Chapter one

Introduction

(1.1) Introduction:

In 1887 the experiments of Hertz confirmed the prediction of Max well's theory, that all optics seemed to be explainable by electromagnetic theory. Only two small but unfortunately two phenomena cannot described by the wave theory. One of them is the spectrum of light emitted by a hot body, beside the photoelectric effect [1].

The solution of these two problems was shown by max well, who proposed that the energy of light is quantized; it consists of discrete bundles of energy called light quanta photons [1].

Quantum theory is the name given to the explanation of the discrete properties of light and electromagnetic wave [1].This discovery motivates de Broglie to propose that particles can also behave like waves. The wave function for de Broglie waves must satisfy an equation developed by Schrödinger in 1926. One of the methods of the quantum mechanics is to determine a solution to this equation, which in turn yields the allowed wave function and energy levels of the system under consideration. Proper manipulation of the wave function allows calculation of all measurable features of the system [2], the Schrödinger equation accounts for the motion of particle moving in free space or in a certain field, unfortunately it does not accounts for the motion of particles in a resistive medium or medium having viscosity.

The viscosity is resistance to shearing motion in a form the internal friction, this viscosity arises because of a friction force between adjacent layers of the fluid as they slid past one another [2].

(1.2) Problem of the thesis:

The research problem is related to the fact that There is no law relates the viscosity and Schrödinger equation directly this makes Schrödinger equation uncap able of describing resistive media

(1.3) Aims of the thesis:

Deriving the Schrödinger equation for medium has viscosity,so one tried to find a relationship between them through driving the equation from the friction force law and law of the total energy.

(1.4) literature review:

Attempts were made by anthers to derive quantum equation that takes into account the effect of friction, some of them use the expression of energy for friction medium [3].And Others use Maxwell's equations [4].

(1.5) Presentation of the thesis:

Beside the introduction the thesis includes two chapters; also the second chapter includes the deriving of the Schrödinger equation, while chapter three is concerned with Schrödinger equation for medium having viscosity.

Chapter two

Schrödinger equation and viscosity

(2.1) Introduction:

The Schrödinger equation plays a big role in the development of quantum mechanics and solving some of the phenomena that did not be explained the classical mechanics such as description behavior of the wave and the particle together, which we will address in this chapter.

(2.2) Quantum mechanics:

Quantum mechanics is asset of physical theories that emerged in the twentieth century, to unexplained phenomena at the level of atoms and subatomic particle. It have been merged between particulate and wave properties to appears the team wave-particle duality, thus becoming responsible for the interpretation of quantum mechanics physics at the atomic level .It also applies to classical mechanics but showed no impact at this level. So quantum mechanics is a generalization of the classical physics atomic levels. Rename to quantum mechanics goes back to the importance of quantum in their construction (a term used to describe the smallest physical amount of energy can be exchanged between particles and is used to indicate the quantities of specific energy that emits intermittently, not continuously).

Often uses the terms quantum physics and quantum theory as synonyms for quantum mechanics to refer to non-relativistic quantum mechanics [5, 6]

(2.3) Planck discovery:

In 1900 Planck discovered a formula for blackbody radiation that is in complete agreement with experiment at all wavelengths [2, 5, 6, & 7]. Planck's analysis led to the curve shown in figure (2.3.1) below

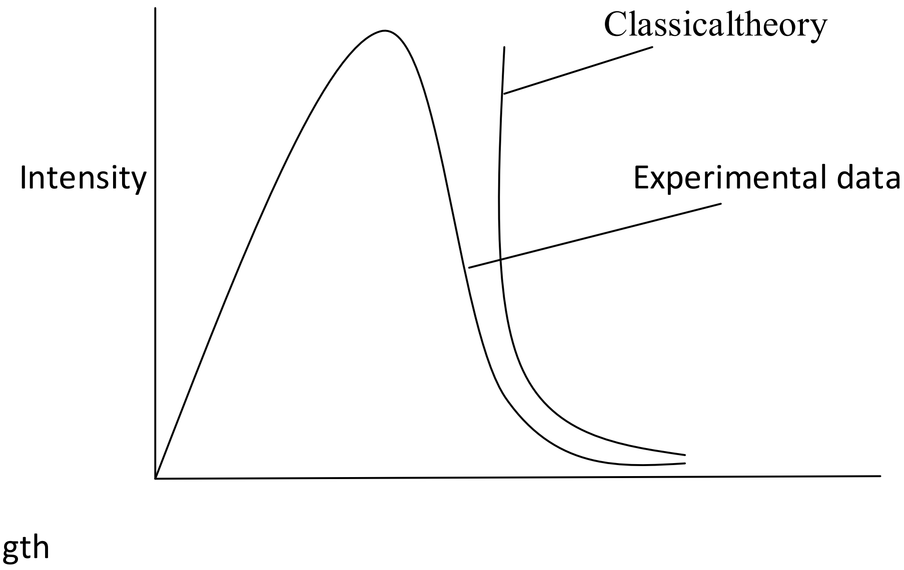

Figure (2.3.1) comparison of the experimental results with the curve predicted by the Rayleigh Jeans classical model for the distribution of the blackbody radiation, [2 and 8].

As known the concept of quantization developed by Planck in 1900 the quantization model assumes that the energy of light wave is present in bundles of energy called photons, hence, the energy is said to be quantized[7,8,9]. (Any quantity that appears in discrete bundles is said to be quantized, just as charge and other properties are quantized.) [1, 2 &9].

According to Planck's theory, the energy of photon is proportional to the frequency of electromagnetic wave

$$E = h f \qquad (2.3.1)$$

Where E is the energy, f is frequency and h is Planck constant.

It is important to note that this theory retains some features of the particle theory of light.

Planck mode two bold and controversial assumptions concerning the nature of the oscillating molecules at the surface of the blackbody [1, 2&7].

The molecules can have only discrete units of energy E_n

$$E_n = nhf \quad (2.3.2)$$

n=1, 2, 3……

Where n is a positive integer called a quantum number and f is frequency of the vibration of the molecules. Because the energy of molecules can have only discrete values given by equation (2.3.1), the energy is quantized; each discrete energy value represents a different quantum state [1, 2].

The molecules emit or absorb energy in discrete packets called photons [1, 2&8].

The key points in Planck's theory is radical a assumption of quantized energy state. This development the birth of quantum theory when Planck presented his theory, most scientists (include Planck) did not consider the quantum concept to be realistic. Hence, Planck and other continued to search for a more rational explanation of blackbody radiation. However, subsequent development showed that a theory based on the quantum concept (rather

Than on classical concepts) had to be used to explain many other phenomena at atomic level. [2]

(2.4) wave function for particle having a definite momentum:

In this paragraph one begin investigating how wave function can be found, considering the simple case of free particles. The experiments exhibiting the corpuscular nature of the electromagnetic radiation require that with the electromagnetic field one associates a particle, the photon, whose energy E and magnitude p of momentum are related to the frequency f and wavelength λ of the electromagnetic radiation by [10&11]:

$$E = hf \quad , \; P = \frac{h}{\lambda} \quad (2.4.1)$$

On the other hand the de Broglie was led to associate matter waves with particles in such a way that the frequency f and the wavelength λ of the wave were linked with the particle energy E and magnitude P of momentum by the same relation (2.4). The de Broglie relation is given by

$$\lambda = \frac{h}{P} \quad (2.4.2)$$

Was confirmed by the results of number of experiment exhibiting the wave nature of matte, following de Broglie, one shall assume that the relation (2.4.1) hold for a types of particles and field quanta. Introducing the angular frequency

$$\omega = 2\pi f \quad (2.4.3)$$

The wave number κ is given by

$$\kappa = \frac{2\pi}{\lambda} \quad (2.4.4)$$

And the reduced Planck constant $\hbar$ is given by

$$\hbar = \frac{h}{2\pi} \quad (2.4.5)$$

Substituting equation (2.4.3), (2.4.4), (2.4.5) in equation (2.4.1) we obtain

$$\mathrm{E} = 2\pi\hbar \frac{\omega}{2\pi} = \hbar\omega \quad (2.4.6)$$

$$\mathrm{P} = 2\pi\hbar \frac{\kappa}{2\pi} = \hbar\kappa \quad (2.4.7)$$

Therefore

$$\mathrm{E} = \hbar\omega \quad , \quad \mathrm{P} = \hbar\kappa \quad (2.4.8)$$

Consider now a free particle of mass m, moving along the x-axis with a definite momentum P, and corresponding energy E. assuming that the particle moves in the positive x- direction one associate with this particle a wave travelling in the same direction with a fixed wave number k [11]. such a wave is a plane wave and can be written as

$$\psi(x,t) = \mathrm{A}\,\ell^{\{i[\kappa x - \omega(\kappa)t]\}} \quad (2.4.9)$$

Where A is a constant, this plane wave has a wavelength λ and a frequency f since from equation (2.4.8)

$$\kappa = \frac{\mathrm{P}}{\hbar} \quad \text{And} \quad \omega = \frac{\mathrm{E}}{\hbar} \quad (2.4.10)$$

Substituting equation (2.4.10) in equation (2.4.9)

$$\psi(x,t) = A\ell^{\frac{i}{\hbar}(Px - Et)} \quad (2.4.11)$$

We note that the wave function equation (2.13) satisfies the two relations

$$-i\hbar\frac{\partial}{\partial x}\psi = P\psi \quad (2.4.12)$$

And[11]

$$i\hbar\frac{\partial}{\partial t}\psi = E\psi \quad (2.4.13)$$

(2.5) Schrödinger wave equation:

To formulate a wave equation expressing the wave property and particles duality for atomic objects the formula (2.4.11)

$$\psi(x,t) = A\ell^{\frac{i}{\hbar}(Px - Et)} \quad (2.5.1)$$

Be inappropriate because the wave function depends on the momentum p and energy E and two magnitudes mending for some waves and mending for some particles [11, 12 &13]. It is therefore appropriate to formulate ψ in terms of physical quantities suitable for some particle and wave together [12].

The energy E and the momentum p of classical system are related according to the relation

$$E = \frac{p^2}{2m} + V \quad (2.5.2)$$

Where V is stands for the potential energy of the particle. To incorporate the wave function ψ , one can multiply both sides of equation (2.5.2) by ψ to get

$$E\psi = \frac{p^2}{2m}\psi + V\psi \, (2.5.3)$$

Differentiate ψ with respect to time, to get

$$\frac{d\psi}{dt} = \frac{\partial\psi}{\partial t} = -\frac{i}{\hbar}\mathrm{E}\mathrm{A}\ell^{\frac{i}{\hbar}(\mathrm{P}x - \mathrm{E}t)} = -\frac{i}{\hbar}\mathrm{E}\psi \, (2.5.4)$$

On the other hand, differentiating ψ twice with respect to x, we find that

$$\frac{d\psi}{dx} = \frac{\partial\psi}{\partial x} = \frac{i}{\hbar}\mathrm{P}\mathrm{A}\ell^{\frac{i}{\hbar}(\mathrm{P}x - \mathrm{E}t)} = \frac{i}{\hbar}\mathrm{P}\psi \, (2.5.5)$$

And

$$\frac{\partial^2\psi}{\partial x^2} = \frac{i^2}{\hbar^2}\mathrm{P}^2\psi = -\frac{\mathrm{P}^2}{\hbar^2}\psi \, (2.5.6)$$

And hence, we find that

$$-\frac{\hbar}{i}\frac{\partial\psi}{\partial t} = \mathrm{E}\psi \, (2.5.7)$$

Though [9, 10&11]

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} = \frac{\mathrm{P}^2}{2m}\psi \, (2.5.8)$$

Substituting the equation (2.5.7) and equation (2.5.8) in equation (2.5.3) one find that [2, 12&13]

$$-\frac{\hbar}{i}\frac{\partial\psi}{\partial t}=-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x}+V\psi \quad (2.5.9)$$

The general Schrödinger's equation in three dimensions takes the form [11, 12&13]

$$-\frac{\hbar}{i}\frac{\partial\psi}{\partial t}=-\frac{\hbar^2}{2m}\nabla^2\psi+V\psi \quad (2.5.10)$$

(2.6) viscosity:

The speed of spreading, and of losing the original shape under a deforming force, is governed by the property of a fluid known asviscosity. Viscosity essentially governs the speed of fluid motion but can never stop it entirety. One further definition of viscosity is often used in fluid mechanics. It measures the friction between two successive layers having thicknesses dz and relative velocity dv through the expression of the frictional force F given by:

$$F=\eta\, {dv}/{dz} \quad (2.6.1)$$

The viscosity varies from fluid to fluid and how given fluid it varies with temperature. The viscosity is reduced by an increase in temperature. [14,15]

A fluid does not support shearing stress. However, fluids do offer some degree of resistance to shearing motion. This resistance to shearing motion is a form of internal friction called viscosity. The viscosity arises because of a friction force between adjacent layers of the fluid as they slid past one another. The degree of viscosity of fluid can be understood with following example. If two plates of glass are separated by layer of fluid such as oil, with one plate fixed in position. It is easy to side one plate over the other (figure 2.2 below) [2].

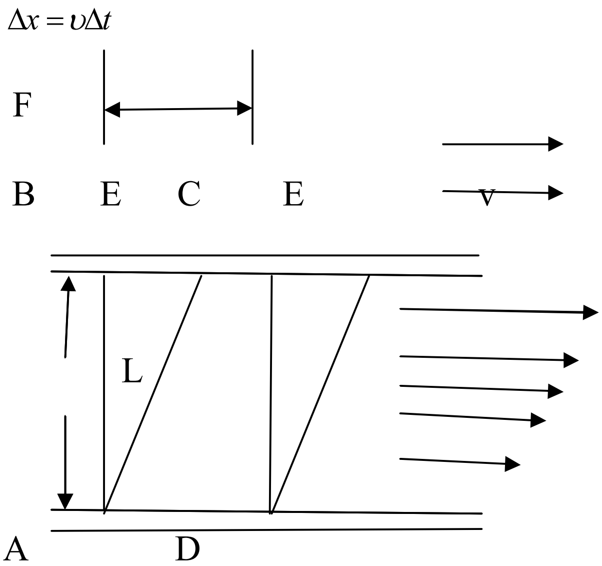

Figure (2.2)

Figure (2.2): a layer of liquid between two solid surfaces in which the lower surface is fixed and upper surface moves to the right with a viscosity ι.

However, if the fluid separating the plates is tar, the task of sliding one plate over the other becomes much more difficult. Thus, we would conclude that tar has a higher viscosity than oil [2].

In figure (2.2) note the speed of successive layer of fluid increases linearly from 0 to ι, one move from a layer adjacent to the fixed plate to layer adjacent to the moving plate.

Consider two parallel layers, one fixed and one moving to the right under the action of an external force F as in figure (2.2). Because of this motion, apportion of fluid is distorted from it is original shape [2&14]

The fluid has undergone a shear strain [2]. By definition, the shear stress on the fluid is equal to the ratio F/A, while the shear strain is defined by the ratio $\nabla x/L$:

$$\text{Shear stress} = F/A \quad (2.34)$$

$$\text{Shear strain} = \nabla x/L \quad (2.35)$$

Where F is force that is plate moving, A is area of plate and Δx is a distance.

Shear stress, sometimes called fluid friction [2&14].

The upper plate moves with speed V, and the fluid adjacent to this plate has the same speed. Thus, in a time Δt, the fluid at the moving plate travels adjacent $\Delta x = V\Delta t$, and we can express the shear strain per unit time as

$$\frac{shear \cdot strain}{\Delta t} = \frac{\Delta x/L}{\Delta t} = \frac{V}{L} \quad (2.36)$$

This equation states that the rate of change of shear strain is $\frac{V}{L}$ [2].

The coefficient of viscosity η, for the fluid is defined as the ratio of the shearing stress to the rate change of the shear strain.

$$\eta = \frac{F/A}{V/L} = \frac{FL}{AV} \quad (2.37)$$

The SI unit of coefficient of viscosity is $N \cdot S/m^2$ [2].

The expression for η given equation (2.37) is valid only if the fluid speed varies linearly with position, in this case, it is common to say that the speed gradient, $\frac{V}{L}$ is uniform, we must express η in the general form [2]:

$$\eta = \frac{F/A}{dV/dy} \quad (2.38)$$

Where the speed gradient dV/dy is the change in speed with position as measured perpendicular to the direction of viscosity [2&14].

Chapter 3

Schrödinger equation by viscous medium

(3.1) introduction:

Viscosity plays an important role in many application, thus it is very important to study it is effect on a quantum system. This will be done in his chapter.

(3.2) force and viscosity:

The viscous force F that opposes the motion of a fluid having viscosity η is given in terms of the change of speed v , in a direction z perpendicular to the flow direction x is given by

$$F = \eta \frac{dv}{dz} \quad (3.2.1)$$

Whereas F is a friction force, η is coefficient of viscosity

Multiply equation (3.2.1) in $\frac{dx}{dx}$ one gets

$$F = \eta \frac{dv}{dx}\frac{dx}{dz} \quad (3.2.2)$$

Assuming that $\frac{dx}{dz} = c_\circ$ = constant

$$F = c_\circ \eta \frac{dv}{dx} \quad (3.2.3)$$

Thus

$$Fdx = c_\circ \eta dv \quad (3.2.4)$$

Integrating both sides of equation (3.2.4) yields

$\int F dx = c_{\circ}\eta \int dv$ (3.2.5)

Hence the energy of the friction E_r is equal to

$E_r = \int F dx = c_{\circ}\eta \int dv$ (3.2.6)

$E_r = c_{\circ}\eta v$ (3.2.7)

The total energy E is equal to

$$E = \frac{p^2}{2m} + V - E_r \quad (3.2.8)$$

Where's p is the momentum, m is the mass and V is the total potential energy

Substitute's equation (3.2.7) in equation (3.2.8) yields

$$E = \frac{p^2}{2m} + V - c_{\circ}\eta v \quad (3.2.9)$$

The momentum p is equal to

$p = mv$ (3.2.10)

Where v is velocity

$$v = \frac{p}{m} \quad (3.2.11)$$

Substitutes (3.2.11) in equation (3.2.9) results in

$$E = \frac{p^2}{2m} + V - c_{\circ}\eta \frac{p}{m} \quad (3.2.12)$$

(3.3) Schrödinger equation by viscous medium for a quantum system:

The wave function is written by

$$\psi = A\ell^{i/\hbar(Px-Et)} \quad (3.3.1)$$

By partially differentiating equation (3.3.1) with respect to x, one gets

$$\frac{\partial\psi}{\partial x} = \frac{i}{\hbar} pA\ell^{i/\hbar(Px-Et)} = \frac{i}{\hbar} p\psi \quad (3.3.2)$$

Differentiating again yields

$$\frac{\partial^2\psi}{\partial x^2} = \frac{1}{\hbar^2} p^2\psi \quad (3.3.3)$$

$$-\hbar^2 \frac{\partial^2\psi}{\partial x^2} = p^2\psi \quad (3.3.4)$$

The wave function ψ can be also differentiated with respect to t to get

$$\frac{\partial\psi}{\partial t} = \frac{i}{\hbar} EA\ell^{i/\hbar(Px-Et)} = -\frac{i}{\hbar} E\psi \quad (3.3.5)$$

Hence

$$\frac{\partial\psi}{\partial t} = -\frac{i}{\hbar} E\psi \quad (3.3.6)$$

Multiply the right side of equation (3.3.6) by $\frac{i}{i}$ to get

$$\frac{\partial\psi}{\partial t} = -\left(\frac{i}{i}\right)\left(\frac{i}{\hbar}\right)E\psi = \frac{1}{i\hbar}E\psi \quad (3.3.7)$$

Thus

$$i\hbar\frac{\partial\psi}{\partial t} = E\psi \quad (3.3.8)$$

Multiply equation (3.2.12) by ψ result in

$$E\psi = \frac{p^2}{2m}\psi + V\psi - \frac{c_\circ\eta}{m}p\psi \quad (3.3.9)$$

Substitute's equation (3.3.2), equation (3.3.4) and equation (3.3.8) in equation (3.3.9) yields

$$i\hbar\frac{\partial\psi}{\partial x} = \frac{-\hbar^2}{2m}\frac{\partial^2\psi}{\partial x} + V\psi - \frac{c_\circ\eta}{m}\frac{\hbar}{i}\frac{\partial\psi}{\partial x} \quad (3.3.10)$$

This equation called the Schrödinger equation for viscous medium for a quantum system.

(3.4) Discussion:

According to equation (3.3.1)and (3.3.7) the viscosity energy lost can be arranged to be directly proportional to v. thus the total energy in equation (3.3.9) consists of additional term corresponding to energy lost by viscosity. Using this expression for energy one finds Schrödinger equation in equation (3.3.10).this equation resembles that of k Haroun [3].

(3.5) Conclusion:

Schrödinger equation can describe motion of particles in viscous fluid. This is important since many applications in physics are concened with viscosity medium.

(3.6) Recommendations:

1- The need for more research in development of Schrödinger equation to account for the effect of viscosity and bulk matter.

2- The non-linearity of this new Schrödinger equation requires linearization by perturbation or numerical analysis.

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