



قال تعالى في محكم تنزيله:

(لَا يُكَلِّفُ اللَّهُ نَفْسًا إِلَّا وُسْعَهَا لَهَا مَا كَسَبَتْ وَعَلَيْهَا مَا اكْتَسَبَتْ رَبَّنَا لَا تُؤَاخِذْنَا إِنْ
نَسِينَا أَوْ أَخْطَأْنَا رَبَّنَا وَلَا تَحْمِلْ عَلَيْنَا إَصْرًا كَمَا حَمَلْتَهُ عَلَى الَّذِينَ مِنْ قَبْلِنَا رَبَّنَا
وَلَا تُحَمِّلْنَا مَا لَا طَاقَةَ لَنَا بِهِ وَاعْفُ عَنَّا وَارْحَمْنَا أَنْتَ مَوْلَانَا فَانصُرْنَا
عَلَى الْقَوْمِ الْكَافِرِينَ)

صدق الله العظيم

[سورة البقرة الآية 286]

**TO MY PARENTS
TO MY HUSBAND
AND
MY SON
ABD ELRAHMAN
I DEDICATE THIS WORK**

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First of all thanks for *God* who help me to complete this work.

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Abstract:

In this research, a Total Lagrangian Formulation for Geometric Nonlinear Beams problems based on green strains was developed.

Geometric Nonlinear Total Lagrangian formulations applied on two-dimensional beam finite elements were used. The formulations were implemented by using MATLAB program. The solution of linear and nonlinear equations was obtained by the Incremental method. The program was applied to obtain the displacements, reactions and stress resultants resulting from incremental strains. The accuracy of the results was checked by using two numerical examples. The linear results are in close agreement with known results.

Acceptable nonlinear results are obtained for pure incremental solutions only. Incremental iterative solutions are obtained for some specific cases only.

مستخلص

في هذا البحث تم تطبيق تقنين لاجرانج (Lagrange) الكلي للاخطية الهندسية المبنية على انفعال قرين (Green) تطبيقاً على المرونة ثنائية الابعاد لعنصر عارضه محدد. تمت الحوسبة باستخدام برنامج (MATLAB) و تم الحصول على حل للمعادلات اللاخطية باستخدام طريقة التطبيق التدريجي للحمل (Incremental method).

طبق البرنامج للحصول على الازاحات ، ردود الافعال ومحصلات الاجهادات الناتجة عن قياس الانفعالات التدريجية ، اختبرت دقة النتائج بناءً على مثالين عددين.

ووجد ان نتائج التحليل الخطي تماثل النتائج المنشورة، وأن نتائج التحليل اللاخطي مقبولة فقط في حالة تطبيق الحمل تدريجياً بدون تصحيح الاتزان لكل جزء من الحمل. ويتم الحصول على حلول تكرارية لبعض الحالات الخاصة فقط.

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List of Abbreviations (Notations)

Symbol	Notation
$[K_T]$	Tangent stiffness matrix
$\{\Delta u\}$	Displacement increment
$\{F\}$	External load vector
$\{F^{nr}\}$	Internal force vector
E	Direct strain
X	Curvature
$\{\varepsilon\}$	Infinitesimal strains
$\{\varepsilon_0\}$	linear strain
$\{\varepsilon_l\}$	Nonlinear strain
$\{a\}$	Nodal variables
$[B_L(a)]$	linear strain matrix
$\delta\{\varepsilon\}$	Variation in strain
$[B]$	Strain matrix
$\{S\}$	Stress resultants
$[D]$	Modulus matrix
F	Axial force
M	Bending moment
E	Young's modulus of beam
A	Area of beam
I	Second moment of area
$\{R\}$	Vector of equivalent applied nodal loads
$[K\sigma]$	Initial stress stiffness matrix
$[P_l]$	Initial stress matrix
$\{a\}^{i+1}$	Nodal displacements for iteration $(i + 1)$
$\{\delta a\}^i$	Displacement increments
$\{\delta \varepsilon\}^i$	Incremental strains
$\{\delta S\}^i$	Increment of the stress resultants
$\{S\}^{i+1}$	Total stress resultants
$-\{\psi\}^{i+1}$	Nodal residual forces for the next iteration

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