

Appendix (A)

The components of the internal forces for calculation of residuals are:

$$\{R^\varphi\} = \int_{-\frac{l}{2}}^{\frac{l}{2}} [B]^T \{S\} dl = \{R_1^\varphi \dots R_6^\varphi\}^T$$

And

$$R_1^\varphi = \frac{-F_1}{2} \left(1 + \frac{\partial u}{\partial x}\right)_1 + \frac{M_1}{2} \left(-\frac{\partial^2 v}{\partial x^2}\right)_1 - \frac{F_2}{2} \left(1 + \frac{\partial u}{\partial x}\right)_2 + \frac{M_2}{2} \left(-\frac{\partial^2 v}{\partial x^2}\right)_2$$

$$R_2^\varphi = \frac{-F_1}{2} \left(\frac{\partial v}{\partial x}\right)_1 - \frac{3M_1}{2l} \left(1 + \frac{\partial u}{\partial x}\right)_1 - \frac{F_2}{2} \left(\frac{\partial v}{\partial x}\right)_2 + \frac{3M_2}{2l} \left(1 + \frac{\partial u}{\partial x}\right)_2$$

$$R_3^\varphi = -\frac{5M_1}{4} \left(1 + \frac{\partial u}{\partial x}\right)_1 + \frac{M_2}{4} \left(1 + \frac{\partial u}{\partial x}\right)_2$$

$$R_4^\varphi = \frac{F_1}{2} \left(1 + \frac{\partial u}{\partial x}\right)_1 - \frac{M_1}{2} \left(-\frac{\partial^2 v}{\partial x^2}\right)_1 + \frac{F_2}{2} \left(1 + \frac{\partial u}{\partial x}\right)_2 - \frac{M_2}{2} \left(-\frac{\partial^2 v}{\partial x^2}\right)_2$$

$$R_5^\varphi = \frac{F_1}{2} \left(\frac{\partial v}{\partial x}\right)_1 + \frac{3M_1}{2l} \left(1 + \frac{\partial u}{\partial x}\right)_1 + \frac{F_2}{2} \left(\frac{\partial v}{\partial x}\right)_2 - \frac{3M_2}{2l} \left(1 + \frac{\partial u}{\partial x}\right)_2$$

$$R_6^\varphi = -\frac{M_1}{4} \left(1 + \frac{\partial u}{\partial x}\right)_1 + \frac{5M_2}{4} \left(1 + \frac{\partial u}{\partial x}\right)_2$$

Appendix (B)

Linear displacement stiffness matrix (K_{0L}):

$$K_0 + K_L =$$

$$\int_{\frac{-l}{2}}^{\frac{l}{2}} [B_0]^T [D][B_0] dx + \int_{\frac{-l}{2}}^{\frac{l}{2}} [B_0]^T [D][B_L] dx + \int_{\frac{-l}{2}}^{\frac{l}{2}} [B_L]^T [D][B_0] dx + \\ \int_{\frac{-l}{2}}^{\frac{l}{2}} [B_L]^T [D][B_L] dx$$

$$K_0 + K_L = \int_{\frac{-l}{2}}^{\frac{l}{2}} [B_0]^T [D][B_0] dx + \int_{\frac{-l}{2}}^{\frac{l}{2}} [B_0]^T [D][A\theta][G] dx + \\ \int_{\frac{-l}{2}}^{\frac{l}{2}} [G]^T [A\theta]^T [D][B_0] dx + \int_{\frac{-l}{2}}^{\frac{l}{2}} [G]^T [A\theta]^T [D][A\theta][G] dx$$

Where:

$$K_{0L11} = \frac{EA}{2l} \left(\left(1 + \frac{\partial u}{\partial x} \right)_2^2 + \left(1 + \frac{\partial u}{\partial x} \right)_1^2 \right) + \frac{EI}{2l} \left(\left(\frac{\partial^2 v}{\partial x^2} \right)_2^2 + \left(\frac{\partial^2 v}{\partial x^2} \right)_1^2 \right)$$

$$K_{0L12} = K_{0L21} = \frac{EA}{2l} \left(\left(1 + \frac{\partial u}{\partial x} \right) \frac{\partial v}{\partial x} \right)_2 + \left(\left(1 + \frac{\partial u}{\partial x} \right) \frac{\partial v}{\partial x} \right)_1 +$$

$$\frac{3EI}{2l^2} \left(\left(1 + \frac{\partial u}{\partial x} \right) \frac{\partial^2 v}{\partial x^2} \right)_1 - \left(\left(1 + \frac{\partial u}{\partial x} \right) \frac{\partial^2 v}{\partial x^2} \right)_2$$

$$K_{0L13} = K_{0L31} = \frac{EA}{8} \left(\left(1 + \frac{\partial u}{\partial x} \right) \frac{\partial v}{\partial x} \right)_2 - \left(\left(1 + \frac{\partial u}{\partial x} \right) \frac{\partial v}{\partial x} \right)_1 +$$

$$\frac{EI}{4l} \left(5 \left(1 + \frac{\partial u}{\partial x} \right) \frac{\partial^2 v}{\partial x^2} \right)_1 - \left(\left(1 + \frac{\partial u}{\partial x} \right) \frac{\partial^2 v}{\partial x^2} \right)_2$$

$$K_{0L14} = K_{0L41} = -\frac{EA}{2l} \left(\left(1 + \frac{\partial u}{\partial x} \right)_2^2 + \left(1 + \frac{\partial u}{\partial x} \right)_1^2 \right) - \frac{EI}{2l} \left(\left(\frac{\partial^2 v}{\partial x^2} \right)_2^2 + \left(\frac{\partial^2 v}{\partial x^2} \right)_1^2 \right)$$

$$K_{0L15} = K_{0L51} = -\frac{EA}{2l} \left(\left(\left(1 + \frac{\partial u}{\partial x} \right) \frac{\partial v}{\partial x} \right)_2 + \left(\left(1 + \frac{\partial u}{\partial x} \right) \frac{\partial v}{\partial x} \right)_1 \right) +$$

$$\frac{3EI}{2l^2} \left(\left(\left(1 + \frac{\partial u}{\partial x} \right) \frac{\partial^2 v}{\partial x^2} \right)_2 - \left(\left(1 + \frac{\partial u}{\partial x} \right) \frac{\partial^2 v}{\partial x^2} \right)_1 \right)$$

$$K_{0L16} = K_{0L61} = \frac{EA}{8} \left(\left(1 + \frac{\partial u}{\partial x} \right) \frac{\partial v}{\partial x} \right)_1 - \left(\left(1 + \frac{\partial u}{\partial x} \right) \frac{\partial v}{\partial x} \right)_2 +$$

$$\frac{EI}{4l} \left(\left(1 + \frac{\partial u}{\partial x} \right) \frac{\partial^2 v}{\partial x^2} \right)_1 - \left(5 \left(1 + \frac{\partial u}{\partial x} \right) \frac{\partial^2 v}{\partial x^2} \right)_2$$

$$K_{0L22} = -\frac{3EA}{5l} \left(\left(\frac{\partial v}{\partial x} \right)_2^2 + \left(\frac{\partial v}{\partial x} \right)_1^2 \right) + \frac{6EI}{l^3} \left(\left(1 + \frac{\partial u}{\partial x} \right)_2^2 - \left(1 + \frac{\partial u}{\partial x} \right)_1^2 \right)$$

$$K_{0L23} = K_{0L32} = \frac{EA}{160} \left(23 \left(\frac{\partial v}{\partial x} \right)_2^2 - 7 \left(\frac{\partial v}{\partial x} \right)_1^2 \right) +$$

$$\frac{3EI}{2l^2} \left(\left(1 + \frac{\partial u}{\partial x} \right)_2^2 + 3 \left(1 + \frac{\partial u}{\partial x} \right)_1^2 \right)$$

$$K_{0L24} = K_{0L42} = -\frac{EA}{2l} \left(\left(1 + \frac{\partial u}{\partial x} \right) \frac{\partial v}{\partial x} \right)_2 + \left(\left(1 + \frac{\partial u}{\partial x} \right) \frac{\partial v}{\partial x} \right)_1 +$$

$$\frac{3EI}{2l^2} \left(\left(1 + \frac{\partial u}{\partial x} \right) \frac{\partial^2 v}{\partial x^2} \right)_2 - \left(\left(1 + \frac{\partial u}{\partial x} \right) \frac{\partial^2 v}{\partial x^2} \right)_1$$

$$K_{0L25} = K_{0L52} = -\frac{3EA}{5l} \left(\left(\frac{\partial v}{\partial x} \right)_2^2 + \left(\frac{\partial v}{\partial x} \right)_1^2 \right) - \frac{6EI}{l^3} \left(\left(1 + \frac{\partial u}{\partial x} \right)_2^2 + \left(1 + \frac{\partial u}{\partial x} \right)_1^2 \right)$$

$$K_{0L26} = K_{0L62} = \frac{EA}{160} \left(23 \left(\frac{\partial v}{\partial x} \right)_1^2 - 7 \left(\frac{\partial v}{\partial x} \right)_2^2 \right) +$$

$$\frac{3EI}{2l^2} \left(3 \left(1 + \frac{\partial u}{\partial x} \right)_2^2 + \left(1 + \frac{\partial u}{\partial x} \right)_1^2 \right)$$

$$K_{0L33} = \frac{EAl}{480} \left(17 \left(\frac{\partial v}{\partial x} \right)_2^2 + 47 \left(\frac{\partial v}{\partial x} \right)_1^2 \right) + \frac{EI}{2l} \left(\left(1 + \frac{\partial u}{\partial x} \right)_2^2 + 7 \left(1 + \frac{\partial u}{\partial x} \right)_1^2 \right)$$

$$K_{0L34} = K_{0L43} = \frac{EA}{8} \left(\left(\left(1 + \frac{\partial u}{\partial x} \right) \frac{\partial v}{\partial x} \right)_1 - \left(\left(1 + \frac{\partial u}{\partial x} \right) \frac{\partial v}{\partial x} \right)_2 \right) +$$

$$\frac{EI}{4l} \left(\left(\left(1 + \frac{\partial u}{\partial x} \right) \frac{\partial^2 v}{\partial x^2} \right)_2 - 5 \left(\left(1 + \frac{\partial u}{\partial x} \right) \frac{\partial^2 v}{\partial x^2} \right)_1 \right)$$

$$K_{0L35} = K_{0L53} = -\frac{EA}{160} \left(23 \left(\frac{\partial v}{\partial x} \right)_2^2 + 7 \left(\frac{\partial v}{\partial x} \right)_1^2 \right) - \frac{3EI}{2l^2} \left(\begin{array}{c} \left(1 + \frac{\partial u}{\partial x} \right)_2^2 \\ + 3 \left(1 + \frac{\partial u}{\partial x} \right)_1^2 \end{array} \right)$$

$$K_{0L36} = K_{0L63} = -\frac{EAl}{60} \left(\left(\frac{\partial v}{\partial x} \right)_2^2 + \left(\frac{\partial v}{\partial x} \right)_1^2 \right) + \frac{EI}{l} \left(\begin{array}{c} \left(1 + \frac{\partial u}{\partial x} \right)_2^2 \\ - \left(1 + \frac{\partial u}{\partial x} \right)_1^2 \end{array} \right)$$

$$K_{0L44} = \frac{EA}{2l} \left(\left(1 + \frac{\partial u}{\partial x} \right)_2^2 + \left(1 + \frac{\partial u}{\partial x} \right)_1^2 \right) + \frac{EI}{2l} \left(\left(\frac{\partial v}{\partial x} \right)_2^2 + \left(\frac{\partial v}{\partial x} \right)_1^2 \right)$$

$$K_{0L45} = K_{0L54} = \frac{EA}{2l} \left(\left(\left(1 + \frac{\partial u}{\partial x} \right) \frac{\partial v}{\partial x} \right)_2 + \left(\left(1 + \frac{\partial u}{\partial x} \right) \frac{\partial v}{\partial x} \right)_1 \right) +$$

$$\frac{3EI}{2l^2} \left(\left(\left(1 + \frac{\partial u}{\partial x} \right) \frac{\partial^2 v}{\partial x^2} \right)_1 - \left(\left(1 + \frac{\partial u}{\partial x} \right) \frac{\partial^2 v}{\partial x^2} \right)_2 \right)$$

$$K_{0L46} = K_{0L64} = \frac{EA}{8} \left(\left(\left(1 + \frac{\partial u}{\partial x} \right) \frac{\partial v}{\partial x} \right)_2 - \left(\left(1 + \frac{\partial u}{\partial x} \right) \frac{\partial v}{\partial x} \right)_1 \right) +$$

$$\frac{EI}{4l} \left(5 \left(\left(1 + \frac{\partial u}{\partial x} \right) \frac{\partial^2 v}{\partial x^2} \right)_2 - \left(\left(1 + \frac{\partial u}{\partial x} \right) \frac{\partial^2 v}{\partial x^2} \right)_1 \right)$$

$$K_{0L55} = \frac{3EA}{5l} \left(\left(\frac{\partial v}{\partial x} \right)_2^2 - \left(\frac{\partial v}{\partial x} \right)_1^2 \right) + \frac{6EI}{l^3} \left(\left(1 + \frac{\partial u}{\partial x} \right)_2^2 + \left(1 + \frac{\partial u}{\partial x} \right)_1^2 \right)$$

$$K_{0L56} = K_{0L65} = \frac{EA}{160} \left(7 \left(\frac{\partial v}{\partial x} \right)_2^2 - 23 \left(\frac{\partial v}{\partial x} \right)_1^2 \right) - \frac{3EI}{2l^2} \left(\begin{array}{c} 3 \left(1 + \frac{\partial u}{\partial x} \right)_2^2 \\ + \left(1 + \frac{\partial u}{\partial x} \right)_1^2 \end{array} \right)$$

$$K_{0L66} = \frac{EAl}{480} \left(47 \left(\frac{\partial v}{\partial x} \right)_2^2 + 17 \left(\frac{\partial v}{\partial x} \right)_1^2 \right) + \frac{EI}{2l} \left(7 \left(1 + \frac{\partial u}{\partial x} \right)_2^2 + \left(1 + \frac{\partial u}{\partial x} \right)_1^2 \right)$$

Appendix (C)

```
clc
clear all
close all
g = ' cantilever beam '
%%%%%%%%%%%%% CANTILEVER
BEAM%%%%%%%%%%%%%
E = input('Enter Youngs Modulus of beam:');
j = input('Enter 1 for circular c/s, 2 for
rectangular c/s or 3 to enter value of I
manually:');
if(j==1)
d = input('Enter Diameter:');
I = pi*(d/2)^4/4
elseif(j==2)
b = input('Enter width of beam:');
h= input('Enter height of beam:');
I = b*h^3/12
elseif(j==3)
I = input('Enter value of I manually:');
end
n = input('Enter number of elements:');
%L = length of beam
L = input('Enter Length of the beam: \n');
%input length if each element, 11, 12, ..., ln
%l = [3; 3];
for i=1:n
%fprintf('element %g', i);
%l(i)=input('Enter length of element :');
l(i)=L/n;
end
%k = [E*A/le 0 0 -E*A/le 0 0;0 0 12*E*I/le^3
6*E*I/le^2 0 -12*E*I/le^3
%6*E*I/le^2;0 6*E*I/le^2 4*E*I/le 0 -6*E*I/le^2
2*E*I/le;-E*A/le 0 0 E*A/le
%0 0 0 -12*E*I/le^3 -6*E*I/le^2 0 12*E*I/le^3 -
6*E*I/le^2;0 6*E*I/le^2
%2*E*I/le 0 -6*E*I/le^2 4*E*I/le];
%enter boundary condition at each node
%K2 = zeros(3*(n+1),3*(n+1));
%K = zeros(3*(n+1),3*(n+1));
%K11 = zeros(3*(n+1),3*(n+1));
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%DB = zeros(4,6);
%B = zeros(4,6);
%G = zeros(8,6);
K = zeros(3*(n+1),3*(n+1));
K11 = zeros(3*(n+1),3*(n+1));
K2 = zeros(3*(n+1),3*(n+1));
ELOAD=zeros(6,n);
DB0 = zeros(4,6);
DB1 = zeros(4,6);
DB = zeros(4,6);
B = zeros(4,6);
G = zeros(4,6);
ds = zeros((3*(n+1)),1);
dds = zeros((3*(n+1)),1);
drs = zeros((3*(n+1)),1);
for i = n+1:-1:3
fprintf('Node %g \n', i);
lc =0;
if (lc == 1)
K2((i-1)*3+1,:) = [];
K2(:,(i-1)*3+1) = [];
ds((i-1)*3+1) = 1;
end
end
drs(1)=1;
drs(2)=1;
drs(3)=1;
%BC

r = size(K2,1);

ASLOD=zeros(3*(n+1),1);
RLOD=zeros(1,3*(n+1));
Fr=zeros(3*(n+1),1);
SR1=zeros(4,n);
deltaSR1=zeros(4,n);
Theta=zeros(8,n);
RINT=zeros(6,n);
ddEs=zeros(6,n);
dEs=zeros(6,n);
dEsi=zeros(6,n);
Ks = zeros(3*(n+1),3*(n+1));
K01=zeros(3*(n+1),3*(n+1));
Ksigma=zeros(3*(n+1),3*(n+1));

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fprintf('Enter forces position');
IP = input('');
fprintf('Enter forces value');
RLOD(IP) = input('');
INC=input('INC=');
FACTO=1/INC ;
niter=input('niter=');
Res1=zeros(3*(n+1),1);
DEPSI1=zeros(1,n);
DEPSI2=zeros(1,n);
DEX1=zeros(1,n);
DEX2=zeros(1,n);
TRINT=zeros(3*(n+1),1);

for jnc=1:INC
    LINC = RLOD'*FACTO;
    ASLOD=ASLOD+LINC;
    for iter =1:niter
        if NTYPE==1
            ASLOD1=LINC;
        else
            ASLOD1=ASLOD-TRINT;
        end
    K = zeros(3*(n+1),3*(n+1));
    K01=zeros(3*(n+1),3*(n+1));
    Ksigma=zeros(3*(n+1),3*(n+1));

    if n == 1
    le = l(n);
    k = [(E*A)/le 0 0 (-E*A)/le 0 0;0 (12*E*I)/le^3
    (6*E*I/le^2) 0 (-12*E*I)/le^3 (6*E*I)/le^2;
    0 (6*E*I)/le^2 (4*E*I)/le 0 (-6*E*I)/le^2
    (2*E*I)/le;(-E*A)/le 0 0 (E*A)/le 0 0;
    0 (-12*E*I)/le^3 (-6*E*I/le^2) 0
    (12*E*I)/le^3 (-6*I*E)/le^2;
    0 (6*E*I)/le^2 (2*E*I)/le 0 (-6*E*I)/le^2
    (4*E*I)/le];
    K = k;
    else
        for i = 1:n-1
            if (i == 1)
                le = l(i);

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k = [(E*A)/le 0 0 (-E*A)/le 0 0;0 (12*E*I)/le^3
(6*E*I/le^2) 0 (-12*E*I)/le^3 (6*E*I)/le^2;
0 (6*E*I)/le^2 (4*E*I)/le 0 (-6*E*I)/le^2
(2*E*I)/le;(-E*A)/le 0 0 (E*A)/le 0 0;
0 (-12*E*I)/le^3 (-6*E*I/le^2) 0
(12*E*I)/le^3 (-6*I*E)/le^2;
0 (6*E*I)/le^2 (2*E*I)/le 0 (-6*E*I)/le^2
(4*E*I)/le];

K = wextend(2,'zpd',k,3, 'r');
else
K = wextend(2,'zpd',K,3, 'r');
end
le = l(i+1);
k = [(E*A)/le 0 0 (-E*A)/le 0 0;0 (12*E*I)/le^3
(6*E*I/le^2) 0 (-12*E*I)/le^3 (6*E*I)/le^2;
0 (6*E*I)/le^2 (4*E*I)/le 0 (-6*E*I)/le^2
(2*E*I)/le;(-E*A)/le 0 0 (E*A)/le 0 0;
0 (-12*E*I)/le^3 (-6*E*I/le^2) 0
(12*E*I)/le^3 (-6*I*E)/le^2;
0 (6*E*I)/le^2 (2*E*I)/le 0 (-6*E*I)/le^2
(4*E*I)/le];
K1 = wextend(2,'zpd',k,3*i, 'l');
K= K+K1;
end
end
% BC
%K2 = K;
if n == 1
le = l(n);
ksigma=
[(SR1(1,n)/(2*le))+(SR1(3,n)/(2*le)), (3/(2*le^2))
*((SR1(2,n))-(SR1(4,n))), (1/(4*le))*(
5*SR1(2,n)+SR1(4,n)), (
1/(2*le))*(SR1(1,n)+SR1(3,n)), (3/(2*le^2))*(SR1(4
,n)-SR1(2,n)), (1/(4*le))*((SR1(2,n))-5*SR1(4,n));
(3/(2*le^2))*(SR1(2,n))-
(SR1(4,n))), (3/(5*le))*(SR1(1,n) +
SR1(3,n)), (1/160)*(23*(SR1(3,n))-
7*SR1(1,n)), (3/(2*le^2))*(SR1(4,n))-
(SR1(2,n))), (-
3/(5*le))*(SR1(1,n)+SR1(3,n)), (1/160)*(23*(SR1(1,
n))-7*(SR1(3,n)));

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(1/(4*1e))*(-
5*SR1(2,n)+SR1(4,n)),(1/160)*(23*(SR1(3,n))-_
7*SR1(1,n)),(1e/480)*(47*SR1(1,n)+17*SR1(3,n)),(1/
(4*1e))*((-SR1(4,n))+(5*SR1(2,n))),(-
1/160)*(23*(SR1(3,n))-7*(SR1(1,n))),(-
1e/60)*((SR1(1,n))+(SR1(3,n)));
(-
1/(2*1e))*(SR1(1,n)+SR1(3,n)),(3/(2*1e^2))*((SR1(
4,n))-(SR1(2,n))), (1/(4*1e))*(-
(SR1(4,n))+(5*SR1(2,n))), (1/(2*1e))*((SR1(1,n)+SR1(
3,n)),(3/(2*1e^2))*((SR1(2,n))-
(SR1(4,n))), (1/(4*1e))*((5*(SR1(4,n))-(SR1(2,n))) ;
(3/(2*1e^2))*((SR1(4,n)-SR1(2,n)), (-
3/(5*1e))*((SR1(1,n)+SR1(3,n)), (-
1/160)*(23*(SR1(3,n))-
7*(SR1(1,n))), (3/(2*1e^2))*((SR1(2,n))-
(SR1(4,n))), (3/(5*1e))*((SR1(3,n)+SR1(1,n)), (-
1/160)*(23*(SR1(1,n))-7*(SR1(3,n)));
(1/(4*1e))*((SR1(2,n))-
5*SR1(4,n)),(1/160)*(23*(SR1(1,n))-
7*(SR1(3,n))),(-
1e/60)*((SR1(1,n))+(SR1(3,n))), (1/(4*1e))*((5*(SR1(
4,n))-(SR1(2,n))), (-1/160)*(23*(SR1(1,n))-
7*(SR1(3,n))), (1e/480)*(47*(SR1(3,n))+17*(SR1(1,n)))
);

Ksigma = ksigma;
k01=[((E*A)/(2*1e))*(((1+(Theta(5,n)))^2)+(1+(The
ta(1,n)))^2)+((E*I)/(2*1e))*((Theta(7,n))^2+(The
ta(3,n))^2), ((E*A)/(2*1e))*((1+(Theta(5,n)))*(The
ta(6,n))+(1+(Theta(1,n)))*(Theta(2,n)))+((3*E*I)/(
2*1e^2))*(((1+(Theta(1,n)))*(Theta(3,n)))-
((1+(Theta(5,n)))*(Theta(7,n)))), ((E*A)/8)*(((1+(
Theta(5,n)))*(Theta(6,n)))-
((1+(Theta(1,n)))*(Theta(2,n))))+((E*I)/(4*1e))*(
5*((1+(Theta(1,n)))*(Theta(3,n)))-
((1+(Theta(5,n)))*(Theta(7,n))), ((-(
E*A)/(2*1e))*((1+(Theta(5,n)))^2+(1+(Theta(1,n)))^2)-
((E*I)/(2*1e))*((Theta(7,n))^2+(Theta(3,n))^2), ((
-
E*A)/(2*1e))*(((1+(Theta(5,n)))*(Theta(6,n)))+((1+(
Theta(1,n)))*(Theta(2,n))))+((3*E*I)/(2*1e)^2)*(
(1+(Theta(5,n)))*(Theta(7,n)))-
((1+(Theta(1,n)))*(Theta(3,n)))), ((E*A)/8)*(((1+(

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Theta(1,n))) *Theta(2,n))-
((1+(Theta(5,n))) *Theta(6,n)))+((E*I)/(4*le))*(((1+
(Theta(1,n)))* (Theta(3,n))))-
(5*(1+(Theta(5,n)))* (Theta(7,n)))) ;
((E*A)/(2*le))*((1+(Theta(5,n)))* (Theta(6,n))+ (1+
(Theta(1,n)))* (Theta(2,n)))+((3*E*I)/(2*le^2))*((1+
(Theta(1,n)))* (Theta(3,n))-
((1+(Theta(5,n)))* (Theta(7,n)))), ((-
3*E*A)/(5*le))*((Theta(6,n))^2+ (Theta(2,n))^2)+((6*E*I)/le^3)*((1+(Theta(5,n)) )^2-
(1+(Theta(1,n)) )^2), ((E*A)/160)*((23*(Theta(6,n)) ^2)-
7*(Theta(2,n)) ^2)+((3*E*I)/(2*le^2))*((1+(Theta(5 ,n)) )^2+3*(1+(Theta(1,n)) )^2), ((-
E*A)/(2*le))*(((1+(Theta(5,n)))* (Theta(6,n)))+((1+
(Theta(1,n)))* (Theta(2,n)))+((3*E*I)/(2*le^2))*(((1+(Theta(5,n)))* (Theta(7,n)))-((1+(Theta(1,n)))* (Theta(3,n)))), ((-
3*E*A)/(5*le))*((Theta(6,n))^2+ (Theta(2,n))^2)-
((6*E*I)/le^3)*((1+(Theta(1,n)) )^2+ (1+(Theta(5,n)) )^2), ((E*A)/160)*((23*(Theta(2,n)) ^2)-
7*(Theta(6,n)) ^2)+((3*E*I)/(2*le^2))* (3*(1+(Theta(5,n)) )^2+ (1+(Theta(1,n)) )^2);
((E*A)/8)*(((1+(Theta(5,n)))* (Theta(6,n)))-((1+(Theta(1,n)))* (Theta(2,n))))+((E*I)/(4*le))*(
5*((1+(Theta(1,n)))* (Theta(3,n))-
((1+(Theta(5,n)))* (Theta(7,n)))), ((E*A)/160)*((23*(Theta(6,n)) ^2)-
7*(Theta(2,n)) ^2)+((3*E*I)/(2*le^2))*((1+(Theta(5 ,n)) )^2+3*(1+(Theta(1,n)) )^2), ((E*A*le)/480)*(17*((Theta(6,n)) ^2)+47*(Theta(2,n)) ^2)+((E*I)/(2*le))*((1+(Theta(5,n)) )^2+7*(1+(Theta(1,n)) )^2), ((E*A)/8)*(((1+(Theta(1,n)))* (Theta(2,n)))-((1+(Theta(5,n)))* (Theta(6,n)))+((E*I)/(4*le))*((1+(Theta(5,n)))* (Theta(7,n))-
5*((1+(Theta(1,n)))* (Theta(3,n)))), ((-
E*A)/160)*((23*(Theta(6,n)) ^2)+7*(Theta(2,n)) ^2)-((3*E*I)/(2*le^2))*((1+(Theta(5,n)) )^2+3*(1+(Theta(1,n)) )^2), ((-
E*A*le)/60)*(((Theta(2,n)) ^2)+(Theta(6,n)) ^2)+((E*I)/le)*((1+(Theta(5,n)) )^2+ (1+(Theta(1,n)) )^2); ((-
E*A)/(2*le))*((1+(Theta(5,n)) )^2+ (1+(Theta(1,n)) )^2)-

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((E*I)/(2*le))*((Theta(7,n))^2+(Theta(3,n))^2), ((-
E*A)/(2*le))*(((1+(Theta(5,n)))*(Theta(6,n)))+((1+
(Theta(1,n)))*(Theta(2,n))))+((3*E*I)/(2*le^2))*(
((1+(Theta(5,n)))*(Theta(7,n)))-
((1+(Theta(1,n)))*(Theta(3,n)))), ((E*A)/8)*(((1+
Theta(1,n)))*(Theta(2,n))-
((1+(Theta(5,n)))*(Theta(6,n)))+((E*I)/(4*le))*((
(1+(Theta(5,n)))*(Theta(7,n)))-
5*((1+(Theta(1,n)))*(Theta(3,n)))), ((E*A)/(2*le)
)*((1+(Theta(5,n)))^2+(1+(Theta(1,n)))^2)+((E*I)/
(2*le))*((Theta(7,n))^2+(Theta(3,n))^2), ((E*A)/(2
*le))*(((1+(Theta(5,n)))*(Theta(6,n)))+((1+(Theta
(1,n)))*(Theta(2,n))))+((3*E*I)/(2*le^2))*(((1+(T
heta(1,n)))*(Theta(3,n)))-
((1+(Theta(5,n)))*(Theta(7,n)))), ((E*A)/8)*(((1+
Theta(5,n)))*(Theta(6,n))-
((1+(Theta(1,n)))*(Theta(2,n)))+((E*I)/(4*le))*((
5*(1+(Theta(5,n)))*(Theta(7,n)))-
((1+(Theta(1,n)))*(Theta(3,n)))) ;
((-_
E*A)/(2*le))*(((1+(Theta(5,n)))*(Theta(6,n))+((1+
(Theta(1,n)))*(Theta(2,n))))+((3*E*I)/(2*le)^2)*(
(1+(Theta(5,n)))*(Theta(7,n)))-
((1+(Theta(1,n)))*(Theta(3,n))), ((-
3*E*A)/(5*le))*((Theta(6,n))^2+(Theta(2,n))^2)-
((6*E*I)/le^3)*((1+(Theta(1,n)))^2+(1+(Theta(5,n)
))^2), ((-
E*A)/160)*((23*(Theta(6,n))^2)+7*(Theta(2,n))^2)-
((3*E*I)/(2*le^2))*((1+(Theta(5,n)))^2+3*(1+(Thet
a(1,n)))^2), ((E*A)/(2*le))*(((1+(Theta(5,n)))*(Th
eta(6,n)))+((1+(Theta(1,n)))*(Theta(2,n))))+((3*E
*I)/(2*le^2))*(((1+(Theta(1,n)))*(Theta(3,n)))-
((1+(Theta(5,n)))*(Theta(7,n)))), ((3*E*A)/(5*le))*
((Theta(6,n))^2-
(Theta(2,n))^2)+((6*E*I)/le^3)*((1+(Theta(1,n)))^
2+(1+(Theta(5,n)))^2), ((E*A)/160)*((7*(Theta(6,n)
))^2)-23*(Theta(2,n))^2)-
((3*E*I)/(2*le^2))*((3*(1+(Theta(5,n)))^2+(1+(Thet
a(1,n)))^2) ;
((E*A)/8)*(((1+(Theta(1,n)))*Theta(2,n))-(
(1+(Theta(5,n)))*Theta(6,n)))+((E*I)/(4*le))*(((1+
(Theta(1,n)))*(Theta(3,n)))-
(5*(1+(Theta(5,n)))*(Theta(7,n)))), ((E*A)/160)*((

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```

23*(Theta(2,n))^2)-
7*(Theta(6,n))^2)+((3*E*I)/(2*le^2))*(3*(1+(Theta
(5,n)))^2+(1+(Theta(1,n)))^2),((-*
E*A*le)/60)*(((Theta(2,n))^2)+(Theta(6,n))^2)+((E
*I)/le)*((1+(Theta(5,n)))^2+(1+(Theta(1,n)))^2),(
(E*A)/8)*(((1+(Theta(5,n)))*(Theta(6,n))-
((1+(Theta(1,n)))*(Theta(2,n)))+((E*I)/(4*le))*((
5*(1+(Theta(5,n)))*(Theta(7,n))-
((1+(Theta(1,n))))*(Theta(3,n)))),((E*A)/160)*((7
*(Theta(6,n))^2)-23*(Theta(2,n))^2)-
((3*E*I)/(2*le^2))*(3*(1+(Theta(5,n)))^2+(1+(Theta
(1,n)))^2),((E*A*le)/480)*(47*((Theta(6,n))^2)+1
7*(Theta(2,n))^2)+((E*I)/(2*le))*((7*(1+(Theta(5,n
)))*((1+(Theta(1,n)))^2));
K01 = k01;
else
for i = 1:n-1
if (i == 1)
le = l(i);

ksigma=
[(SR1(1,i)/(2*le)+(SR1(3,i)/(2*le)),(3/(2*le^2))
*((SR1(2,i)-(SR1(4,i))),,(1/(4*le))*(-
5*SR1(2,i)+SR1(4,i)),,-
1/(2*le))*(SR1(1,i)+SR1(3,i)),,(3/(2*le^2))*((SR1(4
,i)-SR1(2,i)),,(1/(4*le))*((SR1(2,i))-5*SR1(4,i));
(3/(2*le^2))*((SR1(2,i))-
(SR1(4,i))),,(3/(5*le))*((SR1(1,i) +
SR1(3,i)),,(1/160)*(23*(SR1(3,i))-
7*SR1(1,i)),,(3/(2*le^2))*((SR1(4,i))-
(SR1(2,i))),,-
3/(5*le))*((SR1(1,i)+SR1(3,i)),,(1/160)*(23*(SR1(1,
i))-7*(SR1(3,i)));
(1/(4*le))*(-
5*(SR1(2,i))+SR1(4,i)),,(1/160)*(23*(SR1(3,i))-
7*SR1(1,i)),,(le/480)*(47*SR1(1,i)+17*SR1(3,i)),,(1
/(4*le))*(-(SR1(4,i))+((5*SR1(2,i))),,-
1/160)*(23*(SR1(3,i))-7*(SR1(1,i))),,-
le/60)*((SR1(1,i))+(SR1(3,i)));
(-
1/(2*le))*(SR1(1,i)+SR1(3,i)),,(3/(2*le^2))*((SR1(
4,i))-(SR1(2,i))),,(1/(4*le))*(-
(SR1(4,i))+((5*SR1(2,i))),,(1/(2*le))*((SR1(1,i)+SR1

```

```

(3,i)), (3/(2*le^2))*((SR1(2,i))-  

(SR1(4,i))), (1/(4*le))* (5*(SR1(4,i))-(SR1(2,i)));  

(3/(2*le^2))* (SR1(4,i)-SR1(2,i)), (-  

3/(5*le))* (SR1(1,i)+SR1(3,i)), (-  

1/160)*(23*(SR1(3,i))-  

7*(SR1(1,i))), (3/(2*le^2))*((SR1(2,i))-  

(SR1(4,i))), (3/(5*le))* (SR1(3,i)+SR1(1,i)), (-  

1/160)*(23*(SR1(1,i))-7*(SR1(3,i)));  

(1/(4*le))* ((SR1(2,i))-  

5*SR1(4,i)), (1/160)*(23*(SR1(1,i))-  

7*(SR1(3,i))), (-  

le/60)* ((SR1(1,i))+(SR1(3,i))), (1/(4*le))* (5*(SR1  

(4,i))-(SR1(2,i))), (-1/160)*(23*(SR1(1,i))-  

7*(SR1(3,i))), (le/480)*(47*(SR1(3,i))+17*(SR1(1,i  

))))];  

Ksigma = wextend(2, 'zpd', ksigma, 3, 'r');  

%for ir = 1:6
    %for ic = 1:6
        %ick = 3*(n-1)+ ic;
        %icr = 3*(n-1)+ ir;
        %Ksigma(ick,icr)=Ksigma(ick,icr)+  

ksigma(ic,ir);
    %end
%end
k01 =
[((E*A)/(2*le))*(((1+(Theta(5,i)))^2)+(1+(Theta(1  

,i)))^2)+((E*I)/(2*le))*((Theta(7,i))^2+(Theta(3,  

i))^2), ((E*A)/(2*le))*((1+(Theta(5,i)))*(Theta(6,  

i))+(1+(Theta(1,i)))*(Theta(2,i)))+((3*E*I)/(2*le  

^2))*(((1+(Theta(1,i)))*(Theta(3,i)))-  

((1+(Theta(5,i)))*(Theta(7,i)))), ((E*A)/8)*(((1+  

Theta(5,i)))*(Theta(6,i))-  

((1+(Theta(1,i)))*(Theta(2,i))))+((E*I)/(4*le))*(  

5*((1+(Theta(1,i)))*(Theta(3,i)))-  

((1+(Theta(5,i)))*(Theta(7,i)))), ((-  

E*A)/(2*le))*((1+(Theta(5,i)))^2+(1+(Theta(1,i)))  

^2)-  

((E*I)/(2*le))*((Theta(7,i))^2+(Theta(3,i))^2), ((  

-  

E*A)/(2*le))*(((1+(Theta(5,i)))*(Theta(6,i))+(1+  

(Theta(1,i)))*(Theta(2,i)))+((3*E*I)/(2*le)^2)*(  

1+(Theta(5,i)))*(Theta(7,i)))-  

((1+(Theta(1,i)))*(Theta(3,i)))), ((E*A)/8)*(((1+  

Theta(1,i)))*Theta(2,i))-
```

```

((1+(Theta(5,i)))*Theta(6,i)))+((E*I)/(4*le))*(((1+(Theta(1,i)))*Theta(3,i)))-
(5*(1+(Theta(5,i)))*Theta(7,i)));
((E*A)/(2*le))*((1+(Theta(5,i)))*Theta(6,i))+((1+(Theta(1,i)))*Theta(2,i))+((3*E*I)/(2*le^2))*((1+(Theta(1,i)))*Theta(3,i))-
((1+(Theta(5,i)))*Theta(7,i))),((-3*E*A)/5)*((Theta(6,i))^2+(Theta(2,i))^2)+((6*E*I)/le^3)*((1+(Theta(5,i)))^2-
(1+(Theta(1,i)))^2),((E*A)/160)*((23*(Theta(6,i))^2)-
7*(Theta(2,i))^2)+((3*E*I)/(2*le^2))*((1+(Theta(5,i)))^2+3*(1+(Theta(1,i)))^2),((-E*A)/(2*le))*(((1+(Theta(5,i)))*Theta(6,i))+((1+(Theta(1,i)))*Theta(2,i)))+((3*E*I)/(2*le^2))*(((1+(Theta(5,i)))*Theta(7,i))-
((1+(Theta(1,i)))*Theta(3,i))),((-3*E*A)/(5*le))*((Theta(6,i))^2+(Theta(2,i))^2)-
((6*E*I)/le^3)*((1+(Theta(1,i)))^2+(1+(Theta(5,i)))^2),((E*A)/160)*((23*(Theta(2,i))^2)-
7*(Theta(6,i))^2)+((3*E*I)/(2*le^2))*((3*(1+(Theta(5,i)))^2+(1+(Theta(1,i)))^2));
((E*A)/8)*(((1+(Theta(5,i)))*Theta(6,i))-
((1+(Theta(1,i)))*Theta(2,i)))+((E*I)/(4*le))*((5*((1+(Theta(1,i)))*Theta(3,i))-
((1+(Theta(5,i)))*Theta(7,i))),((E*A)/160)*((23*(Theta(6,i))^2)-
7*(Theta(2,n))^2)+((3*E*I)/(2*le^2))*((1+(Theta(5,i)))^2+3*(1+(Theta(1,i)))^2),((E*A*le)/480)*(17*((Theta(6,i))^2)+47*(Theta(2,i))^2)+((E*I)/(2*le))*((1+(Theta(5,i)))^2+7*(1+(Theta(1,i)))^2),((E*A)/8)*(((1+(Theta(1,i)))*Theta(2,n))-
((1+(Theta(5,i)))*Theta(6,i)))+((E*I)/(4*le))*((1+(Theta(5,i)))*Theta(7,i))-
5*((1+(Theta(1,n)))*Theta(3,i))),((-E*A)/160)*((23*(Theta(6,i))^2)+7*(Theta(2,n))^2)-
((3*E*I)/(2*le^2))*((1+(Theta(5,i)))^2+3*(1+(Theta(1,i)))^2),((-E*A*le)/60)*(((Theta(2,i))^2)+(Theta(6,i))^2)+((E*I)/le)*((1+(Theta(5,i)))^2+((1+(Theta(1,i)))^2);
((-E*A)/(2*le))*((1+(Theta(5,i)))^2+(1+(Theta(1,i)))^2)-
((E*I)/(2*le))*((Theta(7,i))^2+(Theta(3,i))^2),((E*I)/(2*le))*((Theta(7,i))^2+(Theta(3,i))^2),

```

$$\begin{aligned}
& - \\
& \frac{E^*A}{(2*le)} * (((1+(\Theta(5,i)))*(\Theta(6,i)))+((1+(\Theta(1,i)))*(\Theta(2,i))))+((3*E^*I)/(2*le^2))* \\
& (((1+(\Theta(5,i)))*(\Theta(7,i)))- \\
& ((1+(\Theta(1,i)))*(\Theta(3,i)))), ((E^*A)/8)*(((1+ \\
& \Theta(1,i)))*(\Theta(2,i)))- \\
& ((1+(\Theta(5,i)))*(\Theta(6,i)))+((E^*I)/(4*le))*((\\
& (1+(\Theta(5,i)))*(\Theta(7,i)))- \\
& 5*((1+(\Theta(1,i)))*(\Theta(3,i)))), ((E^*A)/(2*le) \\
&)*((1+(\Theta(5,i)))^2+(1+(\Theta(1,i)))^2)+((E^*I)/ \\
& (2*le))*((\Theta(7,i))^2+(\Theta(3,i))^2), ((E^*A)/(2 \\
& *le))*(((1+(\Theta(5,i)))*(\Theta(6,i)))+((1+ \\
& \Theta(1,i)))*(\Theta(2,i))))+((3*E^*I)/(2*le^2))*(((1+ \\
& \Theta(1,i)))*(\Theta(3,i)))- \\
& ((1+(\Theta(5,i)))*(\Theta(7,i))), ((E^*A)/8)*(((1+ \\
& \Theta(5,i)))*(\Theta(6,i)))- \\
& ((1+(\Theta(1,i)))*(\Theta(2,i)))+((E^*I)/(4*le))*((\\
& 5*(1+(\Theta(5,i)))*(\Theta(7,i)))- \\
& ((1+(\Theta(1,i)))*(\Theta(3,i)))); \\
& ((- \\
& E^*A)/(2*le))*(((1+(\Theta(5,i)))*(\Theta(6,i)))+((1+ \\
& (\Theta(1,i)))*(\Theta(2,i))))+((3*E^*I)/(2*le^2))* \\
& ((1+(\Theta(5,i)))*(\Theta(7,i)))- \\
& ((1+(\Theta(1,i)))*(\Theta(3,i))), ((- \\
& 3*E^*A)/(5*le))*((\Theta(6,i))^2+(\Theta(2,i))^2)- \\
& ((6*E^*I)/le^3)*((1+(\Theta(1,i)))^2+(1+(\Theta(5,i) \\
&))^2), ((- \\
& E^*A)/160)*((23*(\Theta(6,i))^2)+7*(\Theta(2,i))^2)- \\
& ((3*E^*I)/(2*le^2))*((1+(\Theta(5,i)))^2+3*(1+ \\
& (\Theta(1,i)))^2), ((E^*A)/(2*le))*(((1+(\Theta(5,i)))*(\Theta \\
& (6,i)))+((1+(\Theta(1,i)))*(\Theta(2,i))))+((3*E^*I)/(2*le^2))* \\
& (((1+(\Theta(1,i)))*(\Theta(3,i)))- \\
& ((1+(\Theta(5,i)))*(\Theta(7,i)))), ((3*E^*A)/(5*le))* \\
& ((\Theta(6,i))^2- \\
& (\Theta(2,i))^2)+((6*E^*I)/le^3)*((1+(\Theta(1,i)))^ \\
& 2+(1+(\Theta(5,i)))^2), ((E^*A)/160)*((7*(\Theta(6,i) \\
&)^2)-23*(\Theta(2,i))^2)- \\
& ((3*E^*I)/(2*le^2))*((3*(1+(\Theta(5,i)))+2+(1+ \\
& (\Theta(1,i)))^2); \\
& ((E^*A)/8)*(((1+(\Theta(1,i)))*(\Theta(2,i))- \\
& ((1+(\Theta(5,i)))*(\Theta(6,i))))+((E^*I)/(4*le))*((\\
& (1+(\Theta(1,i)))*(\Theta(3,i)))- \\
& (5*(1+(\Theta(5,i)))*(\Theta(7,i)))), ((E^*A)/160)*((\\
& 23*(\Theta(2,i))^2)- \\
&
\end{aligned}$$

```

7*(Theta(6,i))^2+((3*E*I)/(2*le^2))* (3*(1+(Theta
(5,i)))^2+(1+(Theta(1,i)))^2),((-*
E*A*le)/60)*(((Theta(2,i))^2)+(Theta(6,i))^2)+((E
*I)/le)*((1+(Theta(5,i)))^2+(1+(Theta(1,i)))^2),(
(E*A)/8)*(((1+(Theta(5,i)))*(Theta(6,i))-
((1+(Theta(1,i)))*(Theta(2,i)))+((E*I)/(4*le))*((
5*(1+(Theta(5,i)))*(Theta(7,i)))-(
(1+(Theta(1,i))))*(Theta(3,i)))),((E*A)/160)*((7
*(Theta(6,i))^2)-23*(Theta(2,i))^2)-
((3*E*I)/(2*le^2))* (3*(1+(Theta(5,i)))^2+(1+(Theta
(1,i)))^2),((E*A*le)/480)*(47*((Theta(6,i))^2)+1
7*(Theta(2,i))^2)+((E*I)/(2*le))* (7*(1+(Theta(5,i
)) )^2+(1+(Theta(1,i)))^2)];
K01 = wextend(2,'zpd',k01,3, 'r');
else
    Ksigma = wextend(2,'zpd',Ksigma,3, 'r');
K01 = wextend(2,'zpd',K01,3, 'r');
end
le = l(i+1);
ksigma=
[(SR1(1,n)/(2*le)+(SR1(3,n)/(2*le)),(3/(2*le^2))
*((SR1(2,n)-(SR1(4,n))), (1/(4*le))*(-
5*SR1(2,n)+SR1(4,n)),(-
1/(2*le))*(SR1(1,n)+SR1(3,n)),(3/(2*le^2))* (SR1(4
,n)-SR1(2,n)),(1/(4*le))*((SR1(2,n))-5*SR1(4,n));
(3/(2*le^2))*((SR1(2,n))-
(SR1(4,n))), (3/(5*le))* (SR1(1,n) +
SR1(3,n)),(1/160)*(23*(SR1(3,n))-
7*SR1(1,n)),(3/(2*le^2))* ((SR1(4,n))-
(SR1(2,n))),(-
3/(5*le))* (SR1(1,n)+SR1(3,n)),(1/160)*(23*(SR1(1,
n))-7*(SR1(3,n)));
(1/(4*le))*(-
5*SR1(2,n)+SR1(4,n)),(1/160)*(23*(SR1(3,n))-
7*SR1(1,n)),(le/480)*(47*SR1(1,n)+17*SR1(3,n)),(1
/(4*le))*(-(SR1(4,n))+ (5*SR1(2,n))),(-
1/160)*(23*(SR1(3,n))-7*(SR1(1,n))),(-
le/60)*((SR1(1,n))+(SR1(3,n)));
(-
1/(2*le))*(SR1(1,n)+SR1(3,n)),(3/(2*le^2))* ((SR1(
4,n))-(SR1(2,n))), (1/(4*le))*(-
(SR1(4,n))+(5*SR1(2,n))), (1/(2*le))* (SR1(1,n)+SR1
(3,n)),(3/(2*le^2))* ((SR1(2,n))-
(SR1(4,n))), (1/(4*le))* (5*(SR1(4,n))-(SR1(2,n)));

```

```

(3/(2*le^2))*(SR1(4,n)-SR1(2,n)), (-
3/(5*le))*(SR1(1,n)+SR1(3,n)), (-
1/160)*(23*(SR1(3,n))-
7*(SR1(1,n))), (3/(2*le^2))*(SR1(2,n))-(
SR1(4,n))), (3/(5*le))*(SR1(3,n)+SR1(1,n)), (-
1/160)*(23*(SR1(1,n))-7*(SR1(3,n)));
(1/(4*le))*(SR1(2,n))-
5*SR1(4,n)), (1/160)*(23*(SR1(1,n))-
7*(SR1(3,n))), (-
le/60)*((SR1(1,n)+(SR1(3,n))), (1/(4*le))*(5*(SR1
(4,n))-(SR1(2,n))), (-1/160)*(23*(SR1(1,n))-
7*(SR1(3,n))), (le/480)*(47*(SR1(3,n))+17*(SR1(1,n
))))];

```

k01

```

=[((E*A)/(2*le))*(((1+(Theta(5,n)))^2)+(1+(Theta(
1,n)))^2)+((E*I)/(2*le))*((Theta(7,n))^2+(Theta(3
,n))^2), ((E*A)/(2*le))*((1+(Theta(5,n)))*(Theta(6
,n))+((1+(Theta(1,n)))*(Theta(2,n)))+((3*E*I)/(2*le^2))*(((1+(Theta(1,n)))*(Theta(3,n)))-((1+(Theta(5,n)))*(Theta(7,n)))), ((E*A)/8)*(((1+(Theta(5,n)))*(Theta(6,n)))-((1+(Theta(1,n)))*(Theta(2,n)))+((E*I)/(4*le))*(
5*((1+(Theta(1,n)))*(Theta(3,n)))-((1+(Theta(5,n)))*(Theta(7,n))), ((-E*A)/(2*le))*((1+(Theta(5,n)))^2+(1+(Theta(1,n)))^2)-
((E*I)/(2*le))*((Theta(7,n))^2+(Theta(3,n))^2), ((-
E*A)/(2*le))*(((1+(Theta(5,n)))*(Theta(6,n)))+((1+(Theta(1,n)))*(Theta(2,n)))+((3*E*I)/(2*le^2))*((1+(Theta(5,n)))*(Theta(7,n)))-((1+(Theta(1,n)))*(Theta(3,n))), ((E*A)/8)*(((1+(Theta(1,n)))*Theta(2,n))-
((1+(Theta(5,n)))*Theta(6,n))+((E*I)/(4*le))*(((1+(Theta(1,n)))*(Theta(3,n)))-(
5*((1+(Theta(5,n)))*(Theta(7,n))))));
((E*A)/(2*le))*((1+(Theta(5,n)))*(Theta(6,n))+((1+(Theta(1,n)))*(Theta(2,n)))+((3*E*I)/(2*le^2))*((1+(Theta(1,n)))*(Theta(3,n)))-((1+(Theta(5,n)))*(Theta(7,n))), (((-3*E*A)/(5*le))*((Theta(6,n))^2+(Theta(2,n))^2)+((6*E*I)/le^3)*((1+(Theta(5,n)))^2-
(1+(Theta(1,n)))^2), ((E*A)/160)*(23*(Theta(6,n)))

```

$$\begin{aligned}
& \hat{\theta}_2 = \\
& 7 * (\Theta_2(n))^2 + ((3 * E * I) / (2 * 1e^2)) * ((1 + (\Theta_5(n)))^2 + 3 * (1 + (\Theta_1(n)))^2), ((- \\
& E * A) / (2 * 1e)) * (((1 + (\Theta_5(n))) * (\Theta_6(n))) + ((1 + (\Theta_1(n))) * (\Theta_2(n)))) + ((3 * E * I) / (2 * 1e^2)) * \\
& (((1 + (\Theta_5(n))) * (\Theta_7(n))) - ((1 + (\Theta_1(n))) * (\Theta_3(n)))), ((- \\
& 3 * E * A) / (5 * 1e)) * ((\Theta_6(n))^2 + (\Theta_2(n))^2) - \\
& ((6 * E * I) / 1e^3) * ((1 + (\Theta_1(n)))^2 + (1 + (\Theta_5(n)))^2), ((E * A) / 160) * ((23 * (\Theta_2(n))^2) - \\
& 7 * (\Theta_6(n))^2 + ((3 * E * I) / (2 * 1e^2)) * (3 * (1 + (\Theta_5(n)))^2 + (1 + (\Theta_1(n)))^2); \\
& ((E * A) / 8) * (((1 + (\Theta_5(n))) * (\Theta_6(n))) - \\
& ((1 + (\Theta_1(n))) * (\Theta_2(n)))) + ((E * I) / (4 * 1e)) * \\
& 5 * ((1 + (\Theta_1(n))) * (\Theta_3(n))) - \\
& ((1 + (\Theta_5(n))) * (\Theta_7(n))), ((E * A) / 160) * ((23 * (\Theta_6(n))^2) - \\
& 7 * (\Theta_2(n))^2 + ((3 * E * I) / (2 * 1e^2)) * ((1 + (\Theta_5(n)))^2 + 3 * (1 + (\Theta_1(n)))^2), ((E * A * 1e) / 480) * (17 * \\
& ((\Theta_6(n))^2) + 47 * (\Theta_2(n))^2) + ((E * I) / (2 * 1e)) * ((1 + (\Theta_5(n)))^2 + 7 * (1 + (\Theta_1(n)))^2), ((E * A) / 8) * \\
& (((1 + (\Theta_1(n))) * (\Theta_2(n))) - ((1 + (\Theta_5(n))) * (\Theta_6(n))) + ((E * I) / (4 * 1e)) * ((1 + (\Theta_5(n))) * (\Theta_7(n))) - \\
& 5 * ((1 + (\Theta_1(n))) * (\Theta_3(n))), ((- E * A) / 160) * ((23 * (\Theta_6(n))^2) + 7 * (\Theta_2(n))^2) - \\
& ((3 * E * I) / (2 * 1e^2)) * ((1 + (\Theta_5(n)))^2 + 3 * (1 + (\Theta_1(n)))^2), ((- E * A * 1e) / 60) * (((\Theta_2(n))^2) + (\Theta_6(n))^2) + ((E * I) / 1e) * \\
& ((1 + (\Theta_5(n)))^2 + (1 + (\Theta_1(n)))^2); \\
& ((- E * A) / (2 * 1e)) * ((1 + (\Theta_5(n)))^2 + (1 + (\Theta_1(n)))^2) - \\
& ((E * I) / (2 * 1e)) * ((\Theta_7(n))^2 + (\Theta_3(n))^2), ((- \\
& E * A) / (2 * 1e)) * (((1 + (\Theta_5(n))) * (\Theta_6(n))) + ((1 + (\Theta_1(n))) * (\Theta_2(n)))) + ((3 * E * I) / (2 * 1e^2)) * \\
& (((1 + (\Theta_5(n))) * (\Theta_7(n))) - ((1 + (\Theta_1(n))) * (\Theta_3(n)))), ((E * A) / 8) * (((1 + (\Theta_1(n))) * (\Theta_2(n))) - \\
& ((1 + (\Theta_5(n))) * (\Theta_6(n))) + ((E * I) / (4 * 1e)) * ((1 + (\Theta_5(n))) * (\Theta_7(n))) - \\
& 5 * ((1 + (\Theta_1(n))) * (\Theta_3(n))), ((E * A) / (2 * 1e)) * ((1 + (\Theta_5(n)))^2 + (1 + (\Theta_1(n)))^2) + ((E * I) /
\end{aligned}$$

```

(2*le)) * ((Theta(7,n))^2 + (Theta(3,n))^2), ((E*A)/(2
*le)) * (((1+(Theta(5,n))) * (Theta(6,n))) + ((1+(Theta
(1,n))) * (Theta(2,n)))) + ((3*E*I)/(2*le^2)) * (((1+(T
heta(1,n))) * (Theta(3,n))) -
((1+(Theta(5,n))) * (Theta(7,n)))), ((E*A)/8) * (((1+
Theta(5,n))) * (Theta(6,n))) -
((1+(Theta(1,n))) * (Theta(2,n))) + ((E*I)/(4*le)) * ((
5*(1+(Theta(5,n))) * (Theta(7,n))) -
((1+(Theta(1,n)))) * (Theta(3,n)))) ;
((-E*A)/(2*le)) * (((1+(Theta(5,n))) * (Theta(6,n))) + ((1+
Theta(1,n))) * (Theta(2,n)))) + ((3*E*I)/(2*le)^2) * (
(1+(Theta(5,n))) * (Theta(7,n))) -
((1+(Theta(1,n))) * (Theta(3,n))), ((-3*E*A)/(5*le)) * ((Theta(6,n))^2 + (Theta(2,n))^2) -
((6*E*I)/le^3) * ((1+(Theta(1,n)))^2 + (1+(Theta(5,n))
))^2, ((-E*A)/160) * ((23*(Theta(6,n))^2) + 7*(Theta(2,n))^2) -
((3*E*I)/(2*le^2)) * ((1+(Theta(5,n)))^2 + 3*(1+(Thet
a(1,n)))^2), ((E*A)/(2*le)) * (((1+(Theta(5,n))) * (Th
eta(6,n))) + ((1+(Theta(1,n))) * (Theta(2,n)))) + ((3*E
*I)/(2*le^2)) * (((1+(Theta(1,n))) * (Theta(3,n))) -
((1+(Theta(5,n))) * (Theta(7,n)))), ((3*E*A)/(5*le)) *
((Theta(6,n))^2) + ((6*E*I)/le^3) * ((1+(Theta(1,n)))^
2 + (1+(Theta(5,n)))^2), ((E*A)/160) * ((7*(Theta(6,n)
))^2) - 23*(Theta(2,n))^2) -
((3*E*I)/(2*le^2)) * (3*(1+(Theta(5,n)))^2 + (1+(Thet
a(1,n)))^2) ;
((E*A)/8) * (((1+(Theta(1,n))) * Theta(2,n)) -
((1+(Theta(5,n))) * Theta(6,n))) + ((E*I)/(4*le)) * (((1+
Theta(1,n))) * (Theta(3,n))) -
((5*(1+(Theta(5,n))) * (Theta(7,n)))), ((E*A)/160) * ((23*
(Theta(2,n))^2) -
7*(Theta(6,n))^2) + ((3*E*I)/(2*le^2)) * (3*(1+(Theta
(5,n)))^2 + (1+(Theta(1,n)))^2), ((-E*A*le)/60) * (((Theta(2,n))^2) + (Theta(6,n))^2) + ((E
*I)/le) * ((1+(Theta(5,n)))^2 + (1+(Theta(1,n)))^2), ((E*A)/8) * (((1+(Theta(5,n))) * (Theta(6,n))) -
((1+(Theta(1,n))) * (Theta(2,n))) + ((E*I)/(4*le)) * ((5*
(1+(Theta(5,n))) * (Theta(7,n))) -
((1+(Theta(1,n))) * (Theta(3,n)))), ((E*A)/160) * ((7*
(Theta(6,n))^2) - 23*(Theta(2,n))^2) -
((3*E*I)/(2*le^2)) * (3*(1+(Theta(5,n)))^2 + (1+(Thet

```

```

a(1,n))^2),((E*A*le)/480)*(47*((Theta(6,n))^2)+17*(Theta(2,n))^2)+((E*I)/(2*le))*(7*(1+(Theta(5,n))))^2+(1+(Theta(1,n)))^2)];
K11 = wextend(2,'zpd',ksigma,3*i, 'l');
%for ir = 1:6
    %for ic = 1:6
        %ick = 3*(n-1)+ ic;
        %icr = 3*(n-1)+ ir;
        % K11(ick,icr)=K11(ick,icr)+ksigma(ic,ir);
    %end
%end
Ksigma= Ksigma+K11;
K111 = wextend(2,'zpd',k01,3*i, 'l');
K01=K01+K111;
end
end
if NTYPE==1
    K2=K;
else
    K2=Ksigma+K;
end
K3=K2;

K2(1:3,:) = [];
K2(:,1:3) = [];
jnc

ASLOD1(1:3)=[];
K2;

dsi=ds;
dds = zeros((3*(n+1)),1);
a = 1;
%displacements

dis = inv(K2)* ASLOD1;
for id = 1:3*(n+1)
if(drs(id)==1)
dds(id)=0;
else
dds(id)=dis(a);
a=a+1;
end

```

```

end

Fr = Fr+ K3*dds;
ds = ds+dds;
for j=1:n
    for Ie =1:6;
        ddEs(Ie,j)=dds((3*(j-1))+Ie);
        dEs(Ie,j)=ds(Ie+(3*(j-1)));
        dESi(Ie,j)=dsi(Ie+(3*(j-1)));
    end
    end
    ddEs;
    dEs;
    deltaSR1=zeros(4,n);
    for j=1:n
        le=l(j);
        G=[-1/le 0 0 1/le 0 0;0 0 1 0 0 0;0 0 (-6/le^2)
        (-4/le) 0 (6/le^2) (-2/le);0 0 0 0 0 0;
        -1/le 0 0 1/le 0 0;0 0 0 0 0 1;0 (6/le^2)
        (2/le) 0 (-6/le^2) (4/le);0 0 0 0 0 0];
        Theta=G*dESi;
    Theta1=G*ddEs;

    DB0 = [(-E*A/le),0,0,(E*A/le),0,0;0,((-
    6*E*I)/le^2),((-4*E*I)/le),0,((6*E*I)/le^2),((-
    2*E*I)/le);
    (-
    E*A/le),0,0,(E*A/le),0,0;0,((6*E*I)/le^2),((2*E*I)
    /le),0,((-6*E*I)/le^2),((4*E*I)/le)];
    DB1=[((-E*A)/le)*(Theta(1,j)) 0 E*A*(Theta(2,j))
    ((E*A)/le)*(Theta(1,j)) 0 0;
    ((-E*I)/le)*(Theta(3,j)) ((-
    6*E*I)/le^2)*(Theta(1,j)) ((-
    4*E*I)/le)*(Theta(1,j)) ((E*I)/le)*(Theta(3,j))
    ((6*E*I)/le^2)*(Theta(1,j)) ((-
    2*E*I)/le)*(Theta(1,j));
    ((-E*A)/le)*(Theta(5,j)) 0 0
    ((E*A)/le)*(Theta(5,j)) 0 (E*A)*(Theta(6,j));
    ((-E*I)/le)*(Theta(7,j))
    ((6*E*I)/le^2)*(Theta(5,j))
    ((2*E*I)/le)*(Theta(5,j)) ((E*I)/le)*(Theta(7,j))
    ((-6*E*I)/le^2)*(Theta(5,j))
    ((4*E*I)/le)*(Theta(5,j))];

```

```

DB11=.5*[((-E*A)/le)*(Theta1(1,j)) 0
E*A*(Theta1(2,j)) ((E*A)/le)*(Theta1(1,j)) 0 0;
((-E*I)/le)*(Theta1(3,j)) ((-
6*E*I)/le^2)*(Theta1(1,j)) ((-
4*E*I)/le)*(Theta1(1,j)) ((E*I)/le)*(Theta1(3,j))
((6*E*I)/le^2)*(Theta1(1,j)) ((-
2*E*I)/le)*(Theta1(1,j));
((-E*A)/le)*(Theta1(5,j)) 0 0
((E*A)/le)*(Theta1(5,j)) 0 (E*A)*(Theta1(6,j));
((-E*I)/le)*(Theta1(7,j))
((6*E*I)/le^2)*(Theta1(5,j))
((2*E*I)/le)*(Theta1(5,j))
((E*I)/le)*(Theta1(7,j)) ((-
6*E*I)/le^2)*(Theta1(5,j))
((4*E*I)/le)*(Theta1(5,j))];
if NTYPE==1
    DB=DB0;
else
    DB=DB0+DB1+DB11;
end
for is=1:4
    for js=1:6
        deltaSR1(is,j)=DB(is,js)*ddEs(js,j);
        SR1(is,j)= SR1(is,j)+deltaSR1(is,j);
    end
end
SR1;
for j=1:n
    SR11(1,j)=SR1(1,j);
    SR11(2,j)=SR1(3,j);
    SR12(1,j)=SR1(2,j);
    SR12(2,j)=SR1(4,j);
end
M1=SR12(1,:);
M2=SR12(2,:);
F1=SR11(1,:);
F2=SR11(2,:);
for i=1:n
    for j=1:1
        if i==1&j==1
            Mav(j,i)=M1(j,i);
            Fav(j,i)=F1(j,i);

```

```

    else
Mav(j,i)=0.5*(M1(j,i)+M2(j,i-1));
Fav(j,i)=0.5*(F1(j,i)+F2(j,i-1));
end

end
Mav(j,i+1)=M2(j,n);
Fav(j,i+1)=F2(j,n);
end
Mav;
Fav;
RINT=zeros(6,n);
for j=1:n
    le=1(j);
    G =[-1/le 0 0 1/le 0 0;0 0 1 0 0 0;0 (-6/le^2)
(-4/le) 0 (6/le^2) (-2/le);0 0 0 0 0 0;
    -1/le 0 0 1/le 0 0;0 0 0 0 0 1;0 (6/le^2)
(2/le) 0 (-6/le^2) (4/le);0 0 0 0 0 0];
Theta=G*dEs;
    RINT(1:6,j)=[-.5*SR1(1,j)-.5*SR1(3,j);
    -
(3/(2*le))*SR1(2,j)+(3/(2*le))*SR1(4,j);
    -(5/4)*SR1(2,j)+.25*SR1(4,j);
    .5*SR1(1,j)+.5*SR1(3,j);
    (3/(2*le))*SR1(2,j)-
(3/(2*le))*SR1(4,j);
    -.25*SR1(2,j)+(5/4)*SR1(4,j)];
%RINT(1:6,j)=[-.5*Fav(1,j)-.5*Fav(1,j+1);
% -
(3/(2*le))*Mav(1,j)+(3/(2*le))*Mav(1,j+1);
% -(5/4)*Mav(1,j)+.25*Mav(1,j+1);
% .5*Fav(1,j)+.5*Fav(1,j+1);
% (3/(2*le))*Mav(1,j)-
(3/(2*le))*Mav(1,j+1);
% -.25*Mav(1,j)+(5/4)*Mav(1,j+1)];
end
RINT;
TRINT=zeros(3*(n+1),1);
for j=1:n
    for Ie =1:6;
        TRINT((3*(j-1))+Ie) =TRINT((3*(j-
1))+Ie)+RINT(Ie,j);
    end
end

```

```

TRINT;
ASLOD1=ASLOD-TRINT;

RNORM=((transpose(ASLOD1)*ASLOD1)^0.5)/ASLOD;
end
ASLOD;
fprintf(' the displacements at the nodes
are: ')
ds
fprintf('the reactions at the nodes are:')
Fr;
fprintf('the stresses in the elements are:')
SR1 ;
TRINT;
ASLOD1;
Mav;
Fav;
end

for j=1:n
dEs(1,j)=dEs(1,j);
dEs(2,j)=dEs(2,j);
dEs(3,j)=dEs(3,j);
dEs(4,j)=dEs(4,j);
dEs(5,j)=dEs(5,j);
dEs(6,j)=dEs(6,j);
end
dEs1=dEs(2,:);
dEs2=dEs(5,:);
dEs3=dEs(3,:);
dEs4=dEs(6,:);
dEs5=dEs(1,:);
dEs6=dEs(4,:);

for i=1:n
for j=1:1
if i==1&&j==1
dEsv(j,i)=dEs1(j,i);
dEstheta(j,i)=dEs3(j,i);
dEsu(j,i)=dEs5(j,i);
else
dEsv(j,i)=0.5*(dEs1(j,i)+dEs2(j,i-1));
dEstheta(j,i)=0.5*(dEs3(j,i)+dEs4(j,i-1));
dEsu(j,i)=0.5*(dEs5(j,i)+dEs6(j,i-1));
end
end

```

```

    end

    end
dEsv(j,i+1)=dEs2(j,n);
dEstheta(j,i+1)=dEs4(j,n);
dEsu(j,i+1)=dEs6(j,n);
end
dEsv;
dEstheta;
dEsu;
ee=0:1e:L;
plot(ee,dEsv,'k',ee,dEstheta,'c',ee,dEsu,'m','Lin
eWidth',2.27)
dEsvmax=max(dEsv);
ind=find(dEsv==dEsvmax);
dEsthetamax=dEstheta(ind);
legend('v','theta','u')
xlabel('le(m)')
ylabel('ds')
title('displacements')
grid on
figure
for j=1:n
    SR11(1,j)=SR1(1,j);
    SR11(2,j)=SR1(3,j);
    SR12(1,j)=SR1(2,j);
    SR12(2,j)=SR1(4,j);
end
M1=SR12(1,:);
M2=SR12(2,:);

for i=1:n
    for j=1:1
        if i==1&&j==1
            M(j,i)=M1(j,i);
        else
            M(j,i)=0.5*(M1(j,i)+M2(j,i-1));
        end
    end
    M(j,i+1)=M2(j,n);
end
M ;
ee=0:1e:L;

```

```
plot(ee,M,'m','LineWidth',2.27)
xlabel('Length(m)')
ylabel('M')
title('bending moment')
grid on
figure
```

Appendix (D)

cantilever beam

Enter Youngs Modulus of beam:200000000

Enter 1 for circular c/s, 2 for rectangular c/s or 3 to enter value of I manually:2

Enter I of beam:8.33*10^-5

I =

e-058.3300

Enter area of beam:.1

solution type1

Enter number of elements:2

:Enter Length of the beam

2.5

Node 3

Enter forces position8

Enter forces value-13328

INC=1

FACTO =1

niter=1

LINC =

0

0

0

0

0

0

0

-13328

0

ASLOD=

0

0

0

0

0

0

0

-13328

0

ASLOD1=

0

0

0

0

0

0

0

-13328

0

Jnc=

1

Mav=

* e+041.0

0.0000 -1.6660 -3.3320

Fav=

0 0 0

:the displacements at the the nodes are

ds=

0

0

0

0

-1.3021

-1.8750

0

-4.1667

-2.5000