## Chapter three Mathematical models

### 3.1. Introduction

In order to study the effect of bracing types in multi-storey steel braced buildings we studied two different basic models to make two types of comparisons:

- comparison (1):
study the effect of bracing types on the stiffnesses of multi-storey steel braced frame buildings in form of the ratio of the maximum deflections at the top of the building to the total height (drift index) and compare these ratios with the drift index of the rigid frame.
- comparison (2):
study the effect of bracing types on multi-storey steel braced frame buildings in form of the weights of members and comparing them with the weights of rigid frame members.


### 3.2. Frames information

Table (3.1): Frame information of models:

| Case |  | comparison (1) | comparison (2) |
| :---: | :---: | :---: | :---: |
| Number of stories |  | 30 | 20 |
| Storey height(m) |  | 3 m for all stories |  |
| Number of spans (bays) |  | 3 bays in each direction for all bays |  |
| Length of bay ( m ) |  | 4 m for all bays |  |
| Member Sections | Columns | I And H | H |
|  | Beams | I |  |
|  | Composite Beams | I And H |  |
|  | Bracing | T |  |
| Types of connections | Columns | Rigid for all | Pinned for all |
|  | Beams | Rigid for all |  |
|  | Composite beams | Pinned for all |  |
|  | Bracing | Pinned for all types. |  |

### 3.3. Models



Figure (3.1): Plane of models


Figure (3.2):3-D rigid frame


Figure (3.3): Elevation of rigid frame


Figure (3.4): Elevation of k braced frame


Figure (3.5): Elevation of $x$ braced frame


Figure (3.6): Elevation of eccentric frame $\{$ eccentric is equal to 1 m$\}$


Figure (3.7): Elevation of v braced frame


Figure (3.8): Elevation of single diagonal braced frame


Figure (3.9): Elevation of quadrangular braced frame

### 3.4. Analysis information

### 3.4.1. Loading

### 3.4.1.1. Dead and imposed load

- Dead load:
$\mathrm{DL}=1.5 \mathrm{kN} / \mathrm{m}^{2} \quad$ \{Ceilings, partitions and finishing \}
Deck self - weight $=0.11 \mathrm{kN} / \mathrm{m}^{2}$
- Imposed load:
$\mathrm{LL}=2 \mathrm{kN} / \mathrm{m}^{2} \quad$ \{BS 6399 Part -1:1996 Table: 1 (specific use is public) $\}$
3.4.1.2. Wind load calculations: \{accordance with B.S 6399-

2:1997\}

- Building location: KHARTOUM - ALSAHAFA.
- Basic wind speed $\left(v_{b}\right): \quad v_{b}=45 \mathrm{~m} / \mathrm{s}$ (Khartoum city).
- site altitude in meters above mean sea level ( $\Delta$ ):

$$
\Delta=412.090 \mathrm{~m} \quad \text { [In Khartoum city]. }
$$



| Cl:2.2.2.3 | $\Delta=$ site altitude in meters above mean sea level $\begin{gathered} \Delta=412.090 \mathrm{~m}[\text { Khartoum city }] \\ \mathrm{s}_{\mathrm{a}}=1+0.001 * 412.090 \\ =1.412 \end{gathered}$ | $\begin{gathered} \mathrm{s}_{\mathrm{a}} \\ =1.412 \end{gathered}$ |
| :---: | :---: | :---: |
|  | Directional factor ( $\mathbf{s}_{\mathrm{d}}$ ) <br> Direction $(Ø) \rightarrow 0^{\circ}$ North |  |
| T:3 | $\therefore \mathrm{S}_{\mathrm{d}}=.78$ | $\mathrm{S}_{\mathrm{d}}=.78$ |
| Cl:2.2.2.4 | Seasonal factor ( $\mathbf{S}_{\mathbf{s}}$ ) |  |
| Cl :2.2.2.5 | Permanent buildings $\rightarrow S_{s}=1$ | $S_{\text {s }}=1$ |
|  | Probability factor ( $\mathbf{S}_{\mathbf{p}}$ ) |  |
|  | Normal design $\rightarrow \mathrm{S}_{\mathrm{p}}=1$ | $S_{p}=1$ |
|  | $\rightarrow$ standard value of risk $\mathrm{Q}=0.02$ $\therefore \mathrm{V}_{\mathrm{S}}=45 \times 1.412 \times .78$ | $\begin{gathered} \mathrm{Q}=0.02 \\ \mathrm{~V}_{\mathrm{S}} \end{gathered}$ |
|  | $=49.562 \mathrm{~m} / \mathrm{s}$ | $=49.562$ |
| Cl :1.7.3 | Stage 5: | $\mathrm{m} / \mathrm{s}$ |
|  | $\rightarrow$ Terrain categories effective |  |
|  | height ( $\mathrm{H}_{\mathrm{e}}$ ) |  |
|  | The categories of terrain is B (country) (the building is in |  |
|  | Khartoum). |  |
|  | $\rightarrow$ Reference height ( $\mathrm{H}_{\mathrm{r}}=90 \mathrm{~m}$ ) (conservatively). | $\begin{array}{r} \mathrm{H}_{\mathrm{r}} \\ =90 \mathrm{~m} \end{array}$ |
|  | $\rightarrow$ The effective height He may be conservatively taken as the |  |
|  | reference height $H_{r}\left(H_{e}=H_{r}=\right.$ | $\begin{array}{r} \mathrm{H}_{\mathrm{e}} \\ =90 \mathrm{~m} \end{array}$ |




Table (3.2): Calculations of wind loads:

| story | height | D | $v_{b}$ | $v_{s}$ | $s_{b}$ | $v_{e}$ | $q_{s}$ | a | $c_{a i}$ | $p_{i}$ | $c_{p e}$ | $h_{l a}$ | a | $c_{a e}$ | $p_{e}$ | $p_{\text {total }}$ | $\mathrm{P}(\mathrm{kN})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 30 | 90 | 12 | 45 | 49.5612 | 2.046 | 101.4022 | 6303.117 | 75.595 | 0.795 | 1002.196 | 0.6 | 3 | 12.36932 | 0.939 | 3551.176 | 2548.98 | 91.7633 |
| 29 | 87 | 12 | 45 | 49.5612 | 2.0388 | 101.0454 | 6258.833 | 75.595 | 0.795 | 995.1544 | 0.6 | 6 | 13.41641 | 0.933 | 3503.695 | 2508.54 | 90.30745 |
| 28 | 84 | 12 | 45 | 49.5612 | 2.0316 | $1^{`} 00.6885$ | 6214.705 | 75.595 | 0.795 | 988.1381 | 0.6 | 9 | 15 | 0.919 | 3426.788 | 2438.65 | 87.79141 |
| 27 | 81 | 12 | 45 | 49.5612 | 2.0244 | 100.3317 | 6170.733 | 75.595 | 0.795 | 981.1466 | 0.6 | 12 | 16.97056 | 0.915 | 3387.732 | 2406.586 | 86.63709 |
| 26 | 78 | 12 | 45 | 49.5612 | 2.0172 | 99.97485 | 6126.917 | 75.595 | 0.795 | 974.1799 | 0.6 | 15 | 19.20937 | 0.899 | 3304.859 | 2330.679 | 83.90446 |
| 25 | 75 | 12 | 45 | 49.5612 | 2.01 | 99.61801 | 6083.258 | 75.595 | 0.795 | 967.238 | 0.6 | 18 | 21.63331 | 0.891 | 3252.11 | 2284.872 | 82.25538 |
| 24 | 72 | 12 | 45 | 49.5612 | 2.0028 | 99.26117 | 6039.754 | 75.595 | 0.795 | 960.3209 | 0.6 | 21 | 24.18677 | 0.88 | 3188.99 | 2228.669 | 80.2321 |
| 23 | 69 | 12 | 45 | 49.5612 | 1.9956 | 98.90433 | 5996.407 | 75.595 | 0.795 | 953.4287 | 0.6 | 24 | 26.83282 | 0.878 | 3158.907 | 2205.478 | 79.39722 |
| 22 | 66 | 12 | 45 | 49.5612 | 1.9884 | 98.54749 | 5953.216 | 75.595 | 0.795 | 946.5613 | 0.6 | 27 | 29.54657 | 0.869 | 3104.007 | 2157.445 | 77.66803 |
| 21 | 63 | 12 | 45 | 49.5612 | 1.9812 | 98.19065 | 5910.18 | 75.595 | 0.795 | 939.7187 | 0.6 | 30 | 32.31099 | 0.86 | 3049.653 | 2109.934 | 75.95764 |
| 20 | 60 | 12 | 45 | 49.5612 | 1.974 | 97.83381 | 5867.301 | 75.595 | 0.795 | 932.9009 | 0.6 | 33 | 35.1141 | 0.852 | 2999.364 | 2066.464 | 74.39269 |
| 19 | 57 | 12 | 45 | 49.5612 | 1.9668 | 97.47697 | 5824.578 | 75.595 | 0.795 | 926.108 | 0.6 | 36 | 37.94733 | 0.85 | 2970.535 | 2044.427 | 73.59937 |
| 18 | 54 | 12 | 45 | 49.5612 | 1.9596 | 97.12013 | 5782.012 | 75.595 | 0.795 | 919.3399 | 0.6 | 39 | 40.80441 | 0.842 | 2921.072 | 2001.732 | 72.06237 |
| 17 | 51 | 12 | 45 | 49.5612 | 1.9524 | 96.76329 | 5739.601 | 75.595 | 0.795 | 912.5966 | 0.6 | 42 | 43.68066 | 0.838 | 2885.871 | 1973.275 | 71.03789 |
| 16 | 48 | 12 | 45 | 49.5612 | 1.94 | 96.14873 | 5666.926 | 75.595 | 0.795 | 901.0413 | 0.6 | 45 | 46.57252 | 0.834 | 2835.73 | 1934.689 | 69.64879 |
| 15 | 45 | 12 | 45 | 49.5612 | 1.925 | 95.40531 | 5579.632 | 75.595 | 0.795 | 887.1615 | 0.6 | 48 | 49.47727 | 0.83 | 2778.657 | 1891.495 | 68.09383 |

Table (3.2): Calculations of wind loads \{continuous\}:

| 14 | 42 | 12 | 45 | 49.5612 | 1.91 | 94.66189 | 5493.016 | 75.595 | 0.795 | 873.3895 | 0.6 | 51 | 52.39275 | 0.825 | 2719.043 | 1845.653 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 13 | 39 | 12 | 45 | 49.5612 | 1.895 | 93.91847 | 5407.077 | 75.595 | 0.795 | 859.7252 | 0.6 | 54 | 55.31727 | 0.82 | 2660.282 | 1800.557 |
| 13 | 36 | 12 | 45 | 49.5612 | 1.88 | 93.17506 | 5321.815 | 75.595 | 0.795 | 846.1686 | 0.6 | 57 | 58.24946 | 0.817 | 2608.754 | 1762.585 |
| 12 | 36 | 63.45307 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 11 | 33 | 12 | 45 | 49.5612 | 1.865 | 92.43164 | 5237.232 | 75.595 | 0.795 | 832.7198 | 0.6 | 60 | 61.18823 | 0.81 | 2545.295 | 1712.575 |
| 61.65269 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 10 | 30 | 12 | 45 | 49.5612 | 1.85 | 91.68822 | 5153.325 | 75.595 | 0.795 | 819.3787 | 0.6 | 63 | 64.13267 | 0.8107 | 2506.68 | 1687.302 |
| 60.74286 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 9 | 27 | 12 | 45 | 49.5612 | 1.826 | 90.49875 | 5020.485 | 75.595 | 0.795 | 798.2571 | 0.6 | 66 | 67.08204 | 0.803 | 2418.87 | 1620.612 |
| 58.34205 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 8 | 24 | 12 | 45 | 49.5612 | 1.802 | 89.30928 | 4889.379 | 75.595 | 0.795 | 777.4112 | 0.6 | 69 | 70.03571 | 0.8 | 2346.902 | 1569.491 |
| 7 | 21 | 12 | 45 | 49.5612 | 1.778 | 88.11981 | 4760.007 | 75.595 | 0.795 | 756.8412 | 0.6 | 72 | 72.99315 | 0.798 | 2279.091 | 1522.25 |
| 74.80101 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 6 | 18 | 12 | 45 | 49.5612 | 1.746 | 86.53386 | 4590.21 | 75.595 | 0.795 | 729.8434 | 0.6 | 75 | 75.95393 | 0.795 | 2189.53 | 1459.687 |
| 52.54873 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 5 | 15 | 12 | 45 | 49.5612 | 1.71 | 84.74965 | 4402.875 | 75.595 | 0.795 | 700.0571 | 0.6 | 78 | 78.91768 | 0.792 | 2092.246 | 1392.189 |
| 40.1188 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 4 | 12 | 12 | 45 | 49.5612 | 1.656 | 82.07335 | 4129.189 | 75.595 | 0.795 | 656.5411 | 0.6 | 81 | 81.88406 | 0.79 | 1957.236 | 1300.695 |
| 46.825 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 3 | 9 | 12 | 45 | 49.5612 | 1.586 | 78.60406 | 3787.481 | 75.595 | 0.795 | 602.2095 | 0.6 | 84 | 84.85281 | 0.788 | 1790.721 | 1188.512 |
| 42.78642 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 2 | 6 | 12 | 45 | 49.5612 | 1.484 | 73.54882 | 3315.98 | 75.595 | 0.795 | 527.2408 | 0.6 | 87 | 87.82369 | 0.785 | 1561.827 | 1034.586 |
| 37.24509 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 1 | 3 | 12 | 45 | 49.5612 | 1.3233 | 65.58434 | 2636.7 | 75.595 | 0.795 | 419.2353 | 0.6 | 90 | 90.79648 | 0.741 | 1172.277 | 753.0415 |
| 27.1095 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

### 3.5. Procedure

- Linear analysis will be adopted.
- We will assume there is no reversal wind load.
3.5.1. Comparison (1): Drift index and displacement of storeys comparison

1. Drawing the frame using a deck and the other sections(composite beams, beams, columns and the first bracing type ) with auto sections and then connecting etch storey floor joints with a diaphragm and assigning deck with the dead ,imposed and wind loading.
2. Run analysis and design for the frame sections.
3. Choosing the sections which are satisfy to resist the forces by making auto select section null, and then redrawing the failed once until all members pass the stress/capacity check and analysis/design check with noting reconnecting the diaphragms in each time of this steps.
4. Moving the previous bracing type and drawing the next bracing type and reconnecting the diaphragms.
5. Repeat the same steps from (2 to 4) above using the other bracing type and rigid frame until we reach to the last bracing type.
6. The last frame obtained by using the last bracing type is the constant frame for drift index displacement of storeys comparison.

And subsequently we can say we are fixed all variables (sections and loading conditions) and change one variable (displacements) and this is the basic concept of comparisons.
7. From the constant frame we can get the displacement of rigid frame directly and we can get also the displacement of the braced frames by drawing the different bracing shapes in it (with
constant section of bracing which is the greater bracing section obtained from the previous design of frames) and reconnecting the stories floor joints with a diaphragm and then run analysis.

### 3.5.2. Comparison (2): Weight comparison

The sections which used in comparing of the weight of models members are chosen automatically by Etabs programs (by drawing them by auto select section).

In this comparison we draw any full-model alone, and then analysis and design the model until all members pass the stress/capacity check and analysis/design check and thereby we can get all complete models ready for comparing.

