

Muon Decay Width and Lifetime in the Standard Model

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Abstract

In this thesis we studied the muon decay $\mu \rightarrow e\bar{\nu}_e\nu_\mu$ according to the standard model (SM). We computed the decay width Γ_μ and lifetime τ_μ for leptonic muon, We found their values $\Gamma_\mu = \frac{1}{2.19 \times 10^{-6}} \text{ Second}^{-1}$ and $\tau_\mu = 2.19 \times 10^{-6}$ second respectively, these results showed a good agreement with the experimental results. We Used the obtained value of τ_μ with the given muon mass to determine the Fermi coupling of weak interaction G_f . In addition, We have calculated the value of g_2 , this is the weak coupling constant in the standard model, we found that: $\alpha_2 = \frac{1}{29}$ which is larger than the fine structure constant of the electromagnetic interaction, that is: $\alpha_1 = \frac{1}{137}$. Consequently, the weak interaction is actually found to be not "weak", as the word "weak" reflect meaning.

Dedication

I dedicate my work to my family. A special feeling of gratitude to my loving mom Fatima (Horah), who taught me the best kind of knowledge and also taught me that even the largest task can be accomplished if it is done one step at a time. My darling wife Azza who has never left my side.

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I am, of course, very grateful to all my family members for their permanent love and support, especially my wife and son, they were very patient a lot during my work.This thesis is dedicated to them.

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Chapter 1

Introduction

The interaction between cosmic rays and atoms and molecules present in the air produces a shower of particles that include protons, neutrons, pions (both charged and neutral), kaons, photons, electrons and positrons. These secondary particles then these particle interact via electromagnetic and nuclear interactions to produce an additional particles in a cascade process. Of particular interest is the fate of the charged pions produced in the cascade. Some of these will interact via the strong force with atmospheric molecule nuclei but others will decay via the weak force into a muon plus a neutrino or antineutrino. The muon decay is of most great important in studying the weak interactions. One of the fundamental force in nature. In this thesis we shall study the muon decay width and its lifetime in the standard model of particle physics at length.

1.0.1 The importance of the research

The muon is considered to be the simplest and cleanest process which used to determine the left handed V-A structure of the weak charged currents . The decay rate of the muon has been studied for decades and continuously to test the standard model of particle physics and search for new physics .

1.0.2 The main objectives of the thesis

We proposed to calculate the decay width of muon and its lifetime τ_μ , then we shall use the calculated muon mass and lifetime to determine the Fermi coupling constant. Furthermore we also use the measurement of W mass to determine the g_2 (the weak coupling constant) . As well as the fine structure for weak interaction α .

1.0.3 Literature review

Muon is an elementary particle, muon is very smaller to electron mass. Muon decay is governed by charged weak interaction . One can use the measurements

of τ and m_μ to measure G_f (strength of weak interaction)for more details on this we refer interested reader to the following book:introduction to high energy- D. H.parkin.

Cosmic rays are the source of muons in experiment, cosmic rays interact with upper atmosphere produces many secondary elementary particles, mostly the muon. See for example T-suzaki, Total nuclear capture rates for negative muon-physical Review C35 (1987)2212.

1.1 Outline of the thesis

The thesis is structured as follows: In chapter 2 we briefly reviewed the SM of particle physics. The decay width and lifetime are studied in chapter 3, specifically we discussed the calculation in details. We presented results and discussion in chapter 4. Our conclusions will be presented in chapter 5.

Chapter 2

The Standard Model (SM)

This chapter shall study the structure of the standard model and its mathematical formulation, then we discuss the Higgs mechanism to see how particles can obtained their masses.

2.1 What is the SM

The SM is currently accepted theory and has been verified to high level of accuracy. The SM is a theory that describes the interactions between elementary particles. It combines the strong interaction known as Quantum Chromodynamics (QCD), based on the group $SU(3)_C$ [2, 3, 7], and the Glashow-Weinberg-Salam theory of the electroweak interaction (that unified the weak and electromagnetic interactions), based on the group $SU(2)_L \times U(1)_Y$ [2, 3, 7]. Thus the SM is the product of $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge theory. The spectrum of SM fermion has the following assignment.

$$q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L ; \quad u_R, d_R, \quad (2.1)$$

$$\ell_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L ; \quad e_R, \quad (2.2)$$

where the colour index have been omitted for quarks and we show here only one generation for simplicity. Note that the left handed and right handed fermions transform differently. For example, the doublet shown in Eq.(2.2) is assumed to transform in the fundamental representation of an $SU(2)_L$ group, whereas the right handed partners are singlet under this group, and the neutrinos are assumed to be left handed only. The neutrino will not acquire mass because its right handed partner does not exist in this theory [8].

2.2 SM Lagrangian

The Lagrangian of SM can be written as:

$$\mathcal{L}_{SM} = \mathcal{L}_{Gauge} + \mathcal{L}_{fermion} + \mathcal{L}_{Higgs} + \mathcal{L}_{Yukawa} + \mathcal{L}_{Guage.fixing} + \mathcal{L}_{Ghost}. \quad (2.3)$$

We will briefly introduce each sector of the above Lagrangian:

2.2.1 Guage Sector

The gauge sector is consist of 12 gauge fields which mediate the interactions among the fermion fields; the photon (γ , mediates the electromagnetic interactions), the three weak gauge bosons (W^\pm and Z , mediate the weak interactions) and eight gluons ($g_\alpha, \alpha = 1, 2, 3, \dots, 8$, mediate the strong interactions). The gauge field dynamics are written in the Lagrangian in terms of field strength tensors as

$$\mathcal{L}_{Gauge} = -\frac{1}{4}G_{\mu\nu}^A G^{A\mu\nu} - \frac{1}{4}W_{\mu\nu}^a W^{a\mu\nu} - \frac{1}{4}B_{\mu\nu} B^{\mu\nu} \quad (2.4)$$

where repeated indices imply summ over that index, and μ, ν , takes the values of 0,1,2,3, where the field strength tensors for non-Abelian theories are given by:

$$G_{\mu\nu}^A = \partial_\mu G_\nu^A - \partial_\nu G_\mu^A - ig_s f^{ABC} G_\mu^B G_\nu^C, \quad (2.5)$$

being the $SU(3)_C$ field strength, g_s is the coupling strength of the strong interaction, A,B,C run from 1 to 8 and f^{ABC} are the(antisymmetric) structure constants of $SU(3)$, which satisfies the Lie algebra for the group generator t^A

$$[t^A, t^B] = if^{ABC} t^C. \quad (2.6)$$

$$W_{\mu\nu}^a = \partial_\mu W_\nu^a - \partial_\nu W_\mu^a - ig\epsilon^{abc} W_\mu^b W_\nu^c, \quad (2.7)$$

is the $SU(2)_L$ field strength, a,b,c run from 1 to 3 and ϵ^{abc} is the totally antisymmetric three-index tensor with $\epsilon^{123} = 1$, g is the coupling strength of the weak interaction. The field strength of the $U(1)_Y$ gauge boson which has the same form as electromagnetism is given by:

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu. \quad (2.8)$$

2.2.2 Fermion Sector

Fermions are elementary particles have a spin (1/2) and consists of leptons and quarks. Quarks and leptons are the elementary particles that build up matter. This means that they are the smallest building blocks that have been discovered so far. There exist six different types of quarks, called different flavors, which are divided into three generations depending on how they occur in pairs. The generations have similar properties but increasing mass. In contrast to the quarks in the first generation, which are stable, quarks in the second and third generation are heavier and are therefore unstable and decay.

The SM contains three copies of chiral fermions (generations) with different gauge transformations. The fermionic Lagrangian has the usual covariant Dirac form

$$\mathcal{L}_{Fermions} = \sum \bar{f} \gamma_\mu D^\mu f, \quad (2.9)$$

The covariant derivatives to be read as:

$$D_\mu \begin{pmatrix} u \\ d \end{pmatrix}_L = \left(\partial_\mu - ig_s \frac{\lambda^a}{2} G_\mu^a - ig \frac{\sigma^a}{2} W_\mu^a - ig' \frac{1}{6} B_\mu \right) \begin{pmatrix} u \\ d \end{pmatrix}_L, \quad (2.10)$$

$$D_\mu u_R = \left(\partial_\mu - ig_s \frac{\lambda^a}{2} G_\mu^a - ig' \frac{2}{3} B_\mu \right) u_R, \quad (2.11)$$

$$D_\mu d_R = \left(\partial_\mu - ig_s \frac{\lambda^a}{2} G_\mu^a + ig' \frac{1}{3} B_\mu \right) d_R, \quad (2.12)$$

$$D_\mu \begin{pmatrix} \nu \\ e \end{pmatrix}_L = \left(\partial_\mu - ig \frac{\sigma^a}{2} W_\mu^a + ig' \frac{1}{2} B_\mu \right) \begin{pmatrix} \nu \\ e \end{pmatrix}_L, \quad (2.13)$$

and

$$D_\mu e_R = (\partial_\mu + ig' B_\mu) e_R. \quad (2.14)$$

Where γ_μ are the usual Dirac matrices, g' is the coupling strength of the hypercharge interaction, Y is the hypercharge, σ^a are the generators of $SU(2)_L$ (simply the Pauli matrices), and λ^a are the generators of $SU(3)_C$ (the Gell-Mann matrices).

Note that gauge symmetry forbids a mass term for fermions (quarks and leptons) and gauge bosons. A mass term would break the gauge invariance $SU(2)_L \times U(1)_Y$. But, we observed the mass of gauge bosons W and Z and the fermions experimentally [11], so we need to give mass to these particles. The masses in the SM are generated through a different mechanism, the Higgs mechanism, which will be discussed at length in next section.

2.2.3 The Higgs Mechanism

As was mentioned in the previous section, a Dirac mass term will break the gauge symmetry. So we need a mechanism that gives mass to the SM particles and keeps the Lagrangian invariant under gauge symmetries. This can be done through the mechanism of spontaneous gauge symmetry breaking also known as the Higgs mechanism. This mechanism added a new complex scalar field Φ which is a doublet with respect to $SU(2)_L$ group, and singlet under $SU(3)_C$ and has hypercharge $Y_\Phi = 1$ [12–15].

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}, \quad (2.15)$$

where ϕ_1, ϕ_2, ϕ_3 and ϕ_4 are real scalars. This new scalar Φ adds extra terms to the SM Lagrangian:

$$\mathcal{L}_{Higgs} = (D_\mu \Phi)^\dagger (D^\mu \Phi) - V(\Phi), \quad (2.16)$$

where the covariant derivative D_μ is defined as

$$D_\mu = \partial_\mu - i\frac{g'}{2}B_\mu - ig\frac{\sigma^a}{2}W_\mu^a. \quad (2.17)$$

The general gauge invariant renormalizable potential involving Φ is

$$V(\Phi) = -\frac{1}{2}\mu^2\Phi^\dagger\Phi + \frac{\lambda}{4}(\Phi^\dagger\Phi)^2. \quad (2.18)$$

The above equation.(2.18) describes the Higgs potential, which involves two new real parameters μ and λ . We demand $\lambda > 0$ for the potential to be bounded; otherwise the potential is unbounded from below and there is no stable vacuum state. μ takes the following two values:

- $\mu^2 > 0$ then the vacuum corresponds to $\Phi = 0$, the potential has a minimum at the origin (see Fig.2.1 right panel).
- $\mu^2 < 0$ then the potential develops a non-zero Vacuum Expectation Value (VEV) and the minimum is along a circle of radius $\frac{v}{\sqrt{2}} = \frac{246}{\sqrt{2}}$ (see Fig.2.1 left panel). Minimizing the potential one gets

$$\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2 = -\frac{\mu^2}{\lambda} = v^2. \quad (2.19)$$

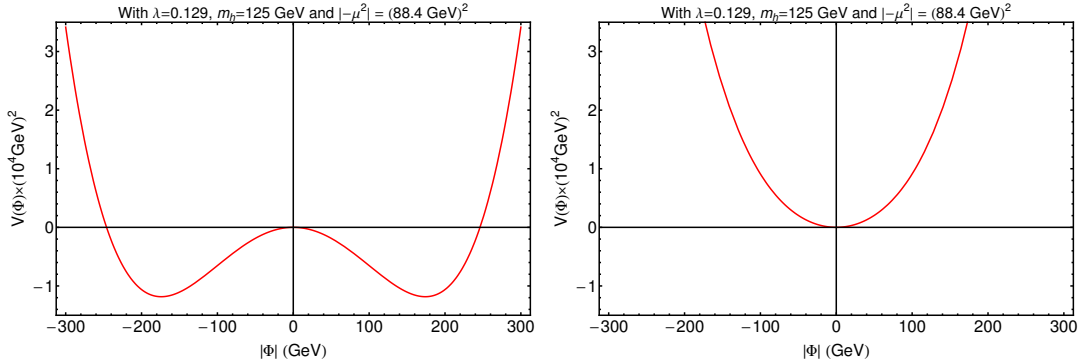


Figure 2.1: The Higgs potential $V(\Phi)$ with: in the left panel, the case $\mu^2 < 0$; and the right panel for the case $\mu^2 > 0$ as a function of $|\Phi| = \sqrt{\Phi^\dagger\Phi}$.

$$\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} . \quad (2.20)$$

As such, we have to choose one of these minima as the ground state ($\phi_3 = v$ and $\phi_1 = 0$, $\phi_2 = 0$ and $\phi_4 = 0$). Therefore the vacuum does not have the original symmetry of the Lagrangian, and therefore the symmetry spontaneously broken [15]. In other words, the Lagrangian is still invariant under the $SU(2)_L \times U(1)_Y$, but the ground state is not. We choose the VEV in the neutral direction as the photon is neutral, so Φ becomes

$$\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} . \quad (2.21)$$

With this particular choice of the ground state, the electroweak gauge group $SU(2)_L \times U(1)_Y$ is broken to electromagnetism one, $U(1)_{em}$,

$$SU(2)_L \times U(1)_Y \xrightarrow{\langle \Phi \rangle} U(1)_{em} . \quad (2.22)$$

2.2.4 Gauge Boson Masses

The gauge boson masses can be obtained from the kinetic term of the Higgs field [2]. Expanding the Lagrangian around the VEV yields:

$$\mathcal{L}_{Higgs} = \frac{1}{2} \begin{pmatrix} 0 & v \end{pmatrix} \left(g \frac{\sigma^a}{2} W_\mu^a + \frac{1}{2} g' B_\mu \right) \left(g \frac{\sigma^b}{2} W_\mu^b + \frac{1}{2} g' B_\mu \right) \begin{pmatrix} 0 \\ v \end{pmatrix} . \quad (2.23)$$

Using the definition of $W_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \pm W_\mu^2)$, $Z_\mu = W_\mu^3 \cos \theta_W - B_\mu \sin \theta_W$ and $A_\mu = W_\mu^3 \sin \theta_W + B_\mu \cos \theta_W$, we obtained three massive gauge bosons

$$m_W^2 = \frac{1}{4} g^2 v^2 , \quad m_Z^2 = \frac{1}{4} (g'^2 + g^2) v^2 , \quad (2.24)$$

and one massless gauge boson (identified as the photon)

$$m_A^2 = 0 . \quad (2.25)$$

The Weinberg angle θ_W is defined by

$$\cos \theta_W = \frac{g}{\sqrt{g'^2 + g^2}} , \quad (2.26)$$

$$\sin \theta_W = \frac{g'}{\sqrt{g'^2 + g^2}} . \quad (2.27)$$

2.2.5 Fermion Masses

Fermion masses are originating from Yukawa interactions, which are the couplings between the fermion doublets and the scalar field Φ [2]. These Yukawa couplings are uniquely fixed by gauge invariance and the Lagrangian, as given by:

$$\mathcal{L}_{Yukawa} = Y_{ij}^d \bar{q}_L^i \Phi d_R^j + Y_{ij}^u \bar{q}_L^i \tilde{\Phi} u_R^j + Y_{ij}^e \bar{l}_L^i \Phi e_R^j + \text{h.c.} , \quad (2.28)$$

where the Y 's are 3×3 complex matrices, the so called Yukawa coupling constants, h.c. indicates the Hermitian conjugate and $\tilde{\phi}$ is defined by

$$\tilde{\Phi} = \begin{pmatrix} -\phi_2^* \\ \phi_1^* \end{pmatrix} . \quad (2.29)$$

When the Higgs doublet acquires a non vanishing VEV, Eq.(2.28) gives the mass terms for the fermions as follows:

$$\mathcal{L}_{Yukawa} = m_u \bar{u}_L u_R + m_d \bar{d}_L d_R + m_e \bar{e}_L e_R , \quad (2.30)$$

with $m_u = \frac{1}{\sqrt{2}} y_u v$; $m_d = \frac{1}{\sqrt{2}} y_d v$; $m_e = \frac{1}{\sqrt{2}} y_e v$.

Note that neutrinos are massless and will never acquire mass, this is due to the absent of its chiral partner ν_R .

When we consider all the generations of quarks, there are possibilities for their mixing. This mixing is described by the Cabbibo Kobayatchi Masakawa, which has four observable parameters, including three mixing angles and one phase [16]. It appears upon the diagonalisation of Yukawa matrices by using two unitary matrices U and V , where

$$UY_u^\dagger Y_u U^\dagger = \text{diag}(f_u^2, f_c^2, f_t^2); \quad VY_d^\dagger Y_d V^\dagger = \text{diag}(h_d^2, h_s^2, h_b^2) . \quad (2.31)$$

The CKM matrix is given by

$$V_{CKM} = UV^\dagger . \quad (2.32)$$

The form of the CKM matrix that describes the quark sector mixing is parametrised as

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} ,$$

and the standard parametrisation in terms of the three mixing angles and one phase can have the form

$$V_{CKM} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} , \quad (2.33)$$

where $s_{12} = \sin \theta_{12}$, $c_{12} = \cos \theta_{12}$ etc. are the sines and cosines of the three mixing angles θ_{12} , θ_{23} and θ_{13} , and δ is the CP violating phase.

2.2.6 The Higgs Boson

As was discussed in section 2.2.3, the spontaneous symmetry breaking predicted a new particle: the Higgs Boson, which must be a scalar and neutral. The Lagrangian for this new scalar comes from the kinetic term of Eq.(2.16) expanded around the VEV

$$\mathcal{L}_{Higgs.Boson} = \frac{1}{2}(\partial_\mu h)(\partial^\mu h) - \frac{1}{2}m_h^2 h^2 + \text{interactions} , \quad (2.34)$$

Here $m_h^2 = \sqrt{\lambda}v$ is the Higgs boson mass. The interaction terms contain both Higgs self-interactions and interactions with gauge bosons and fermions. Note that as a consequence of the Higgs mechanism all the Higgs couplings are completely determined in terms of the coupling constants and masses. The Higgs boson, which was the last missing piece of the SM, has been discovered by the ATLAS and CMS experiments, and this is compatible with the SM Higgs expectations with a mass of about 126 GeV [5,6].

2.2.7 Gauge Fixing and Ghosts

Gauge fixing and ghost fields are necessary when the gauge fields are quantised. These are important subjects when dealing with higher loops order. Note that our calculation is three level decay so we just write down the Lagrangian for gauge fixing and ghost fields and refer interested reader on this subject to look at [2,17].

$$\mathcal{L}_{Gauge.fixing} = -\frac{\zeta}{2}(\partial_\mu A^\mu)^2 , \quad (2.35)$$

and

$$\mathcal{L}_{Ghost} = \bar{c}_b \partial^\mu D_\mu^{ab} c_a . \quad (2.36)$$

Thus, we are now in the right position to write the full SM Lagrangian

$$\mathcal{L}_{SM} = \mathcal{L}_{Gauge} + \mathcal{L}_{Fermions} + \mathcal{L}_{Yukawa} + \mathcal{L}_{Higgs} + \mathcal{L}_{Gauge.fixing} + \mathcal{L}_{Ghost} . \quad (2.37)$$

Chapter 3

Decay Widths and Scattering Cross Sections

We are now ready to calculate the rates of some simple scattering and decay processes. The cross section, σ , is a measure of the probability of a specific scattering process under some given set of initial and final conditions, such as momenta and spin polarization. The lifetime, τ , or, equivalently, decay width, $\Gamma = (\frac{1}{\tau})$, which is a measure of the probability of a specific decay process occurring within a given amount of time in the particles rest frame [16]. The calculation involves two steps:

Calculate the amplitude M

It is often referred to as the matrix element, and denoted by $M_{f,i}$, to indicate that in a matrix representation of the transformation process, with the initial and final states as bases, this is the element that connects a particular final state f to a given initial state i . A process can be a combination of subprocesses, in which case, the total amplitude is the sum of the subprocess amplitudes. Each simple (sub) process is represented by a unique Feynman diagram. Its amplitude is a point function in the phase space of all the particles involved, including any intermediate propagator, and depends on the nature of the coupling at each vertex (of the diagram). For a given diagram, the amplitude can be obtained by using the Feynman rules for combining the elements a factor for each external line (representing a free particle in the initial or final state), one for each internal line (representing a virtual propagator particle), and one for each vertex point where the lines do meet.

Integrate the amplitude

Integrating the amplitude over the allowed phase space to get the σ or Γ . The integral can be constructed, easily in principle, by utilizing Fermis golden rule. This section will describe the above rules and use them to calculate the decay rates.

3.1 Physical meaning of decay width

One of the most important characteristics of a particle is the lifetime [16]. It depends, of course, on the available decay modes or channels, which are subject to conservation laws for appropriate quantum numbers, coupling strength of the decay process, and kinematic constraints. The decay rate is the probability per unit time that a given particle will decay. The probability that a single unstable entity will cease to exist as such after an interval is proportional to that interval. In elementary particles, for an ensemble of $N \rightarrow \infty$ identical particles, the change in the number after a time dt is given by:

$$dN = -\Gamma N dt. \quad (3.1)$$

Therefore, the expected number surviving after time t is:

$$N(t) = N(0) \exp^{-\Gamma t} \quad (3.2)$$

The time after which the ensemble is expected to reduced to $\frac{1}{e}$ of its original size is called the lifetime:

$$\tau = \frac{1}{\Gamma}. \quad (3.3)$$

If multiple decay modes are available, as is often the case, then one can associate a decay rate for each mode, and the total rate, will be the sum of the rates of the individual modes.

$$\Gamma_{total} = \sum_{i=1}^n \Gamma_i. \quad (3.4)$$

The particles lifetime is given by

$$\tau = \frac{1}{\Gamma_{total}}. \quad (3.5)$$

In such cases, we are often interested in the branching fractions, i.e. the probabilities of the decay by individual modes. The branching fraction of mode i is:

$$B_i = \frac{\Gamma_i}{\Gamma_{total}} \quad (3.6)$$

Since the dimension of Γ is the inverse of time, in our system of natural units, it has the same dimension as mass (or energy). When the mass of an elementary particle is measured, the total rate shows up as the irreducible width of the shape of the distribution. Hence the name decay width

3.1.1 Physical meaning of scattering cross section

Consider the $2 \rightarrow n$ scattering process

$$ab \rightarrow cd\dots \quad (3.7)$$

The system of incoming particles labeled a, b constitute the initial state $|i\rangle$, and that of the outgoing particles labeled c, d, \dots constitute the final state $|f\rangle$. If a packet of a particles is made to pass head-on through a packet of b particles with overlap area is A , and the number of particles swept by that overlap area in the two packets are N_a and N_b respectively, then the number of scatterings, N_S is directly proportional to N_a and N_b , and inversely to A . The overall constant of proportionality is called the cross section, σ :

$$N_s = \sigma \frac{N_a N_b}{A} \quad (3.8)$$

Thus, the cross section must have the same dimension as area. Cross sections in contemporary High Energy Physics experiments are typically measured in units of nanobarn (nb) to femtobarn (fb), where a barn is defined as follow:

$$1b = 10^{-24} cm^2 = 2.568 GeV^{-2} \quad (3.9)$$

As for decays, one is often more interested in various differential (or exclusive) cross sections, σ_i rather than the total (or inclusive) cross section, σ_{total}

$$\sigma_{total} = \sum_{i=1}^n \sigma_i \quad (3.10)$$

For example, the total cross section of proton-antiproton collisions at a center-of-mass energy (\sqrt{s}), as in Tevatron Run 2, is huge,

$$\sigma(p\bar{p} \rightarrow X) \approx 75mb, \quad (3.11)$$

where X represents anything, but that for the most highly sought-after processes are small (*duh!*), e.g.

$$\sigma(p\bar{p} \rightarrow t\bar{t}X) \approx 75pb. \quad (3.12)$$

3.2 Calculation of widths and cross sections

The matrix element between the initial state $|i\rangle$ and the final state $|f\rangle$ is called the S matrix:

$$S_{fi} = (2\pi)^4 \delta^4(p_f - p_i) M_{fi} \quad (3.13)$$

where p_i is the total initial momentum, p_f the total final momentum, and the 4-dimensional δ function expresses the conservation of 4-momentum (E, \vec{p}). The quantity M_{fi} , called the (reduced) matrix element or amplitude of the process, contains the non-trivial physics of the problem, including spins and couplings. It is usually calculated by perturbative approximation [16]. The probability of the transition from $|i\rangle$ to $|f\rangle$ is given by

$$P_{i \rightarrow f} = \frac{S_{fi}}{\langle f|f \rangle \langle i|i \rangle} \quad (3.14)$$

The rate of the transition is determined by Fermi's Golden Rule:

$$\text{transitionrate} = 2\pi |M|^2 \times (\text{phasespace}) \quad (3.15)$$

3.2.1 The Golden Rule for Decays

For an n -body decay

$$i \rightarrow f_k; k = 1, \dots, n \quad (3.16)$$

the differential decay rate is given by

$$d\Gamma = |M|^2 \frac{S}{2m_i} \left(\prod_{k=1}^n \frac{d^3 \vec{p}_k}{(2\pi)^3 2E_k} \right) \times (2\pi)^4 \delta^4(p_i - \sum_{(k=1)}^n p_k) \quad (3.17)$$

where p_k is the 4-momentum of the k^{th} particle, and S is a product of statistical factors: $\frac{1}{m!}$ for each group of m identical particles in the final state. Usually we are not interested in specific momenta of the decay products. So, the total decay rate is obtained by integrating the above. For a general 2-body decay, the total width is given by

$$\Gamma = \frac{S |\vec{p}|}{8\pi m_i} |\vec{M}|^2 \quad (3.18)$$

where $|\vec{p}|$ is the magnitude of the momentum of either outgoing particle in the particle rest frame (this is fully determined by the masses of the 3 particles involved in the process), and M is evaluated at the momenta required by the conservation laws.

3.2.2 The Golden Rule for Scattering

Just as for the decay rate, for a $2 \rightarrow n$ scattering process

$$ij \rightarrow f_k; k = 1, \dots, n \quad (3.19)$$

the differential cross section is given by

$$d\sigma = |M|^2 \frac{S}{4\sqrt{(p_i p_j)^2 - (m_i m_j)^2}} \left(\prod_{(k=1)}^n \frac{d^3 \vec{p}_k}{(2\pi)^3 2E_k} \right) \times (2\pi)^4 \delta^4(p_i + p_j - \sum_{k=1}^n p_k) \quad (3.20)$$

For a 2×2 process in the CM frame, this leads to

$$d\sigma = \frac{S}{64\pi^2 E_{CM}^2} \frac{|\vec{p}_f|}{|\vec{p}_i|} |M|^2 d\Omega, \quad (3.21)$$

where $|\vec{p}_f|$ is the magnitude of the momentum of either outgoing particle, $|\vec{p}_i|$ is the magnitude of the momentum of either incoming particle, and

$$d\Omega = \sin\theta d\theta d\phi \quad (3.22)$$

is the solid-angle element in which the final state particles scatter.

3.3 Feynman rules for calculating the amplitude

In the previous sections, the formula for decay rates and scattering cross sections are given in terms of the amplitude M_{fi} . Here we give the recipe to calculate iM_{fi} for a given Feynman diagram for tree-level processes:

3.3.1 External lines

(a) For an incoming electron, positron, or photon, associate a factor u, \bar{v} , or e_μ , respectively. (b) For an outgoing electron, positron, or photon, associate a factor \bar{u}, ν , or e_μ^* respectively.

3.3.2 Vertices

For each vertex, include a factor of $ig\gamma^\mu$ for an electron or $ig\gamma^\mu$ for a positron. Care must be exercised to get the overall sign for fermions correct.

3.4 Internal lines

(a) For an gauge boson connecting two vertices, include a term

$$\frac{ig_{\mu\nu} - q_\mu q_\nu / m_W^2}{q^2 - m_W^2 + i\epsilon} \quad (3.23)$$

(b) Integrate over all undetermined internal momenta.

3.5 Muon life time

This will be the most important calculation as most weak decays of particle are calculated in the same manner [16]. We firstly draw a Feynman diagram where the muon first decays into a neutrino and W immediately followed by the decay of the W^- in to an electron and anti-electron neutrino as:

$$\mu(p) \longrightarrow \nu_\mu + (W^- \longrightarrow e^-(q) + \bar{\nu}_e(k)) \quad (3.24)$$

The matrix element is given by,

$$M = \frac{g_2}{2} (\bar{\nu}_\mu \gamma^\lambda p_{L\mu}) \times \frac{1}{(q^2 - m_w^2)} \times \frac{g_2}{2} (\bar{e} \gamma_\lambda p_L \nu_e) \quad (3.25)$$

where $P_L = (1 - \gamma^5)$. $\frac{1}{(q^2 - m_w^2)}$ is the propagator of intermediate W since the maximum value of $q^2 \sim m_W^2$ is much smaller than m_W^2 , then the propagator to a very good approximation equal to $\frac{1}{m_W^2}$. if we neglect the spinor complication and remember that the wave function are basically just given by the square root of energy , we set m_μ for each vertex and the full matrix element is :

$$M = \frac{g_2^2}{2} \frac{m_\mu^2}{m_W^2} = 2\sqrt{2}G_f m_\mu^2 \quad (3.26)$$

now we can write down the decay rate differential in the momenta of the incoming and outgoing particle :

$$d\Gamma_\mu = (2\pi)^4 \delta^4(k + q + \acute{k} - p) |M|^2 \frac{d^3k}{2E_K (2\pi)^3} \frac{d^3q}{2E_q (2\pi)^3} \frac{d^3\acute{k}}{2E_{\acute{k}} (2\pi)^2} \quad (3.27)$$

$$d\Gamma_\mu = \frac{1}{(2\pi)^5} \frac{8G_f^2 m_\mu^4}{2m_\mu} \frac{1}{2} \delta^4(k + q + \acute{k} - p) \frac{d^3k}{2E_k} \frac{d^3q}{2E_q} \frac{d^3\acute{k}}{2E_{\acute{k}}}, \quad (3.28)$$

where the Extra factor one half comes from the fact that only left handed moun are involved . In the rest frame moun $p = (m_\mu, 0, 0, 0)$ so that the differential can be written as:

$$\rho = \delta^3(k + q + \acute{k}) \delta(E_K + E_P + E_{\acute{k}} - m_\mu) \frac{d^3k}{2E_k} \frac{dq}{2E_q} \frac{d\acute{k}}{2E_{\acute{k}}}. \quad (3.29)$$

if we neglect the mass of $m_e, m_{\bar{\nu}_e}, m(\nu_\mu)$ we get:

$$\rho = \delta(|k| + |q| + |k + q| - m_\mu) \frac{d^3k d^3\acute{k}}{8|k| \cdot |q|} \frac{d^3q}{|k + q|} \quad (3.30)$$

now go to polar coordinates.

$$\delta(|k| + |q| + \sqrt{(k^2 q^2 + 2kq \cos \theta_q)} - m_\mu) \frac{k^2 d|k| d\Omega_k q^2 d|q| d\Omega_q}{8|k| |q| \sqrt{k^2 + q^2 + 2|k||q| \cos \theta_q}} \quad (3.31)$$

we can always do the integral over $d\Omega_k = 4\pi$ and over azimuth angle of q around k direction $d\Omega_q = 2\pi$ we get:

$$\delta(|k| + |q| + \sqrt{k^2 + q^2 + 2kq \cos \theta_q} - m_\mu) \frac{(4\pi |k| d|k| - 2\pi |q| d|q| d \cos \theta_q)}{8\sqrt{k^2 + q^2 + 2|k||q| \cos \theta_q}} \quad (3.32)$$

integrating also over $\cos \theta_q$ the delta function gives :

$$\sqrt{k^2 + q^2 + 2|k||q| \cos \theta_q} = m_\mu - |k| - |q| \quad (3.33)$$

and the Jacobian

$$\frac{\sqrt{k^2 + q^2 + 2|k||q| \cos \theta_q}}{|k||q|} = \pi^2 d|k|d|q|, \quad (3.34)$$

therefore we get the differential decay width

$$d\Gamma_\mu = \frac{2G_f^2 m_\mu^3}{(2\pi)^5} (\pi^2 d|k|d|q|) \quad (3.35)$$

which can be integrated ($|k| < \frac{m_\mu}{2}$; $|q| < \frac{m_\mu}{2}$; $m_k - |k| - |q| < \frac{m_\mu}{2}$) thus:

$$d\Gamma_\mu = \frac{2G_f^2 m_\mu^5}{128\pi^3} \quad (3.36)$$

or if you could do the spin and kinematics correctly as we will do next we get,

$$\Gamma_\mu = \frac{G_f^2}{192\pi^3} (m_\mu)^5 \quad (3.37)$$

so by measuring Γ_μ and m_μ we can get a good value for g_2 (and indirectly $\sin^2 \theta_W$)

3.6 Decay rate of muon in the SM

Note that our calculation will relies heavily on previous sections. We start by drawing Feynman diagram for muon decay in the SM.

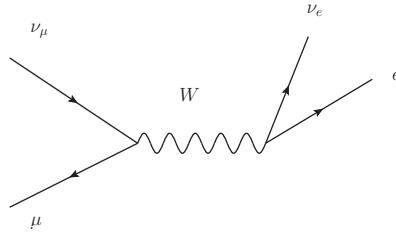


Figure 3.1: Feynman diagram for muon decay.

The Feynman rules for this process is:

The vertex vector = $\frac{-i}{2\sqrt{2}} g_2 \gamma^\mu (1 - \gamma^5)$

The propagator = $-i \frac{g_{\mu\nu} - \frac{q_\mu q_\nu}{(m_W)^2}}{q^2 - (m_W)^2}$

. The amplitude of this process is given by the matrix element :

$$M = i[\bar{u}_3(-\frac{ig_2}{2\sqrt{2}}\gamma^\mu(1-\gamma^5)u_1)][-i\frac{g_{\mu\nu}-\frac{q_\mu q_\nu}{(m_W)^2}}{q^2-(m_W)^2}][\bar{u}_4(-\frac{ig_2}{2\sqrt{2}})\gamma^\nu(1-\gamma^5)v_2] \quad (3.38)$$

for low momentum transfer $q^2 \ll (m_W)^2$ this means ,

$$M = \frac{g_2^2}{8(m_W)^2} [(\bar{u}_3 \gamma^\mu (1 - \gamma^5) u_1) (\bar{u}_4 \gamma_\mu (1 - \gamma^5) \nu_2)] \quad (3.39)$$

averaging over initial state spin and summing over final state spin we obtained :

$$\begin{aligned} \sum_{(spin)} |M|^2 &= \frac{1}{2} \left(\frac{g_2^2}{8(m_W)^2} \right)^2 Tr(\gamma^\mu (1 - \gamma^5) (p_1 + m_e) \gamma^\nu (1 - \gamma^5) p_3) \\ &\quad \times Tr(\gamma_\mu (1 - \gamma^5) p_2 \gamma_\nu (1 - \gamma^5) (p_4 + m_\mu)) \end{aligned} \quad (3.40)$$

use trace technology to evaluate the trace; first bring the $(1 - \gamma^5)$ factors together :

$$(1 - \gamma^5) p_2 \gamma_\nu (1 - \gamma^5) = (1 - \gamma^5) p_2 (1 + \gamma^5) \gamma_\nu = (1 - \gamma^5) (1 - \gamma^5) p_2 \gamma_\nu = 2(1 - \gamma) p_2 \gamma_\nu \quad (3.41)$$

the mass dependent terms do not contribute, so we get:

$$\begin{aligned} |M|^2 &= \frac{g_2^4}{2m_W^4} [p_1^\mu p_3^\nu + p_1^\nu p_3^\mu - (p_1 p_3) g^{\mu\nu} - \epsilon^{\mu\nu\lambda\sigma} p_{1\lambda} p_{3\sigma}] \\ &\quad \times [p_{2\mu} p_{4\nu} + p_{2\nu} p_{4\mu} - (p_2 - p_4) g_{\mu\nu} - i \epsilon_{\mu\nu k\tau} p_2^k p_4^\tau] \\ &= 2 \frac{g_2^4}{m_W^4} [(p_1 p_2) (p_3 p_4)] \end{aligned} \quad (3.42)$$

where :

$$\epsilon^{\mu\nu\lambda\sigma} \epsilon_{\mu\nu k\tau} = -2(\delta_k^\lambda \delta_\tau^\sigma - \delta_\tau^\lambda \delta_k^\sigma) \quad (3.43)$$

in the muon rest frame; we have

$$p_1 \cdot p_2 = m_\mu E_e; p_1 = (m_\mu, 0, 0, 0), p_2 = (E_e, 0, 0, 0) \quad (3.44)$$

and,

$$\begin{aligned} p_3 \cdot p_4 &= \frac{(p_3 + p_4)^2 - (p_3^2 - p_4^2)}{2} = \frac{(p_1 - p_2)^2 - 0 - 0}{2} \\ &= \frac{p_1^2 + p_2^2 - 2p_1 p_2}{2} = \frac{m_\mu(m_\mu - 2E_e)}{2} \end{aligned} \quad (3.45)$$

then the spin averaged squared matrix element simplifies to :

$$|M|^2 = \frac{g_2^4}{m_W^4} m_\mu^2 E_e (m_\mu - 2E_e) \quad (3.46)$$

Note that $|M|^2$ depend non-trivially on θ ; we should use Fermi's Golden rules :

$$d\Gamma_\mu = \frac{|M|^2}{2m_\mu} \frac{d^3 p_2}{(2\pi)^3 2E_2} \frac{d^3 p_3}{(2\pi)^3 2E_3} \frac{d^3 p_4}{(2\pi)^3 2E_4} \quad (3.47)$$

the derivation of the matrix element yields valuable kinematic information

$$\max E_2, E_4 < \frac{1}{2}m_\mu < (E_2 + E_4) \implies E_2 < \frac{1}{2}m_\mu E_4 < \frac{1}{2}m_\mu E_2 + E_4 > \frac{1}{2}m_\mu \quad (3.48)$$

since all three final state particles are assumed to be massless ,Energy and three momentum are the same . The sum of three momentum for the 3-final state particle must be zero ,therefore no-single particle can have more than half of the available energy and no two particle can have less than half of the available energy ;

$$\frac{d\Gamma_\mu}{dE} = \left(\frac{g_2}{m_W}\right)^4 \frac{(m_\mu)^2 E^2}{2(4\pi)^3} \left(1 - \frac{4E}{3m_\mu}\right) \quad (3.49)$$

integrate above formula over the electron energy we obtain the muon decay rate :

$$\Gamma_\mu = \frac{g_2^4}{m_W^4} \frac{(m_\mu)^2}{2(4\pi)^3} \int_0^{\frac{m_\mu}{2}} dE E^2 \left(1 - \frac{4E}{3m_\mu}\right) = \frac{1}{6144\pi} \left(\frac{g_2}{m_W}\right)^4 m_\mu^5 \quad (3.50)$$

we can define Fermi coupling constant G_f by $G_f = \frac{\sqrt{2}g_2^2}{8m_W^2}$. this allows us to write the muon decay rate as :

$$\Gamma_\mu = \frac{G_F^2 m_\mu^5}{192\pi^3} \implies \tau_\mu = \frac{1}{\Gamma_\mu} = \frac{192\pi^3}{G_f^2 m_\mu^5} \quad (3.51)$$

we can use the measurement of the muon mass and lifetime τ to calculate the Fermi constant G_f , the muon mass and lifetime (2007PDG)

$$M_\mu = 105.658369 \pm 0.000009 \text{ Mev}$$

$$\tau_\mu = (2.19703 \pm 0.0004) \times 10^{-6} \text{ second}$$

$$G_f = \frac{192\pi^3}{\tau_\mu m_\mu^5} = 1.6637 \times 10^{-5} (\text{Gev})^{-2}$$

we can also use w mass measurement ; $M_w = 80.4 \text{ GeV}$ to determine g_2 ,

$$g_2 \approx 0.65 \implies \alpha_2 = \frac{g_2^2}{4\pi} = \frac{1}{29} \quad (3.52)$$

this means that weak interaction is stronger than the electromagnetic interaction $\alpha_{em} = \frac{1}{137}$ the calculation given

$$\tau = \frac{\hbar}{\Gamma} = \frac{192\pi^3 \hbar}{G_f^2 m_\mu^5} \quad (3.53)$$

This can be improved by including the effect of electron mass ; if we define $\left(\frac{m_e}{m_\mu}\right)^2 = x$ and add the correction from electron mass then we get:

$$\Gamma = \frac{G_f^2 m_\mu^5}{192\pi^3} (1 - 8x) \quad (3.54)$$

If x is positive will give smaller decay constant correspondingly longer lifetime

Chapter 4

Methodology and Findings

4.1 Methodology

To obtain our results we computed by hand the decay rate of muon. These calculations need to be performed numerically by using dedicated numerical packages. Here in this thesis we used MATHEMATICA program version 9 to obtain the figures 4.1 and 4.2. WE used equation (3.49) to generate the figure 4.1.

4.2 Result and Discussion

In this chapter we present our result for the muon decay

Figure 4.1, tells us the energy distribution of the electrons emitted in muon decay. We can see that the maximum energy for the electron or any individual outgoing particle should be half of muon mass $E_{Max} = \frac{m_\mu}{2}$ and minimal total for any pair. We show in this figure typically the fraction of energy carried off by the electron which is about 53 MeV.

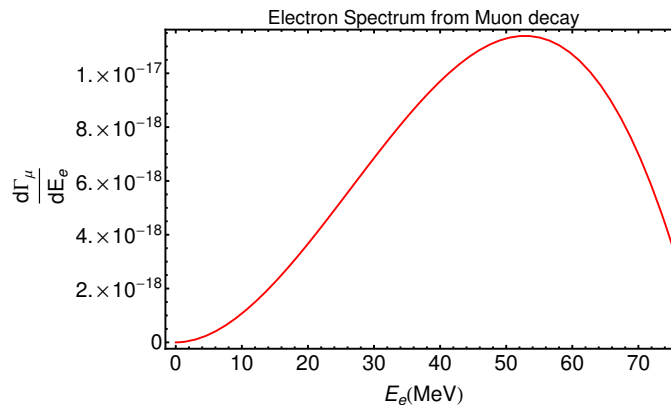


Figure 4.1: *The Electron Energy spectrum from muon decay.*

As depicted in figure 4.2, the decay rate of muon as function of its mass is presented: the red line is muon decay as function of its mass, while the blue line is the decay rate of muon for $m_\mu = 105.6$ MeV. We observed that the muon decay rate into $e\nu_e\nu_\mu$ increase with muon mass increases.

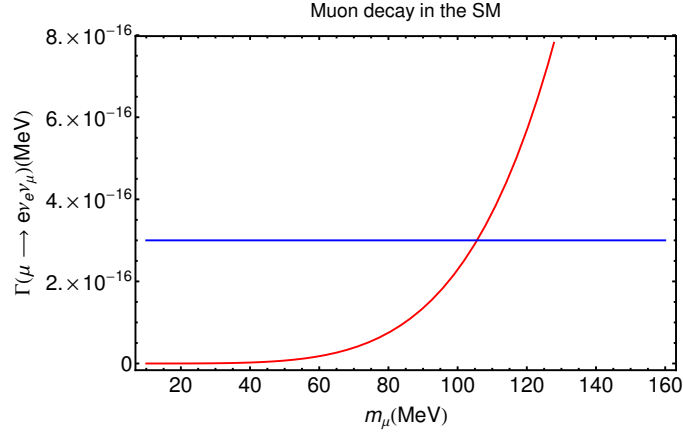


Figure 4.2: *decay rates of muon as function of its mass.*

4.3 Conclusion

In this thesis we studied some aspects of leptonic muon decay and its lifetime in the standard model of particle physics. We also used the measured muon mass and calculated life time of muon to determine the weak coupling strength G_f . Our results are in a good agreement with the experimental result. We also determined the value of Fermi coupling for weak interaction G_f by knowing the lifetime of muon and its mass, as well as the weak coupling constant g_2 and fine structure constant for the weak interaction α . We summarized our results in the following:

characteristic properties of muon

Rest mass : $m_\mu = 105.658398 \frac{MeV}{c^2}$

Meanlife : $\tau_\mu = (2.19703 \pm 0.0004) \times 10^{-6} s$.

Fermi constant: $G_f = 1.66 \times 10^{-5} GeV^{-2}$

Weak coupling: $\alpha_2 = \frac{1}{29}$.

This work may be extended by including the effects from the loop contribution such as one-loop and two-loop effects.

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