Comparison Between Different Compensating Methods

A Thesis Submitted in Partial Fulfillment of the Requirements for the Degree of M.Sc. in Electrical Engineering (Control and Microprocessor)

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الآية

قال تعالى:

وَإِلَّا قَالَ رَبُّكَ لِلْمَلَائِكَةِ إِنَّي جَاعِلٌ في الأَرْضِ خَلِيَّةً قَالُواْ أُتْجِّعُلُ فِيهَا مَن يُفْسَدُ فِيهَا يَسْفِكُ الدَّمَاءَ وَنَحْنُ نُسَبِّحُ بِحَمْدِكَ وَنُقَدِّسُ لَكَ قَالَ إِنَّي أَعْلَمُ مَا لَا تَعْلَمُونَ إِلَّا آدَمَ الأَسْمَاءَ كُلَّهَا ثُمَّ عَرَضَهُمْ عَلَى الْمَلَائِكَةِ فَقَالَ آتِنِي بِأَسْمَاءِ هَؤُلَاءِ إِن كُنتُمْ صَادِقِينَ قَالُواْ سَبَحَانَكَ لَا عَلَمُ لَنَا إِلَّا مَا عَلَمْتُنَا إِنَّكَ أَنتَ الْعَلِيمُ الْحَكِيمُ.

صدق الله العظيم

سورة البقرة الآية (30-32)
Dedication

To the mother........
She is one of the most inspiring and strong women I ever know. She gave me and taught me unconditional love and acceptance.

To my father........
Who has offered me every think without taking any think?

To our families........
Those offered me unconditional love and support during the course of this thesis.
Acknowledgement

First of all thank Allah for completion of this research.

I would like to express my deep and great appreciation to my supervisor Dr: Award Allah ta’ifour for support guidance throughout this research.

Finally would like to thank the staff of electrical engineering department, college of engineering who helped me in different ways.
Abstract

The idea of this thesis is the comparison between different compensation methods that are used in the control systems in order to enhance performance in terms of speed of response, stability, and the flexibility of the system.

The comparison is mainly between the lead compensation, lag compensation and the lead–lag compensation to be able to know what best one is choosing for specific analog control systems.

The comparison is made by using the frequency response approach and the root locus approach in addition to use MATLAB to compare the response between the compensated and uncompensated systems.
مستخلص

تقوم فكرة البحث على المقارنة بين المعوضات المختلفة المستخدمة في نظم التحكم من أجل تحسين الأداء من ناحية سرعة الإستجابة واستقرار النظام ومدي مرونته. المقارنة بصورة أساسية بين معوض التقدم، معوض التأخير ومعوض التقدم/التأخير من أجل معرفة ما هو المعوض الأمثل لكل نوع من أنظمة التحكم التماثلية. أجريت المقارنة عن طريق الإستجابة الترددية والمحل الهندسي للجزور.

بالإضافة إلى استخدام برنامج المثال لمقارنة الاستجابة بين الأنظمة المعوضة والأنظمة غير المعوضة.
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CHAPTER ONE

INTRODUCTION
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INTRODUCTION

1.1 Introduction

Compensators are used to alter the response of a control system in order to accommodate set design criteria by introducing additional poles and/or zeros to a system. The response of the system will change significantly. One must use proper design procedures in order to ensure that the added poles and/or zeros have the desired effect the designer is seeking.

This project discusses the use of mathematical analysis and MATLAB to implement three types of compensators lead, Lag, and lag-lead. Lead compensation alters the transient response of System. Lag compensation alters steady-state error of systems.

Generally the purpose of the Lead-Lag compensator is to create a controller which has an overall magnitude of approximately 1. The lead-lag compensator is largely used for phase compensation rather than magnitude. A pole is an integrator above the frequency of the pole. A zero is a derivative above the frequency of the zero.

1.2 Problem Statement

Compensating networks are used in order to obtain the desired performance of the system, use compensating networks. Compensating networks are applied to the system in the form of feed forward path gain adjustment.

The lag compensating networks increase the steady state accuracy of the system. An important point to be noted here is that the increase in the steady state accuracy brings instability to the system. Compensating networks also introduces poles and zeros in the system thereby causes changes in the transfer function of the system.
1.3 Objectives

The main objectives of this study are to:

- Design control system using lead compensator.
- Design control system using lag compensator.
- Design control system using lag-lead compensator.
- Compare the results of all proposed compensators.

1.4 Methodology

- Study of all previous works.
- Design of lead, lag, lead-lag compensators using MATLAB Toolbox.

1.5 Layout

In thesis, Chapter One include introduction to the work. Chapter Two gives theoretical background and literature review. Chapter Three includes the procedure of designing lead compensator, lag compensator, and lag-lead compensator. Chapter Four shows the simulation results. Chapter Five discusses conclusion and recommendation.
CHAPTER TWO

THEORETICAL BACKGROUND AND LITERATURE REVIEW
CHAPTER TWO
THEORITICAL BACKGROUND AND LITERATURE REVIEW

2.1 Introduction

When designing control systems first step is to identify the requirements of the specifications and then optimize the performance expected from the control system (such as stability and speed response). Specifications are essential prerequisites for the design of any control system must be cared before the start the designing.

For routine design problems and performance specifications (related to accuracy and relative stability, and speed of response) may be given in terms of numerical values are accurate. In other cases, they may be offered partly in terms of precise numerical values, and partly in terms of data quality. In the latter case specifications may be modified during the design, since that may never be satisfied with certain specifications (due to conflicting needs), or may lead to a system very expensive.

Overall, it should not be performance standards more stringent than is necessary to perform a specific task. If the resolution in the process of steady state is importance in light of a particular control system then do not require performance specifications unnecessarily rigid since the transient response of these specifications will require the cost of components. Remember that the most important part in the design of the control system is the state of the performance specifications carefully so they will not result in optimal control system for a particular purpose.

Any control system it is important to get the best possible performance provided by to adjust the gain to improve the performance of the system and always be used method
cannot be relied upon as an absolute because of the imbalance caused by increased gain in the stability and flexibility of the system which makes the system for access to the specifications required to this dilemma is to resort to add amendments to the control and that will help to improve the performance of these amendments and that is known as compensation.

2.2 Compensators

Compensators are used to alter the response of a control system in order to accommodate set design criteria by introducing additional poles and/or zeros to a system. The response of the system will change significantly. One must use proper design procedures in order to ensure that the added poles and/or zeros have the desired effect the designer is seeking.

2.2.1 Type of compensation

There are many different kinds of compensators listed as follow:

i. Analog Compensation
ii. Digital Compensation

a) Series Analog Compensation
   - Lead Compensation
   - Lag Compensation
   - Lag-lead Compensation

b) Parallel Compensation

2.2.1.1 Lead compensation

Response First, a lead compensator is a device that provides phase lead in frequency
\[
\frac{E(s)}{Et(s)} = \frac{R4R2}{R3R1} \frac{(R1C1s + 1)}{(R2C2s + 1)} = \frac{R4C1}{R3C2} \cdot \frac{S + \frac{1}{R1C1}}{S + \frac{1}{R2C2}}
\]

\[
= K_c \alpha \frac{(T_0s + 1)}{(\alpha T_0s + 1)} = K_c \alpha \frac{S + \frac{1}{\alpha T_0}}{S + \frac{1}{\alpha T}} \quad (2.1)
\]

\[
T = R_1 C_1 \quad \alpha T = R_1 C_1 \quad K_c = \frac{R4C1}{R3C2}
\]

This network has a dc gain of \( K_c \alpha = R_2 / (R1R_3) \)

From equation (2.1) see that this network is a lead network if \( R_1C_1 > R_2C_2 \), or \( \alpha < 1 \). It is a lag network if \( R_1C_1 < R_2C_2 \) the pole-zero configurations of this network when \( R_1C_1 > R_2C_2 \) and \( R_1C_1 < R_2C_2 \) are shown in Figure (2.2) (a) and (b), respectively [1]

Figure (2.1) shows an electronic circuit using operational amplifiers

![Figure (2.1) Electronic circuit of a lead network](image)
2.2.1.2 Lag compensation

Electronic lag compensator using operational amplifiers, the configuration of the electronic lag compensator using operational amplifiers is the same as that for the lead compensator. If \( R_2C_2 > R_1C_1 \) in the circuit shown in Figure (2.1), it becomes lag compensator. Referring to Figure (2.1), the transfer function of the lag compensator is given by

\[
\frac{E_o(s)}{E_i(s)} = K_C \beta \frac{(T_s + 1)}{(\beta T_s + 1)} = K_C \frac{s^{1+1}}{s^{1+1}} \quad (2.2)
\]

Where

\[
T = R_1C_1 \quad \beta T = R_2C_2 \quad \beta = \frac{R_2C_2}{R_1C_1} > 1 \quad K_C = \frac{R_4C_1}{R_3C_2}
\]

Note that use \( \beta \) instead of \( \alpha \) in the above expressions. (In the lead compensator used \( \alpha \) indicate the ratio \( R_1C_1 / R_2C_2 \) which was less than 1, or \( 0 < \alpha < 1 \). In this Chapter always assume that \( 0 < \alpha < 1 \) and \( \beta > 1 \).
2.2.1.3 Lag-lead compensation

Lag-lead compensation combines the advantages of lag and lead compensations. Since the lag-lead compensator possesses two poles and two zeros, it increases the order of the system by 2, unless cancellation of pole(s) and zero(s) occurs in the compensated system [8].

![Figure (2.3) Lag-lead network](image)

Function for the compensator in Figure (2.3) is obtained as follows: The complex impedance $z_1$ is given by
\[
\frac{1}{Z_1} = \frac{1}{R_1 + \frac{1}{C_1S}} + \frac{1}{R_3} \quad (2.3)
\]

OR

\[
Z_1 = \frac{(R_1C_1s+1)R_3}{(R_1+R_3)C_1s+1} (2.4)
\]

Similarly Complex impedance \(Z_2\) is given by

\[
Z_2 = \frac{(R_2C_2s+1)R_4}{(R_2+R_4)C_2s+1} \quad (2.5)
\]

Hence, we have

\[
\frac{E(s)}{E_t(s)} = \frac{Z_2}{Z_1} = \frac{R_4}{R_3} \cdot \frac{(R_2C_2s+1)R_3}{(R_1C_1s+1)} \cdot \frac{(R_2C_2s+1)R_3}{(R_2+R_4)C_2s+1} \quad (2.6)
\]

The single inverter has the transfer function

\[
\frac{E_0(s)}{E(s)} = -\frac{R_6}{R_5} \quad (2.7)
\]

Thus the transfer function of the compensator shown in Figure (2.3) is

\[
\frac{E_0(s)}{E_t(s)} = \frac{E(s)}{E(s)} \cdot \frac{E_0(s)}{E_0(s)} = \frac{R_4R_6}{R_3R_5} \cdot \frac{(R_2C_2s+1)}{(R_1+R_3)C_1s+1} \cdot \frac{(R_2C_2s+1)}{(R_2+R_4)C_2s+1} \cdot \frac{(R_1C_1s+1)}{(R_1C_1s+1)} \quad (2.8)
\]

\[
G_C(s) = \frac{\beta (T_1s+1)(T_2s+1)}{(\gamma + 1)(\beta T_2s+1)} = K_C \frac{S+\frac{1}{T_1}}{S+\frac{1}{T_1}} \cdot \frac{S+\frac{1}{T_2}}{S+\frac{1}{T_2}} \quad (2.9)
\]

2.2.4 Parallel compensation

Thus far series compensation techniques using lead, Lag, or lag-lead compensators are presented this section discusses parallel compensation technique in the parallel compensation design where the controller for compensator is in a minor loop. The
design may seem to be more complicated than in the series compensation case. It is, however, not complicated if the characteristic equation is of the same form as the characteristic equation for the series compensated system in this section present a simple design problem involving parallel compensation [7]

a. Basic Principle for Designing Parallel Compensated System:

![Figure (2.4) Parallel compensator](image)

Referring to Figure (2.4)

The closed-loop transfer function for the system with series compensation is:

\[
\frac{C}{R} = \frac{G_1G_2}{1 + G_1G_2 + G_2GcH} \tag{2.10}
\]

The characteristic equation is

\[
1 + G_1G_2 + G_2GcH = 0 \tag{2.11}
\]

Given G and H, the design problem becomes that of determining the compensator G that satisfies the given specification. The closed-loop transfer functions for the system with parallel compensator.
2.3 Control System Design Approaches

The purpose of compensator design generally is to satisfy both transient and steady-state specifications. Compensators are used to alter the response of a control system in order to accommodate set design specifications. By introducing additional poles and/or zeros to a system the response of the system will change significantly. Here there are two basic control design approaches the root locus design and frequency response control design approach.

2.3.1 Root-locus approach

The root-locus method is a graphical method for determining the locations of all closed-loop poles from knowledge of the locations of the open-loop poles and zeros as some parameter (usually the gain) is varied from zero to infinity. The method yields a clear indication of the effects of parameter adjustment [2].

In practice the root-locus plot of a system may indicate that the desired performance cannot be achieved just by the adjustment of gain. In fact in some cases the system may not be stable for all values of gain. Then it is necessary to reshape the root locus to meet the performance specifications.

In designing a control system, if other than a gain adjustment is required, modify the original root locus by inserting a suitable compensator. Once the effects on the root locus of the addition of poles and/or zeros are fully understood, we can readily determine the locations of the pole(s) and zero(s) of the compensator that will reshape the root locus as desired. In essence, in the design by the root-locus method the root loci of the system are reshaped through the use of a compensator so that a pair of dominant
closed-loop poles can be placed at the desired location (Often, the damping ratio and undamped natural frequency of a pair of dominant closed-loop poles are specified).

2.3.1.1 Lead compensation techniques based on root-locus approach

The root-locus approach to design is powerful when the specifications are given in terms of time-domain quantities, such as the damping ratio and undamped natural frequency of the desired dominant closed-loop poles, maximum overshoot, rise time, and settling time.

Consider a design problem in which the original system either is unstable for all values of gain or is stable but has undesirable transient-response characteristics. In such a case, the reshaping of the root locus is necessary in the broad neighborhood of the \((j\omega)\) axis and the origin in order that the dominant closed-loop poles be at desired locations in the complex plane. This problem may be solved by entering an appropriate lead compensator in cascade with the feed forward transfer function.

2.3.1.2 Lag compensation techniques based on the root-locus approach

Consider the problem of finding a suitable compensation network for the case where the system exhibits satisfactory transient-response characteristics but unsatisfactory steady-state characteristics.

Compensation in this case essentially consists of increasing the open loop gain without appreciably changing the transient-response characteristics. This means that the root locus in the neighborhood of the dominant closed-loop poles should not be changed appreciably but the open-loop gain should be increased as much as needed.

This can be accomplished if a lag compensator is put in cascade with the given. Feed forward transfer function.
To avoid an appreciable change in the root loci, the angle contribution of the lag network should be limited to a small amount. Say 5°. To assure this, place the pole and zero of the lag network relatively close together and near the origin of the s plane. Then the closed-loop poles of the compensated system will be shifted only slightly from their original locations. Hence, the transient-

Consider a lag compensator $G(s)$ where

$$G_C(s) = K_C \beta \frac{(T_s + 1)}{(\beta T_s + 1)} K_c \frac{S^{\frac{1}{T}}}{S + \frac{1}{T}}$$

If place the zero and pole of the lag compensator very close to each other, then at $s = s_1$ is one of the dominant closed-loop poles, the magnitudes $S_1 + (1/T)$ and $S_1 + [I / (\beta T_1)]$ are almost equal, or

$$|G_c(s_1)| = |K_c \frac{(T_s + 1)}{(\beta T_s + 1)}| \approx K_c$$

To make the angle contribution of the lag portion of the compensator to be small, require

$$-5^0 < \text{angle of} \left( \frac{S^{\frac{1}{T}}}{S + \frac{1}{T}} \right) > 0$$

This implies that if gain $K_c$ of the lag compensator is set equal to 1, then the transient response characteristics will not be altered. (This means that the overall gain of the open-loop Transfer function can be increased by a factor of $\beta$ where $\beta > 1$.) If the pole and zero are placed very close the origin. Then the value of $\beta$ can be made large (A large value of $\beta$ may be used, provided physical realization of the lag compensator is possible.)

It is noted that the value of $(T_1)$ must be large, but its exact value is not critical.

However, it should not be too large in order to avoid difficulties in realizing the phase lag compensator by physical components.
An increase in the gain means an increase in the static error constants. If the open loop transfer function of the uncompensated system is \( G(s) \), then the static velocity error constant \( K_r \) of the uncompensated system is

\[
K_r = \lim_{s \to 0} s G_c(s) \tag{2.16}
\]

If the compensator is chosen as given by Equation (2.2), then for the compensated system with the open-loop transfer function \( G_c(s) G(s) \) the static velocity error constant \( k_r \) becomes

\[
\bar{K}_r = \lim_{s \to 0} s G_c(s) \\
= \lim_{s \to 0} s G_c(s) K V \\
= \bar{K}_c \beta K_C \tag{2.17}
\]

Thus if the compensator is given by Equation (2.2), then the static velocity error constant is increased by a factor of \( k_c \beta \), where \( K_c \) is approximately unity.

The main negative effect of the lag compensation is that the compensator zero that will be generated near the origin creates a closed-loop pole near the origin. This closed loop pole and compensator zero will generate a long tail of small amplitude in the step response thus increasing the settling time.

2.3.1.3 Lag – lead compensation techniques based on the root-locus approach

Consider the system shown in Figure (2.4). Assume that we use the lag-lead compensator:

\[
G_c(s) = K_c \frac{\beta (T_1 s + 1)(T_2 s + 1)}{\gamma (\gamma s + 1)(\beta T_2 s + 1)} = K_c \left( \frac{s + \frac{1}{T_1}}{s + \frac{1}{\gamma}} \right) \left( \frac{s + \frac{1}{T_2}}{s + \frac{1}{\beta T_2}} \right) \tag{2.9}
\]
Here $\beta > 1$ and $Y > 1$ (Consider $K_c$ to belong to the lead portion of the lag-lead compensator.)

In designing lag-lead compensators, consider two cases where $y \neq \beta$ and $y = \beta$. Case 1, $Y \neq \beta$. In this case, the design process is a combination of the design of the lead compensator and that of the lag compensator.

### 2.3.2 Frequency-response

It is important to note that in a control system design, transient-response performance is usually most important.

In the frequency-response approach, specify the transient-response performance in an indirect manner. That is, the transient-response performance is specified in terms of the phase margin, gain Margin, resonant peak magnitude (they give a rough estimate of the system damping); the gain crossover frequency, resonant frequency, bandwidth (they give a rough estimate of the speed transient response); and static error constants (they give the steady-state accuracy). Although the correlation between the transient response and frequency response is indirect the frequency domain specifications can be conveniently met in the Bode diagram approach [2].

After the open loop has been designed by the frequency-response method the closed loop poles and zeros can be determined. The transient-response characteristics must be checked to see whether the designed system satisfies the requirements in the time domain.

If it does not, then the compensator must be modified and analysis repeated until a satisfactory result is obtained.

Design in the frequency domain is simple and straightforward. The frequency-response plot indicates clearly the manner in which the system should be modified. Although the
exact quantitative prediction of the transient-response characteristics cannot be made, the frequency-response approach can be applied to systems or components whose dynamic characteristics are given in the form of frequency-response data. Note that because of difficulty in deriving the equations governing certain components such as pneumatic and hydraulic components. The dynamic characteristics of such components are usually determined experimentally through frequency-response tests. The experimentally obtained frequency-response plots can be combined easily with other such plot when the Bode diagram approach is used. Note also that in dealing with high-frequency noises find that the frequency-response approach is more convenient than other approaches.

There are basically two approaches in the frequency-domain design. One is the polar approach and the other is the Bode diagram approach. When a compensator is added the polar plot does not retain the original shape and therefore, need to draw a new polar plot, which will take time and is thus inconvenient. On the other hand, A Bode diagram of the compensator can be simply added the original Bode diagram, and thus plotting the complete Bode diagram is a simple matter. Also the open-loop gain is varied, the magnitude curve is shifted up or down without changing the slot of the curve and the phase curve remains the same. For design purposes, it is best to work with the Bode diagram.

A common approach to the Bode diagram is that first adjust the open-loop gain so that requirement on the steady-state accuracy is met. Then the magnitude and phase curve of uncompensated open loop (with the open-loop gain just adjusted) is plotted. If the specification the phase margin and gain margin are not satisfied, then a suitable compensator that will reshape open-loop transfer function is determined. If there are any other requirements to be met, try to satisfy them, unless some of Contradictory.
Basic Characteristics of Lead, Lag and Lag-Lead Compensation:

Lead compensation essentially yields an appreciable improvement in transient response and a small change in steady-state accuracy. It may accentuate high-frequency noise effects, lag compensation, on the other hand yields an appreciable improvement in steady-state accuracy at the expense of increasing the transient-response time. Lag compensation will suppress the effects of high-frequency noise signals. Lag-lead compensation combines the characteristics of both lead compensation and lag compensation. Is use of a lead or lag compensator raises the order of the system by 1 (unless cancellation occurs between the zero of the compensator and a pole of the uncompensated open-loop transfer function). The use of a lag-lead compensator raises the order of the system by 2 (unless cancellation occurs between zero(s) of the lag-lead compensator and pole(s) of the uncompensated open-loop transfer function). which means that the system becomes more complex and it is more difficult to control the transient response behavior. The particular situation determines the type of compensation to be used.

2.3.2.1 Lead compensation techniques based on the frequency-response approach

First examine the frequency characteristics of the lead compensator. Then present a design technique for the lead compensator by use of the Bode Diagram.

Consider a lead compensator having the following transfer function:

\[ \frac{K_C \alpha (T_s + 1)}{(\alpha T_s + 1)} = K_C \frac{S + \frac{1}{T}}{S + \frac{1}{\alpha T}} (0 < \alpha > 1) \]  

(2.1)

Where \( \alpha \) is called the attenuation factor of the lead compensator. It a zero at \( S = -1/T \) and a pole at \( s = -1/(\alpha T) \). Since \( 0 < \alpha < 1 \), we see that the zero is always located to the right of the pole in the complex plane.
Notes that for a small value of $\alpha$ pole is located far to the left. The minimum value of $\alpha$ is limited by the physical construction of lead compensator. The minimum value of $\alpha$ is usually taken to be about 0.05 (This means that the maximum phase lead that may be produced by a lead compensator is about 65°).

2.3.2.2 Lag compensation techniques based on the frequency- response approach

The primary function of a lag compensator is to provide attenuation in the high frequency range to give a system sufficient phase margin. The phase-lag characteristic is of no consequence in lag compensation.

2.3.2.3 Lag-lead compensation based on the frequency-response approach

The design of lag-lead compensator by the frequency- response approach is based on the combination of the design techniques under lead compensation and lag compensation let us assume that the lag-lead compensator is of the following form

$$G_C(s) = K \frac{(T_1s+1)(T_2s+1)}{(T_1s+1)(\beta T_2s+1)}$$ (2.9)

Here $\beta>1$. The phase lead portion of the lag-lead compensator (the portion involving $T_1$) alters the frequency-response curve by adding phase-lead angle and increasing the phase margin at the gain crossover frequency. The phase-lag portion (the portion involving $T_1$) provides attenuation near and above the gain crossover frequency and thereby allows an increase of gain at the low-frequency range to improve the steady-state performance [3]
CHAPTER THREE

CONTROL SYSTEM DESIGN
CHAPTER THREE
CONTROL SYSTEM DESIGN

3. 1 Introduction
Specifically consider the design of lead compensator, lag compensator, and lag-lead compensator. In such design problems, place a compensator in series with UN alterable transfer function $G(s)$ to obtain desirable behavior. The main problem then involves the judicious choice of the poles ($s$) and zero ($s$) of the compensator $G_C(s)$ to alter the root-locus (or frequency response) so that the performance specifications will be met.

3.2 Design of Lead Compensation Based on the Frequency-Response Approach

![Figure (3.1) Control System]

The procedures for designing lead compensator for the system in Figure (3.1) response is stated as follows

i. Assume the following lead compensator:

$$G_c(s) = K_C \frac{T_s + 1}{\alpha T_s + 1} = K_C \frac{S + \frac{1}{\alpha}}{S + \frac{1}{\alpha T_s}}$$

(3.1)

Define

$$K_C \alpha = K$$
The open-loop transfer function of the compensated system is
\[
G_C(s) = K \frac{T_s + 1}{aT_s + 1}
\]
(3.2)

The open-loop transfer function of the compensated system is
\[
G_C(s) G(s) = \frac{T_s + 1}{aT_s + 1} \cdot KG(s) = \frac{T_s + 1}{aT_s + 1} G_1(s)
\]
(3.3)

Where:

ii. Determine gain K to satisfy the requirement on the given static error constant.

Using the gain K thus determined, draw a Bode diagram of \(G_1(j\omega)\), the gain adjusted but uncompensated system. Evaluate the phase margin [4]

iii. Determine the necessary phase-lead angle to be added to the system. Add an additional 5\(^\circ\) to 12\(^\circ\) to the phase-lead angle required, because the addition of the lead compensator shifts the gain crossover frequency to the right and decreases the phase margin

iv. Determine the attenuation factor \(a\) by use of Equation:
\[
\sin \phi = \frac{1-a}{1+a}
\]

v. Determine the frequency where the magnitude of the uncompensated system \(G_1(j\omega)\) is equal to \(-20\log(1/\sqrt{\alpha})\) select this frequency as the new gain crossover frequency.

This frequency corresponds to \(\omega_m=1/\sqrt{\alpha T}\) and the maximum phase shift \(\phi_m\) occurs at this frequency.

vi. Determine the corner frequencies of the lead compensator as follows:

Zero of lead compensator \(\omega = 1/T\)

Pole of lead compensator \(\omega = 1/\alpha T\)
vii. Using the value of $K$ determined in step 1 and that of $a$ determined in step 4, calculate constant $K_C$ from $K_C = k/\alpha$

viii. Check the gain margin to be sure it is satisfactory. If not repeat the design process by modifying the pole–zero location of the compensator until a satisfactory result is obtained [1].

**3.3 Design Lead Compensation Based on the Root-Locus Approach**

The procedure for designing lead compensator for the system in Figure (3.1) by the root locus is stated as follows:

i. From the performance specifications, the desired location for the dominant closed-loop poles are determined.

ii. By drawing the root-locus plot of the uncompensated system (original system) ascertain whether or not the gain adjustment alone can yield the desired closed-loop poles. If not, calculate the angle deficiency. This angle must be contributed by the lead compensator if the new root locus is to pass through the desired location for the dominant closed-loop poles.

iii. Assume the lead compensator $G_c(s)$ to be

$$G_c(s) = K_C \frac{\alpha T s + 1}{\alpha T s + 1} = K_C \frac{S + \frac{T}{\alpha}}{S + \frac{T}{\alpha}} (0 < \alpha < 1) \quad (3.4)$$

Here $\alpha$ and $T$ are determined from the deficiency, $K_C$ is determined from the requirement the open-loop gain.

If static error constants are not specified, determine the location of the pole and zero of the compensator so that the lead compensator will contribute the necessary angle $\phi$ if no other requirements are imposed on the system, try to make the value of $\alpha$ as small possible. Larger value of $\alpha$ generally results in a larger value of $K_C$ which is desirable (}
if a particular static error constant is specified, it is generally simpler to use the frequency–response approach) determine the open-loop gain of the compensated system from the magnitude condition.

Once a compensator has been designed check to see whether all performance specifications have been met. If the compensated system does not meet the performance specification then repeat the design procedure by adjusting the compensator pole and zero until all such specifications are met. If a large static error constant is required, cascade a lag network or alter the lead compensator a lag–lead compensator.

Note that if the selected dominant closed-loop poles are not really dominant, it will be necessary to modify the response obtained from the dominant closed-loop poles alone. The amount of modification depends on the location of these remaining closed-loop poles. Also, the closed-loop zeros affect the response if they are located near the origin [5].

### 3.4 Design Lag Compensation Based on the Frequency–Response Approach

The procedures for designing lag compensator for the system in Figure (3.1) by the frequency response is stated as follows

i. Assume the following lag compensator:

$$G_c(s) = K_c \beta \frac{T_s + \frac{1}{T}}{\beta T s + 1} = K_c \frac{s + \frac{1}{T}}{s + \frac{1}{\beta T s}} \quad (\beta > 1) \quad (3.5)$$

Define $K_c \beta = K$

Then $G_c(s) = K \frac{T_s + 1}{\beta T s + 1} \quad (3.6)$

Where $G_1(s) = KG(s)$

The open-loop transfer function of the compensated system is
\[ S(s) \cdot G_c(s) = G_1(s) \frac{Ts+1}{\beta Ts+1} \]  

(3.7)

ii. Determine gain K to satisfy the requirement on the given static velocity error constant.

iii. If the gain – adjusted but uncompensated system \( G_1(J\omega) \), does not satisfy the specification on the phase and gain margins, then find the frequency point where the phase angle of the open – loop transfer function is equal to \( -180^\circ \) plus the required phase margin. The required phase margin is the specified phase margin plus \( 5^\circ \) to \( 12^\circ \) (the addition of \( 5^\circ \) to \( 12^\circ \) compensated for the phase lag of the lag compensator) Choose this frequency as the new gain crossover frequency.

iv. To prevent detrimental effects of phase lag compensator, the pole and zero of the lag compensator must be located substantially lower than the new gain crossover frequency. Therefore, choose the corner Frequency \( \omega = 1/T \) (corresponding to the zero of the lag compensator) 1 octave to 1 decade below the new gain crossover frequency (if the time constants of the lag compensator do not become too large the corner frequency \( 1/T \) may be chosen 1 decade below the new gain crossover frequency.

Notice that choose the compensator pole and zero sufficiently small thus the phase lag occurs at the low – frequency region so that it will not affect the phase margin.

v. Determine the attenuation necessary to bring the magnitude curve down to 0 dB at the new gain crossover frequency. Noting that this attenuation is - 20 log \( \beta \) determine the value of \( \beta \). Then the other corner frequency (corresponding to the pole of the compensator) is determined from

\[ \omega = \frac{1}{\beta T} \]
Using the value of $K$ determined in step 1 and that of $\beta$ Determined in step 4. Calculate constant $K_c$ from

$$K_c = \frac{\kappa}{\beta} \quad (3.8)$$

### 3.5 Design Lag Compensation Based on the Root–Locus Approach

draw the root – locus plot for the uncompensated system open – loop transfer function is $G(s)$ Based on the transient – response specifications locate the dominant closed – loop poles on the root - locus.

The procedure for designing lag compensator for the system in Figure (3.1) by the root locus is stated as follows:

i. Assume the transfer function of the lag compensator to be

$$G_c(s) = K_c \beta \frac{Ts+1}{\beta Ts+1} = K_c \frac{S+\frac{1}{T}}{S+\frac{1}{\beta T}} \quad (\beta > 1) \quad (3.9)$$

Then the open – loop transfer function of the compensated system becomes $G_c(s) \cdot G(s)$.

ii. Evaluate the particular static error constant specified in the problem.

iii. Determine the amount of increase in the error constant necessary to satisfy the specifications.

iv. Determine the pole and the zero of the lag compensator produce the necessary increase in the particular static error constant without altering the original root loci (note that the ratio of the value of gain required in the specifications and the gain found in the uncompensated.)
System is the required ratio between the distance of the zero from the origin and that of the pole from the origin.

v. Draw a new root – locus plot for the compensated system. Locate the desired dominant closed loop poles on the root locus (if the angle contribution of the lag network is very small, that is, a few Degrees, then the original and new root loci are almost identical, otherwise there will be a slight discrepancy between them. Then locate on the new locus the desired dominant closed – loop poles based on the transient response specifications).

vi. Adjust gain $K_C$, of the compensator from the magnitude condition so that the dominant closed – loop poles lie at the desired location.

3.6 Design of Lag– Lead Compensation Based on the Frequency – Response Approach

Design of a lag – lead compensator by the frequency – response is based on the combination of the design techniques discussed under lead compensation and lag compensation let us assume that the lag lead compensator is of following form:

$$G_C(s) = K_C \frac{(T_1s+1)(T_2s+1)}{(\beta s+1)(\beta T_2s+1)} = K_C \frac{(S+\frac{1}{T_1})(S+\frac{1}{T_2})}{(S+\frac{\beta}{T_1})(S+\frac{\beta}{T_2})}$$  (3.10)

Where $\beta > 1$ the phase lead portion of the lag – lead compensator (the portion involving $T_1$ alters the frequency – response curve by adding phase lag portion (The portion involving $T_2$) Provides attenuation near and above the gain crossover frequency and thereby allows an increase of gain at the low frequency range to improve the steady – state performance.

3.7 Design Lag – Lead Compensation Based on the Root – Locus Approach
Consider the system shown in Figure (3.1) assume that we use the lag – lead compensator [4].

\[
G_{C}(s) = K_{C} \frac{\beta (T_{1}s+1)(T_{2}s+1)}{\gamma (T_{1}s+1)(\beta T_{2}s+1)} = \frac{K_{C}(S+\frac{1}{T_{1}})(S+\frac{1}{T_{2}})}{(S+\frac{\gamma}{T_{1}})(S+\frac{1}{\beta T_{2}})}
\]  

(3.11)

Where \(\beta > 1\) and \(\gamma > 1\)

(Consider \(K_{c}\) to belong to the lead portion of the lag – lead compensator.)

In designing lag – lead compensator, consider two cases where \(y \neq \beta\) and \(y = \beta\)

3.7.1 Case 1\((Y \neq \beta)\)

In this case the design process is a combination of the design of the lead compensator and that of the lag compensator. The design procedure for the lag – lead compensator follows:

i. From the given performance specifications determine the desired location for the dominant closed – loop poles.

ii. Using the uncompensated open – loop transfer function \(G(s)\) determine the angle deficiency \(\varnothing\) if the dominant closed – loop are to be at the desired location the phase 0 lead portion of the lag – lead compensator must contribute this angle \(\varnothing\)

iii. Assuming that later choose \(T_{2}\) sufficiently large so that the magnitude of the lag portion

\[
\left| \frac{S+\frac{1}{T_{2}}}{S+\frac{1}{\beta T_{2}}} \right|
\]  

(3.12)

Is approximately unity where \(S = S_{1}\) is one of the dominant closed – loop poles choose the values of Tandy from the requirement that.
Angle of $K_c \frac{s^1 + \frac{1}{T_1}}{s^1 + \frac{1}{T_2}} = \emptyset$

The choice of $T_1$ and $y$ is not unique (infinitely many sets of $T_1$, and $y$ are possible then determine the value if $K_C$ from the magnitude condition

$$\left| K_c \frac{s^1 + \frac{1}{T_1}}{s^1 + \frac{1}{T_2}} G(S1) \right| = 1 \quad (3.13)$$

iv. If the static velocity error constant $k_v$ is specified determine value of $\beta$ to satisfy the requirement for $k_v$. The static velocity error constant $k_v$ is given by

$$K_v = \lim_{s \to 0} G_c(S) G(S)$$

$$= \lim_{s \to 0} S Kc \left( \frac{s^1 + \frac{1}{T_1}}{s^1 + \frac{1}{T_2}} \right) \left( \frac{s^1 + \frac{1}{T_2}}{s^1 + \frac{1}{\beta T_2}} \right) G(s)$$

$$= \lim_{s \to 0} S Kc \frac{\beta}{y} G(s)$$

Where $K_c$ and $y$ are already determined in step 3. Hence given the value of $K_v$, the value of $\beta$ can be determined from this last equation. Then using the value of $\beta$ thus determined choose the value of $T_2$ such that

$$\frac{s^1 + \frac{1}{T_2}}{s^1 + \frac{1}{\beta T_2}} = 1 \quad (3.14)$$

$$-50 < \text{angle of } \frac{s^1 + \frac{1}{T_2}}{s^1 + \frac{1}{\beta T_2}} < 0 \quad (3.15)$$
3.7.2 Case 2 (Y = β)

i. If Y = β is required in Equation (3.6) then the proceeding design procedure for the lag-lead compensator may be modified as follows:

ii. From the given performance specifications determine the desired location for the dominant closed-loop poles.

The lag-lead compensator given by Equation (3.6) is modified to

$$G_C(s) = K_C \frac{(T_1s+1)(T_2s+1)}{(β^s+1)(βT_2s+1)} = K_C \frac{(S+\frac{1}{T_1})(S+\frac{1}{T_2})}{(S+\frac{β}{T_1})(S+\frac{1}{βT_2})} \quad (3.16)$$

Here β >1. The open loop transfer function of the compensated system is GC(s) G(s). If the static velocity error constant is specified determine the value of constant Kc from the following equation:

$$K_v = \lim_{s \to 0} G_C(s) G(S)$$

iii. To have the dominant closed-loop poles at the desired location, calculate the angle contribution Ø needed from the phase lead portion of the lag-lead compensator.

iv. For the lag-lead compensator later choose T_2 sufficiently large so that

$$\left| \frac{S1+\frac{1}{T_2}}{S1+\frac{β}{βT_2}} \right| \quad (3.17)$$

Is approximately unity, where S = S_1 is one of the dominant closed-loop poles. Determine the values of (T_1) and β from the magnitude and angle conditions.

$$\left| K_c \frac{S1+\frac{1}{T_1}}{S1+\frac{β}{T_1}} G(S1) \right| = 1 \quad (3.18)$$
Angle of $\frac{S_1 + \frac{1}{T_2}}{S_1 + \frac{1}{\beta T_2}} = 0$

v. Using the value of $\beta$ just determined, choose $T_2$ so that

$$\left| \frac{S_1 + \frac{1}{T_2}}{S_1 + \frac{1}{\beta T_2}} \right| = 1$$  \hspace{1cm} (3.19)

$$-5^0 \leq \arg \left( \frac{S_1 + \frac{1}{T_2}}{S_1 + \frac{1}{\beta T_2}} \right) < 0$$  \hspace{1cm} (3.20)

The value of $\beta T_2$ the largest time constant of the lag – lead compensator should not be too large to be physically realized.

**3.8 Application in MATLAB**

All the above systems were then put to test in mat lab in order to observe and study their responses to a unit step input. The results and the feeding techniques described in detail in the next chapter.
CHAPTER FOUR

SIMULATION RESULTS
CHAPTER FOUR

SYSTEM SIMULATION RESULTS

4.1 Introduction

This section demonstrates the simulation result using mat lab models of system under study with all three compensators in order to observe and study the responses to a unit step input for all compensation, then comparing the results which obtained from the mathematical derivation with the results of uncompensated system. Comparing has been made between the two methods of design (root – locus and frequency response).

4.2. Simulation of Lead compensation Based on the Frequency Response Approach

By choosing UN compensated system

\[ G(s) = \frac{16}{s(s+2)} \]  \hspace{1cm} (4.1)

The close –loop transfer function is

\[ \frac{C(s)}{R(s)} = \frac{16}{s^2+2s+16} \]  \hspace{1cm} (4.2)

After designing obtained \( G_c(s) = \frac{16.5}{s+4.02} \frac{1}{s+20.1} \)  \hspace{1cm} (4.3)

By using mat lab in appendix A-1 the unit step response of this system is shown in Figure (4.1)
Figure (4.1) Unit step response of lead compensation in frequency response

From Figure (4.1) obtained the results in Table (4.1)

Table (4.1): Unit step response specifications of lead compensator in frequency response

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Uncompensated system</th>
<th>Compensated system</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_r$</td>
<td>0.315 Second</td>
<td>0.102 Second</td>
</tr>
<tr>
<td>$M_p$</td>
<td>44.4 %</td>
<td>21.5%</td>
</tr>
<tr>
<td>$t_s$</td>
<td>3.53 Second</td>
<td>0.461 Second</td>
</tr>
<tr>
<td>Steady state</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Figure (4.2) Root-locus for the lead Uncompensated system
Figure (4.3) Root-locus for the Lead Compensated System

Figure (4.4) Bode Diagram for the Lead Uncompensated System
From Table (4.1) it notes that:

- The speed of the system increases.
- Maximum overshoot of the system increases according to the gain.
- Steady state error is not improved.

### 4.3 Simulation of Lead compensation Based on the Root-locus Approach

By choosing UN compensated system (type one) \( G(s) = \frac{16}{s(s+2)} \) \hspace{2cm} (4.4)

The close-loop transfer function is \( \frac{C(s)}{R(s)} = \frac{16}{s^2 + 2s + 16} \) \hspace{2cm} (4.5)

After designing obtained \( G_c(s) = 1.17 \frac{s + 2.93}{s + 5.46} \) \hspace{2cm} (4.6)

By using mat lab: in appendix A-2 the unit step response of this system is shown in Figure (4.6)
Figure (4.6) Unit step response of lead compensation in root-locus

From Figure (4.6) obtained the results in Table (4.2)

Table (4.2): Unit step response specifications of lead compensator in root-locus

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Uncompensated system</th>
<th>Compensated system</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t_r )</td>
<td>0.82 Second</td>
<td>0.375 Second</td>
</tr>
<tr>
<td>( M_p )</td>
<td>16.3%</td>
<td>20.8%</td>
</tr>
<tr>
<td>( t_s )</td>
<td>4.04 Second</td>
<td>2.05 Second</td>
</tr>
<tr>
<td>Steadystate</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>
Figure (4.7) Root-locus for the lead Uncompensated system

Figure (4.8) Root-locus for the Lead Compensated System
From Table (4.2) it notes that:

- The speed of the system increases.
- Maximum overshoot of the system increases according to the gain.

### 4.4 Simulation of Lag Compensation Based on the Frequency

By choosing an uncompensated system (Type zero) \( G(s) = \frac{2400(s+4)}{(s+2)(s+6)(s+8)} \) \((4.7)\)

After designing, obtained \( G_c(s) = \frac{162.8(s+4)(s+1.1)}{(s+2)(s+6)(s+8)(s+0.0746)} \) \((4.8)\)
By using mat lab in appendix A-3 the unit step response of this system is shown in Figure (4.11)

![Unit Step Responses of Compensated and Uncompensated Systems](image)

Figure (4.11) Unit step response of lag compensation in frequency response

From Figure(4.11) obtained the results in Table (4.3)

Table (4.3): Unit step response specifications of lag compensator in frequency response

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Uncompensated system</th>
<th>Compensated system</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_r$</td>
<td>0.0984 Second</td>
<td>0.122 Second</td>
</tr>
<tr>
<td>$M_p$</td>
<td>11%</td>
<td>14%</td>
</tr>
<tr>
<td>$t_s$</td>
<td>0.56 Second</td>
<td>1.17 Second</td>
</tr>
<tr>
<td>Steadystate</td>
<td>0.107 (1-0.893)</td>
<td>0.01 (1-0.99)</td>
</tr>
</tbody>
</table>
Figure (4.12) Root-locus for the lag Uncompensated system

Figure (4.13) Root-locus for the lag Compensated system
From table (4.3) it notes that:

- The speed of the system slow.
These compensating networks increase the steady state accuracy of the system. An important point to be noted here is that the increase in the steady state accuracy brings instability to the system.

- Maximum overshoot of the system increases.
- In general the lag compensator improve the steady state error.

4.5 Simulation of Lag compensation Based on the Root-locus Approach

By choosing UN compensated system (Type zero)

\[ G(s) = \frac{164.6}{(s+1)(s+2)(s+10)} \]  \hspace{1cm} (4.9)

After designing obtained \( G_c(s) = \frac{158.1(s+0.111)}{(s+1)(s+2)(s+10)(s+0.01)} \) \hspace{1cm} (4.10)

By using mat lab in appendix A-4 the unit step response of this system is shown in Figure (4.16)

![Unit step response](image)

Figure (4.16) Unit step response of lag compensation in root-locus

From Figure (4.16) obtained the results in Table (4.4)

Table (4.4): Unit step response specifications of lag compensator in root-locus
<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Uncompensated system</th>
<th>Compensated system</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_r$</td>
<td>0.32 Second</td>
<td>0.351 Second</td>
</tr>
<tr>
<td>$M_p$</td>
<td>37%</td>
<td>40.2%</td>
</tr>
<tr>
<td>$t_s$</td>
<td>5.14 Second</td>
<td>14.8 Second</td>
</tr>
<tr>
<td>Steady state</td>
<td>0.108 (1-0.893)</td>
<td>0.02 (1- 0.98)</td>
</tr>
</tbody>
</table>

Figure (4.17) Root-locus for the lag Uncompensated system
Figure (4.18) Root-locus for the Lag Compensated System

Figure (4.19) Bode Diagram for the Lag Uncompensated System
From Table (4.4) it notes that:

- The speed of the system slow.
- Maximum over shoot of the system increases.
- The steady state accuracy increases.

4.6 Simulation of Lag-Lead compensation based on the Frequency response Approach

By choosing UN compensated system (Type one) \( G(s) = \frac{4}{s(s+0.5)} \) \hspace{1cm} (4.11)

After designing obtained \( G_c(s) = \frac{10(2s+1)(5s+1)}{(0.1992s+1)(8.019s+1)} \) \hspace{1cm} (4.12)

With mat lab in appendix A-5 the unit step response of this system is shown in Figure (4.21)
Figure (4.21) Unit step response of lag-lead compensation in frequency response

From Figure (4.21) obtained the results in Table (4.5):

Table (4.5): Unit step response specification of lag-lead compensator in frequency response

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Compensated system</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_r$</td>
<td>0.684 Second</td>
</tr>
<tr>
<td>$M_p$</td>
<td>16.2%</td>
</tr>
<tr>
<td>$t_s$</td>
<td>13.5 Second</td>
</tr>
<tr>
<td>Steady state</td>
<td>1</td>
</tr>
</tbody>
</table>
Figure (4.22) Root-locus for the lag-Lead Uncompensated system

Figure (4.23) Root-locus for the lag-Lead Compensated system
From Table 4.5 it notes that:

- The speed of the system increases.
- Maximum overshoot of the system decreases.
- The steady state accuracy increases.
- The uncompensated system is unstable after compensated is became stable.
4.7 Simulation of Lag- Lead compensation Based on the Root- locus Approach

By choosing UN compensated system (Type one)  
\[ G(s) = \frac{40}{s(s+1)(s+4)} \]  
(4.13)

After designing  obtained  
\[ Gc(s)= \frac{(s+0.4)(s+0.2)}{(s+4)(s+0.02)} \]  
(4.14)

By using mat lab in appendix A-6 the unit step response of this system is shown in Figure (4.26)

![Unit Step Responses of Compensated and Uncompensated Systems](image)

Figure (4.26) Unit step response of lag- lead compensation in root-locus

From Figure (4.26) obtained the results in Table (6)

Table (4.6): Unit step response specification of lag- lead compensator in root-locus

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Uncompensated system</th>
<th>Compensated system</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t_r )</td>
<td>0.565 Second</td>
<td>0.317 Second</td>
</tr>
<tr>
<td>( M_p )</td>
<td>67.3%</td>
<td>21.1%</td>
</tr>
<tr>
<td>( t_s )</td>
<td>14.7 Second</td>
<td>3.42 Second</td>
</tr>
<tr>
<td>Steady state</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>
Figure 4.27 Root-locus for the lag-Lead Uncompensated system

Figure 4.28 Root-locus for the lag-Lead Compensated system

Figure 4.29 Bode Diagram for the Lag-Lead Uncompensated System
From Table (4.6) it notes that:

- The speed of the system increases.
- Maximum overshoot of the system decreases.
- The steady state accuracy increases.
CHAPTER FIVE

CONCLUSION AND RECOMMENDATION
Chapter FIVE

Conclusions and Recommendations

5.1 Conclusions

- Lead compensation basically speeds up the response and increases the stability of the system
- Lag compensation improves the steady-state accuracy of the system, but reduces the speed of the response.
- If both fast responses and good static accuracy are desired, a lag – lead compensator may be employed. By use of the lag – lead compensator, the low frequency gain can be increased (which means an improvement in steady-state accuracy) while at the same time the system bandwidth and stability margins can be increased.

5.2 Recommendations and Future Work

- Implementation of proposed compensators.
- Comparing the analog compensation with the digital compensation.
- Apply different compensators to non linear systems.
References


Mason University, Fairfax , Virginia .
Appendix
Appendix

CODE A-1: lead compensation in frequency response and UN compensation

clc
clearall
closeall

% Lead Compensator Frequency
% unit step
numc = [0 0 264 1056];
denc = [1 22 304 1056];
um = [0 0 16];
den = [1 2 16];

% Lead Compensator Frequency
% unit step
numc = [0 0 264 1056];
denc = [1 22 304 1056];
um = [0 0 16];
den = [1 2 16];

t = 0 : 0.01 : 8;
[c1,X1,t] = step(num,den,t);
[c2,X2,t] = step(numc,denc,t);
plot(t,c1,'--m',t,c2,'g-','LineWidth',2)
gridon
title('Unit Step Responses of Compensated and Uncompensated Systems')
xlabel('t (sec)')
ylabel('Outputs')
text(0.35,1.3,'Compensated System')
text(1.55,0.88,'Uncompensated System')
CODE A: 2: lead compensation in root locus and UN compensation

clc
clearall
closeall

% Lead Compensator
% unit step
numc = [0 0 18.7 54.23];
denc = [1 7.4 29.5 54.23];
num = [0 0 16];
den = [1 2 16];

t = 0 : 0.05 :8;

[c1,X1,t] = step(numc,denc,t);
[c2,X2,t] = step(num,den,t);

plot(t,c1,'-r',t,c2,'-')
gridon
title('Unit Step Responses of Compensated and Uncompensated Systems')
xlabel('t (sec)')
ylabel('Outputs c1 and c2')
text(0.6,1.32,'Compensated System')
text(1.3,0.68,'Uncompensated System')
CODE A-3: lag compensation in frequency response and UN compensation

clc
        clear all
close all

% Lag Compensator
% unit step

numc = [162.8  830.3  716.3];
denc = [1 16.07 240 931.9  723.5];
um = [   200  800];
den = [  1 16  276  896];
t = 0 : 0.001 :3;
[c1,X1,t] = step(num,den,t);
[c2,X2,t] = step(numc,denc,t);
plot(t,c1,'-r',t,c2,'b-','LineWidth',2)
hold on
title('Unit Step Responses of Compensated and Uncompensated Systems')
xlabel('t (sec)')
ylabel('Outputs c1 and c2')
text(1.7,1.1,'Compensated System')
text(1.7,0.8,'Uncompensated System')
CODE A: 4: lag compensation in root locus and UN compensation
clc
clear all
close all
% Lag Compensator
% unit step
numc = [158.1 17.55];
denc = [1 13.01 32.13 178.4 17.75];
num = 164.6;
den = [1 13 32 184.6];
t = 0 : 0.01 :15;
[c1,X1,t] = step(numc,denc,t);
[c2,X2,t] = step(num,den,t);
plot(t,c1,'-','LineWidth',2)
hold on
plot(t,c2,'.r','LineWidth',0.1)
plot([0 15],[1 1],':k')
% grid on
title('Unit Step Responses of Compensated and Uncompensated Systems')
xlabel('t (sec)')
ylabel('Outputs c1 and c2')
text(11,1.1,'Compensated System')
text(11,0.8,'Uncompensated System')
CODE A-5: Lag - lead compensation in frequency response and UN compensation

clc

clear all

close all

% Lag-Lead Compensator Frequency
% unit step
numc = [0 0 0 40 24 3.2];
denc = [1 9.02 24.18 56.48 24.32 3.2];
num = [0 0 0 40];
den = [1 5 4 40];
t = 0 : 0.2 :40;
[c1,X1,t] = step(numc,denc,t);
plot(t,c1,'LineWidth',2)
grid on
title('Unit Step Responses of Compensated System')
xlabel('t (sec)')
ylabel('Outputs')
CODE A: 6: Lag - lead compensation in root locus and UN compensation

clc
clear all
close all

% Lag-Lead Compensator
% unit step
numc = [25.04 5.008];
denc = [1 5.032 25.1 5.008];
num = [0 0 4];
den = [1 0.5 4];
t = 0 : 0.02 :8;
[c1,X1,t] = step(numc,denc,t);
[c2,X2,t] = step(num,den,t);
plot(t,c1,t,c2,'.')
grid on
title('Unit Step Responses of Compensated and Uncompensated Systems')
xlabel('t (sec)')
ylabel('Outputs c1 and c2')
text(1,0.4,'Compensated System')
text(2.5,1.5,'Uncompensated System')
CODE A: 7: Lag - lead compensation in root locus and UN compensation

clc
clear all
close all

% Lead Compensator using Frequency response method
% Uncompensated system
zeros = [];
poles = [0 -2];
sys = zpk(zeros,poles,1);
rlocus(sys)
title('Root Locus for the Uncompensated system')
figure
bode(sys)
title('Bode Diagram for the Uncompensated system')

% System with Compensator
zeros = -4.02;
poles = [0 -2 -20.1];
sys = zpk(zeros,poles,1);
figure
rlocus(sys)
title('Root Locus for the Lead-compensated system')
figure
bode(sys)
title('Bode Diagram for the Lead-compensated system')
CODE A: 8 lead compensation in root locus and UN compensation

clc
clear all
close all

% Lead Compensator using Root Locus
% Uncompensated system
zeros = [];
poles = [0 -2];
sys = zpk(zeros,poles,1);
rlocus(sys)
title('Root Locus for the Uncompensated system')
figure
bode(sys)
title('Bode Diagram for the Uncompensated system')

% System with Compensator
zeros = -2.93;
poles = [0 -2 -5.46];
sys = zpk(zeros,poles,1);
figure
rlocus(sys)
title('Root Locus for the Lead-compensated system')
figure
bode(sys)
title('Bode Diagram for the Lead-compensated system')
CODE A-9: lag compensation in frequency response and UN compensation

clc
clear all
close all

% Lag Compensator using Frequency Response method
% Uncompensated system
zeros = -4;
poles = [-2 -6 -8];
sys = zpk(zeros,poles,1);
rlocus(sys)
title('Root Locus for the Uncompensated system')
figure
bode(sys)
title('Bode Diagram for the Uncompensated system')

% System with Compensator
zeros = [-4 -1.1];
poles = [-0.0746 -2 -6 -8];
sys = zpk(zeros,poles,1);
figure
rlocus(sys)
title('Root Locus for the Lag-compensated system')
figure
bode(sys)
title('Bode Diagram for the Lag-compensated system')
CODE A: 10: lag compensation in root locus and UN compensation

clc
clear all
close all

% Lag Compensator using root-locus
% Uncompensated system
zeros = [];
poles = [-1 -2 -10];
sys = zpk(zeros,poles,1);
rlocus(sys)
title('Root Locus for the Uncompensated system')
figure
bode(sys)
title('Bode Diagram for the Uncompensated system')

% System with Compensator
zeros = -0.111;
poles = [-0.01 -1 -2 -10];
sys = zpk(zeros,poles,1);
figure
rlocus(sys)
title('Root Locus for the Lag-compensated system')
figure
bode(sys)
title('Bode Diagram for the Lag-compensated system')
CODE A-11: Lag - lead compensation in frequency response and UN compensation

clc
clear all
close all
% Lag-Lead Compensator using frequensy response
% Uncompensated system
zeros = [];
poles = [0 -0.5];
sys = zpk(zeros,poles,1);
figure
rlocus(sys)
title('Root Locus for the Uncompensated system')
figure
bode(sys)
title('Bode Diagram for the Uncompensated system')
% System with Compensator
zeros = [-0.2 -0.5];
poles = [0 -0.012 -0.5 -5];
sys = zpk(zeros,poles,1);
figure
rlocus(sys)
title('Root Locus for the Lag-Lead-compensated system')
figure
bode(sys)
title('Bode Diagram for the Lag-Lead-compensated system')
CODE A: 12: Lag - lead compensation in root locus and UN compensation

clc
clear all
close all
% Lag-Lead Compensator using root-locus
% Uncompensated system
zeros = [];
poles = [0 -1 -4];
   sys = zpk(zeros,poles,1);
rlocus(sys)
title('Root Locus for the Uncompensated system')
   figure
bode(sys)
title('Bode Diagram for the Uncompensated system')
% System with Compensator
% Uncompensated system
zeros = [-0.4 -0.2];
poles = [0 -0.02 -1 -4 -4];
   sys = zpk(zeros,poles,1);
figure
rlocus(sys)
title('Root Locus for the Lag-Lead-compensated system')
   figure
bode(sys)
title('Bode Diagram for the Lag-Lead-compensated system')