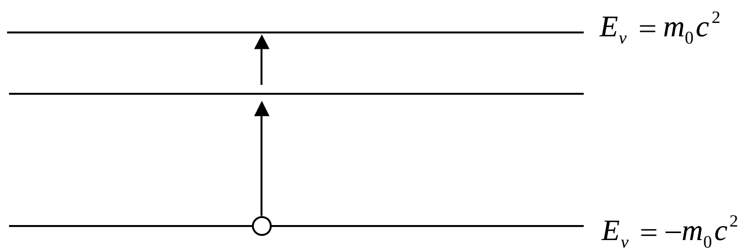


Figure (4.3.2) : vacuum energy levels

(4.4) Generation of particle and anti particle on the basis of conservation law :

In the work done by Mubarak [ ] it was shown that the energy conservation requires

$$E = mc^2 + 2m\Phi \tag{4.4.1}$$

The GSR mass was proposed by some authors to be

$$m = \frac{m_0}{\sqrt{(1 + \frac{2\Phi}{c^2} - \frac{v^2}{c^2})}} \tag{4.4.2}$$

From (4.4.1) and (4.4.2) one gets

$$E = \frac{m_0 c^2}{\sqrt{(1+\frac{2\Phi}{c^2}-\frac{v^2}{c^2})}} + \frac{2m_0 c^2}{\sqrt{(1+\frac{2\Phi}{c^2}-\frac{v^2}{c^2})}}$$

$$E = m_0 c^2 (1+\frac{2\Phi}{c^2}-\frac{v^2}{c^2})^{-\frac{1}{2}} + 2m_0 c^2 (1+\frac{2\Phi}{c^2}-\frac{v^2}{c^2})^{-\frac{1}{2}}$$

To find vacuum state , the energy E need to be minimized w.r.t to Φ , to get

$$\frac{dE}{d\Phi} = -\frac{1}{2} m_0 c^2 (1+\frac{2\Phi}{c^2}-\frac{v^2}{c^2})^{-\frac{3}{2}} \times \frac{2}{c^2} + 2m_0 \left[\Phi \times \frac{-1}{2}(1+\frac{2\Phi}{c^2}-\frac{v^2}{c^2})^{-\frac{3}{2}} \times \frac{2}{c^2} + (1+\frac{2\Phi}{c^2}-\frac{v^2}{c^2})^{-\frac{1}{2}} \right]$$

Thus

$$\frac{dE}{d\Phi} = \frac{-m_0}{(1+\frac{2\Phi}{c^2}-\frac{v^2}{c^2})^{\frac{3}{2}}} - \frac{2m_0\Phi/c^2}{(1+\frac{2\Phi}{c^2}-\frac{v^2}{c^2})^{\frac{3}{2}}} + \frac{2m_0}{(1+\frac{2\Phi}{c^2}-\frac{v^2}{c^2})^{\frac{1}{2}}}$$

$$= \frac{-m_0 - 2m_0\Phi/c^2 + 2m_0(1+\frac{2\Phi}{c^2}-\frac{v^2}{c^2})}{(1+\frac{2\Phi}{c^2}-\frac{v^2}{c^2})^{\frac{3}{2}}} \qquad (4.4.3)$$

This equation can be satisfied , when

$$2m_0(1+\frac{2\Phi}{c^2}-\frac{v^2}{c^2}) = \frac{2m_0\Phi}{c^2} + m_0$$

$$1+\frac{2\Phi}{c^2}-\frac{v^2}{c^2} = \frac{\Phi}{c^2} + \frac{1}{2}$$

$$\frac{\Phi}{c^2} = \frac{v^2}{c^2} - \frac{1}{2}$$

$$\Phi = v^2 - \frac{c^2}{2} \qquad (4.4.4)$$

If vacuum particles are at rest $v = 0$, thus equation (4.4.4) become

$$\Phi = -\frac{c^2}{2} \qquad (4.4.5)$$

Substituting this value in (4.4.1) and (4.4.2) yield

$$m=\frac{m_0}{0}=\infty \qquad (4.4.6)$$

Thus from (4.4.1)

$$E=\infty \qquad (4.4.7)$$

Thus condition (4.4.3) is the condition for maximum E.

Now one can use equation (4.4.1)

$$E=mc^2+2m\Phi$$

But the mass term is in the form

$$m=\frac{(1+\frac{2\Phi}{c^2})m_0}{\sqrt{1+\frac{2\Phi}{c^2}-\frac{v^2}{c^2}}} \qquad (4.4.8)$$

Inserting (4.4.8) in (4.4.1) yields

$$E=\frac{(1+\frac{2\Phi}{c^2})m_0c^2}{\sqrt{1+\frac{2\Phi}{c^2}-\frac{v^2}{c^2}}}+\frac{2m_0\Phi(1+\frac{2\Phi}{c^2})}{\sqrt{1+\frac{2\Phi}{c^2}-\frac{v^2}{c^2}}}$$

$$E=m_0c^2(1+\frac{2\Phi}{c^2})(1+\frac{2\Phi}{c^2}-\frac{v^2}{c^2})^{-\frac{1}{2}}+2m_0(\Phi+\frac{2\Phi^2}{c^2})(1+\frac{2\Phi}{c^2}-\frac{v^2}{c^2})^{-\frac{1}{2}}$$

The differentiation of E w.r.t to Φ requires

$$\frac{dE}{d\Phi}=m_0c^2\left[(1+\frac{2\Phi}{c^2})\times\frac{-1}{2}(1+\frac{2\Phi}{c^2}-\frac{v^2}{c^2})^{-\frac{3}{2}}\times\frac{2}{c^2}+(1+\frac{2\Phi}{c^2}-\frac{v^2}{c^2})^{-\frac{1}{2}}\times\frac{2}{c^2}\right]$$
$$+2m_0\left[(\Phi+\frac{2\Phi^2}{c^2})\times\frac{-1}{2}(1+\frac{2\Phi}{c^2}-\frac{v^2}{c^2})^{-\frac{3}{2}}\times\frac{2}{c^2}+(1+\frac{2\Phi}{c^2}-\frac{v^2}{c^2})^{-\frac{1}{2}}\times(1+\frac{4\Phi}{c^2})\right]$$
$$=\frac{-m_0(1+\frac{2\Phi}{c^2})}{(1+\frac{2\Phi}{c^2}-\frac{v^2}{c^2})^{\frac{3}{2}}}+\frac{2m_0}{(1+\frac{2\Phi}{c^2}-\frac{v^2}{c^2})^{\frac{1}{2}}}-\frac{2m_0/c^2(\Phi+\frac{2\Phi^2}{c^2})}{(1+\frac{2\Phi}{c^2}-\frac{v^2}{c^2})^{\frac{3}{2}}}+\frac{2m_0(1+\frac{4\Phi}{c^2})}{(1+\frac{2\Phi}{c^2}-\frac{v^2}{c^2})^{\frac{1}{2}}} \qquad (4.4.9)$$

Where for minimum energy one has

$$\frac{dE}{d\Phi} = 0 \qquad (4.4.10)$$

$$\frac{-m_0(1+\frac{2\Phi}{c^2}) + 2m_0(1+\frac{2\Phi}{c^2}-\frac{v^2}{c^2}) - 2m_0/c^2(\Phi+\frac{2\Phi^2}{c^2}) + 2m_0(1+\frac{4\Phi}{c^2})(1+\frac{2\Phi}{c^2}-\frac{v^2}{c^2})}{(1+\frac{2\Phi}{c^2}-\frac{v^2}{c^2})} = 0$$

$$(1+\frac{2\Phi}{c^2}) + 2(1+\frac{2\Phi}{c^2}) - \frac{2v^2}{c^2} - \frac{2\Phi}{c^2} - \frac{4\Phi^2}{c^4} + 2(1+\frac{4\Phi}{c^2})(1+\frac{2\Phi}{c^2}-\frac{v^2}{c^2}) = 0 \qquad (4.4.11)$$

$$(1+\frac{2\Phi}{c^2}) - \frac{2\Phi}{c^2} - \frac{2v^2}{c^2} - \frac{4\Phi^2}{c^4} + 2(1+\frac{2\Phi}{c^2}) - \frac{2v^2}{c^2} + 4\Phi(1+\frac{2\Phi}{c^2}-\frac{v^2}{c^2}) = 0$$

$$3(1+\frac{2\Phi}{c^2}) - \frac{2\Phi}{c^2} - \frac{4v^2}{c^2} - \frac{4\Phi^2}{c^4} + \frac{4\Phi}{c^2} - \frac{8\Phi^2}{c^4} - \frac{4v^2\Phi}{c^4} = 0$$

$$3 + \frac{8\Phi}{c^2} - \frac{4v^2}{c^2} + \frac{4\Phi^2}{c^4} - \frac{4v^2\Phi}{c^4} = 0 \qquad (4.4.12)$$

For stationary vacuum constituents

$$v = 0 \qquad (4.4.13)$$

Equation (4.4.12) reads

$$\frac{4\Phi^2}{c^4} + \frac{8\Phi}{c^2} + 3 = 0$$

$$4\Phi^2 + 8\Phi c^2 + 3c^4 = 0$$

Solving for Φ

$$\Phi = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad (4.4.14)$$

$$\Phi = \frac{-8c^2 \pm \sqrt{16c^4 - 48c^4}}{8} \qquad (4.4.15)$$

Substituting (4.4.15) in the energy E relation (4.4.1) gives

$$E = \pm\sqrt{-2}\, mc^2 \qquad (4.4.16)$$

Consider now a vacuum is full of photons . this means that

$$\frac{4\Phi^2}{c^4}+\frac{8\Phi}{c^2}-1=0 \qquad (4.4.17)$$

Using (4.4.14)

$$\Phi=\frac{-4c^2\pm\sqrt{16c^4+16c^4}}{8}$$

$$\Phi=\frac{-4c^2\pm4\sqrt{2}\,c^2}{8}=-\frac{1}{2}c^2\pm\frac{1}{\sqrt{2}}c^2 \qquad (4.4.18)$$

Inserting equation (4.4.18) in equation (4.4.1) the energy is given by

$$E=mc^2+2m(-\frac{1}{2}c^2\pm\frac{1}{\sqrt{2}}c^2)$$

$$E=\pm\sqrt{2}\,mc^2 \qquad (4.4.19)$$

(4.5) Discussion :

Non linear Generalized Lorentz transformation in equations (4.2.6) and (4.2.7) are used to find new generalized SR . This new transformation is non linear in t . This transformations deals with particles moving with constant initial velocity and constant acceleration under the action of a potential field . The spatial displacement is found in terms of initial velocity v_0 and potential per unit mass Φ . By assuming the speed of light is constant in all frames moving in a potential field, the transformation coefficient is found to depend on v_0 as well as Φ as shown by equation (4.2.11) . The new space and time transformations are shown in equations (4.2.6) and (4.2.7) . It is very interesting to note the expressions for γ , x and E reduces to that of SR when no field exists as shown by equations (4.2.13) , (4.2.6) , (4.2.7) and (4.2.18) respectively . Unlike SR , which recognize rest mass and kinetic energy only , the generalized energy expression (4.2.17) recognize potential and reduces to Newton energy relation , as equation (4.2.25) indicates , with kinetic and potential term , beside rest mass term .

The vacuum energy is found by minimizing GGR energy relation (4.3.1) , together with the velocity and potential given by (4.3.11) and (4.3.12) . one here assumes the vacuum consisting of photons , where we assume v to be equal to c in (4.3.11) . In this case vacuum energy is given by equation (4.3.14) to be

$$E_v = 2m_0c^2 \qquad (4.5.1)$$

According to figure (4.3.1) , one can assume vacuum as consisting of photon of energy

$$E_p = hf = 2m_0c^2$$

Beside matter and anti matter particle with energies

$$E_m = m_0c^2 \quad , \quad E_a = -m_0c^2$$

Thus the total vacuum energy is

$$E_v = E_p + E_m + E_a = 2m_0c^2 \qquad (4.5.2)$$

Where the particle exists in the energy state $= m_0c^2$

This is equivalent to the existence of the particle and anti particle representing this lower state , together with non interacting photon . When the photon is incident on this particle in the lower state $(-m_0c^2)$ it gives it an energy $(2m_0c^2)$. This the particle transfer itself to the state of energy (m_0c^2) appearing as a particle and leaving a vacancy in the state $(-m_0c^2)$ which appears as an anti particle , with numerical masses m_0c^2 . The photon of energy $2m_0c^2$ disappear producing a particle and anti particle pair of total numerical mass $2m_0c^2$.

Using also Mubarak energy conservation expression (4.4.1) , vacuum energy is again obtained by minimizing E and assuming vacuum to constitute stationary particles to get relation (4.4.16)

$$E = \sqrt{2}\, m_0c^2 i = \hbar w = i\hbar w_0$$

$$w = iw_0 \qquad w_0 = \sqrt{2}\, m_0c^2 \qquad (4.5.3)$$

The wave function of vacuum particles takes the form

$$\psi = Ae^{\frac{iE}{\hbar}t} = Ae^{iwt} = Ae^{-w_0t} \qquad (4.5.4)$$

Thus the number of vacuum particles is given by

$$n \sim |\psi|^2 \sim A^2\, e^{-2w_0t} \qquad (4.5.5)$$

This means that vacuum particles decay and transforms to other forms like elementary particles . This result conforms with what proposed by cosmology physists at the early universe .

(4.6) Conclusion :

The generalized Lorentz transformation and generalized SR can describe successfully the motion of particles in a field . It reduces to SR when no field exist . It also satisfy Newtonian limit by consisting of kinetic and potential term .

The energy expressions of GSR shows that vacuum fluctuation produces particles and anti particles and photon periodically . It shows also that vacuum decays to produce matter .

(4.7) Future work and recommendation :

The future work can be suggested to be in the following topics :

1) Description of electron hole electric and physical interactions on the bases of matter and antimatter model .
2) Make use of non linear Lorentz transformation to construct new particle physics model .
3) Promote complete model to describe matter – antimatter interaction on the basis of repulsive gravity model .

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